# A Stand Level Growth and Yield Model for Red Oak/Sweetgum Forests in Southern Bottomlands 

John Clinton Iles

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# A STAND LEVEL GROWTH AND YIELD MODEL FOR RED OAK/SWEETGUM FORESTS IN SOUTHERN BOTTOMLANDS 

By<br>John Clinton Iles

A Thesis<br>Submitted to the Faculty of Mississippi State University<br>in Partial Fulfillment of the Requirements for the Degree of Master of Science in Forestry in the Department of Forestry

Mississippi State, Mississippi
August 2008

# A STAND LEVEL GROWTH AND YIELD MODEL FOR RED OAK/SWEETGUM FORESTS IN SOUTHERN BOTTOMLANDS 

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A greater emphasis is being placed on hardwood management, yet there has been relatively little effort to develop growth and yield information for hardwood forest types. Measurements on permanent growth and yield plots collected in 1981, 1988, 1994, and 2006 in minor stream bottoms in Mississippi and Alabama were used to construct a stand level growth and yield model for red oak/sweetgum stands. The model predicts arithmetic mean diameter, quadratic mean diameter, trees per acre, basal area, total tree height, and cubic foot volume per acre for the total stand and by species. Different sets of equations were constructed depending on the amount of information known about a hardwood stand. Models were chosen based on significance of variables, coefficient of determination, index of fit, and biological trends. Predicted stand development patterns are discussed. These models will be base models for a complete diameter distribution growth and yield model.

## DEDICATION

To my Lord and Savior, Jesus Christ. Thank you for giving me the ability, strength, and desire to accomplish this. May everything I do, including this research, bring honor and glory to your name.

To my two best friends in the world: my wife, Amanda, and son, Jack. This would have never been possible without your sacrifice and encouragement. I hope to be as supportive of both of your dreams as you have been of mine.

To my parents, Curt and DeDe Iles. Thank you for teaching me the importance of hard work and accomplishing goals.

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## CHAPTER I

## INTRODUCTION

Most of our understanding of forest stand dynamics and outcomes has come from growth and yield research (Davis et al. 2005). The primary use of a growth and yield model is for decision-making by the forest manager (Rauscher and Young 2000). By quantifying the way a forest changes over time, the forest manager gains a greater understanding of forest growth and is better able to make sound and justifiable decisions. In an attempt to understand forest growth, there has been a large amount of research in growth and yield for different species and forest types. The species of interest in this study are red oak/sweetgum in mixtures in minor streambottoms in the South. Growth and yield models for this forest type will be very useful for future management.

## Background

There are three general categories of growth and yield models: whole stand models, diameter class models, and individual tree models (Davis et al. 2005). Whole stand models are based on characteristics such as age or basal area per acre and are categorized as either density free or variable density. Density free models assume normalized yields (USDA Forest Service 1929). Variable density models estimate volume strictly by growth functions (Davis et al. 2005). Diameter class models differ from whole stand models by placing emphasis on the average tree in each diameter class.

Diameter class models can be based on either empirical projections or known growth functions. A traditional diameter class model is the stand table projection. In this model, the growth-index ratio is determined by dividing the average diameter growth by the diameter class interval for each diameter class. The ratio can be interpreted as the percent of trees in the diameter class that will move up at least one diameter class. If the ratio is greater than one, then every tree in the diameter class will move up one class and the percentage above one will move up two diameter classes. Stand table projections are best suited for short projections in young actively growing stands (Avery and Burkhart 2002). Individual tree models factor in competition as a measure of productivity (Davis et al. 2005). Individual tree models can be either distance dependent or distance independent. Either category of the individual tree model is more complex than whole stand or diameter class models. However, distance dependent models are the most complex because they not only require data from each individual tree, but also require distances from tree to tree. Individual tree models have become more common with highly capable computer systems. While each category of model has different characteristics, it is important to note that most modern growth and yield systems do not neatly fall into a single category but may incorporate methods from several categories (Davis et al. 2005).

## Single Species Models

A majority of growth and yield research has focused on a single species of commercial importance in even-aged stands like loblolly (Pinus taeda L.) and slash (Pinus elliottii Engelm.) pines (Baldwin and Cao 1999). This is due to the simplicity of
modeling single species, even aged stands, and the commercial value of southern pine plantations.

Growth and yield predictions can range from simple to very complicated. Brooks and Wiant (2004) predicted volume in pine stands within 5\% of the actual yield using a simple equation based on uncomplicated inventory parameters:
$V=a * B H$
where
$V=$ volume per acre,
$B=$ average stand basal area per acre,
$H$ = average stand height, and
$a=$ the fitted coefficient.
The equation was then compared to other growth and yield models and to actual yield and proved to be reliable for certain broad applications (Brooks and Wiant 2004). Lenhart $(1972,1973)$ presented a detailed schedule of cubic foot, green weight, and dry weight yields for unthinned old-field loblolly pine plantations in the Interior West Gulf Coastal Plain. The expected yields were within $6 \%$ of the test data.

The Weibull function is a widely used method for quantifying diameter distributions. This method has been successful due to its flexibility and simplicity (Cao 2004). The probability density function for the Weibull function is characterized by a, b, and c parameters, which represent the location, scale, and shape parameters, respectively (Bailey and Dell 1973). Matney and Sullivan (1982) developed a three-parameter Weibull diameter distribution moment recovery model for determining stand and stock tables in thinned and unthinned loblolly pine plantations. The recovery equations were
based on old-field plantation data. First, equations were written to calculate per acre values or surviving trees, projected and current basal area, and projected and current total tree volume from initial stand parameters. The parameters of the Weibull distribution were then calculated so the resulting distribution expected moments equaled the predicted moments (i.e., the diameter distribution produced the same volume and basal area that was calculated in the first phase). The three parameter Weibull distribution method was successful because it was flexible when describing stand density.

The method of Matney and Sullivan (1982) represents a unique and practical technique of estimating diameter distributions over time in thinned and unthinned stands and has been utilized in developing other growth and yield models. Using similar methods, Baldwin and Feduccia (1987) developed a growth and yield model that predicts both weight and volume yields by diameter classes for 10 to 45-year-old thinned or unthinned loblolly pine plantations for the West Gulf region. Plot data were obtained from thinned and unthinned cutover plantation sites located in the western Gulf region. The diameter distribution model assumed the three parameter Weibull function using the parameter recovery technique. The distribution of diameters was recovered from the first percentile, the ninety-third percentile, and the quadratic mean diameter. The model compared favorably with an independent test data set. The computer program COMPUTE_P-LOB (Baldwin and Feduccia 1987) allows the user to select multiple thinned and unthinned stand management intervals. Zarnoch et al. (1991) developed a similar model designed to predict yields of thinned and unthinned slash pine stands in the western Gulf region. This growth and yield system was based on the moment-percentile
method of parameter recovery. The predicted values based on the test data were within 5\% of the observed values.

Matney and Farrar (1992) developed a loblolly pine growth simulator for planted cutover site-prepared land in the middle Gulf South. This simulator predicts growth in thinned and unthinned loblolly pine stands. The simulator used a combination three parameter Weibull distribution dbh moment recovery system in the form of a weighted constrained least squares diameter moment recovery system.

Burkhart et al. (2003) developed a distance dependent growth and yield model for loblolly pine named PTAEDA. It is based on a previous model by Daniels and Burkhart (1975). Each tree is given a coordinate location in a stand and projected annually as a function of size, site quality, and competition from other trees. Using this detailed method, genetic and micro-site variability are represented in the projected growth increments (Burkhart et al. 2003). Mortality is generated stochastically through Bernoulli trials. The most recent version of the PTAEDA simulator takes into account sitepreparation, hardwood competition, thinning, fertilization, pruning and includes economic analysis.

## Mixed Stands

Mixed species stands are more complex in structure and predicting their growth is more difficult than in pure stands (Dale 1970). The complexity is caused by the varying composition and density of a number of species within a given stand. Any growth study in a mixed stand should accomplish three goals: 1) estimate periodic growth by repeated
sampling of tree populations, 2) identify the best model for considering growth differences actually observed, and 3) project the growth of the populations into the future (Grosenbaugh 1970). Nelson et al. (1961) attempted to predict the yield of Virginia pine (Pinus virginiana Mill.) in various densities and in stands with various species composition. Multiple regression techniques were applied based on functions of stand age, density, composition, and site. The study produced an equation for predicting the yield of pure Virginia pine stands. A corresponding composition correction factor could be selected and multiplied by the pure stand value to get the predicted yield of Virginia pine in a mixed stand.

## Southern Bottomland Hardwood Growth and Yield

The North Carolina State University Hardwood Research Cooperative initiated in 1969 established 641 1/5-acre permanent plots spread across nine hardwood forest types. Only 95 of the original plots were measured after five years and only 58 were measured after 10 years (Roeder and Gardner 1984). The data taken from these plots has been used for extensive model building and predicting volumes and weights from hardwood forests (Smith et al. 1975). The predicted volumes were used to determine the economic feasibility of managing Southern hardwoods. The equations predicted yield based on least squares multiple regression. The fit equation required inputs of site type, age, average height of merchantable trees, and basal area. Roeder and Gardner (1984), working with the same plot measurements, constructed compatible growth and yield estimates of mixed hardwood stands. Multiple linear regression techniques were used to
predict volume growth as a function of merchantable basal area and height.
Merchantable basal area and height were effective variables in representing site type and age. The tables based on the predictive equations presented growth in height and merchantable basal area as a percentage of the initial values. Percentage volume growth as a function of the percentage height and merchantable basal area growth was also presented. This approach allowed the user to predict the percentage growth of a current stand into the future a given number of years. Gardner et al. (1982) expanded the study to predict above ground biomass. Site type, age, and basal area were the best overall predictors for total biomass growth. McTague et al. (2006) developed site index curves that represented the influence of site type on productivity using the same NCSU Hardwood Research Cooperative plots. The purpose of this study was to develop site index curves that would be the height growth drivers in the development of future hardwood growth and yield models. The repeated height observations within each plot were correlated. This study represented an advance in the ability to model growth in mixed stands across a range of sites. The NCSU Hardwood Research Cooperative growth and yield data provides very limited bottomland hardwood information because of the total number of plots spread out across nine forest types and the lack of remeasurements.

