International Journal of Applied Research in Mechanical Engineering

Volume 2 | Issue 4 Article 9

April 2013

EULER ANGLE ERROR COMPUTATION AND THRUST VECTOR CONTROL SCHEME FOR INITIAL TURNING PHASE OF A VERTICAL LAUNCH SURFACE TO AIR MISSILE

RAM B. SANKAR
Directorate of Systems, Defence Research and Development Laboratory, Hyderabad, India, sankar.rb@gmail.com

Recommended Citation

SANKAR, RAM B. (2013) "EULER ANGLE ERROR COMPUTATION AND THRUST VECTOR CONTROL SCHEME FOR INITIAL TURNING PHASE OF A VERTICAL LAUNCH SURFACE TO AIR MISSILE," *International Journal of Applied Research in Mechanical Engineering*: Vol. 2 : Iss. 4 , Article 9.
Available at: https://www.interscience.in/ijarme/vol2/iss4/9

EULER ANGLE ERROR COMPUTATION AND THRUST VECTOR CONTROL SCHEME FOR INITIAL TURNING PHASE OF A VERTICAL LAUNCH SURFACE TO AIR MISSILE

RAM B SANKAR[1], AJIT B CHAUDHARY[2], ANIL KUMAR[3]

[1,2,3]Directorate of Systems, Defence Research and Development Laboratory, Hyderabad, India

---

Abstract- A vertical launch missile requires to be turned towards the Predicted Interception Point (PIP) with the target within few seconds from the launch. The autopilot used for the same requires the error in Euler Angles between the missile and the target to be supplied, which is generally computed using Error Quaternions approach [1]. For some kinds of engagement scenarios, say a Near Boundary Low Altitude target, the turn angle required in the pitch plane during the initial phase can become more than 90 degrees. In such cases, the conventional 3-2-1 frame transformation method fails to compute the Euler Angle errors correctly. As an alternative approach the Euler Angle error computation using 2-3-1 frame transformation is proposed in this paper. After computing the Euler Angle Errors, the desired turning is achieved by Thrust Vector Control [2] using jet vanes or thrusters. The initial turning using Aerodynamic control is not desirable because of low dynamic pressure, where as the usage of jet vanes brings more hardware complications for small tactical missiles. The possibility of using thrusters for initial turning phase in a short range tactical missile is studied in this paper with the help of simulation results.

I. INTRODUCTION

A short range, vertical launch missile used against areal kind of targets ranging from very low altitudes to high altitudes requires sharp turning from initial pitch attitude (90^0), to be done within very few seconds from the launch.

Initially the Euler Angle Errors have to be obtained from the missile Euler Angles and the target Euler Angles at PIP. These are obtained from the Error Quaternion approach. Conventionally for the Quaternion [3] computation procedure [4] from the Euler Angles, 3-2-1 rotation sequence [5] is adopted. But for a vertical launch missile, 3-2-1 rotation sequence cannot be adopted. It is because for the cases involving pitch down demand of more than 90 deg, the 3-2-1 rotation sequence fails to compute the Euler Angle Errors correctly. Hence 2-3-1 rotation sequence has to be adopted to obtain the Euler Angle Errors. This possibility is studied and presented in detail.

