

12-15-2007

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COORDINATION OF INVENTORY AND TRANSPORTATION DECISIONS
IN A TWO-STAGE SUPPLY CHAIN

By

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A Dissertation
Submitted to the Faculty of
Mississippi State University
in Partial Fulfillment of the Requirements
for the Degree of Doctor of Philosophy
in Industrial and Systems Engineering
in the Department of Industrial and Systems Engineering

Mississippi State, Mississippi

August 2007

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2007

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DECISIONS IN A TWO-STAGE SUPPLY CHAIN

Pages in Study: 95

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In this research, we focus on a two-stage supply chain with alternative supply sources. We deal with a problem in which the buyers do not trust sellers and ask for timely shipments. In the problem, buyers are located in two regions and each region consists of a single distribution center (*DC*) and multiple retailers. Retailers place orders to *DCs* whereas *DCs* replenish their inventory from outside suppliers. As retailers place orders to *DCs*, *DCs* do not send items immediately to retailers. As a consequence, retailers might have to endure a long and uncertain wait. Two reasons for late deliveries from *DCs* to retailers are shipment consolidation and being out of stock.

We approach to the problem from two different perspectives; retailers' perspective and supply chain perspective. When we approach the problem from a retailer's point of view,, we model the problem as a *DC* selection problem. When a retailer is ready to replenish its inventory, that retailer must decide whether it should replenish from the first or second *DC*. We develop a decision rule that minimizes the expected cost for the retailer that is about

to replenish its inventory; then retailers repeatedly use this decision rule as a heuristic. The decision rule forms the basis of our proposed ordering policy. A simulation study, which compares the proposed policy to three traditional ordering policies illustrates how the proposed policy performs under different conditions. Overall, the numerical analysis shows, over a large set of scenarios, that the proposed policy performs better than the other three policies. The results of parametric analysis indicates that the performance of the proposed policy can be further improved by adjusting the batch sizes and the reorder points.

When we approach the problem from the supply chain perspective, we focus on a shipment consolidation problem together with the alternative sourcing problem. To develop two models, we introduce a “promised latest delivery time” concept that is as follows: The promised latest delivery time is the end of shipment consolidation time that the *DC* provides to retailers at the time of order placement. Numerical experiments suggest that a shipment consolidation policy is more beneficial when cost of transportation high and backlog cost is per unit time is relatively low. In addition, using an alternative supply source is especially beneficial for the supply chain when the cost of serving to both regions is not very high compared to serving to a single region.

Key words: inventory transportation coordination, shipment consolidation, alternative sourcing

DEDICATION

To my parents...

ACKNOWLEDGMENTS

I would like to thank my advisor and dissertation chair, Burak Ekşiođlu, for his suggestions, encouragement and insightful comments throughout my graduate study and dissertation research.

I acknowledge my dissertation committee members, Sandra D. Ekşiođlu, Joseph Geunes, and Mingzhou Jin, for their constructive criticism concerning the material of this dissertation and for all the effort they have devoted to the supervision of this research.

I also would like to verbalize my appreciation to all of my sincere friends in Starkville, especially to Gzde Ađırbaş.

Last but not least, I would like to thank my parents, Glay and Mamo apar, my brother, İbrahim, and sister, İnsaf, for their love, trust, and continuous support throughout my life.

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CHAPTER 1

INTRODUCTION

1.1 Coordination of Inventory and Transportation Decisions

In today's competitive business world, companies face the crucial task of delivering goods on time. In order to benefit from high volumes, distributors (suppliers) try to consolidate deliveries while maintaining a high percentage of on time delivery. Increasing delivery frequencies typically increases suppliers' performance, but it also increases the transportation costs. Finding a suitable middle ground is a challenge confronting many companies throughout various supply chains. This dissertation is a study about the coordination of inventory and transportation decisions in a two-stage supply chain.

1.1.1 Motivation

The problem of coordinating inventory and transportation decisions is observed in many supply chains but our particular experience is with an automotive spare parts distribution company. The company has divided the service area into two regions. Each region has a distribution center (*DC*) and several retailers scattered throughout the region. Retailers place orders to the regional *DC*, who in turn replenish their inventory from outside suppliers. *DCs* do not dispatch orders placed by retailers immediately. As a consequence,

retailers might have to endure a long and uncertain wait. Two reasons for late deliveries are consolidating shipments and being out of stock.

Currently shipment dispatching decisions are based on experience and lack a structured approach to determine a consolidation period. This gives rise to situations in which *DCs* are unable to provide firm delivery dates leading to difficult inventory management problems for retailers. We analyze such an environment in which there are two *DCs* and two retailers, and provide models to improve the decision making and enhance overall performance in terms of both cost and timely delivery.

Our objective is twofold. Firstly, to evaluate a two stage supply chain with alternative supply sources from retailers' point of view and to develop a tool that can be easily employed by retailers to achieve cost and waiting time reduction. Secondly, to analyze the overall supply chain and to develop models that consider dual source of supply opportunity and shipment consolidation simultaneously to coordinate transportation and inventory decisions.

1.1.2 Overview

We approach the problem from two different perspectives. Firstly, we analyze the situation from retailers' point of view. Based on these findings, we focus on the overall supply chain. We assume there is a fixed lead time from *DCs* to retailers and model the problem as a *DC* selection problem. Secondly, we analyze the problem from a supply chain perspective and calculate a consolidation time that minimizes the expected cost per unit time for the whole supply chain. Each chapter can be read on its own and includes motivation, problem

definition, assumptions, and a review of literature where necessary. We conclude each chapter with a discussion of possible future work. The following subsections provide an overview of the chapters.

1.1.2.1 A Decision Rule for Coordination of Inventory and Transportation Decision in a Two-Stage Supply Chain with Alternative Supply Sources

Chapter 2 focuses on the problem from retailers' point of view. We develop a decision rule that minimizes the expected cost for retailers wishing to replenish their inventory. Retailers repeatedly use this decision rule as a heuristic which forms the basis of our proposed ordering policy. Our simulation study illustrates how the proposed policy performs under different conditions. Overall, the numerical analysis shows that the proposed policy performs better than the others over a large set of scenarios.

1.1.2.2 Analysis of Ordering Policies in a Two-Stage Supply Chain with Alternative Supply Sources

Numerical analysis in the previous chapter illustrates that the decision rule based ordering policy outperforms traditional ordering policies. This however, is not the best policy under all circumstances. In chapter 3, we present additional analysis to determine settings in which the decision rule based ordering policy is most beneficial to retailers. By changing parameters such as inventory holding cost, backlog cost, end customer arrival rate, ordering cost, and lead time from distribution centers to retailers; we identify the supply chain settings to employ the proposed decision rule based ordering policy.

1.1.2.3 Coordination of Inventory and Transportation Decisions via Shipment Consolidation

Chapter 4 focuses on the overall supply chain. We suggest two models to coordinate inventory and transportation decisions via shipment consolidation. Each model considers the total expected cost of supply chain and determines the delivery time of shipments from *DCs* to retailers. The first model assumes that a retailer is assigned to a *DC* and can be served by only the assigned *DC*. The second model places no such constraints. Numerical experiments present how the proposed models perform under different conditions compared to two other policies from the literature.

1.1.3 Contribution

Dual or alternative sourcing and shipment consolidation have been studied for decades. Thus, many aspects of these topics have already been covered by earlier work. We outline our contribution in the following paragraphs.

We introduce a decision rule based ordering policy for a retailer which aims to decrease its cost by comparing the expected cost of replenishing from two distribution centers. We analyze a two stage supply chain and derive a decision rule which takes into consideration lead time from distribution centers to retailers. Existing literature either assumes transportation to be instant or chooses single stage supply chain for analysis.

Unlike existing literature, we focus on overall supply chain performance rather than just the distributor side or the retailer side. Recent shipment consolidation publications assume that transportation from outside supplier to distributors takes no time to come up

with tractable algorithms ([4], [11], [13]). We propose a model which handles the lead time issue. Finally, to the best of our knowledge, the problem of shipment consolidation in connection with dual sourcing has not been addressed earlier. By considering both approaches we extend the current knowledge.

CHAPTER 2

A DECISION RULE FOR COORDINATION OF INVENTORY AND TRANSPORTATION DECISIONS

2.1 Introduction

Coordinating inventory and transportation decisions is a challenging problem faced by many companies throughout various supply chains. In this paper, we study the coordination of inventory and transportation decisions in a two-stage supply chain. The primary motivation for this research comes from a real-life problem. One of the authors worked for an automobile manufacturer as a “Logistics Project Engineer,” and he analyzed the existing processes for spare parts distribution. The spare parts were supplied from two distribution centers (DCs) to multiple “service stations.” The company uses the term “service stations” to designate retailers in the supply chain; then the company divided the country into two regions, and service stations in each region were assigned to a specific DC which served only that area. The service stations were not allowed to receive parts from the DC in the other region. Consequently, service stations commonly complained because in some cases they had to wait for their orders for a long time even if the other DC had the items in stock. As a result, the service stations requested the opportunity to place their orders not only

from the DC they were assigned to, but also from the other DC. They were claiming that this process of “order switching” would potentially lower their costs.

We investigate the DC selection problem described above to analyze whether or not order switching is a good decision to improve supply chain performance in the long run. Specifically, we consider a two-stage supply chain with two retailers and two DCs. Each supply chain member uses a (Q, R) inventory policy, and the retailers face independent Poisson end customer demand. Each DC is able to serve all retailers on a first-come-first-served basis, but the transportation cost is smaller if the retailer to be served lies within the DC’s service area. Our goal is to provide an ordering policy for the retailers and to compare the effectiveness of our policy to the current policy used by the company. Our computational results also provides comparisons of our policy to two other traditional ordering policies. The current ordering policy used by the above-mentioned automotive manufacturer will be called OP_1 , and the new, proposed policy will be referred to as OP_4 . We will use OP_2 and OP_3 to represent the other two ordering policies discussed in the literature. The structure of the supply chain and the four ordering policies are described below in more detail.

2.1.1 The ordering policy used by the auto manufacturer - OP_1

Based on the simple rule currently implemented by the above-mentioned auto manufacturer, the retailers cannot make any DC selection decisions. The retailers are assigned to a DC and must replenish their inventories from that DC. This policy of dedicated DCs will be referred to as OP_1 . For the sake of simplicity, assume that two retailers and two DCs

exist and that R_1 is assigned to DC_1 and R_2 to DC_2 as shown in Figure 2.1. This assignment is based on the assumption that the lead time and the transportation cost from DC_1 to R_1 are smaller than the corresponding time and cost from DC_2 to R_1 . Similarly, the lead time and the transportation cost from DC_2 to R_2 are smaller than the corresponding time and cost from DC_1 to R_2 .

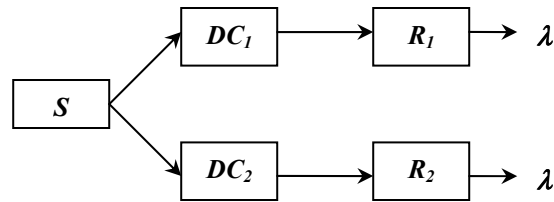


Figure 2.1

Structure of the supply chain under OP_1

2.1.2 The second ordering policy - OP_2

Under the second ordering policy, each retailer is assigned to a DC, like in the first case. However, unlike OP_1 , under OP_2 retailers may switch their orders from one DC to the other. Figure 2.2 shows the structure of the supply chain under the second ordering policy. For example, if DC_1 is out of stock, then R_1 replenishes its inventory from DC_2 , provided that DC_2 has inventory on hand. If DC_2 is also out of stock, then R_1 waits for DC_1 to deliver the products.

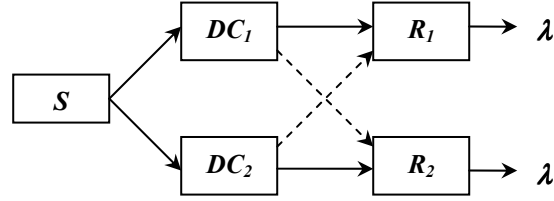


Figure 2.2

Structure of the supply chain under OP_2

2.1.3 The third ordering policy - OP_3

Under the third ordering policy, the retailers are not necessarily assigned to specific DCs. Figure 2.3 shows the structure of the supply chain under the third ordering policy. The retailers make their DC selection decisions based on the earliest delivery time. For example, if R_1 is ready to place an order it will contact both DC_1 and DC_2 and purchase the items from the DC that can deliver earlier. The following example illustrates how OP_3 differs from the DC that can deliver earlier. The following example illustrates how OP_3 differs from OP_2 : Assume that when R_1 is ready to place an order, DC_1 is out of stock and DC_2 has inventory on hand. In such a situation, R_1 would purchase the items from DC_2 under OP_2 . However, under OP_3 , if DC_1 can deliver the items earlier than DC_2 , R_1 would purchase the items from DC_1 although DC_1 is out of stock.

2.1.4 The proposed ordering policy - OP_4

The proposed ordering policy is similar to the third ordering policy discussed above, in the sense that the retailers are not assigned to specific DCs. Therefore the structure of the supply chain can be represented as in Figure 2.3. However, under OP_4 , the retailers make their DC selection decisions based on expected total cost. In other words, the decision rule used by the retailers that select a DC under OP_4 demands a simple comparison of the

expected costs of ordering from DC_1 versus DC_2 . The total cost includes the expected holding cost, the expected backlog cost, the fixed ordering cost, and the transportation cost. The derivation of the expected costs and details of the decision rule appear in Section 2.4.

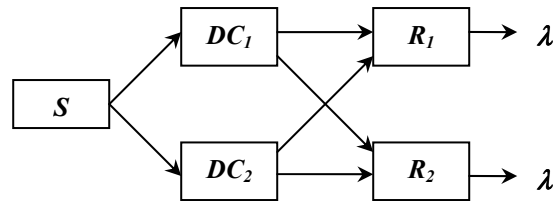


Figure 2.3

Structure of the supply chain under OP_3 and OP_4

2.2 Review of Related Literature

Literature related to this work can be divided into three main sub-categories: i) research related to (Q, R) continuous review inventory policies, ii) research related to lateral transshipments among members of the same echelon within a supply chain, and iii) research related to dual/alternative sourcing. The focus of this study falls under the dual/alternative sourcing category, even though, as discussed later, similarities between lateral transshipment and dual/alternative sourcing research studies do exist.

One of the challenging tasks in continuous review inventory problems is to find the order quantity Q and the reorder point R that minimizes cost, subject to some constraints. Although they are all the same, many different representations of this inventory model exist. For example, [9] use (Q, r) , [23] use (Q, R) , [2, 3] and [29] use (R, Q) to represent

the same inventory model. Some research studies, such as the one by [14], use (R, nQ) because of the assumption that the order quantity (nQ) is a multiple of the minimum batch size (Q), where n is the minimum integer required to increase the inventory position to a level above R . Two of the most distinctive attributes of (Q, R) models are as follows:

1. Types of supply chains: While some studies only consider one entity that uses a (Q, R) policy (e.g. [1, 9, 23]), others consider a multi-echelon inventory system (e.g. [2, 3, 14, 29]).
2. Exact or near-optimal evaluations: The (Q, R) inventory problems are not necessarily easy to solve. Thus, many of the research papers either provide approximate solution approaches or try to find bounds on the optimal solution (e.g. [1, 2, 7, 9, 31]). On the other hand, only a small number of articles give an exact evaluation of a problem specific (Q, R) inventory system (e.g.[3, 17, 33]).

Although the focus of our study is not to find the best Q and R values for the supply chain; our numerical analysis, for a small subset of the problem set, determines the best Q and R values for the retailers via a simulation study.

Research papers that study lateral transshipments among members of the same echelon typically assume that retailers replenish their inventory from a single DC while DCs can procure products either from an outside source or from other DCs located within the supply chain's same stage. The reasons for allowing lateral transshipments include maintaining high service levels, decreasing backlog costs, and reducing lost sales in case of stockouts. [32] studied a periodic inventory system and analyzed the effect of emergency lateral transshipments between two retailers, not DCs, when different inventory pooling policies are used. [19] also analyzed emergency lateral transshipments between retailers in a model with more than two retailers that use the $(S - 1, S)$ inventory policy. They developed a heuristic method to find a near-optimal control parameter S for the model and

then performed a simulation study to verify the results. Like [19], [5] also evaluated a new decision rule for lateral transshipments. Axsater's decision rule minimizes the expected cost, which includes the lateral transshipment cost and future cost differences and uses the assumptions that each player uses a (Q, R) inventory policy and no further lateral transshipments will take place.

The research studies related to the dual/alternative sourcing category have focused on order splitting quantities, lead time reduction opportunities, and alternative sourcing versus single sourcing comparisons. [30] studied a two-stage supply chain with two retailers and two warehouses and assumed that each facility uses a (Q, R) inventory policy. They analyzed three ordering policies, and via a simulation study, they determined, based on total cost, the best Q and R parameters for each supply chain entity. [28, 27] and [15] analyzed alternative sourcing strategies both with dual sources and a single source to compare respective costs. Following these studies, [7] analyzed a (Q, R) inventory system with lead time and an expediting factor to decrease the lead time. [31] worked on topics similar to [7], but they included an order splitting proportion to the problem. [26] analyzed another supply chain environment where only one supplier with alternative transportation modes is available. Based on inventory and transportation costs, [26] proposed a model that helps with transportation mode selection after order placement.

As mentioned above, our study falls under the alternative sourcing category. However, there are similarities between dual/alternative sourcing and lateral transshipments. For example, in the supply chain shown in Figure 2.4.a, R_1 faces an alternative sourcing problem. In other words, every time R_1 needs to replenish its inventory, it decides which

DC to select. On the other hand, Figure 2.4.b reveals that R_1 always replenishes its inventory from DC_1 , but here DC_1 faces a lateral transshipment problem. From a managerial and operational point of view, alternative sourcing and lateral transshipment problems are quite different because of the questions of, “who pays the additional cost?” and “who fulfills the order?” In Figure 2.4.a it is reasonable to assume that R_1 pays the additional cost because it decides if the order will come from DC_1 or DC_2 . Also in Figure 2.4.a, an upstream supply chain member fulfills the order. In Figure 2.4.b, however, it is reasonable to assume that DC_1 pays the additional cost because it receives a lateral transshipment from DC_2 ; i.e. the order is fulfilled from another source within the same stage of the supply chain. From a modeling and overall supply chain point of view, though, alternative sourcing and lateral transshipment problems are very similar because ultimately the demand of R_1 in Figure 2.4 will be satisfied either directly from DC_1 or from DC_2 in the form of a lateral transshipment or a direct transshipment.

This paper analyzes four different ordering policies: OP_1 , OP_2 , OP_3 , and OP_4 , as described in Section 4.1. [30] also analyzed three of these policies: OP_1 , OP_2 , and OP_3 . Their study compared the costs of OP_1 , OP_2 , and OP_3 based on a simulation study, and they showed that none of the policies dominated the others. Our study derives a decision rule that can easily be used in practice, and we propose a new policy, OP_4 , based on this decision rule. Our study is also similar to the study by [5] in the sense that he also provided a decision rule, but the two studies have some major differences. For example, [5] considered a single stage supply chain and assumed that lateral transshipments between DCs are

instant. However, this paper analyzes a two-stage supply chain where transportation times are not zero.