Zhao et al. (2004a) developed a distance independent individual tree diameter growth and mortality model for mixed species in the Lower Mississippi Alluvial Valley (LMAV). Their study was based on continuous forest inventory (CFI) plots. They modeled two forest types in the LMAV: 1) sweet pecan (Carya illinoensis K.), American
sycamore (Platanus occidentalis L.), elm (Ulmus spp.), and eastern cottonwood (Populus deltoides Bartr.) and 2) green ash (Fraxinus pennsylvanica Marsh.), sugarberry (Celtis laevigata Willd.), Nuttall oak (Quercus nuttallii Palmer), and overcup oak (Quercus lyrata Walt.). Diameter at breast height (dbh) and crown class were the only measurements used for model development. $\mathrm{Dbh}^{2}$ and $1 / \mathrm{dbh}$ were used to predict basal area increment and mortality. Relative diameter and relative diameter interaction were important factors for small and large trees compared to the other trees in the same stand. Competition was accounted for by using basal area and the basal area proportion of species. The results of the model demonstrated that 1 ) basal area increments decreased as the competition from increased basal area increased 2) competition was occurring on an inter- and intra-specific level, and 3) smaller diameter trees had a higher mortality rate and the mortality rate rapidly decreases as diameter increases. Most importantly, the model accurately predicted basal area growth. When excluding ingrowth, the model was within $1 \%$ of actual basal area. Including ingrowth, the model overestimated basal area by $4.2 \%$. Using CFI plots to construct models can create a bias because they are implemented on a grid system and are not treated operationally the same as other areas. Zhao et al. (2004b) also developed a matrix model for the LMAV which was designed to provide more growth information and to analyze the development of stands using different management regimes. In a matrix model, a stand state is described by a vector. The transition from state to state is described by a transition matrix. Thus, a future stand diameter distribution can be projected based on the current diameter
distribution and a transition matrix. The overall results from the matrix model were accurate in the short term and became less accurate as time progressed.

Rauscher and Young (2000) tested the accuracy of growth and yield models for Southern hardwood forests. The goal of their project was to test the accuracy of ten publicly available growth and yield models for upland and bottomland hardwoods in the South. The models chosen were designed for upland hardwoods because there were none available for Southern bottomland hardwoods. However, four models did contain enough applicable species to be tested for accuracy in bottomland situations. Although one of the upland models was capable of producing accurate results for bottomland hardwoods, Lee and Coble (2006) discuss the importance of developing and calibrating a model in the region in which they are to be used.

## Prior Research in the Study Area

The red oak/sweetgum (Liquidambar styraciflua L.) forest mixture represents a common and valuable timber resource throughout minor river bottoms in the South. However, there is little growth and yield research on this forest type. Matney et al. (1985) developed equations to predict the height and cubic foot volume to any top diameter for individual sweetgum and cherrybark oak (Quercus pagoda Raf.) trees. The purpose of the research was to provide tree profile-volume predictors necessary for a growth and yield model for this specific forest type. The predictions were based on two equations. One equation predicted diameter at any height above ground or height to any top diameter limit. The second equation predicted a volume ratio that allowed direct
determination of cubic foot volume to any top diameter (Matney et al. 1985). The tree profile equations were a modified form of the Chapman Richards function. The volume ratio equations were calculated using nonlinear regression. Analyses of the results indicated a high degree of precision and accuracy.

In a 1980 joint effort between Mississippi State University and the United States Forest Service (USFS), 150 permanent red oak-sweetgum mixture growth and yield plots were established throughout Mississippi. These plots provide data for ongoing hardwood growth and yield research. The criteria for the establishment of the plots required at least $75 \%$ of the basal area to be red oak and sweetgum and the minimum basal area for either the red oaks or the sweetgums to be $30 \%$ of the total basal area (Perkins et al. 1994).

From the initial measurement of the plots, Franco (1988) produced variabledensity yield tables. The equations employed to provide the yield tables were based on a modified form of Schumacher's equation and used stand age, average height of dominants and codominants, total merchantable basal area, and relative merchantable basal area as independent variables. Tests showed that the equations performed well under a variety of stand conditions.

In addition to volume, timber quality is very important in managing hardwood forests. A prediction system that estimates the grade of individual trees under specific tree and stand characteristics and assigns tree volumes to categories of log grades was developed for red oak-sweetgum mixtures by Belli et al. (1990) using discriminant analysis. Both stand level and tree level information were used to group trees into classes based on their log grade. The discriminator variables that produced the best overall
classification rates were dbh and relative basal area. In a related study, Belli et al. (1993) developed a discriminant analysis method of predicting tree grade in red oak-sweetgum stands in which no tree quality information is available. On test data with grade 5 (cull) trees eliminated, the model predicted the correct log grade within $70 \%, 54 \%, 66 \%$, and $78 \%$ of the time for cherrybark oak, water oak (Quercus nigra L.), willow oak (Quercus phellos L.), and sweetgum, respectively. These studies on prediction of log grade provide excellent information on how grade changes in trees as age increases.

Another important factor in an accurate growth and yield model is survival. A distance-independent individual tree growth and yield model was developed for bottomland hardwoods in the minor stream bottoms of Mississippi. This model predicted basal area growth and mortality (Perkins et al.1994). Basal area growth and survival were predicted using weighted multiple linear regression. Survival was separated by diameter class and species using conditional probability. Mortality was predicted using a three stage approach and then overall mortality was estimated. Mortality was allocated to each diameter class and distributed among species within each diameter class. Finally, overall mortality was predicted using multiple regression.

## Statement of Problem and Justification of Research

There is a great demand for growth and yield models for bottomland hardwoods to support management decisions and economic analysis but there is a lack of research in this area (Rauscher et al. 2000). Growth and yield information is particularly lacking in the minor streambottoms of the South (Perkins et al. 1994). Mixed stands, such as the
red oak/sweetgum mixture, provide many more difficulties when building a growth and yield model. The stands are more complex in nature and multiple variables must be considered, but the benefit of a growth and yield model for bottomland hardwoods will be very valuable to the forestry community. An accurate growth and yield estimate will not only provide a basis for sound management and investment decisions but help ensure the sustainability of the resource.

## Objective

The objective of this study was to develop a stand level growth and yield model for red oak/sweetgum forests in minor stream-bottoms in the Southeast. The model is designed to require stand information that is regularly measured in timber cruises so it can be readily utilized by landowners and forest managers. Yields are predicted on the stand and species-stand level. Model predictions are used to characterize the biological aspects of stand development. The model will become a component of an overall red oak/sweetgum growth and yield support system, and provide the basis for the development of a diameter distribution growth and yield model.

## CHAPTER II

## METHODS

## Study Area

Permanent growth and yield plots were established throughout north and central Mississippi and east Alabama (Figure 1) primarily on old fields in minor bottoms. Minor bottoms are floodplains and terraces that were formed from local soils (Hodges and Switzer 1979). Plots were established in the two very similar species associations of red oak/sweetgum and red oak/white oak/mixed species. Most of the plots were within the red oak/sweetgum species association. These two species associations are very similar. The red oak/white oak/mixed species association occurs later in the succession of bottomland hardwoods. In this species association, hickory replaces sweetgum as the largest non-oak species (Meadows and Stanturf 1997). The original plots were established in 1981, and remeasured in the following measurement periods: 1988, 1992, and 2006. Forty new plots were established in 1993 and 37 new plots were established in 2007 to replace destroyed plots. There are currently 160 valid plots in the study, 86 of which are original plots.


Figure 1.
Mississippi and Alabama counties with growth and yield plots.

The criteria for plot establishment were:

1. Stands should occur in stream bottoms (rivers, creeks, or other streams) but not on lands occurring between the Mississippi river and the levee and not in the loessal hills.
2. Mississippi and Alabama represent the general geographic area.
3. All plots must be in areas which developed as even-aged stands.
4. Stands should be essentially undisturbed from cutting and severe damage (fire, beaver, management, wind, etc.) for at least the last twenty years.
5. The minimum basal area for red oak is $30 \%$ of the total basal area.
6. The minimum age is twenty years; there is no maximum age.
7. Stands must have a minimal basal area of 60 square feet; all basal areas are based on 3.5" dbh and larger trees.
8. Minimum plot size is 0.1 acre.
9. Maximum plot size is 1 acre.
10. Plots and areas immediately adjacent to the plots must have the potential of remaining undisturbed for at least the next 10 years.
11. Stands should be in good condition with little disease, good crowns, and minimal percentage of blow-downs.

Plots were located to capture a wide range of site qualities and ages within these criteria so that the model would be applicable to a variety of sites.

