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# Evaluation of a Product Development Process through Uncertainty Analysis Techniques

Pang Hui Wong

# EVALUATION OF A PRODUCT DEVELOPMENT PROCESS

# THROUGH UNCERTAINTY ANALYSIS TECHNIQUES

By

Pang-Hui Wong

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By

Pang-Hui Wong

Approved:

Susan T. Hudson
Adjunct Assistant Professor of
Mechanical Engineering
(Director of Thesis)

James C. Newman III
Associate Professor of
Aerospace Engineering
(Committee Member)

W. Glenn Steele
Professor and Head of
Mechanical Engineering
(Committee Member)

Rogelio Luck
Graduate Coordinator of
Mechanical Engineering

Carl A. James
Assistant Research Professor of
Mechanical Engineering
(Committee Member)

A. Wayne Bennett
Dean of the College of Engineering

Name: Pang-Hui Wong

Date of Degree: August 2, 2003

Institution: Mississippi State University

Major Field: Mechanical Engineering

Major Professor: Dr. Susan T. Hudson

Title of Study: EVALUATION OF A PRODUCT DEVELOPMENT PROCESS THROUGH UNCERTAINTY ANALYSIS TECHNIQUES

Page in Study: 126

Candidate for Degree of Master of Science

For any product development process, limited time and resources are always a focus for the engineer. However, will the overall program goals be achieved with the provided time and resources? Uncertainty analysis is a tool that is capable of providing the answer to that question. Product development process uncertainty analysis employs previous knowledge in modeling, experimentation, and manufacturing in an innovative approach for analyzing the entire process. This research was initiated with a pilot project, a four-bar-slider mechanism, and an uncertainty analysis was completed for each individual product development step. The uncertainty of the final product was then determined by combining uncertainties from the individual steps. The uncertainty percentage contributions of each term to the uncertainty of the final product were also

calculated. The combination of uncertainties in the individual steps and calculation of the percentage contributions of the terms have not been done in the past. New techniques were developed to evaluate the entire product development process in an uncertainty sense. The techniques developed in this work will be extended to other processes in future work.

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LIST OF VARIABLES AND SYMBOLS

| VARIABLE | | DEFINITION |
|---|---|---|
| $B_i$ | = | Systematic uncertainty of variable "i" |
| $B_{ik}$ | = | Correlated systematic uncertainties of variables "i" and "k" |
| $d$ | = | Piston displacement |
| $d_{av}$ | = | Actual average experiment data points |
| $d_{cr}$ | = | Connecting rod diameter |
| $d_{cs}$ | = | Crankshaft diameter |
| $d_{exp}$ | = | Equivalent experiment data points |
| $d_p$ | = | Piston diameter |
| $d_{total}$ | = | Total displacement |
| $E$ | = | Comparison Errors |
| $l_{cr}$ | = | Connecting rod Length |
| $l_{cs}$ | = | Crankshaft length |
| $l_p$ | = | Piston length |
| $l_1$ | = | Inner length of the connecting rod |
| $l_2$ | = | Outer length of the connecting rod |
| $P_i$ | = | Random uncertainty of variable "i" |

| | | |
|---|---|---|
| $R_x$ | = | Random uncertainty of variable “x” |
| $S_x$ | = | Systematic uncertainty of variable “x” |
| $s_x$ | = | Slop |
| $t_x$ | = | Manufacture tolerance of variable “x” |
| $U_x$ | = | Total uncertainty of variable “x” |
| $\theta$ | = | Crank angle |
| $\theta_i$ | = | Partial derivative of data reduction equation with respect the variable “i” |
| $\theta_0$ | = | Reference angle |
| $\omega$ | = | Engine speed |
| $\Delta t$ | = | Difference of elapsed time and initial time |

# CHAPTER I

# INTRODUCTION

Uncertainty has always been a part of a product development process; however, uncertainty analysis is still an evolving field. Uncertainty analysis application in experimentation is well established, and uncertainty analysis is capable of giving a promising result in any specific area of a product development process. However, new challenging tasks will be to find the uncertainty of the final manufactured product including the uncertainties in all of the steps in a product development process and to show the relationship or connection of each step in a product development process. This project will show a way of handling the uncertainties in each step of a product development process so that the overall final uncertainty of the final manufactured product as well as the percentage contribution of each step to the overall final uncertainty of the final comparison error can be determined. Generalization of this methodology will enable application of the methodology to other different product development processes.

## Research Objectives

The steps in a product development process can be generalized as follows: modeling, experimentation, manufacturing, and comparison. Experimental uncertainty analysis is well established, and uncertainties due to manufacturing alone are fairly well understood. Uncertainty due to modeling is an evolving area. However, uncertainty analysis for comparisons between steps in a product development process is new. Combining the uncertainties of all of the steps to give the overall uncertainty of the final manufactured product that represents the entire product development process is a new challenge in the field of uncertainty analysis. The main objectives of this research are to determine the performance and the overall uncertainty of the final manufactured product, determine the relationships between each product development step, and determine the relative contributions of each step to the overall uncertainty of the final product for a single, well-defined case. This has not been done in the past. However, the final goal of the overall research in this area is to outline a general methodology that is applicable to other product development processes.

The first objective stated in the previous paragraph is to determine the performance and the overall uncertainty of the final manufactured product. This can be accomplished by using information from the model, experiment, and manufacturing and making comparisons incorporating uncertainty analysis ideas. The degree of goodness of the product will be the main focus when drawing conclusions based on the comparisons. Therefore, uncertainty analysis will be the tool that best suits in performing such a task. By referring to the uncertainty of the final product, the engineer can determine if the product's performance meets program goals and requirements with a certain degree of confidence.

The second objective of this research is to determine the relationship between each step in the product development process. This is new in the uncertainty analysis field because, although the uncertainty in each individual step (modeling, experimentation, manufacturing, and comparisons) may be calculated, there is no well-defined relationship between the steps. Also, different product development processes will have different specific individual steps, and, therefore, different relationships between the steps. The route to generate the overall uncertainty of the final product will be different for different processes. There is no general data reduction equation to combine all of the steps.

The third objective of the research is to determine the relative contributions of each step to the overall uncertainty of the product. Knowing the relative contributions of each step will identify the controlling steps where improvements are needed. The understanding of this third objective will lead to a more efficient and reliable product development process in terms of cost and time.

## Uncertainty Analysis Overview

Uncertainty can be defined as the interval around a result from an experiment or a design calculation where the "true" value is expected to lie with a certain degree of confidence. In every experiment, one question arises, "*How do the uncertainties in the individual variables propagate through a data reduction equation into result?*" The answer can be found through uncertainty analysis. An overview of the uncertainty analysis methods employed for this research is given below. Further detailed information on these uncertainty analysis methods can be found in Coleman and Steele.[1]

**General Uncertainty Analysis**

During the planning phase of the experimentation, only general uncertainties will be considered in each measured variable rather than separate systematic and random uncertainties. For a general uncertainty analysis, the result, *r*, is determined by a data reduction equation and is a function of *J* measured variables.

$$r = f(X_1, X_2, \ldots, X_J) \tag{1.1}$$

The uncertainty of the result, $U_r$, is a function of the uncertainties in the measured variables.

$$U_r = f(U_{X1}, U_{X2}, \ldots, U_{XJ}) \tag{1.2}$$

Equation (1.2) can be expressed in the following form:

$$U_r^2 = \left(\frac{\partial r}{\partial X_1}\right)^2 U_{X1}^2 + \left(\frac{\partial r}{\partial X_2}\right)^2 U_{X2}^2 + \ldots + \left(\frac{\partial r}{\partial X_J}\right)^2 U_{XJ}^2 \tag{1.3}$$

Equation (1.3) assumes the measured variables are independent of one another and the uncertainties in the measured variables are also independent of one another. The first order derivatives of the data reduction equation with respect to each of the measured variables are defined as the sensitivity coefficients. In the planning phase, all the uncertainties in the measured variables should be expressed with a level of confidence. A 95% confidence level is often used. Thus the uncertainty in the result is also being expressed at 95% confidence.