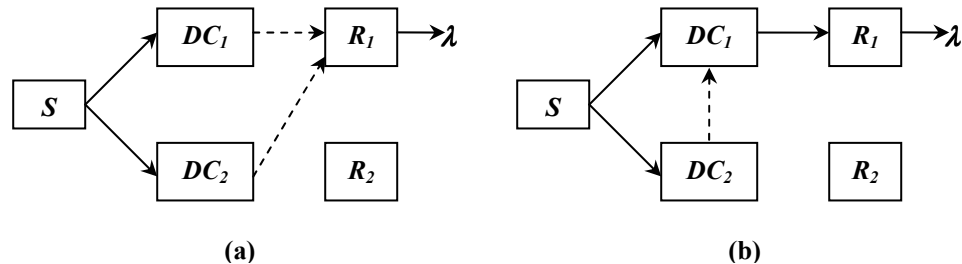


Figure 2.4

Comparison of dual/alternative sourcing to lateral transshipment

The rest of the paper is organized as follows: Section 2.3 presents the details of the problem description and formulation; Section 2.4 provides the derivation of the decision rule; Section 4.3 presents the results of our extensive numerical study; and finally, Section 4.4 gives some concluding remarks and directions for further research.

2.3 Problem Description and Formulation

The supply chain analyzed is a two-stage supply chain with two distribution centers (DCs) and two retailers, as shown in Figure 2.3. The DCs replenish their inventory from an outside supplier that has infinite capacity. The supplier delivers the items after L time units. When the inventory position, i.e. inventory on hand plus outstanding orders minus backorders, at a DC declines to or below R , the DC orders a batch of Q items such that the resulting inventory position is larger than R and not larger than $R + Q$. The retailers are identical and face independent Poisson end customer arrivals. Each customer arrives

with a unit demand. Every time the inventory position at a retailer declines to or below r , the retailer orders a batch of q items, such that the resulting inventory position is between $[r+1, r+q]$. The retailer must decide whether the items will be ordered from DC_1 or DC_2 . Note that under OP_1 , as described in Section 2.1.1, the retailer does not have to make a decision. In other words, R_1 always orders from DC_1 and R_2 from DC_2 . Under OP_2 , R_1 always orders from DC_1 as long as DC_1 has inventory on hand. The only time R_1 would order from DC_2 is when DC_1 is out of stock and DC_2 has inventory on hand. Under OP_3 , the retailers will order from the DC that promises an earlier delivery time. Based on the assumption made by the auto manufacturer mentioned above, the lead time from DC_1 to R_1 is shorter than the lead time from DC_2 to R_1 . However, the actual delivery time from DC_1 to R_1 could be longer if DC_1 is out of stock.

Our goal is to develop a new ordering policy, named OP_4 , based on a decision rule in which expected costs of ordering from DC_1 and DC_2 are calculated respectively. Since the new policy will be used by the retailers, the supply chain is analyzed from the view point of R_1 . The analysis is analogous for R_2 . Under OP_4 , every time that R_1 needs to order a new batch of items, it calls both DCs to find out when each one will be able to deliver the items, and makes a DC selection. In our problem formulation, e will be used to denote the early delivery option and l will be used for the late delivery option. For example, DC_e (DC_l) will denote the distribution center that promises the earlier (later) delivery time. Assume that R_1 needs to make a DC selection decision at time t . Also assume, without loss of generality, that $t = 0$. Based on this information and using the

following notation, the expected cost of ordering the items from DC_e is calculated and compared to the expected cost of ordering the items from DC_l .

- q = batch quantity at R_1 ,
- r = reorder point at R_1 ,
- λ = customer arrival rate at R_1 ,
- h = inventory holding cost per item per unit time at R_1 ,
- b = backlog cost per item per unit time at R_1 ,
- T_e = arrival time of the new batch to R_1 if ordered from DC_e ,
- T_l = arrival time of the new batch to R_1 if ordered from DC_l ,
- T_j = arrival times of the previously ordered batches to R_1 ,
- m = number of previously ordered batches that will arrive to R_1 at or before T_e ,
- n = number of previously ordered batches that will arrive to R_1 at or before T_l but after T_e ,
- IL = inventory level (inventory on hand minus backorders) at R_1 at the time R_1 is ready to place an order,
- s_e = fixed cost of ordering and transporting a batch of q items from DC_e to R_1 ,
- s_l = fixed cost of ordering and transporting a batch of q items from DC_l to R_1 ,
- $inv_e(te_i)$ = expected inventory holding cost associated with an item that reaches R_1 at time te_i to satisfy the i^{th} customer demand when the early delivery option is chosen,
- $back_e(te_i)$ = expected backlog cost associated with an item that reaches R_1 at time te_i to satisfy i^{th} customer demand when the early delivery option is chosen,
- $inv_l(tl_i)$ = expected inventory holding cost associated with an item that reaches R_1 at time tl_i to satisfy the i^{th} customer demand when the late delivery option is chosen,
- $back_l(tl_i)$ = expected backlog cost associated with an item that reaches R_1 at time tl_i to satisfy i^{th} customer demand when the late delivery option is chosen.

2.4 Derivation of the Decision Rule

Our decision rule makes an informed DC selection decision using the available information. Since our analysis uses the viewpoint of R_1 , Figure 2.5 shows the batch arrival times at R_1 . When R_1 is ready to order a new batch of items, it knows the arrival times of all previously ordered items, as well as the arrival time of the new batch that will be ordered either from DC_e or DC_l . Clearly, all previously ordered items must arrive at or before T_l .

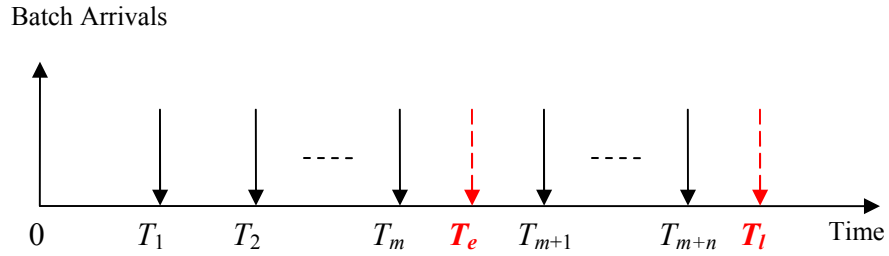


Figure 2.5

Batch arrivals at R_1

Using the batch arrival information and equations (2.1) and (2.2) below, R_1 can calculate the expected inventory holding and backlog costs associated with all the items that are either on hand or will arrive at or before T_l :

$$inv_e(te_i) = h \left[\int_{te_i}^{\infty} (u - te_i) f^i(u) du \right], \quad (2.1)$$

$$back_e(te_i) = b \left[\int_0^{te_i} (te_i - u) f^i(u) du \right], \quad (2.2)$$

where u is the arrival time of the i^{th} customer to R_1 and $f^i(u)$ is the corresponding probability density function. Similar equations can be developed to calculate $inv_l(tl_i)$ and $back_l(tl_i)$. Since customer arrivals to R_1 follow a Poisson distribution, the time when the i^{th} customer arrives at R_1 has an Erlang(λ, i) distribution with the following density and cumulative distribution functions, respectively:

$$f^i(t) = \frac{\lambda^i t^{i-1} e^{-\lambda t}}{(i-1)!}, \quad (2.3)$$

$$F^i(t) = \int_0^t f^i(u) du = 1 - \sum_{k=0}^{i-1} \frac{(\lambda t)^k e^{-\lambda t}}{k!}. \quad (2.4)$$

It is also convenient at this point to introduce the following equation that will be used in expected cost calculations. The equation's derivation appears in the Appendix A.

$$G^i(t) = \int_0^t u f^i(u) du = \frac{i}{\lambda} F^{i+1}(t). \quad (2.5)$$

Cost calculations will change depending on the sign of IL . If $IL \geq 0$ then R_1 has inventory on hand, and if $IL < 0$ then R_1 has backorders. Therefore, these two cases will be analyzed separately.

2.4.1 Cost calculations when $IL \geq 0$

If $IL \geq 0$, then R_1 has inventory that can be used to satisfy the demands of the first IL customers that will arrive at R_1 . Thus, items already at R_1 will have no backlog costs

associated with them. However, the q items that R_1 will order at time $t = 0$ and the $(m + n)q$ previously ordered items will either have backlog or holding costs associated with them depending on when the item and its corresponding demand arrive at R_1 . Let Δ be the difference in the expected inventory and backlog costs of ordering the new batch of q items from DC_l versus DC_e where

$$\Delta = \sum_{i=1}^{IL} [inv_l(tl_i) - inv_e(te_i)] + \sum_{i=IL+1}^{IL+(m+n+1)q} [inv_l(tl_i) - inv_e(te_i)] + \sum_{i=IL+1}^{IL+(m+n+1)q} [back_l(tl_i) - back_e(te_i)]. \quad (2.6)$$

The expression for Δ can be simplified to the following equation in which $T_e = T'_0$, $T_{m+1} = T'_1$, $T_{m+2} = T'_2$, \dots , $T_{m+n} = T'_n$, and $T_l = T'_{n+1}$. The details are provided in the Appendix B.

$$\Delta = qhT'_0 - qhT'_{n+1} + (h + b) \sum_{j=0}^n \sum_{i=IL+mq+jq+1}^{IL+mq+jq+q} \left(\frac{i}{\lambda} F^{i+1}(T'_j) - \frac{i}{\lambda} F^{i+1}(T'_{j+1}) + T'_{j+1} F^i(T'_{j+1}) - T'_j F^i(T'_j) \right). \quad (2.7)$$

2.4.2 Cost calculations when $IL < 0$

If $IL < 0$, then R_1 has backorders which means that the first $|IL|$ items R_1 will receive will satisfy the demand that has occurred prior to the current time, $t = 0$. Thus, there will be backlog costs associated with the first $|IL|$ items. If $|IL| \geq (m + n + 1)q$, then the previously ordered items, as well as the new items that will be ordered by R_1 , will all be used to satisfy past demand. Otherwise, either backlog or holding costs may be associated

with previously ordered items and the new items that R_1 will order. The equation for Δ in this case will be as follows:

$$\Delta = \sum_{k=1}^{|IL|} [btl'_k - bte'_k] + \sum_{i=1}^{IL+(m+n+1)q} [inv_l(tl_i) - inv_e(te_i)] + \sum_{i=1}^{IL+(m+n+1)q} [back_l(tl_i) - back_e(te_i)], \quad (2.8)$$

where tl'_k and te'_k respectively show the arrival times of item k to R_1 in relation the “late” and “early” options. As also shown in the appendix, equation 2.8 can be rewritten as follows:

$$\Delta = b \sum_{k=1}^{|IL|} [tl'_k - te'_k] + \sum_{i=1}^{IL+(m+n+1)q} [h \frac{i}{\lambda} - h \frac{i}{\lambda} F^{i+1}(tl_i) - htl_i + htl_i F^i(tl_i)] - \sum_{i=1}^{IL+(m+n+1)q} [h \frac{i}{\lambda} - h \frac{i}{\lambda} F^{i+1}(te_i) - hte_i + hte_i F^i(te_i)] + \sum_{i=1}^{IL+(m+n+1)q} [btl_i F^i(tl_i) - b \frac{i}{\lambda} F^{i+1}(tl_i)] - \sum_{i=1}^{IL+(m+n+1)q} [bte_i F^i(te_i) - b \frac{i}{\lambda} F^{i+1}(te_i)]. \quad (2.9)$$

The expression for Δ given by (2.9) can be further simplified and is provided in the appendix. However, this simplification depends on the value of IL .

DECISION RULE: If $(s_l - s_e + \Delta) < 0$, then order the new batch from DC_l ; otherwise order it from DC_e .

In the proposed ordering policy, OP_4 , the above decision rule is used repeatedly as a heuristic every time a retailer has to select a DC to replenish inventory.

2.5 Numerical Analysis

Our ordering policy, OP_4 , has been evaluated and compared to OP_1 , OP_2 , and OP_3 in a simulation study. The supply chain system represented in Figure 2.3 has been simulated for 10,000 time units using two sets of parameters as shown in Table 2.1. Most of these values are similar to the ones used previously in the literature (e.g. NLC01, Axs03).

Table 2.1

The low and high values for the parameters

	q	r	λ	h	b	s_1	s_2	L_1	L_2	Q	R	H	B	L	O
Low	4	2	1	3	6	30	50	2	5	8	0	1.2	3.5	12	50
High	8	6	5	5	10	45	80	4	9	24	16	2.8	5.9	24	100

The definitions for q , r , λ , h , b , s_1 , and s_2 appeared in Section 2.3. Note, however, that in our analysis, R_1 and R_2 are identical. Also, definitions for s_e and s_l were provided in Section 2.3, but s_e and s_l are essentially the same as s_1 and s_2 . For example, if DC_1 promises an early delivery, then $s_e = s_1$, otherwise $s_e = s_2$. The rest of the parameters in Table 2.1 are defined as follows:

- L_1 = lead time from DC_1 to R_1 (and from DC_2 to R_2),
- L_2 = lead time from DC_2 to R_1 (and from DC_1 to R_2),
- Q = batch quantity at the DCs,
- R = reorder point at the DCs,
- H = inventory holding cost per item per unit time at the DCs,
- B = backlog cost per item per unit time at the DCs,
- L = lead time from the outside supplier to the DCs,
- O = fixed cost of ordering and transporting a batch of q items from the outside supplier, S , to DC_1 (and from S to DC_2).

By keeping track of the events throughout the simulation, the costs incurred by DC_1 , DC_2 , R_1 , and R_2 are individually calculated. Events such as arrival of a customer to R_1 or R_2 , arrival of a retailer order to DC_1 or DC_2 , and arrival of a batch of items to a DC or a retailer will change the state of the system. For each entity in the supply chain, all costs are initialized to zero and each time the state of the system changes, holding, backlog, ordering, and transportation costs are updated. For example, if the last two events occurred at times t_0 and t_1 , then the inventory holding cost of R_1 , $HC_{R_1}(\cdot)$ can be updated by $HC_{R_1}(t_1) = HC_{R_1}(t_0) + IL_{R_1}(t_0) * (t_1 - t_0)h$ if $IL_{R_1}(t_0) \geq 0$, where $IL_{R_1}(t_0)$ is the inventory level at R_1 at time t_0 . If on the other hand, $IL_{R_1}(t_0) < 0$, then the backlog cost of R_1 , $BC_{R_1}(\cdot)$ can be updated by $BC_{R_1}(t_1) = BC_{R_1}(t_0) - IL_{R_1}(t_0) * (t_1 - t_0)b$. The ordering and transportation cost of R_1 , $OTC_{R_1}(\cdot)$ will also be updated as follows if the event that occurred at time t_1 triggered a replenishment: $OTC_{R_1}(t_1) = OTC_{R_1}(t_0) + s_1$ if R_1 replenishes its inventory from DC_1 , and $OTC_{R_1}(t_1) = OTC_{R_1}(t_0) + s_2$ if R_1 replenishes its inventory from DC_2 . Once all costs are calculated, then the inventory level is updated. If the last event is a customer arrival to R_1 , then $IL_{R_1}(t_1) = IL_{R_1}(t_0) - 1$. On the other hand, if the last event is the arrival of a batch to R_1 , then $IL_{R_1}(t_1) = IL_{R_1}(t_0) + q$. The cost and inventory level calculations for R_2 , DC_1 , and DC_2 are analogous to those of R_1 .

2.5.1 Performance Analysis

To evaluate the performance of the proposed ordering policy, OP_4 , a large number of scenarios have been generated. As given in Table 2.1, 15 parameters affect the total cost

of the analyzed supply chain. Using the low and high values identified we generated $2^{15} = 32,768$ scenarios. Some of the combinations among the 32,768 scenarios may not be realistic but, they provide a large set of problems to compare OP_4 to OP_1 , OP_2 , and OP_3 . Each scenario has been replicated 30 times, and within each replication the same sequence of random numbers have been used to calculate the total costs of each studied policy. Since the retailers are identical, the average retailer cost under OP_4 has been compared to average retailer costs under the remaining three policies. Similarly, since the DCs are identical, average DC cost under OP_4 has been compared to average DC cost under OP_1 , OP_2 , and OP_3 . More specifically, paired t-tests have been performed to test the hypotheses given in Table 2.2. In Table 2.2, μ_{ip} under columns TC , RET , and DC respectively denote the mean total cost for the supply chain, the mean total cost for the retailers, and the mean total cost for the DCs in scenario i that use policy p where $i = 1, \dots, 32,768$ and $p = 1, \dots, 4$.

Table 2.2

Paired t-tests performed to compare OP_4 to OP_1 , OP_2 , and OP_3

	TC	RET	DC
OP_1	$H_0 : \mu_{i1} \geq \mu_{i4}$	$H_0 : \mu_{i1} \geq \mu_{i4}$	$H_0 : \mu_{i1} \geq \mu_{i4}$
	$H_1 : \mu_{i1} < \mu_{i4}$	$H_1 : \mu_{i1} < \mu_{i4}$	$H_1 : \mu_{i1} < \mu_{i4}$
OP_2	$H_0 : \mu_{i2} \geq \mu_{i4}$	$H_0 : \mu_{i2} \geq \mu_{i4}$	$H_0 : \mu_{i2} \geq \mu_{i4}$
	$H_1 : \mu_{i2} < \mu_{i4}$	$H_1 : \mu_{i2} < \mu_{i4}$	$H_1 : \mu_{i2} < \mu_{i4}$
OP_3	$H_0 : \mu_{i3} \geq \mu_{i4}$	$H_0 : \mu_{i3} \geq \mu_{i4}$	$H_0 : \mu_{i3} \geq \mu_{i4}$
	$H_1 : \mu_{i3} < \mu_{i4}$	$H_1 : \mu_{i3} < \mu_{i4}$	$H_1 : \mu_{i3} < \mu_{i4}$

The results of the paired t-tests are summarized in Table 2.3. All tests were performed at the 5% level of significance. From Table 2.3, in most of the scenarios the proposed policy, OP_4 , performed statistically better than the other three policies. For example, with respect to total cost, OP_4 was better than or equal to OP_1 in 69.18% of the 32,768 scenarios. In other words, the null hypothesis shown in Table 2.2 could not be rejected in 69.18% of the cases. When compared to OP_3 with respect to retailer cost, OP_4 had better or equal performance in 91.36% of the scenarios.