## Measurements

All plots from the 1981 and 1993 establishments were visited to determine its remeasurement viability. If a plot was still viable (not harvested, thinned, or disturbed), it was re-monumented, its GPS coordinates recorded, and remeasured. Once existing plots were located and remeasured, new plots were established. The goal was to have at least 150 study plots. Specific age classes were targeted for new plots to fill in gaps in the existing data. All trees were tagged facing plot center, with a recorded azimuth and distance from plot center. The information recorded for all trees was:

1. Species.
2. Dbh to 0.1 inch.
3. Crown class.
4. Log grade.
5. Azimuth and distance from plot center.

Data taken on ten trees which represented the range of dbh's was:

1. Total height.
2. Merchantable height.
3. Height to an 8 " top.
4. Height to a 4 " top.

The following measurements were taken on all ingrowth trees:

1. Tag all trees 4 inch dbh and above that were not recorded in the last remeasurement.
2. Azimuth, distance, total height, and dbh.

Site index data taken on six dominant or codominant red oak trees included:

1. Age at dbh.
2. Total height.

## Calculations

The stand level attributes of trees per acre (TPA), arithmetic mean diameter (AD), quadratic mean diameter (QD), basal area, and cubic foot volume were calculated from the observed data. A single plot age at dbh for each measurement period was determined. Each age had to be consistent with the age of the same plot in a different measurement period. This was done by subtracting all plot ages for all measurement periods from one selected year prior to the first measurement in 1981. The mean was calculated at this year and this served as the base age. The age for each measurement period was then determined by adding the difference between the base year and the measurement year.

TPA was calculated by summing the trees per plot and dividing by the plot size.
$T P A=\sum_{i=1}^{n} t p p_{i} /$ plotsize
where
TPA = trees per acre,
tpp $=$ trees per plot, and
plotsize $=$ area of plot size in acres.

AD was calculated by using the simple arithmetic mean formula

$$
\begin{equation*}
A D=\sum_{i=1}^{n} d b h_{i} / t p p \tag{3}
\end{equation*}
$$

where
$A D=$ arithmetic mean diameter in inches and
$d b h=$ diameter at breast height in inches.
QD is an important stand level attribute because it has a direct relationship to
basal area. QD is the diameter that corresponds to the average tree basal area in the stand (Curtis and Marshall 2000). QD was calculated by using the quadratic mean formula

$$
\begin{equation*}
Q D=\sqrt{\sum_{i=1}^{n} d b h_{i}^{2} / t p p} \tag{4}
\end{equation*}
$$

where
$Q D=$ quadratic mean diameter in inches.
Basal area per acre was calculated from QD and TPA.
$B A=0.005454 * Q D^{2} * T P A$
where
$B A=$ basal area per acre in $\mathrm{ft}^{2}$.
Stand level volume was calculated using the basal area per acre and average total
height in the stand. The stand level volume calculation estimates a surrogate volume used at this point in the study to determine how well the models interact with each other. The stand level volume does not represent actual merchantable volume but the final growth and yield model will predict merchantable volume

Volume $=B A^{*} \operatorname{avgHT} *(2 / 3)$
where
Volume $=$ volume per acre in $\mathrm{ft}^{3}$ and
$\operatorname{avgHT}=$ average total height of all trees in the stand.

## Species Groups

The stand was categorized into six species groups. Species groups were determined based on commercial importance and frequency as follows:

1. red oak
2. white oak
3. sweetgum
4. hickory (Carya spp.)
5. other commercial
6. non commercial

Cherrybark oak, water oak, and willow oak were the primary red oak group species observed. Swamp chestnut (Quercus michauxii Nutt.), white oak (Quercus alba L.), and overcup oak (Quercus lyrata Walt.) were the primary white oak group species observed. Species that had commercial value but did not occur frequently enough in the stand to comprise their own group were categorized as other commercial. Yellow poplar (Liriodendron tulipifera L.), green ash (Fraxinus pennsylvanica Marsh.), and sugarberry (Celtis laevigata Willd.) are common examples of species in the other commercial group. Species that had no commercial value were categorized as non commercial. Common species grouped into the non commercial group were American hornbeam (Carpinus caroliniana Walt.) and eastern hophornbeam (Ostrya virginiana (Mill.) K. Koch). AD, QD, TPA, and basal area were calculated for each species group within each plot.

## Regression Analysis

Regression analysis was used to build equations to predict stand level attributes. The first step was to plot each dependent stand level variable over all possible independent variables and transformations of the independent variables to establish initial model forms and test linear regression assumptions. Linear and nonlinear regression techniques were attempted. Selected linear regression models met the following underlying assumptions:

1. Model chosen is appropriate for the data.
2. The error variance is constant.
3. The errors are independent random variables
4. There are no outliers.
5. The errors are normally distributed random variables.
6. No important independent variables were omitted from the model.

The entire dataset was used to construct both linear and nonlinear regression models. First, linear regression was attempted, and if the model was not satisfactory, then nonlinear regression was utilized. A variety of functional relationships were tested for each model which were selected largely by trial and error testing based on significance of variables, fit statistics, homogeneity of variance, biological trends, and sensitivity analysis. Coefficient of Determination $\left(R^{2}\right)$ was calculated for the linear models, and represents the amount of variation in the dependent variable that can be explained by the independent variables. $R^{2}$ was calculated as one minus the quantity of the error sum of squares divided by the corrected total sum of squares.
$R^{2}=1-S S E / S S T C$
where
$0 \leq R^{2} \leq 1$,
$R^{2}=$ coefficient of determination,
SSE = error sum of squares, and
SSTC = corrected total sum of squares.
Index of Fit $\left(I^{2}\right)$ was calculated for the nonlinear models as one minus the error sum of squares divided by the corrected total sum of squares.
$I^{2}=1-$ SSE $/$ SSTC
where
$-\infty \leq I^{2} \leq 1$, and
$I^{2}=$ index of fit.
Homogeneity of variance was verified by observing residual plots of the predicted and independent variables. For a model to be accepted, the residual values had to be scattered about zero with no trends across the range of the data. The model with the most significant variables, best fit statistic, and homogenous variance was then plotted over the data to ensure biological accuracy. A model could have a good fit statistic but may not provide an adequate prediction of the stand level attributes. Variables were manipulated until the model displayed acceptable biological trends. These five factors: significant variables, best fit statistic, homogenous variance, biological accuracy, and sensitivity analyses, were used to form a consensus on the best model for each stand level attribute.

## Model Scenarios

Different models for each stand level attribute were constructed that depend on varying amounts of initial stand input information. A Visual Basic/Microsoft Excel growth and yield application program was developed to run three different projection scenarios: "bare ground", "stand density", and "existing inventory". The "bare ground" scenario is appropriate when there is no existing stand, or no knowledge of the existing stand. The only inputs required to run the model are age and site index. Models were constructed to predict all stand level attributes based only on these two independent variables. Site index can be collected from a county soil survey, the Baker Broadfoot site evaluation method (Baker and Broadfoot 1979), or other sources. All measurements from each measurement period were used for this model. Each measurement for each period was counted as one observation, totaling 638 observations for the "bare ground" scenario. The "stand density" scenario is appropriate when there is no inventory data for the existing stand, but TPA and/or height of the dominant and codominant red oaks is known. The only inputs required to run this model are age, site index or height of the dominant and codominant red oaks, and TPA. Site index is computed if height and age of the dominant and codominant red oaks are known. All 638 observations were also used to construct equations for the "stand density" scenario. The "existing inventory" scenario is appropriate when all current stand level attributes (current age, site index or height of the dominant and codominant red oaks, existing AD, QD, and TPA) are known. This model projects the stand level attributes to a future age. Each observation consisted
of two measurements for one plot, totaling 382 observations for the "existing inventory" scenario.

An individual total tree height equation was also constructed for use with a diameter distribution growth and yield model that is the next objective in the larger research program. The height equation was based on the 9,294 observations from all measurement periods.

## CHAPTER III

## RESULTS

## Observed Data

Model data covered a wide range of sites, ages, and densities (Table 1). A wide range of site indices and ages were important to the study because: 1 ) site index is a variable in virtually all equations and 2) models derived from the data need to be applicable to as wide an area and as many growth stages as possible.

Table 1.
Range of the observed data from all measurement periods.

| Variable | Minimum | Lower <br> Quartile | Mean | Upper <br> Quartile | Maximum |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Site index (50 years) <br> in feet | 67 | 99 | 105 | 111 | 133 |
| Age <br> in years | 15 | 40 | 52 | 63 | 92 |
| Trees <br> per acre | 85 | 167 | 262 | 328 | 742 |
| Arithmetic mean diameter <br> in inches | 4.9 | 7.5 | 9.3 | 10.9 | 15.2 |
| Quadratic mean diameter <br> in inches | 5.2 | 8.4 | 10.6 | 12.7 | 17.8 |

## Site Index

Age and height of the dominant and codominant red oaks were the drivers for the site index model. Red oaks are the most commercially important hardwood species in the South and they comprise the majority of the overstory in many stands. First, age and height of the dominant and codominant red oaks were used to construct a site index
equation. A weighted nonlinear regression equation was constructed to predict the height of the dominant and codominant red oaks with red oak age as the independent variable. A weight of $1 /$ Age $^{2}$ was used to ensure the variance was homogenous. The nonlinear regression (Chapman - Richards function) model was
$H D=a^{*}\left(1-e^{b^{*} \text { Age }}\right)^{c}$
where
$H D=$ Height of the dominant and codominant red oaks in feet, Age $=$ Age of the dominant and codominant red oaks, $e=$ base of natural logarithm, and $a, b, c=$ parameters to be estimated from the data.

This equation was converted into an anamorphic site index equation with the base age of 50
$S I=\left(H D^{*} a^{*}\left(1-e^{b^{* 50}}\right)^{c}\right) /\left(a^{*}\left(1-e^{b^{*} A g e}\right)^{c}\right)$
where
$S I=$ Site index (base age 50) of red oaks in feet and $a, b, c=$ parameters from Equation 9.

