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ARTICLES

GAMING REGULATION AND MATHEMATICS: A MARRIAGE OF NECESSITY

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INTRODUCTION

Probability is at the foundation of the gaming business. Every wager in a casino is designed and calibrated according to the laws of chance to exact a certain percentage of the players' money. This is how the casino makes money.[1] In the short run, a player may win or lose, but in the long run, the gods of probability will catch up. Mathematicians call this the law of large numbers. For casinos, this mathematical law ostensibly guarantees revenues at virtually no risk. As those on the dealing side of the table know, there is no luck when it comes to the business of casino gaming—"it is all mathematics."[2]

Not surprisingly, then, mathematical issues arise in gaming law and regulation. A game based on bad mathematics—that is, one whose house advantage is misrepresented (deliberate or otherwise), or whose house advantage is too large or small—may

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1. One popular author put it this way: "A casino is a mathematics palace set up to separate players from their money." NICHOLAS PILEGGI, CASINO 14 (1995).

2. DAVID SPANIER, INSIDE THE GAMBLER'S MIND 71-75 (1994) (quoting Nico Zographos, a dealer and gambler extraordinaire who dealt for the Greek syndicate baccarat games in France during the decades after World War I).

not satisfy standards of fairness and honesty, qualities that are paramount to maintaining the integrity of gaming. Integrity is an important regulatory concern for two reasons. First, the government has an interest in protecting patrons from being cheated or taken advantage of by unscrupulous operators. In most cases, only the government has the ability to have access to the information necessary to assure fairness and honesty. Second, the well being of the entire industry is dependent on the public perception that it is fair and honest.

In Nevada, a primary regulatory policy objective is to "instill public confidence and trust that the games are honest and fair,"[3] and on the Nevada Gaming Control Board's list of gaming scams is "altering the random selection process of the gambling game."[4] Thus, gaming regulators require evidence that slot combinations are selected randomly, cards are dealt from a shuffled deck, the rolls of the dice result in random outcomes, the house advantage is within acceptable limits, and the casino is keeping only its fair share of the money wagered. These considerations are inherently mathematical—establishing randomness (or the lack thereof) requires statistical proof; determining a game's advantage involves mathematical arguments.

Despite this obvious reliance on the mathematics, little has been written about the role of quantitative evidence in the regulatory process and requirements relating to devices and procedures used in the games. The purpose of this Article is to examine the role of mathematics and statistics in the laws and regulatory process related to casino games, devices, and procedures.

I. FAIRNESS AND HONESTY

Regulation attempts to achieve many goals, but most gaming regulatory systems share common objectives: keep the games fair and honest and assure that players are paid if they win.[5] Fairness and honesty are two different concepts—a casino can be honest but not fair. Honesty refers to whether the casino offers games whose chance elements are random. Randomness may be defined here as "the observed unpredictability and absence of pattern in a set of

3. Keith Copher, Chief of Enforcement, Nevada Gaming Control Board, Address at the Casino Regulatory Compliance Conference in Las Vegas, Nevada (Aug. 17, 2000).

4. *Id.*

5. A recent British government-sponsored, wide-ranging review cites three reasons to regulate gambling: (1) "[k]eeping gambling crime free[,]" (2) "[f]airness to the punter[,]" and (3) "[p]rotecting the vulnerable." GAMBLING REVIEW BODY, GAMBLING REVIEW REPORT, 2001, Cm. 5206, at 2-4 [hereinafter GAMBLING REVIEW REPORT].

elements or events that have definite probabilities of occurrence."[6] For example, a slot machine is honest if the outcome of each play is not predetermined in the casino's favor.

Fairness refers to the game advantage. How much of each dollar wagered should the casino be able to keep? For example, a slot machine is not fair if it retains, on average, 90% of every dollar bet. In economic terms, regulators attempt to ensure fairness by setting the maximum (and minimum) price a casino can charge players for the gambling experience. Since the price of a casino game is a function of the odds the casino offers to the player, regulatory price controls are a function of controlling the game odds.

A. *Mathematical Evidence in Gaming Regulation*

Two major regulatory issues relating to fairness and honesty—ensuring random outcomes and controlling the house advantage—are inextricably tied to mathematics. Most regulatory bodies routinely require some type of mathematical analysis to demonstrate the game advantage and/or confirm that game outcomes are random.

B. *Probability Versus Statistical Analyses*

We use the term "mathematical evidence" in a general sense here. A formal distinction exists between two types of such evidence: (1) arguments made using only laws of probability theory, and (2) statistical analyses of empirical data (whose inferential conclusions depend on the laws of probability). The distinction is important because the former can establish results with 100% certainty while the latter cannot.[7] For example, assuming random outcomes, the house advantage in roulette and craps can be determined exactly solely by applying the laws of probability. The determination is based on standard probability logic: If X is true (random outcomes), then Y must be true (the house edge is 5.3%). If the assumptions are true and the math is valid, the conclusion is without question. This type of probability-only analysis, however, is not possible or feasible for some games

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6. NEV. ADMIN. CODE ch. 14, § 14.010(10) (2002). Similar language is used in other states. In Colorado, for example, "'randomness' means the unpredictability and absence of pattern in the outcome of an event or sequence of events[,]" 1 COLO. CODE REGS. § 12-47.1-1221(5) (2001) and "[a] random event has a given set of possible outcomes, each with a given probability or occurrence." 1 COLO. CODE REGS. § 12-47.1-1224.

7. Probability is primarily a theoretical branch of mathematics that studies the consequences of mathematical definitions in predicting the likelihood of future events. Statistics is primarily an applied branch of mathematics that tries to make sense of observations in the real world by analyzing observed (past) events.

and situations—such as establishing that game outcomes are indeed random, or analyzing games with an extremely large number of possible outcomes. For these cases, a well-designed and conducted statistical analysis of observed sample data might be appropriate.

Unlike probability arguments, statistical evidence consists of the analysis of empirical data. This data might be, for example, the outcomes of a large number of rolls of a die, or data generated from a computer simulation model.[8] In statistical terms, this observed data is a sample from the population. Because the entire population is not observed, attaching a 100% confidence level to the conclusions of such a statistical study is not possible. Rather, the results might be reported with a 95% or 99% level of confidence. Thus, for example, we may not be 100% sure that a particular die is honest, but observing a large enough number of rolls may convince us that the rolls of the die are random with a high degree of confidence. Results and conclusions based on statistical studies with high confidence levels—typically 95% or 99%—have long been accepted in many disciplines, including the hard sciences, psychology, social sciences, and business.[9] Courts also admit and use statistical evidence with results carrying similar confidence levels in a wide variety of legal cases.[10] The level of confidence that is appropriate for a given situation depends on the issue at hand—certainly a higher confidence level is needed when human lives are at stake as opposed to, say, a researcher's reputation—and is open to some debate, but 95% and 99% confidence levels are common in many settings.

The gaming regulatory process often uses both probability theory arguments and statistical studies based on analysis of empirical sample data. These mathematical analyses are primarily relied on as evidence to confirm odds, house advantage, or randomness. In the case of statistical evidence, standards vary with respect to the type of tests and the level of confidence required. More specific details about these standards are discussed below.