Table 2.3

Summary results of the paired t-tests (% times H_0 could not be rejected)

	<i>TC</i>	<i>RET</i>	<i>DC</i>
OP_1	69.18	63.21	81.82
OP_2	67.77	66.88	71.37
OP_3	76.12	91.36	64.72

In addition to the paired t-tests, average percent improvements have been calculated as reported in Table 2.4. The values in Table 2.4 under columns *TC*, *RET*, and *DC* show the average improvement achieved in total cost, retailer cost, and DC cost respectively; they are calculated using the following equation:

$$\% \text{ cost improvement} = \frac{1}{2^{15}} \sum_{i=1}^{2^{15}} \frac{\mu_{ip} - \mu_{i4}}{\mu_{ip}} \times 100.$$

As can be seen from Table 2.4, the proposed ordering policy leads to reductions in the total supply chain cost and the average retailer cost. For example, using OP_4 instead of OP_1 reduced the supply chain cost by an average of 3.11%, the retailer cost by 3.42%, and

the DC cost by 2.61%. When OP_4 is compared to OP_3 with respect to DC cost, on average, OP_3 actually performed better, which is not surprising because under OP_4 , retailers make DC selections by trying to minimize their expected total cost. On the other hand, under OP_3 , retailers make their DC selections based on earliest delivery times. This indicates that a retailer under OP_4 can order a batch of items from a DC that will not necessarily have the earliest delivery time. Thus, the longer waiting time for a retailer indicates a higher backlog cost for the DC.

Table 2.4

Average percent improvement achieved using OP_4

	<i>TC</i>	<i>RET</i>	<i>DC</i>
OP_1	3.11	3.42	2.61
OP_2	3.02	4.53	0.89
OP_3	0.11	2.64	-3.76

Although OP_4 generally performed better than OP_1 , OP_2 , and OP_3 , we performed more detailed analyses to see if further improvements could be achieved.

2.5.2 Parametric Analysis

By parametrically changing the batch size q and the reorder point r at the retailers, we tried to find q and r values that would lead to better results. We also tried to identify the parameters that have a relatively larger impact on the expected cost.

Adjusting the retailers' q and r values: Based on Table 2.3, OP_4 performs statistically better than OP_1 , OP_2 , and OP_3 in most of the 32,768 scenarios. However, in a number of

scenarios, OP_4 performs relatively poorly. Among the scenarios in which OP_4 performed considerably poorly, four of them were selected. Table 2.5 shows the parameters for the selected problems.

Table 2.5

Parameter values for the selected problems

Prob.	q	r	λ	h	b	s_1	s_2	L_1	L_2	Q	R	H	B	L	O
1	8	6	5	5	6	45	50	4	5	24	0	2.8	3.5	12	100
2	4	2	5	3	10	45	50	4	5	24	16	2.8	3.5	12	50
3	4	6	5	5	10	45	50	4	5	24	16	1.2	5.9	12	50
4	4	6	1	5	6	30	50	4	5	8	16	2.8	5.9	24	50

For the problems given in Table 2.5, in an effort to minimize costs, q and r have been parametrically changed over a large set of values while keeping other parameter values constant. This has been done separately for each ordering policy because the q and r values that minimize the costs under OP_4 are not necessarily the same for OP_1 , OP_2 , or OP_3 . The first half of Table 2.6 shows the average percent improvement achieved for the four selected problems using OP_4 and using the original values given in Table 2.5. For example, the average DC cost for the selected problems was 15.03% worse when OP_4 was used instead of OP_3 . However, when the same comparison is made after parametric analysis, using OP_4 led to an improvement of 2.51% in DC costs. Thus, by comparing the left half of Table 2.6 to the right half, we can see that the performance of the proposed ordering policy can be improved via a parametric analysis. Note that in the left half of Table 2.6, OP_4 is compared to OP_1 , OP_2 , and OP_3 using the same parameters (i.e. those

given in Table 2.5), but in the right half, the best of OP_4 is compared to the bests of OP_1 , OP_2 , and OP_3 .

Table 2.6

Average percent improvement achieved before and after parametric analysis

	Before Parametric Analysis			After Parametric Analysis		
	<i>TC</i>	<i>RET</i>	<i>DC</i>	<i>TC</i>	<i>RET</i>	<i>DC</i>
OP_1	-4.52	-3.12	-4.85	5.94	1.96	10.91
OP_2	-5.69	-1.61	-9.25	7.58	5.21	9.57
OP_3	-6.32	0.88	-15.03	10.20	11.60	2.51

Identifying higher impact parameters: Based on the initial set of 32,768 scenarios, we observed that Q , R , L , and λ had higher impacts on the expected costs of the retailers, the DCs, and the overall supply chain. Note that Q is DCs' batch size, R is DCs' reorder point, L is the lead time from the outside supplier to the DCs, and λ is the customer arrival rate at the retailers. Percent improvement achieved in costs using OP_4 at the low level of all 15 parameters were compared to the corresponding improvement values at the high level, and based on this comparison Q , R , L , and λ have been identified as having the most impact on costs. Further analysis have been performed on these four parameters to examine how the performance of the proposed ordering policy OP_4 changes.

Figure 2.6 depicts how the performance of OP_4 changes with respect to the other three ordering policies as the customer arrival rate changes. The vertical axis shows the percent improvement achieved in retailer costs by using OP_4 rather than using OP_1 , OP_2 , or OP_3 . As can be seen from Figure 2.6, the proposed policy, OP_4 , performed better for smaller

values of λ . For larger λ values the performance of all four ordering policies seem to be equivalent. Intuitively, as λ increases the retailers are simply trying to keep up with the high demand, so regardless of which ordering policy they use the performance is expected to be the same. In general, spare parts are relatively slow moving items. Thus, the λ value will typically be low for auto spare parts. Hence, the decision rule developed may be a useful tool for the auto manufacturer discussed earlier.

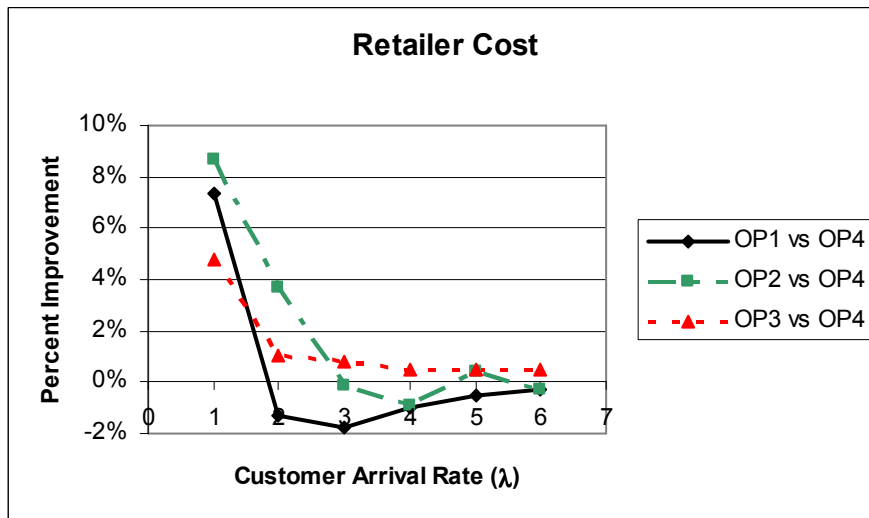


Figure 2.6

Percent improvement achieved as the arrival rate changes

Figure 2.7 summarizes the results of $6 * 2^{14}$ scenarios and shows that the performance of the proposed policy, OP_4 , was better for those scenarios in which Q was larger. The observation that OP_4 performs better as Q increases can be explained by the fact that OP_4 considers only the retailers' information in making a decision. This may have a negative

impact on the DC costs. Thus, if Q is large, then the DCs will have enough cushion to offset any negative impact on their inventory levels. In other words, when Q is small R_1 's DC selection decision may have a higher negative impact on R_2 's DC selection decision and consequently increase the cost.

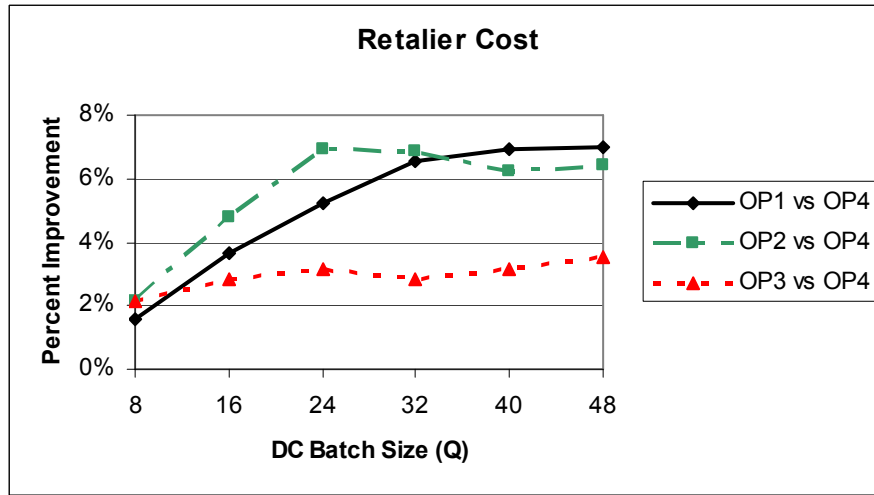


Figure 2.7

Percent improvement achieved as the batch size changes

Figure 2.8 shows the effect of R on the performance of OP_4 . For example, when compared to OP_3 , the benefit of using OP_4 increased for scenarios in which R was larger. When R is large, like with large Q , retailers will have more flexibility in the sense that R_1 's DC selection decision will typically have a smaller impact on R_2 's future DC selection decisions. However, when compared to OP_1 and OP_2 , the performance of OP_4 decreased slightly with increasing R . This decrease was more evident in total cost. This is perhaps due to the fact that as R increases the chances that a DC will be out of stock decreases.

Thus, using OP_4 will more frequently result in R_1 choosing DC_1 and R_2 choosing DC_2 . In other words, the DC selection decisions in OP_4 and OP_1 will be the same in most cases because the auto manufacturer mentioned earlier made the DC-retailer assignments based on the assumptions that $L_1 < L_2$ and $s_1 < s_2$.

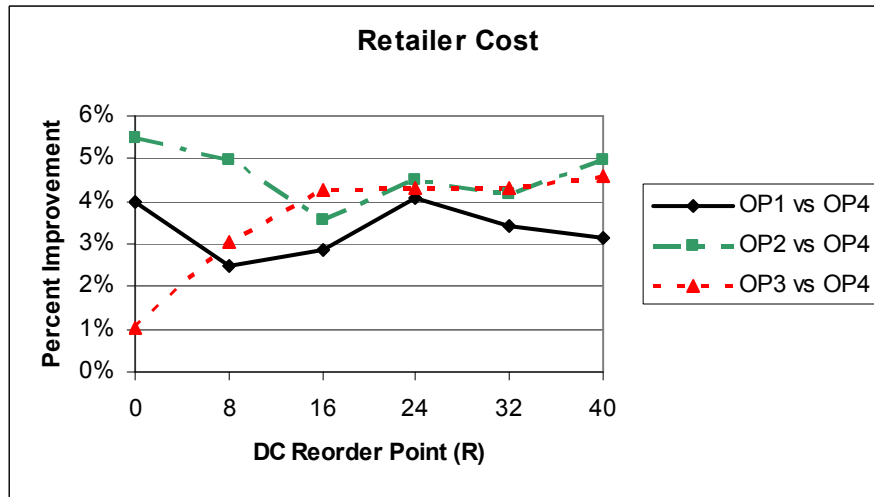


Figure 2.8

Percent improvement achieved as the reorder point changes

Based on Figure 2.9 as L increases the benefit of using OP_4 seems to slightly increase first but then starts to decrease. When L is too large the DCs are out of stock for longer periods of time which means the backlog costs is the dominant cost component. Therefore, the benefit of order switching diminishes in the long run. So, OP_2 , OP_3 , and OP_4 all tend to become like OP_1 .

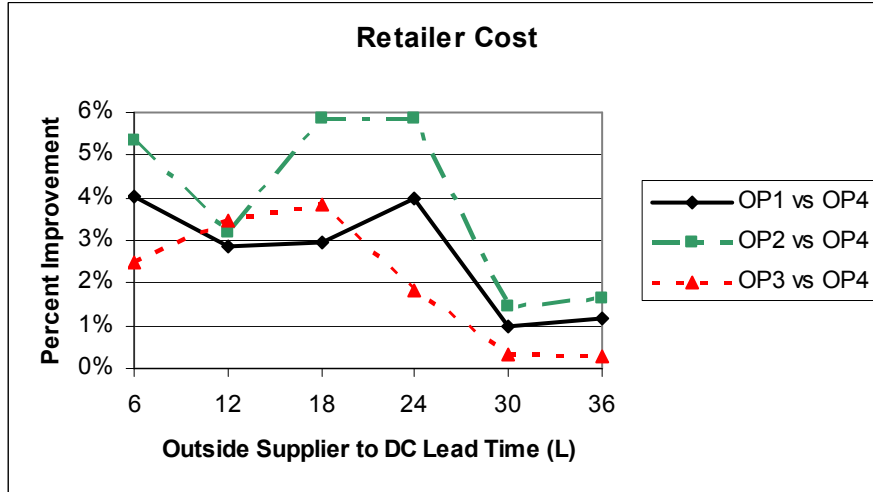


Figure 2.9

Percent improvement achieved as the lead time changes

2.6 Conclusions and Research Directions

We have derived and evaluated a decision rule for DC selection in a two-stage supply chain with two retailers and two DCs, and the decision rule has been developed for the retailers to use each time they need to replenish inventory. This decision rule minimizes the expected total cost for the retailer. The decision rule has been evaluated in a simulation study, and the results support that the rule performs well. The retailers repeatedly use the decision rule as a heuristic which is the basis of our proposed ordering policy, OP_4 . Our policy is compared to three other policies, OP_1 , OP_2 , and OP_3 , in which DC selection decisions are respectively: i) always select the same DC, ii) select the DC with inventory on hand, and iii) select the DC that promises an earlier delivery. From a managerial point of view, OP_1 , OP_2 , and OP_3 may seem to be easier to use and implement; however, OP_4

is also easy to use. Although OP_4 may be complicated to derive mathematically, it can very easily be incorporated into a decision support system.

For the automobile manufacturer discussed in Section 4.1, the service stations, i.e. the retailers, currently use OP_1 as their ordering policy, and their request from the headquarters was to be allowed to use OP_3 to improve their performance. By deriving a new decision rule and developing an ordering policy based on this rule, we show through our numerical study that the retailers' performance can be improved more by OP_4 than by OP_3 . The proposed ordering policy, OP_4 , also improved the performance of the whole supply chain for most of the scenarios considered. For the scenarios in which OP_4 performed poorly, we demonstrated that the performance could be improved via a parametric analysis. Using these results, the retailers may convince the headquarters to allow them to begin "order switching." Based on our analysis, OP_4 performs particularly well for supply chains with slow moving items and high inventories at the DCs. In other words, if λ is small and Q and R are large OP_4 performs better than OP_1 , OP_2 , and OP_3 with respect to both retailer and DC costs.

There are many possible future research directions related to this study. In this study, we assume that if one of the retailers is using the proposed decision rule, then the other is doing the same. An interesting extension would be to look at the case when the retailers compete to minimize their costs by using different ordering policies. Analyzing a supply chain with more than two DCs and two retailers could also be another research study. Finally, developing decision rules for the DCs and/or the system as a whole would contribute to the literature.

CHAPTER 3

ANALYSIS OF ORDERING POLICIES IN A TWO-STAGE SUPPLY CHAIN WITH ALTERNATIVE SUPPLY SOURCES

3.1 Introduction

In the previous chapter, we introduced a decision rule based ordering policy for a DC selection problem. Numerical experiments have shown that the decision rule outperforms the traditional ordering policies in terms of retailers cost. However, the decision rule based policy is not necessarily the best for the supply chain in all cases. In this chapter, we present additional analysis to determine settings where OP_4 is most beneficial to retailers.

In our experiment design we have decreased the number of problem instances from 32,768 to 21. The small number of experiments has allowed us to gain more insight about OP_4 and identify the combination of parameters under which OP_4 can outperform the other three policies.

The rest of the chapter is organized as follows. Section 4.3 presents the results of numerical analysis and we conclude the chapter in section 3.3.

3.2 Numerical Analysis

Performance of the four ordering policies (OP_1 , OP_2 , OP_3 , and OP_4) are tested on a relatively small number of cases compared to the previous chapter. In total, we consider 21

problem instances to evaluate the performance of the decision rule based ordering policy compared to the other policies. For each problem instance a simulation study for 20,000 time units is run and each instance is replicated 10 times. Data related to average total cost at retailers and *DCs* together with average waiting time of retailers to receive a shipment from *DCs* is collected.

We used following set of parameters to create problem instances:

$$\begin{array}{lll}
 h=1 & L_1 = 2 & H=0.8 \\
 B=5 & L =24 & O=200 \\
 \lambda \in \{0.5, 1, 1.5, 2.0, 2.5\} & \frac{L_2}{L_1} \in \{1.1, 1.3, 1.5, 1.7, 1.9\} & \frac{h}{b} \in \{1, 0.2, 0.05, 0.0125, 0.01\} \\
 \frac{s_1}{b} \in \{1, 3, 5, 7, 9\} & \frac{s_2}{s_1} \in \{1.1, 1.3, 1.5, 1.7, 1.9\} &
 \end{array}$$

$\lambda=1.5$, $L_2=3$, $b=20$, $s_1=100$, and $s_2=150$ were chosen to create the base scenario. We changed each parameter in the base scenario to find out the effect of the parameters on costs and average waiting time.

To determine reorder points and order quantities, we assumed that *DCs* determine the best Q and R . This is a meaningful assumption. In the automotive industry under consideration, *DCs* are authorized by the automotive manufacturer to set order batch sizes for retailers. We found values of Q and R by selecting the parameter pair that minimizes *DCs'* cost through a simulation study. Once *DCs* establish their Q and R values and order batch size for retailers, retailers pick the reorder point that minimizes their average cost assuming all other information is known. Table 3.1 shows the parameters and their respective values for 21 problem instances.

When all retailers use the same ordering policy, their costs and the average waiting time does not vary significantly from each other because the retailers are assumed to be identical. The same observation is valid for the DCs (i.e. the costs of the two DCs are close to each other), since they are assumed to be identical. The objective of the analysis is to determine how the cost and the average waiting time vary with respect to the parameters.

Figure 4.38 depicts the normalized cost and delivery performance of the four policies. To normalize the values, we assume that performance of OP_1 is 100%. The following formula is then used to determine normalized performance of the other policies:

$$N_{TC_p} = \frac{TC_p}{TC_1} * 100. \quad (3.1)$$

In the above formula, N_{TC_p} stands for normalized total cost of ordering policy p . We normalize average cost at retailer and DC together with the average waiting time for retailers to fulfill their inventory from a DC . We denote normalized average cost of retailers and DCs by N_{RC_p} and N_{DCC_p} respectively for a policy p . Normalized average waiting time of retailers is shown by N_{WT_p} .

Based on our numerical experiments, OP_2 performs poorly compared to OP_1 with respect to total cost of supply chain, retailers' cost, and average retailers order waiting time. This is beneficial from a DCs ' perspective. Choosing a DC with inventory will decrease backlog and inventory holding costs. It might seem considering inventory instead of delivery time might appear to increase average waiting time for retailers.