Uncertainty magnification factor (UMF) and uncertainty percentage contribution (UPC) are two nondimensionlized factors derived from Equation (1.3) that are extremely beneficial to the planning phase uncertainty analysis. To obtain the UMFs from Equation (1.3), each term in that equation is divided by $r^2$, and only the right-hand side of the equation is multiplied by $(X_{.i} / X_i)^2$, which is equal to 1. Hence Equation (1.3) will then be transformed into the following form:

$$\frac{U_r^{\ 2}}{r^2} = \left(\frac{X_1}{r}\frac{\partial r}{\partial X_1}\right)^2 \left(\frac{U_{X1}}{X_1}\right)^2 + \left(\frac{X_2}{r}\frac{\partial r}{\partial X_2}\right)^2 \left(\frac{U_{X2}}{X_2}\right)^2 + \ldots + \left(\frac{X_J}{r}\frac{\partial r}{\partial X_J}\right)^2 \left(\frac{U_{XJ}}{X_J}\right)^2 \quad (1.4)$$

Here $U_r$ / $r$ is the relative uncertainty of the result and $U_{Xi}$ / $X_i$ is the relative uncertainty for each variable. The UMFs are the factors in parentheses that multiply the relative uncertainties of the variables, which can be defined as

$$UMF_i = \frac{X_i}{r}\frac{\partial r}{\partial X_i} \quad (1.5)$$

The UMF for a given $X_i$ indicates the influence of the uncertainty in that variable on the uncertainty in the result. A UMF value greater than 1 indicates that the influence of the uncertainty in the variable is magnified as it propagates through the data reduction equation into the result and vice-versa. However, since the UMFs are squared in Equation (1.4), their signs are not important and only the absolute values of the UMFs will be considered. UMF is sometimes called *normalized sensitivity coefficient*.

Uncertainty Percentage Contribution (UPC) will be the second nondimensionlized form of Equation (1.3) and is found by dividing the equation by $U_r^2$, as shown in the following equation:

$$1 = \frac{\left(\frac{\partial r}{\partial X_1}\right)^2 U_{X1}^2 + \left(\frac{\partial r}{\partial X_2}\right)^2 U_{X2}^2 + \ldots + \left(\frac{\partial r}{\partial X_J}\right)^2 U_{XJ}^2}{U_r^2} \tag{1.6}$$

UPC for each variable is then defined as

$$UPC_i = \frac{\left(\frac{\partial r}{\partial X_i}\right)^2 (U_{Xi})^2}{U_r^2} \tag{1.7}$$

The UPC for a given $X_i$ gives the percentage contribution of the uncertainty in that variable to the squared uncertainty in the result. This is a very useful and powerful tool in the planning phase before proceeding to design an experiment using detailed uncertainty analysis.

## Detailed Uncertainty Analysis

Detailed uncertainty analysis is a more complex approach compared to general uncertainty analysis that is used in the planning phase of an experiment. The primary reason for applying a more complex approach is that it is very useful in the design, construction, debugging, data analysis, and reporting phases of an experiment to consider separately the systematic and random components of uncertainty. The following paragraphs will outline the consideration of systematic and random errors in each measured variable and the propagation of the systematic and random uncertainties into the experimental result.

As shown by Equation (1.1), the result, $r$, is determined by a data reduction equation and is a function of $J$ measured variables.

$$r = f(X_1, X_2, \ldots, X_J) \tag{1.1}$$

Each individual variable $X_i$ is influenced by two main types of errors, which are the systematic errors and random errors. These errors in the measured variables then propagate through the data reduction equation and yield the systematic and random errors in the final experiment result. The procedure of detailed uncertainty analysis is to first obtain the estimates of both the systematic and random uncertainties for each measured variable, and then use the uncertainty analysis expression to obtain the values for the systematic and random uncertainties of the experimental result. The detailed uncertainty analysis expressions for the experimental result are

$$U_r^2 = B_r^2 + P_r^2 \tag{1.8}$$

where

$$B_r^2 = \sum_{i=1}^{J} \theta_i^2 B_i^2 + 2\sum_{i=1}^{J-1} \sum_{k=i+1}^{J} \theta_i \theta_k B_{ik} \tag{1.9}$$

and

$$P_r^2 = \sum_{i=1}^{J} \theta_i^2 P_i^2 \tag{1.10}$$

with an assumption that there are no correlated random uncertainties in the final experiment result. $U_r$, $B_r$, and $P_r$ are the overall uncertainty, systematic uncertainty, and random uncertainty of the result given by Equation (1.8). $B_i$ and $P_i$ are the systematic uncertainty and the random uncertainty of each measured variable $X_i$ in Equation (1.1).

$B_{ik}$ is the correlated systematic uncertainty for the measured variables that share common elemental error sources and will be discussed further in the following sections. $\theta_i$ is the first-order derivative of the data reduction equation of the result with respect to the measured variable $X_i$.

$$\theta_i = \frac{\partial r}{\partial X_i} \tag{1.11}$$

If the measurements in Equation (1.9) share no common elemental error source, then the correlated systematic uncertainty terms are zero, and Equation (1.9) becomes

$$B_r^2 = \left(\frac{\partial r}{\partial X_1}\right)^2 B_1^2 + \left(\frac{\partial r}{\partial X_2}\right)^2 B_2^2 + \ldots + \left(\frac{\partial r}{\partial X_J}\right)^2 B_J^2 \tag{1.12}$$

On the other hand, if the measurements in Equation (1.9) do share common elemental error sources, then the correlated systematic uncertainty terms will not be zero, and there are certain procedures that need to be followed to obtain the correlated systematic uncertainty estimates. Since correlated systematic uncertainties are not independent of each other, the $B_{ik}$ term must be approximated using the following equation:

$$B_{ik} = \sum_{\alpha=1}^{L} (B_i)_\alpha (B_k)_\alpha \tag{1.13}$$

The term $L$ represents the number of elemental systematic error sources that are common for measurements of variables $X_i$ and $X_k$.

The random uncertainties for each variable will be determined using the same procedure as the systematic uncertainties. Equation (1.10) can be represented in the following form:

$$P_r^{\,2} = \left(\frac{\partial r}{\partial X_1}\right)^2 P_1^{\,2} + \left(\frac{\partial r}{\partial X_2}\right)^2 P_2^2 + \ldots + \left(\frac{\partial r}{\partial X_J}\right)^2 P_J^2 \tag{1.14}$$

The individual random uncertainties of the variables can be determined by

$$P_{Xi} = 2S_{Xi} = 2\left\{\frac{1}{N_i - 1}\sum_{k=1}^{N_i}\left[(X_i)_k - \overline{X}_i\right]^2\right\}^{1/2} \tag{1.15}$$

Equation (1.15) assumes a 95% confidence level with a large sample size, $N_i \geq 10$. The random uncertainties of the variables are the standard deviations of the sample population multiplied times 2 for a large sample size experiment.

Equation (1.9) and Equation (1.10) will determine both the systematic uncertainties and random uncertainties for the result. The overall uncertainty of the final result will then be the root-sum-square of both the systematic and random uncertainties as shown in Equation (1.8).

## Product Development Process Uncertainty Analysis Overview

From the experimental uncertainty analysis techniques, the result and the uncertainty of the result are given by Equation (1.1) and Equation (1.2) where the result and the uncertainty of the result are functions of the measured variables and the uncertainties in those measured variables. The product development process is analogous to the experiment. Each step in the product development process is unique and independent of each other. Therefore, the final product, $P$, is a function of the $m$ steps in the process as shown below:

$$P = f\left(Step_1, Step_2, Step_3, \ldots, Step_m\right) \tag{1.16}$$

The uncertainties associated with each step are also independent of each other. Using the same analogy to the experimental uncertainty analysis, the uncertainty of the final product, $U_p$, is a function of the uncertainties in the $m$ steps in the process.

$$U_p = f\left(U_{Step1}, U_{Step2}, U_{Step3}, \ldots, U_{Stepm}\right) \tag{1.17}$$

Evaluation of the uncertainty in each step in the product development process is well defined. However, the relationship between the uncertainties in the steps of a product development process is not currently clearly defined. The only way to determine that relationship is to fully understand the uncertainties in each of the individual steps and the interactions between the steps to produce the final product. Only through total understanding of the process is one able to determine the uncertainty of the final product. This determination will also help to identify the critical steps that contribute the highest uncertainties to the uncertainty of the final product. Then improvements can be made regarding those critical steps. This will also help to evaluate the overall product development process and determine if one can meet the research goals.