8. A simulation study is a form of statistical analysis where the observed data is generated from a computer model. Simulations are often used to determine house advantages and optimal strategies for games that are difficult or impossible to analyze using other methods.

9. It is common to express such results as statistically significant at the 5 percent (or 1 percent) level of significance.

10. Examples include Title VII discrimination, deceptive advertising, trademark infringement, mass torts, environmental protection, and identification evidence. *See generally*, 2 JOSEPH GASTWIRTH, STATISTICAL REASONING IN LAW AND PUBLIC POLICY (1988) (illustrating the use of high confidence levels in statistical evidence in various legal settings and noting cases where courts use statistics to analyze public policy concerns).

II. REGULATIONS RELATING TO FAIRNESS

Fairness refers to the games being designed so that they do not take unreasonable advantage of the player. Slot machines commonly pay back, on average, 95% of all wagers accepted.[11] The 5% retained by the casino pays for its capital costs and operating expenses, and also provides a fair profit. Setting the machines to retain 40%, however, may not be reasonable.

A. Price Setting

Unlike most other consumer purchases where the price of a product is fixed, the price of a casino game depends on the rules of the game, the payoffs for winning wagers, the amount of time the player plays, the speed of the game, and possibly the skill of the player. Although all these factors may contribute to how much a player ultimately spends, the inherent cost of the game is set by the "odds," or more precisely, the house advantage. All else equal, playing a 2% house edge game will cost the player twice as much, on average, as playing a game with a 1% house edge. Casinos set prices on games by (consciously or unconsciously) setting the odds. This may be accomplished through rule variations that are more or less favorable to the player, by altering the payoffs on certain wagers, or by offering a different type of game product. Regardless of the method, the house advantage determines how much it costs a player to play a particular game.

1. Rule Variations

Examples of games in which rule variations can affect the odds are blackjack and craps. In blackjack, the dealer hitting a soft seventeen increases the house advantage 0.2% compared to a similar game where the dealer must stand on a soft seventeen. The number of decks used, no soft doubling, and no re-splitting of pairs are other examples of rule variations in blackjack that affect the overall price of the game to the player. In craps, the "free odds" bet can vary the price of the game product by the amount of odds that can be taken—the greater the odds taken, the lower the overall house advantage.[12] A player who bets the pass line and takes single odds is at a 0.85% disadvantage, but only a 0.61% with double odds, and 0.47% with triple odds. This means for every $100 wagered on the pass line with single odds ($50 pass line and $50 odds), on average, the player will pay a price of about

11. This is about average for 25-cent slots in Nevada. *See* NEV. STATE GAMING CONTROL BD., GAMING REVENUE REPORT 1 (Nov. 2001) (listing the win percent for 25-cent slot machines at approximately 5% [indicating a 95% pay back] during November, the previous three months, and for the previous year).

12. On the total wager: original plus odds.

$0.85; while $100 bet on the pass line with triple odds ($25 pass line and $75 odds) will cost about $0.47.

2. Altering Payoffs

Another way for casinos to set prices is by altering the payoffs on wagers. The field bet in craps, for example, typically offers even money on the 3, 4, 9, 10, and 11, and a 2-to-1 payoff for the 2 and 12. The house advantage for this payoff structure is 5.56%. If either the 2 or 12 (but not both) were to pay triple, the house advantage would be reduced to 2.78%.[13] Similarly, the usual casino commission on winning banker bets in baccarat is 5%. Some casinos attempt to attract and keep players by lowering this commission to 4%, or even 3%.[14] Such a reduction is equivalent to raising the payoff on this bet from 95% to 96% or 97%, with the effect of reducing the house advantage from the usual 1.06% to 0.60% (with the 4% fee) or 0.14% (3% fee). Other examples where payoffs can be easily manipulated to raise or lower the price of a game include keno, video poker, and slots.

3. Different Game Products

Variations on standard games, or completely new games, are part and parcel of the product mix for many casinos. Casinos can compete on the price of roulette by offering the single-zero game, which costs the player about $0.27 for every $10 wagered, compared to the double-zero wheel with a price of about $0.53 per $10 bet. Games like Double Exposure and Spanish 21 are just twists on blackjack, while Three Card Poker, Let It Ride, and Caribbean Stud are variations on poker. These differing game products offer players a wide variety of prices to pay for their gaming experience.

B. Fairness Standards and Regulatory Price Controls

In markets where the government or other factors do not create a monopoly or oligopoly, usually a market is competitive based on four factors. The first factor is whether the products supplied by competitors are identical or homogenous. If the products are homogenous, players can base their decisions to buy solely on price. The second factor is whether any seller is so large that it can effect the decisions of other sellers by increasing or decreasing output. In a competitive economy, no seller can effect the output decisions of other sellers. The third factor is whether sellers have the same access to input, such as raw materials or workers. The fourth factor is whether all buyers have accurate and immediate information on pricing and output.

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13. If both the 2 and 12 paid 3-to-1, the house advantage would be zero.
14. Setting the commission at 2% would give the player the advantage.

Until the middle of the 1960s, the casino gaming market was homogenous. Casinos offered essentially the same product—casino games—and competed for players based primarily on the price of the games. This situation was similar to other industries except that the nature of its pricing was different—casino games are modified to change the odds, and hence the price, to the player.

The modern casino industry is no longer homogenous, having evolved so that now, rather than simply paying for the opportunity to gamble, the player is buying a "casino experience." Some players are willing to pay more to play in one casino as opposed to another because the favored casino is more opulent, has childcare facilities, is a resort, or a variety of other factors. The idea of creating a "casino experience" was both a response to player demand and an attempt by casino owners to differentiate their product from competitors. A casino experience extends beyond the games offered to include casino ambiance, service, and amenities, such as rooms, food, pools, entertainment, and other attractions. More frequently, players are paying for a casino experience product package. Not only are the packages less homogenous, but also the pricing of the package is more complex because it includes the cost of gaming and other costs, such as room, food, beverage, and entertainment. Despite this, the pricing of casino games remains an important factor in the pricing of the casino experience, particularly for premium and experienced players.

1. Restricting Minimum Price

In the casino industry, minimum price setting would require a casino to theoretically retain at least a set amount of each wager made. For example, government regulation could require that the payout on gaming devices cannot exceed 95% of the amount wagered. Therefore, the casino must charge the player a minimum average price to play a gaming device of $0.05 for every dollar played. The law precludes a casino from lowering the price below the minimum to benefit the player. With casino games, like blackjack, it would prevent the casino from applying rules to the game such that players' theoretical disadvantage is at least a certain percentage.