Table 3.1

Parameters and respective values of each test problems

Prob. #	q	r	λ	h	b	s_1	s_2	L_1	L_2	Q	R	H	B	L	O
1	14	4	1.5	1	20	100	150	2	3	28	28	0.8	5	24	200
2	14	3	1.5	1	20	100	110	2	3	28	28	0.8	5	24	200
3	14	3	1.5	1	20	100	130	2	3	28	28	0.8	5	24	200
4	14	3	1.5	1	20	100	170	2	3	28	28	0.8	5	24	200
5	14	3	1.5	1	20	100	190	2	3	28	28	0.8	5	24	200
6	14	3	1.5	1	20	20	30	2	3	28	28	0.8	5	24	200
7	14	4	1.5	1	20	60	90	2	3	28	28	0.8	5	24	200
8	14	3	1.5	1	20	140	210	2	3	28	28	0.8	5	24	200
9	14	2	1.5	1	20	180	270	2	3	28	28	0.8	5	24	200
10	14	4	1.5	1	20	100	150	2	2.2	28	28	0.8	5	24	200
11	14	3	1.5	1	20	100	150	2	2.6	28	28	0.8	5	24	200
12	14	3	1.5	1	20	100	150	2	3.4	28	28	0.8	5	24	200
13	14	3	1.5	1	20	100	150	2	3.8	28	28	0.8	5	24	200
14	14	-4	1.5	1	1	100	150	2	3	28	28	0.8	5	24	200
15	14	0	1.5	1	5	100	150	2	3	28	28	0.8	5	24	200
16	14	7	1.5	1	80	100	150	2	3	28	28	0.8	5	24	200
17	14	7	1.5	1	100	100	150	2	3	28	28	0.8	5	24	200
18	18	0	0.5	1	20	100	150	2	3	18	0	0.8	5	24	200
19	25	3	1	1	20	100	150	2	3	25	0	0.8	5	24	200
20	17	5	2	1	20	100	150	2	3	34	34	0.8	5	24	200
21	19	9	2.5	1	20	100	150	2	3	38	38	0.8	5	24	200

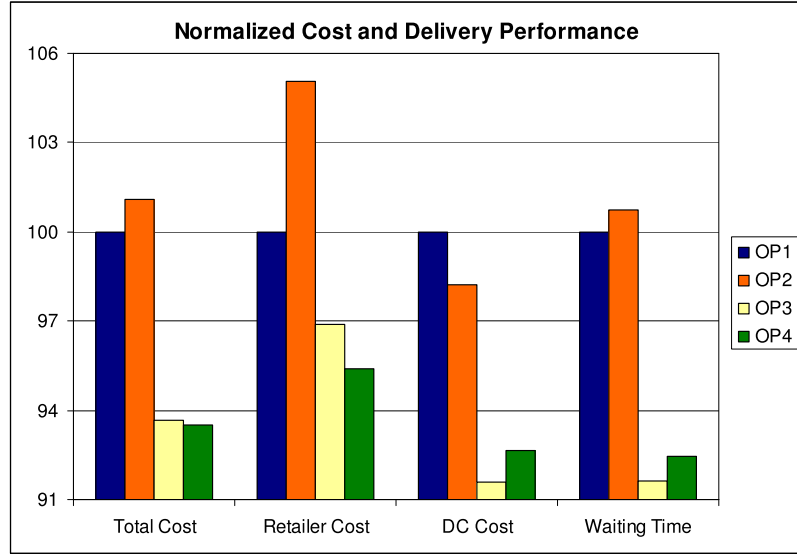


Figure 3.1

Normalized cost and delivery performance of four policies

For example, R_1 needs to place an order. DC_1 does not have inventory, but DC_2 does. The retailer places an order with DC_2 under OP_2 . Selecting DC_2 does not necessarily mean that retailer will receive the shipment earlier from DC_2 . It is possible for DC_1 to replenish inventory and deliver the item to R_1 before DC_2 (knowing that $L_1 < L_2$).

In addition, numerical analysis shows that on average OP_3 is better than OP_1 with respect to cost and delivery performance. It is easy to show that OP_3 will always yield lower cost at DCs because of the fact that selecting a DC that delivers an order earlier means less backlog cost and less inventory holding cost at DCs . Luckily, retailers' greedy approach in terms of early delivery benefited themselves on average as well. As detailed analysis depicts OP_3 may not be a good policy for retailers under every circumstance in terms of average cost of retailers. Nevertheless, it is the best policy to decrease average waiting time for retailers as well as decreasing DCs ' cost.

OP_4 is designed to decrease retailers cost. Based on numerical experiments we can say that for all problem instances we achieved improvement in retailers' average cost. Although OP_4 does not consider any information related to DCs other than delivery times from DCs , similar to OP_2 and OP_3 being able to select the DC in the other region decreased DCs ' cost by allowing less backlog and inventory holding cost at DCs . Delivery performance of OP_4 is slightly less than OP_3 , however, using OP_4 decreased retailers cost more than OP_3 on average.

It may be difficult to catch a trend in total cost since it is the sum of DCs ' and retailers' average costs. Thus, we focused on costs at retailers and DCs separately. Table 3.2 presents normalized cost of retailers and DCs as well as total cost supply chain. Based on our observation, performances of OP_2 and OP_3 with respect to DCs ' average cost do not get affected from parameters except from end customer arrival rate, λ . This has an intuitive explanation, as long as the number of arrivals does not change, retailers' decisions will not be affected since OP_2 and OP_3 do not consider any information regarding inventory holding, backlog, and ordering costs. On the other hand, when λ increases, there will be more opportunities for retailers to replenish their inventory from the alternative DC , DC located in the other region. Hence, DCs will gain more benefits when customer arrival rate gets higher. On the other hand, unlike OP_2 and OP_3 , OP_4 is sensitive to any parameter change at retailers.

When λ becomes larger both OP_3 and OP_4 create more cost reduction opportunities for retailers and DCs . In addition, being able to replenish their inventory from both DCs is important for retailers especially when replenishing from the alternative DC is not very

expensive compared to replenishing from the assigned DC , the one located in the same region. For example when s_2/s_1 ratio is 1.1 (not very expensive case) retailers benefit is 2.17%. However, when the ratio is 1.9 (very expensive case) using OP_3 increases retailers cost by 1.44%. Similar observations hold as backlog cost increases. While backlog cost per unit time increases, the opportunity of replenishing from alternative supply source becomes more critical for retailers. As a consequence, advantage of using OP_3 and OP_4 increase as backlog cost increases (i.e. h/b ratio decreases).

When cost of ordering from both DC s is relatively cheap (i.e. s_1/b ratio is small) retailers will have more opportunity by being able to use both DC s as their supplier. It is clear that as s_1/b ratio increases (note that s_2/s_1 ratio is fixed) alternative sourcing becomes less attractive since the gap between replenishing from DC s increases as well ($s_2 - s_1$ gets larger).

As costs of replenishing from DC s increase, at a certain point, using OP_3 increases retailers' cost opposite to retailers' expectation to decrease their cost by reducing the backlog cost. On the other hand, using an expected cost based approach will allow retailers to gain benefit from alternative sourcing as much as possible without increasing their cost. For example, when s_1/b ratio is equal to 9 ($s_1=180$ and $s_2=270$) using OP_3 increases retailers cost by 0.69% compared to 4.14% cost decrease when $s_1=20$. For a similar change in s_1 , even though benefit decreases from 5.23% to 0.58% OP_4 still continued to decrease the average cost of retailers. Meanwhile, when s_2/s_1 ratio gets larger, allowing retailers to replenish from both DC s under OP_4 bring less benefit to DC s. Because of the high cost of alternative sourcing retailers prefer replenishing from the assigned DC . It is easy

Table 3.2

Change in Normalized Average Costs

		N_{TC_2}	N_{RC_2}	N_{DCC_2}	N_{TC_3}	N_{RC_3}	N_{DCC_3}	N_{TC_4}	N_{RC_4}	N_{DCC_4}
λ	0.5	102.63	103.49	101.69	99.51	100.29	98.63	99.05	98.99	99.12
	1.0	103.02	104.97	100.89	98.96	102.17	95.45	98.58	99.42	97.67
	1.5*	100.91	102.84	99.20	98.73	100.21	97.43	98.20	98.25	98.14
	2.0	101.51	106.01	97.24	95.38	98.14	92.76	95.20	96.12	94.33
	2.5	104.57	117.38	92.12	85.53	93.85	77.45	88.95	92.55	85.46
L_2/L_1 ratio	1.1	100.81	102.63	99.20	99.19	100.78	97.78	98.03	97.99	98.06
	1.3	100.55	102.18	99.13	98.50	99.66	97.48	97.73	97.65	97.79
	1.7	101.44	103.78	99.40	98.68	99.95	97.57	98.08	98.18	97.99
	1.9	101.72	104.59	99.20	98.79	100.04	97.69	98.27	98.40	98.16
h/b ratio	1.0000	101.08	104.18	99.10	99.51	102.81	97.40	99.49	99.03	99.79
	0.2000	101.05	103.39	99.25	99.24	101.51	97.49	98.73	99.00	98.52
	0.0125	101.30	103.31	99.26	98.13	98.77	97.47	97.54	96.92	98.17
	0.0010	101.43	103.55	99.25	97.63	97.80	97.46	97.08	96.12	98.06
s_1/b ratio	1	99.51	100.16	99.18	96.87	95.86	97.37	96.52	94.77	97.39
	3	100.26	101.83	99.16	98.14	99.06	97.49	97.75	97.49	97.92
	7	101.35	103.36	99.21	98.97	100.43	97.41	98.15	98.25	98.04
	9	101.68	103.58	99.26	99.37	100.69	97.69	98.85	99.42	98.13
s_2/s_1 ratio	1.1	99.51	99.92	99.15	97.61	97.83	97.42	97.15	96.79	97.47
	1.3	100.28	101.55	99.17	98.04	98.79	97.39	97.47	97.40	97.54
	1.7	101.41	104.02	99.13	98.81	100.49	97.35	98.06	98.10	98.02
	1.9	101.85	105.02	99.08	99.29	101.44	97.41	98.32	98.42	98.23

to show that decrease in performances of OP_2 and OP_3 will continue as s_2/s_1 gets larger. The reason is both OP_2 and OP_3 do not take cost into account. In contrast, OP_4 will always balances the conflicting objectives of replenishing early and replenishing with less cost. Figure 3.2 illustrates how much a retailer should pay additionally ($s_2 - s_1$) to receive the early shipment over selecting the DC that delivers later.

As can be seen from the Figure 3.2, a retailer, i.e. R_1 , considers paying more for a very early delivery (time close to zero) to avoid high backlog cost. In other words, as delivery times get closer to each other, the benefit of receiving an early delivery decreases for the retailer. For example in Figure 3.2, we assume that late delivery is at time 10 and there is no scheduled shipment to the retailer when problem number 1 is run. Under

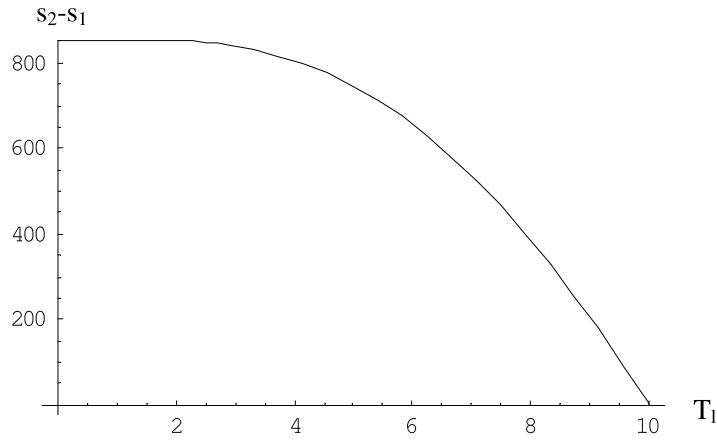


Figure 3.2

Normalized cost and delivery performance of four policies

these assumptions, the retailer can pay up to 853.22 monetary units more for an early delivery that is going to arrive at time 0.1. The retailer considers paying up to 839.21, 656.69, 206.72, and 21.80 for a shipment that will be delivered to the retailer at times 3, 6, 9, and 9.9. For instance, the retailer chooses early delivery, employing the decision rule based ordering policy, if DC_e delivers the shipment before time 9.7689201 ($s_l - s_e=50$), otherwise, it select DC_l . On the other hand under OP_3 , for the same example, the retailer selects DC_e even when the early delivery time is 9.99 which has the expected benefit of 2.19 monetary units while its additional cost is 50.

So far we have only analyzed the performance of ordering policies with respect to cost savings opportunities. In the following paragraphs, we evaluate the performance of the policies in terms of retailers' average waiting time to replenish their inventory from DC_s .

Table 3.3 presents result of average waiting time for the test problems. As seen earlier with respect to cost evaluation, OP_2 's performance is not better than OP_1 in most cases.

Even though retailers choose the *DC* that has inventory, the *DC* may not be able to replenish retailers' inventory earlier than the other *DC* because of the lead time difference. Whereas employing OP_3 and OP_4 shorten the average waiting time as λ increases, utilizing OP_2 causes longer waiting time for the retailers. Increase in the average waiting time is also observed when L_2/L_1 ratio becomes larger. The performance of OP_2 does not seem to be affected by other parameters.

As mentioned earlier, OP_3 is the best policy to decrease the retailers' average waiting time. Similar to OP_2 , OP_3 is not also very sensitive to changes in parameters other than λ and L_2/L_1 ratio. While the more frequent customer arrivals reveals more opportunity for retailers, higher L_2/L_1 ratio decreases the chances of average waiting time reduction. This is due to the fact that longer lead time from the alternative *DC* decreases the number of occasions that the alternative *DC* can replenish earlier than the assigned *DC*. The similar observations hold when we examine the time reduction opportunities when OP_4 is employed in lieu of OP_1 .

3.3 Conclusion

A two-stage supply chain with two retailers and two distribution centers has been analyzed with respect to the average costs at *DC* and retailers, and the average waiting time for retailers to receive a shipment from *DCs*.

When the policies are compared, numerical analysis indicates that OP_4 has improved the retailers' cost in all instances whereas utilizing OP_2 and OP_3 have increased the cost of retailers in some problem instances. Based on numerical experiments, an inventory

Table 3.3

Change in Normalized Delivery Performance

		N_{WT_2}	N_{WT_3}	N_{WT_4}
λ	0.5	105.26	99.21	99.59
	1.0	104.11	99.03	99.42
	1.5*	100.77	98.27	98.35
	2.0	100.84	96.12	96.35
	2.5	99.37	87.83	91.74
L_2/L_1 ratio	1.1	96.47	94.72	96.93
	1.3	98.57	96.68	97.19
	1.7	103.14	99.40	98.89
	1.9	105.13	99.22	99.65
h/b ratio	1.0000	100.61	98.19	99.76
	0.2000	100.81	98.25	98.61
	0.0125	100.88	98.26	98.35
	0.0010	100.87	98.28	98.28
s_1/b ratio	1	100.82	98.21	98.05
	5	100.70	98.32	98.24
	7	100.81	98.16	98.24
	9	100.67	98.29	98.26
s_2/s_1 ratio	1.1	100.66	98.22	98.06
	1.3	100.72	98.16	97.95
	1.7	100.77	98.12	98.26
	1.9	100.56	98.17	98.36

based policy, OP_2 , will most likely increase the cost of retailers and the average time of replenishment. Moreover, except for the change in replenishment lead times, it is not clear that under which parameter setting OP_2 will be more cost effective strategy than OP_1 . If replenishment lead times from DCs are close to each other, employing OP_2 will create an opportunity for retailers to receive the orders earlier than the case of employing OP_1 .

Utilizing OP_3 and OP_4 brings an improvement chance to retailers with respect to both the average cost and the average waiting time. OP_3 is the best ordering policy to reduce the average waiting time whereas OP_4 is the best ordering policy in terms retailers average cost.

An interesting result is that allowing retailers to use an alternative supplier does not increase the average cost at DCs and actually helps DCs to decrease their inventory holding and backlog costs as well as improving their delivery performance. Based on our numerical experiments, in a supply chain with an alternative supply source, alternative sourcing improves the supply chain performance both in regard to cost and delivery performance. In addition, employing a cost based DC selection approach brings mutual benefits to retailers and DCs .

Some future extensions of research can be as follows, in this supply chain we assume that there are only two retailers, it could be an interesting extension to look at a supply chain with more than two retailers. Moreover, considering a compound Poisson distribution could be important since in some situation each customer arrival may results more than one unit of demand. Finally, solving the problem from supply chain perspective will contribute to the related literature.

CHAPTER 4
COORDINATION OF INVENTORY AND TRANSPORTATION DECISIONS VIA
SHIPMENT CONSOLIDATION

4.1 Introduction

In today's very competitive business environment, timely delivery of items to the customers is a crucial task that many companies face. In order to benefit from the economies of scale, distributors (suppliers) typically prefer to consolidate deliveries while trying to maintain a high percentage of on time delivery. These conflicting objectives make suppliers' job extremely difficult since finding a good delivery time while considering both objectives is quite challenging. For example, increasing the delivery frequency increases suppliers' performance in general, but it also raises the transportation costs.

Coordination of inventory and transportation decisions using shipment consolidation has been studied intensively. Shipment consolidation (also referred to as freight consolidation) allows companies to combine many small customer orders to achieve economies of scale in transportation costs as well as reduce inventory holding costs ([6], [10], [18], and [25]).

Shipment consolidation has been studied under many different settings such as consolidation schemes, demand characteristics, and vehicle attributes. The most distinctive

attribute of shipment consolidation is the dispatching scheme employed. One of the first papers in this area, [25], discusses three dispatching schemes used in industry to decide when a shipment is to be dispatched; quantity based, time based, and hybrid practices. Under the time based practice, all consolidated orders are dispatched at a predetermined time at once, whereas under the quantity based scheme a shipment is dispatched whenever a threshold quantity is reached. A hybrid policy is a combination of quantity and time based policies such that a shipment is dispatched either when a threshold quantity or preset time is reached. Many researchers have compared performances of dispatching policies to find out how different consolidation policies perform under different settings ([12], [13], [16], [22], [24], and [25]).

Although interest in integrated inventory replenishment and shipment dispatching decisions has increased over the last decade, researchers (such as [4], [8], [11], and [12]) still follow an approach that only considers the distributor's side of the problem.

In addition, in order to decrease operations regarding management of inventories on the buyer's side and to use the economies of scale on the supplier's side, many different approaches have been employed, such as vendor managed inventory (VMI). Under a VMI agreement, supplier has control of the buyer's inventory. By being able to control the buyer's inventory, the supplier enjoys the opportunity to control the delivery quantities and time. Due to its easy applicability, researchers have considered shipment consolidation in connection with VMI to increase the benefit of VMI ([4], [11], and [20]).

