Funding Higher Education and Wage Uncertainty: Income Contingent Loan versus Mortgage Loan

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Abstract

In a world where graduate incomes are uncertain (observation of the UK graduate wages from 1993 to 2003) and the higher education is financed through governmental loan (UK Higher Education Reform 2004), we build a theoretical model to show which scheme between an income contingent loan and a mortgage loan is preferred for higher level of uncertainty. Assuming a single lifetime shock on graduate incomes, we compare the individual expected utilities under the two loan schemes, for both risk neutral and risk averse individuals. We extend the analysis for graduate people working in the public sector and private sector, to stress on the extreme difference on the level of uncertainty. To make the model more realistic, we allow for the effects of the uncertainty each year for all the individual working life, assuming that the graduate income grows following a geometric Brownian motion. In general, we find that an income contingent loan is preferred for low level of the starting wage and high uncertainty.

JEL Classification: D81, I22, H80.

Keywords: Education Choice; Risk Aversion; Uncertainty.

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1 Introduction

The investment in education is risky: an individual making schooling decisions is likely to be only imperfectly aware of her abilities, the probability of success, the job and the earnings that may be obtained after completing an education. The literature has mainly investigated the riskiness of the investment in education from the point of view of the effects on the returns.

One of the first analysis in this topic is the Weiss (1974) lifecycle model with completely imperfect capital markets. He studies the risk adjusted average rate of return to schooling, which is the subjective discount rate at which the individual would be indifferent between acquiring a certain level of education and having no education at all. Weiss finds that the risk adjusted average returns to education sharply decrease as the risk aversion increases. Weiss' model has been extended by Hause (1974) and Levahari and Weiss (1974). Particularly interesting is the work of Olson, White and Shefrin (1979). They follow the traditional literature focusing on the returns to education under uncertainty. However there is an innovation with respect to the past works because they deal for the first time with the graduate income uncertainty, and they take into account the way higher education is funded. They assume that consumption equals income in each period after schooling and educated individuals get a random stream of income that varies according to the level of education achieved. Olson, White and Shefrin allow borrowing to finance education, and in particular they consider a mortgage loan that is paid back only after the completion of schooling. They find that the estimated real returns of college are large, and the estimated risk adjustments for college are small but positive.

The recent literature considers the wage uncertainty without taking into account the education financing systems. As in Pistaferri and Padulla (2001) that extend the Olson, White and Shefrin's model to consider two types of risk: employment risk and wage uncertainty, within an imperfect credit market framework. Or Hartog and Serrano (2003) that analyze the effects of stochastic post schooling earnings on the optimal schooling length, and show a negative effect of risk on investment.

In our paper we combine the problem of the uncertainty on the graduate incomes with the optimal choice of the higher education funding system.

We use the same framework of Hartog and Serrano but incorporating the possibility that students can take out a loan to finance their education costs. Our interest for the combination wage uncertainty and student loans starts from two empirical facts:

- observation of the level of uncertainty in the real wages of graduate people in the UK from 1993 to 2003;
- reform of the higher education financing system in the UK, with the introduction of an income contingent loan scheme.

Our intuition is that in the fields where the graduate income is more uncertain, if the higher education is financed through a loan scheme, an income contingent loan is the preferred system. If the individuals expect a high variance in their graduate income, they feel more protected by a loan system that allow them to repay the educational debt only when they have the financial resources to do it. An income contingent loan is a guarantee against high uncertainty, and reduce the disincentive to not invest in higher education.
We try to support our intuition with a theoretical model, in which we assume stochastic graduate incomes and a higher education financing system based on government loans. In particular, we study an income contingent loan and a mortgage loan comparing the individual expected utilities before and after the repayment of the educational debt. The individual utilities are function of the graduate income, which is affected by a single lifetime shock and it remains constant for all the working life. We consider both risk neutral and risk averse individual, the government instead is always assumed risk neutral and there is no default. The model is tested undertaking some simulations, where the parameters are calibrated on the real data. The findings confirm our expectations, for increasing level of the income uncertainty an income contingent loan is preferred to a mortgage loan.

We extend our analysis comparing the two loan schemes for individuals working in the private sector and in the public sector. We consider the two extreme cases, no income uncertainty in the public sector and highest uncertainty in the private sector. Our findings show that an income contingent loan is more preferred in the private sector.

We finally try to make the model more realistic changing the assumption on the income. We assume that it grows following a geometric Brownian

motion. This choice allow us to consider the variation of the income in each year after the graduation, and for all the working life. In the first part of the model the randomness is only on the starting level of the wage that after the shock remains fixed. With the new assumption we do not know the starting level as well, and we set it according to the empirical data. However, each initial level of the income generates many paths that grow following a deterministic trend. In a single path the incomes change year by year according to the intensity of the volatility of the Brownian motion.
In this way we are closer to the reality where the individual wage can be affected by many shocks during the working life, e.g. change of job, firing, temporary unemployment, career improvement, wage bonus etc.
To deal with the Brownian motion we develop a numerical iterative solution to compare the two financing schemes. Our findings show that if the individuals start their working life with low wages an income contingent is more beneficial, and the extent of the preference is increasing in the level of the wage uncertainty. Instead for high initial wages a ML is more advantageous; the individuals in fact prefer smoothing their repayment in more years, instead an income contingent loan imply larger percentage of payment and in few years.

We now briefly describe the empirical evidence that justifies our theoretical analysis.

The current UK higher education financing system is based on an up-front fee fixed across universities and courses. Only students whose family income exceeds a given amount pay the fee in full, the others are exempted. The Higher Education Reform (approved in 2004 and that will be effective from 2006) increases the tuition, enlarges the number of students liable and universities can set their fees up to a maximum £3000 p.a. Fees will be covered by a system of subsidized loans. The innovation is the introduction of an income-contingent scheme to repay the loans. Graduates start to pay back only when their incomes are above $\pounds 15,000$ per year and at 9 per cent fixed repayment rate. There is a zero real interest rate and repayments are made through the tax system as a payroll deduction. A similar higher education system is effective in Australia since 1989, the difference is the presence of increasing thresholds of income and increasing repayment rates.

1.1 Empirical Observation

We use a dataset built merging the quarterly Labour Force Surveys since 1993 to 2003. From a total of $136,839$ individuals in the sample, we restrict our attention to those whose highest qualification is a degree. We end up with around $19,957$ graduate people, divided in seven group degrees, and observed in three periods of 3 years each: 1993-1995, 1996-1999, 2000-20003. See Table 1.

According to our theoretical intuition, the preference for an Income Contingent Loan with respect to a Mortgage Loan, depends on the uncertainty of the graduate wage, the starting wage and its growth rate. We expect that the higher is the uncertainty of the income the more an income contingent loan is preferred.

In Table 2 we report the average hourly wage (pounds per hour, in real January 2000 prices) and the respective standard deviation for the different degrees in all the sample. We consider all the graduate people, regardless their age. In general, the Health and Science sectors provide higher wages, while higher wage uncertainty comes from the Arts & Humanity degree and Other Degrees. Observing (Table 2) the same degree courses in the three-year groups, we notice that in Health the standard deviation of the income strongly increases from $1993-1995$ to $1996-1999$. This is probably due to the inclusion of the nurses among the official Health degrees since 1996. In Science the s.d of the wage is almost constant, instead in Engineering it sharply increases making this degree the one with the most uncertain income in the triennium $2000-2003$. In Social Science the s.d of the wage slowly decreases, making the outcome of this degree the less uncertain during the period $2000-2003$ (excluding the Other Degrees). In Arts & Humanity the s.d of the wage almost doubles from $1993-1995$ to $1996-1999$ but then it decreases in the next 3 years, although remaining at a significant high level. In the Combined Degree the s.d of the income rises considerably in the triennium $2000-2003$.
We have to mention that these results are valid if there is no selectivity, that could be generated by the higher education institution or by the individuals when choosing their occupation.

From the observation of Table 2, we can infer that an income contingent loan would be the repayment system preferred by the graduate people in Engineering and Combined Degree since the uncertainty of their income has been growing since 1993 to 2003. The graduate people in Science and Arts & Humanity would prefer an ICL because they face the highest uncertainty. Instead the graduates in Social Science and in Other Degrees would be more advantaged by a ML since the uncertainty of their income is decreasing.

We consider now the starting wages (Table 3). We select in the sample the people who complete their degree at 21-22 years old, and receiving a wage at the age of 22-23 years old. In Figure are reported the real hourly wage (in January 2000 prices) for the different degrees, for all the the sample period. The highest starting wage is in Health, followed by Social Science which has also the highest standard deviation. Looking at the wages in the three-year groups, we notice in Health an increasing starting wage but in the same time a strongly increasing standard deviation, the highest in the period $2000-2003$. In Science and Arts & Humanity the wage rises slowly and the standard deviation first rises and then goes down. In Engineering the trend is the opposite, slowly reduction of the starting wage and a considerable decline of the s.d, the lowest in the period $2000-2003$. In Social Science the wage is increasing and the standard deviation sharply decreasing, almost halved from 1993 to 2003. In the Combined Degree and Other Degree there is an increase in the starting wages and a strong increase in their standard deviation.

We have to stress the point that the wages showed in Table 3 in the different period do not refer to the same individuals. The LFS is a rotating cross section and we can follow the same individual only for 4 quarters. This implies that the trend of the income growth here showed cannot be applied to a model where wage grows along all the working life of each individual. However, we can get useful information for people that start repaying their educational debt in the different periods. According to our intuition that people with low initial wages prefer an ICL since they take longer to pay off the debt and exploit an implicit subsidy granted by this scheme. In the period $1993-1995$ those graduated in Arts & Humanity and Other Degrees would certainly prefer an ICL, since they have the lowest starting wages. Social Science graduate have a medium income but highest standard deviation, so in this case they could prefer an ICL. For the Engineering graduated a ML is the best repayment system, since they have the highest starting wage. The situation changes if we consider the period $2000-2003$, now the graduate in Social Science have the highest starting wage and low standard deviation, therefore they could find more advantageous a ML. In Health the graduates get the highest income but they have also the highest standard deviation, so if the effect of the high wage prevails on the uncertainty they should prefer a ML. In Arts & Humanity and Other Degrees an ICL should be the preferred system.

2 The Individual Decision Problem

Consider the individual at the end of compulsory school deciding whether investing in more education or starting to work. This choice involves two levels of potential income and is represented by a binary variable

$$d = \begin{cases} 0 & \text{if she does not go to college} \\ 1 & \text{if she goes to college} \end{cases}$$

If the individual does not go to college she receives at the beginning of period zero and for all her working life a deterministic income $X < 1$. This is a strong assumption, since the non-graduate income could be also random, but for the purposes of our model the uncertainty affects only the graduate income.

Education is costly and people going to college have zero income during that period. We assume the existence of a simple capital market where individuals can borrow only to finance their fees and living expenses. Upon graduation, as in Hartog and Serrano (2003), income is uncertain because subject to a random shock. For simplicity, the shock has a single lifetime realization, after which the income remains constant at the new level reached. Let y be the shock with $E(y) = 1$ and $Var(y) = \sigma^2$.

In this model individuals cannot insure the wage risk and seek to maximize the expected lifetime utilities. Utility is defined over the individuals' income stream. We assume, as in Olson, White and Shefrin (1979), that consumption, c, is always equal to income for people not going to school and in each period after school, since individuals cannot borrow and lend.

$$d = 0 \implies c = X \quad \text{for ever}$$

$$d = 1 \implies c = \begin{cases} y & \text{for ever after college} \\ 0 & \text{during college} \end{cases}$$

As stated earlier, students take out loans to avoid negative consumption while at school. In general, if persons do not invest in education they have the following expected utility:

$$V(0) = \int_0^{\infty} e^{-\rho t} u(X)\, dt. \tag{1}$$

People attending college start to repay their loan after s years of school and for T years. Assuming a general repayment schemes, we define R as the general per-period payment. The expected utility is:

$$V(1) = E\left\{ \int_s^{T+s} e^{-\rho t} u(y - R)\, dt + \int_{T+s}^{\infty} e^{-\rho t} u(y)\, dt \right\} \tag{2}$$

3 Comparing Mortgage and Income Contingent Loans under Risk-Neutrality

The Government finances the investment in higher education issuing debt that is paid back only with the graduates' repayments. The students take out a loan of fixed size that cover all the costs of the university, that are equal for all the courses and subjects. The loan is repaid according to two financing systems: Mortgage Loan and Income Contingent Loan. The scheme is fully funded and the participation is obligatory, there is no opting out choice. We assume that the Government is risk neutral, and therefore it does not have any preference over the two funding systems. As noted by Olson, White and Shefrin (1979), under a mortgage loan scheme is possible to escape through bankruptcy. However, in our model we assume that all the debt is paid off and there is no default[1].

[1] The case of the students' default is analyzed in another future work.

3.1 Mortgage Loan

The individuals take out a loan equal to C and repay through T equal, fixed and periodical instalments φ, at a certain real interest rate, r. For simplicity, (and UK relevance) $r = 0$. In our model the formula[2] for the instalment that we use is: $\varphi = C/T$. The repayment period is therefore just

$$T = C/\varphi. \tag{3}$$

3.2 Income Contingent Loan

The individuals borrow an amount equal to the total cost of education, C, and start to pay back their loan after graduation according to level of their income. Under this scheme if the wage is below a minimum threshold no payment is due. If the wage increases, a greater portion of the debt is repaid and all the loan is paid off in less time. Therefore, the main difference with a mortgage loan is that the repayment period, $\tilde{T}$, is **random**. In our model, for simplicity, we assume no initial threshold and the total cost of schooling is given by a fixed percentage (γ) of the random graduate income.

$$C = \gamma \int_{s}^{\tilde{T}+s} y \, dt \tag{4}$$

We solve the integral and work out the repayment period:

$$\tilde{T} = \frac{C}{\gamma y} \quad . \tag{5}$$

Substituting this parameter in the equation (2) we obtain the expected utility under an income contingent loan.

3.3 Risk Neutrality and Expected Costs

We assume first that individuals are risk neutral, i.e. $u(y) = y$. So we can consider only the costs to compare the two repayment schemes. We work

[2] We assume the instalments are worked out applying a French Amortization method, therefore they include capital and interests. This method is useful because we can obtain single payments that correspond to a fixed percentage of the total cost. If we consider a total loan C, a real interest r and a repayment period of T years, the formula for a single instalment is: $\varphi = \frac{C\,r}{[1-(1+r)^{-T}]}$. Taking the limit and applying L'Hopital's rule: $\lim_{r\to 0} \varphi = \lim_{r\to 0} \frac{C}{T(1+r)^{-T-1}} = \frac{C}{T}$

out the present value of the costs, substituting for each scheme the respective repayment period, T and $\tilde{T}$, and discounting to $t = 0$.

Proposition 1. *The utility from ICL is greater than the utility from ML*

$$V(1)_{ICL} > V(1)_{ML}.$$

Proof. See Appendix A.