This product development process uncertainty methodology is unique and different than the traditional approach. The traditional approach has separate groups for models, experiments, and manufacture. Each group may use uncertainty analysis at the end of each individual step for comparison purposes but not for the overall uncertainty of the product development process. Therefore, this product development process uncertainty method will bring together the computational work, experimental work, and manufacturing work. The overall product development process will be modeled so that the uncertainty is built into the product development process as well as each individual step in the process. Thus the engineer knows what to expect for the uncertainty in the product development process and the controlling factors for the uncertainty in the process. The most important conclusion about this method is that uncertainty will be built into the product development process rather than simply used at the end for comparisons.

# CHAPTER II

# LITERATURE SURVEY

## Objectives

Uncertainty analysis is a relatively new field of study, and it was conceived as an experimental strategy. Recently, researchers have begun to apply experimental uncertainty analysis techniques in the product development process with the hope of improving the current methodology. Efforts and new ideas are required to get the uncertainty analysis methodology updated and improved.

In a product development process, the process generally consists of the following individual steps: modeling (computation), experimentation, manufacturing, and comparisons. From an experimental uncertainty technique point of view, each of the individual steps has uncertainties associated with it, and the uncertainty analysis of each step could be treated as a complete uncertainty analysis just for that particular step. However, the relationship between the uncertainties in the steps of a product development process is not currently defined. Therefore, the effect of the inherent uncertainties of each step in the product development process on the uncertainty of the final manufactured product is unknown until the relationship between the steps is determined.

One of the research goals is to develop a final product uncertainty and to identify the main controlling uncertainties for the product development process. Through the literature survey conducted, there are several tools or methods that can aid in identifying the controlling parameter(s) in a product development process and also forming a linkage between the uncertainties of each step. This establishes a relationship between the uncertainties to give the overall final uncertainty of the process. The literature survey focused on modeling, multidisciplinary design optimization (MDO), robust design, and the design of experiments (DoE). The primary focus was to determine how these current areas may contribute to the current research. These areas are summarized and discussed relative to the research goal in the following paragraphs.

## Key Areas

### Modeling

Modeling is one of the general steps in any product development process. In most engineering designs, modeling is often related to the model simulation, and the results obtained through the model simulation will later be used in the validation analysis. Improving the uncertainty of the model results will improve the uncertainty of the final product because uncertainty analysis in the model step will first highlight the controlling input parameter during the computational simulation prior to the execution of experiment and manufacturing.

Uncertainty exists in modeling due to variations in design conditions, numerical accuracy, simplifying assumptions, structure of the model, etc.[1-6] Most of the researchers agreed that uncertainties existing in any computational simulation prediction are greatly affected by the input parameter uncertainty. The input parameters serve solely to provide a more physically meaningful model equation, but each input parameter has an individual probability distribution that equivalently represents the uncertainty associated with the particular input parameter.[2] Different approaches have been used to estimate the propagations of the input parameter uncertainties, such as Taylor series approximation, vector uncertainty approach, Monte Carlo simulation, etc.[2-5]

In the general methodology, the first order Taylor series expansion is used to estimate the propagation of the input parameter uncertainties, and the uncertainty of the result is shown by Equation (1.3). If there are correlated uncertainties among the uncertainties of the input parameters, then the covariance matrix will be included in the uncertainty analysis as shown in Equation (1.9).[1] However, this method is best for the least number of input parameters. For a larger number of input parameters, Taylor series approximation will not be suitable in handling the large covariance matrix.[4] NASA Ames Research Center has developed an alternative approach, called the vector approach, to solve the propagation of the input parameters with correlated uncertainties. The independent input parameter uncertainties are modeled as vectors, and these uncertainties are propagated through a data reduction equation in vector form. The effects of the correlated uncertainties are implicitly included when two uncertainty vectors having the correlated terms are added. The NASA Ames Research Center has proven that this vector uncertainty approach is mathematically equivalent to the first order Taylor series expansion approach,[4] but the determination of which methodology is to be used will depend on which methodology is more convenient.

Monte Carlo simulation is another general technique used by researchers in model simulation. Monte Carlo simulation relies on the probability distribution of each of the input parameters, and it generates an estimate of the overall uncertainty in the prediction due to all the input parameter uncertainties regardless of the quantity of the parameters.[5] In most cases, any input parameters with large sample size are assumed to have Gaussian distributions. However, Los Alamos National Laboratory[2] has a different approach in handling the propagations of the input parameter uncertainties. The Bayesian approach and the concept of hierarchy of experiments were applied to merge the uncertainties associated with the input parameters and give the uncertainty of the final result. This concept is similar to the experiment uncertainty technique methodology, which was explained in Chapter I, and the final uncertainty was expressed by Equation (1.3).

How valid is the final output uncertainty obtained through the model simulation? According to an American Institute of Aeronautics and Astronautics (AIAA) committee[7], the key to establishing credibility for a computational simulation is through verification and validation (V&V). *Verification is the process determining that a model implementation accurately represents the developer's conceptual description of the model and the solution to the model. Validation is the process of determining the degree of which a model is an accurate representation of the real world from the perspective of* *the intended uses of the model*.[7] The AIAA has developed a general guide for the verification and validation process.[7-8] In a nutshell, the validity of the model simulation result is based on the direct comparison between the model simulation result and the most trustworthy experiment data. *However, this validation strategy does not imply that the experimental measurements are more accurate than the computational result. This strategy only asserts that experimental measurements are the most faithful reflections of reality for the purposes of validation*.[8] Therefore, the validation analysis will only be applied after the experiment was conducted and then only the credibility of the model simulation can be determined. An understanding of the V&V analysis definitely is beneficial to this research because comparison between model and experiment is an important step in the product development process.

MSU researchers have developed a step-size independent technique for determining multidisciplinary sensitivity derivatives.[9] It is an expansion of the Taylor series function using a complex step. This multidisciplinary sensitivity analysis technique is an advantage compared to other numerical methods like the central-finite difference theorem because it is not subject to cancellation errors.[9] Sensitivity analysis is performed in connection with uncertainty analysis for modeling with the aim of determining the uncertainty of the model results and identifying the controlling parameters contributing to the uncertainty. Coherent to sensitivity analysis, experimental uncertainty analysis techniques use uncertainty magnification factor (UMF) and uncertainty percentage contribution (UPC) to gain insight into the uncertainty distribution among the parameters.[1] The UMF for a given variable indicates the possible influence of the uncertainty in that variable on the overall uncertainty in the final result based solely on the data reduction equation for the result. UPC for a given variable gives the percentage contribution of the uncertainty in that variable to the squared uncertainty in the final result. The UPC includes both the UMF term and the magnitude of the uncertainty for the variable. Applying similar analyses to modeling will allow the controlling parameters for uncertainty to be identified and the model results to be enhanced improving the uncertainty of the final product.

Modeling uncertainty analysis should be applied early in a product development process because uncertainty analysis on model simulation results will set up a solid foundation before researchers advance to further development steps of the research. Researchers will be able to plan an experiment setup that meets model validation requirements, and the direct comparisons can be made between the experiment result and the model simulation result during the later process.

## Multidisciplinary Design Optimization

Multidisciplinary design optimization (MDO) is a design tool that can be used to obtain the best design parameter(s) of the overall design.[10-15] With the defined objective function, there are optimization techniques that can either maximize or minimize the objective function as preferred.[10] Optimization can be applied on various conditions such that the function or sub-function can either be constrained or unconstrained for single or multiple variables. For the unconstrained optimization, there are no limitations on the design optimization process, and the only goal is to achieve the best design parameter(s) that fulfills the objective function. The constrained optimization is the opposite of the unconstrained optimization, and, in reality, all designs will be subjected to constraints. Each design parameter is associated with constraint, and the best design result will not violate any of the constraints. MDO techniques are often used for design, but they may also be adapted to aid in determining the uncertainty of the final manufactured product using information from various steps in the product development process.

An optimization research that incorporated uncertainty analysis was conducted at Rice University.[11] The issue was to select the least expensive combination of experiment equipment that would give the desired accuracy of results. The basic idea behind this research was that the uncertainty analysis would give reliable results and the design optimization techniques would give the best optimization results. The data reduction equations of the allowable uncertainty were treated as the constraining equations, and the cost of the experimental equipment was the objective function of the optimization process. Thus the optimization technique was performed, and the optimized result was obtained without any constraint violation. The result of the optimization process gave the minimum cost of the experimental equipment with the allowable uncertainty constraints.

