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Minimization of Harmonics Noise Using Wavelet Transformation Technology

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Abstract: The field programmable gate array technology can design high performance system at low cost for wavelet analysis. Wavelet transform has gained the reputation of being a very effective signal analysis tool for much practical application. Implementation of transform needs the meeting of real-time processing for most application. The objectives of this paper are to compare the Haar and Daubeches technology and to calculate the bit error rate (BER) between the input audio signal and reconstructed output signal. It is seen that the BER using Daubechies wavelet technology is less than Haar wavelet. The design procedure is explained using the stat of art electronic design. Automation tools for system design on FPGA, simulation, synthesis and implementation on the FPGA technology has been carried out. The power hovmoller, cross wavelet spectra and coherence are described. A Practical step-up-step guide to wavelet analysis is given with examples taken from time series. The guide includes a comparison to the windowed Fourier transform. New statistical significance test for wavelet power spectra are developed by deriving theoretical wavelet spectra for white and red noise. Empirical formula is given for the effect of smoothing on significance levels and filtering. The notion of orthogonal no separable trivet wavelet packets, which is the generation of orthogonal university wavelet packets is introduced. A de-noising method based on wavelet packet shrinkage is developed. The principle of wavelet packet shrinkage for de-noising and the section of thresholds and threshold function are analyzed.

1. Introduction

Wavelet analysis is a common tool for analyzing the local variation of power within a time series by decomposing a time series into time frequency space. The process is able to measure both the dominant modes of variability and how those modes vary in time. Unfortunately, many studies using wavelet analysis have suffered from an apparent lack of quantitative results. The diffuseness has been exacerbated by the use of arbitrary normalization test. The transform of a signal is just another form of representing the signal. The wavelet transform provides a time frequency representation of the signal. It was developed to overcome the short coming of the short time Fourier transform, which can also be used to analyze no stationary signals. The STFT gives a constant resolution of frequencies for wavelet transform that uses multi-resolution technique by which different frequencies are analyzed with different resolution.

The energy with wavelet is concentrated in time or space are suited for analysis of transcend signals. The wavelet transform uses wave with finite energy. The analysis is done similar to the STFT analysis. The signal to be analyzed is multiplied with a wavelet function just as it is multiplied with a window function in STFT and then the transform is computed for each segment of generation. However unlike STFT in wavelet transform the width of the wavelet function changes with special component .The wavelet transform at high frequencies gives good time resolution and poor frequencies. Many signals like music, speech and images can be efficiently represented by wavelet that are translations and dilations of a single function called mother wavelet with band pass property. The motion of orthogonal wavelet packets which are used for shingling the generalized concept of orthogonal wavelet packets can be applied to the case of the spine wavelets and so on. Since majority of information is multidimensional many researchers interest themselves in the investigation into multivariate wavelet theory. The classical method for constructing multivariate wavelets is that separable multivariate wavelets may be obtained by means of tensor product of some unvaried wavelets. But there exist a lot of defects in this method, such as scarcity of designing freedom. Therefore it is significant to investigate no separable multivariate wavelet theory. The definition for no separable orthogonal trivariate wavelet packets is given and the procedure for their construction is described. Next, the orthogonally property of non separable trivariate wavelet packets is investigated. The wavelet transform is an emerging signal processing technique that can be used to represent real life non stationary signals with high efficiency. The wavelet transform is an alternate tool to traditional time frequency representative techniques such as the discrete Fourier transform and the discrete cosine transform. Applications such as transient signal analysis, numerical analysis, and computer vision and image compression among many other audiovisual can be solved by wavelet transform. Wavelet transform is mostly needed to be embedded in consumer electronics and thus a single chip hardware implementation is more desirable than a multi chip parallel implementation.

2. Wavelet Analysis:

Wavelet analysis includes a discussion of different wavelet functions and gives in detail the analysis of wavelet power spectrum. Reviews are adapted for discrete notation from the continuous formulas given in Daubechies (1990). Practical details in applying wavelet analysis are taken from Farge (1992), Wengand Lau (1994), and Meyers et al. (1993).