The first theoretical result found in our analysis is that under an ICL the expected utility is lower than under a ML. This result could sound strange because individuals are risk neutral, but it depends on the expected costs of education that are greater with a ML than an ICL. If we assume a general repayment method R, the present value of the education cost is:

$$PVC = \int_0^T R\, e^{-\rho t} dt = \frac{R}{\rho}\, [1 - e^{-\rho T}]$$

Taking the derivatives of PVC with respect to T, we can easily observe that this function is concave [3]. Consider now a loan with a certain repayment period of 10 years, and another loan with two even probability repayment periods of 5 or 15 years. The concavity property implies that the expected present value of the cost of an uncertain repayment is lower than the present value of the expected cost of a certain repayment:

$$E(PVC) = \frac{1}{2}\, PVC(5) + \frac{1}{2}\, PVC(15) < PVC(10).$$

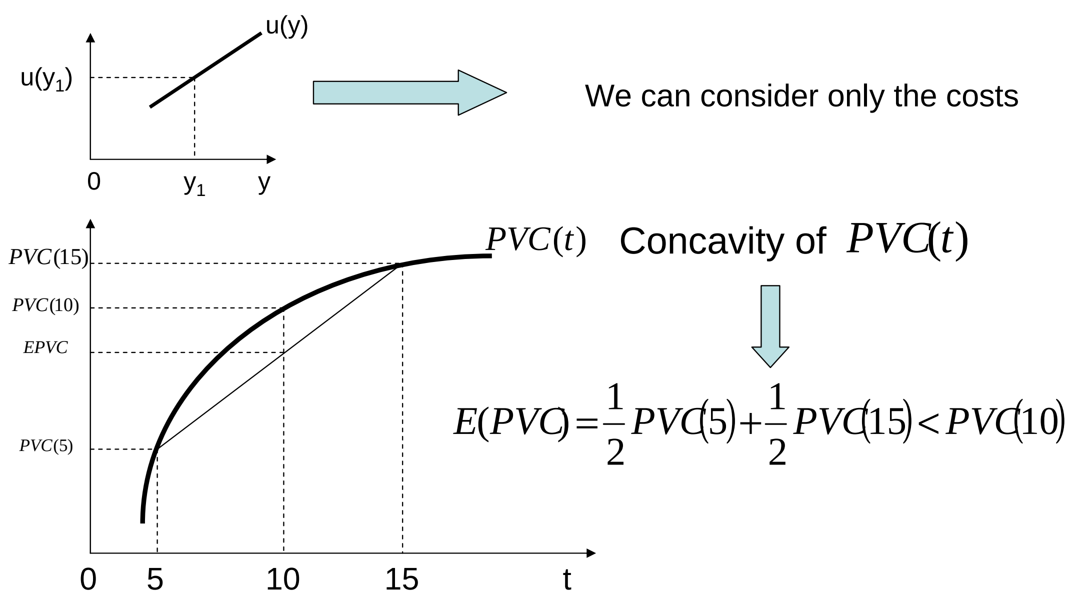
See Figure 1.

4 Comparing Mortgage and Income Contingent Loans under Risk Aversion

In this Section we consider individuals who are risk averse and we work out their expected utility (represented by the equation (2)), under a mortgage loan and an income contingent loan system. We consider the assumptions

[3] $\frac{\vartheta^2 PVC}{\vartheta T^2} = -R\rho\, e^{-\rho T} < 0$ for all T.

Figure 1: Expected Cost and Risk Neutrality



$$E(PVC)=\frac{1}{2}PVC(5)+\frac{1}{2}PVC(15)<PVC(10)$$

stated in Section 2 and we develop the analysis using two types of utility function: constant absolute risk aversion (CARA) and constant relative risk aversion (CRRA). We omit the majority of calculations that are showed in more detail in the Appendices B and C.

4.1 Expected Utility with a Mortgage Loan

Under a mortgage loan, the expected utility is obtained substituting $R = \varphi$ in equation (2):

$$V_{ML} = \int_{s}^{T+s} e^{-\rho t} E\left[u\left(y-\varphi\right)\right] dt + \int_{T+s}^{\infty} e^{-\rho t} E\left[u\left(y\right)\right] dt. \tag{6}$$

To get a closer-form solution for V_{ML}, we use a second order Taylor expansion around the mean $E[y-\varphi] = 1-\varphi$ [4] for the utility during the repayment period, and around $E[y] = 1$ for the utility after the repayment period:

$$E[u(y-\varphi)] \simeq u\left(1-\varphi\right) + \frac{1}{2} u^{''}\left(1-\varphi\right)\sigma^2. \tag{7}$$

$$E[u(y)] \simeq u(1) + \frac{1}{2} u^{''}(1)\sigma^2. \tag{8}$$

We develop our analysis using both a CARA and CRRA utility function:

$$u(y) = -\frac{1}{a} e^{-ay} \quad \text{CARA}$$

and

$$u(y) = \frac{y^b}{b} \quad CRRA.$$

where a is the risk aversion parameter and $b = 1-a$.

After simplifying [5], we get:

$$V_{ML_{CARA}} = \frac{e^{-\frac{\rho C + a\varphi + as + s\rho\varphi}{\varphi}}}{2a\rho} \left[-2 + a^2\sigma^2 + e^{a\varphi}\left(2 + a^2\sigma^2\right) - e^{\frac{\rho C}{\varphi} + a\varphi}\left(2 + a^2\sigma^2\right)\right]. \tag{9}$$

[4]See Pistaferri and Padula (2001) and Hartog and Serrano (2003).
[5]see Appendix B for the proof.

And if we use a CRRA utility function the expected utility is:

$$\begin{aligned} V_{ML_{CRRA}} = & \frac{e^{-\rho s}}{\rho} \{ \left(1 - e^{-\frac{\rho C}{\varphi}}\right) \left[\frac{(1-\varphi)^b}{b} + \frac{1}{2}(b-1)(1-\varphi)^{b-2}\sigma^2 \right] \\ & + e^{-\frac{\rho C}{\varphi}} \left[\frac{1}{b} + \frac{1}{2}(b-1)\sigma^2 \right] \}. \end{aligned} \tag{10}$$

4.2 Expected Utility with the Income Contingent Loan

Under an income contingent loan we do not know how long people take to repay their education debt, therefore in the general equation of the expected utility the random income appears twice. First in the integral's bounds as random repayment period, second as argument of the utility function.

$$V_{ICL} = E \left\{ \int_{s}^{\frac{C}{\gamma y}+s} e^{-\rho t} u \left[y \left(1-\gamma\right) \right] dt + \int_{\frac{C}{\gamma y}+s}^{\infty} e^{-\rho t} u\left(y\right) dt \right\} \tag{11}$$

Solving the integral we get the following equation:

$$V_{ICL} = \frac{e^{-\rho s}}{\rho} E \left\{ \left[1 - e^{-\frac{\rho C}{\gamma y}} \right] u \left[y\left(1-\gamma\right)\right] + \left[e^{-\frac{\rho C}{\gamma y}} \right] u\left(y\right) \right\}. \tag{12}$$

To simplify the calculations we define all the expression included in the expected value operator as $g(y)$. This trick allows us to apply a second order Taylor expansion of $E[g(y)]$ around the mean $E\left[y\right] = 1$. Then, the equation (12) becomes:

$$V_{ICL} = \frac{e^{-\rho s}}{\rho} \left[g(1) + g''(1) \frac{\sigma^2}{2} \right]. \tag{13}$$

The remaining procedure consists of calculating the value of $g(1)$ and $g''(1)$, in general and with CARA and CRRA utility functions in particular Finally, we substitute the expressions found in equation (13), and we obtain the following results. If we use a CARA utility function, we get after simplifying:

$$\begin{aligned} V_{ICL_{CARA}} = & \frac{e^{-\frac{a\gamma+(c+\gamma s)\rho}{\gamma}}}{2a\rho} \{ -2[1 - e^{a\gamma} + e^{a\gamma + \frac{\rho C}{\gamma}}] \\ & + \frac{1}{\gamma^2} [(-a^2\gamma^2[1 - e^{a\gamma}(\gamma-1)^2 + e^{a\gamma+\frac{\rho C}{\gamma}}(\gamma-1)^2] \\ & + 2\rho C[1 + e^{a\gamma}(\gamma-1)]a\gamma + \rho C(e^{a\gamma}-1)(\rho C - 2\gamma))\sigma^2] \}. \end{aligned} \tag{14}$$

And using a CRRA utility function the expected utility [6] is:

$$V_{ICL_{CRRA}} = \frac{e^{-\left(s+\frac{C}{\gamma}\right)\rho}}{2b\gamma^2\rho}\{e^{\frac{\rho C}{\gamma}}(1-\gamma)^b\gamma^2[2+(b-1)b\sigma^2] - [(1-\gamma)^b - 1] \cdot [2\gamma^2 + ((b-1)b\gamma^2 + 2(b-1)C\gamma\rho + C^2\rho^2)\sigma^2]\}. \quad (15)$$

5 Simulations

In this Section we use the equations of the expected utility derived previously and we compare them through simulations. We assign numerical values to each parameter, and see how the variations affect the difference between the expected utility under a mortgage loan and the expected utility under an income contingent loan. The analysis is developed using both a CRRA utility function and a CARA utility function. In particular, for a mortgage loan we use the equations (10) and (9), and for an income contingent loan the equations (15) and (14).

We consider the following vectors of parameters:

$\sigma = [0 \quad 0.2 \quad 0.4 \quad 0.6 \quad 0.8 \quad 1.2]$. In our equations σ is the standard deviation of the wage. Therefore, the higher is σ, the higher is the uncertainty around the level of the income. In the UK Labour Force Survey of 2002, the average income of graduate people is around £25000, with a standard deviation $\sigma = 0.64$. Taking into account this value, we assign six values to σ: from the lowest case with no uncertainty to a standard deviation of 120% of the income.

$\varphi = [500 \quad 600 \quad 800 \quad 1000 \quad 1200 \quad 2400]$, we set six possible installments under a ML[7].

$C = [6000 \quad 8000 \quad 10000 \quad 12000]$, we consider six possible costs of the education, assuming a $s = 4$ years full-time degree. Therefore, we obtain the following repayment periods under a ML: $T = [24 \quad 20 \quad 15 \quad 10 \quad 5]$.

[6] The expected utility with an income contingent loan is equal to the expected utility with a mortgage loan if $\varphi = \gamma$ and the variance of the income is zero.

[7] The values in the program are calibrated assuming ten thousand as unit of measure of the expected income, which is equal to 1 in the model.

γ = [0.05 0.09 0.15 0.25 0.5], we assign 5 values to the rate of repayment under an ICL. The value of 9% is the one chosen in the UK Reform.

ρ = [0.02 0.08 0.15 0.25 0.5], we assign 5 increasing values to the subjective discount rate.

a = [0 0.25 0.5 0.75], we set 4 increasing values for the risk aversion parameter. In the model we used $b = 1 - a$ for the CRRA utility function. These values have been chosen following the literature.

The simulations are performed using MATLAB, and we compute the differences $V_{ICL} - V_{ML}$. We analyze all the possible combinations of the above parameters and we obtain a database of 5760 observations.

We consider the following cases.

- σ and a increasing: $C =$ £10000, $\varphi =$ £1000, $T_{ML} = 10$ years, $\rho = 8\%$, $\gamma = 9\%$, CRRA utility function. Given a and for σ increasing the difference $V_{ICL} - V_{ML}$ is positive and increasing. This means that the higher is the uncertainty on the income, the most an ICL is preferred. Given σ, for increasing risk aversion $V_{ICL} - V_{ML}$ is increasing. However, if we decrease φ in order to have a repayment period of $T_{ML} = 20$ years, a ML is preferred and it becomes more advantageous for increasing a. (Table 4). If we use a CARA utility function, we observe that $V_{ICL} - V_{ML}$ is positive and almost constant for increasing level of uncertainty. Given σ, for higher level of a $V_{ICL} - V_{ML}$ first decreases and then increases. If the repayment period augments there is no change in the trend and in the sign of $V_{ICL} - V_{ML}$. (Table 5)

- σ and ρ increasing: $C =$ £10000, $\varphi =$ £1000, $T_{ML} = 10$ years, $a = 0.5$, $\gamma = 9\%$, CRRA utility function. If we keep σ constant, $V_{ICL} - V_{ML}$ is positive and increasing up to $\rho = 8\%$; for higher values of ρ it is decreasing. If we increase the ML repayment period to $T_{ML} = 20$ years, the sign and the trend of $V_{ICL} - V_{ML}$ changes. ML is the preferred systems (except when $\sigma = 1.2$) and the difference $V_{ICL} - V_{ML}$ first decreases, and then when ρ is higher than 8% it starts to increases. We can notice that $V_{ICL} - V_{ML}$ converges to zero when ρ is very high. If we keep ρ constant, the difference $V_{ICL} - V_{ML}$ increases for σ increasing. In particular when $T_{ML} = 20$ years, the sign of $V_{ICL} - V_{ML}$ changes from negative to positive if σ is very high. Therefore, an ICL becomes the most advantageous system. (Table 6). Using a CARA utility function and keeping σ constant the trend of $V_{ICL} - V_{ML}$ is increasing. For low level of ρ a ML is preferred, for higher ρ $V_{ICL} - V_{ML}$ sharply increases and then it remains constant. A higher repayment period under a ML does not affect this behavior. (Table 7)

- σ and φ increasing : $C =$ £10000, $\rho = 8\%$, $a = 0.5$, $\gamma = 9\%$, CRRA utility function. Given σ, if φ increases the repayment period under ML decreases and an ICL becomes more beneficial. The trend of $V_{ICL} - V_{ML}$ is increasing, its sign instead is negative for low φ and positive for high φ. Keeping φ constant, if σ is increasing the trend of $V_{ICL} - V_{ML}$ is always increasing, instead its sign depends on the level of φ. (Table 8). When we use a CARA utility function we obtain similar results, however the sign of $V_{ICL} - V_{ML}$ is always positive (Table 9).