Although arguments for minimum price setting are common in other industries,[15] from a policy perspective, minimum price setting in the casino industry is rarely, if ever, justified. The casino industry can use it to fix prices and, in the worst situation, such restrictions may harm the jurisdiction's ability to compete with other jurisdictions. Minimum price setting can disrupt a

15. *See* ANTHONY N. CABOT, CASINO GAMING: POLICY, ECONOMICS AND REGULATION 424-28 (1996) [hereinafter CASINO GAMING] (noting use of minimum price setting in consumer based industries).

competitive market, helping established firms because it eliminates competitive pricing. In a local market, setting a minimum price may be good for casinos (but bad for players) that do not have to compete based on price, if the price set is above market price. In a national or regional market, minimum price setting may harm both the players and the casinos. If the cost of traveling is lower than the "price" differences, players will seek better deals at casinos that compete based on odds.

2. Restricting Maximum Price

In most industries, regulatory price controls mean the government sets prices for services. A state public service commission may set basic telephone rates at $9 per month. In the casino industry, government sets rates by dictating odds of the games. For example, a jurisdiction may require casinos to offer no more than double odds on craps,[16] and their roulette table must both have 0 and 00. These regulations set prices in the casino industry; so do requirements that a casino use multiple decks in blackjack. With slot machines, these standards are usually defined by setting a minimum payout, i.e., prohibiting the casino from theoretically retaining more than a set amount of each dollar wagered. In Nevada for example, all gaming devices "must theoretically pay out a mathematically demonstrable percentage of all amounts wagered, which must not be less than 75% for each wager available for play on the device."[17] This means that the maximum average price that the casino operator can charge is $0.25 for each dollar played.[18] Similar maximum price requirements exist in other jurisdictions.[19] Demonstrating compliance with such regulatory controls is largely a mathematical exercise.[20]

Three rationales are argued as the basis for setting maximum price controls. The first rationale is based on the premise that players generally are incapable of obtaining information about the

16. A bet on the pass line with double odds faces a 0.61% house edge. The greater the odds take, the lower the edge. For example, the house advantage is 0.47% with triple odds, 0.33% with 5X odds, 0.18% with 10X odds, and 0.02% with 100X odds.

17. NEV. ADMIN. CODE ch. 14, § 14.040(1) 9 (2002).

18. As such, Nevada regulations mandate that "[g]aming devices that may be affected by player skill must meet this standard when using a method of play that will provide the greatest return to the player over a period of continuous play." NEV. ADMIN. CODE ch. 14, § 14.040(1)(a).

19. In Mississippi and Colorado, for example, the minimum payout for gaming devices is 80%. *See* Miss. Gaming Comm'n Reg. IV § 4(b) *available at* http://www.ms.gov; 1 COLO. CODE REGS. § 12.47.1-1242(1) (2001).

20. Colorado regulations state that "[t]he theoretical payout percentage is determined using standard methods of probability theory." 1 COLO. CODE REGS. § 12.47.1-1242(1).

true price of casino games and therefore cannot make rational decisions on whether to gamble or to comparison shop. In contrast, casinos that have perfect knowledge of pricing can exploit the player. In most other retail transactions, consumers provide the best protection in assuring the transaction is fair. This occurs because consumers can view the product or services and compare the cost relative to other identical, similar, or substitute products. In these situations consumers have knowledge of price and other market information. In the casino industry, however, consumer knowledge about pricing ranges from sophisticated to oblivious. Moreover, even sophisticated gamblers may have difficulty in appreciating the price of certain bets. In principle, and assuming random results, players can determine the price of the game based on its rules and method of play. Even unsophisticated players can buy any of a myriad of books that explain both odds and optimum play. For many gaming devices, however, the player is incapable of determining the odds of winning and the frequency of payouts. For example, a video slot machine can have a virtually unlimited number of stops on each reel. The number of such stops is a matter of programming the software that drives the gaming device. A regulatory response to the lack of consumer information about pricing is to set a price in an attempt to assure that the games are fair to the player.[21]

A second rationale for maximum price restrictions is to protect the players from losing too much money. In a jurisdiction that adopts an overriding policy to protect the public in all gambling transactions, regulators may attempt to set high minimum payouts to protect the gambler because it would prevent

21. A related consumer information issue has recently surfaced with the increase of gaming devices involving a component of skill. In a game involving skill, the return depends on the player's decisions. When the player chooses poorly, the return will be less than when he plays well. Video poker is the classic example, but new gaming devices requiring an optimal strategy to obtain the maximum return are being introduced. With these games, consider the kind of information the game must disclose so players may avail themselves of higher returns using an optimal strategy. The New Jersey Casino Control Commission has proposed that games of skill provide enough information to enable a player to figure out an optimal strategy. Whether a particular game provides sufficient disclosure under this proposed test is a matter of mathematical proof, available to manufacturers and regulators alike. Typically the only data required for deriving an optimal strategy are the probability of each award occurring and the amount each award pays. In some cases, however, that derivation may involve complicated mathematics. Nonetheless, in New Jersey, so long as the player has sufficient information to derive an optimal strategy, even if that derivation requires complicated mathematics, there is an appropriate degree of fairness. Another suggestion is that skill games be required to have an "autoplay" feature—one that shows the player the optimal choice on any given play. *See* Lloyd Levenson and Michael Gross, *Now Playing: Reel Games of Skill*, INTERNATIONAL GAMING AND WAGERING BUSINESS, July 2001, at 36-37.

the player from losing too much money during the time that the casino can open in a given day.

A third rationale for maximum price controls applies to monopoly markets. If the casino exists in a monopoly (or oligopoly) market, setting maximum prices may be necessary to prevent the monopolist from using its market power to exploit players. Government ordinarily does not regulate prices in competitive economies. The reason is simple. Suppose that, in a perfect competitive environment, the competitive price for a homogenous product like milk is $1.00 a gallon. The competitor who attempts to charge a higher price for milk will not sell its product. If casino players have perfect knowledge of the odds in every casino game, then they can make decisions on where to gamble based on the information. In contrast, a monopoly casino can still charge monopoly prices. It would do this by increasing table minimum bets or minimum denominations of its gaming devices. This would raise the average cost that a player would pay to play in the casino. To compensate for both factors, a jurisdiction needs to set maximum theoretical wins on gaming devices and table games based on minimum bets for these games as well as the mix of devices per denomination. A monopoly casino also can raise the cost of non-casino products or services. For example, it could charge high parking rates, admission to the casino, or raise the price of food and beverages.

C. *What Is Fair?*

While good policy reasons may exist in some jurisdictions to set maximum prices, the government may have difficulty in setting fair standards. The word "fair" is subject to consideration of several factors, many of which are difficult, if not impossible, to define in regulations. Regulations that set maximum price restrictions tend to be overly general and can present practical problems. For example, regulations can require casinos to pay back at least 80% on slot machines.[22] This type of required payback may or may not be fair to the player depending on the minimum wager. If an 80% payback applies to a nickel gaming device, the average cost per hour for the player to play this machine with one coin-in and 325 pulls-per-hour is $3.25. This may be fair to the player, but unacceptable to the casino operator if the cost of maintaining a gaming device exceeds $3.25. As a result, the casino may not offer five-cent slot machines if it is limited to a 20% advantage. In contrast, if the player is playing a dollar machine at a 20% advantage with three coins in, the

22. This is an example of a usual payback percentage. *See, e.g.*, 1 COLO. CODE REGS. § 12.47.1-1242(1).

average price is $195 per hour.[23] This is arguably an unfair advantage over the player.