On the other hand, implementing VMI may not always be possible especially if there is a trust issue between the supplier and the buyer. In the absence of trust, buyer avoids giving

control of inventory assuming that the supplier will push its inventory to the buyer's facilities instead of keeping inventory at the best possible location in terms of supply chain's overall performance. The primary motivation of this research is a product of such an environment of mistrust where the buyer does not want to give the control of inventory, however, demands higher on-time delivery performance to decrease its cost and planning efforts due to late deliveries.

In the following paragraphs, we introduce a problem that we observed in an automotive spare parts distribution setting in which the buyer does not trust the seller and asks for timely shipments. In the problem we observe, customers are located in two regions and each region consists of a single distribution center (*DC*) and multiple service stations. The company uses the term "service stations" to denote authorized repair shops. Throughout this chapter, we will use the term retailers to indicate service stations. Retailers and *DCs* use a (Q, R) continuous review inventory policy. Retailers place orders to *DCs* whereas *DCs* replenish their inventory from outside suppliers. As retailers place orders to *DCs*, *DCs* do not send items to retailers immediately. As a consequence, retailers might have to endure a long and uncertain wait. Two reasons for late deliveries from *DCs* to retailers are shipment consolidation and being out of stock.

Shipment consolidation is used as a tool to decrease transportation cost at *DCs*. Currently, shipment consolidation decisions at the company are made based on experience without a structured approach about how long the order consolidation period should be. Thus, this gives rise to situations in which *DCs* are unable to provide firm delivery dates and consequently retailers have difficulties managing their inventories.

Although shipment consolidation is one of the main reasons for delays, there are cases in which *DCs* delay shipments to retailers because they are out of stock due to the extended lead time from outside suppliers. For example, some items come from outside suppliers that are located in other countries and the lead time from these suppliers to *DCs* can exceed a month. These give rise to the following question. When a retailer places an order is it possible to give him/her an accurate shipment or delivery time considering the future demand, the lead time from the outside suppliers, and the inventory holding, backlog, and transportation costs?

We provide two models that partially answer the above question and to improve whole supply chain performance in terms of both total cost and timely delivery. In our models, we assume that *DCs* and retailers are in an agreement. Such that the retailers will share their demand and inventory information with the *DC* that provides information regarding shipment dispatching time at the time of order placement. Although this approach could lead to certain contractual issues such as sharing the monetary benefit, these issues are not included in the present study. To come up with a solution to our research question, we introduce a “promised latest delivery time” concept that is as follows: The promised latest delivery time is the end of shipment consolidation time that the *DC* provides to retailers at the time of order placement. It is to be noted that a *DC*'s promised latest delivery time is the latest time that the *DC* will dispatch the orders, but, may actually dispatch the shipment earlier.

Our approach to the consolidation problem can be considered as a hybrid policy which has been found to be very promising, [12]. We also use a reevaluation approach that is

employed repeatedly to determine a promised latest delivery time whenever a *DC* needs to calculate a new promised latest delivery time. The reevaluation approach allows *DCs* to calculate a more precise shipment consolidation time by taking into consideration the latest information available. This approach is similar to the recurrent approach proposed by [21]. While [21] evaluates the decision when a customer arrives, we evaluate the situation only at the beginning of shipment consolidation. However, it is easy to reevaluate the decision as new orders are placed to *DCs* as well.

The rest of the paper is organized as follows. Section 4.2 presents the problem description, details related to four consolidation policies, and derivation of the models. Section 4.3 presents the numerical analysis. Lastly, section 4.4 provides some concluding remarks and presents directions for future research.

4.2 Problem Description and Formulation

Consider a two-stage supply chain with two distribution centers (*DC*) and two retailers. Figure 4.1 shows the structure of the supply chain. Each supply chain member uses a (Q, R) continuous review inventory policy. When the inventory position at a facility drops down to the reorder point that facility places an order of size Q . It is assumed that *DCs*' order quantity, Q , and reorder point, R , are multiples of retailers' order quantity q . Retailers face independent Poisson end customer demand. It is also assumed that there is a fixed transportation cost of serving retailers. When a *DC* serves only one retailer a transportation cost of F_1 is incurred. If a *DC* serves both retailers in a single shipment, a fixed transportation cost of F_2 is incurred. It is assumed that serving a single retailer is less

costly than serving both retailers at the same time ($F_1 < F_2$). Furthermore, there is a fixed lead time, L , from the outside supplier to DCs . Transportation from DCs to retailers takes no time. The other notation used in the formulation are given below:

- q = order quantity at retailers,
- r = reorder point at retailers,
- λ = customer arrival rate at retailers,
- h = inventory holding cost per item per unit time at retailers,
- b = backlog cost per item per unit time at retailers,
- IL_j^R = inventory level (inventory on hand - backorders) at retailer j ,
- IP_j^R = inventory position (inventory on hand - backorders + outstanding orders) at retailer j ,
- inv_j^i = expected inventory holding cost per unit time associated with the i^{th} customer demand at retailer j ,
- $back_j^i$ = expected backlog cost per unit time associated with the i^{th} customer demand at retailer j ,
- Q = order quantity at DCs ,
- R = reorder point at DCs ,
- H = inventory holding cost per item per unit time at DCs ,
- B = backlog cost per item per unit time at DCs ,
- IL_l^D = inventory level at DC_l ,
- IP_l^D = inventory position at DC_l ,
- In_l = expected inventory holding cost per unit time at DC_l ,
- Ba_l^k = expected backlog cost per unit time associated with the k^{th} retailer order at DC_l .

We propose two new consolidation policies; single source consolidation, CP_3 , and dual source consolidation, CP_4 . In order to compare the performances of proposed consolidation policies, we consider two other policies from the literature: no consolidation, (CP_1), and time based consolidation, (CP_2). More details are given about these policies in the following subsections. For all consolidation policies, we assume that retailers share demand and inventory information with DCs .

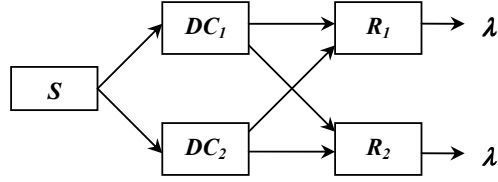


Figure 4.1

Structure of the supply chain

4.2.1 No Consolidation - CP_1

Under the first policy, (CP_1), we assume that there is no shipment consolidation. That is, either a DC has inventory on hand and the DC dispatches the shipment as soon as it receives an order from the retailer, or the DC is out of stock so it replenishes its own inventory first then dispatches the shipment. CP_1 is studied by many researchers, ([14] and [30]), so we do not present any details in this study.

4.2.2 Time Based Consolidation - CP_2

Under the second consolidation policy, (CP_2), we consider the model proposed by [13]. According to this model, the authors calculate the optimum consolidation time for a warehouse under the assumption that the shipment from the outside supplier is instant and that the warehouse faces the end customer demand directly (no retailers in the system). Therefore, CP_2 does not consider cost at retailers and lead time issues. We choose [13] for comparison because, the authors assume that the distributor uses a (Q, R) continuous review inventory policy which is similar to our problem setting. We find the optimal shipment consolidation time assuming that there is no lead time from outside supplier to DC_s using the formula given by [13]. However, in the simulation study, we measure CP_2 's per-

formance in a supply chain with a positive lead time from the outside supplier to DCs together with costs at retailers. In order to improve the performance of the model, we introduce retailer cost to the model under the assumption that DCs backlog cost is the sum of backlog costs at DCs and retailers.

4.2.3 Single Source Consolidation - CP_3

Under the proposed single source consolidation policy, CP_3 , DCs utilize a hybrid consolidation scheme to determine the promised latest delivery time, T . The time period between the declaration of the promised latest delivery time and the actual time of shipment is used as an order consolidation interval. However, a DC can ship product earlier if the number of consolidated order reaches to the number of items that the DC has secured for the retailer.

We assume that the promised latest delivery time cannot be longer than the lead time from outside supplier to DCs ($T \leq L$), as this might result in retailers enduring a longer wait and losing their trust in DCs . If $T > L$, a retailer would wait longer to receive a shipment from a DC , and the DC would have the incentive to keep no inventories, but replenish its inventory based on actual orders received from the retailer.

The main idea behind CP_3 is to investigate and coordinate demand arrival and item arrival to retailers and DCs . For this purpose, a DC may evaluate different number of batches to find the option with the least expected cost per unit time. Let m be the number of batches available for consolidation just after the order that triggers the consolidation at the DC . m is defined as follows:

$$m = \begin{cases} (IL_t^D + sQ)/q, & IL_t^D > 0; \\ sQ/q - 1, & IL_t^D \leq 0. \end{cases}$$

In the above formulation let s be the index of the scheduled shipment that arrives the DC at time t_s .

Suppose DC_1 has IL_1^D/q batches in inventory just after the order that triggers the consolidation at the DC and the DC is scheduled to receive a shipment of size Q from the outside supplier at time t_1 . In this case, DC_1 can calculate the expected costs associated with $m = IL_1^D/q$ batches that are ready for consolidation at time zero and $m = (IL_1^D + Q)/q$ batches that would be available for consolidation at time t_1 . Note that the number of scheduled shipments could be more than one. For example, if there is a second shipment scheduled to arrive at the DC , the DC considers $m = (IL_1^D + 2Q)/q$ batches for consolidation as well. After computing the expected costs of supply chain and the corresponding latest delivery times for different number of batches, DC_1 makes a decision based on the expected cost per unit time and uses the corresponding latest delivery time as the promised latest delivery time, T .

For example, assume at time 0, R_1 receives a customer demand and its inventory position, IP_1^R , drops down to or below the reorder point r thus R_1 requests an order of size q from DC_1 . Upon receipt of information regarding IL_1^R , DC_1 considers the overall expected supply chain cost which includes inventory holding and backlog costs at R_1 and DC_1 ; and transportation cost.

Suppose the expected costs of the supply chain are $U^{IL_1^D/q}$ and $U^{(IL_1^D+Q)/q}$ when $m = IL_1^D/q$ and $m = (IL_1^D + Q)/q$ batches are considered for consolidation. In addition, assume that $T^{IL_1^D/q}$ and $T^{(IL_1^D+Q)/q}$ are the respective latest delivery times. If $U^{IL_1^D/q} < U^{(IL_1^D+Q)/q}$, DC_1 chooses to consider only IL_1^D/q batches and the promised latest delivery time, T , is equal to $T^{IL_1^D/q}$. Otherwise, DC_1 considers $(IL_1^D + Q)/q$ batches for consolidation ($T = T^{(IL_1^D+Q)/q}$). Note that under the second alternative, DC_1 cannot dispatch the shipment before time t_1 , whereas it is possible to ship earlier when IL_1^D/q batches are considered for shipment consolidation. The next paragraph explains the details regarding the time of the actual dispatch.

When R_1 places an order which initiates the shipment consolidation at DC_1 , DC_1 allocates one of the batches for this new order leaving the remaining batches for future orders of R_1 . Let \bar{t} be the earliest time that DC_1 can dispatch a shipment. For the example given in the previous paragraphs, \bar{t} will be equal to zero or t_1 , depending on whether IL_1^D/q or $(IL_1^D + Q)/q$ batches are considered for the next shipment consolidation cycle. Suppose, R_1 chooses to consider $(IL_1^D + Q)/q$ batches during the consolidation cycle, hence, \bar{t} will be equal to t_1 . Considering the promised latest delivery time T , there can be three different cases with respect to the actual shipment dispatching time: i) DC_1 receives less than m batch requests from R_1 within time T , so DC_1 dispatches the shipment at time T ; ii) DC_1 receives the m^{th} order before time T and after \bar{t} , so the shipment is dispatched upon receipt of the m^{th} order; iii) DC_1 receives the m^{th} order before time \bar{t} , so the shipment is dispatched at time \bar{t} .

Following example is used to clarify the information given above. Assume $IL_1^D=8$, $q=2$, $R=6$, $L=10$, and $Q=4$ just after the order (at time zero) that triggers the shipment consolidation at DC_1 . Assume that after the expected cost calculation, T is determined as 7. In addition, assume there are no scheduled shipments to DC_1 . In this case, DC_1 dispatch the shipment either at time T or earlier. If DC_1 receives $m = IL_1^D/q = 4$ orders before time T , then DC_1 dispatches a shipment. Suppose the retailer places its 10 future orders at times 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, and 1.2. As a response to the retailer's order, DC_1 places orders to the outside supplier and those orders will be delivered at times 10.3 (0.3+10), 10.5, 10.7, 10.9, and 11.1. Based on this example and our assumptions, DC_1 dispatches a shipment at time 0.6. Figure 4.2 presents the information available to the DC at time 0.6.

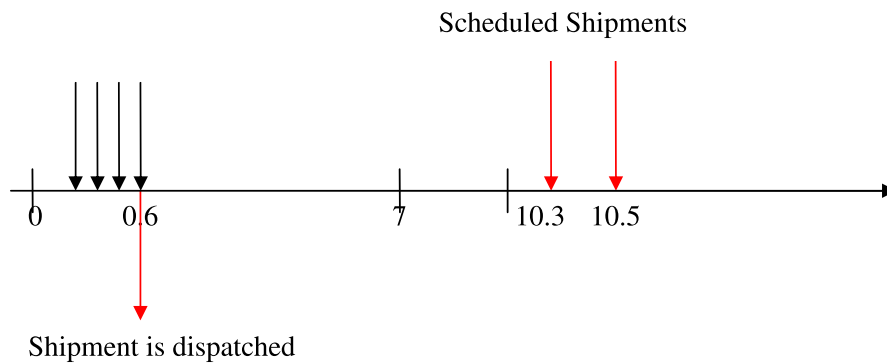


Figure 4.2

The scheduled shipments and arrival of orders to the DC and the DC's dispatching decision at time 0.6.

When DC_1 receives the next order from the retailer, i.e. the fifth order, it needs to calculate the promised latest delivery time again. Note that DC_1 receives the fifth order from the retailer at time 0.7. At this time there are three previously scheduled shipments that will be received at times 10.3, 10.5, and 10.7 respectively. Therefore, s will be 1, 2, or 3 which indicates that m will be 1, 3, or 5. At time 0.7 when the retailers places its order, we re-initialize the time index to zero. In other words, when the fifth order is received at time 0.7, we set the clock back to zero. Since $IL_1^D = -2$ at time zero and there are three outstanding orders we calculate three T values; one corresponding to each m value. In the calculation of the first T , $\bar{t}=9.6$. In the calculation of the second T , $\bar{t}=9.8$. In the calculation of the third T , $\bar{t}=10$. The promised latest delivery time will be in one of these three $[\bar{t}, L]$ intervals that gives the minimum expected cost per unit time. Assume that when $m = 1$, $T=9.7$ corresponds to the minimum expected cost per unit time. In this case a shipment will be dispatched at time 9.6, due to the fact that the DC is going to receive the sixth order at time 0.1. Figure 4.3 depicts the arrival of the sixth order from the retailer and time of dispatch together with the promised latest delivery time, T .

The process will restart by resetting the clock back to zero when the next order is received which will be the seventh order in our example. Figure 4.4 shows the situation at the DC when the retailer places the seventh order. The DC will match the demand with the scheduled shipments that will arrive at time 9.6 or later, since the scheduled shipment at time 9.4 is already matched with the retailer's previous orders.

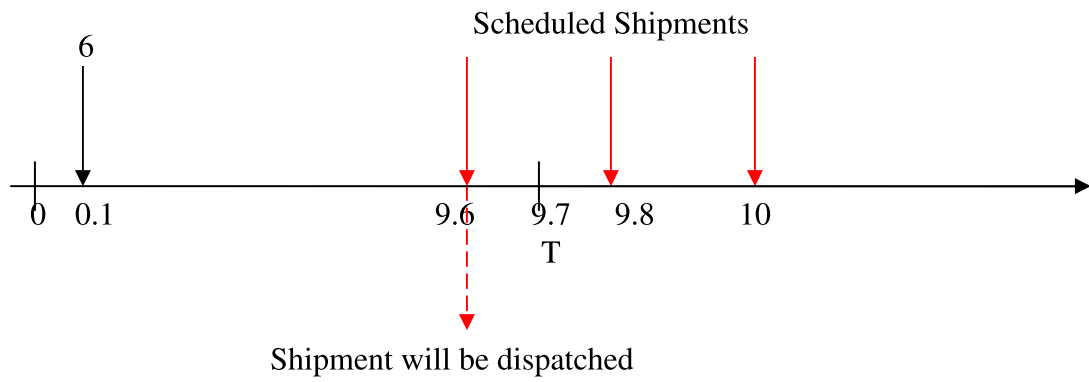


Figure 4.3

Arrival of the sixth order to the DC and corresponding events at the DC

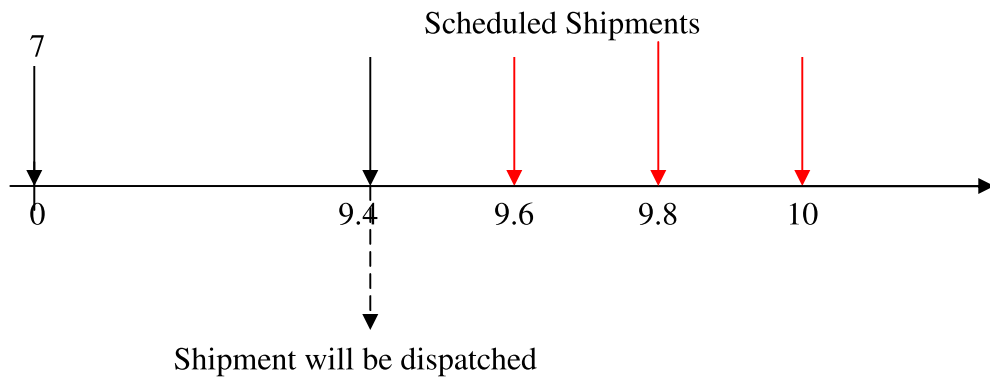


Figure 4.4

Arrival of the seventh order to the DC and corresponding events at the DC

When customer arrivals to retailer follow a Poisson distribution, the time when the i^{th} customer arrives at R_j has an Erlang(λ, i) distribution with the following density and cumulative distribution functions, respectively:

$$f_j^i(t) = \frac{\lambda^i t^{i-1} e^{-\lambda t}}{(i-1)!}. \quad (4.1)$$

$$F_j^i(t) = \int_0^t f_j^i(u) du = 1 - \sum_{k=0}^{i-1} \frac{(\lambda t)^k e^{-\lambda t}}{k!}. \quad (4.2)$$

It is also convenient at this point to introduce the following equation which will be required in expected cost calculations.

$$\int_0^t u f_1^i(u) du = \frac{i}{\lambda} F_1^{i+1}(t). \quad (4.3)$$

We analyze the system in two parts; the expected cost per unit time at retailers and the expected cost per unit time at DC_s . Then, we combine the expected costs to compute the total expected cost per time unit. To present formulation more clearly, we show the formulation for $DC_1 - R_1$ pair. It will be the same for $DC_2 - R_2$ pair.