- σ and γ increasing: $C =$ £10000, $\varphi =$ £1000, $T_{ML} = 10$ years, $a = 0.5$, $\rho = 8\%$, CRRA utility function. Given σ, the trend of $V_{ICL} - V_{ML}$ is decreasing as γ increases, since an ICL becomes worse. We observe that for increasing σ the trend is increasing, and its sign is positive for low level of γ and always negative if γ is very high. (Table 8). The results are the same with a CARA utility function.(Table 9)

- φ and γ increasing: $C =$ £12000, $\sigma = 0.6$, $a = 0.5$, $\rho = 8\%$, CRRA utility function. Given $\gamma = 5\%$, $V_{ICL} - V_{ML}$ is increasing as φ increases. The sign of $V_{ICL} - V_{ML}$ is negative for low φ: a ML has longer repayment period and it is more beneficial. For high φ, the repayment under a ML is short and an ICL become more advantageous. Given φ, for higher values of γ the trend is decreasing because the utility of an ICL reduces. (Table 8.) With a CARA utility function, the previous results are confirmed, but $V_{ICL} - V_{ML}$ decreases more slowly for increasing φ. (Table 9)

The uncertainty on the level of the income affects strongly the trend of $V_{ICL} - V_{ML}$. In fact, if the standard deviation is increasing $V_{ICL} - V_{ML}$ is always increasing, and an ICL becomes more advantageous. The sign of $V_{ICL} - V_{ML}$ depends on the length of repayment period. The individuals prefer the system with the longest repayment period, since the real interest rate is zero they can exploit the advantages of an implicit subsidy, due to the decreasing present value of the payments. Therefore, when φ is low the repayment period under a ML is long and this system becomes more attractive. The same happens for low levels of the ICL repayment rates, this system gives higher expected utility then it is preferred. The effect of the risk aversion is affected by the presence of the uncertainty, therefore increasing a does not change the trend and the sign of $V_{ICL} - V_{ML}$, it just augments the size of $V_{ICL} - V_{ML}$ in absolute value. So if a system is preferred to the other, the higher risk aversion strengthens this preference. The subjective discount rate instead affects the trend of $V_{ICL} - V_{ML}$, which converges to zero for high values of ρ, therefore discounting a lot reduces the differences between the two systems.

6 Public Sector vs Private Sector

In this section we compare the two funding systems distinguishing between graduate people working in the public sector and in the private sector. The typical difference is the absence of uncertainty on the level of the income in the public sector, and the higher variance but also the higher level of the income in the private sector.

In our model, the graduate income in the public sector y_{pu} is constant and with $\sigma = 0$. Instead, the graduate income in the private sector y is random with $E(y) = 1$ and $\sigma > 0$. An important assumption is $y_{pu} < 1$, implying that on average people working in the public sector have a lower income than those working in the private one. We compute now the expected utilities of the individuals under a ML and an ICL for both sectors.

We consider first a mortgage loan. The repayment period is always $T = C/\varphi$ regardless the sector. Starting from the general equation of the expected utility, we observe that in the public sector all the variables are deterministic, and the equation (2) becomes[8]

$$V_{MLpu} = \frac{1}{\rho}\left[1 - e^{-\frac{\rho C}{\varphi}}\right] u(y_{pu} - \varphi) + \frac{1}{\rho} e^{-\frac{\rho C}{\varphi}} u(y_{pu)} \tag{16}$$

[8]For simplicity we consider the repayments from period zero to T.

Using a CRRA utility function, we obtain

$$V_{MLpu} = \frac{e^{-\frac{\rho C}{\varphi}}}{b\rho}\left[y_{pu}^b + \left(e^{\frac{\rho C}{\varphi}} - 1\right)(y_{pu} - \varphi)^b\right] \tag{17}$$

For the private sector the equation () remains unchanged, we just redefine it as

$$\begin{aligned} V_{MLpr} = & \frac{1}{\rho}\{\left(1 - e^{-\frac{\rho C}{\varphi}}\right)\left[\frac{(1-\varphi)^b}{b} + \frac{1}{2}(b-1)(1-\varphi)^{b-2}\sigma^2\right] \\ & + e^{-\frac{\rho C}{\varphi}}\left[\frac{1}{b} + \frac{1}{2}(b-1)\sigma^2\right]\}. \end{aligned} \tag{18}$$

We analyze now an income contingent loan. The repayment period in the public sector is $T = \frac{C}{\gamma y_{pu}}$, and the general equation of the expected utilities

$$V_{ICLpu} = \frac{1}{\rho}\left[1 - e^{-\frac{\rho C}{\gamma y_{pu}}}\right] u(y_{pu}(1-\gamma)) + \frac{1}{\rho}e^{-\frac{\rho C}{\varphi}} u(y_{pu}) \tag{19}$$

Using a CRRA utility function we get

$$V_{MLpu} = \frac{e^{-\frac{\rho C}{\gamma y_{pu}}}}{b\rho}\left[y_{pu}^b + \left(e^{\frac{\rho C}{\gamma y_{pu}}} - 1\right)(y_{pu}(1-\gamma))^b\right]. \tag{20}$$

In the private sector we have the same equation (11) just redefined

$$\begin{aligned} V_{ICLpr} = & \frac{e^{-\frac{\rho C}{\gamma}}}{2b\gamma^2\rho}\{e^{\frac{\rho C}{\gamma}}(1-\gamma)^b\gamma^2[2 + (b-1)b\sigma^2] - [(1-\gamma)^b - 1] \\ & \cdot [2\gamma^2 + ((b-1)b\gamma^2 + 2(b-1)C\gamma\rho + C^2\rho^2)\sigma^2]\}. \end{aligned} \tag{21}$$

6.1 Simulations

We perform numerical simulations using the equations derived above, and we calibrate each parameter involved with same values used in Section(4). We have just to include a new vector for the income in the public sector. We consider $y_{pu} = [0.2 \quad 0.5 \quad 0.9 \quad 1.2]$ where each element is expressed as percentage of the income in the private sector, assumed on average equal to one. We compute first the difference between the expected utilities for both systems in the public sector, for increasing levels of y_{pu} and risk aversion. We compare these results with the difference between the expected utilities in the private sector, for increasing level of uncertainty and risk aversion. We keep all the other parameters constant and equal for both the public and the private sector.

Observing the top graph in Figure 2, in the public sector a mortgage loan is always preferred when the income is lower than the income in the private sector. The difference $V_{MLpu} - V_{ICLpu}$ is positive but decreasing for higher y_{pu}. Higher levels of risk aversion strengthen the preference for a ML when y_{pu} is low. Conversely, looking at the bottom graph in Figure 2, in the private sector an income contingent loan is always preferred for increasing level of uncertainty. The difference $V_{MLpr} - V_{ICLpr}$ is negative and decreasing for higher σ and higher risk aversion.

The same conclusions are confirmed when we change the repayment periods under a ML. Observing Figure 3, we consider 3 decreasing repayments periods that makes a ML less advantageous. However, in the public sector a ML is always preferred for low level of income, also when the ML repayment takes just five years. Instead in the private sector, an income contingent loan is greatly preferred when the ML repayment period is short.

Finally, we compute the difference of expected utilities under the same funding system in the two sectors: $V_{MLpu} - V_{MLpr}$ and $V_{ICLpu} - V_{ICLpr}$. We fix two levels of uncertainty and the income in the public sector is increasing. Looking at Figure 4, the differences are negative for low levels of public income, implying that in the private sector the utility is higher. For the same level of low income, we observe that a mortgage loan is preferred, and the gap with respect to an ICL remains unchanged when we increase the uncertainty in the private sector. The gap shrinks when the public income increases, and an ICL becomes more advantageous for level of y_{pub} bigger than the average private income.

In conclusion, an ICL is preferred in the private sector where the income is random, because this funding system offers an implicit insurance against uncertainty. In the public sector the individuals cannot exploit all the advantages of an ICL, since their income is not affected by uncertainty and on average it is lower. Only when the public income is higher than the private income it is convenient to switch from a ML to an ICL.

7 Increasing Income

In the second part of our work we change the assumptions on the income. After graduating the individuals receive a wage that is not affected by a single lifetime shock but it increases following a geometric Brownian motion. We first consider the case of a constant growth rate of the income during the working life, then we add a stochastic component and we compute the individual expected utilities of the individuals under the two repayment schemes.

7.1 Constant Growth Rate

We assume that $y(t)$ is the value of £1 of graduate income after time t increasing at a constant rate λ for all the individual working life, ending in period T_{max} . Then $y(t)$ satisfies the ordinary differential equation (ODE)

$$dy(t)/dt = \lambda y(t). \tag{22}$$

The solution of the ODE gives the level of income $y(t) = e^{\lambda t} y_0$, where y_0 is the initial wage after graduating. We consider a logarithmic utility function $\log u = \log y_0 + \lambda t$ and we work out the expected utilities under the two higher education funding systems.

Under a mortgage loan the repayment period is given by equation(3) $T = C/\varphi$. We assume for simplicity that the initial wage y_0 is higher than the instalment φ, in the next section we consider a more general case. Using equation (6) we substitute T and $\log(u)$. Noticing that all the components are deterministic, we obtain the following expression for the expected utility

$$\begin{aligned} EU_{ML} &= \int_s^{(C/\varphi)+s} e^{-\rho t} \left(\log(y_0 - \varphi) + \lambda t\right) dt + \int_{(C/\varphi)+s}^{T_{\max}} e^{-\rho t} \left(\log y_0 + \lambda t\right) dt \\ &= \frac{1}{\rho^2} (e^{-(s+T_{\max})\rho} [e^{\rho T_{\max}} \lambda (1 + s\rho) - e^{\rho s} \lambda (1 + T_{\max}\rho) + \\ &\rho(-e^{\rho s} \log y_0 + e^{\rho(T_{\max} - \frac{C}{\varphi})} (\log y_0 - \log(y_0 - \varphi)) + e^{\rho T_{\max}} \log(y_0 - \varphi))]). \end{aligned} \tag{23}$$

The cost of education with an income contingent loan is given by (4), and under the new assumptions on income it is

$$C = \gamma \int_s^{\widetilde{T}+s} e^{\lambda t} y_0 \, dt. \tag{24}$$

Solving equation(24) for $\widetilde{T}$, we get the repayment period

$$\widetilde{T} = \frac{\log[e^{\lambda s} + \frac{\lambda C}{\gamma y_0}]}{\lambda} - s. \tag{25}$$

The expected utility under an ICL is given by

$$EU_{ICL} = E\left\{\int_s^{\widetilde{T}} e^{-\rho t} u\left[y\left(1-\gamma\right)\right] dt + \int_{\widetilde{T}}^{T_{\max}} e^{-\rho t} u\left(y\right) dt\right\}. \tag{26}$$

Substituting the expression for $\widetilde{T}$ and the log utility we get

$$\begin{aligned} EU_{ICL} = & \int_s^{\frac{\log[e^{\lambda s}+\frac{\lambda C}{\gamma y_0}]}{\lambda}} e^{-\rho t}\left(\log(y_0(1-\gamma)) + \lambda t\right) dt + \\ & \int_{\frac{\log[e^{\lambda s}+\frac{\lambda C}{\gamma y_0}]}{\lambda}}^{T_{\max}} e^{-\rho t}\left(\log y_0 + \lambda t\right) dt \\ = & \frac{1}{\rho^2}\{e^{-(s+T_{\max})\rho t}(e^{\rho T_{\max}}\lambda(1+s\rho) - e^{\rho s}\lambda(1+T_{\max}\rho)) + \\ & \rho[-e^{\rho s}\log y_0 + e^{-\rho s}\log(-(1-\gamma)y_0) + \\ & (\log y_0 - \log(-(1-\gamma)y_0))(e^{\lambda s} + \frac{\lambda C}{\gamma y_0})^{-\frac{\rho}{\lambda}}]\}. \end{aligned} \tag{27}$$

The two expressions of the expected utilities found in equations (23) and (27) can be used for numerical simulations, in order to see which funding system is more profitable. However our main task in this work is to compare the two schemes under uncertainty. Therefore, we will see later that the constant wage growth is a particular case of the stochastic income growth.

8 Stochastic Income

We assume that the growth rate of the income is affected by a white noise process, formally defined as the derivative of the standard Brownian motion, or standard Wiener process, $W(t)$

$$\epsilon = dW(t)/dt.$$

The derivative does not exist in the usual sense, since the Brownian motion is nowhere differentiable. However this process is used with the convention that its meaning is given by integral representation. If $\sigma(y,t)$ is the intensity of the noise at point y at time t, then it is common agreement that $\int_0^T \sigma(y(t),t)\epsilon(t)dt = \int_0^T \sigma(y(t),t)dW(t)$. In our case, adding a white noise $\sigma\epsilon(t)$ to the constant growth rate of the income in equation (22), we obtain the stochastic differential equation, SDE

$$dy(t)/dt = (\lambda + \sigma\epsilon)y(t). \tag{28}$$

where $\sigma \geq 0$, $\lambda > 0$, are some constants, and $y(0)$ is the deterministic component of the income.
This means that $y(t)$ satisfies

$$dy(t)/y(t) = \lambda dt + \sigma dW(t). \tag{29}$$

This expression can be interpreted heuristically as expressing the relative or percentage increment dy/y in y during an instant of time dt. Then, the expected instantaneous growth rate is λ, and the standard deviation of the instantaneous growth rate is σ.

To solve the SDE we introduce the Itô process $R(t)$ given by
$dR(t) = \lambda dt + \sigma dW(t)$. We rewrite the SDE as

$$dy(t) = y(t)dR(t) \tag{30}$$

this means that $y(t)$ is the stochastic exponential, $\varepsilon(R)$, of $R(t)$. The solution of equation(30) is

$$\begin{aligned} y(t) &= y(0)\varepsilon(R)(t) \\ &= y(0)\exp[R(t) - R(0) - \frac{1}{2}[R,R](t)\,]. \end{aligned} \tag{31}$$

$R(t)$ is easily found to be $R(t) = \lambda t + \sigma W(t)$, $R(0) = 0$, and its quadratic variation $[R,R](t)$ is the quadratic variation of an Itô process and equal to

$$[R,R](t) = \int_0^t \sigma^2 ds = \sigma^2 t.$$

Substituting these expressions in equation(31) we get

$$\begin{aligned} y(t) &= y(0)\exp[\lambda t + \sigma W(t) - \frac{1}{2}\,\sigma^2 t\,] \\ &= y(0)\exp[(\lambda - \frac{1}{2}\,\sigma^2)t + \sigma W(t)\,]. \end{aligned} \tag{32}$$

This process is a geometric brownian motion. Since $W(t)$ is Normally distributed with $E(W(t)) = 0$ and $Var(W(t)) = t$, the transformation function $f(y) = \ln y$ is also Normally distributed with

$$E(\ln y(t)) = \ln y_0 + (\lambda - \frac{1}{2}\,\sigma^2)t \tag{33}$$

and

$$Var(\ln y(t)) = \sigma^2 t. \tag{34}$$

Therefore, $y(t)$ has a lognormal distribution with

$$E(y(t)) = y_0\, e^{\lambda t}$$

$$Var(y(t)) = y_0^2\, e^{2\lambda t}(e^{\sigma^2 t} - 1).$$

9 Expected Utilities with Brownian motion

Our target as in the previous section is to find the individual expected utilities under both a mortgage loan system and an income contingent loan. However this task is not very easy when the income follows a geometric brownian motion. We can find an algebraic solution for the mortgage loan scheme, but not for the income contingent loan; therefore we develop a numerical method and we compare the expected utilities through some simulations.