By predicting all red oak heights of the dominants and codominant at age 50, stands at different ages can be compared.

## Stand Level Models

Trees per Acre
Two equations for estimations of TPA were constructed. The first equation was designed to predict total TPA with age and site index as the independent variables.

Weighted nonlinear regression was used because of the strong inverse J-shaped trend of

TPA over age. This equation is only for the "bare ground" scenario because TPA is known for the other scenarios. A weight of $\mathrm{Age}^{2}$ was used to ensure homogeneity of variance. The nonlinear regression model was
$T P A=\left(a * S I^{b} *\right.$ Age $\left.^{c}\right) /\left(\left(1+e^{d^{*} \text { Age }}\right)^{g}\right)$
where
$T P A=$ trees per acre and $a, b, c, d, g=$ parameters to be estimated from the data.

A linear regression equation was constructed to predict future TPA with existing age, future age, and existing TPA as the independent variables. This equation is used in the "existing inventory" scenario. The dependent variable was a ratio of future TPA to current TPA.

$$
\begin{equation*}
T P A_{\text {ratio }}=T P A_{1} / T P A_{0} \tag{12}
\end{equation*}
$$

where
$T P A_{\text {ratio }}=$ ratio of future trees per acre to current trees per acre,
$T P A_{0}=$ existing trees per acre, and
$T P A_{1}=$ future trees per acre.
$\ln T P A_{\text {ratio }}=a+b^{*}\left(\right.$ Age $_{1} /$ Age $\left._{0}\right)+c /$ Age $_{0}+d / T P A_{0}$
where
ln = natural logarithm,
$a, b, c, d=$ parameters to be estimated from the data.

## Arithmetic Mean Diameter and Quadratic Mean Diameter

Three linear regression equations were constructed to predict AD and QD . For all cases, the same model was used for both AD and QD . The first equation predicted mean
diameter with only age and site index as the independent variables. This equation is used for the "bare ground" scenario. The linear regression model was
$\ln A D_{\text {Total }}=a+b * \ln$ Age $+c /$ Age $+d^{*} S I^{2 / 3}+g * S I /$ Age $^{1 / 8}$
where
$A D_{\text {Total }}=$ total arithmetic mean diameter in inches, and $a, b, c, d, g=$ parameters to be estimated from the data.
$\ln Q D_{\text {Total }}=a+b * \ln$ Age $+c /$ Age $+d * S I^{2 / 3}+g * S I /$ Age $^{1 / 8}$
where
$Q D_{\text {Total }}=$ quadratic mean diameter in inches and $a, b, c, d, g=$ parameters to be estimated from the data.

The second model predicted mean diameter with age, site index, and TPA as the independent variables. This equation will be used for the "stand density" scenario. The linear regression model was
$\ln A D_{\text {Total }}=a+b * \ln$ Age $+c /$ Age $+d * S I^{2 / 3}+g * S I /$ Age $^{1 / 8}+h * T P A$
where
$a, b, c, d, g, h=$ parameters to be estimated from the data.
$\ln Q D_{\text {Total }}=a+b^{*} \ln$ Age $+c /$ Age $+d * S I^{2 / 3}+g * S I /$ Age $^{1 / 8}+h * T P A$
where
$a, b, c, d, g, h=$ parameters to be estimated from the data.
The third equation predicted mean diameter annual percentage growth with existing age, future age, and existing TPA as the independent variables. The growth
model is used for the "existing inventory" scenario. The dependent variable of mean diameter annual percentage growth was calculated by

$$
\begin{equation*}
A D_{\text {growth }}=100 *\left[\left(A D_{1}-A D_{0}\right) /\left(\left(A g e_{1}-A g e_{0}\right) * A D_{0}\right)\right] \tag{18}
\end{equation*}
$$

where
$A D_{\text {growth }}=$ arithmetic mean diameter annual percentage growth, $A D_{0}=$ existing arithmetic mean diameter,
$A D_{1}=$ future arithmetic mean diameter,
Age $_{0}=$ existing age, and
Age $_{1}=$ future age.
$Q D_{\text {growth }}=100 *\left[\left(Q D_{1}-Q D_{0}\right) /\left(\left(A g e_{1}-A g e_{0}\right) * Q D_{0}\right)\right]$
where
$Q D_{\text {growth }}=$ quadratic mean diameter annual percentage growth,
$Q D_{0}=$ existing quadratic mean diameter, and
$Q D_{1}=$ future quadratic mean diameter.
The linear regression model was

$$
\begin{equation*}
A D_{\text {growth }}=a+b^{*} A g e_{1}+c^{*} A g e_{0}+d / A g e_{0}+g * \sqrt{\text { Age }_{0}}+h *\left(1 / T P A_{0}\right) \tag{20}
\end{equation*}
$$

where
$T P A_{0}=$ existing trees per acre, and
$a, b, c, d, g=$ parameters to be estimated from the data.

$$
\begin{equation*}
Q D_{\text {growhth }}=a+b^{*} A g e_{1}+c^{*} A g e_{0}+d / A g e_{0}+g * \sqrt{A g e_{0}}+h^{*}\left(1 / T P A_{0}\right) \tag{21}
\end{equation*}
$$

where
$a, b, c, d, g, h, i=$ parameters to be estimated from the data.

## Height

A weighted nonlinear individual total tree height equation was constructed with height of the dominant and codominant red oaks, dbh , and QD as the independent variables. A weight of $1 / H D^{2}$ was used to ensure homogeneity of variance. The total tree height prediction equation was

$$
\begin{equation*}
\left.H T=\left(a+b^{*} H D^{c}\right) *\left(1-e^{-d^{*}(d b h / Q D)}\right)^{g}\right) \tag{22}
\end{equation*}
$$

where
$H T=$ individual tree total height in feet,
$d b h=$ diameter breast height in inches, and $a, b, c, d, g=$ parameters to be estimated from the data.

## Basal Area and Volume

Basal area and volume were estimated from known equations and predicted using regression. QD and TPA predictions were used to estimate basal area, using the equation

$$
\begin{equation*}
B \hat{A}=\left(0.005454 * Q \hat{D}^{2}\right) * T \hat{P} A \tag{23}
\end{equation*}
$$

where
$B \hat{A}=$ estimated basal area per acre in $\mathrm{ft}^{2}$,
$Q \hat{D}=$ predicted quadratic mean diameter in inches, and
$T \hat{P} A=$ predicted trees per acre.
QD, TPA, and tree height predictions were used to estimate volume using the equation
Vol̂ume $=B \hat{A}^{*} \operatorname{avg} H T^{*}(2 / 3)$
where
Vol̂ume $=$ estimated volume per acre in $\mathrm{ft}^{3}$ and $a v \hat{g} H T=$ predicted average total height of all trees in the stand.

## Species Models

Stand level equations were also constructed by six species groups. Each stand level model was first constructed for red oaks. The model chosen for red oaks was also used for the other species groups. The dependent variable was a ratio of the species stand level attribute to the total stand level attribute. All independent variables for each species model were total stand level attributes. No species variables were included based on inspection of data trends and findings from other studies. The percentage of each species group out of the total composition of the stand was plotted over AD, QD, and TPA. These plots indicated no trends that would suggest inclusion of species variables in the model. Johnson and Krinard (1988) observed this same result over 29 years in two cutover red oak/sweetgum stands. They found that initial red oak/sweetgum stand composition varied widely but stands typically had very similar composition by the end of the study period. Based on this research and observation of the data, it was concluded that regardless of the current species composition, the future species composition can be predicted by total stand level variables. This observation results in a simpler model requiring fewer inputs. The species models that performed best were:

Trees per Acre

$$
\begin{equation*}
T P A_{\text {species }} / T P A_{\text {total }}=a+b^{*} \text { Age }+c * Q D+d * T P A \tag{25}
\end{equation*}
$$

where
$T P A_{\text {species }}=$ trees per acre for a species group (ex. red oak),
$T P A_{\text {total }}=$ total stand trees per acre, and
$a, b, c, d=$ parameters to be estimated from the data.
$\ln \left(A D_{\text {species }} / A D_{\text {total }}\right)=a+b^{*} \ln A g e+c^{*} \ln A D_{\text {Total }}+d^{*} S I$
where
$A D_{\text {species }}=$ arithmetic mean diameter for a species groups (ex. red oak) in inches, $A D_{\text {total }}=$ stand arithmetic mean diameter in inches, and $a, b, c, d=$ parameters to be estimated from the data.
$\ln \left(Q D_{\text {species }} / Q D_{\text {total }}\right)=a+b^{*} \ln A g e+c^{*} \ln Q D_{\text {Total }}+d * S I$
where
$Q D_{\text {species }}=$ quadratic mean diameter for a species groups (ex. red oak) in inches, $Q D_{\text {total }}=$ stand quadratic mean diameter in inches, and $a, b, c, d=$ parameters to be estimated from the data.

## Parameter Estimates and Fit Statistics

The parameter estimates and fit statistics for all stand level models are shown in Table 2. The $R^{2}$ and $I^{2}$ for all models for the "bare ground" and "stand density" scenarios were greater than 0.50 . There is a large increase in $R^{2}$ for AD and QD from the "bare ground" scenario to the "stand density" scenario. This indicates that the addition of TPA to the models greatly contributes to the explanation of average stand diameter. The lower $R^{2}$ "existing inventory" models can be explained based on the form of dependent variables. The dependent variables were ratios of current and future stand level attributes. The dependent ratio variables become very close to a constant and are therefore not well related to the parameters. This results in a simpler model. The purpose of the "existing inventory" models was to predict the ratio change of current and future stand level attributes based on a change in age and stand density. Even though
there is little relationship between the dependent ratio and independent variables, when that ratio is multiplied by the existing stand level variable, the prediction is reliable. The models chosen demonstrated the best combination of coefficient of determination, significant variables, and homogeneity of variances. A low coefficient of determination on a ratio variable should not decrease confidence in the models. $I^{2}$ was calculated based on the actual observed future values and the predicted future stand level attributes. The $I^{2}$ for the "existing inventory" model are very high.