4.2.3.1 Expected Cost at Retailer

If a demand arrives after the corresponding item, there will be an associated inventory holding cost. If a demand arrives before the corresponding item there will be an associated backlog cost.

Inventory Holding Cost:

If $IL_1^R > 0$ then there is an inventory holding cost associated with the first IL_1^R items.

Initial Inventory Holding Cost When $IL_1^R > 0$:

In this case, the initial IL_1^R demand could be satisfied from inventory on hand. For a promised latest delivery time T , if demand arrives before time T then there will be an inventory holding cost until demand arrival. First integral in (4.4) presents the mode of calculation of the expected inventory cost when demand arrives before time T . When the demand arrives after time T , there is a need to retain the item in the inventory until time T and this calculation is shown in the second integral in (4.4).

$$\sum_{i=1}^{IL_1^R} inv_1^i = \frac{h}{T} \sum_{i=1}^{IL_1^R} \left[\int_0^T u f_1^i(u) du + T \int_T^\infty f_1^i(u) du \right]. \quad (4.4)$$

Using the information provided regarding Erlang Distribution and equation (4.3), the expected cost calculation per unit time given by (4.4) can be re-written as:

$$\sum_{i=1}^{IL_1^R} inv_1^i = hIL_1^R - \frac{h}{T} \sum_{i=1}^{IL_1^R} \left[T F_1^i(T) - \frac{i}{\lambda} F_1^{i+1}(T) \right]. \quad (4.5)$$

Future Inventory Holding Cost When $IL_1^R > 0$:

In addition to initial inventory holding cost, if R_1 receives the shipment before time T , there will be an associated inventory holding cost for the items received before time T .

$$\sum_{i=\max\{mq, IL_1^R\}+1}^{(m+1)q+IL_1^R} inv_1^i = \frac{h(IL_1^R + q)}{T} \left[\int_0^{\bar{t}} (T - \bar{t}) f_1^{mq}(u) du + \int_{\bar{t}}^T (T - u) f_1^{mq}(u) du \right]$$

$$-\frac{h}{T} \sum_{i=\max\{mq, IL_1^R\}+1}^{(m+1)q+IL_1^R} \left[\int_0^{\bar{t}} (T - \bar{t}) f_1^i(u) du + \int_{\bar{t}}^T (T - u) f_1^i(u) du \right]. \quad (4.6)$$

Above formula can be represented as follows after necessary calculations.

$$\sum_{i=\max\{mq, IL_1^R\}+1}^{(m+1)q+IL_1^R} inv_1^i = \frac{h(IL_1^R + q)}{T} [(T - 2\bar{t})F_1^{mq}(\bar{t}) + TF_1^{mq}(T) - \frac{mq}{\lambda}(F_1^{mq+1}(T) - F_1^{mq+1}(\bar{t}))]$$

$$-\frac{h}{T} \sum_{i=\max\{mq, IL_1^R\}+1}^{(m+1)q+IL_1^R} [(T - 2\bar{t})F_1^i(\bar{t}) + TF_1^i(T) - \frac{i}{\lambda}(F_1^{i+1}(T) - F_1^{i+1}(\bar{t}))]. \quad (4.7)$$

Backlog Cost:

There is no backlog cost if $IL_1^R > 0$ during the consolidation cycle at the retailer for the first IL_1^R demand. However, if $IL_1^R < 0$, there will be an initial backlog cost associated with the order placed by the retailer which triggers the consolidation period at the *DC*.

Initial Backlog Cost When $IL_1^R < 0$:

In this case there is a backlog associated with the already realized demand (because of the order that triggers the consolidation period) and these demands will be backlogged until the shipment is dispatched. The expected backlog cost associated with them will be:

$$\sum_{i=1}^q back_1^i = b * \min\{q, -IL_1^R\}. \quad (4.8)$$

In addition to the initial backlog cost, there would also be a backlog cost associated with future demand, if the demand exceeds the number of items that the retailer has. We

describe the method to calculate the associated backlog cost at the retailer in the following paragraphs.

Future Backlog Cost:

There is a backlog cost associated with demand between IL_1^R+1 and mq , if $0 \leq IL_1^R < mq$. The duration of backlog time will vary based on the arrival time of m^{th} order placed by R_1 . If m^{th} order arrives before time \bar{t} , then there will be a backlog cost associated with the end customer demands between $IL_1^R + 1$ and $mq - 1$ until the time \bar{t} .

$$back_1^i = \frac{b}{\bar{t}} \int_0^{\bar{t}} \left(\int_0^v (\bar{t} - u) f_1^i(u) du \right) f_1^{mq}(v) dv. \quad (4.9)$$

If m^{th} order arrives before time T and after time \bar{t} , then there will be a backlog cost associated with the end customer demands between $IL_1^R + 1$ and $mq - 1$ until the arrival time of the m^{th} order to DC_1 . The corresponding expected backlog cost per unit time for an end customer demand realized at R_1 will be:

$$back_1^i = b \int_0^T \left(\int_0^v f_1^i(u) du \right) \frac{1}{v} f_1^{mq}(v) dv. \quad (4.10)$$

However, if m^{th} order is not placed until time T , then all previously placed orders will only arrive at time T to the retailer. This will result in a backlog of each realized end customer demand until time T . The following formula is used to calculate the associated backlog cost:

$$back_1^i = \frac{b}{T} \int_T^\infty \left(\int_0^T f_1^i(u) du \right) f_1^{mq}(v) dv. \quad (4.11)$$

After combining (4.10) and (4.11), we arrive at the following formula by calculating the integrals to find the associated expected backlog cost per unit time.

$$\begin{aligned}
& \sum_{i=\max\{1, IL_1^R+1\}}^{mq-1} \text{back}_1^i = \frac{b(1 - F_1^{mq}(T))}{T} \sum_{i=\max\{1, IL_1^R+1\}}^{mq-1} [TF_1^i(T) - \frac{i}{\lambda}F_1^{i+1}(T)] \\
& + b \sum_{i=\max\{1, IL_1^R+1\}}^{mq-1} [F_1^{mq}(T) - \frac{i}{mq-1}F_1^{mq-1}(T) + \sum_{j=0}^i \frac{i(mq+j-2)!F_1^{mq+j-1}(2T)}{2^{mq+j-1}j!(mq-1)!} \\
& + \sum_{j=0}^{i-1} \frac{(mq+j-1)!F_1^{mq+j}(2T)}{2^{mq+j}j!(mq-1)!}] + \frac{bF_1^{mq}(\bar{t})}{\bar{t}} \sum_{i=\max\{1, IL_1^R+1\}}^{mq-1} [\bar{t}F_1^i(\bar{t}) - \frac{i}{\lambda}F_1^{i+1}(\bar{t})]. \quad (4.12)
\end{aligned}$$

If the retailer receives more than $\max\{mq, IL_1^R\} + 1$ and less than $(m+1)q + IL_1^R$ end customer demand before time \bar{t} , these demands will be backlogged until time \bar{t} . If these demands are realized after time \bar{t} , then there will be only inventory cost associated with these demands and inventory holding cost is already calculated using (4.7).

$$\sum_{i=\max\{mq, IL_1^R+1\}}^{(m+1)q+IL_1^R} \text{back}_1^i = \frac{b}{\bar{t}} \sum_{i=\max\{mq, IL_1^R+1\}}^{(m+1)q+IL_1^R} [\int_0^{\bar{t}} (\bar{t} - u) f_1^i(u) du]. \quad (4.13)$$

After necessary calculations (4.13) will be simplified to:

$$\sum_{i=\max\{mq, IL_1^R+1\}}^{(m+1)q+IL_1^R} \text{back}_1^i = \frac{b}{\bar{t}} \sum_{i=\max\{mq, IL_1^R+1\}}^{(m+1)q+IL_1^R} [\bar{t}F_1^i(\bar{t}) - \frac{i}{\lambda}F_1^{i+1}(\bar{t})]. \quad (4.14)$$

In addition, we know that if the total demand at R_1 exceeds $(m+1)q + IL_1^R$, then all the exceeding demand will be backlogged until time T as DC_1 will not be left with any items to fulfill R_1 's demand before time T .

$$\sum_{i=\max\{1, IL_1^R+(m+1)q+1\}}^{\infty} back_1^i = \frac{b}{T} \sum_{i=\max\{1, IL_1^R+(m+1)q+1\}}^{\infty} \left[\int_0^T (T-u) f_1^i(u) du \right]. \quad (4.15)$$

(4.15) can be simplified to

$$\sum_{i=\max\{1, IL_1^R+(m+1)q+1\}}^{\infty} back_1^i = \frac{b}{T} \sum_{i=\max\{1, IL_1^R+(m+1)q+1\}}^{\infty} \left[T F_1^i(T) - \frac{i}{\lambda} F_1^{i+1}(T) \right]. \quad (4.16)$$

The expected cost of R_1 per unit time will be as follows:

$$U_1^R = \sum_{i=1}^{\infty} inv_1^i + back_1^i. \quad (4.17)$$

4.2.3.2 Expected Cost at Distribution Center

The expected cost calculation at DC_1 will be similar to the one in the retailer's case.

Inventory Holding Cost:

The inventory holding cost rate will be $H \cdot IL_1^D$, if DC_1 does not have a scheduled shipment arrival and $IL_1^D > 0$. However, if there is a scheduled shipment arrival at DC_1 which is before the time of shipment dispatch, then there is a need to determine the average inventory holding cost per unit time.

Assuming DC_1 will receive the shipment at time t and the latest dispatching time is T , the average inventory on hand will be:

$$IL_1^a = (T IL_1^D + (T-t)[(1 - F_1^{mq}(T))Q + \frac{Qmq}{\lambda}(F_1^{mq+1}(T) - F_1^{mq+1}(t))]). \quad (4.18)$$

In the formula above if $IL_1^D < 0$, the first part of the summation is equal to zero.

Expected inventory holding cost per unit time will be:

$$In_1 = H.IL_1^a. \quad (4.19)$$

Backlog Cost:

In addition to the inventory holding cost, there will be a backlog cost associated with each order placed by R_1 . To calculate the expected backlog cost of DC_1 , we match the items at the DC with R_1 's orders. By considering the fact that every q many end customer demands will result in a new retailer order, (4.12) is used (with some modifications) to calculate the expected backlog cost per unit time at DC_1 . The final formula is given below:

$$\begin{aligned} \sum_{k=1}^m ba_1^k &= \frac{Bq(1 - F_1^{mq}(T))}{T} \sum_{k=1}^{m-1} [TF_1^{kq}(T) - \frac{kq}{\lambda} F_1^{kq+1}(T)] \\ &+ Bq \sum_{k=1}^{m-1} [F_1^{mq}(T) - \frac{kq}{mq-1} F_1^{mq-1}(T) + \sum_{j=0}^k \frac{kq(mq+j-2)! F_1^{mq+j-1}(2T)}{2^{mq+j-1} j! (mq-1)!} \\ &+ \sum_{j=0}^{k-1} \frac{(mq+j-1)! F_1^{mq+j}(2T)}{2^{mq+j} j! (mq-1)!}] + \frac{Bq F_1^{mq}(\bar{t})}{\bar{t}} \sum_{k=1}^{m-1} [\bar{t} F_1^{kq}(\bar{t}) - \frac{kq}{\lambda} F_1^{kq+1}(\bar{t})]. \quad (4.20) \end{aligned}$$

Finally, if the number of orders placed by R_1 is greater than m , there is no chance for DC_1 to replenish R_1 's additional requests until time T . Thus, all those orders that are placed after the m^{th} order will be backlogged until time T .

$$\sum_{k=m+1}^{\infty} ba_1^k = \frac{Bq}{T} \sum_{k=m+1}^{\infty} [TF_1^{kq}(T) - \frac{kq}{\lambda} F_1^{kq+1}(T)]. \quad (4.21)$$

The total expected cost of DC_1 per unit time will be:

$$U_1^D = [[\sum_{k=0}^{\infty} Ba_1^k] + In_1] + \frac{F_1}{T}. \quad (4.22)$$

4.2.3.3 Expected Cost of Supply Chain

The total expected cost of supply chain per unit time will simply be the sum of the expected costs per unit time at DC_1 and R_1 .

$$U_1^{SC} = U_1^R + U_1^D. \quad (4.23)$$

Now the problem reduces to:

$$\text{Min } U_1^{SC}$$

s.t.

$$T \geq \bar{t}$$

$$T \leq L$$

As mentioned earlier, in the above formula \bar{t} could either be zero or the arrival time of shipment to the DC . If the DC chooses to consider $m = IL_1^D/q$ batches then $\bar{t}=0$. If DC_1 considers $m = (IL_1^D + sQ)/q$ batches then $\bar{t} = t_s$.

Since we have not been able to prove that U_1^{SC} is a convex function of time for all possible parameters, we search the interval between \bar{t} and T . We choose the value of T which minimizes the total expected cost of DC - retailer pair per unit time.

4.2.4 Dual Source Consolidation - CP_4

So far, under the three previous consolidation policies, we assume that each retailer is assigned to a single DC that is located in the same region and compute the consolidation time for a supply chain for the DC -retailer pair. When we derive the model for dual source consolidation, we relax the assumption that a retailer can only be served by a single DC located in the same region. Hence, under CP_4 , a retailer can be served by two DC s. A DC can serve two retailers at the same time or one at a time. To find the best alternative with the lowest cost, we compare three alternative DC -retailer assignments. We then follow a similar approach to CP_3 to compute the expected cost of alternatives.

$$\text{Argmin} \begin{cases} D_1^{SC}, & \text{Assign } R_1 \text{ to } DC_1 \text{ and } R_2 \text{ to } DC_2; \\ D_2^{SC}, & \text{Assign } R_1 \text{ and } R_2 \text{ to } DC_1; \\ D_3^{SC}, & \text{Assign } R_1 \text{ and } R_2 \text{ to } DC_2. \end{cases}$$

D_z^{SC} denotes the expected unit cost of supply chain when option z is chosen under the dual source consolidation policy, CP_4 .

We have already derived the formula required for D_1^{SC} . Under CP_3 , calculation is performed for a single DC -retailer pair. Under CP_4 , we calculate the expected cost of $DC_1 - R_1$ and $DC_2 - R_2$ assignments, and then find the total expected cost of D_1^{SC} .

Calculations will be exactly the same for D_2^{SC} and D_3^{SC} . The only difference is the number of batches available for consolidation at DC_1 and DC_2 and the time \bar{t} at which those batches will be available for consolidation. If there are two or three options that have the exact expected cost value, we choose one of them randomly. For example, if $D_2^{SC} = D_3^{SC}$ and $D_1^{SC} > D_2^{SC}$, we randomly select DC_1 or DC_2 , and assign both retailers' orders to that DC .

In the following section, we present cost calculation when both retailers are assigned to DC_1 , D_2^{SC} . As mentioned earlier, the same calculation steps will be used for DC_2 , D_3^{SC} .

In a manner parallel to the cost calculation adopted in CP_3 , we compute the expected cost at retailers and DCs and then combine them to obtain the total expected cost of the system under CP_4 . Note that all formulas presented for cost calculation of R_1 , need to be repeated for R_2 under subsection 4.2.4.1.

4.2.4.1 Expected Cost at Retailers

The formula is applicable when both retailers are assigned to DC_1 , D_2^{SC} . The analysis is analogous for D_3^{SC} . We assume that, at time 0, just after receiving a new order from one of the retailers, DC_1 has m batches left for consolidation. In addition, retailers have k_j many outstanding orders including the order that initiated the shipment consolidation at DC_1 .

Each retailer may get a certain number of batches from the remaining m batches available at DC_1 . The number of batches that R_j can be promised, m_j , by DC_1 is between 0 and m . There can be two situations that need to be considered. Firstly, R_j may not receive

enough end customer demand to place more than m_j orders before time T . Secondly, although R_j places more than m_j orders of size q before time T , DC_1 cannot deliver them due to the constraint that all the remaining batches have been promised to the other retailer. In the first case, R_j may receive the shipment at time T or earlier depending on the number of orders placed by the other retailer. If the total number of orders placed by both retailers is greater than or equal to m , shipment will arrive before time T . Else the retailers will have to wait until time T to receive the shipment. Finally, if the number of orders exceeds the number of available batches, the m^{th} order received by DC_1 will trigger the decision on shipment dispatch. Thus, the m^{th} order is important for the expected cost calculation. Based on the above scenario, we calculate the expected cost of the retailers.

We assume that R_1 will place a new order after it receives y_1 many end customer demand (current $IP_1^R = r + y_1$). If $IL_1^R < 0$ then there is a backlog cost associated with the backordered demand at a rate of $b \cdot IL_1^R$.

$$back_1^0 = bIL_1^R. \quad (4.24)$$

Dispatch shipment at time T :

The total number of orders received by DC_1 should be less than the total number of batches available, m . For a given m_1 , R_2 can place a maximum of $m_2 = m - m_1 - 1$ orders during $(0, T)$ time interval. For this condition to be realized, R_2 should receive less than $y_2 + (m - m_1 - 1)q$ end customer demand. Note that, the probability that an event occurs a certain number of times within a fixed period of time (T) will follow Poisson

distribution with parameter λT . Poisson distribution has the following probability density function:

$$P(X = x) = \frac{e^{-\lambda T} (\lambda T)^x}{x!}. \quad (4.25)$$

Let n_1 denote the number of end customer demand that R_1 will receive during the time interval $(0, T)$. \bar{n}_1 is the upper limit of n_1 , and \underline{n}_1 is the lower limit of n_1 . For the expected cost calculation of R_1 , lower and upper bounds are $\bar{n}_1 = y_1 + m_1 q - 1$, $\underline{n}_1 = y_1 + (m_1 - 1)q$, $\bar{n}_2 = y_2 + (m_2)q - 1$, and $\underline{n}_2 = 0$

Moreover, the probability that R_1 will receive at most \bar{n}_1 units of demand during the time interval $(0, T)$ can be calculated by using (4.25) as follows:

$$P(X \leq \bar{n}_1) = \sum_{x=0}^{\bar{n}_1} P(X = x). \quad (4.26)$$

If R_1 places an order for only m_1 batches, the number of end customer arrivals to R_1 should be between $y_1 + (m_1 - 1)q$ and $y_1 + m_1 q - 1$.

For $ILL_1^R \geq n_1$, (similar to $ILL_1^R \geq mq$ case for R_1 under CP_3), there will only be an inventory holding cost at R_1 . The expected inventory holding cost can be computed using (4.27). On the other hand, if $0 \leq ILL_1^R < n_1$, inventory holding and backlog costs will occur at R_1 . While the expected cost of inventory can be computed after modifying the formula given in (4.5), (4.28) is needed to evaluate the expected backlog cost at R_1 .

$$\sum_{i=1}^{\bar{n}_1} inv_1^i = P(X \leq \bar{n}_2) [hILL_1^R - \frac{h}{T} \sum_{i=1}^{\bar{n}_1} [TF_1^i(T) - \frac{i}{\lambda} F_1^{i+1}(T)]]. \quad (4.27)$$

$$\sum_{i=\max\{1, LL_1^R+1\}}^{\bar{n}_1} \text{back}_1^i = \frac{P(X < \bar{n}_2)b}{T} \sum_{i=\max\{1, LL_1^R+1\}}^{\bar{n}_1} [TF_1^i(T) - \frac{i}{\lambda}F_1^{i+1}(T)]. \quad (4.28)$$

Dispatch shipment before time T :

The shipment is dispatched when the total number of orders placed by R_1 and R_2 is equal to the number of batches available at DC_1 , ($m_1 + m_2 = m$). The order that triggers dispatching decision can be released by either one of the retailers. In the following subsections, we present how cost calculations change at R_1 based on the order that triggers the shipment dispatching decision.