9.1 Approach Chosen

Assuming a logarithmic utility function we have the following expression

$$\ln y = \ln(y_0) + (\lambda - \frac{1}{2}\,\sigma^2)t + \sigma W(t).$$

Under a mortgage loan, we can work out the expected utility applying the general equation(10) and using the equation(33) for the expected value of a log income. We obtain

$$
\begin{aligned}
V_{ML} = & \int_{s}^{(C/\varphi)+s} e^{-\rho t}\left(\log(y_0-\varphi)+(\lambda-\frac{1}{2}\sigma^2)t\right)dt+ \\
& \int_{(C/\varphi)+s}^{T\max} e^{-\rho t}\left(\log y_0+(\lambda-\frac{1}{2}\sigma^2)t\right)dt \\
= & \frac{1}{2\rho^2}(e^{-(s+T_{\max})\rho}[(e^{\rho T_{\max}}(1+s\rho)-e^{\rho s}(1+T_{\max}\rho))(2\lambda-\sigma^2)+ \\
& 2\rho(-e^{\rho s}\log y_0+e^{\rho(T_{\max}-\frac{C}{\varphi})}(\log y_0-\log(y_0-\varphi))+ \\
& e^{\rho T_{\max}}\log(y_0-\varphi))]).
\end{aligned}
\tag{35}
$$

Under an income contingent loan the repayment period is not fixed but it depends on the annual income, which is stochastic in our case. In the previous section, with a non stochastic income, we got the repayment period solving for $\widetilde{T}$ the equation (24) of the education cost. In this case the cost is given by the following expression

$$
C=\gamma\int_{s}^{\widetilde{T}+s} y_0\exp[(\lambda-\frac{1}{2}\sigma^2)t+\sigma W(t)]dt \tag{36}
$$

To obtain $\widetilde{T}$ we should solve the integral of the exponential of a brownian motion, and according to the recent literature on this field it is a very complex task. Therefore to overcome this obstacle we adopt a numerical method.

9.2 Numerical Method

Our objective is to compare the value of the utility under a mortgage loan and an income contingent loan scheme, for a generated path of stochastic income. We developed the method in several steps.

1. We generate a path of annual incomes for an individual working life. Since the problem requires a discrete solution, we prefer to apply the Euler-Maruyama method to the SDE (29), instead of using the close form in equation(32). The Euler-Maruyama method takes the form

$$
y_j = y_{j-1} + y_{j-1}\lambda\Delta t + y_{j-1}\sigma(W(\tau_j)-W(\tau_{j-1})). \tag{37}
$$

   To generate the increments $W(\tau_j)-W(\tau_{j-1})$ we compute discretized Brownian motion paths, where $W(t)$ is specified at discrete t values.

As explained in Higham (2001) we first discretize the interval $[0, I]$. We set $dt = I/N$ for some positive integer N, and let W_j denote $W(t_j)$ with $t_j = jdt$. According to the properties of the standard Brownian motion $W(0) = 0$ and

$$W_j = W_{j-1} + dW_j \tag{38}$$

where dW_j is an independent random variable of the form $\sqrt{dt}N(0,1)$. The discretized brownian motion path is a 1-by-N array, where each element is given by the cumulative sum in equation(38). To generate equation(37), we define $\Delta t = I/L$ for some positive integer L, and $\tau_j = \Delta t$. As in Higham (2001) we choose the stepsize Δt for the numerical method to be an integer multiple $R \geq 1$ of the Brownian motion increment dt: $\Delta t = Rdt$. Finally, we get the increment in equation(37) as cumulative sum:

$$W(\tau_j) - W_(\tau_{j-1}) = W(jRdt) - W((j-1)Rdt) = \sum_{h=jR-R+1}^{jR} dW_h. \tag{39}$$

2. Income contingent loan. We work out the yearly repayments as fixed percentage of the stochastic incomes generated. We then built a vector whose elements are the cumulative sum of the repayments, in order to see the amount of loan repaid. To obtain the repayment period, we observe the years in which the cumulative sum of the payments is equal[9] to the cost of education. We work out the individual utility as discounted sum of the net incomes during and after the repayment period, up to the end of the working life. We use a Logarithmic and CRRA utility function.

3. Mortgage loan. We set the fixed repayment period as the ratio between the cost of education and the annual instalment. The individual utility is given by the discounted sum of the net incomes during and after the repayment period. We use a Logarithmic and CRRA utility function. However, it can happen that the annual income is lower than the instalment, in a usual mortgage loan the individual repays in the subsequent years at a higher interest rate. Or in case of default he will

[9] Since it is almost impossible to get a value equal to the cost, when the repayment is greater than it, we infer with certainty that debt has been paid off.

have a damage of his credit reputation. In our model to highlight a loss of utility in case of no repayment in one year, we compute the level of the utility for that year as negative percentage[10] of the annual income.

4. In point (2) and (3) we obtain a single value of the utility for individual income path generated in point (1). We generalize our method generating a high number of income paths and for each path we compute a level of utility. We then work out the average utility under both financing scheme and the difference of the average in order to compare the two systems.

5. We let the various parameters change and we repeat steps from (1) to (4), observing the trend of the difference of the average utility under the two funding schemes.

10 Simulations

The numerical method previously explained is implemented through a program built in MATLAB, that allows us to do all the simulations required. We consider a CRRA and a Log utility function and seven vectors of parameters:

$Y_0 = [8000 \quad 15000 \quad 30000]$, we consider three levels of initial income. During a a working period of 40 years these incomes generates different paths according to the volatility of the Brownian motion and the deterministic growth rate.

$\sigma = [0 \quad 0.02 \quad 0.05 \quad 0.09 \quad 0.15]$, we assign 5 values to the standard deviation, in order to have different intensities of the effect of the stochastic shock on income. When $\sigma = 0$ there is no stochastic growth; and e.g. $\sigma = 0.05$ means that the maximum annual variation of the income can be 5%, with respect to the case with $\sigma = 0$.

$\lambda = [0.5 \quad 1 \quad 1.5]$, we assign three values to the deterministic growth rate. Applying these rates to the case with no uncertainty, i.e. $\sigma = 0$, we obtain for $\lambda = 0.5$ a total increase of the initial income in 40 years of around 40%, meaning a constant increase of 1% per year. If $\lambda = 1$ the

[10] We set this percentage equal to the average-low interest rate for a typical mortgage loan e.g. around 5%.

total increase of the income at the end of the working life is 63%, that is 2.4% p.a. Finally, $\lambda = 1.5$ corresponds to 77% increase of the initial income after 40 years, that is 4% p.a.

$a = [0 \quad 0.25 \quad 0.5 \quad 0.75]$, we set 4 increasing values for the risk aversion parameter; in the program we use $b = 1 - a$ as in the CRRA utility function considered in the first part of this work. These values have been chosen following the literature.

$\rho = [0.02 \quad 0.08 \quad 0.15 \quad 0.25 \quad 0.5]$, we assign 5 increasing values to the subjective discount rate.

$\varphi = [500 \quad 600 \quad 800 \quad 1000 \quad 1200 \quad 2400]$, we set 6 possible installments under a mortgage loan and two levels of the cost of education $C = [12000 \quad 10000]$. When $\varphi = £1000$ the program applies a cost of £10000 and the resulting repayment period under ML is 10 years. For the other values of φ the cost applied is always £12000, and we get 5 repayment periods under ML, respectively: $T = [24 \quad 20 \quad 15 \quad 10 \quad 5]$.

$\gamma = [0.05 \quad 0.09 \quad 0.15 \quad 0.25 \quad 0.5]$, we assigned 5 values to the rate of repayment under an ICL. The value of 9% is the one chosen in the UK Reform.

The Brownian motion of equation (38) is produced setting $I = 1$ and $N = 160$ in order to have a small value of dt. Using a random number generator we produce 160 "pseudorandom" numbers from the N(0,1) distribution. The increments of equation (39) are computed setting $R = 4$, in order to have 40 annual incomes. A single value of the utility is given by a unique income path, and to obtain a more precise average utility we generated 1000 income paths. Then, we computed all the possible combinations among the parameters above, and we built a database with 27000 different values of the difference between the average utilities under an ICL and a ML. In particular, 21600 values are with a CRRA utility function, and 5400 with a Log utility. Among all the possible cases generated we identified the most relevant. For simplicity, we define $A_{ICL} - A_{ML}$ the difference between the average utilities under ICL and ML.

1. Risk Aversion Changing . We assume low risk aversion $a = 0.25$, low deterministic growth $\lambda = 1\%$, a repayment period under ML $T_{ML} = 10$ years, $\rho = 8\%$ and $\gamma = 9\%$. We consider σ and Y_0 increasing.

If the initial income is low, for increasing σ $A_{ICL} - A_{ML}$ is positive, therefore an ICL is preferred to a ML; however the trend is decreasing. For higher levels of Y_0 $A_{ICL} - A_{ML}$ becomes negative, meaning that a ML is favorite, but the trend is slowly increasing. Keeping the same parameters but increasing the risk aversion to $a = 0.75$, the sign of $A_{ICL} - A_{ML}$ and its trend are the same as before, however the size of $A_{ICL} - A_{ML}$ in absolute value is sharply reduced. For example, for the low level of the income the size of $A_{ICL} - A_{ML}$ reduces of around 99%. (Table 11)

2. Deterministic Growth Rate changing. We assume the same parameters of the previous case with low risk aversion, and we increase λ from 1% per year to 4% per year. For a low Y_0 we observe that $A_{ICL} - A_{ML}$ is positive and the trend increasing for higher σ. Therefore, an ICL becomes more advantageous if the uncertainty is increasing. Comparing this case with the one with low deterministic growth, we notice that a higher λ changes completely the trend of $A_{ICL} - A_{ML}$ if Y_0 is low. Instead with higher levels of the initial income, the effect of higher λ is weak. (Table 11)

3. Discount Rate Changing . We use a CRRA utility function and we assume $a = 0.5$, $\gamma = 9\%$, $T_{ML} = 10$ years, $\lambda = 1\%$ and $Y_0 = £8000$. Given σ, for values of ρ up to 15% $A_{ICL} - A_{ML}$ is positive and increasing, for higher ρ the trend is decreasing. Given ρ if σ increases the trend is decreasing. Using a Log utility function, for higher values of ρ the trend is almost constant, given σ. If we consider a high Y_0, $A_{ICL} - A_{ML}$ is always negative and the trend is first decreasing and then increasing, both using CRRA and Log utility functions. We then keep the same parameters as in the first case but we increase λ to 4%. Given σ, the trend of $A_{ICL} - A_{ML}$ is increasing up to $\rho = 25\%$ and then it decreases. However, given ρ, for increasing σ $A_{ICL} - A_{ML}$ is increasing. (Table 12)

4. Repayment Period ML Changing . We assume $a = 0.5$, $\rho = 8\%$, $\lambda = 1\%$, $\gamma = 9\%$ and a repayment period under ML $T_{ML} = 24$ years. In this case $A_{ICL} - A_{ML}$ is always negative for any level of Y_0. The individuals, having the opportunity to repay in a long period and with zero real interest rate, prefer a ML to an ICL. The trend of $A_{ICL} - A_{ML}$ is slowly increasing for higher σ. If we reduce the repayment period to

$T_{ML} = 5$ years, an ICL is always preferred for low and medium levels of Y_0. $A_{ICL} - A_{ML}$ is negative only if $Y_0 = £30000$, in this case in fact a fixed installment of £2400 is preferred to 9% of the wage that will imply to pay back the in less than 5 years. (Table 13)

5. Repayment Rate under ICL changing. We assume $a = 0.5$, $\rho = 8\%$, $\lambda = 1\%$, $\sigma = 5\%$ and $Y_0 = £8000$. We let γ and ϕ increase. If $\gamma = 5\%$ an ICL is always preferred, for $\gamma = 9\%$ an ICL is more advantageous if $T_{ML} < 15$ years, that is $\phi > 800$. For higher γ a ML is preferred. We increase λ to 4% keeping equal the other parameters. If $T_{ML} = 24$ years a ML is always the most advantageous for any γ. In the other cases we observe a decreasing trend of $A_{ICL} - A_{ML}$ as γ and ϕ are higher. The same results are confirmed if we use a Log utility function. (Table 13)

The increase of the initial income affects strongly the sign of $A_{ICL} - A_{ML}$: keeping constant all the other parameters, for low level of Y_0 $A_{ICL} - A_{ML}$ is positive and the individuals prefer an ICL. Instead if the individuals receive high initial wages $A_{ICL} - A_{ML}$ is negative, and a ML is the most advantageous system. The trend of $A_{ICL} - A_{ML}$ depends on the ratio λ/σ. If λ is low, for increasing level of uncertainty $A_{ICL} - A_{ML}$ is decreasing. Instead, keeping constant all the other parameters, if λ is high, for increasing σ the trend of $A_{ICL} - A_{ML}$ is increasing. The effects of higher λ are more evident when Y_0 is low: an ICL is the most advantageous system when the uncertainty is increasing. When the risk aversion increases the sign of $A_{ICL} - A_{ML}$ and its trend remain unchanged. The risk aversion affects the absolute value of $A_{ICL} - A_{ML}$, which declines sharply when a increases. Also in this case the economic effect is clearer when Y_0 is low: the benefits of an ICL reduce when the risk aversion is higher. For small values of the subjective discount rate, keeping constant all the other parameters, $A_{ICL} - A_{ML}$ increases. When ρ is high the trend of $A_{ICL} - A_{ML}$ is decreasing. Finally, the effect of a reduction of the repayment period under a ML is to increase the average utility under an ICL. It becomes the most advantageous system above all for low and medium level of Y_0. The opposite effect is realized when the repayment rate under an ICL is increased.

11 Conclusion

In this work we started from two empirical facts: the different levels of uncertainty in the wages of graduate people in the UK from 1993 to 2003, and the introduction of an income contingent loan to finance higher education in the UK from 2006. We expected that for higher uncertainty in the graduate incomes, if the higher education is financed through a loan system, the income contingent is the one preferred by the individuals.

We built a model to give a theoretical base to our intuition. We tried to show the superiority of an income contingent loan with respect to a mortgage loan, when the graduate incomes are stochastic and the level of uncertainty is increasing. We assumed a single lifetime shock affecting the income after graduation, and we compared the expected utilities of the individuals before and after the repayment of the educational debt under the two loan schemes. The main result found is that for risk neutral individuals the expected costs of education under an income contingent loan are lower than the expected costs under a mortgage loan scheme. If the individuals are risk averse, an income contingent loan is preferred when the level of uncertainty is increasing. The extent of the preference of one system over the other is strongly related to the repayment period. The system that allows a longer repayment period is more preferred.

We compared then the two loan schemes for people working in the public sector and in the private sector. We focused on two extreme cases of no uncertainty and highest uncertainty on the graduate income. An income contingent loan is generally preferred in the private sector because it offers an implicit insurance against uncertainty.

To make our model closer to the reality of the job market, we assumed that the income grows following a geometric Brownian motion. The uncertainty affects the income each year during the individual working life and not only once. We compared the average utilities under the two financing schemes, developing a numerical iterative method. We found that the level of the initial income strongly affects the preference of one system over the other. If the individuals receive a low initial wage, they prefer an ICL above all for increasing level of uncertainty. Instead, if they are very risk averse, and still getting a low initial wage, a ML becomes more beneficial. The size of the deterministic growth rate is also very important, in fact when this rate is high the individuals find an ICL more advantageous for increasing uncertainty. Finally, reducing the repayment period under a ML makes always an ICL more profitable.