Optimization may consist of multiple individual “modules,” and the NASA Glenn Research Center in Cleveland has developed a general optimization tool, COMETBOARDS. COMETBOARDS optimizes each module of the design process individually then uses the best design obtained from each module to give the overall optimum design.[12] Each module can be defined with a different objective function, design variables, and design constraints, which leads to multiple steps of optimization over a design process.[12] The product development process, analogously, may also have different "modules" with different objective functions. The "modules" here would be the various steps in the product development process. In short, the aim of performing the uncertainty analysis is to get the overall uncertainty for the final manufactured product and to understand the relative contributions of each step to this overall uncertainty. Proper methods to determine the uncertainty of each step will help in determining the overall final uncertainty. Also, minimizing the highest uncertainty contributed by one of the product development process steps will help to minimize the overall uncertainty of the final product.

With the analogies between product development uncertainty analysis and design optimization, design optimization techniques offer great promise for development methods to determine the uncertainty of the final manufactured product as well as the contributions of each step in the product development process to this overall uncertainty.

## Robust Design

Robustness means the state where the technology, product, or process performance is minimally sensitive to factors causing variability and aging at the lowest manufacturing cost unit.[16] Robust engineering concentrates on identifying the ideal function for a specific process design and selectively choosing the best nominal values of design parameters that optimizes the performance reliability at lower cost. In robust design, Signal-to-Noise Ratio is an index of robustness. Higher ratios will improve the level of performance of the desired function to the variability of the desired function. The final result obtained from the robust design will be the least sensitive to the noise, which means the final result is robust. The robust final result also gives the smallest overall uncertainty of the design because the final result is least sensitive to the noise.

Robust design can be applied on both static and dynamic problems.[17-18] Static systems are defined as the final output of the system with a fixed target value, and the dynamic systems have a target value that depends on the input signal set by the operator. For dynamics systems, the relationship between the signal and the response will determine the final output result. Thus, any deviation from the relationship will deviate to the final output result from the ideal target value. Taguchi proposed a two-step procedure to identify the "optimal" factor settings that minimized the average loss, but McCaskey and Tsui[17] showed Taguchi's two-step procedure was only appropriate under multiplicative model. McCaskey and Tsui[17] proposed another two-step procedure, which adopted the same methodology but was more convenient to apply to other dynamic problems.

Depending on the objective defined for the robust design model, robust design also can handle uncertainties that exist in each subsystem of the design model. Multiple subsystems may exist under a robust engineering system design, and the evaluation of the system will be burdened by the uncertainties that exist in each subsystem. Georgia Institute of Technology, University of Illinois, and University of Waterloo have conducted research on incorporating the uncertainty analysis into robust design.[19-21] Robust design is capable of improving the quality of individual components in a complex engineering system, and MDO is a useful tool for designing the complex system. The University of Illinois[20] developed a method that integrates the robust design concept and the MDO framework in designing complex systems through uncertainty analysis. A complex system had subsystems in which all the subsystems were related to each other. The errors associated with those subsystems were formulated in terms of the input parameters, and the subsystems' outputs were subjected to robust multidisciplinary optimization to reduce the variability of the parameters. Uncertainty analysis was used to evaluate the means and the variances of the system outputs. A robust multidisciplinary design procedure was developed, and the MDO algorithm was used to inspect the uncertainty propagation with the objective of increasing the robust feasibility. Georgia Institute of Technology[21] proposed a robust design simulation (RDS) framework that used the Monte Carlo simulation's result to generate a desired probability distribution function of the parameters. The dependency of the objective on the parameters' uncertainty would be clearly exposed. Thus, the amount of design evaluation can be reduced significantly at the system level, and the robust design can be achieved.

As for the product development process, the product development steps may be analogous to the subsystems in robust design. By referring to the method used to form the relationship among the uncertainties of each subsystem in robust design, it may be possible to apply the same ideas to the product development process to determine the uncertainty of the final manufactured product.

## Design of Experiments

Design of Experiments (DoE) is a test or a series of tests to identify the variations on input parameters of a process or a system required to produce changes in output response.[22] The application of DoE is broad because DoE can serve as a tool to evaluate issues from material alternatives to the evaluation and comparison of entire design configurations.[22] DoE plays an important role in the development process and troubleshooting process to improve the performance of a process or a system. DoE also can determine the key product design parameters that have the most impact on product performance so that the product is affected minimally by the external sources of variability. Generally, DoE is capable of performing the optimization task to give the robust result and also the sensitivity analysis to determine the propagation of the parameters in the overall design.

The NASA Langley Research Center performed research concerning the influence that the order of setting the inputs variables has on the quality of an experiment result.[23] There were uncertainties associated with those independent variables, and the order of setting these independent variables would determine the systematic errors' propagation during the experiment. Previously, the random errors that were associated with the variables were the primary focus. However, recently, research has found that the systematic errors did have significant effects on the outcome of the experiment. Systematic errors are hard to detect compared to the random errors, and a different approach is needed to handle these systematic errors. Researchers found that systematic errors would be significantly large if the independent variables were in sequential order.

Thus, randomizing the order of the independent variables would decrease the systematic errors.[23]

In the product development process, the functions of DoE may have a significant impact on the uncertainty analysis. DoE may be useful in determining the key steps (input parameters) that contribute the most to the uncertainty[24] and also in rearranging the design configuration to give the best estimate of the uncertainties of the steps in the product development process. This will give the best estimate of the overall uncertainty.

Based on the literature survey, there are several areas of research that may contribute to research on uncertainty in a product development process. Since each product development process step has uncertainty associated with it, it is the engineer's concern to be able to determine the overall uncertainty for the final product, to understand the contribution of each step in the product development process to this overall uncertainty, and to minimize the uncertainty so that program goals are achieved. Combining experimental uncertainty analysis techniques with other methods available will allow the research goals to be obtained. However, the process of incorporating the uncertainty analysis techniques with the other methods will be complex due to the complexity that already exists in individual methods.

# CHAPTER III

# PILOT PROJECT

## Background

To initiate this uncertainty analysis research, a four-bar-slider mechanism, which consists of a single cylinder engine, was chosen to be the pilot project.[25] This pilot project included the four general steps in a product development process: model, experiment, manufacture, and comparisons. The four-bar-slider mechanism is a tool or linkage that is used to convert the rotational energy to the translation energy or vice versa. The components of this four-bar-slider mechanism are the crankshaft, connecting rod, single-cylinder engine, and the piston from a reciprocal internal combustion engine, which is shown in Figure 3.1.


Figure 3.1: Single-Cylinder Engine

The process of this pilot project was defined to achieve the objectives of the research. The objectives of the project are listed in Table 3.1.

The first step was to develop a model or mathematical equation to represent the four-bar-slider mechanism and complete the model uncertainty analysis. The mathematical equation was a kinematical equation for determining the piston displacement. For manufacture, the connecting rod was selected for redesign and remanufacture. The connecting rod had two different diameters; the baseline design case was 0.75-inch diameter and another case was 0.85-inch diameter. The manufacture's tolerance was included in the manufacture uncertainty analysis. The experimentation was planned to measure the piston displacement and an uncertainty analysis was completed on the experimental data. With all these uncertainty analyses available, a relationship between each step had to be defined so that the performance of the final product could be determined. The final objective was to determine the uncertainty of the final product and the relative contributions of each step to the overall final product uncertainty. This was the most important goal of the pilot project. Initial work regarding this project is documented in reference 25. The work presented here is a continuation of that work to complete the project objectives. Objectives (4) through (6) define new research in this field; similar work has not been done in the past.