If a jurisdiction wishes to set prices, it must consider the gaming device denomination or table limits in deciding what is fair to both the player and the casino. The result of placing broad maximum price restrictions that apply regardless of denomination or whether the game is a table or gaming device risks being unfair to both the player and casino. Maximum price restrictions may actually increase the price of playing if the casino must eliminate low denomination machines or table games that become unprofitable to offer to players because of such price requirements. This can be partially addressed by having different maximum pricing for different denominations.

Another factor affecting the game price that could be considered in price controls is the pace of the game. The government may want to take into account the average number of plays per hour and allow the casino to have a higher theoretical win for the slower games. Moreover, any regulation on maximum price generally ignores that gaming is no longer homogeneous. The comparable costs to operate a five-star casino resort are considerably higher than a generic four-wall casino. Yet, setting a maximum price would apply equally to both types of establishments. If the maximum price is set too low, it may discourage potential casino operators from investing in the more capital-intense casino/resorts.

D. What Fairness Standards Should Apply?

1. Theoretical Win

Most frequently, regulators set prices in the gaming industry by requiring the casinos to theoretically pay out a mathematically demonstrable percentage of all amounts wagered, e.g., not less than 80%. Using this theoretical win test provides an average price per play, but may not provide meaningful comparisons between different types of games or an easy standard for calculating costs. For example, a casino may charge more "per hand" for blackjack than a gaming device, but make more per hour on a gaming device because the player can play two to six times more plays per hour.

23. Concededly, individual players often play for the jackpot, which is substantially higher in a dollar machine. Where players are only gambling for entertainment, pay for that service, and know the cost of playing nickel versus dollar machines, most players would choose the less expensive nickel machines and not play the other gaming devices.

2. *Hold Percentage*

The most obvious alternative to setting maximum prices based on a theoretical win is to use the average "hold" from the game as experienced over a specified time period. Attempting to set minimum prices by dictating the maximum "hold" for casino games, however, raises other problems. Although hold gives the casino operator a rough estimate of how the table games are performing, it is not an accurate tool to test the fairness of the games. Casino operators use hold for casino games only because no better method of evaluating performance is available. Unlike gaming devices, no method exists to calculate handle on casino games, and casinos rely on hold percentages to give them basic information on how the games are performing. Many factors other than theoretical win can influence hold percentage; game interest, dealer conduct, and theft are a few. Regulators should not rely on hold as a substitution for theoretical win percentage in deciding whether the game meets minimum standards.[24]

3. *Price Per Hour*

A better standard is average price per hour. Price per hour is a function of average price per play multiplied by the average number of plays per hour.

4. *Problems with Applying Any Methodology*

While governments can easily set maximum theoretical wins based on denomination for gaming devices, an inherent difficulty exists with setting maximum prices on table games. Except in certain jurisdictions with maximum bet restrictions, table games often allow a range of bets, such as from $5 to $100. Thus, one player at the table may be betting $5 per play while another player is betting $500 per play (five different crap bets at $100 per bet).

Another practical consideration is how to treat a game, such as craps, that allows for different bets with widely varying house advantages. Should the government ban the bets with a house advantage over the maximum amount, or consider a higher average of all bets on the table? If you set a maximum theoretical win of 5%, does the typical craps game qualify? An informed player can play the game with a theoretical win of less than 1%, but uninformed players might make some bets, such as a "hard way," that have a theoretical win of over 5%.

Equally problematic are games involving elements of skill. The odds on blackjack are radically different depending upon the

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24. *See, e.g.*, ROBERT C. HANNUM AND ANTHONY N. CABOT, PRACTICAL CASINO MATH 43-48 (2001) for further details on the role and limitations of hold percentage.

skill level of the player, which may range from a slight advantage for a skilled card counter,[25] to a substantial disadvantage for a new player with no knowledge of even basic strategy. A typical blackjack player pays about 1.5% more than a player using what is known as "basic strategy," i.e., making the decision in every instance with the highest expected return for the player. In setting the maximum theoretical win in these circumstances, should the regulators consider the most skilled or least skilled players?[26]

E. Other Approaches and Their Problems

1. Maximum Bet Restrictions

In some states, the government seeks to protect players from losing too much money in any given session. A maximum loss restriction can assure that players do not continue to chase their losses, an activity that over time will assure that they intensify their losses. For example, Missouri imposes a "maximum loss [of] five hundred dollars per individual player per gambling excursion."[27] Enforcement of such maximum loss restrictions can be problematic. One state, for example, attempted to do this by requiring players to buy script and use it to gamble. The casino could only issue a maximum amount of script to each player in a given session. The script became more valuable as a black market item than its face value and placed problem gamblers at an even greater disadvantage in their attempts to chase losses.

Another method to protect players from losing too much is to put maximum bet limits. Colorado has a $5 maximum bet, as did South Dakota until recently.[28] While theoretically this limits the potential loss that a player may sustain, it may also lower the casino's volatility and assure that more players end up with a net loss.[29]

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25. Card counters recently attempted to persuade the New Jersey Casino Control Commission to require casinos to post signs informing patrons of the casinos' card counter countermeasures. The Commission voted down the proposed requirement, claiming it would be disruptive, costly, and time-consuming, and asserted that while the rules in question may affect the outcome of the game, they also reflect the Commission's duty to "provide patrons with fair odds" while allowing casinos "some reasonable return." As one card counter noted "simply put, the card counters want the average gambler to know that the casinos can—and do—count cards, too." Amy S. Rosenberg, *Raw Deal, Say Card Counters*, PHILADELPHIA INQUIRER, July 19, 2001, at A01.

26. *See also* 1 COLO. CODE REGS. § 12.47.1-1242(1) (2001).

27. MO. REV. STAT. § 313.805(3) (2001).

28. South Dakota raised the maximum amount to $100. *See* S.D. CODIFIED LAWS § 42-7B-14(3) (2001).

29. *See* HANNUM & CABOT, *supra* note 24, at 180.

III. REGULATIONS RELATING TO HONESTY

Regulations relating to honesty attempt to assure that the games produce random results and *offer players an equal opportunity to win each prize on any given play*, and that these results conform to the approved or represented odds of the game being offered. Regulations can range from standards for random number generators in gaming devices to methodologies to test whether table games and gaming devices are performing as mathematically expected.

A. *What Is Random?*

The concept of random is elusive and its precise meaning has long been debated among experts in probability, statistics, and the philosophy of science.[30] A standard dictionary might define "random" in a general sense as "[h]aving no specific pattern, purpose, or objective"[31] or "made, done, or happening without method or conscious decision."[32] The same dictionaries also might provide a meaning in a more specific statistical sense such as "a phenomenon that does not yield the same results every time it occurs under identical circumstances,"[33] or "an event having a relative frequency of occurrence that approaches a stable limit as the number of observations of the event increases to infinity,"[34] or even "governed by or involving equal chances for each item."[35]

All of these definitions are deficient for purposes of establishing the necessary criteria for randomness in games of chance. The implication that random refers to events necessarily governed by equal chances is particularly misleading. Consider, for example, the following simple game. A weighted coin with probability of heads equal to .30 is tossed. A player betting on heads will win a 2-to-1 payoff if heads occurs. The house will in the long run enjoy a 10% edge over the player,[36] an advantage that will hold true assuming heads really occurs 30% of the time and it is not possible to predict on which particular tosses these 30% heads will occur.[37] The "game" is honest, and the events, heads and tails, can be considered "random" if the outcomes adhere to

30. For an excellent discussion on the history of attempts to formally define criteria for randomness, see DEBORAH J. BENNETT, RANDOMNESS 152-73 (1998).