R_1 initiates the shipment:

We assume that R_1 places his m_1^{th} order (R_1 receives $n_1 = y_1 + (m_1 - 1)q$ many end customer demand) which is the m^{th} order received by DC_1 . Under previous assumptions, R_2 has already placed m_2 ($m_2 = m - m_1$) many orders before R_1 's m_1^{th} order. If R_2 has only placed m_2 many orders before R_1 's m_1^{th} order then $\bar{n}_2 = y_2 + m_2q - 1$ and $\underline{n}_2 = y_2 + (m_2 - 1)q$.

At first, we find the probability that R_1 receives n_1^{th} end customer demand before R_2 receives $(\bar{n}_2 + 1)^{th}$ demand but after R_2 receives \underline{n}_2^{th} end customer demand. Probability that R_1 will receive n_1^{th} demand after \underline{n}_2 is:

$$Z(X_2^{\underline{n}_2} < X_1^{n_1}) = \int_0^T \left(\int_0^v f_2^{\underline{n}_2}(u) du \right) f_1^{n_1}(v) dv. \quad (4.29)$$

Likewise, the probability that R_1 receives the n_1^{th} end customer demand before R_1 receives $(\bar{n}_2 + 1)^{th}$ demand can be found by applying (4.29). Consequently, the probability

that DC_1 will receive R_1 's m_1^{th} order after R_2 's m_2^{th} order but before R_2 's $(m_2 + 1)^{th}$ order will be:

$$Z(X_2^{n_2} < X_1^{n_1} \& X_1^{n_1} < X_2^{n_2}) = Z(X_2^{n_2} < X_1^{n_1}) * Z(X_1^{n_1} < X_2^{n_2}). \quad (4.30)$$

After finding the associated probability, $Z(\cdot)$, using (4.30), to calculate the expected cost the formulae (4.12) through (4.7) should be multiplied with probabilities.

$$\begin{aligned} \sum_{i=\max\{1, IL_1^R+1\}}^{m_1q-1} \text{back}_1^i &= \frac{Z(\cdot)b(1 - F_1^{m_1q}(T))}{T} \sum_{i=\max\{1, IL_1^R+1\}}^{m_1q-1} [TF_1^i(T) - \frac{i}{\lambda}F_1^{i+1}(T)] \\ + Z(\cdot) \sum_{i=\max\{1, IL_1^R+1\}}^{m_1q-1} &[F_1^{m_1q}(T) - \frac{i}{m_1q-1}F_1^{m_1q-1}(T) + \sum_{j=0}^i \frac{i(m_1q+j-2)!F_1^{m_1q+j-1}(2T)}{2^{m_1q+j-1}j!(m_1q-1)!} \\ + \sum_{j=0}^{i-1} \frac{(m_1q+j-1)!F_1^{m_1q+j}(2T)}{2^{m_1q+j}j!(m_1q-1)!}] &+ \frac{Z(\cdot)bF_1^{m_1q}(\bar{t})}{\bar{t}} \sum_{i=\max\{1, IL_1^R+1\}}^{m_1q-1} [\bar{t}F_1^i(\bar{t}) - \frac{i}{\lambda}F_1^{i+1}(\bar{t})]. \end{aligned} \quad (4.31)$$

In addition, the expected inventory holding cost that will incur is calculated in a similar manner to (4.7) :

$$\begin{aligned} \sum_{i=(m_1-1)q+k_1+IL_1^R+1}^{m_1q+k_1+IL_1^R} \text{inv}_1^i &= \frac{Z(\cdot)h(IL_1^R + k_1 + q)}{T} [(T - 2\bar{t})F_1^{m_1q}(\bar{t}) \\ + TF_1^{m_1q}(T) - \frac{m_1q}{\lambda}(F_1^{m_1q+1}(T) - F_1^{m_1q+1}(\bar{t}))] \end{aligned}$$

$$-\frac{Z(\cdot)h}{T} \sum_{i=(m_1-1)q+k_1+LL_1^R+1}^{m_1q+k_1+LL_1^R} [(T-2\bar{t})F_1^i(\bar{t}) + TF_1^i(T) - \frac{i}{\lambda}(F_1^{i+1}(T) - F_1^{i+1}(\bar{t}))]. \quad (4.32)$$

R_2 initiates the shipment:

R_2 places the order which triggers the shipment dispatching decision at DC_1 . Subsequent to calculation of $Z(X_1^{n_1} < X_2^{n_2} \& X_2^{n_2} < X_1^{\bar{n}_1})$, (4.32) and (4.31) are used again after replacing $F_1^{m_1q}$ with $F_2^{n_2}$. The reason for the replacement is that n_2^{th} demand at R_2 triggers the dispatching decision instead of m_1q^{th} demand at R_1 . Calculation of inventory cost is similar. Final expected cost at R_1 will be:

$$U_1^R = \sum_{i=1}^{\infty} inv_1^i + back_1^i. \quad (4.33)$$

4.2.4.2 Expected Cost at Distribution Center

We calculate the expected inventory holding cost at DC_1 by using (4.19). Initial backlog cost is calculated by considering the total rate of backlog as a result of currently consolidated orders at DC_1 .

$$Ba_0 = (k_1 + k_2)qB. \quad (4.34)$$

To calculate the expected backlog cost associated with future demand, we follow the same steps given above for R_1 .

Dispatch shipment at time T :

As stated earlier, sum of m_1 plus m_2 has to be less than m . We consider each retailer separately and calculate the associated expected cost at DC_1 by matching retailers' orders with the items at DC_1 .

R_1 either may not place any order during the time interval $(0, T)$ or may have to place $m-1$ orders to DC_1 . For a given m_1 , R_2 can place at most $m - m_1 - 1$ orders. So, the total expected backlog cost as a result of R_1 's orders can be found by:

$$\sum_{m_1=1}^{m-1} ba_1^{m_1} = \frac{Bq}{T} \sum_{m_1=1}^{m-1} P(X \leq \bar{n}_2) [TF_1^{m_1q}(T) - \frac{m_1q}{\lambda} F_1^{m_1q+1}(T)]. \quad (4.35)$$

Dispatch shipment before time T :

After calculating $Z(\cdot)$ and changing m with m_1 or m_2 depending on which order initiates the dispatching decision, (4.20) is used together with corresponding probability to compute the expected backlog cost at DC_1 .

Finally, if the number of orders placed by R_1 is more than m there is no chance for DC_1 to replenish R_1 's inventory before time T , and (4.21) will be used to calculate corresponding backlog cost until time T .

The total expected cost of DC_1 will be:

$$U_1^D = \left[\left[\sum_{i=1}^2 \sum_{k=0}^{\infty} Ba_1^k \right] + In_1 \right] + \frac{F_2}{T}. \quad (4.36)$$

4.2.4.3 Expected Cost at the other Distribution Center

In the previous subsections, we presented the expected cost calculation at the retailers and DC_1 . The last component of the expected total supply chain cost is the expected cost at DC_2 . With the shipment consolidation taking place at DC_1 , it is clear that the only cost at DC_2 will be the inventory holding cost during the shipment consolidation period at DC_1 .

Similar to DC_1 case, the initial expected inventory cost at DC_2 will be calculated by using (4.19).

4.2.4.4 Expected Cost of Supply Chain

The total expected cost of the supply chain will be the sum of the DCs ' cost and the retailers' cost.

$$D_2^{SC} = \sum_{i=1}^2 U_i^D + U_i^R. \quad (4.37)$$

Although plot of the total expected cost of supply chain versus time is very similar to plot drawn under CP_3 , we were not able to prove that the expected cost function is convex under CP_4 as well. Therefore, in the numerical experiments, we search the range of $[\bar{t}, L]$ irrespective of the increase in cost rate.

4.3 Numerical Experiments

In this section, we investigate the performances of the four consolidation policies via a numerical experiment. This experiment is used to identify the ideal settings under which

the proposed consolidation policies, CP_3 and CP_4 , significantly improve the total cost and average delivery time compared to the other two policies.

We use the following set of parameters to come up with instances:

$$\begin{array}{lll} h = 1 & H = 0.8 & \lambda = 1 \\ \frac{h}{b} \in \{0.2, 0.05, 0.01\} & \frac{F_1}{b} \in \{9, 49, 99\} & \frac{F_2}{F_1} \in \{1.1, 1.5, 1.9\} \\ L \in \{8, 16, 24\} & & \end{array}$$

Initial analysis has shown that, for arbitrary order quantities and reorder points at the retailers and the DCs , the performance of CP_1 is significantly less than the other policies. Hence, by performing a simulation study, we change Q , R , q , and r values parametrically and determine the best set of reorder points and order quantities that minimizes the total cost of supply chain when CP_1 is used. Later, during the performance comparison, the same Q , R , q , and r values are used for all consolidation policies.

We consider 81 problem instances in total. For each problem instance, we run a simulation study for 20,000 time units and each instance is replicated 10 times. Data related to average inventory holding and backlog costs at the retailers and the DCs together with total transportation cost at the DCs , and the average retailers waiting time to receive an order from DCs is collected. All results are normalized before comparison. In order to normalize a performance measure, we used the following formula presented for the average total cost.

$$N_{TC_p} = \frac{TC_p}{TC_1} 100. \tag{4.38}$$

TC_p denotes the average total cost of consolidation policy p and N_{TC_p} denotes the normalized total cost performance of consolidation policy p . For all performance measures, lower normalized value means higher performance. Figure 4.5 shows the normalized costs and delivery performance of four policies.

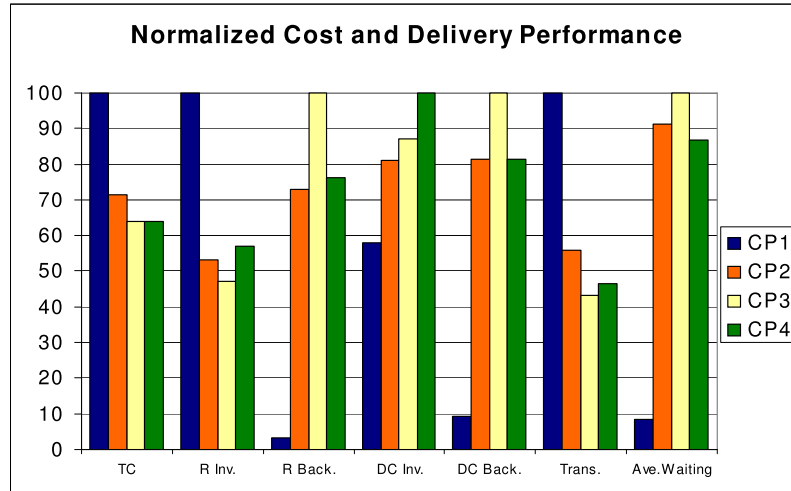


Figure 4.5

Normalized cost and delivery performance of the policies

Based on the numerical experiment, although CP_1 decreases the average waiting time of the retailers to receive their orders from the DCs , CP_1 is the most expensive alternative for the supply chain. Employing CP_2 decreases the overall cost of supply chain by 27%. CP_2 has a delivery performance between CP_3 and CP_4 .

Figure 4.5 shows that the difference in average total cost of supply chain under CP_3 and CP_4 is negligible. Utilizing a dual source consolidation allows DCs to reduce the average waiting time for the retailers without having a negative affect on the total cost, as seen in most of the problem instances.

The following examples elucidate how a dual source can increase the supply chain Performance. Based on the information given in Figure 4.5, CP_2 has a better delivery performance than CP_3 . However, by introducing CP_4 , the average waiting time is decreased by 5% compared to CP_2 and there is no significant increase in the total cost of supply chain.

We present Table 4.1 to depict the improvement in average costs as a consequence of using CP_2 , CP_3 , CP_4 over CP_1 . We use (4.39) to compute the percentage improvement of policy p with respect to total cost, PI_{TC_p} . The results related to delivery performance are present in Table 4.2.

$$PI_{TC_p} = \frac{TC_1 - TC_p}{TC_1} 100. \quad (4.39)$$

Table 4.1

Percent improvements in total cost with respect to the parameters

		CP_2 vs. CP_1	CP_3 vs. CP_1	CP_4 vs. CP_1
L	8	37.71	38.39	40.80
	16	22.53	35.08	34.42
	24	22.55	33.97	31.19
h/b	0.2	22.25	26.58	28.27
	0.05	26.80	34.31	37.34
	0.01	33.74	46.54	40.81
$\frac{F_1}{b}$	9	13.64	17.51	23.32
	49	30.86	40.40	37.12
	99	38.29	49.52	45.98
$\frac{F_2}{F_1}$	1.1	28.201	36.11	40.40
	1.5	27.985	36.21	35.29
	1.9	26.600	35.11	30.73

In the following paragraphs, we examine in detail, the effect of using CP_1 , CP_2 , CP_3 , and CP_4 at the DCs , and their impact on the total cost and average waiting time.

The average waiting time of the retailers' is the least when CP_1 is employed for all cases. This could be due to the fact that the DCs do not consolidate the retailers' orders under CP_1 but rather ship immediately. On the other hand, even a short waiting time could increase the total cost of supply chain enormously.

Table 4.2

Delivery performance of consolidation policies

		CP_1	CP_2	CP_3	CP_4
L	8	0.10	3.97	3.59	3.05
	16	0.54	4.08	4.59	3.96
	24	0.47	4.08	5.08	4.50
h/b	0.2	0.49	4.28	4.84	4.17
	0.05	0.37	4.08	4.36	3.75
	0.01	0.25	3.77	4.06	3.59
$\frac{F_1}{b}$	9	0.28	1.92	3.09	2.84
	49	0.41	4.24	4.69	4.10
	99	0.43	5.96	5.49	4.57
$\frac{F_2}{F_1}$	1.1	0.36	4.03	4.39	3.50
	1.5	0.39	4.05	4.38	3.60
	1.9	0.36	4.05	4.49	4.41

An increase in lead time from the outside supplier to the DCs , diminishes the benefits of shipment consolidation. Due to the delay in receiving shipment from the outside supplier and overdue consolidated orders pending distribution, the DCs are forced to deliver the items to the retailers at the earliest.

Long lead time has more adverse effects on CP_2 than CP_3 and CP_4 with respect to total cost. However, unlike the decrease in delivery performances observed in CP_3 and CP_4 , the performance of CP_2 is not significantly affected by the change in lead time.

Increase in backlog cost as compared to inventory holding cost at the retailers, increases the performances of CP_2 , CP_3 and CP_4 in terms of total cost of supply chain.

CP_2 does not take inventory holding cost into consideration. Thus as backlog cost becomes the dominant cost factor CP_2 performance gets better. Since F_1/b ratio is fixed, as backlog cost increases, so does transportation cost. Hence, consolidation brings more opportunities to supply chain with higher cost of transportation. There is an additional issue for CP_4 's performance, as backlog cost gets higher (F_1/b and F_2/F_2 are fixed), cost of serving both retailers by a DC gets expensive. Thus, alternative supply option becomes less attractive to improve the supply chain's performance.

It is clear that a consolidation program may not bring a lot of cost reduction chances to supply chains that have very expensive backlog cost or relatively less transportation cost (i.e. F_1/b ratio is lower). For example, when $F_1/b = 9$ all three consolidation policies are able to reduce total cost of supply chain less than 19% on average, whereas when $F_1/b = 99$ cost reductions are 38.29%, 49.52% and 45.98% respectively under the policies CP_2 , CP_3 and CP_4 . The numerical experiments also confirm that when transportation cost is less, the DCs tend to dispatch shipments more frequently. Consequently, retailers wait for a shorter period of time.

When transportation cost ratio of serving both retailers at the same time versus serving a single retailer is investigated, it is clear that both CP_2 and CP_3 do not get adversely

affected by the high cost of serving both regions. However, as F_2 gets expensive, performance of CP_4 approaches CP_3 in terms of average waiting time of retailers to replenish their inventory. Even though it is a big opportunity for a supply chain to have alternative supplier in the shipment consolidation settings, the benefit diminishes very fast as F_2/F_1 ratio gets larger.

4.4 Conclusions and Research Directions

In this research, we focused on a two-stage supply chain with two retailers and two distribution centers with four possible shipment consolidation policies available for implementation. We proposed two new consolidation schemes which consider overall supply chain performance rather than taking only distribution centers into account. We found the promised latest delivery time which is used as consolidation time by taking the available information into consideration. We then repeatedly used this approach to improve two performance metrics: Total cost of supply chain and average waiting time of retailers to receive a shipment from a distribution center. We compared the suggested consolidation policies, CP_3 and CP_4 with two other policies from the literature. Based on our numerical experiments, we can say that policy that is proven to be optimal in the long run under the assumption that instant shipment from the outside supplier to distributor (i.e. CP_2), CP_2 may not be a good approximation in a supply chain oriented environment with a positive lead time.

By utilizing CP_3 and CP_4 , we were able to reduce the total cost of supply chain more than 10% compared to CP_2 and more than 35% compared to CP_1 . Although utilizing

an immediate shipment policy decreases the average waiting time of retailer, the high percentage of cost improvements compared to CP_1 shows that immediate shipment may not be the best way of serving retailers in terms of overall supply chain performance.

In the future, it may be meaningful to evaluate supply chain with compound poisson end customer arrivals. In addition, during the numerical experiments we used q and r values found via simulation. It would contribute to the existing literature if we determine the optimal inventory control parameters under CP_3 and CP_4 for supply chains. Currently, we assume that there is no vehicle capacity constraint. Adding a vehicle capacity to the problem would be an interesting extension of this research.