Table 2.
Parameter estimates and fit statistics for stand level equations for the three model scenarios and an individual tree height equation.

|  | Parameter Estimate |  |  |  |  |  | Fit Statistic |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $a$ | $b$ | $c$ | $d$ | $g$ | $h$ | $\mathrm{R}^{2}$ | $\mathrm{I}^{2}$ |
| Bare Ground |  |  |  |  |  |  |  |  |
| TPA | 4806.0000 | 0.0900 | 0.2469 | -0.3501 | -0.0481 | - | - | 0.51 |
| $A D$ | -1.5116 | 0.6033 | 8.1933 | 0.1037 | -0.0174 | - | 0.59 | - |
| $Q D$ | -2.0700 | 0.6578 | 10.1896 | 0.1522 | -0.0274 | - | 0.67 | - |
| Stand Density |  |  |  |  |  |  |  |  |
| SI | 127.2000 | 0.0317 | 0.8481 | - | - | - | - | 0.61 |
| $A D$ | 1.1556 | 0.1147 | 0.3485 | 0.0802 | -0.0135 | -0.0012 | 0.81 | - |
| $Q D$ | 0.6936 | 0.1516 | 2.0613 | 0.1278 | -0.0233 | -0.0013 | 0.87 | - |
| Existing Inventory |  |  |  |  |  |  |  |  |
| $A D_{\text {growth }}$ | -19.0356 | 0.0690 | -0.3814 | 119.1337 | 4.7010 | -121.0499 | 0.32 | 0.92 |
| $Q D_{\text {growth }}$ | -12.4211 | 0.0400 | -0.2614 | 90.0047 | 3.2642 | -125.8141 | 0.35 | 0.93 |
| $T P A_{\text {ratio }}$ | 0.9817 | -1.2504 | 10.4978 | 25.4132 | - | - | 0.51 | 0.95 |
| Individual Trees |  |  |  |  |  |  |  |  |
| Height | 43.0572 | 0.0744 | 1.4796 | 1.5109 | 1.317 | - | - | 0.85 |

The parameter estimates and fit statistics for all species models are given in Table 3. The $R^{2}$ is lower than the stand level predictions for all species groups for which there are a variety of explanations. Most importantly, the models are ratio models and
therefore the dependent variable is close to a constant. Also, the number of observations for each species group was reduced. Not all species groups were present in every observation. Also, the number of trees within a species group used to calculate the stand level attributes were highly variable for some groups. For example, a plot may have contained only two white oaks. These two white oaks would have been used to calculate white oak TPA, AD, and QD. The primary species of interest are red oak and sweetgum. These models were fit for red oak first and then applied to all the other species. The $R^{2}$ and $I^{2}$ for red oak TPA, AD, and QD indicate that the fit was acceptable for ratio models. The sweetgum TPA model had the best $R^{2}$ and $I^{2}$ of all species models. This indicates a good trend for sweetgum TPA. The sweetgum AD and $\mathrm{QD} R^{2}$ were very low but the $I^{2}$ is acceptable. This indicates a high level of variability for sweetgum mean diameter across the range of the independent variables and is probably because sweetgum is a very adaptable species that can tolerate a wide range of soil and site conditions (Burns and Honkala. 1990). Sweetgum is typically a dominant species early in the life of the stand; however, sweetgum can also survive in the midstory in closed canopy stands. Because of the high sweetgum component in all of the plots, but a wide range of sizes, modeling sweetgum mean diameter was challenging.

Table 3.

Parameter estimates and fit statistics for species stand level equations.

|  | Parameter Estimate |  |  |  | Fit Statistic |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{T P A}$ | $a$ | $b$ | $c$ | $d$ | $\mathrm{R}^{2}$ | $\mathrm{I}^{2}$ |
|  |  |  |  |  |  |  |
| Red Oak | 0.43410 | -0.00470 | 0.01469 | -0.00040 | 0.20 | 0.42 |
| White Oak | 0.02120 | -0.00030 | 0.00480 | 0.00004 | 0.01 | 0.04 |
| Sweetgum | -0.09477 | 0.00237 | 0.00550 | 0.00136 | 0.44 | 0.84 |
| Hickory | 0.32286 | 0.00117 | -0.01580 | -0.00043 | 0.07 | 0.01 |
| Other Commercial | 0.31316 | -0.00111 | -0.00493 | -0.00034 | 0.05 | 0.06 |
| Non Commercial | 0.29816 | 0.00091 | -0.00976 | -0.00043 | 0.11 | -0.02 |
| $\boldsymbol{A D}$ |  |  |  |  |  |  |
| Red Oak | -1.38920 | 0.72240 | -0.79400 | 0.00740 | 0.41 | 0.66 |
| White Oak | 1.05295 | 0.33638 | -0.94963 | -0.00355 | 0.14 | 0.02 |
| Sweetgum | -0.14572 | -0.04904 | 0.19161 | -0.00157 | 0.02 | 0.51 |
| Hickory | 2.24402 | -0.45493 | -0.07004 | -0.00659 | 0.16 | 0.15 |
| Other Commercial | 0.65783 | 0.05641 | -0.59258 | 0.00147 | 0.10 | 0.03 |
| Non Commercial | 1.74453 | -0.10989 | -0.59106 | -0.00535 | 0.43 | 0.10 |
| $\boldsymbol{Q D}$ |  |  |  |  |  |  |
| Red Oak | -1.14700 | 0.59030 | -0.62780 | 0.00660 | 0.32 | 0.71 |
| White Oak | 1.20306 | 0.23036 | -0.77636 | -0.00457 | 0.12 | 0.01 |
| Sweetgum | 0.19370 | -0.17040 | 0.23990 | -0.00231 | 0.02 | 0.54 |
| Hickory | 2.65896 | -0.62126 | 0.08769 | -0.00866 | 0.19 | 0.13 |
| Other Commercial | 0.84795 | -0.05403 | -0.43852 | 0.00030 | 0.10 | 0.05 |
| Non Commercial | 1.92068 | -0.15508 | -0.55690 | -0.00634 | 0.44 | 0.11 |

## Percent Difference of Observed and Predicted

Models which were applied to the observed data and predicted attribute values were then compared to the observed attribute values. The percent difference between the predicted and observed values was calculated as
\% Difference $=($ Observed-Predicted $) /$ Predicted.
The mean percent difference was calculated for all observations and by specific age classes to identify any specific data range problem areas in model application. The total mean percent difference, percent difference within age classes, and numbers of
observations for each model are given in Tables 4 through 13. A negative mean percent difference indicates the equation overestimated the stand level attribute and a positive mean percent difference indicates the equation underestimated the stand level attribute.

Table 4 shows that the "bare ground" scenario slightly underestimates $\mathrm{AD}, \mathrm{QD}$, and TPA and slightly overestimates basal area and cubic foot volume. It is possible for basal area and cubic foot volume to be overestimated while QD and TPA underestimated because the models do not necessarily have the same percent difference in the same ranges of the data. TPA has a very low total mean percent difference but the mean percent differences at both young and old ages are large.

Table 4.
Mean percent difference of observed and predicted stand level attributes for the "bare ground" scenario.

| \% Difference |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Bare Ground Scenario |  |  |  |  |  |  |
| Age Class | Number of Observations | Arithmetic Mean Diameter | Quadratic Mean Diameter | Trees per acre | Basal Area | Cubic Foot Volume |
| 15-19 | 5 | 1.9618\% | 2.0657\% | -24.6630\% | -22.3693\% | -19.7242\% |
| 20-29 | 42 | 3.1599\% | 4.3307\% | -5.8444\% | -3.9196\% | -4.5868\% |
| 30-39 | 103 | -1.3496\% | -1.8662\% | 6.7769\% | -2.7632\% | -1.3927\% |
| 40-49 | 150 | 0.8074\% | 0.9099\% | 2.1916\% | -2.7532\% | -2.4716\% |
| 50-59 | 139 | 2.7686\% | 3.5661\% | -5.6869\% | -3.6293\% | -4.7962\% |
| 60-69 | 100 | 3.7279\% | 3.9469\% | -3.8376\% | 0.3372\% | 0.9673\% |
| 70-79 | 72 | 0.0002\% | -0.6133\% | 2.7659\% | -1.3805\% | -0.2957\% |
| 80+ | 27 | -4.4001\% | -4.6281\% | 11.8915\% | 0.1090\% | -0.1565\% |
| Total | 638 | 1.197\% | 1.344\% | 0.006\% | -2.416\% | -2.196\% |

Table 5 lists the mean percent difference for the "stand density" scenario. Mean percent difference for height of the dominant and codominant red oaks is included in the "stand density" scenario. The mean percent difference for this equation is very low
across all ranges of the data. Mean percent difference between AD and QD are within $\pm 1 \%$. TPA is not included in Table 5 because it is an input in the model and not a predicted variable. The "stand density" model slightly underestimates basal area and cubic foot volume.

Table 5.