2.1. Windowed Fourier Transform

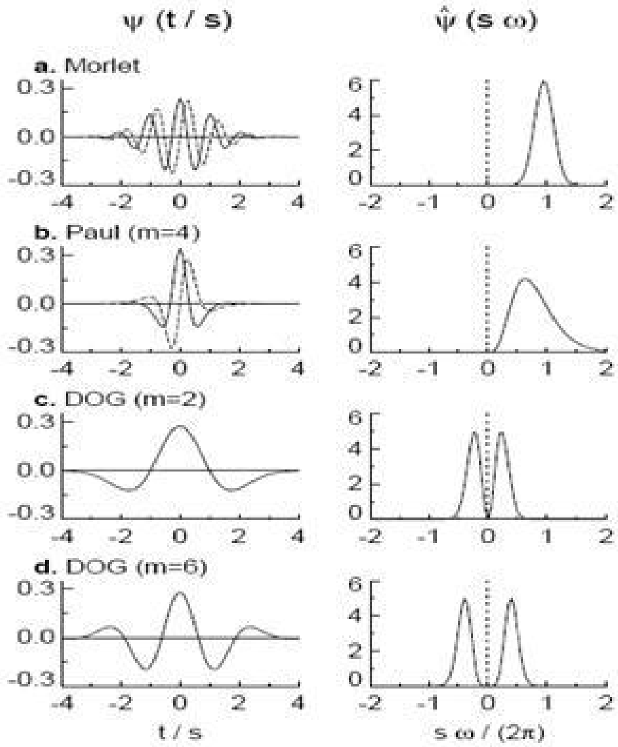
The WFT represents an analysis tool for extracting local frequency information from a signal. The Fourier transform is performed on a sliding segment of length T from a time series of time step δt and total length $N\delta t$, thus returning frequencies from T^{1} to $(2\delta t)^{1}$ at each time step. The segments are windowed with an arbitrary function like boxcar or like Gaussian window. According to Kaiser (1994), the WFT is an inaccurate and inefficient method of time–frequency localization, as it imposes a scale or response interval T for analysis. The inaccuracy is due to the aliasing of high and low frequency components that do not fall within the frequency range of window. The inefficiency comes from the $T/(2\delta t)$ frequencies. In addition several window lengths must usually be analyzed to determine the most appropriate choice. Predetermined scaling is not appropriate for wide range of dominant frequencies. The method of time–frequency localization which is scale independent for wavelet analysis may be employed.

2.2. Wavelet Transform

The wavelet transform is used to analyze time series that contain non stationary power at many different frequencies (Daubechies 1990). If a function has a time series x_n with equal time spacing δt and other has a wavelet function $\Psi_0(\eta)$ that depends on a non dimensional time parameter η. Then the function is admissible as a wavelet function with zero mean and is localized in both time and frequency space (Farge 1992).Morlet wavelet consisting of a plane wave and modulated by Gaussian system with non dimensional given frequency w_0 is by

$$\Psi_0(\eta) = \pi^{-\frac{1}{4}} e^{i\omega_0 \eta} e^{-\frac{\eta^2}{2}} \qquad \text{……………………….. (1)}$$

and its wavelet is shown in Fig. 1a.



Wavelet function is used generically to refer either orthogonal or non orthogonal wavelets but wavelet basis refers only to an orthogonal set of functions. Orthogonal basis implies the use of the discrete wavelet transform while non orthogonal wavelet function is used either in discrete or in continuous wavelet transform (Farge 1992). This paper contains continuous transform although all of the results for significance testing, smoothing in time and scale, and cross wavelets are applicable to the discrete wavelet transform. The continuous wavelet transform of a discrete sequence x_n is defined as the convolution of x_n with a scaled and translated version of $\Psi_0(\eta)$

$$\hat{x}_k = \frac{1}{N}\sum_{n=0}^{N-1} x_n e^{-\frac{2\pi kn}{N}} \quad \text{……………………………. (2)}$$

Where the (*) indicates the complex conjugate. The variation of wavelet scale "s" and mapping along localized time index "n" give the amplitude versus scale and how this amplitude varies with time. The calculations of wavelet transform are easy using equation (2) but it is considerably faster to do the calculations in Fourier space. The approximation of continuous wavelet transform by convolution (2) is done *N* times for each scale, where *N* is the number of points in the time series (Kaiser 1994). The convolution theorem for N points allows doing all *N* convolutions simultaneously in Fourier space using a discrete Fourier transform (DFT). The DFT of x_n is

$$\hat{x}_k = \frac{1}{N}\sum_{n=0}^{N-1} x_n e^{-\frac{2\pi kn}{N}} \quad \text{……………….. (3)}$$

Where k=1….N-1 is the frequency index.