Table 3.1: Pilot Project Objectives

| | |
|---|---|
| (1) | Develop computation model, design mechanism, and complete uncertainty analysis of model |
| (2) | Plan and execute experiment and complete uncertainty analysis of experimental data |
| (3) | Manufacture the product and complete uncertainty analysis for manufacturing |
| (4) | Evaluate relationships between steps |
| (5) | Define performance and uncertainty of final product |
| (6) | Determine relative contributions to uncertainty of final product |

## Model

The model was based on the four-bar-slider mechanism. Detailed information on this mechanical linkage can be found in Shigley and Vicker.[26] The kinematical model equation for this four-bar-slider mechanism is

$$d(\theta) = l_{cs}\cos(\theta) + \sqrt{{l_{cr}}^2 - l_{cs}^2 \sin^2(\theta)} + l_p + s_x \tag{3.1}$$

Equation (3.1) can be used to determine the piston displacement as a function of the crank angle, $\theta$. Equation (3.1) must be written in terms of the measurable variables according to the rules of uncertainty analysis. However, the center-to-center distance of the connecting rod, $l_{cr}$ was not measured directly. Therefore, the outer length, $l_2$, and the inner length, $l_1$, of the connecting rod were measured to give the overall center-to-center distance of the connecting rod, and Equation (3.1) was rewritten as

$$d(\theta) = l_{cs}\cos(\theta) + \sqrt{\left(\frac{l_1 + l_2}{2}\right)^2 - l_{cs}^2 \sin^2(\theta)} + l_p + s_x \tag{3.2}$$

Equation (3.2) was the new data reduction equation for the model to determine the piston displacement. Figure 3.2 shows the schematic drawing of the four-bar-slider mechanism with the three main components: the crankshaft length, $l_{cs}$, connecting rod length, $l_{cr}$, and piston length, $l_p$. Both the diameters of the crankshaft, $d_{cs}$, and connecting rod, $d_{cr}$, are also shown in Figure 3.2. These two members were connected through a pin joint connection. Ideally the pin joint connection only allows 2-D movement and no movement in the z-plane. The dimensions of both the diameters define the "fit" of the pin joint. For a perfect fit, both the diameters are equivalent. Otherwise, slop, $s_x$, will exist in the pin joint connection. The model assumed that the slop was negligible. However, the slop term was included in the model data reduction equation since the slop will contribute to the uncertainty of the model results.

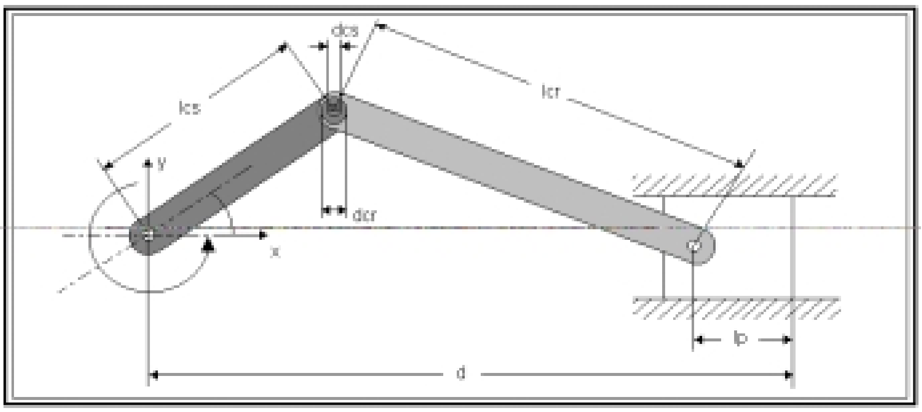


Figure 3.2: Schematic Drawing of Four-Bar-Slider Mechanism

**Results**

For the baseline design, both the lengths and the diameters of the existing parts were measured, including the primary dimensions of the original connecting rod. Ten measurements were made, and the mean and standard deviation of each measurement were calculated. Table 3.2 shows the mean values of the baseline design variables.

Table 3.2: Baseline Design Variables

| Crankshaft Length (Inch) | Piston Length (Inch) | Inner Length (Inch) | Outer Length (Inch) | Crankshaft Diameter (Inch) | Connecting Rod Diameter (Inch) |
|---|---|---|---|---|---|
| 0.777 | 1.101 | 2.577 | 3.902 | 0.746 | 0.754 |

The nominal values of the connecting rod length and diameter were 3.25 inch and 0.75 inch respectively, for the baseline design case. The measurements in Table 3.2 were made using a micrometer. These measured values were used as input for the model.

All the measured variables in Table 3.2 were substituted back into Equation 3.2 to produce a plot of displacement versus the crank angle, $\theta$, from the model. Figure 3.3 shows the model result for the baseline design.

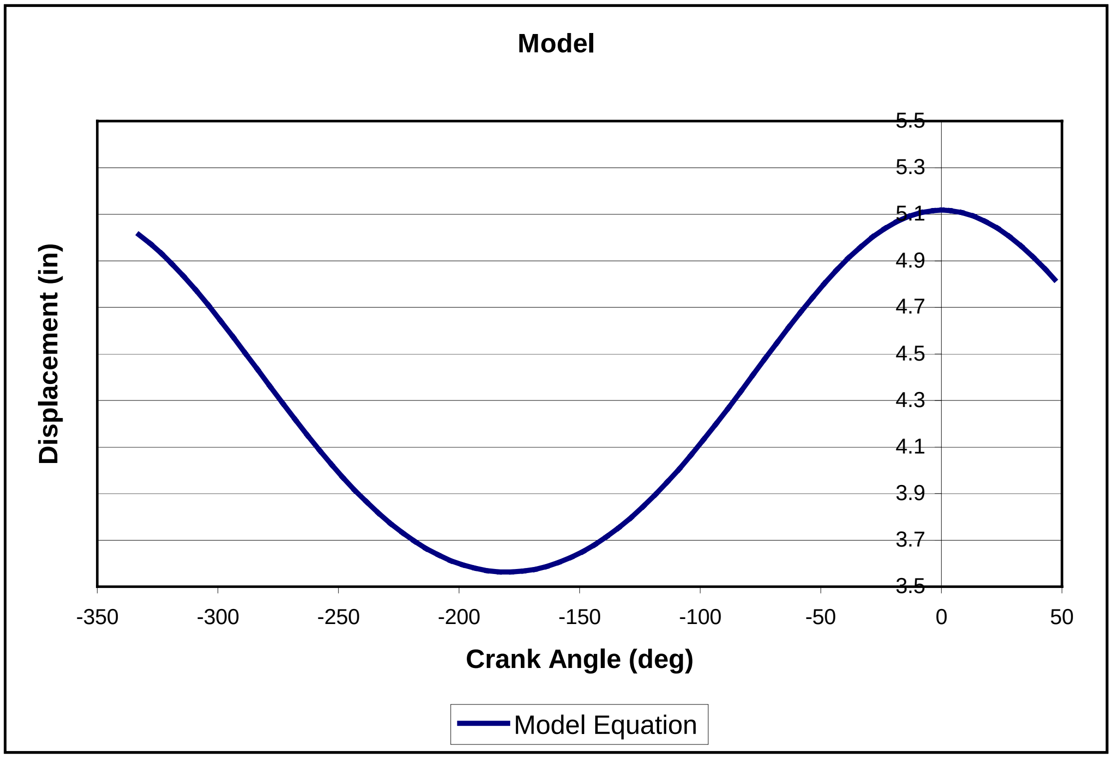


Figure 3.3: Model Result of Baseline Design

The baseline design assumed that there was a perfect fit for the pin joint connection. However, when the diameter of the connecting rod is not equivalent to the diameter of the crankshaft, there will be slop in that pin joint. Therefore, for the increased diameter design, the diameter of the connecting rod was increased to 0.85 inch to exaggerate the slop as may be seen with increased wear on the engine. The measurements of the increased diameter variables are shown in Table 3.3. Ten measurements were made for the new connecting rod diameter. The other pieces had the same dimensions as the baseline design.

Table 3.3: Increased Diameter Design Variables

| Crankshaft Length (Inch) | Piston Length (Inch) | Inner Length (Inch) | Outer Length (Inch) | Crankshaft Diameter (Inch) | Connecting Rod Diameter (Inch) |
|---|---|---|---|---|---|
| 0.777 | 1.101 | 2.577 | 3.902 | 0.746 | 0.842 |

Since the only difference between these two designs was the connecting rod diameter, the model result was the same for both cases. (Remember that the model assumed a perfect fit and did not depend on the connecting rod diameter as shown in Equation (3.2)). The only ”measured” variables in Equation (3.2) were the crankshaft length, connecting rod length, and piston length.