Mean percent difference of observed and predicted stand level attributes for the "stand density" scenario.

| \% Difference <br> Stand Density Scenario |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Number of | Height of | Arithmetic | Quadratic | Basal | Cubic Foot |
| Age Class | Observations | Dom. And Codom. | Mean Diameter | Mean Diameter | Area | Volume |
| $15-19$ | 5 | $2.8847 \%$ | $-8.0106 \%$ | $-6.4344 \%$ | $-11.2556 \%$ | $-8.6414 \%$ |
| $20-29$ | 42 | $-1.5412 \%$ | $0.4954 \%$ | $3.4768 \%$ | $7.9303 \%$ | $6.9525 \%$ |
| $30-39$ | 103 | $1.1844 \%$ | $-0.2836 \%$ | $1.0498 \%$ | $3.0754 \%$ | $4.3057 \%$ |
| $40-49$ | 150 | $0.0174 \%$ | $-1.2226 \%$ | $0.1918 \%$ | $1.1550 \%$ | $1.3095 \%$ |
| $50-59$ | 139 | $-1.3222 \%$ | $-1.2611 \%$ | $0.2726 \%$ | $1.4102 \%$ | $0.2776 \%$ |
| $60-69$ | 100 | $0.8633 \%$ | $1.5375 \%$ | $2.4421 \%$ | $5.7207 \%$ | $6.7369 \%$ |
| $70-79$ | 72 | $1.1539 \%$ | $0.4369 \%$ | $0.4678 \%$ | $1.9226 \%$ | $3.3042 \%$ |
| $80+$ | 27 | $-0.7631 \%$ | $-0.9723 \%$ | $-0.5559 \%$ | $-0.2341 \%$ | $-0.6540 \%$ |
| Total | $\mathbf{6 3 8}$ | $\mathbf{0 . 0 6 2 \%}$ | $\mathbf{- 0 . 3 8 9 \%}$ | $\mathbf{0 . 8 6 4 \%}$ | $\mathbf{2 . 6 1 3 \%}$ | $\mathbf{2 . 8 5 5 \%}$ |

In the "existing inventory" model, the mean percent difference for AD, QD, TPA, basal area, and cubic foot volume are very low (Table 6). Because this model accepts inputs of current stand level attributes, predicted future stand level attributes are very close to the observed stand level attributes.

Table 6.

Mean percent difference of observed and predicted stand level attributes for the "existing inventory" scenario.

|  | \% Difference |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Number of | Arithmetic | Quadratic | Trees | Basal |
| Existing Inventory Scenario | Cubic Foot |  |  |  |  |  |
| Age Class | Observations | Mean Diameter | Mean Diameter | per acre | Area | Volume |
| $15-19$ | 5 | $2.0278 \%$ | $2.5961 \%$ | $3.0488 \%$ | $8.5240 \%$ | $42.4861 \%$ |
| $20-29$ | 26 | $-1.2897 \%$ | $-0.9175 \%$ | $1.8447 \%$ | $-0.4905 \%$ | $13.6353 \%$ |
| $30-39$ | 84 | $-0.1793 \%$ | $-0.0262 \%$ | $0.0883 \%$ | $-0.5220 \%$ | $4.4094 \%$ |
| $40-49$ | 100 | $0.6876 \%$ | $0.7113 \%$ | $-0.7409 \%$ | $0.5729 \%$ | $0.9332 \%$ |
| $50-59$ | 73 | $-0.5494 \%$ | $-0.9861 \%$ | $2.0415 \%$ | $-0.4172 \%$ | $-4.4626 \%$ |
| $60-69$ | 60 | $0.6210 \%$ | $0.1471 \%$ | $-0.1910 \%$ | $-0.2161 \%$ | $1.6125 \%$ |
| $70-79$ | 34 | $-0.3725 \%$ | $0.1928 \%$ | $2.7645 \%$ | $2.4306 \%$ | $-13.2538 \%$ |
| $80+$ | 0 | - | - | - | - | - |
| Total | $\mathbf{3 8 2}$ | $\mathbf{0 . 0 3 8 7 \%}$ | $\mathbf{0 . 0 0 3 8 \%}$ | $\mathbf{0 . 5 9 7 1 \%}$ | $\mathbf{0 . 2 1 6 0 \%}$ | $\mathbf{0 . 9 1 8 9 \%}$ |

The mean percent difference for the individual tree height model is given in Table
7. Mean percent difference was calculated for the entire dataset as well as ranges of dbh.

Overall, the model slightly underestimates tree height.

Table 7.
Mean percent difference of observed and predicted heights for the individual tree height model.

| \% Difference <br> Individual Tree Height |  |  |
| :---: | :---: | :---: |
|  | Number of | Individual |
| DBH | Observations | Tree Height |
| $3.5-9.0$ | 4846 | $-0.4844 \%$ |
| $10.0-19.9$ | 3347 | $1.1541 \%$ |
| $20.0-29.9$ | 954 | $1.4422 \%$ |
| $30.0+$ | 147 | $0.4423 \%$ |
| Total | $\mathbf{9 2 9 4}$ | $\mathbf{0 . 3 1 8 \%}$ |

The mean percent differences for each species group are included in Tables 8
through 13. In general, these mean percent differences are higher than the stand level
models. However, the mean percent difference for red oak and sweetgum, the two primary species of interest, are very similar to the mean percent differences of the total stand level attributes. Table 8 indicates that the difference between observed and predicted values is very low for red oaks. The mean percent difference for all stand level attributes is highest in the 15-19 year age class. Early in the life of the stand, there is high variation in species composition, which makes it difficult to model red oak development (Johnson and Krinard 1988). As age increases and red oaks emerge as the dominant species, the mean percent difference falls to acceptable levels.

## Table 8.

Mean percent difference of observed and predicted stand level attributes for the red oak group.

|  | \% Difference <br> Red Oak |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Number of | Arithmetic <br> Age Class | Quadratic <br> Observations | Trees <br> Mean Diameter <br> Mean Diameter | Basal <br> Area | Cubre Foot <br> Volume |
| $15-19$ | 5 | $3.4158 \%$ | $-0.1060 \%$ | $23.1596 \%$ | $21.1281 \%$ | $21.0249 \%$ |
| $20-29$ | 42 | $4.2287 \%$ | $4.0496 \%$ | $15.4835 \%$ | $18.8191 \%$ | $20.5580 \%$ |
| $30-39$ | 102 | $-1.1058 \%$ | $-1.0177 \%$ | $8.7679 \%$ | $-0.9102 \%$ | $-2.3182 \%$ |
| $40-49$ | 148 | $3.3637 \%$ | $2.9015 \%$ | $1.1188 \%$ | $-0.7218 \%$ | $-0.7341 \%$ |
| $50-59$ | 135 | $3.1812 \%$ | $2.8332 \%$ | $-0.2962 \%$ | $-2.3471 \%$ | $-2.2511 \%$ |
| $60-69$ | 99 | $1.9911 \%$ | $1.4639 \%$ | $4.6826 \%$ | $0.6624 \%$ | $0.4465 \%$ |
| $70-79$ | 72 | $-1.2189 \%$ | $-1.5624 \%$ | $14.2721 \%$ | $4.3502 \%$ | $3.0953 \%$ |
| $80+$ | 27 | $6.0339 \%$ | $4.7853 \%$ | $-0.9147 \%$ | $1.6983 \%$ | $2.1947 \%$ |
| Total | $\mathbf{6 3 0}$ | $\mathbf{2 . 0 3 4 1 \%}$ | $\mathbf{1 . 6 4 9 7 \%}$ | $\mathbf{5 . 1 6 2 7 \%}$ | $\mathbf{1 . 2 7 6 5 \%}$ | $\mathbf{1 . 0 2 5 2 \%}$ |

The total mean percent difference for sweetgum is low (Table 9). The mean percent difference is high at both ends of the age classes. This is due both to high variation and a low number of observations.

Table 9.

Mean percent difference of observed and predicted stand level attributes for the sweetgum group.

| \% Difference <br> Sweetgum |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Age Class | Number of Observations | Arithmetic Mean Diameter | Quadratic <br> Mean Diameter | Trees per acre | Basal <br> Area | Cubic Foot Volume |
| 15-19 | 5 | 10.4892\% | 7.1421\% | -2.9684\% | 10.7154\% | 15.2279\% |
| 20-29 | 42 | 8.9252\% | 6.6493\% | -9.5165\% | -3.9120\% | -1.2551\% |
| 30-39 | 101 | 0.4245\% | 1.4228\% | -1.6383\% | 1.8291\% | 4.1915\% |
| 40-49 | 149 | -1.3546\% | -1.2534\% | 1.6762\% | -2.3714\% | -2.1352\% |
| 50-59 | 138 | 3.1566\% | 2.5216\% | -6.7192\% | -5.9169\% | -3.0914\% |
| 60-69 | 100 | 1.8641\% | 1.1770\% | 2.0902\% | 3.4438\% | 6.5327\% |
| 70-79 | 72 | 9.0806\% | 7.9188\% | -8.8291\% | 1.2273\% | 7.5282\% |
| 80+ | 27 | 10.7458\% | 8.4820\% | -3.9109\% | 8.2565\% | 14.6707\% |
| Total | 634 | 2.8932\% | 2.4239\% | -2.8230\% | -0.6943\% | 2.0401\% |

The observed and predicted mean percent differences for white oak, hickory, other commercial, and non commercial species groups are very high (Tables 10-13). These high mean percent observations are caused by low numbers of observations and low numbers of trees within each observation. This results in a high variation and, therefore, a high mean percent difference between observed and predicted stand level attributes.