In the continuous limit, the Fourier transform of a function $\Psi(\tilde{t}/s)$ is given by $\Psi_0(s\omega)$

By the convolution theorem, the wavelet transform is the inverse Fourier transform of the product.

$$W_n(s) = \sum_{n=0}^{N-1} \hat{x}_k \Psi^*(s\omega_k) e^{i\omega_k n\delta t} \quad \text{……………… (4)}$$

Where the angular frequency is defined as

$$\omega_k = \begin{cases} \frac{2\pi k}{N\delta t} : & k \le \frac{N}{2} \\ \frac{2\pi k}{N\delta t} : & k > \frac{N}{2} \end{cases} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

Equation (4) and standard Fourier transform routine is suitable for calculating the continuous wavelet transform for a given *s* at all *n* simultaneously and efficiently.

2.3. Normalization

The wavelet transforms obtained by equation (4) for every "s " are directly comparable to each other and with the transforms of other time series, the wavelet function at each scale *s* is normalized to have unit energy

$$\Psi(s\omega_k) = \left(\frac{2\pi s}{\delta t}\right)^{1/2} \Psi_0(s\omega_k) \qquad \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

$$\int_{-\infty}^{+\infty} \left|\hat{\Psi}_0(\omega')\right|^2 d\omega' = 1$$

That is, they have been normalized to have unit energy. Using these normalizations, at each scale *s* one has

$$\sum_{k=0}^{N-1} \left|\Psi(s\omega_k)\right|^2 = N \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

Where N is the number of points. The wavelet transform is weighted only by the amplitude of the Fourier coefficients $\hat{x}_k$ and not by the wavelet function. If one is using the convolution formula the normalization is

$$\psi\left[\frac{(n' - n)\delta t}{s}\right] = \left(\frac{\delta t}{s}\right)^{1/2} \psi_0\left[\frac{(n' - n)\delta t}{s}\right] \qquad \ldots\ldots\ldots\ldots\ldots (8)$$

Where $\Psi_0(\eta)$ is normalized to have unit energy.

2.4. Wavelet Function.

The problem of wavelet analysis is the arbitrary choice of the wavelet function $\Psi_0(\eta)$ Same arbitrary choice is also made in using one of the more traditional transforms such as the Fourier, Bessel and Legendre etc. In choosing the wavelet function, there are several factors which should be considered.

(i) Orthogonal or Non orthogonal.

The number of convolutions at each scale is proportional to the width of the wavelet basis at that scale in orthogonal wavelet analysis. This produces a wavelet spectrum that contains discrete blocks of wavelet power and is useful for signal processing as it gives the most compact representation of the signal. Periodic shift in the time series produces a different wavelet spectrum. Conversely, a non orthogonal analysis is highly redundant at large scales, where the wavelet spectrum at adjacent times is highly correlated. The non orthogonal transform is useful for time series analysis, where smooth continuous variations in wavelet amplitude are expected.

(ii) Complex or Real

A complex wavelet function provides information about both amplitude and phase and is suitable for adapting oscillatory behavior. A real wavelet function returns only a single component and can be used to isolate peaks or discontinuities.

(iii) Width.

For concreteness, the width of a wavelet function is defined as the *e*-folding time of the wavelet amplitude. The resolution is obtained by the balance between the width in real space and the width in Fourier space. A function with narrow time period will have good time resolution but poor frequency resolution, while a broad time period function will have poor time resolution, but good frequency resolution.

(iv) Shape.

For time series with sharp jumps or steps, a boxcar like function such as Harr, while for smoothly varying time series a smooth function such as a damped cosine are considered. For wavelet power spectra the choice of wavelet function is not critical, and every function will give the same qualitative results as other.

2.5. Reconstruction

Since the wavelet transform is a band pass filter with a known response function, it is possible to reconstruct the original time series using either deconvolution or the inverse filter. This is easy for orthogonal wavelet transform with orthogonal basis, but for continuous wavelet transform it is complicated by the redundancy in time and scale. The redundancy will make it possible to reconstruct the time series using completely different (δ) function (Farge 1992). This reconstructed time series is just the sum of the real part of the wavelet transform over all scale

$$x_n = \frac{\delta_j \delta_t^{1/2}}{c_\delta \Psi_0(0)} \sum_{j=0}^{J} \frac{\Re\{W_n(s_j)\}}{s_j^{1/2}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