## Uncertainty

Experimental uncertainty analysis techniques from Coleman and Steele[1] were applied to evaluate the uncertainty of the model. The systematic and random uncertainties of each measured variable in Equation (3.2) were considered. Several assumptions were made for the model. First, it was assumed that no slop existed in the pin joint connection between the crankshaft and the connecting rod. All other connections were also assumed to be a perfect fit. Second, the engine was assumed to run at constant speed. Third, the crank angle, $\theta$, was assumed to be a known constant with zero uncertainty. The last assumption was that there was zero displacement in the z-plane direction; all the mechanism movements were in 2-D only.

In this research, only the connecting rod was customized and manufactured. Measurements were made for all the parts including the customized connecting rod, and each variable was measured ten times. The mean value of the ten measurements of each variable was substituted into Equation (3.2), and the standard deviation for each measurement variable was also determined. According to Coleman and Steele,[1] ten measurements for each variable can be considered a large sample size, and thus the large sample assumption was applied ($t$ = 2). The random uncertainty of each measured variable with 95% confidence level was determined using Equation (1.15).

Since all the dimensions of the parts were measured including the connecting rod, the random uncertainties of these parts due to the measurements were fixed once the mean values were used in the model. Therefore, they were treated as fossilized systematic uncertainties for the model uncertainty analysis. These fossilized systematic uncertainties were referred to as the second elemental uncertainty source with the first systematic uncertainty source for these variables being one-half the least count of the micrometer used to measure the dimensions. The total systematic uncertainty for these variables was the root-sum-square of the two elemental sources. Details for the model uncertainty analysis can be found in the MathCAD Worksheets in the Appendix.

In Equation (3.2), the slop term, $s_x$, has a zero nominal value, but this term will contribute to the overall uncertainty of the model. If the diameters of both the crankshaft and the connecting rod are not equivalent, then there will be slop in the pin joint connection. The exact position of the slop in the pin joint cannot be determined at every instant, and this random uncertainty in the pin joint connection will be constrained geometrically by the diameters of both the crankshaft and connecting rod.

$$R_{Sx} = \frac{(d_{cr} - d_{cs})}{2} \tag{3.3}$$

If both the diameters of the crankshaft and the connecting rod are not manufactured according to the specified nominal values, then the uncertainty of the slop will also vary. This systematic uncertainty of the slop is dependant on the uncertainties of both diameters.

$$S_{Sx} = \sqrt{\left(\frac{U_{dcr}}{2}\right)^2 + \left(\frac{U_{dcs}}{2}\right)^2} \tag{3.4}$$

The uncertainties for all of the variables were calculated and Equation (1.12) and Equation (1.14) were applied to determine the overall systematic and random uncertainties of the model result. With the calculated values for the overall systematic and random uncertainties of the model result, the overall total uncertainty of the model result was calculated using Equation (1.8). After all the terms were substituted into the appropriate equations, the final form of the uncertainty equations for the model result was

$$U_d(\theta) = \sqrt{(S_d(\theta))^2 + (R_d(\theta))^2} \tag{3.5}$$

where

$$S_d(\theta) = \sqrt{\begin{array}{l}\left(\frac{\partial d(\theta)}{\partial l_{cs}}\right)^2 (S_{lcs})^2 + \left(\frac{\partial d(\theta)}{\partial l_1}\right)^2 (S_{l1})^2 + \left(\frac{\partial d(\theta)}{\partial l_2}\right)^2 (S_{l2})^2 + \ldots \\ + \left(\frac{\partial d(\theta)}{\partial l_p}\right)^2 (S_{lp})^2 + \left(\frac{\partial d(\theta)}{\partial S_x}\right)^2 (S_{Sx})^2\end{array}} \tag{3.6}$$

and

$$R_d(\theta) = \sqrt{\left(\frac{\partial d(\theta)}{\partial S_x}\right)^2 \left(R_{Sx}\right)^2} \tag{3.7}$$

Since the correlation terms were negligible for this model analysis, there were no correlated terms in Equation (3.6). The random uncertainty for the overall model result was solely due to the slop term because other random uncertainties were treated as the fossilized systematic uncertainties. The calculated uncertainty was plotted as the uncertainty bands around the model result. Figure 3.4 and Figure 3.5 below show the plots for the baseline design and the increased diameter design. The uncertainty bands of the increased diameter case were wider than the baseline design because the random uncertainty of the overall model results solely depended on the difference between both the diameters as shown in Equation (3.3). Increasing the diameter of the connecting rod did not affect the model result because the slop had a zero nominal value in the model equation, but the model uncertainty increased due to uncertainty contribution of the slop.