The Euler Angle Errors obtained from the Error Quaternion approach directly indicate the required turn angles (roll, pitch and yaw) for the missile to be aligned with the target PIP at the end of turning phase. These required turn angles are achieved by attitude control using a set of thrusters. The control scheme using thrusters are preferred over both the Aerodynamic Control as well as the Jet Vane Control [6]. Aerodynamic Control is not preferred in the turning phase because of less dynamic pressure in the launch phase. Jet Vane Control is not preferred because jet vane control surfaces need extra actuator for control deflection or they need to be coupled with the existing actuators used for deflecting Aerodynamic control surfaces. Also, once the turning phase is over the presence of jet vanes is not preferred because it induces drag. Hence either the material for jet vane control surfaces should be chosen such that it will erode over a period of time in the presence of hot gases or a mechanism should be designed to fold or throw the jet vane mechanism once the turning phase is over. Since both the options are tedious, thrusters are preferred for Thrust Vector Control. The thrusters need only firing commands from OBC (On Board Computer) and no extra mechanical components are required. When thrusters are used, the missile can be either cold launched or hot launched. The study in this paper has been carried out under the assumption that the missile is cold launched. The required turn angles are achieved using thrusters after which the rocket motor is fired. The study for using thrusters for controlling the attitude to achieve the desired turn angles is presented in detail. The method for evolving the specifications of the thrusters to be used is also explained with the help of examples. It was understood that for the initial missile turning phase control, four thrusters are required with same specifications – two each for roll and pitch channels. The method is established, implemented and verified in a Six DOF simulation model, the results for which are presented in detail. It was seen that with the turning scheme using thrusters, the maximum roll angle and lateral angle demands were achieved with in a small duration itself and it was made sure that the body rates were within allowed limits.

II. EULER ANGLE ERROR COMPUTATION USING 2-3-1 ROTATION SEQUENCE

Conventionally for the Euler Angle Error computation 3-2-1 frame transformation is used. But for a SAM system with Near Boundary, Low Altitude target engagement requirement, it was seen from the 6-DOF simulations that the pitch down demand is as high as 110 degrees (which is more than 90 degrees). In such cases the conventional 3-2-1 rotation sequence method [7] computes the Euler Angle Errors incorrectly, which can be demonstrated based on Equation (1).

$Psi = atan2(2*(q_3*q_0 + q_1*q_2),(q_0^2 + q_1^2 - q_2^2 - q_3^2))$
$Phi = atan2(2*(q_2*q_3 + q_0*q_1),(q_0^2 - q_1^2 - q_2^2 + q_3^2))$
$Theta = -asin[2*(q_1*q_3 - q_0*q_2)]$ (1)

where

Phi, Theta, Psi - Euler Angle Errors
q_0, q_1, q_2, q_3 - Error Quaternions

It can be seen from Equation (1) [8] that the computation of Pitch angle (Theta) from Quaternions involves ‘asin’ function, which returns values between –π/2 and +π/2 radians only. Hence for Pitch down demand more than 90 deg, 3-2-1 Frame Transformation method cannot be used. The computation of Roll angle (Phi) and Yaw angle (Psi) are done by using ‘atan2’ function, which returns values between -π and +π. To resolve the above problem, instead of 3-2-1 rotation sequence, 2-3-1 rotation sequence is adopted to compute the Euler Angle Errors. Equation (2) shows the computation of Euler Angle Errors from the Error Quaternions for 2-3-1 frame transformation [9].

$Phi = atan2(2*(q_1*q_0 - q_2*q_3),(q_0^2 - q_1^2 + q_2^2 - q_3^2))$
$Theta = atan2(2*(q_2*q_0 - q_1*q_3),(q_0^2 + q_1^2 - q_2^2 - q_3^2))$
$Psi = asin[2*(q_1*q_2 + q_3*q_0)]$ (2)

where

Phi, Theta, Psi - Euler Angle Errors
q_0, q_1, q_2, q_3 - Error Quaternions

It can be seen from Equation (2) that the computation of Pitch angle (Theta) from Quaternions involves ‘atan2’ function instead of ‘asin’ function, which returns values between –π and +π radians. Hence for Pitch angle difference more than 90 deg, 2-3-1 Frame Transformation method can be adopted. However, the computation of Yaw angle (Psi) is done by using ‘asin’ function, which returns values between –π/2 and +π/2 radians only. This constraint is acceptable for our requirement because of the following reason. In order to achieve the Euler Angle Errors, the missile rolls first towards the crossing parameter of the target PIP and then a pure pitch down will align the missile towards the target PIP. Hence the possibility of high yaw demand will not come to meet the requirements during initial missile turning phase. Thus, Euler Angle Error computation using 2-3-1 rotation sequence is more suitable for Vertical Launch SAM systems.