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APPENDIX A
DERIVATION OF $G^I(T)$

$$G^i(t) = \int_0^t u f^i(u) du = \int_0^t \frac{\lambda^i u^i e^{-\lambda u}}{(i-1)!} du$$

Let $y = e^{-\lambda u}$ and $x = -\frac{\lambda^{i-1} u^i}{(i-1)!}$ then $dy = -\lambda e^{-\lambda u} du$ and $dx = -\frac{i \lambda^{i-1} u^{i-1}}{(i-1)!} du$.

$$\begin{aligned} G^i(t) &= \int_0^t \frac{\lambda^i u^i e^{-\lambda u}}{(i-1)!} du = \int_0^t x dy = xy - \int_0^t y dx \\ &= -\frac{\lambda^{i-1} u^i e^{-\lambda u}}{(i-1)!} + \int_0^t \frac{i \lambda^{i-1} u^{i-1} e^{-\lambda u}}{(i-1)!} du \\ &= -\frac{\lambda^{i-1} u^i e^{-\lambda u}}{(i-1)!} + \frac{i}{(i-1)} \int_0^t \frac{\lambda^{i-1} u^{i-1} e^{-\lambda u}}{(i-2)!} du \\ &= -\frac{\lambda^{i-1} u^i e^{-\lambda u}}{(i-1)!} + \frac{i}{(i-1)} \left[-\frac{\lambda^{i-2} u^{i-1} e^{-\lambda u}}{(i-2)!} + \frac{i-1}{(i-2)} \int_0^t \frac{\lambda^{i-2} u^{i-2} e^{-\lambda u}}{(i-3)!} du \right] \\ &\quad \vdots \\ &= \left[-\frac{i}{(i)} \frac{\lambda^{i-1} u^i e^{-\lambda u}}{(i-1)!} - \frac{i}{(i-1)} \frac{\lambda^{i-2} u^{i-1} e^{-\lambda u}}{(i-2)!} \dots - \frac{i}{(1)} \frac{\lambda^0 u^1 e^{-\lambda u}}{(0)!} \right. \\ &\quad \left. - i \frac{\lambda^{-1} u^0 e^{-\lambda u}}{1} + \frac{i}{\lambda} \right]_0^t \\ &= \frac{i}{\lambda} \left[1 - \frac{\lambda^0 t^0 e^{-\lambda t}}{(0)!} - \frac{\lambda^1 t^1 e^{-\lambda t}}{(1)!} \dots - \frac{\lambda^{i-1} t^{i-1} e^{-\lambda t}}{(i-1)!} - \frac{\lambda^i t^i e^{-\lambda t}}{(i)!} \right] \\ &= \frac{i}{\lambda} \left[1 - \sum_{k=0}^i \frac{(\lambda t)^k e^{-\lambda t}}{k!} \right] = \frac{i}{\lambda} F^{i+1}(t) \end{aligned}$$

APPENDIX B
CALCULATION OF Δ

Calculation of Δ when $IL \geq 0$

To simplify the expression for Δ , given by equation (2.6), equations (2.1) and (2.2) from Section 2.4 will be used. So, let us calculate equations (2.1) and (2.2) first.

$$inv_e(te_i) = h \left[\int_{te_i}^{\infty} (u - te_i) f^i(u) du \right] = h \int_{te_i}^{\infty} u f^i(u) du - hte_i \int_{te_i}^{\infty} f^i(u) du$$

Since $\int_0^{\infty} f^i(u) du = 1$ and $\int_0^{\infty} u f^i(u) du = i/\lambda$ (mean for Erlang(λ, i)) then

$$inv_e(te_i) = h \left[\frac{i}{\lambda} - \int_0^{te_i} u f^i(u) du \right] - hte_i \left[1 - \int_0^{te_i} f^i(u) du \right]$$

Now using equations (4.2) and (2.5) from Section 2.4 we have

$$inv_e(te_i) = h \frac{i}{\lambda} - h \frac{i}{\lambda} F^{i+1}(te_i) - hte_i + hte_i F^i(te_i) \quad (\text{B.1})$$

Similarly, $back_e(te_i)$ can be calculated as follows:

$$back_e(te_i) = b \left[\int_0^{te_i} (te_i - u) f^i(u) du \right] = bte_i \int_0^{te_i} f^i(u) du - b \int_0^{te_i} u f^i(u) du$$

$$back_e(te_i) = bte_i F^i(te_i) - b \frac{i}{\lambda} F^{i+1}(te_i) \quad (\text{B.2})$$

The calculations are analogous for $inv_l(tl_i)$ and $back_l(tl_i)$, so substituting (B.1) and (B.2) into (2.6) we get

$$\Delta = \sum_{i=1}^{IL} \left[h \frac{i}{\lambda} - h \frac{i}{\lambda} F^{i+1}(tl_i) - htl_i + htl_i F^i(tl_i) \right]$$

$$\begin{aligned}
& - \sum_{i=1}^{IL} \left[h \frac{i}{\lambda} - h \frac{i}{\lambda} F^{i+1}(te_i) - hte_i + hte_i F^i(te_i) \right] \\
& + \sum_{i=IL+1}^{IL+(m+n+1)q} \left[h \frac{i}{\lambda} - h \frac{i}{\lambda} F^{i+1}(tl_i) - htl_i + htl_i F^i(tl_i) \right] \\
& - \sum_{i=IL+1}^{IL+(m+n+1)q} \left[h \frac{i}{\lambda} - h \frac{i}{\lambda} F^{i+1}(te_i) - hte_i + hte_i F^i(te_i) \right] \\
& + \sum_{i=IL+1}^{IL+(m+n+1)q} \left[btl_i F^i(tl_i) - b \frac{i}{\lambda} F^{i+1}(tl_i) - bte_i F^i(te_i) + b \frac{i}{\lambda} F^{i+1}(te_i) \right] \quad (\text{B.3})
\end{aligned}$$

Since the first IL items are already at R_1 , by definition $te_i = tl_i = 0$ for $i = 1, 2, \dots, IL$. Thus, (B.3) reduces to

$$\begin{aligned}
\Delta & = \sum_{i=IL+1}^{IL+(m+n+1)q} \left[-h \frac{i}{\lambda} F^{i+1}(tl_i) - htl_i + htl_i F^i(tl_i) \right] \\
& - \sum_{i=IL+1}^{IL+(m+n+1)q} \left[-h \frac{i}{\lambda} F^{i+1}(te_i) - hte_i + hte_i F^i(te_i) \right] \\
& + \sum_{i=IL+1}^{IL+(m+n+1)q} \left[btl_i F^i(tl_i) - b \frac{i}{\lambda} F^{i+1}(tl_i) - bte_i F^i(te_i) + b \frac{i}{\lambda} F^{i+1}(te_i) \right] \quad (\text{B.4})
\end{aligned}$$

From Figure 2.5 we can see that $te_i = tl_i$ for $i = IL + 1, IL + 2, \dots, IL + mq$. So,

(B.4) simplifies to

$$\begin{aligned}
\Delta & = \sum_{i=IL+mq+1}^{IL+(m+n+1)q} \left[-h \frac{i}{\lambda} F^{i+1}(tl_i) - htl_i + htl_i F^i(tl_i) \right] \\
& - \sum_{i=IL+mq+1}^{IL+(m+n+1)q} \left[-h \frac{i}{\lambda} F^{i+1}(te_i) - hte_i + hte_i F^i(te_i) \right] \\
& + \sum_{i=IL+mq+1}^{IL+(m+n+1)q} \left[btl_i F^i(tl_i) - b \frac{i}{\lambda} F^{i+1}(tl_i) - bte_i F^i(te_i) + b \frac{i}{\lambda} F^{i+1}(te_i) \right] \quad (\text{B.5})
\end{aligned}$$

From Figure 2.5 we can also see that

$$te_i = \begin{cases} T_e & \text{if } IL + mq + 1 \leq i \leq IL + mq + q \\ T_{m+1} & \text{if } IL + mq + q + 1 \leq i \leq IL + mq + 2q \\ \vdots & \\ T_{m+n} & \text{if } IL + mq + nq + 1 \leq i \leq IL + mq + nq + q \end{cases}$$

and

$$tl_i = \begin{cases} T_{m+1} & \text{if } IL + mq + 1 \leq i \leq IL + mq + q \\ T_{m+2} & \text{if } IL + mq + q + 1 \leq i \leq IL + mq + 2q \\ \vdots & \\ T_l & \text{if } IL + mq + nq + 1 \leq i \leq IL + mq + nq + q \end{cases}$$

Let $T_e = T'_0$, $T_{m+1} = T'_1$, $T_{m+2} = T'_2$, ..., $T_{m+n} = T'_n$, $T_l = T'_{n+1}$, and

$\Delta = \sum_{j=0}^n \Delta_j$ where

$$\begin{aligned} \Delta_j = & \sum_{i=IL+mq+jq+1}^{IL+mq+jq+q} \left(-h \frac{i}{\lambda} F^{i+1}(T'_{j+1}) - hT'_{j+1} + hT'_{j+1} F^i(T'_{j+1}) \right. \\ & \left. + h \frac{i}{\lambda} F^{i+1}(T'_j) + hT'_j - hT'_j F^i(T'_j) \right. \\ & \left. + bT'_{j+1} F^i(T'_{j+1}) - b \frac{i}{\lambda} F^{i+1}(T'_{j+1}) \right. \\ & \left. - bT'_j F^i(T'_j) + b \frac{i}{\lambda} F^{i+1}(T'_j) \right) \end{aligned} \quad (\text{B.6})$$

By rearranging the terms (B.6) can be written as

$$\begin{aligned} \Delta_j = & qhT'_j - qhT'_{j+1} + \sum_{i=IL+mq+jq+1}^{IL+mq+jq+q} \left(\frac{i}{\lambda} F^{i+1}(T'_j)(h+b) \right. \\ & \left. - \frac{i}{\lambda} F^{i+1}(T'_{j+1})(h+b) + T'_{j+1} F^i(T'_{j+1})(h+b) - T'_j F^i(T'_j)(h+b) \right) \end{aligned} \quad (\text{B.7})$$

Using (B.7), the expression for Δ can be simplified to

$$\begin{aligned} \Delta = qhT'_0 - qhT'_{n+1} + (h+b) \sum_{j=0}^n \sum_{i=IL+mq+jq+1}^{IL+mq+jq+q} & \left(\frac{i}{\lambda} F^{i+1}(T'_j) \right. \\ & \left. - \frac{i}{\lambda} F^{i+1}(T'_{j+1}) + T'_{j+1} F^i(T'_{j+1}) - T'_j F^i(T'_j) \right) \end{aligned} \quad (\text{B.8})$$

Calculation of Δ when $IL < 0$

Substituting (B.1) and (B.2) into (2.8) will give

$$\begin{aligned} \Delta = b \sum_{k=1}^{|IL|} [tl'_k - te'_k] + \sum_{i=1}^{IL+(m+n+1)q} & \left[h \frac{i}{\lambda} - h \frac{i}{\lambda} F^{i+1}(tl_i) - htl_i + htl_i F^i(tl_i) \right] \\ - \sum_{i=1}^{IL+(m+n+1)q} & \left[h \frac{i}{\lambda} - h \frac{i}{\lambda} F^{i+1}(te_i) - hte_i + hte_i F^i(te_i) \right] \\ + \sum_{i=1}^{IL+(m+n+1)q} & [btl_i F^i(tl_i) - b \frac{i}{\lambda} F^{i+1}(tl_i)] \\ - \sum_{i=1}^{IL+(m+n+1)q} & [bte_i F^i(te_i) - b \frac{i}{\lambda} F^{i+1}(te_i)] \end{aligned} \quad (\text{B.9})$$

Equation (B.9) can be further simplified. However, unlike (B.3), the simplification of (B.9) depends on the size of IL . To simplify the notation let z be the number of full batches that will be used for backorders; i.e. $IL + zq \leq 0$ and $IL + zq + q > 0$.

Case 1: $z < m$

If $z < m$ then the first customer demand that R_1 receives will be satisfied by an item that gets to R_1 at or before T_m . Therefore, $tl'_k = te'_k$ for $k = 1, \dots, |IL|$. Also, $tl_i = te_i$ for $i = 1, \dots, IL + mq$. Thus, (B.9) reduces to

$$\Delta = \sum_{i=IL+mq+1}^{IL+(m+n+1)q} \left[-h \frac{i}{\lambda} F^{i+1}(tl_i) - htl_i + htl_i F^i(tl_i) \right]$$

$$\begin{aligned}
& - \sum_{i=IL+mq+1}^{IL+(m+n+1)q} \left[-h \frac{i}{\lambda} F^{i+1}(te_i) - hte_i + hte_i F^i(te_i) \right] \\
& + \sum_{i=IL+mq+1}^{IL+(m+n+1)q} \left[btl_i F^i(tl_i) - b \frac{i}{\lambda} F^{i+1}(tl_i) - bte_i F^i(te_i) + b \frac{i}{\lambda} F^{i+1}(te_i) \right] \quad (\text{B.10})
\end{aligned}$$

Note that (B.10) is identical to (B.5). Therefore, following the same steps shown above the expression for Δ will become as given in (B.8).

Case 2: $z = m$

If $z = m$ then the first customer demand that R_1 receives will be satisfied by an item that reaches R_1 either at time T_e if the early delivery option is selected or at time T_{m+1} if the late delivery option is selected.

$$te_i = \begin{cases} T_e & \text{if } 1 \leq i \leq IL + zq + q \\ T_{z+1} & \text{if } IL + zq + q + 1 \leq i \leq IL + zq + 2q \\ \vdots & \\ T_{m+n} & \text{if } IL + mq + nq + 1 \leq i \leq IL + mq + nq + q \end{cases}$$

and

$$tl_i = \begin{cases} T_{z+1} & \text{if } 1 \leq i \leq IL + zq + q \\ T_{z+2} & \text{if } IL + zq + q + 1 \leq i \leq IL + zq + 2q \\ \vdots & \\ T_l & \text{if } IL + mq + nq + 1 \leq i \leq IL + mq + nq + q \end{cases}$$

Note that $tl'_k = te'_k$ for $k = 1, \dots, mq$, $te'_k = T_e$ for $k = mq + 1, \dots, |IL|$, and

$$tl'_k = T_{m+1} \text{ for } k = mq + 1, \dots, |IL|$$

Given the above definitions for te_i , tl_i , te'_k , and tl'_k , (B.9) can be written as

$$\Delta = b \sum_{k=zq+1}^{|IL|} [T_{z+1} - T_e]$$

$$\begin{aligned}
& +(h+b) \sum_{i=1}^{IL+zq+q} \left(\frac{i}{\lambda} F^{i+1}(T_z) - \frac{i}{\lambda} F^{i+1}(T_{z+1}) + T_{z+1} F^i(T_{z+1}) - T_z F^i(T_z) \right) \\
& +(h+b) \sum_{j=z+1}^{m+n} \sum_{i=IL+jq+1}^{IL+jq+q} \left(\frac{i}{\lambda} F^{i+1}(T'_{j-m}) - \frac{i}{\lambda} F^{i+1}(T'_{j-m+1}) \right. \\
& \quad \left. + T'_{j-m+1} F^i(T'_{j-m+1}) - T'_{j-m} F^i(T'_{j-m}) \right) \tag{B.11}
\end{aligned}$$

Case 3: $m < z < m + n$

If $m < z < m + n$ then the first customer demand that R_1 receives will be satisfied by

an item from the $(z+1)^{th}$ batch. Therefore,

$$te_i = \begin{cases} T_z & \text{if } 1 \leq i \leq IL + zq + q \\ T_{z+1} & \text{if } IL + zq + q + 1 \leq i \leq IL + zq + 2q \\ \vdots & \\ T_{m+n} & \text{if } IL + mq + nq + 1 \leq i \leq IL + mq + nq + q \end{cases}$$

and

$$tl_i = \begin{cases} T_{z+1} & \text{if } 1 \leq i \leq IL + zq + q \\ T_{z+2} & \text{if } IL + zq + q + 1 \leq i \leq IL + zq + 2q \\ \vdots & \\ T_l & \text{if } IL + mq + nq + 1 \leq i \leq IL + mq + nq + q \end{cases}$$

In the above equation for tl_i when $z = m + n - 1$ then $T_{z+2} = T_{m+n+1} = T_l$. Also

$tl'_k = te'_k$ for $k = 1, \dots, mq$ and

$$te'_k = \begin{cases} T_e & \text{if } mq + 1 \leq k \leq mq + q \\ T_{m+1} & \text{if } mq + q + 1 \leq k \leq mq + 2q \\ \vdots & \\ T_z & \text{if } zq + 1 \leq k \leq |IL| \end{cases}$$

and

$$tl'_k = \begin{cases} T_{m+1} & \text{if } mq + 1 \leq k \leq mq + q \\ T_{m+2} & \text{if } mq + q + 1 \leq k \leq mq + 2q \\ \vdots & \\ T_{z+1} & \text{if } zq + 1 \leq k \leq |IL| \end{cases}$$

Given the above definitions for te_i , tl_i , te'_k , and tl'_k , (B.9) can be written as

$$\begin{aligned} \Delta = & b \sum_{j=m}^{z-1} \sum_{k=jq+1}^{jq+q} [T'_{j-m+1} - T'_{j-m}] + b \sum_{k=zq+1}^{|IL|} [T_{z+1} - T_z] \\ & + (h+b) \sum_{i=1}^{IL+zq+q} \left(\frac{i}{\lambda} F^{i+1}(T_z) - \frac{i}{\lambda} F^{i+1}(T_{z+1}) + T_{z+1} F^i(T_{z+1}) - T_z F^i(T_z) \right) \\ & + (h+b) \sum_{j=z+1}^{m+n} \sum_{i=IL+jq+1}^{IL+jq+q} \left(\frac{i}{\lambda} F^{i+1}(T'_{j-m}) - \frac{i}{\lambda} F^{i+1}(T'_{j-m+1}) \right. \\ & \quad \left. + T'_{j-m+1} F^i(T'_{j-m+1}) - T'_{j-m} F^i(T'_{j-m}) \right) \end{aligned} \quad (\text{B.12})$$

Case 4: $z = m + n$

If $z = m + n$ then $z = m + n$ then the first customer demand that R_1 receives will be satisfied by an item from the $(m+n)^{th}$ batch if early option is selected or at time T_l if the late delivery option is chosen.

$$te_i = \begin{cases} T_{m+n} & 1 \leq i \leq IL + (m+n)q + q \end{cases}$$

and

$$tl_i = \begin{cases} T_l & 1 \leq i \leq IL + (m+n)q + q \end{cases}$$

$$\begin{aligned} \Delta = & b \sum_{k=1}^{|IL+(m+n)q|} [tl'_k - te'_k] + \sum_{i=1}^{IL+(m+n+1)q} \left[h \frac{i}{\lambda} - h \frac{i}{\lambda} F^{i+1}(tl_i) - htl_i + htl_i F^i(tl_i) \right] \\ & - \sum_{i=1}^{IL+(m+n+1)q} \left[h \frac{i}{\lambda} - h \frac{i}{\lambda} F^{i+1}(te_i) - hte_i + hte_i F^i(te_i) \right] \end{aligned}$$

$$\begin{aligned}
& + \sum_{i=1}^{IL+(m+n+1)q} [btl_i F^i(tl_i) - b \frac{i}{\lambda} F^{i+1}(tl_i)] \\
& - \sum_{i=1}^{IL+(m+n+1)q} [bte_i F^i(te_i) - b \frac{i}{\lambda} F^{i+1}(te_i)] \tag{B.13}
\end{aligned}$$

Case 5: $z > m + n$

If this is the case then neither the shipment from DC_e nor DC_l to R_l will be used to fulfill customer demand during this order cycle. Thus, Δ will simply be as follows:

$$\Delta = b \sum_{k=1}^q [T_l - T_e] = bq[T_l - T_e].$$