Table 10.
Mean percent difference of observed and predicted stand level attributes for the white oak group.

| \% Difference <br> White Oak |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Age Class | Number of Observations | Arithmetic Mean Diameter | Quadratic Mean Diameter | Trees per acre | Basal <br> Area | Cubic Foot Volume |
| 15-19 | 0 | - | - | - | - | - |
| 20-29 | 5 | 15.9687\% | 18.4512\% | 28.3213\% | 63.9962\% | 77.1113\% |
| 30-39 | 21 | 2.5845\% | 4.2268\% | 5.3267\% | 23.5556\% | 33.6732\% |
| 40-49 | 49 | 6.7231\% | 7.9914\% | 37.0276\% | 57.6954\% | 76.4501\% |
| 50-59 | 77 | 12.2966\% | 14.9329\% | 23.4833\% | 83.4157\% | 121.6197\% |
| 60-69 | 58 | 10.1448\% | 12.5052\% | 32.5801\% | 109.2244\% | 161.8755\% |
| 70-79 | 45 | 3.5787\% | 3.3883\% | -5.2554\% | 19.7822\% | 41.2645\% |
| 80+ | 22 | 12.0590\% | 15.6368\% | 20.1961\% | 110.7758\% | 167.0065\% |
| Total | 277 | 8.7549\% | 10.6290\% | 21.5650\% | 71.2167\% | 105.1382\% |

Table 11.
Mean percent difference of observed and predicted stand level attributes for the hickory group.

\left.| \% Difference |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Hickory |  |  |  |  |  |  |$\right]$

Table 12.

Mean percent difference of observed and predicted stand level attributes for the other commercial group.

| \% Difference <br> Other Commercial |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Number of | Arithmetic | Quadratic | Trees | Basal | Cubic Foot |
| Age Class | Observations | Mean Diameter | Mean Diameter | per acre | Area | Volume |
| $15-19$ | 5 | $-2.4202 \%$ | $-3.9067 \%$ | $-48.0317 \%$ | $-56.3143 \%$ | $-58.1074 \%$ |
| $20-29$ | 39 | $-1.9570 \%$ | $-2.6837 \%$ | $-15.5402 \%$ | $-16.9245 \%$ | $-15.8641 \%$ |
| $30-39$ | 75 | $6.4983 \%$ | $8.0200 \%$ | $\mathbf{- 1 0 . 4 4 0 0 \%}$ | $14.8129 \%$ | $28.4540 \%$ |
| $40-49$ | 130 | $8.5860 \%$ | $10.0386 \%$ | $-7.5686 \%$ | $42.2881 \%$ | $72.7613 \%$ |
| $50-59$ | 131 | $11.0275 \%$ | $11.1724 \%$ | $\mathbf{- 1 3 . 4 0 0 5 \%}$ | $40.5689 \%$ | $79.5175 \%$ |
| $60-69$ | 89 | $9.0027 \%$ | $9.7862 \%$ | $-12.9007 \%$ | $12.2324 \%$ | $36.5801 \%$ |
| $70-79$ | 63 | $8.1425 \%$ | $9.3612 \%$ | $-2.6062 \%$ | $25.5030 \%$ | $56.5329 \%$ |
| $80+$ | 26 | $3.8851 \%$ | $6.1552 \%$ | $5.6489 \%$ | $48.6535 \%$ | $96.6651 \%$ |
| Total | $\mathbf{5 5 8}$ | $\mathbf{7 . 8 4 0 4 \%}$ | $\mathbf{8 . 7 2 1 6 \%}$ | $\mathbf{- 9 . 9 1 7 7 \%}$ | $\mathbf{2 6 . 7 7 7 2 \%}$ | $\mathbf{5 4 . 5 3 6 0 \%}$ |

Table 13.
Mean percent difference of observed and predicted stand level attributes for the non commercial group.

| \% Difference <br> Non Commercial |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Age Class | Number of Observations | Arithmetic Mean Diameter | Quadratic Mean Diameter | Trees per acre | Basal <br> Area | Cubic Foot Volume |
| 15-19 | 5 | -10.3246\% | -12.7251\% | -11.0516\% | -37.2975\% | -44.1170\% |
| 20-29 | 35 | -5.3868\% | -7.8069\% | -15.5275\% | -31.6556\% | -34.5395\% |
| 30-39 | 72 | 7.8449\% | 7.7293\% | -7.6568\% | 26.8564\% | 48.7891\% |
| 40-49 | 118 | 2.8443\% | 2.8478\% | -17.9663\% | -2.8657\% | 8.0624\% |
| 50-59 | 124 | 6.2636\% | 6.7792\% | 4.8654\% | 30.6106\% | 50.4248\% |
| 60-69 | 89 | 0.4874\% | -0.4946\% | -9.7100\% | -4.2623\% | 3.2675\% |
| 70-79 | 64 | 1.4256\% | -0.2050\% | -0.9093\% | -3.5370\% | 0.1335\% |
| 80+ | 26 | -1.7505\% | -2.8558\% | -0.2507\% | -3.1791\% | 1.1995\% |
| Total | 533 | 2.8632\% | 2.3732\% | -6.7460\% | 6.3948\% | 18.0449\% |

While the predictions for white oak, hickory, other commercial, and non commercial appear to be less reliable, red oak and sweetgum predictions were very close to the observed data.

## CHAPTER IV

## DISCUSSION

The behavior of each equation was tested by plotting the equation over the observed data. The plots display how well the model behaved within the range of the data as well as outside the range of data. These plots also provide valuable information about stand development and species interactions.

## Site Index

The red oak dominant and codominant height equation (Equation 10) which was used to construct the site index equation is shown in Figure 2. A nonlinear regression equation was chosen because of the strong curvilinear relationship between height of the dominant and codominant red oaks and age. A summary of five site indices are given representing the complete range of site qualities. Height growth is known not to be a linear function that continues to increase at the same rate over age. There is a biological limit to tree height regardless of age. The site index equation (Equation 10) effectively models that nonlinear relationship between height and age. Figure 2 also demonstrates the wide range of site qualities and ages used to construct the models.


Figure 2.
Predicted average height of the dominant and codominant red oaks.

## Stand Level Models

## Trees per Acre

Figure 3 represents the "bare ground" TPA model. The decrease in TPA as age increases indicates a high level of competition. There are fewer TPA on higher quality sites. This indicates that on higher quality sites, there is a higher mortality rate over the life of the stand. The "existing inventory" surviving TPA model is shown in Figure 4. A TPA ratio of greater than one indicates a gain in TPA between periods due to ingrowth. A TPA ratio of less than one indicates a loss of TPA between periods. The TPA ratio plot was constructed with an existing age of 48 years and a projected age of 58 years. This graph reveals the same decreasing TPA over time as the "bare ground" model
(Figure 3). Future TPA is predicted to be less than current TPA for all TPA except at extremely low TPA. Site index was not a significant variable in the "existing inventory" TPA model because it was already accounted for through existing TPA in the dependent variable.


Figure 3.
Predicted trees per acre based on age and site index.


Figure 4.
Predicted ratio of future trees per acre to existing trees per acre based on existing age (48 years), future age (58 years), and existing trees per acre.

## Arithmetic Mean Diameter and Quadratic Mean Diameter

Mean diameter equations for both AD and QD equations for each of the growth and yield projection scenarios, "bare ground", "stand density", and "existing inventory", are plotted in Figures 5 through 10, respectively. The higher the site index, the higher the mean diameter predicted for the "bare ground" and "stand density" models (Figures 5 through 8). Figures 5 and 6 are for the "bare ground" model. Figures 7 and 8 are the "stand density" model, which has a much higher $R^{2}$ value and visually fit the data better. The prediction of both scenario models is very similar until age 80 or greater, which accounts for the majority of data. This comparison indicates that TPA (a variable in the
"stand density" and "existing inventory" models) has a major effect on mean diameter at the extremes of the data.


Figure 5.
Predicted arithmetic mean stand diameter based on age and site index.


Figure 6.
Predicted quadratic mean diameter based on age and site index.


Figure 7.
Predicted arithmetic mean diameter based on age, site index, and trees per acre.


Figure 8.
Predicted quadratic mean diameter based on age, site index and trees per acre.

The "existing inventory" model predicts percent mean diameter percent growth per year. This model was designed to project mean diameter to a future age based on the current mean diameter. Figures 9 and 10 show that mean diameter percent growth decreases as age increases and becomes negative at approximately age 70 to 75 . Site index was not a significant variable in the "existing inventory" mean diameter percent growth per year model, however it was accounted for. Existing mean diameter is an input into the dependent variable and has already been demonstrated to be correlated with site index (Figures 5 through 8). Existing TPA is an independent variable in the model and has
already been demonstrated to be correlated with site index (Figure 3). Therefore, the inputs in the model already have site index built into them.

Mean diameter percent growth per year was very similar regardless of the inputs of existing mean diameter and TPA. There are several explanations for this observation. The initial mean diameter input into the model should increase with higher quality sites. The same percent growth per year applied to different existing mean diameters will result in different amounts of growth. A high quality site will be putting on more diameter growth than the lower quality site and still have a very similar annual percent growth. Because larger mean diameters are being input in the model at older ages, the percent growth per year should be less. The average projection age used to construct the model was 8 years. It is important to understand that the "existing inventory" model was not designed to compound percent growth per year across the entire life of the stand. Future mean diameter is projected by a simple percentage based on the average projection.


Figure 9.
Predicted arithmetic mean stand diameter growth based on initial age, future age, initial arithmetic mean diameter, initial trees per acre, and site index.


Figure 10.
Predicted quadratic mean stand diameter growth based on initial age, future age, initial quadratic mean diameter, initial trees per acre, and site index.

## Height

The individual tree height model (Figure 11) displays a good fit based on many observations. Height steeply increases as dbh increases until the tree reaches approximately 100 to 120 feet tall. At this point, depending on the quality of the site, height growth begins to taper regardless of increased dbh.