The factor Ψ_0 (0)) removes the energy scaling, while the $s_j^{\frac{1}{2}}$ converts the wavelet transform to an energy density. The factor $C_\delta$comes from the reconstruction of a δ function from its wavelet transform using the function Ψ_0 (η) and C_δ is a constant for each wavelet function. The sum of complex $W_n(s)$ is used for original time series if complex. For new wavelet function C_δ is derived assuming a time series with a δ function at time $n = 0$ and given by $x_n = \delta_{n0}$. This time series has a Fourier transform $\hat{x}_k = N^{-1}$, constant for all *k*. Substituting $\underline{x_k}$ in equation (4), at time $n = 0$, the wavelet transform becomes

$$W_\delta(s) = \frac{1}{N} \sum_{k=0}^{N-1} \Psi^*(s\omega_k) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

The reconstruction for equation (9) gives

$$c_\delta = \frac{\delta_j \delta_t^{1/2}}{\Psi_0(0)} \sum_{j=0}^{J} \frac{\Re\{W_\delta(s_j)\}}{s_j^{1/2}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (11)$$

The C_δ is scale independent and is a constant for each wavelet function.

3. Theoretical Spectrum And Significance Level:

To determine significance levels for either Fourier or Wavelet spectra an appropriate background spectrum is required. This spectrum is either white noise or red noise. The theoretical white or red noise spectra are derived and compared. These spectra are used to establish a null hypothesis for the significance of a peak in the wavelet power spectrum.

3.1. Fourier red noise spectrum

Many geophysical time series can be modeled as either white noise or red noise. A simple model for red noise is the univariate lag-1 autoregressive [AR (1) or Markov] process:

$$x_n = \alpha x_{n-1} + z_n \quad \text{.......................... (12)}$$

Where α is the assumed lag-1 autocorrelation, $x_0 = 0$, and z_n is taken from Gaussian white noise. Following Gilman et al. (1963), the discrete Fourier power spectrum of equation (12), after normalizing, is

$$P_k = \frac{1 - \alpha^2}{1 + \alpha^2 - 2\alpha \cos\left(2\Pi k / N\right)} \quad \text{.......................... (13)}$$

Where k = 0 … *n/2* is the frequency index. Thus, by choosing an appropriate lag-1 autocorrelation, one can use equation (13) to model a red-noise spectrum. Note that α= 0 in equation (13) gives a white-noise spectrum. The Fourier power spectrum for the Niño3 SST is shown by the thin line in Fig. 2.

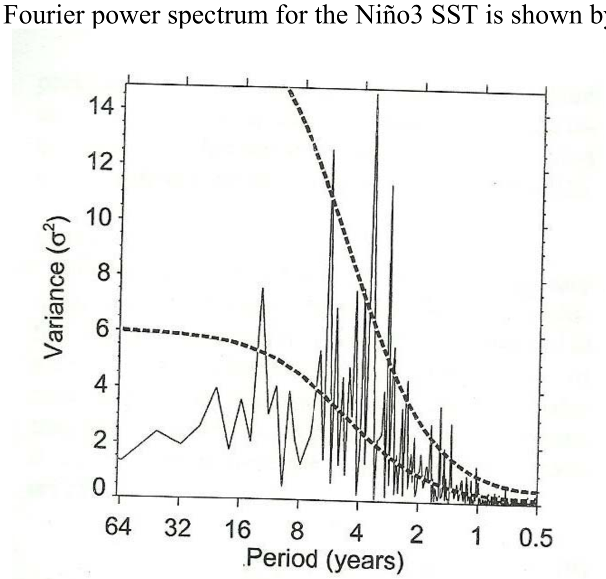


Fig. 2

The spectrum has been normalized by $N/2\sigma^2$, where *N* is the number of points, and σ^2 is the variance of the time series. Using this normalization, white noise would have an expectation value of 1 at all frequencies. The red-noise background spectrum for α= 0.72 is shown by the lower dashed curve in Fig. 2. This red-noise was estimated from $(\alpha_1 + \sqrt{\alpha_2})/2$, where α_1 and α_2 are the lag-1 and lag-2 autocorrelations of the Niño3 SST. One can see the broad set of ENSO peaks between 2 and 8 yr, well above the background spectrum.