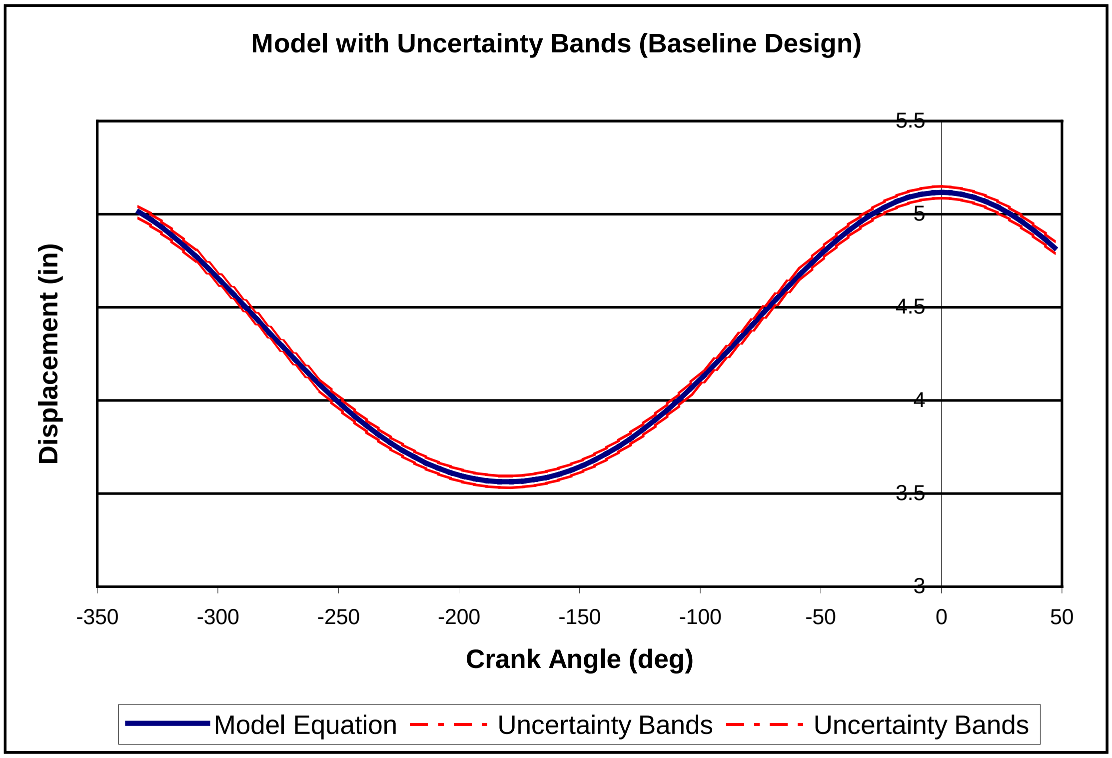


Figure 3.4: Model Result of Baseline Design with Uncertainty Bands

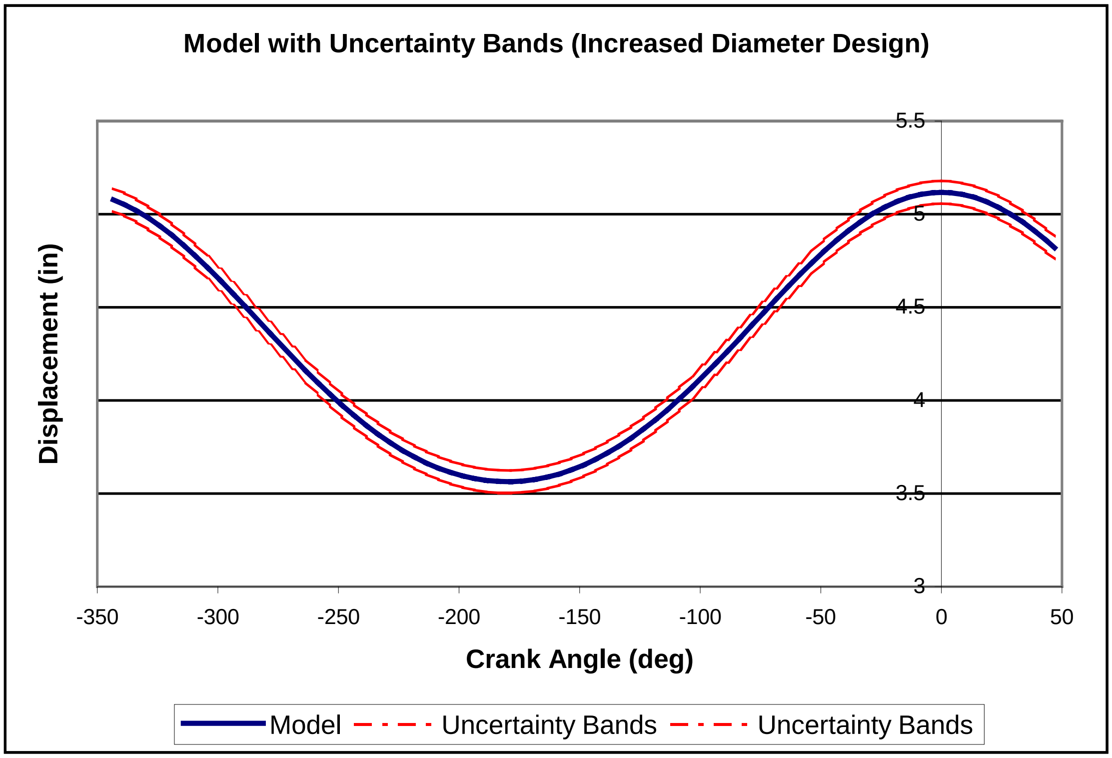


Figure 3.5: Model Result of Increased Diameter Design with Uncertainty Bands

## Manufacture

Manufacture is one of the important steps in a product development process, and, in this pilot project, the connecting rod was customized and then manufactured. The length of the connecting rod was specified to be 3.25 inches with two different diameters, 0.75 inch and 0.85 inch, as shown in Table 3.4.

Table 3.4: Specified Connecting Rod Dimensions

| | Length (Inch) | Diameter (Inch) |
|---|---|---|
| Baseline Design | 3.25 | 0.75 |
| Increased Diameter Design | 3.25 | 0.85 |

As mentioned previously, the connecting rod length was specified in terms of inner and outer length, $l_1$ and $l_2$ respectively. The geometry of the connecting rod was shown in Figure 3.6. The data reduction equations for $l_1$ and $l_2$ were determined from the geometry of the connecting rod.

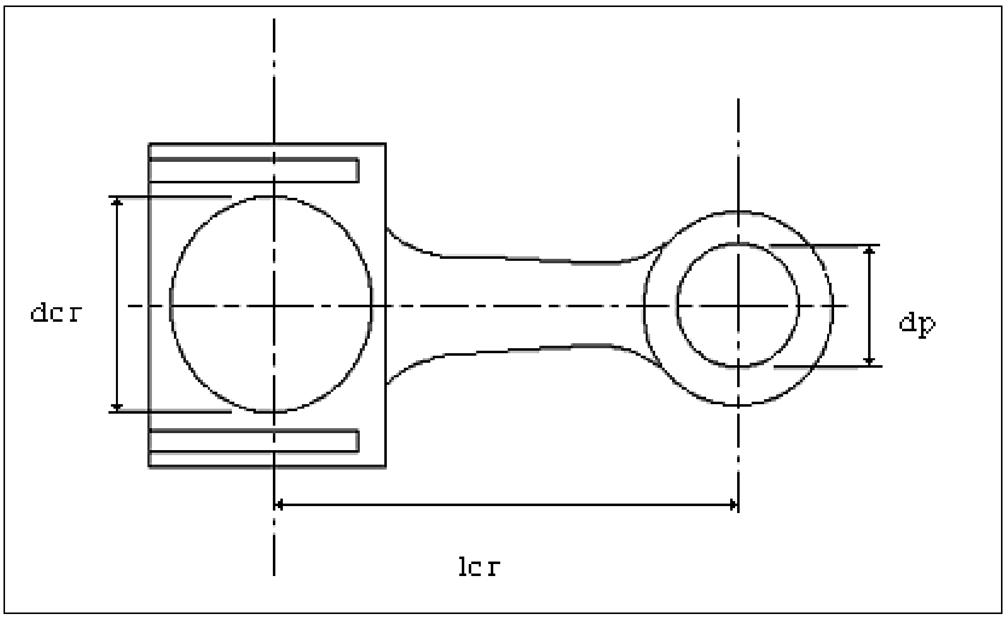


Figure 3.6: Geometry of Connecting Rod

$$l_1 = l_{cr} - \frac{d_{cr} + d_p}{2} \tag{3.8}$$

and

$$l_2 = l_{cr} + \frac{d_{cr} + d_p}{2} \tag{3.9}$$

For this product development process, the manufacturing uncertainty sources solely came from the manufacturing tolerances. The tolerances of the machine capabilities are presented in Table 3.5. With the aid of Equation (3.8) and Equation (3.9), experimental uncertainty analysis techniques were again applied to determine the systematic uncertainties of the inner and the outer lengths (Equation (1.12)).

Table 3.5: Manufacture Tolerances

| Sources | $t_{lcr}$ (Inch) | $t_{dcr}$ (Inch) |
|---|---|---|
| Manufacture Tolerances | 0.01 | 0.05 |

The terms $t_{lcr}$ and $t_{dcr}$ were the tolerances of the connecting rod length and the connecting rod diameter. The connecting rod length had the tolerance of 0.01 inch and the connecting rod diameter had the tolerance of 0.05 inch. These were the only manufacturing uncertainties that were included in the uncertainty analysis.

$$S_{l1} = \sqrt{t_{lcr}^{\ 2} + \frac{t_{dcr}^{\ 2}}{4} + \frac{U_{dp}^{\ 2}}{4}} \tag{3.10}$$

and

$$S_{l2} = \sqrt{t_{lcr}{}^2 + \frac{t_{dcr}{}^2}{4} + \frac{U_{dp}{}^2}{4}} \tag{3.11}$$

Since the piston was not manufactured, the uncertainty with respect to the piston diameter was determined based on the measurements made. Tens measurements of the piston diameter were made, and the random uncertainty of the piston diameter was determined by applying Equation (1.15) with the large sample assumption. The systematic uncertainty of the piston diameter was one-half the least count of the micrometer used to make the measurements. The detailed analysis on the manufacture uncertainty is included in the Appendix MathCAD Worksheets, and further discussion of the manufacturing uncertainty effects on the model will be included in the following chapter.

As for the manufacturing uncertainty effects on the experiment, the experiment was conducted on the manufactured connecting rod. Therefore, the manufacturing uncertainty was implicitly included in the randomness of piston head displacement during the experiment. The uncertainty of the experimental data points thus included the manufacturing uncertainty effects.

## Experiment

An experiment was conducted to simulate the actual piston displacement. The piston displacement from the experiment was then used for comparison with the model result and a subsequent discussion of comparisons of the steps in the product development process. The measurement in this experiment was the piston head displacement, and the displacement data was recorded with respect to time. The data was recorded every 0.005 seconds through a data acquisition system, and the duration of each experiment was about 10 seconds.

An equation was needed to convert the time to the crank angle because the model equation was a function of crank angle. The conversion of time to the crank angle is important for making a direct comparison between the model result prediction and the actual experimental data. The conversion equation used was

$$\theta = \theta_0 + \omega(\Delta t) \tag{3.12}$$

where $\theta_0$ is the reference angle, $\omega$ is the engine speed, and $\Delta t$ is the difference between the elapsed time and the initial time. A proximity sensor calibration was used to determine the reference angle, $\theta_0$, which was the crank angle where the model and the experiment matched. The crank angle depended on Top-Dead-Center of the piston and the angle where the proximity sensor "turned on". The reference angle was the difference between the "turned on" crank angle and the crank angle at Top-Dead-Center. The angle at Top-Dead-Center was measured when the piston was in a stationary position at the furthest point away from the crankshaft. The "turn on" angle was obtained when the data acquisition system showed the initial forming of a square wave for the proximity sensor voltage. Detail proximity sensor setup can be found in reference 25.