III. THRUSTERS FOR ATTITUDE CONTROL

The Euler Angle Errors computed forms the input for thruster based initial turning scheme for the missile. The feasibility of using thrusters [10] was studied for this attitude control during the initial turning phase and it was understood that two thrusters each with same impulses are required for performing roll correction and lateral attitude correction. First thruster is fired to get turning in the desired direction and the second thruster is fired to maintain the angle to a desired constant value. The first thruster is fired at a constant time instant and the second thruster is fired at an instant depending on the desired turn angle. The timing of the second thruster firing directly determines the turn angle achieved. Hence a total of four thrusters are required to perform both roll and lateral attitude corrections. The roll correction is planned to be done from 0.3 – 0.8 seconds (0.5 seconds duration) and the pitch correction is planned to be done from 0.8 – 1.5 seconds (0.7 seconds duration). The maximum permissible body rates were taken as 500 deg/s for roll channel and 200 deg/s for lateral channel, due to INS limitations. The specifications of the thrusters to be used are obtained from the following calculations.

IV. THRUSTER SPECIFICATIONS

A. Equations

In order to obtain thruster specifications the relationship between thruster forces and the achieved turn angle have to be obtained. The equations for roll attitude control can be shown as follows.

The entire roll attitude control can be divided into three stages as shown in Figure 1.

(1) Thruster T1 is fired and only T1 is present for duration of t_1 seconds.

(2) Thruster T2 is fired after t_1 seconds and both thrusters T1 and T2 will be present simultaneously but in opposite directions for t_2 seconds.

(3) Thruster T1 is completed and only thruster T2 is present for duration of t_3 seconds.

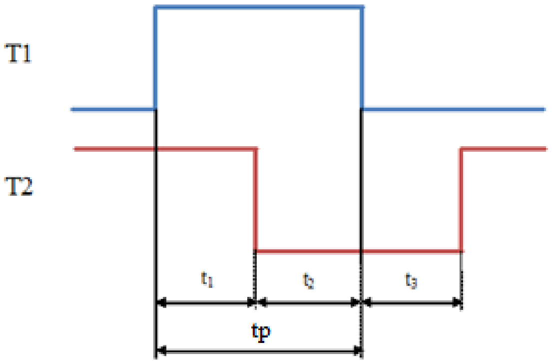

Figure 1 – Thruster profiles

The above three stages completes the roll attitude control. Assuming the duration of one thruster pulse to be t_p, the following equations can be written.

$t_3 = t_1; t_2 = t_p - t_1$

The roll angles achieved at the end of each of three stages are given by

$$\mathrm{Phi}_{\mathrm{achvd1}} = \frac{1}{2} * \mathrm{p}_{\mathrm{dot}} * \mathrm{t}_1^2$$

$$\mathrm{Phi}_{\mathrm{achvd2}} = \mathrm{p}_{\mathrm{dot}} * \mathrm{t}_1 * \mathrm{t}_2$$

$$\mathrm{Phi}_{\mathrm{achvd3}} = \mathrm{p}_{\mathrm{dot}} * \mathrm{t}_1 * \mathrm{t}_3 - \frac{1}{2} * \mathrm{p}_{\mathrm{dot}} * \mathrm{t}_3^2$$

where

p_{dot} is Roll acceleration

The total angle achieved at the end of roll attitude control can be obtained by adding the above equations.