3.2. Wavelet red noise spectrum

The wavelet transform in equation (4) is a series of band pass filters of the time series. If this time series can be modeled as a lag-1 AR process, then it seems reasonable that the local wavelet power spectrum, defined as a vertical slice as in equation (13). To test this hypothesis, 100 000 Gaussian white-noise time series and 100 000 AR (1) time series were constructed, along with their corresponding wavelet power spectra. The local wavelet spectra were constructed by taking vertical slices at time $n = 256$. The lower smooth curves in Figs. 3a and 3b show the theoretical spectra from (13). The dots show the results from the Monte Carlo simulation. On average, the local wavelet power spectrum is identical to the Fourier power spectrum given by (13).Therefore, the lower dashed curve in Fig. 2 also corresponds to the red-noise local wavelet spectrum. The average of all the local wavelet spectra tends to approach the (smoothed) Fourier spectrum of the time series.

4. **Extensions to wavelet analysis**:

4.1. Filtering

The wavelet transform in (4) is essentially a band pass filter of uniform shape and varying location and width. By summing over a subset of the scales in (9), one can construct a wavelet filtered time series .

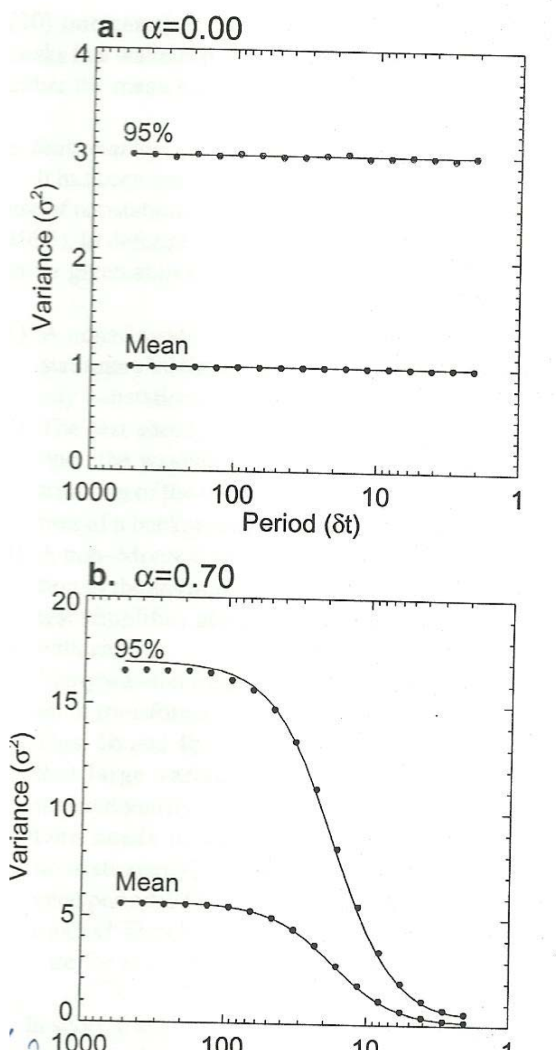


Fig.3(a,b)

$$x'_n = \frac{\delta_j \delta_t^{1/2}}{C_\delta \Psi_0(0)} \sum_{j=j_1}^{j_2} \frac{\Re\{W_n(s_j)\}}{s_j^{1/2}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (13)$$

This filter has a response function given by the sum of the wavelet functions between scales j_1 and j_2. This filtering can also be done on both the scale and time simultaneously by defining a threshold of wavelet power. This "denoising" removes any low-amplitude regions of the wavelet transform, which are presumably due to noise. This technique has the advantage over traditional filtering in that it removes noise at all frequencies and can be used to isolate single events that have a broad power spectrum or multiple events that have varying frequency. A more complete description including examples is given in Donoho and Johnstone (1994). Another filtering technique involves the use of the two dimensional wavelet transform. An example can be found in Farge et al. (1992), where two-dimensional turbulent flows are "compressed" using an orthogonal wavelet packet. This compression removes the low-amplitude "passive" components of the flow, while retaining the high-amplitude "dynamically active" components.

5. Conclusion:

Wavelet analysis is a useful tool for analyzing time series with many different timescales or changes in variance. The steps involved in using wavelet analysis are as follows: Find the Fourier transform of the (possibly padded) time series. Choose a wavelet function and a set of scales to analyze. For each scale, construct the normalized wavelet function using (6).Find the wavelet transform at that scale using (4). Determine the cone of influence and the Fourier wavelength at that scale. After repeating the steps for all scales, remove any padding and contour plot the wavelet power spectrum. Assume a background Fourier power spectrum (e.g., white or red noise) at each scale, then use the chi-squared distribution to find the 95% confidence (5% significance) contour.

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