One further extension, that we will face in another work, is the case for students to be unable to pay off their loan, and the government can choose to impose a default premium to keep its budget balanced.

In conclusion, the wage uncertainty, the level of the starting wage and its randomness affect the choice of the funding system. The government to really improve the individual welfare should not stick to a single scheme, but it should allow the graduates to choose the preferred repayment systems according to their type of degree and its perceived riskiness. The better mechanism should be very flexible, in order to switch from one loan scheme to the other according to the graduates' earning characteristics.

A Appendix: Proof Proposition 1

Under risk neutrality equation (2) becomes

$$V(1) = E(\int_{s}^{\infty} e^{-\rho t} y\ dt) - E(\int_{s}^{T+s} e^{-\rho t} R\ dt) \tag{40}$$

So we can compare only the expected costs. Under ML the present value of the cost of size C is:

$$\begin{aligned} PVC_{ML} &= \int_{s}^{T+s} \varphi\, e^{-\rho t}\, dt \\ &= e^{-\rho s}\, \frac{\varphi}{\rho}\, [1 - e^{-\rho \frac{C}{\varphi}}]\,. \end{aligned} \tag{41}$$

Under ICL the present value of the cost of size C is:

$$\begin{aligned} PVC_{ICL} &= \int_{s}^{\tilde{T}+s} y\, \gamma\, e^{-\rho t}\, dt \\ &= e^{-\rho s}\, \frac{\gamma\, y}{\rho} \Big[1 - e^{-\rho \frac{C}{\gamma y}}\Big] \quad . \end{aligned} \tag{42}$$

Knowing that $E(y) = 1$, we take the expected value of both the equations above.

$$\begin{aligned} E(PVC_{ML}) &= \frac{\varphi}{\rho}\, [1 - e^{-\rho \frac{C}{\varphi E(y)}}\,] e^{-\rho s} \\ E(PVC_{ICL}) &= E\, \big[\frac{\gamma\, y}{\rho}\, (1 - e^{-\rho \frac{C}{\gamma y}}\,) e^{-\rho s}\, \big] \end{aligned} \tag{43}$$

Assuming that the instalment under a mortgage loan is equal to the repayment rate under an income contingent loan: $\varphi = \gamma$, we can easily observe that the expected values can be written:

$$E(PVC_{ML}) = f[E(y)]$$
$$E(PVC_{ICL}) = Ef(y)$$

Since $f(y) = \frac{\gamma\, y}{\rho}\, \big(1 - e^{-\frac{\rho\, C}{\gamma\, y}}\big) e^{-\rho s}$ is a concave function [11], we obtain that the expected costs under ICL are lower than the expected cost under ML: $E(PVC_{ICL}) < E(PVC_{ML})$. According to equation (40) the expected utility under ICL is higher than the expected utility under ML.

[11] $f''(y) = -\frac{\rho C^2 e^{-\left(s + \frac{C}{\gamma y}\right)\rho}}{\gamma y^3}$. It is reasonable to assume that γ, ρ and C are all greater or equal than zero. Therefore, the second derivative of $f(y)$ is always negative when the shock on income is positive: $f''(y) < 0, \quad \forall\, y > 0$.

B Appendix: Expected Utility with a Mortgage Loan

The Taylor approximation in equation (7) is the following

$$\begin{aligned} E[u(y-\varphi)] &= E\left\{u\left(1-\varphi\right)+u^{'}\left(1-\varphi\right)\left(y-1\right)+\frac{1}{2}u^{''}\left(1-\varphi\right)\left(y-1\right)^{2}\right\} \\ &= u\left(1-\varphi\right)+u^{'}\left(1-\varphi\right)E(y-1)+\frac{1}{2}u^{''}\left(1-\varphi\right)E(y-1)^{2} \\ &= u\left(1-\varphi\right)+\frac{1}{2}u^{''}\left(1-\varphi\right)\sigma^{2}. \end{aligned} \tag{44}$$

Plugging the equations (7) and (8) in the equation (6), substituting $T = C/\varphi$ and solving the integral, we obtain:

$$\begin{aligned} V_{ML} = &\frac{e^{-\rho s}}{\rho}\left(1-e^{-\frac{\rho C}{\varphi}}\right)\left[u\left(1-\varphi\right)+\frac{1}{2}u^{''}\left(1-\varphi\right)\sigma_s^2\right] \\ &+\frac{e^{-\rho s}}{\rho}e^{-\frac{\rho C}{\varphi}}\left[u\left(1\right)+\frac{1}{2}u^{''}\left(1\right)\sigma_s^2\right]. \end{aligned} \tag{45}$$

Finally, substituting the CARA and CRRA utility functions in equation(45) and simplifying we get equations (9) and (10).

C Appendix: Expected Utility with an Income Contingent Loan

In Section (4.2) we defined a new function $g(y)$ as:

$$g(y) = \left[1-e^{-\frac{\rho C}{\gamma y}}\right]u\left[y\left(1-\gamma\right)\right]+\left[e^{-\frac{\rho C}{\gamma y}}\right]u\left(y\right) \tag{46}$$

We rewrite the equation (12)

$$V_{ICL} = \frac{e^{-\rho s}}{\rho}E[g(y)] \tag{47}$$

and we apply a second order Taylor expansion to $E[g(y)]$, around the mean $E\left[y\right]=1$, then:

$$\begin{aligned} E[g(y)] &= E\left\{g(1)+g'(1)(y-1)+g''(1)\frac{(y-1)^2}{2}\right\} \\ &= g(1)+g'(1)E(y-1)+\frac{g''(1)}{2}E\left(y-1\right)^2 \\ &= g(1)+g''(1)\frac{\sigma_s^2}{2}. \end{aligned} \tag{48}$$

The equation (47) becomes

$$V_{ICL}=\frac{e^{-\rho s}}{\rho}\left[g(1)+g''(1)\frac{\sigma_s^2}{2}\right] \tag{49}$$

From now on we follow this procedure:

1. we work out the value of g(1), in general and with a CARA and CRRA utility functions;

2. we work out the first derivative and the second derivative of g(y), both in general and with a CARA and CRRA utility functions;

3. we calculate $g'(1)$ and $g''(1)$ using both CARA and CRRA utility function;

4. we substitute the equations of $g(1)$ and $g''(1)$, using a CARA and CRRA utility function, in the equation (49) and we obtain equations (59), (14), (15).

- **Value of g(1)**

  In general,

$$g(1)=\left[1-e^{-\frac{\rho C}{\gamma}}\right]u\left[(1-\gamma)\right]+\left[e^{-\frac{\rho C}{\gamma}}\right]u\left(1\right) \tag{50}$$

Using a CARA utility function we have

$$g(1)_{CARA}=-\frac{1}{a}e^{-\frac{a\gamma+\rho C}{\gamma}}\left[1-e^{a\gamma}+e^{\frac{a\gamma^2+\rho C}{\gamma}}\right] \tag{51}$$

If we use a CRRA utility function we have

$$g(1)_{CRRA} = \frac{1}{b}[-e^{-\frac{\rho C}{\gamma}}((1-\gamma)^b - 1) + (1-\gamma)^b]. \tag{52}$$

- **Value of g'(y)**

  In general,

$$\begin{aligned} g'(y) = {} & u'[y(1-\gamma)](1-\gamma)\left[1 - e^{-\frac{\rho C}{\gamma y}}\right] + u[y(1-\gamma)]\left[\frac{-\rho C e^{-\frac{\rho C}{\gamma y}}}{\gamma y^2}\right] \\ & + u'(y)[e^{-\frac{\rho C}{\gamma y}}] + u(y)\left[\frac{\rho C e^{-\frac{\rho C}{\gamma y}}}{\gamma y^2}\right] \end{aligned} \tag{53}$$

using a CARA utility function:

$$\begin{aligned} g'(y)_{CARA} = {} & (1-\gamma)e^{-ay(1-\gamma)}\left[1 - e^{-\frac{\rho C}{\gamma y}}\right] + \left[\frac{\rho C e^{-ay(1-\gamma)-\frac{\rho C}{\gamma y}}}{a\gamma y^2}\right] \\ & - \frac{1}{a}e^{-ay-\frac{\rho C}{\gamma y}}\left(\frac{\rho C}{\gamma y^2} - a\right) \end{aligned} \tag{54}$$

using a CRRA utility function:

$$\begin{aligned} g'(y)_{CRRA} = {} & (y(1-\gamma))^{b-1}(1-\gamma)\left[1 - e^{-\frac{\rho C}{\gamma y}}\right] + (y(1-\gamma))^b\left[\frac{-\rho C e^{-\frac{\rho C}{\gamma y}}}{b\gamma y^2}\right] \\ & + y^{b-1}[e^{-\frac{\rho C}{\gamma y}}] + \left[\frac{y^{b-2}\rho C e^{-\frac{\rho C}{\gamma y}}}{b\gamma}\right]. \end{aligned} \tag{55}$$

- **Value of g''(y)**

$$\begin{aligned} g''(y) = {} & \frac{e^{-\frac{\rho C}{\gamma y}}\rho C(2\gamma y - \rho C)}{y^4\gamma^2}u[y(1-\gamma)] + \frac{e^{-\frac{\rho C}{\gamma y}}\rho C(-2\gamma y + \rho C)}{y^4\gamma^2}u(y) \\ & - \frac{2e^{-\frac{\rho C}{\gamma y}}\rho C(1-\gamma)}{y^2\gamma}u'[y(1-\gamma)] + \frac{2e^{-\frac{\rho C}{\gamma y}}\rho C}{y^2\gamma}u'(y) \\ & + \left[1 - e^{-\frac{\rho C}{\gamma y}}\right](1-\gamma)^2u''[y(1-\gamma)] + \left[e^{-\frac{\rho C}{\gamma y}}\right]u''(y). \end{aligned} \tag{56}$$

Now we work out $g''(y)$ using a CARA utility function and in $y = 1$

$$g''(1)_{CARA} = \frac{e^{-\frac{a\gamma+\rho C}{\gamma}}}{a\gamma^2}\{-a^2[1 - e^{a\gamma}(\gamma - 1^2 + e^{a\gamma+\frac{\rho C}{\gamma}}(\gamma - 1)^2]\gamma^2 \\ + 2ac[1 + e^{a\gamma}(\gamma - 1)]\rho\gamma + \rho C(e^{a\gamma} - 1)(\rho C - 2\gamma)\}. \tag{57}$$

Using a CRRA and evaluating in $y = 1$

$$g''(1)_{CRRA} = \frac{1}{b\gamma^2}\{e^{-\frac{\rho C}{\gamma}}[(b-1)b\gamma^2[1 + (e^{\frac{\rho C}{\gamma y}} - 1)(1-\gamma)^b] \\ + 2\rho C(b-1)\gamma(1 - (1-\gamma)^b) + C^2\rho^2(1 - (1-\gamma)^b)]\}. \tag{58}$$

- **Results**

Substituting $g(1)$ and $g''(1)$ in equation (49) we get the general expected utility under an income contingent loan:

$$V_{ICL} = \left[1 - e^{-\frac{\rho C}{\gamma}}\right] u\left[(1-\gamma)\right] + \left[e^{-\frac{\rho C}{\gamma}}\right] u\left(1\right) \\ + [\frac{e^{-\frac{\rho C}{\gamma}}\rho C(2\gamma - \rho c)}{\gamma^2}u[1-\gamma] + \frac{e^{-\frac{\rho C}{\gamma}}\rho C(-2\gamma + \rho c)}{\gamma^2}u(1) \\ - \frac{2e^{-\frac{\rho C}{\gamma}}\rho C(1-\gamma)}{\gamma}u'[1-\gamma] + \frac{2e^{-\frac{\rho C}{\gamma}}\rho C}{\gamma}u'(1) \\ + \left[1 - e^{-\frac{\rho C}{\gamma}}\right](1-\gamma)^2 u''[1-\gamma] + \left[e^{-\frac{\rho C}{\gamma}}\right] u''(1)]\frac{\sigma_s^2}{2}. \tag{59}$$

Substituting in equation (49) the equations for $g(1)$ and $g''(1)$ with CARA and CRRA utility functions, we obtain equations (14)) and (15).

D Appendix: Difference between Expected Utilities from No schooling and Schooling

The analysis is extended comparing the expected utility of people not going to college with the expected utility from schooling, under the two repayment systems. We modify the general equation (1) of the expected utility from no schooling, using a CRRA and CARA utility function:

$$V_{0_{CRRA}} = \frac{1}{\rho}\frac{X^b}{b}. \tag{60}$$

$$V_{0_{CARA}} = \frac{1}{\rho}\frac{e^{aX}}{a}. \tag{61}$$

We use the same vectors of parameters as in the previous simulations, the only difference in this case is that we set also the level of the non graduate income, X. In the UK Labour Force Survey of 2002, the average annual income for graduate people is £25054 with a standard deviation $\sigma_y = 0.641$, the average income of people with no qualifications is £9928 with $\sigma_X = 0.777$. The average annual income of people with a secondary school qualification[12] is £17353 with $\sigma_X = 0.883$. In our simulations, we calibrate the two non graduate incomes as percentage of the graduate one, and we compute the differences $V_0 - V_{ML}$ and $V_0 - V_{ICL}$, with CRRA and CARA utility functions. Moreover, the level of uncertainty concerns the graduate incomes, therefore the values of σ are referred to y. We generate a database with all the possible combinations of the parameters, and we obtain 57600 observations. We consider the following cases. In the following Tables we show only the results for the ICL function with a CRRA utility.

- X and σ increasing: CRRA utility function, $C =$ £10000, $\varphi =$ £1000, $T_{ML} = 10$ years, $a = 0.5$, $\rho = 8\%$, $\gamma = 9\%$. For a low level of X, both the differences $V_0 - V_{ML}$ and $V_0 - V_{ICL}$ are negative and have almost the same size. For σ increasing not going to college is more advantageous: when the uncertainty on the graduate income is higher, the individuals prefer lower levels of instructions. Considering a high level of X, not schooling gives higher utility under both funding systems. Using a CARA utility function, we observe the same trend as before, but in this case schooling is always preferred.(Table 10)
- X and a increasing: CRRA utility function, $C =$ £10000, $\varphi =$ £1000, $T_{ML} = 10$ years, $\sigma = 0.6$, $\rho = 8\%$, $\gamma = 9\%$. For both high and low

[12]Precisely, we consider individuals with an A-level.