$$\mathrm{Phi}_{\mathrm{reqd}} = \mathrm{Phi}_{\mathrm{achvd1}} + \mathrm{Phi}_{\mathrm{achvd2}} + \mathrm{Phi}_{\mathrm{achvd3}}$$

$$= \frac{1}{2} * \mathrm{p}_{\mathrm{dot}} * \mathrm{t}_1^2 + \mathrm{p}_{\mathrm{dot}} * \mathrm{t}_1 * \mathrm{t}_2 + \mathrm{p}_{\mathrm{dot}} * \mathrm{t}_1 * \mathrm{t}_3 - \frac{1}{2} * \mathrm{p}_{\mathrm{dot}} * \mathrm{t}_3^2$$

$$= \mathrm{p}_{\mathrm{dot}} * (\mathrm{t}_1 * \mathrm{t}_2 + \mathrm{t}_1^2) = \mathrm{p}_{\mathrm{dot}} * \mathrm{t}_1 * \mathrm{t}_\mathrm{p}$$

Hence the start time of second thruster pulse from the instant of first thruster firing is given by

$$\mathrm{t}_1 = \frac{\mathrm{Phi}_{\mathrm{reqd}}}{\mathrm{p}_{\mathrm{dot}} * \mathrm{t}_\mathrm{p}} \qquad (1)$$

where

Phi_{reqd} - Required roll correction (deg)

t_p - Duration of one thruster pulse (s)

The maximum roll rate obtained during this phase is given by

$$\mathrm{p}_{\max} = \mathrm{p}_{\mathrm{dot}} * \mathrm{t}_1 \qquad (2)$$

From Equation (1) and (2), the duration of thruster pulse for roll channel and roll acceleration requirements can be obtained.

The thruster force requirement for roll correction can be obtained by the following Equation (3).

$$\mathrm{F} = \frac{(\mathrm{I_{XX}} * \mathrm{p}_{\mathrm{dot}})}{(\mathrm{Z_{TH}} - \mathrm{Z_{CG}}) * \mathrm{g} * \frac{180}{\pi}} \qquad (3)$$

where

I_{xx} - Moment of Inertia (kg-m^2)

Z_{TH} - Thruster location along missile Z axis (m)

Z_{CG} - Missile CG location along Z axis (m)

g - Acceleration due to gravity (m/s^2)

The equations can be extended to lateral channel also as follows.

The start time of second thruster pulse from the instant of first thruster firing is given by

$$\mathrm{t}_1 = \frac{\mathrm{Theta}_{\mathrm{reqd}}}{\mathrm{q}_{\mathrm{dot}} * \mathrm{t}_\mathrm{p}} \qquad (4)$$

where

$Theta_{reqd}$ - Required pitch correction (deg)

q_{dot} - Pitch acceleration (deg/s^2)

t_p - Duration of one thruster pulse (s)

The maximum pitch rate obtained during this phase is given by

$$\mathrm{q}_{\max} = \mathrm{q}_{\mathrm{dot}} * \mathrm{t}_1 \qquad (5)$$

From Equation (4) and (5), the duration of thruster pulse for lateral channel and pitch acceleration requirements can be obtained.

The thruster force requirement for pitch correction can be obtained by Equation (6).

$$\mathrm{F} = \frac{(\mathrm{I_{YY}} * \mathrm{q}_{\mathrm{dot}})}{(\mathrm{X_{TH}} - \mathrm{X_{CG}}) * \mathrm{g} * \frac{180}{\pi}} \qquad (6)$$

where

I_{YY} - Moment of Inertia (kg-m^2)

X_{TH} - Thruster location along missile X axis (m)

X_{CG} - Missile CG location along X axis (m)

Since the Moment of Inertia and CG location varies during boost phase, their mean values during turning duration are taken for obtaining thruster specifications.

B. Thruster Specifications

Maximum turn angle required in pitch plane (Obtained from simulations for Near Boundary, Low Altitude target engagement)

$$\mathrm{Theta}_{\mathrm{reqd}} = 90 - (-20) = 110 \text{ deg}$$

The maximum pitch turn angle can be written based on Equation (4) as follows.

$$\mathrm{Theta}_{\mathrm{reqd}} = \mathrm{t}_1 * \mathrm{q}_{\mathrm{dot}} * \mathrm{t}_\mathrm{p}$$