31. AMERICAN HERITAGE DICTIONARY, 568 (3d ed. 1992).

32. CONCISE OXFORD DICTIONARY 1185 (10th ed. 1999).

33. WEBSTER'S II NEW COLLEGE DICTIONARY 916 (1995).

34. *Id.*

35. CONCISE OXFORD DICTIONARY 1185 (10th ed. 1999).

36. The player expectation on a 1-unit bet is $(0.30)(2) + (0.70)(-1) = -0.10$.

37. This is essentially the mathematical definition of randomness set forth by Richard von Mises in his landmark book *Probability, Statistics and Truth*. *See* RICHARD VON MISES, PROBABILITY, STATISTICS AND TRUTH 23-29 (2d ed. 1939).

the prescribed probabilities in an unpredictable way.[38]

Two points are important. First, random does not necessarily mean equally likely (although in some situations it does). Second, what really matters with respect to the honesty of a game is that the outcomes conform to the prescribed probabilities in a long sequence—randomness is a function of long-run frequency—and the inability to predict where in a sequence a particular outcome will occur.[39] Although various views exist on the meaning of randomness, few dispute that unpredictability is an intuitively desirable trait of a random sequence,[40] and common to all views is the unpredictability of future events based on past events.[41]

This conceptual essence of randomness is contained in definitions found in gaming regulations in numerous jurisdictions. Phrases such as "observed unpredictability" and "absence of pattern" are common.[42] Excerpts from Colorado's requirements regarding randomness of slot machines illustrate that "'[r]andomness' means the unpredictability and absence of pattern in the outcome of an event or sequence of events."[43] Further,

> Events in slot machines are occurrences of elements or particular combinations of elements that are available on the particular slot machine. A random event has a given set of possible outcomes, each with a given probability of occurrence. The set of these probabilities is called the distribution. Two events are independent if the outcome of one has no influence over the outcome of the other. The

38. Whether these probabilities are acceptable given the payoffs is an issue of fairness.

39. Bennett states:

> Controversy still exists as to whether it is the outcome or the process which should determine randomness, that is, whether randomness is a characteristic of the arrangement itself or the process by which the arrangement was created, or both. [One author has] said that it is "the nature of a certain ultimate arrangement," and not "the particular way in which it is brought about," that should be considered when judging a random arrangement, and the arrangement must be judged by what would be observed in the long run . . . . [This expert] goes on to say that if the arrangement is too small, we must evaluate the nature of the agent that produced it, and often times the agent must be judged from the events themselves.

BENNETT, *supra* note 30, at 165-66; *see also*, JOHN VENN, THE LOGIC OF CHANCE 108 (4th ed. 1962). Bennett further states that "[o]thers, past and present, contend that 'randomness is a property, not of an individual sample [or item], but of the process of sampling,' and thus nonrandom-looking sequences can be generated by a random process." BENNETT, *supra* note 30, at 166; *see also* IAN HACKING, LOGIC OF SCIENTIFIC INFERENCE 123 (1965) (stating that "[r]andom samples are defined entirely in terms of the sampling device").

40. *See* VON MISES, *supra* note 37.

41. *See, e.g.*, BENNETT, *supra* note 30, at 152-73.

42. *See* NEV. ADMIN. CODE ch. 14, 14.010(10) (2002); Miss. Gaming Comm'n Reg. IV.1(f); South Dakota Gaming Comm'n Reg. 20.18.17.01(5) (2002).

43. 1 COLO. CODE REGS. § 12.47.1-1221(5) (2001).

outcome of one event cannot affect the distribution of another event if the two events are independent. The random number generator in a slot machine must produce game plays that are random and independent[.][44]

Although the above-cited regulation explicitly states that game plays must be independent, statistical independence is related to unpredictability and, in a sense, a part of the definition of randomness of games' outcomes. Despite this, it is worthwhile to, and many jurisdictions do, specifically address the issue of statistical independence because technically, random and independent are not equivalent concepts.

In practice, the criteria for randomness and honesty—unpredictability, absence of pattern or regularity, independence, and conforming to prescribed probabilities—are judged in large part using statistical tests. Indeed, a practical definition of randomness is that "a sequence is random by virtue of how many and which statistical tests it satisfies."[45]

B. Testing for Randomness

Disproving randomness can be easy. Proving randomness is not so easy. This is because tests designed to assess randomness assume the phenomenon being tested is random (the null hypothesis), and reject this presumption only if the observed data provides compelling evidence against randomness. The reasoning is similar to the justice system in the U.S., where a criminal defendant is presumed innocent and declared guilty only when there is convincing evidence to the contrary. Because of this "innocent [random] until proven guilty"[46] logic employed in statistical hypothesis testing, and because there are many ways a process or series can exhibit departures from randomness, confirming randomness can be a difficult statistical problem.[47] In more technical terms, no single test is powerful against all types of departures from randomness.

To illustrate, consider the following three sequences, each showing the results in order of 36 rolls of a die:

(A) 123456123456123456123456123456123456

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44. *Id.* § 12.47.1-1224.

45. BENNETT, *supra* note 30, at 169; *see also* 2 DONALD E. KNUTH, THE ART OF COMPUTER PROGRAMMING 41 (3d ed. 1998) (stating: "[i]n practice, we apply about half a dozen different kinds of statistical tests to a sequence, and if it passes them satisfactorily we consider it to be random").

46. 2 KNUTH, *supra* note 45, at 41.

47. Popper states that "there are no tests for the presence or absence of regularity in general, only tests for presence or absence some given or proposed *specific* regularity." KARL R. POPPER, THE LOGIC OF SCIENTIFIC DISCOVERY 359 (1959) (emphasis in original).

(B) 111111222222333333444444555555666666

(C) 352644261523351263514263451226341625

Sequences A and B are not random, but because the frequencies of the six possible outcomes are exactly what you would expect with a "random" die toss, each would pass a chi-squared goodness-of-fit test designed to assess whether the observed frequency of each outcome is consistent with the expected frequencies under the hypothesis of randomness. This particular test would not disprove randomness. Both sequences A and B, however, would fail a runs test (an isolated occurrence of one value constitutes a run) because there are too many runs in A and too few in B. Thus, the sequences demonstrate that they are indeed not random. However, sequence C would pass both tests for randomness.