X, we observe a slowly increasing trend of $V_0 - V_{ML}$ and $V_0 - V_{ICL}$ as a increases. But when the risk aversion is very high the preference for schooling sharply increases, therefore $V_0 - V_{ML}$ and $V_0 - V_{ICL}$ becomes negative. Comparing a ML and an ICL we do not observe large differences, they produce almost the same utility. Using a CARA utility function, the individuals always prefer schooling, and with increasing intensity as a increases.(Table 10)

- X and ρ increasing: CRRA utility function, $C = £10000$, $\varphi = £1000$, $T_{ML} = 10$ years, $\sigma = 0.6$, $a = 0.5\%$, $\gamma = 9\%$. For low levels of ρ, schooling is preferred both with low and high X, but when the individuals discount more the future $V_0 - V_{ML}$ and $V_0 - V_{ICL}$ become positive and slowly increasing. Using a CARA utility function, we have a completely opposite result: for increasing ρ, $V_0 - V_{ML}$ and $V_0 - V_{ICL}$ are negative decreasing, and schooling is always preferred. (Table 10)

- X and C increasing: CRRA utility function, $\varphi = £1000$, $\sigma = 0.6$, $a = 0.5\%$, $\rho = 8\%$, $\gamma = 9\%$. If the cost of education increases the utility of schooling decreases under both funding systems and also using a CARA utility function.

- X and φ increasing: CRRA utility function, $C = £10000$, $\sigma = 0.6$, $a = 0.5\%$, $\gamma = 9\%$. If φ increases means that the repayment period under a ML is shorter, the individuals perceives the education more costly and prefer not schooling. Using a CARA utility function, schooling is preferred and the trend of $V_0 - V_{ML}$ is almost constant.

- X and γ increasing: CRRA utility function, $C = £12000$, $\varphi = £1200$, $T_{ML} = 10$ years, $\sigma = 0.6$, $a = 0.5\%$, $\rho = 8\%$. For higher γ the repayments under an ICL increase, therefore the education is more costly and $V_0 - V_{ICL}$ is increasing. We observe the same trend using a CARA utility function, although schooling is always preferred.

We found that schooling is always more beneficial if the non graduate income is low. Assuming instead a non graduate wage of around 70% of the graduate income, the individuals obtain higher utility not going to college. We observed an increasing preference for not schooling when the graduate incomes are more uncertain. Higher risk aversion and increasing costs of education imply also a preference for not schooling. The effect of the two funding systems on the decision of schooling is almost the same. Instead the use of a CARA utility increases the preference for schooling, although we observed the same trends as with a CRRA utility function.

Table 1: Graduates and Type of Degree in the LFS sample

Year Groups	Quarters 1	2	3	4	Total
	Numer of graduates				
1993 - 1995	921	1,046	1,505	1,473	4,945
1996 -1999	1,983	2,018	2,014	2,067	8,082
2000 -2003	1,803	1,844	1,867	1,416	6,930
Total	4,707	4,908	5,386	4,956	19,957

Type of Degree	Number of graduates
Other Degrees	1,183
%	5.93
Health	548
%	2.75
Science	3,274
%	16.41
Engineer	3,822
%	19.15
Social Science	4,836
%	24.23
Arts & Humanity	1,721
	8.62
Combined Degrees	4,573
%	22.91
Total	19,957
%	100

Table 2: Graduate Real Wages and Uncertainty

Real wage in Jan 2000 prices

Type of Degree	Mean	Std. Dev.	Number of graduates
Other Degrees	12.33174	13.41489	1078
Health	16.26671	11.80191	497
Science	15.01208	12.51473	2978
Engineer	14.74389	11.76977	3481
Social Science	14.92945	11.37423	4377
Arts & Humanity	12.64997	12.40635	1535
Combined Degrees	14.25019	11.60422	4101
Total	14.44071	11.95561	18047

Means, Standard Deviations and Frequencies of Real wage in Jan 2000 prices

		Year Groups			
	Type of Degree	1993 - 1995	1996 -1999	2000 -2003	Total
Other Degree	mean	11.019921	12.375529	13.041323	12.33174
	sd	7.5843386	17.239635	9.9678092	13.41489
	individuals	219	484	375	1078
Health	mean	14.814256	16.040615	17.421173	16.26671
	sd	7.885604	12.649623	12.525105	11.80191
	graduates	102	223	172	497
Science	mean	14.490965	14.945038	15.455694	15.01208
	sd	12.724064	12.649371	12.209983	12.51473
	graduates	758	1155	1065	2978
Engineer	mean	13.992772	14.496828	15.633646	14.74389
	sd	9.6647374	8.8103475	15.737084	11.76977
	graduates	901	1424	1156	3481
Social Science	mean	14.40798	14.765561	15.605251	14.92945
	sd	12.386426	11.386794	10.326181	11.37423
	graduates	1265	1719	1393	4377
Arts & Humanity	mean	11.895496	12.78388	12.972142	12.64997
	sd	7.7992316	15.704237	10.346116	12.40635
	graduates	349	631	555	1535
Combined Degrees	mean	13.407298	14.000984	14.982011	14.25019
	sd	10.247739	9.7621185	13.86765	11.60422
	graduates	863	1674	1564	4101
	Total	13.790482	14.276073	15.093832	14.44071
		10.958598	11.794237	12.763152	11.95561
		4457	7310	6280	18047

Table 3: Graduates Starting Wages and Uncertainty

Real wage in Jan 2000 prices

Type of Degree	Mean	Std. Dev.	Number of graduates
Other degrees	12.12592	7.770354	128
Health	16.91942	16.3115	97
Science	15.24289	10.86905	1089
Engineer	15.62052	10.79661	824
SocialSc	15.78737	16.17494	1083
Arts_Hum	13.12505	11.22077	418
Combined	14.83077	12.65067	1013
Total	15.10569	12.77576	4652

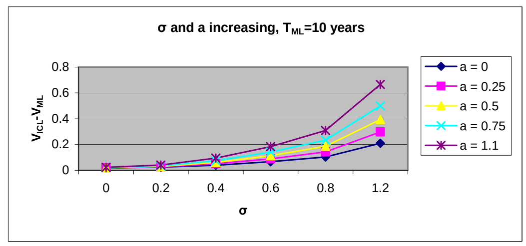
Means, Standard Deviations and Frequencies of Real wage in Jan 2000 prices

		Year Groups			
	Type of Degree	1993 - 1995	1996 -1999	2000 -2003	Total
Other degrees	mean	11.174943	11.568382	13.51111	12.12592
	sd	5.672305	6.5872767	10.098041	7.770354
	graduates	26	60	42	128
Health	mean	14.793301	17.631483	17.424286	16.91942
	sd	7.899203	17.837273	18.612716	16.3115
	graduates	22	43	32	97
Science	mean	14.144273	15.70853	15.546182	15.24289
	sd	7.9690148	12.68903	10.548363	10.86905
	graduates	284	418	387	1089
Engineer	mean	16.231335	15.362984	15.553148	15.62052
	sd	16.368482	9.7340117	7.1109406	10.79661
	graduates	180	350	294	824
Social Science	mean	14.992755	15.999468	16.120447	15.78737
	sd	21.274193	15.276069	12.486533	16.17494
	graduates	273	437	373	1083
Arts & Humanity	mean	11.496981	13.508741	13.575512	13.12505
	sd	9.9817788	12.177202	10.731739	11.22077
	graduates	85	174	159	418
Combined Degrees	mean	13.382515	14.002617	16.299038	14.83077
	sd	9.4655283	8.517458	16.596761	12.65067
	graduates	184	414	415	1013
	Total	14.314292	15.05002	15.6578	15.10569
		14.382085	12.068041	12.461834	12.77576
		1054	1896	1702	4652

Table 4: $V_{ICL} - V_{ML}$ - CRRA - Uncertainty and Risk Aversion

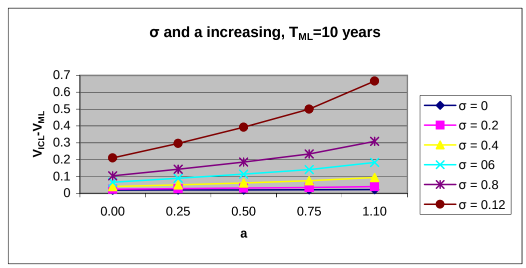
C = £ 10000   φ = £1000 T_{ML} = 10   ρ = 8%   γ = 9%

	σ					
a	0	0.2	0.4	0.6	0.8	1.2
0.00	0.0188	0.0241	0.04	0.0665	0.1037	0.2098
0.25	0.0197	0.0274	0.0504	0.0889	0.1427	0.2966
0.50	0.0206	0.0309	0.0619	0.1137	0.1860	0.3929
0.75	0.0215	0.0348	0.0746	0.1410	0.2339	0.4993
1.10	0.0229	0.0408	0.0944	0.1838	0.3089	0.6664



C = £ 10000   φ = £500 T_{ML} = 20   ρ = 8%   γ = 9%

	σ					
a	0	0.2	0.4	0.6	0.8	1.2
0.00	-0.1189	-0.1136	-0.0976	-0.0711	-0.0339	0.0722
0.25	-0.1222	-0.1156	-0.0959	-0.0630	-0.0170	0.1144
0.50	-0.1256	-0.118	-0.0952	-0.0574	-0.0043	0.1472
0.75	-0.1290	-0.1207	-0.0958	-0.0544	0.0037	0.1696
1.10	-0.1340	-0.1252	-0.0989	-0.0550	0.0064	0.1817



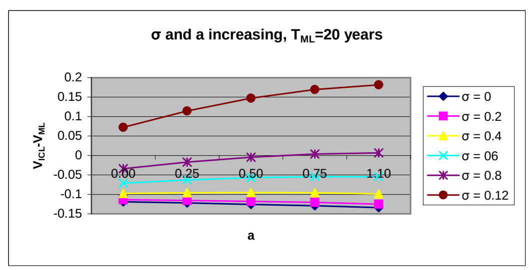
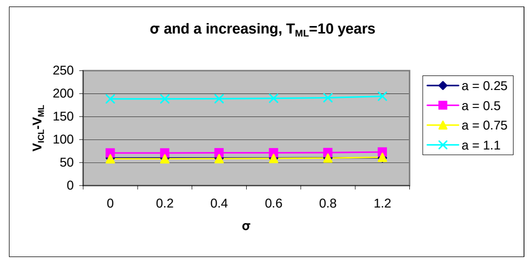


Table 5: $V_{ICL} - V_{ML}$ - CARA - Uncertainty and Risk Aversion

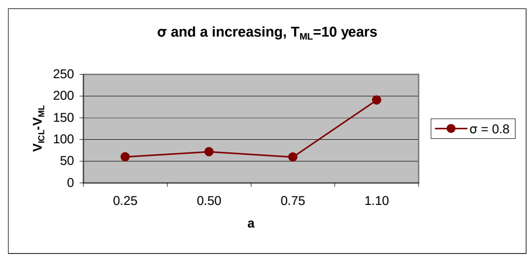
C = £ 10000   φ = £1000 T_{ML} = 10   ρ = 8%   γ = 9%

	σ					
a	0	0.2	0.4	0.6	0.8	1.2
0.25	59.749	59.767	59.823	59.917	60.048	60.421
0.50	71.050	71.098	71.244	71.485	71.824	72.791
0.75	58.260	58.350	58.619	59.068	59.697	61.492
1.10	188.383	188.544	189.028	189.835	190.963	194.189



C = £ 10000   φ = £500 T_{ML} = 20   ρ = 8%   γ = 9%

	σ					
a	0	0.2	0.4	0.6	0.8	1.2
0.25	59.6069	59.6244	59.6771	59.7649	59.8878	60.239
0.50	70.9039	70.9495	71.0863	71.3143	71.6335	72.5456
0.75	58.1099	58.1946	58.449	58.8729	59.4664	61.162
1.10	188.2261	188.3783	188.8348	189.5956	190.6608	193.7043



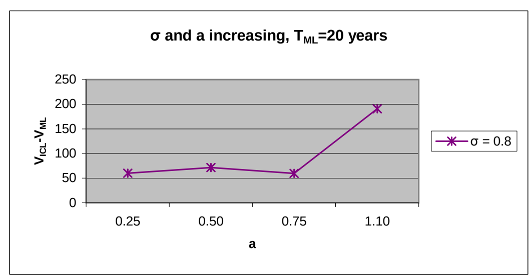
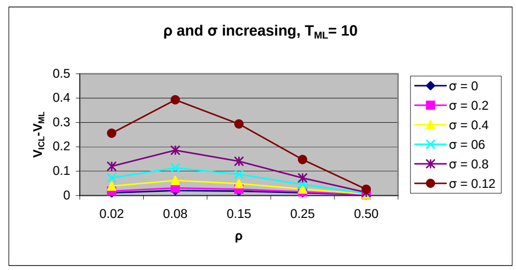


Table 6: $V_{ICL} - V_{ML}$ - CRRA - Uncertainty and Subjective Discount Factor

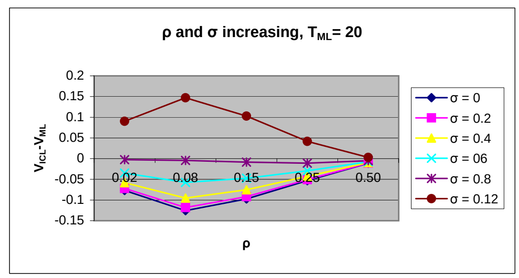
C = £ 10000 φ = £1000 T_{ML} = 10 a = 0.5 γ = 9%

	σ					
ρ	0	0.2	0.4	0.6	0.8	1.2
0.02	0.0114	0.0182	0.0386	0.0726	0.1201	0.2559
0.08	0.0206	0.0309	0.0619	0.1137	0.1860	0.3929
0.15	0.0183	0.0260	0.0489	0.0872	0.1407	0.2937
0.25	0.0115	0.0153	0.0266	0.0455	0.0720	0.1476
0.50	0.0028	0.0034	0.0053	0.0085	0.0129	0.0256



C = £ 10000 φ = £500 T_{ML} = 20 a = 0.5 γ = 9%

	σ					
ρ	0	0.2	0.4	0.6	0.8	1.2
0.02	-0.0767	-0.072	-0.0581	-0.0349	-0.0025	0.0903
0.08	-0.1256	-0.118	-0.0952	-0.0574	-0.0043	0.1472
0.15	-0.0973	-0.0918	-0.0751	-0.0473	-0.0083	0.1029
0.25	-0.0531	-0.0505	-0.0426	-0.0294	-0.0109	0.0418
0.50	-0.0111	-0.0107	-0.0096	-0.0076	-0.0049	0.0029



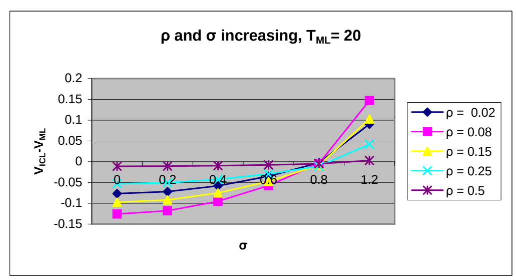
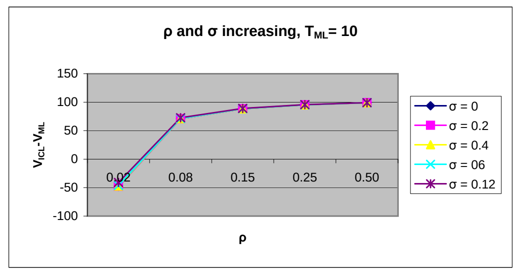