In order to keep pitch rate under 200 deg/s, we have

$$\mathrm{q}_{\mathrm{dot}} * \mathrm{t}_1 \leq 200 \text{ deg/s}$$

From the above equations, duration of thruster pulse (t_p) can be obtained as

$$\mathrm{t}_\mathrm{p} = \frac{110}{200} = 0.55 \text{ seconds}$$

As mentioned earlier, the lateral channel turning phase duration is given by

$$\mathrm{t}_{\mathrm{latetal}} = 0.7 \text{ seconds}$$

Hence t_1 is obtained as

$$\mathrm{t}_1 = \mathrm{t}_{\mathrm{lateral}} - \mathrm{t}_\mathrm{p} = 0.7 - 0.55 = 0.15$$

In order to keep pitch rate under limit, based on Equation (5) pitch acceleration should be

$$\mathrm{q}_{\mathrm{dot}} = \frac{200}{\mathrm{t}_1} = 1333.33 \text{ deg/s}^2$$

For typical values of thruster location, CG location and Moment of Inertia, the thruster force requirement can be obtained based on Equation (6) as

$$\mathrm{F} = 186 \text{ kgf}$$

Hence for pitch down of 110 degrees between 0.8 – 1.5 seconds, two thrusters are required with following specifications.

Thruster, T3	-	186 kgf, 0.55 sec
Thruster, T4	-	186 kgf, 0.55 sec

For a maximum pitch turn of 110 degrees, T3 is fired first and T4 is fired after t_1 seconds. For turn angles less than 110 degrees, the thruster T4 is fired before t_1 seconds. The firing start time of thruster T4 determines the final turn angle obtained.

C. Roll channel Thruster Specifications

The maximum roll angle turn required for the missile during the initial turning phase is 180 degrees. For roll attitude control two more thrusters are required. The procedure followed for obtaining lateral channel thruster specifications can be repeated to obtain the thruster specifications for performing roll correction. Hence for roll correction of 180 degrees between 0.3 – 0.8 seconds, two thrusters are found to be required with following specifications.

Thruster, T1	-	106 kgf, 0.36 sec
Thruster, T2	-	106 kgf, 0.36 sec

The start time of second thruster pulse from the instant of first thruster firing can be obtained as

$$t_1 = 0.14 \text{ seconds}$$

For a maximum roll correction of 180 degrees, T1 is fired first and T2 is fired after t_1 seconds. For turn angles less than 180 degrees, the thruster T2 is fired before t_1 seconds. The firing start time of thruster T2 determines the final turn angle obtained.

D. Comments

The study on thruster specifications can be extended by generalizing the equation for the roll and lateral channels as shown in the following.

$$t_{start} = \frac{\theta_{error}}{(q_{dot} * t_p)}$$

where

t_{start} - Start time of second thruster pulse (seconds)

θ_{error} - Euler Angle Error to be corrected (degrees)

q_{dot} - Body acceleration of the missile (deg/s²)

t_p - Total Duration of the thruster pulse (seconds)

As the thruster with constant force produces constant body acceleration, with the duration of a thruster pulse fixed, it can be noted that the start time of the second thruster pulse is having a linear relationship with the Euler Angle Error to be corrected or the turn angle to be achieved. The argument holds good for both the roll and the lateral channels, which is a very significant output of the study and can be used for the implementation of the autopilot for Thrust Vector Control based on thruster.

It has to be noted the linear relationship is valid only when the thrusters are able to give constant force during its pulse duration. But in reality the thrusters don’t provide constant force during its pulse duration. Hence an autopilot array has to be formed relating the required turn angle for each channel with the start time of second thruster. Based on the thrust profiles produced by the actual thrusters, the array can be tuned.