The preceding simple example illustrates that one test is not sufficient to confirm that a given phenomenon is random. There are many ways that a sequence can be nonrandom—any pattern, predictability, or lack of independence—and there are many statistical tests for detecting non-randomness.[48] Ideally numerous tests should be used, and even if a sequence has passed all available tests, it might only mean that the tests were not sensitive to the particular regularity that existed.[49]

C. Standards to Assure Randomness

For non-computer controlled games, methods for confirming randomness range from inspections and measurements of equipment to statistical analysis of empirical data. To verify the honesty of dice, for example, precise instruments are used to assure that all of the sides of a pair of dice are the same length and width and are parallel. Likewise, roulette tables can be inspected to assure that they are perfectly balanced. Other evidence may also be required, particularly for devices used in new games seeking regulatory approval. When considering the approval of a new table game that used a new spinner device, for example, the Mississippi Gaming Commission requested a statistical analysis of actual spins to confirm, at the 99% confidence level, that the device produced random outcomes.[50]

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48. One author, whose authoritative series of books on computer programming includes a lengthy discussion of random numbers and tests for randomness, notes that the number of tests for randomness is limitless. *See* 2 KNUTH, *supra* note 45, at 41; *see also*, BENNETT, *supra* note 30, at 173.

49. Bennett states that: "Though some tests have become standard, it remains questionable which tests and how many of these tests a sequence must satisfy to be accepted as random." BENNETT, *supra* note 30, at 173.

50. Telephone Conversation between Robert Hannum, Professor, The University of Denver, Dep't of Statistics & Operations Technology, and Vince

Most regulations regarding randomness of gaming devices are directed towards computer-controlled devices.[51] At the heart of all computer-controlled gaming devices is the random number generator (RNG). A RNG is an algorithm (mathematical formula) that produces sequences of numbers—called pseudorandom numbers—that appear to have been generated randomly.[52] Depending on the design parameters, each sequence may correspond to a group of symbols on a reel-type slot machine, a group of cards on a video poker machine, or whatever other outcome symbols are used in the gaming device. The RNG controls the game outcomes and therefore the randomness of the outcomes.[53] While the design of the RNG algorithm may differ between programmers and devices, the RNG should run

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Noce, Special Projects Officer, The Mississippi Gaming Commission (Jan. 5, 2001). This request (along with other measurement and production related evidence of the spinner's integrity) was made after submission and acceptance of a detailed probability analysis showing the house advantages for each possible bet, along with the overall game advantage. *See* Memorandum from Robert Hannum, Professor, The University of Denver Dep't of Statistics & Operations Technology, to Vince Noce, Special Projects Officer, The Mississippi Gaming Commission 2-9 (Nov. 15, 2001) (detailing a mathematical analysis of the *Spin & Win* game to confirm house advantages for different wagers).

51. The legal definition of gaming device is broad. In Nevada, for example, a "gaming device" includes electronic gaming devices such as slot machines, video poker machines, and keno systems that track all tickets and randomly generate the numbers chosen. *See* NEV. REV. STAT. § 463.0155 (2001). A "slot machine" is not limited to the traditional concept of a reel machine, but includes any machine that "upon insertion of a coin, token, or similar object, or upon payment of any consideration," entitles the person to have the opportunity to receive cash or prizes. NEV. REV. STAT. § 463.0191. This definition applies if the opportunity is based on chance or upon the skill of the operator in playing a gambling game. *Id.* For example, if the gaming device emulates the game of blackjack that involves skill, it is still a slot machine because blackjack is a gambling game under Nevada law. *See* NEV. REV. STAT. § 463.0152. Thus, a video poker machine is a "slot machine" for the purposes of the Nevada Gaming Control Act. Further, the definition of gaming device was expanded in 1993 in order to regulate those persons who make a substantial portion of a device but less than all of it. The law now recognizes that unscrupulous persons may make components of a gaming device that can illegally alter the play of a machine and so the definition includes slot machines as well as numerous components of devices. *See* NEV. REV. STAT. § 463.0155.

52. It is not possible to generate truly random numbers on a computer. *See, e.g.*, 2 KNUTH, *supra* note 45, at 2-4.

53. The recent British government-commissioned *Gambling Review Report* recommends that online gambling software, controlled in large part by RNGs, should be tested and inspected by the Gambling Commission to ensure that random generators are in fact random. GAMBLING REVIEW REPORT, *supra* note 5, at 5, 169. For a comprehensive discussion of online gambling issues, see ANTHONY N. CABOT, INTERNET GAMBLING REPORT IV (2001).

continuously,[54] have proper seeding,[55] and the results generated by the algorithm should meet minimum confidence levels that the numbers generated are random.

D. Minimum Confidence Levels and Statistical Tests

Gaming regulatory bodies recognize the importance of statistical tests in confirming randomness, and regulations often specify required confidence levels and particular statistical tests. Some tests that are typically performed as part of the random number generation test are the chi-squared analysis, the runs test, the reel-to-reel correlation test, and the serial correlation test. For example, South Dakota requires the devices to have a random selection process that satisfies a 98% confidence level using standard chi-squared runs and serial correlation tests.[56]

Under Nevada Regulations, a random number generator is a hardware, software, or combination hardware and software device for generating number values that exhibit characteristics of randomness.[57] All gaming devices manufactured and distributed for use in Nevada must meet certain mathematical standards for randomly choosing the game's outcome. These standards provide that the random selection process used by gaming devices "must meet 95% confidence limits using a standard chi-squared test for goodness of fit."[58] Nevada further requires that each possible permutation or combination of game elements which produce winning or losing game outcomes must be available for random selection at the initiation of each play. For gaming devices that are representative of live gambling games, the mathematical

54. The requirement that the RNG runs continuously means that while the player is playing the device or while it sits idle, the RNG is continuously producing new numbers every millisecond until a player-instigated action, such as the acceptance of tokens by the device, the pull of the handle, or the push of a button, signals the RNG to stop processing the pseudorandom numbers. This requirement assures that a player would have almost no chance of being able to stop the microprocessor on a specific number (if, for example, he knew the sequence of numbers that were about to appear) and that persons cannot string together plays of the device to derive patterns. It also prevents a person from observing the outcome of games on one device, finding a second device with the same RNG, waiting until the outcomes are in synch, and playing the second device knowing the outcome.

55. Random seeding means that each device, when initially activated or reset, will start with a different or random number. In a reset, this can be accomplished by having the RNG store the last-known value, and begin at that point when reactivated.

56. *See* S.D. ADMIN. R. 20:18:17:21 (2002); *see also*, DONALD E. GROMER & ANTHONY N. CABOT, *South Dakota, in* INTERNATIONAL CASINO LAW 133 (Anthony N. Cabot, et al. eds., 2d ed. 1993) (discussing South Dakota gaming law).

57. Nev. Gaming Reg. 14, Attachment 1 *Technical Standards for Gaming Devices* R. 1.010(5) *available at* http://gaming.state.nv.us/stats_regs.htm#regs.