Figure 11.
Predicted individual tree total height prediction based on dbh, quadratic mean diameter, and height of the dominant and codominant red oaks.

## Basal Area and Volume

Stand level basal area was predicted based on the QD and TPA equations. Figure 12 shows that basal area increases with age until 50 to 65 years depending on site quality. Stands with higher site indices have a higher basal area that peaks later in the age of the stand. Basal area change over time can be explained by the two components of which it
is comprised, TPA and QD. TPA was observed to decrease as age increased. TPA is decreasing at a slower rate at the same age that basal area begins to decrease, but the trees that are dying later in the life of the stand are larger. At the same time that TPA is decreasing, QD is increasing. As trees die in the stand, the growth potential of the site is being partitioned to fewer trees. At the time when the most trees are dying in the understory, the overall basal area of the stand is increasing the greatest. Therefore, QD has a greater effect on basal area than TPA. Basal area peaks around the same age that QD percent growth per year reaches or drops below zero. At the time the trees lack the physiological efficiency to enable them to capitalize on available growth potential. When an overstory tree dies late in the life of the stand, it is most likely a large tree. Such mortality has a notable impact on stand basal area. The overstory is no longer able to capture the available growth potential, and slower growing shade tolerant species colonize the site. This causes a short-term decrease in basal area.

Trends for cubic foot volume (Figure 13) closely follow those for basal area. Height, TPA, and QD were used to predict cubic foot volume. Basal area plays a much larger role in cubic foot volume than height. Therefore, the cubic foot volume curves (Figure 13) are very similar to the basal area curves (Figure 12).

It is not expected that basal area or cubic foot volume will continue to decrease through the life of the forest. The stands should reach a steady state basal area and volume that continues in the stand until a disturbance occurs. Our model was not able to capture this steady state because of a lack of data at very old ages (100 + years). The red oak/sweetgum species association and red oak/white oak/mixed species association are
phases that occur within natural forest succession (Meadows and Stanturf 1997).
Because the stands in this study have been relatively undisturbed since establishment, a shift to shade tolerant, slower growing species such as red maple, sugarberry, beech, and elms can be expected (Putnam et al. 1960). These species are not as quick as red oaks to capitalize on the available growth due to mortality, but eventually, basal area and volume should begin to level off.


Figure 12.
Predicted basal area based on predicted quadratic mean diameter and predicted trees per acre.


Figure 13.
Predicted cubic foot volume based on predicted quadratic mean diameter, predicted trees per acre, and predicted height.

The surrogate cubic foot volume is a measurement that accounts for all the stand level attributes predicted. It is a good indicator of the interaction between all models developed. For this model, cubic foot volume should not be used as merchantable volume. Merchantable volume will be an output of subsequent work on a diameter distribution model.

## Species Models

## Trees per Acre

Figure 14 shows that the highest percentage of sweetgum stems occurs early in the life of the stand followed by a sharp decline as age increases. It appears that the stand
level TPA model (Figure 3) is highly influenced by sweetgum TPA. Red oak TPA also decreases with time but at a much slower rate. The white oak, hickory, and other commercial species groups TPA remain at low levels. The only species group to gain TPA throughout the life of the stand is the non-commercial group. Non-commercial species such as American hornbeam and eastern hophornbeam are shade tolerant species which colonize the midstory and understory in closed canopy stands.

The species TPA model (Figure 14) predicts a similar stand development pattern to that reported by Johnson and Krinard (1988). They found that red oak stocking relative to total stocking increased with age. They also observed that though both sweetgum and red oak TPA decreased as age increased, sweetgum lost a higher percentage than oak.


Figure 14.
Predicted trees per acre for species groups based on age, stand quadratic mean diameter, and stand trees per acre.

## Arithmetic Mean Diameter and Quadratic Mean Diameter

Mean diameter begins to seperate by species groups around age 15. Figures 15 and 16 clearly demonstrate that red oaks have a much greater diameter growth rate than the other species. Red oak diameter increases almost in a linear trend as age increases. The other species groups also increase in diameter as age increases but at much smaller rates. Clatterbuck and Hodges (1988) found this same trend in diameter growth when examining the development of even-aged cherrybark oak/sweetgum stands. Their study primarily related stand development to height growth patterns of different cherrybark oak/sweetgum mixtures. The diameter growth patterns they observed are very similar to Figures 15 and 16. They predicted that cherrybark oak and sweetgum have similar
diameters at early ages and that cherrybark oak diameter greatly increases compared to sweetgum beginning around age 20. The same trend is predicted in Figures 15 and 16, but at a slightly earlier age. Clatterbuck and Hodges related this diameter growth to height stratification between the species. They found that from 15 to 30 years of age, cherrybark oak began to outgrow sweetgum in height and begin to achieve an equal position in the canopy. From age 30 and greater, cherrybark oak height stratified above sweetgum, gained dominance in the canopy, and spread its crown. This achievement of dominance in the canopy can be directly related to subsequent diameter growth that is observed in Figures 15 and 16. Lockhart et al. (2006) observed this same trend in a planted mixture of cherrybark oak and sweetgum. The purpose of their study was to determine stand development patterns of cherrybark oak and sweetgum across a range of spacings. While the purpose of the study was primarily for hardwood afforestation, it also supports known natural stand development concepts. They found that at age 17, the dbh of the two species was very similar but by age 20, cherrybark oak had surpassed sweetgum in dbh (Lockhart et al. 2006).


Figure 15.
Predicted arithmetic mean diameter for species groups based on age, stand arithmetic mean diameter, and site index.


Figure 16.
Predicted quadratic mean diameter for species groups based on age, stand quadratic mean diameter, and site index.

## Species Basal Area and Volume

Sweetgum has the highest basal area until approximately age 30 (Figure 17). Even though sweetgum has a smaller QD at this age, the high number of sweetgum TPA results in sweetgum still maintaining the highest basal area in a stand until approximately age 30. At age 30, red oaks begin to dominate the stand. This trend concurs with prior research in red oak/sweetgum stand development (Clatterbuck and Hodges 1988, Johnson and Krinard 1988). Even before red oaks establish dominance in the canopy, they are partitioning more of their growth towards diameter. Once they are receiving full sunlight, their crowns spread and photosynthetic activity increases. This increases leaf area and consequently more growth in height and diameter (Clatterbuck 1985). The same trends are observed in cubic foot volume in Figure 18.


Figure 17.
Predicted basal area for species groups based on predicted species quadratic mean diameter and predicted species trees per acre.


Figure 18.
Predicted volume for species groups based on predicted species quadratic mean diameter, predicted species trees per acre, and predicted height equation.

## Biological Explanations for Stand Development

There a variety of reasons for the emergence or red oaks as the dominant species on minor streambottoms over time. Lockhart et al. (2005) discussed four possible factors. Sweetgum has an excurrent crown form that allows red oaks to receive enough sunlight to successfully compete through the early stand development stages. Red oaks have a semi-excurrent crown early in stand development that allows them to compete with sweetgum for the overstory. However, once red oaks surpass sweetgum in height, their crown becomes more decurrent. Red oak twigs are tougher and thicker than sweetgum. During storms, red oak twigs scrape and break the twigs of sweetgum. The high initial TPA of sweetgum does not allow the crowns to differentiate and causes them to stagnate. Red oaks grow basipetally, their buds break from the top of the crown and then downward. Sweetgums grow acropetally, which means that their buds break from the base of the crown and then upward. This early growth of red oaks in the top of the crown helps them to compete (Lockhart et al. 2005).

In summary, the growth and yield model developed supports other stand development research conducted on similar site types and mimics known biological and ecological processes.

## Computer Program

A computer program based on the equations was written in Visual Basic within Microsoft Excel 2002. This program allows the user to select one of the three projection
model scenarios and outputs the predicted stand level and species attributes. Charts predicting the future development of the stand are also displayed.

## Further Research

This growth and yield model will be a very useful hardwood management tool for predicting stand level attributes; however, it does not address individual tree size, product, or quality. Further development of this model will predict diameter distributions based on the stand level predictions discussed here and the Weibull probability function. A tree list with a predicted dbh will be generated and used to produce a stand and stock table by product. A log grade distribution model will also be developed to predict the amount of volume by grade. The stand level equations provide the basic framework upon which the final model will operate.

After the comprehensive hardwood growth and yield model is constructed; the next major step will be to incorporate intermediate stand management such as commercial thinning and timber stand improvement.

## CHAPTER V

## CONCLUSIONS

The hardwood growth and yield model quantifies change in stand level attributes and species composition based on age, site quality, and density. The prediction equation system supports known biological concepts of red oak/sweetgum stand development. The model predicts species stand level attributes that are consistent with previous stand development research in red oak/sweetgum stands. All inputs are measurements that are routinely collected on standard timber cruises. The model receives simple inputs such as only age and site index or more detailed inputs such as inventory data. Total stand level attributes are predicted as well as stand level attributes for each species group.

The hardwood growth and yield model will be a valuable management tool for hardwood managers. Currently, hardwood growth and yield estimates are very general and rely heavily on the experience of the forester. This model will provide sound objective estimates of stand level attributes that can aid in silvicultural and financial decisions. However, it is important to understand the hardwood growth and yield model is a tool that should be used along with other sound silvicultural and financial principles. It is a decision support tool and not an estimator tool.

The stand level growth and yield model will provide the framework for a complete hardwood growth and yield simulator. This simulator will predict volume by
species, size class, product, and grade. It will be the first of its kind, a publicly available growth and yield model for southern hardwoods.

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