V. THRUSTER BASED CONTROL DESIGN FOR ROLL CHANNEL

A simple thruster model has been implemented in a Six DOF simulation model with the specifications computed in the previous section. The thruster T1 was fired from 0.3 seconds. The firing start time of thruster T2 was varied based on the turn angle requirement as per Equation (4). The roll angles achieved for various demands are plotted from Six DOF simulation model. Figure 2 shows the roll dynamics for different roll angle demands (45^0, 90^0, 135^0, and 180^0). It can be seen that maximum roll rate is within permissible value 500 deg/s. The roll angle plot shows that different roll angle demands are getting achieved at end of roll correction phase. Figure 3 shows the thruster force profiles for different roll angle demands. Thruster T1 is fired always fired at a constant instant shown by black colour. The start timing of Thruster T2 is varied according to different roll angle demands.

VI. THRUSTER BASED CONTROL DESIGN FOR LATERAL CHANNEL

A simple thruster model has been implemented in a Six DOF simulation model with the specifications computed in the previous section. The thruster T3 was fired from 0.8 seconds. The firing start time of thruster T4 was varied based on the turn angle requirement as per Equation (1). The pitch angles achieved for various demands are plotted from Six DOF simulation model.

Figure 4 shows the pitch dynamics for different pitch angle demands (30^0, 60^0, 90^0, and 110^0). It can be seen that maximum pitch rate is within permissible value 200 deg/s. The pitch angle plot shows that different pitch angle demands are getting achieved at end of pitch correction phase. Figure 5 shows the thruster force profiles for different pitch angle demands. Thruster T3 is fired always fired at a constant instant shown by black colour. The start timing of Thruster T4 is varied according to different pitch angle demands.

VII. CONCLUSIONS

A computation method for Euler Angle errors between the missile and target at PIP was proposed using 2-3-1 frame transformation for a vertical launch surface to air missile. The inadequacies of the conventional algorithm involving 3-2-1 frame transformation were brought out for cases involving pitch down demand more than 90 degrees. Even though the conventional computation does good work when the pitch down demand is less than 90 degrees for a vertical launch missile, the method fails when the same goes more than 90 degrees. The computation method proposed can be used for any vertical launch missiles, where there are requirements of near boundary and low altitude target engagements. Such cases necessarily demand a pitching down of more than 90 degrees.

The possibility of using thrusters for the initial turning phase was explored. It was found that two thrusters each are required for performing the roll control and the pitch control based on the Euler Angle Errors between the missile and target at PIP. The specifications for the four thrusters were brought out based on the initial turning requirements. It was seen the firing time of the second thruster is having a linear relationship with the turn angle to be achieved in both the channels. By taking this advantage a simple autopilot was implemented in the Six DOF simulation model, which computes the start time of the second thruster based on the turn angle demand. The thruster based turning can be implemented for the vertical launch missiles with scenarios involving Near Boundary target engagements.

ACKNOWLEDGMENT

The authors take this opportunity to express their sincere gratitude to Shri N Prabhakar, Outstanding Scientist, (Associate Director, DRDL) for his encouragement and valuable suggestions for the work. The authors are also grateful to Shri V Srinivasa Rao, Sc 'D' and Shri Sunil Kumar Tata, Sc 'D' for their valuable inputs to the study.

REFERENCES

[1] Sheelu Jose, Dhekane M. V., and Dasgupta. S."Quaternion Implementation in Autopilot," Proceedings of the AlAA International Communications Satellite Systems Conference, San Diego, California, June 11-14, 2006, AlAA Paper No. 2006-5430.

[2] Michael. W. Dierks, and Kevin. A. Wise., "TVC Control for the AIAA Design Challenge Airplane," Proceedings of the AlAA Guidance, Navigation, and Control Conference, 1993, pp. 995-1005, AlAA Paper No. 93-3810-CP.

[3] Hamilton, and Sir W. R., Elements of Quaternions, Longmans, London, Green and Co., 1866.

[4] George. M. Siouris., Missile Guidance and Control Systems, Springer, New York, 2003.

[5] James. R. Wertz., Spacecraft Attitude Determination and Control, Kluwer, Dordrecht, 1978.