58. NEV. ADMIN. CODE ch. 14, § 14.040(2) (2002).

probability of a symbol or other element appearing in a game outcome must be equal to the mathematical probability of that symbol or element occurring in the live gambling game. For other gaming devices, "the mathematical probability of a symbol appearing in a position in any game outcome must be constant."[59] Recognizing the importance of independence, Nevada requires that "[t]he selection process must not produce detectable patterns of game elements or detectable dependency upon any previous game outcome, the amount wagered, or upon the style or method of play."[60]

Mississippi regulations specify "99% confidence limits on the chi-squared goodness of fit" test but are otherwise similar to Nevada's.[61] Iowa requires that the game in each device shall be random and tested to at least a "99 percent confidence level using standard serial correlation analysis."[62] Correlation test or analysis occurs when "each card, number, or stop position is independently chosen without regard to any other card, number, or stop within that game play."[63] Colorado's regulations state that a selection process is considered random if the following specifications are met:

> (1) A reel, card, or ball or other event that determines the outcome of the play satisfies at least 99 percent confidence level using the standard chi-squared analysis. Chi-squared analysis is the sum of the squares of the difference between the expected result and the observed result.[64]
>
> (2) A reel, card, or ball or other event that determines the outcome of the play satisfies at least 99 percent confidence level using the Median Runs Test or any similar pattern checking statistic. The Median Runs test is a mathematical statistic that determines the existence of recurring patterns within a set of data.
>
> (3) A reel, card, or ball is independently chosen without reference to any other event produced during that play. This test is the correlation test. Each pair of events is considered random if they meet at least the 99 percent confidence level using standard correlation analysis.
>
> (4) A reel, card, or ball or other event is independently chosen without reference to the same event in the previous game or games. This test is the serial correlation test. The event is considered

59. *Id.* § 14.040(2)(b).
60. *Id.* § 14.040(2)(c).
61. Miss. Gaming Comm'n Reg. IV § 4(c) *available at* http://www.ms.gov.
62. IOWA ADMIN. CODE r. 491-11.10(2)(d) (2001).
63. *Id.* at r. 491-11.10(2)(c).
64. This is not quite technically correct. The chi-squared test statistic referred to here is based on the sum of the squares of the difference between the expected result and the observed result divided by the expected result.

random if it meets at least 99 percent confidence level using standard serial correlation analysis.[65]

South Dakota regulations contain a similar series of requirements using virtually identical language[66] but specifying a 98% confidence throughout.[67] Regardless of the method, the greatest regulatory concern is that the device selects the cards or symbols within acceptance levels of randomness.

Typical regulations require that devices display an "accurate representation of the game outcome[,]"[68] and that after selection of this outcome, "the gaming device must not make a variable secondary decision which affects the result shown to the player."[69] Devices "[m]ust display the rules of play and payoff schedule[,]"[70] and "[m]ust not automatically alter paytables or any function of the device based on internal computation of the hold percentage."[71]

E. Near-Miss and Ghost Programs

The near miss occurs when the manufacturer programs its machines so that certain jackpot symbols appear in losing combinations more frequently than they would through normal random selection. The existence of "near-miss" programs was brought to prominence in a disciplinary action brought by the Nevada regulators against a licensed manufacturer.[72] In its complaint, the regulators stated that all gaming devices manufactured and distributed for use in Nevada must meet certain mathematical standards for randomness in choosing the game's outcome.[73] The regulators alleged that the manufacturer's devices did not meet the standard for randomness because it contained a "near miss" feature.[74]

Most slot machines independently select the final position of each reel, and then decide whether the combination of reels is a win or a loss. The manufacturer's machines first decided whether

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65. 1 COLO. CODE REGS. § 12.47.1-1241(1)-(4) (2001).

66. The language is somewhat sloppy in requirements (3) and (4). The word "random" is used in (3) where "independent" is the correct term. Similarly, requirement (4) should refer to independent events (plural) and not a single event as random.

67. *See* S.D. ADMIN. R. 20:18:17:21(1)-(4) (2002).

68. NEV. ADMIN. CODE ch. 14, § 14.040(3) (2002).

69. *Id.*

70. *Id.* § 14.040(4).

71. *Id.* § 14.040(5).

72. *See* Complaint for the Nevada State Gaming Control Board, State Gaming Control Board v. Universal Distrib. of Nev., Inc., No. 88-4 (Nev. Gaming Comm'n May 2, 1988).

73. Complaint for the Nevada State Gaming Control Board at 3 ¶ 11, State Gaming Control Board v. Universal Distrib. of Nev., Inc., No. 88-4 (Nev. Gaming Comm'n May 2, 1988).

74. *Id.* at 3-5 ¶¶ 13-14, ¶ 22.

the game result was a specific winning combination (for example, three 7s) or a loss; and then, if a loss, decided the positions of the reels from one of a group of tables of losing combinations. As a result, certain losing combinations that included jackpot symbols appeared more frequently than other losing combinations that did not include jackpot symbols. The effect was to give the player the illusion that he was nearly winning almost every time he lost. This was the origin of the label "near miss."

Regulators brought a disciplinary action that included a demand that the manufacturer replace or modify all "near-miss" machines.[75] In response to the regulators' complaint, the manufacturer admitted the allegations concerning how the machines decided the game outcomes, but denied any wrongdoing. The manufacturer denied that the "near-miss" feature represented an unacceptable manner of play. The manufacturer argued that no law or regulation prohibited its game process, or required a process of first independently selecting the position of each reel and then deciding if the game outcome is a win or a loss.

The manufacturer said the regulators' constantly changing standards for approval of new gaming devices were internal standards that did not rise to the level of law or regulation. At that time, Nevada did not specifically prohibit a "near-miss" feature, nor did it have written minimum standards for gaming devices.[76]

The manufacturer's arguments in support of the suitability of the near-miss feature were threefold. First, it argued that the "near-miss" feature did not decide whether the player won or lost. Instead, the feature only selected the losing combinations after the microprocessor had randomly determined that the player had lost. Second, the manufacturer contended that other manufacturers received similar results by using reel strips with more jackpot symbols on one reel than on other reels. Third, the manufacturer argued that the "near-miss" devices were the most successful and popular devices, and that they had not received a single player complaint.

After receiving extensive testimony from several experts in statistics, the regulators decided that a randomly occurring "near miss," such as that on a traditional reel strip machine, was acceptable. The regulators decided that the generated "near miss" was not a randomly occurring "near miss," and that the

75. *Id.* at 14 ¶ 5.

76. When the Gaming Control Board heard of the near-miss feature in 1988, the state had no written regulations governing the concept of what would become known as "randomness." In the early 1980s, when computer technology was developed for gaming devices, Nevada's gaming rules on slot machines—written in state Regulation 14 in 1969 and amended in 1972—were based on how traditional mechanical slots operated.

manipulation of a losing combination to alter the result shown to the player was unacceptable.