Table 7: $V_{ICL} - V_{ML}$ - CARA - Uncertainty and Subjective Discount Factor

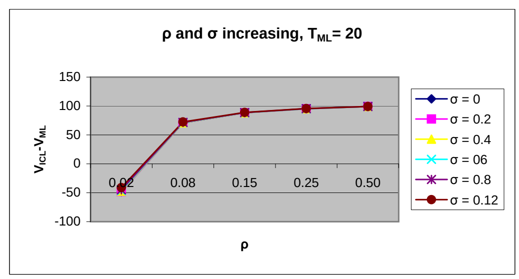
C = £ 10000 φ = £1000 T_{ML} = 10 a = 0.5 γ = 9%

	σ					
ρ	0	0.2	0.4	0.6	0.8	1.2
0.02	-47.956	-47.760	-47.170	-46.186	-44.809	-40.876
0.08	71.050	71.098	71.244	71.485	71.824	72.791
0.15	88.370	88.393	88.461	88.574	88.732	89.183
0.25	95.334	95.343	95.373	95.421	95.489	95.684
0.50	99.143	99.145	99.150	99.159	99.171	99.207



C = £ 10000 φ = £500 T_{ML} = 20 a = 0.5 γ = 9%

	σ					
ρ	0	0.2	0.4	0.6	0.8	1.2
0.02	-48.0444	-47.8499	-47.2663	-46.2937	-44.932	-41.0415
0.08	70.9039	70.9495	71.0863	71.3143	71.6335	72.5456
0.15	88.2546	88.2751	88.3366	88.4391	88.5826	88.9926
0.25	95.2689	95.2775	95.3033	95.3463	95.4064	95.5783
0.50	99.129	99.1305	99.1351	99.1427	99.1535	99.1841



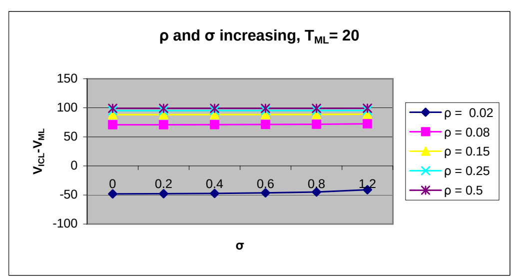
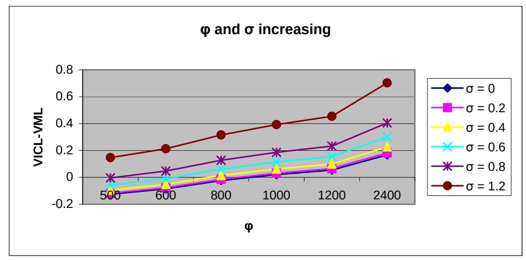


Table 8: $V_{ICL} - V_{ML}$ - CRRA - ICL and ML Parameters

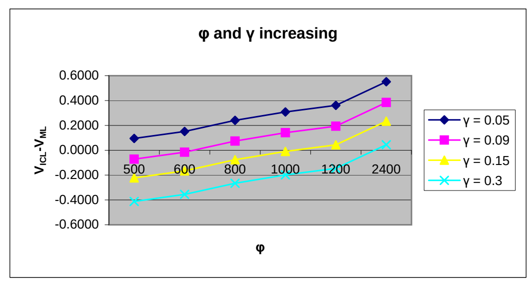
C = £ 10000 a = 0.5 ρ = 8% γ = 9%

	σ					
φ	0	0.2	0.4	0.6	0.8	1.2
500	-0.1256	-0.1180	-0.0952	-0.0574	-0.0043	0.1472
600	-0.0852	-0.0769	-0.0520	-0.0106	0.0474	0.2131
800	-0.0238	-0.0144	0.0139	0.0610	0.1270	0.3156
1000	0.0206	0.0309	0.0619	0.1137	0.1860	0.3929
1200	0.0545	0.0656	0.0990	0.1546	0.2324	0.4548
2400	0.1674	0.1823	0.2270	0.3014	0.4056	0.7034



C = £ 12000 a = 0.5 σ = 0.6 ρ = 8%

	γ			
φ	0.05	0.09	0.15	0.3
500	0.0948	-0.0720	-0.2226	-0.4118
600	0.1516	-0.0153	-0.1658	-0.3550
800	0.2409	0.0740	-0.0765	-0.2658
1000	0.3080	0.1412	-0.0094	-0.1986
1200	0.3609	0.1940	0.0435	-0.1458
2400	0.5514	0.3845	0.2340	0.0447



C = £ 10000 φ = £ 1000 T_{ML} = 10 a = 0.5 ρ = 8%

	σ					
γ	0	0.2	0.4	0.6	0.8	1.2
0.05	0.1461	0.1583	0.1948	0.2557	0.3409	0.5844
0.09	0.0206	0.0309	0.0619	0.1137	0.1860	0.3929
0.15	-0.0726	-0.0653	-0.0433	-0.0065	0.0449	0.1919
0.30	-0.1811	-0.1779	-0.1685	-0.1528	-0.1307	-0.0678

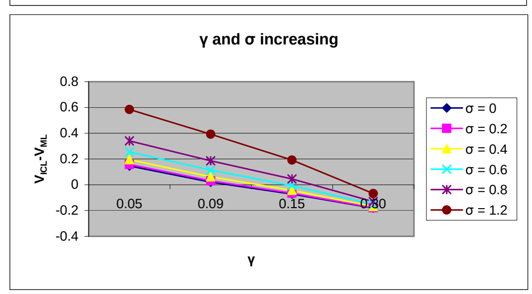
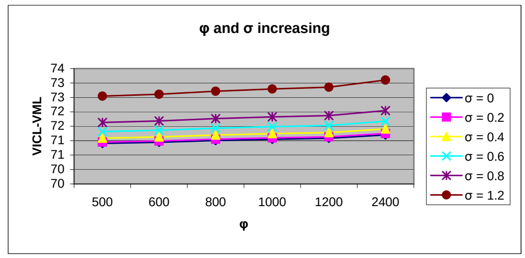


Table 9: $V_{ICL} - V_{ML}$ - CARA - ICL and ML Parameters

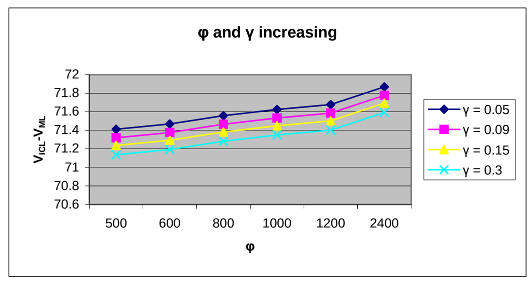
C = £ 10000 a = 0.5 ρ = 8% γ = 9%

	σ					
φ	0	0.2	0.4	0.6	0.8	1.2
500	70.904	70.950	71.086	71.314	71.634	72.546
600	70.944	70.991	71.130	71.361	71.685	72.611
800	71.006	71.053	71.195	71.433	71.765	72.714
1000	71.050	71.098	71.244	71.485	71.824	72.791
1200	71.084	71.133	71.281	71.526	71.870	72.853
2400	71.197	71.250	71.409	71.673	72.044	73.102



C = £ 12000 a = 0.5 σ = 0.6 ρ = 8%

	γ			
φ	0.05	0.09	0.15	0.3
500	71.412	71.3198	71.2366	71.1362
600	71.4687	71.3765	71.2933	71.1929
800	71.558	71.4658	71.3826	71.2822
1000	71.6252	71.533	71.4498	71.3494
1200	71.678	71.5858	71.5026	71.4022
2400	71.8685	71.7763	71.6931	71.5927



C = £ 10000 φ = £ 1000 T_{ML} = 10 a = 0.5 ρ = 8%

	σ					
γ	0	0.2	0.4	0.6	0.8	1.2
0.05	71.126	71.175	71.321	71.564	71.905	72.878
0.09	71.050	71.098	71.244	71.485	71.824	72.791
0.15	70.994	71.041	71.183	71.419	71.749	72.694
0.30	70.931	70.977	71.114	71.342	71.661	72.573

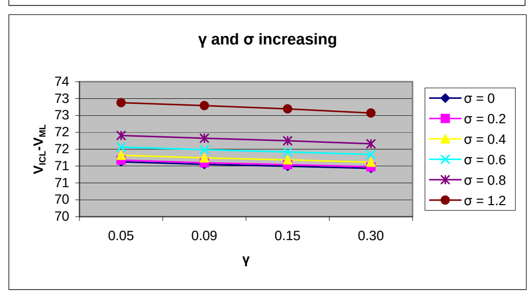
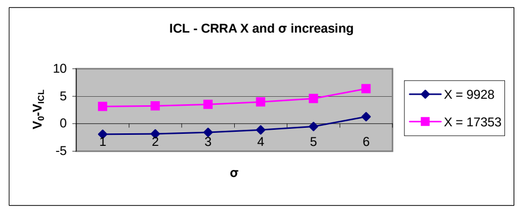


Table 10: Schooling versus No Schooling

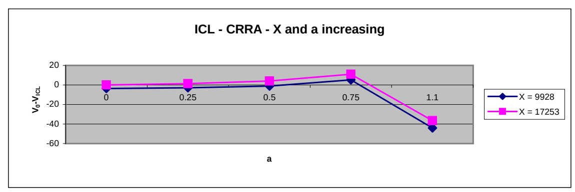
C = £ 10000   φ = £ 1000 T_{ML} = 10   a = 0.5   ρ = 8%   γ = 9%

ICL-CRRA	σ					
X	0	0.2	0.4	0.6	0.8	1.2
9928.00	-1.9252	-1.8362	-1.5693	-1.1243	-0.5014	1.2783
17353.00	3.1446	3.2336	3.5006	3.9455	4.5684	6.3481



φ = £ 1000   γ = 9%
C = £ 10000 T_{ML} = 10   σ = 0.6   ρ = 8%

ICL-CRRA	a				
X	0	0.25	0.5	0.75	1.1
9928.00	-3.6911	-2.9218	-1.1243	5.1021	-43.9691
17353.00	0.0142	1.4089	3.9455	11.0469	-36.5197



T_{ML} = 10   γ = 9%
C = £ 10000   φ = £ 1000   a = 0.5   σ = 0.6

ICL-CRRA	ρ				
X	0.02	0.08	0.15	0.25	0.5
9928.00	-24.2981	-1.1243	1.6527	2.3389	2.0241
17353.00	-4.0188	3.9455	4.3567	3.9612	2.8353

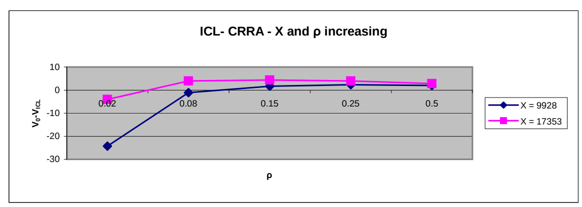


Figure 2: Public Sector Increasing Income and Risk Aversion
—— Private Sector Increasing Uncertainty and Risk Aversion

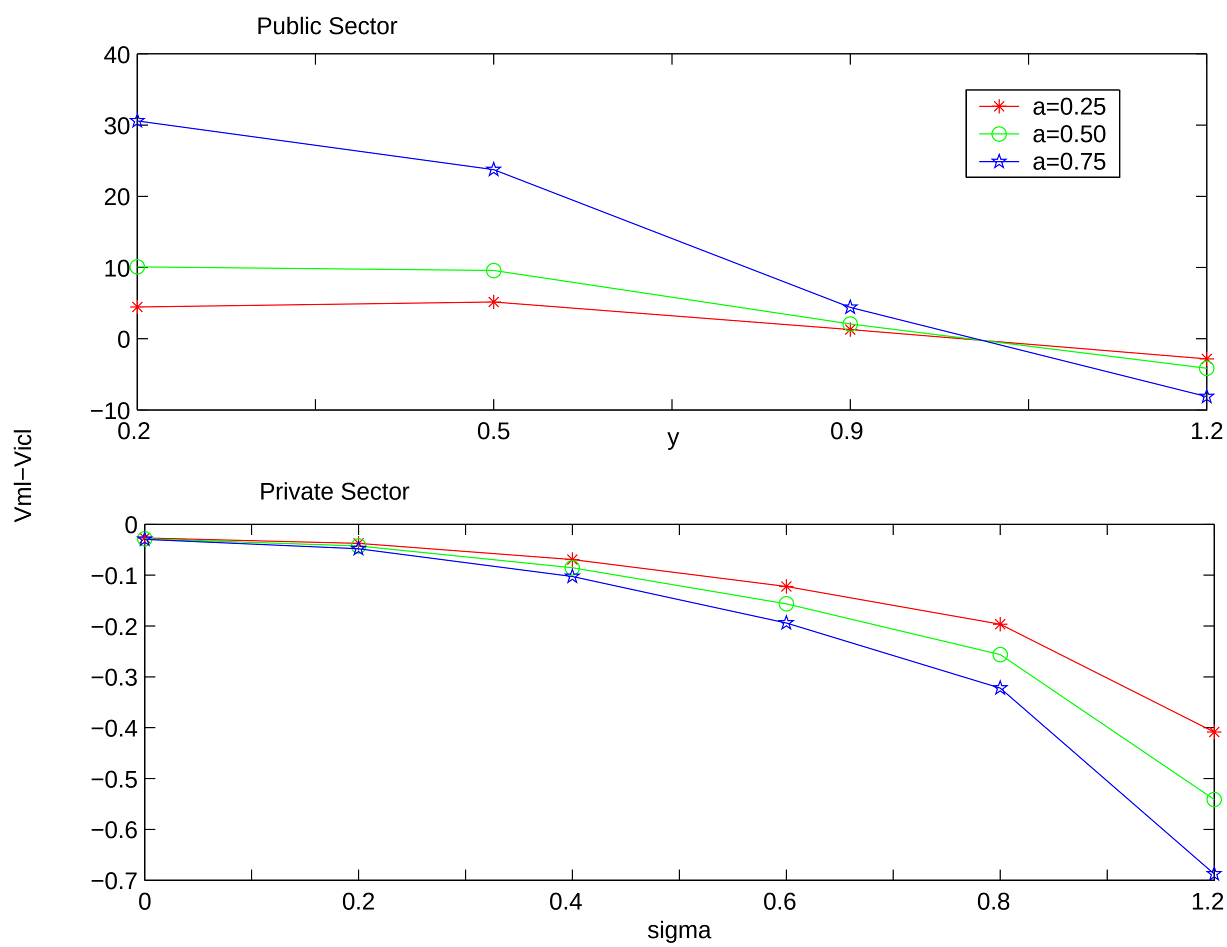


Figure 3: Public Sector Increasing Income and Tml
—— Private Sector Increasing Uncertainty and Tml