[6] Der-Ren Taur, and Jeng-Shing Chern, "Optimal Side Jet Control for Vertically Cold Launched Tactical Missiles," Proceedings of the AIAA Guidance, Navigation and Control Conference, Denver, CO, August 14-17, 2000, AIAA-paper-2000-4164.

[7] George. P. Davailus, and Brett. A. Newman., "The Application of Quaternion Algebra to Gyroscopic Motion, Navigation and Guidance," Proceedings of the AlAA/AAS Astrodynamics Specialist Conference, Keystone, Colorado, August 21-24, 2006, AlAA Paper No. 2006-6657.

[8] Philips, W. F., Hailey, C. E., Gerbert, G. A., Review of Attitude Representation Used for Air Kinematics, Journal of Aircraft, Vol. 38, No. 4, July-August 2001, pp. 718-737.

[9] George. M. Siouris., Aerospace Avionics Systems, Academic Press, London, 1993.

[10] N. Prabhakar, Sunil Kumar Tata, A. K. Sarkar, M. Vaidynathan, "Pitch Down of Vertically Launched Missile using Multiple Single Shot Thrusters", Second Annual Multinational Ballistic Missile Defence Conference, Telaviv, Israel, July 2011.

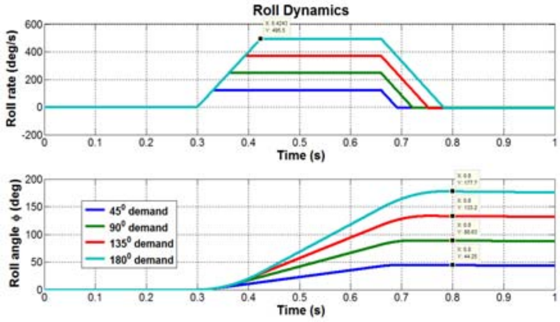

Figure 2 – Roll Dynamics achieved using thrusters for different roll angle demands

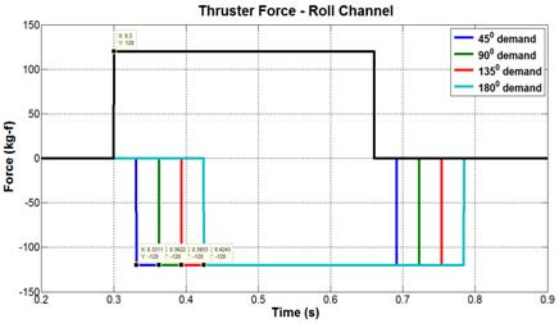

Figure 3 – Thruster profiles to achieve different roll angle demands

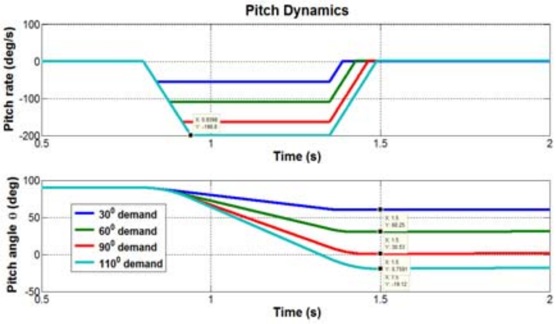

Figure 4 – Pitch Dynamics achieved using thrusters for different pitch angle demands

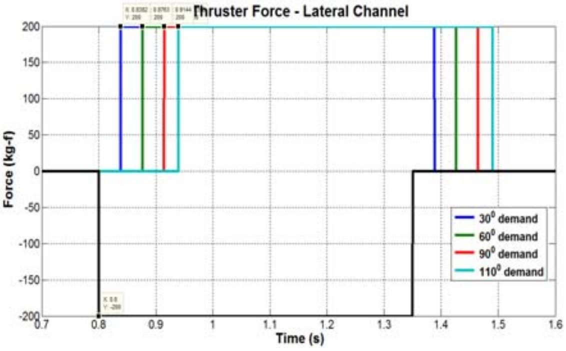

Figure 5 – Thruster profiles to achieve different pitch angle demands

◆◆◆