The final issue was whether a gaming device was random if it manipulated the losing combination to alter the result shown to the player. The regulators decided that such a device was not random.[77] The regulators ordered the retrofitting of all the manufacturer's "near-miss" machines.[78]

Near-miss programs can also be programmed in video poker machines. The program is designed to display an "almost" winning combination (near win) when the player has lost that game.[79] After a player receives the hand and discards unwanted cards, the program begins. Before revealing the player's final hand, the program determines if that hand is a loss. If it is a loss, the program searches the frozen field for a combination close to a winning hand and displays those cards.[80] The result is that it leads the player, although losing that game, to believe that he almost won, and that a winning combination may soon appear.[81] In reality, while a winning combination may be statistically unlikely, the possibility of winning or coming close to winning often induces that player to play again. Many players set a limit upon how much they will play, but after seeing the near miss, they often play more then they would have otherwise.[82]

In another case, Nevada regulators discovered irregularities in certain poker gaming devices assembled by a licensed manufacturer.[83] These devices had computer programs that canceled the draw of a royal flush if the patron placed the maximum wager and, in so doing, prevented the patron from winning the largest jackpot.[84] The regulators found that the manufacturer had created a cheating program that was activated by a sub-routine in the main program. This sub-routine was not ordinarily involved in the selection of a winning combination. However, as modified, the sub-routine would instruct the main

77. In the Matter of the Petition of Universal Co., Ltd. and Universal Distrib. of Nev., Inc. for a Declaratory Ruling, No. 88-8 at 2 (Nev. Gaming Comm'n Mar. 15, 1989) (issuing a declaratory order).

78. *Id.* In July 1989, the Nevada Gaming Commission approved key amendments to Regulation 14 specifying standards for random selection of game outcomes, including statistical testing at the 95% confidence level, various requirements related to independence of game outcomes, and displaying an accurate representation of the game outcome. *See* NEV. ADMIN. CODE ch. 14, § 14.040 (2002).

79. The game determines if the player has lost. If so, the game displays a hand close to a winning combination. *Id.*

80. *Id.*

81. *Id.* The program induces the player to believe that a jackpot may appear within the next few tries. *Id.*

82. *Id.*

83. *See* JEFF BURBANK, LICENSE TO STEAL 35-40 (2000).

84. *Id.* at 35.

program to pick another card to avoid the royal flush.

To combat ghost and near-miss programming: "governments often institute regulations that rely on Government Test Laboratories and the integrity of the manufacturers."[85] In addressing the problems with ghost programs, "regulators run various tests on the device's software before its licensing, and recall the devices for periodic retesting."[86] Further, "[r]egulators also limit and monitor access to the devices since persons with access to the logic board can install ghost programs after inspection."[87] In addition:

> Regulators can also limit the type of medium on which software can be stored in the machine, such as chips with permanent read-only memory, or hard drives using sophisticated data encryption. These requirements make it more difficult to install ghost programs after the devices have been inspected and approved by regulators. Ghost programs are typically not outlawed by name or description, but inferentially through detailed and specific legislation that dictates how proper software should run. Any intentional variations from these narrow legal parameters may carry criminal sanctions.[88]

It is important to note that regulators look to the manufacturers to provide legitimate software because advancing technology makes it more difficult for game-testing technicians to find hidden programs.[89] As a result, "[i]f a game is found to contain deceptive software, the manufacturer may be liable for criminal sanctions similar to those imposed for ghost program violations."[90]

States with legalized video gaming can prevent deceptive programming due to increasingly harsher penalties for using deceptive software.[91] For instance, "[i]f a manufacturer is caught submitting a deceptive program, sanctions include criminal prosecution and government cessation of business relations with that manufacturer."[92] Further, "[i]f an operator were caught trying to install a deceptive program, sanctions include criminal prosecution and the loss of his license."[93] The risks associated with using deceptive gaming practices are unnecessary and "illogical" given that legalized video gambling will generate substantial

85. CABOT, CASINO GAMING, *supra* note 15, at 385. A logic board is the brain of the game. *Id.* It contains all the EPROM's and electrical components necessary to run the game.

86. *Id.*

87. *Id.*

88. *Id.* at 385-86 *citing* Standards on coin-operated video gaming devices 2 (N. Am. Gaming Regulators Ass'n. 1990).

89. *See id.* at 386.

90. CABOT, CASINO GAMING, *supra* note 15, at 386.

91. *See id.*

92. *Id.*

93. *Id.*

revenues for the manufacturer and operator.[94]

IV. CONCLUSION

Mathematics' expanding role in the gaming industry necessarily requires a different level of education and orientation for legislators and gaming regulators. Many in these positions have limited or distant backgrounds in mathematical principles and little or no understanding of these principles as applied to the gaming industry. This creates circumstances that increase the probabilities that important mathematical concepts are misapplied or ignored. As noted in this Article, several areas of gaming regulation are particularly susceptible to concern.

The first area of concern is the fundamental misunderstanding of the value and relevance of common mathematical measurements used in the casino environment that can adversely impact the effectiveness of gambling regulations and decisions. This may be evident in several areas. For example, using hold percentage to assess the fairness of table games may have the unintended effect of excluding games that have a fundamentally fair house advantage but, for other reasons, have a high hold percentage. The misunderstanding of the use of hold percentage in this context results from the failure to understand all the factors that contribute to the hold percentage. Incorrectly assuming that a high hold percentage necessarily equates to an unfair casino advantage may result in fundamentally fair games with a high degree of player loyalty being excluded.

The second area of concern is that most legislators and regulators simply have little understanding of the value of the measurements and confidence tests used for randomness sufficient to understand and set appropriate standards. Good examples are the common benchmarks for assuring randomness of the random number generators that control the modern gaming devices. Most states have requirements that the numbers generated by the random number generator meet minimum confidence levels that satisfy the acceptable definition of random. The benchmarks are generally set using several tests including the standard chi-squared, runs, and serial correlation tests. Whether the nature and limitations of these tests are known and understood by regulators may, in many cases, be doubtful.

Confounding this situation is that states often set different confidence levels, ranging from 95% to 99%, that the devices must satisfy. Here the question becomes whether the regulators understand the differences between 95% and 99% confidence and the impact both have on the regulated casinos and the consumer who picks one benchmark over another. A problem that may occur

94. *Id.*

from a regulatory prospective is that regulators place undue reliance on their technological staff to compensate for their lack of understanding of the standards and benchmarks. This may result in a technically and even mathematically sound decision, but ignore the reason for the regulation—to wit, to meet policy objections by the least intrusive means.

This latter point is tied to the third area of concern, which is whether the legislators and regulators have a firm understanding of the mathematical standards' impact on the casino environment. For example, consider the impact of a five-dollar maximum bet. It may result in a player losing less money in a given amount of time, but it also may result in a higher percentage of all players losing money. Another example is the impact of defining the play of a casino game. It may provide easier regulatory monitoring because the rules are consistent between casinos in a jurisdiction, but it also may be used to fix prices above the competitive price through regulation.

Growing reliance on technology in the casino gaming industry intensifies the importance of mathematics to the regulatory process. For example, mechanical slot machines have a finite number of stops and a limited number of potential combinations. In contrast, new computer-controlled gaming devices have virtually unlimited numbers of combinations and rely on software solutions to provide random results. Table games have also introduced computer elements including linking table jackpots in multiple locations. This increase in technology coupled with the growing proliferation and popularity of casino games requires that legislators and regulators concentrate more heavily on mathematics for sound policy and decision-making.