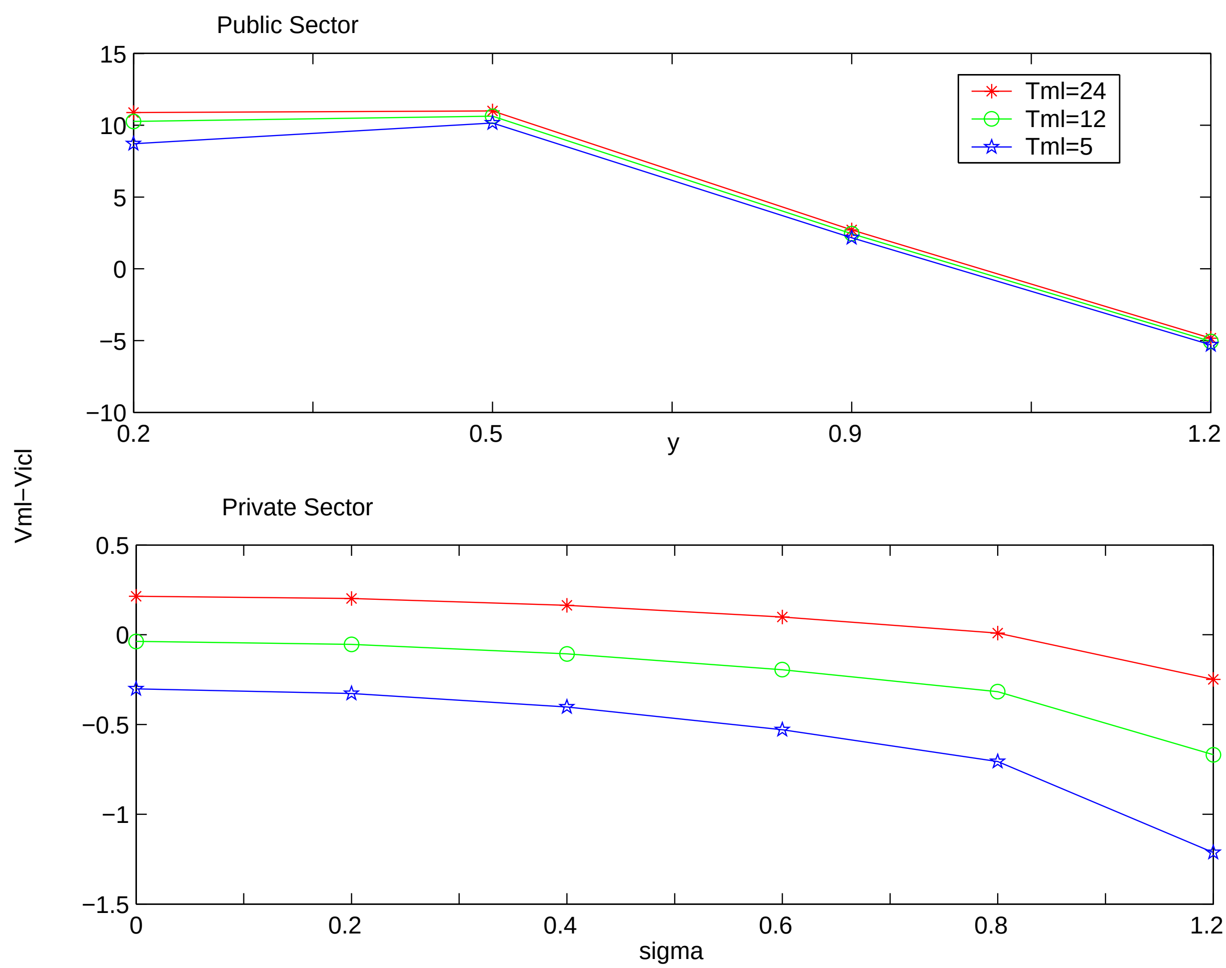


Figure 4: ML vs ICL - Difference in the Two Sectors

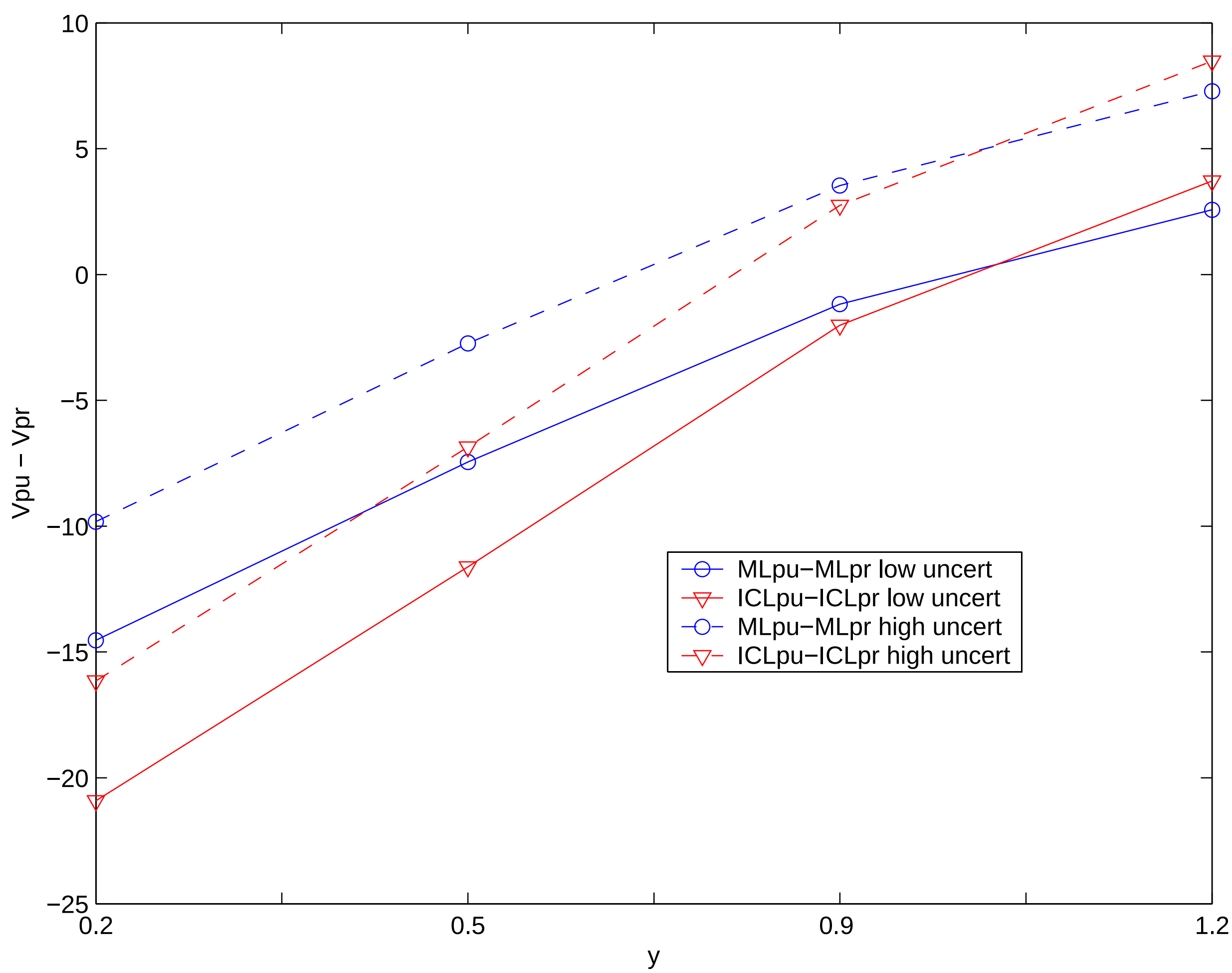
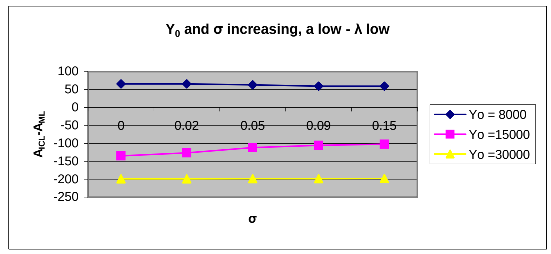


Table 11: $A_{ICL} - A_{ML}$ - Risk Aversion and Deterministic Growth

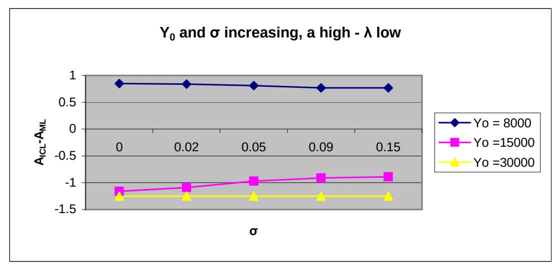
a = 0.25 φ = £1000 ρ = 8% λ = 1%

	σ				
Y_o	0	0.02	0.05	0.09	0.15
8000	65.94	66.06	63.19	59.62	59.41
15000	-134.92	-126.37	-111.66	-105.49	-102.42
30000	-198.84	-198.79	-198.7	-198.55	-198.23



a = 0.75 φ = £1000 ρ = 8% λ = 1%

	σ				
Y_o	0	0.02	0.05	0.09	0.15
8000	0.85	0.84	0.81	0.77	0.77
15000	-1.16	-1.09	-0.97	-0.91	-0.89
30000	-1.25	-1.25	-1.25	-1.25	-1.25



a = 0.25 φ = £1000 ρ = 8% λ = 4%

	σ				
Y_o	0	0.02	0.05	0.09	0.15
8000	7.59	16.22	23.68	26.2	27.48
15000	-128.48	-128.42	-128.29	-128.08	-127.08
30000	-245.57	-245.53	-245.44	-245.29	-244.99

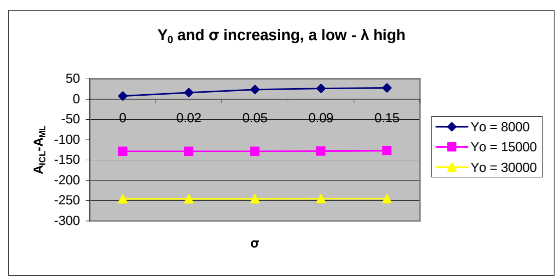
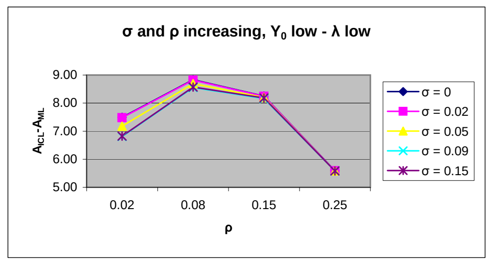


Table 12: $A_{ICL} - A_{ML}$ - Uncertainty and Subjective Discount Factor

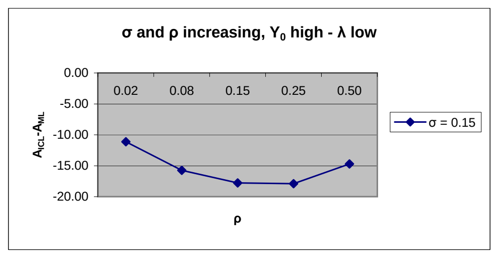
a = 0.5 Y_0 = £ 8000 φ = £1000 λ = 1% γ = 9%

	σ				
ρ	0	0.02	0.05	0.09	0.15
0.02	2.68	2.66	1.99	1.17	1.11
0.08	7.49	7.48	7.18	6.81	6.82
0.15	8.83	8.82	8.69	8.55	8.57
0.25	8.25	8.24	8.20	8.16	8.18
0.50	5.59	5.59	5.58	5.58	5.58



a = 0.5 Y_0 = £ 30000 φ = £1000 λ = 1% γ = 9%

	σ				
ρ	0	0.02	0.05	0.09	0.15
0.02	-11.12	-11.13	-11.13	-11.13	-11.12
0.08	-15.75	-15.76	-15.76	-15.76	-15.75
0.15	-17.79	-17.79	-17.80	-17.80	-17.79
0.25	-17.91	-17.91	-17.91	-17.91	-17.91
0.50	-14.69	-14.69	-14.69	-14.70	-14.70



a = 0.5 Y_0 = £ 8000 φ = £1000 λ = 4% γ = 9%

	σ				
ρ	0	0.02	0.05	0.09	0.15
0.02	-4.32	-2.70	-1.32	-0.89	-0.64
0.08	1.63	2.45	3.14	3.36	3.50
0.15	4.33	4.71	5.03	5.14	5.21
0.25	5.19	5.33	5.45	5.48	5.52
0.50	4.24	4.25	4.26	4.27	4.27

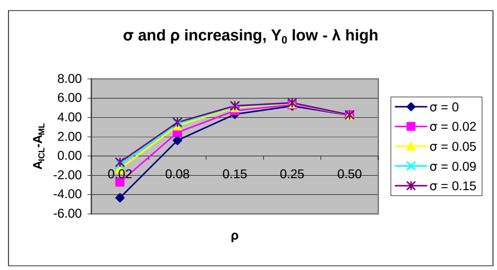


Table 13: $A_{ICL} - A_{ML}$ - ICL and ML Parameters

a = 0.5 φ = £ 500 T_{ML} = 24 ρ = 8% γ = 9% λ = 1%

	σ				
Y_o	0	0.02	0.05	0.09	0.15
8000	-19.65	-18.91	-18.58	-18.50	-18.40
15000	-31.50	-31.50	-31.51	-31.36	-30.96
30000	-35.97	-35.98	-35.98	-35.98	-35.86

a = 0.5 φ = £ 2400 T_{ML} =5 ρ = 8% γ = 9% λ = 1%

	σ				
Y_o	0	0.02	0.05	0.09	0.15
8000	38.08	38.82	39.15	39.22	39.30
15000	7.61	7.60	7.60	7.74	8.12
30000	-9.40	-9.40	-9.41	-9.41	-9.30

a = 0.5 σ =5% ρ = 8% Y_0 = 8000 λ = 1%

	γ			
φ	0.05	0.09	0.15	0.3
500	4.64	-18.58	-40.07	-62.29
600	11.93	-11.29	-32.78	-55.00
800	23.42	0.20	-21.29	-43.51
1000	26.19	7.18	-8.22	-39.75
1200	38.81	15.59	-5.90	-28.12
2400	62.37	39.15	17.66	-4.57

a = 0.5 σ =5% ρ = 8% Y_0 = 8000 λ = 4%

	γ			
φ	0.05	0.09	0.15	0.3
500	-1.61	-22.52	-44.04	-72.18
600	5.52	-15.39	-36.91	-65.06
800	17.02	-3.90	-25.42	-53.56
1000	21.67	3.14	-15.75	-30.60
1200	32.79	11.87	-9.64	-37.79
2400	57.30	36.38	14.87	-13.28

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