A linear transducer was used to measure the piston displacement. The linear transducer was fixed on the top of the cylinder wall, and the follower was screwed into the head of the piston. The experiment data was recorded into the computer through a data acquisition system. This experimental setup measured only the distance between the piston head and the top of the cylinder wall. A discussion on converting the experiment data points to fit the model frame of reference can be found in Chapter IV.

## Results

The reference angle was found to be 47 degrees and Equation (3.12) assumed a constant engine speed, $\omega$. The engine speed was calculated by a Labview program using a once per revolution probe signal. Since a complete revolution was 360 degrees, there were several repeated cycles in each run of the experiment. There were different engine speeds associated with those repeated cycles. However, since the engine speed was calculated from a once per revolution signal, the calculated engine speed did not show the engine speed variation within a cycle. Therefore, the actual engine speed at each data points within a cycle during the experiment could not be determined. Since there were repeated cycles for each run of the experiment, an average piston displacement and average crank angle were used. The average crank angle was calculated based on 5 degrees increment. The average piston displacement was sorted first according to the crank angle and then followed by calculating the average of the piston displacement. The results are shown in Figure 3.7 and Figure 3.8. This method of data analysis fully used the experiment data points collected from each run of the experiment.

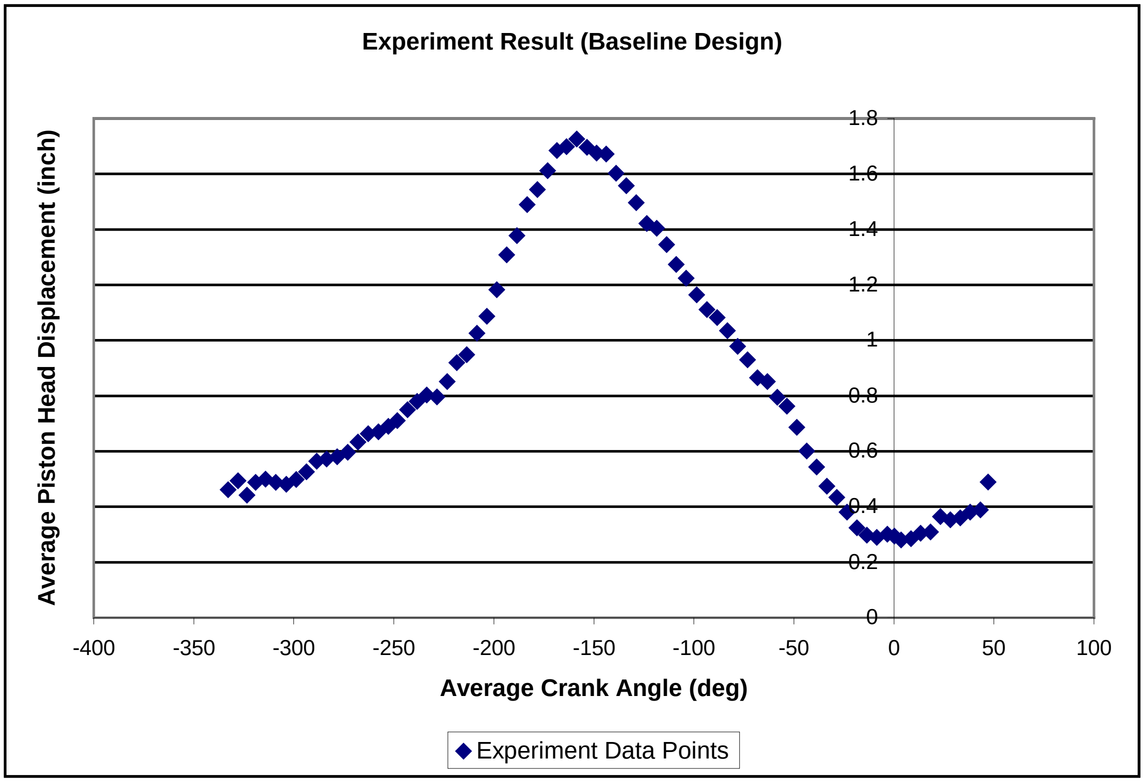


Figure 3.7: Experiment Result of Baseline Design

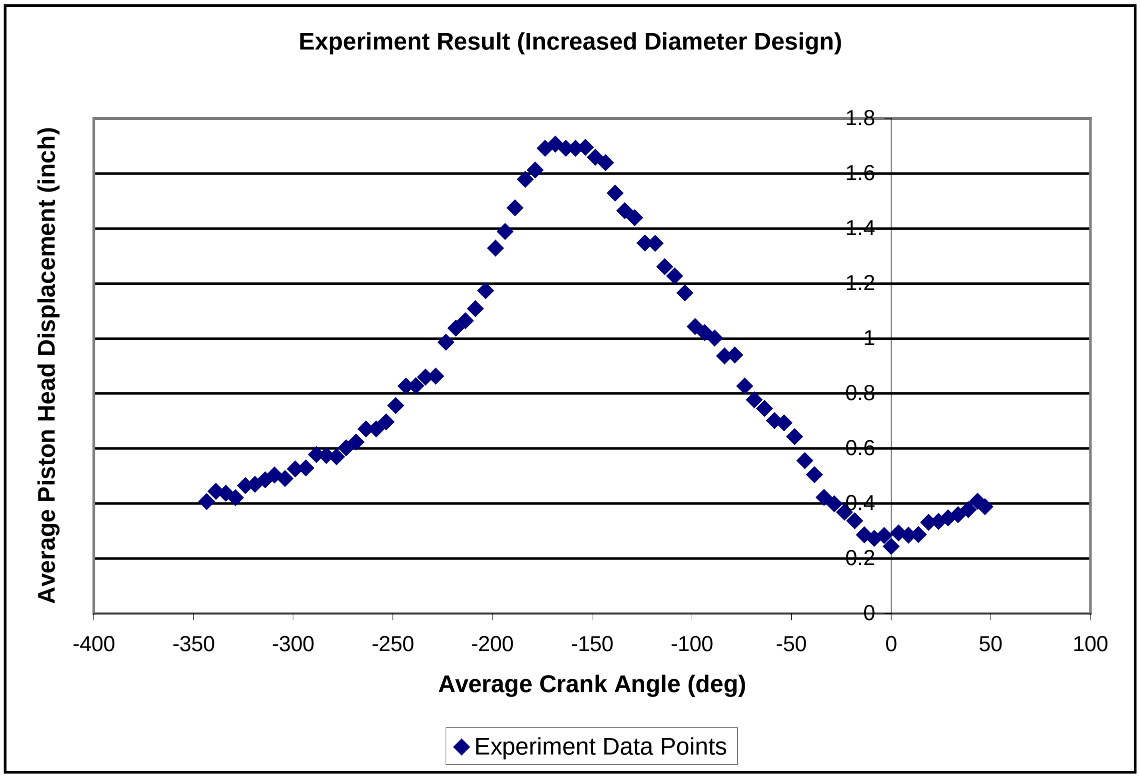


Figure 3.8: Experiment Result of Increased Diameter Design

## Uncertainty

The piston head displacement was measured directly in the experiment, and the average of the displacement from all cycles was used as the experiment result. Therefore, Equation (1.15) was used to calculate the random uncertainty of the piston head displacement with the large sample assumption applied.

$$R_{Xi} = 2S_{Xi} = 2\left\{\frac{1}{N_i - 1}\sum_{k=1}^{N_i}\left[(X_i)_k - \overline{X}_i\right]^2\right\}^{1/2} \qquad (1.15)$$

The random uncertainty of the average experimental data points dominated the systematic uncertainty of the average experimental data points. Therefore, the random uncertainty calculated using Equation (1.15) was treated as the total experimental uncertainty, $Ud_{av}$.

$$Ud_{avi} = R_{xi} \tag{3.13}$$

Figure 3.9 and Figure 3.10 show the plots of the experiment data of the baseline design and the increased diameter design with uncertainty bands. The uncertainty bands of the baseline design were smaller than the increased diameter design because the slop greatly affected the randomness of the piston displacement.

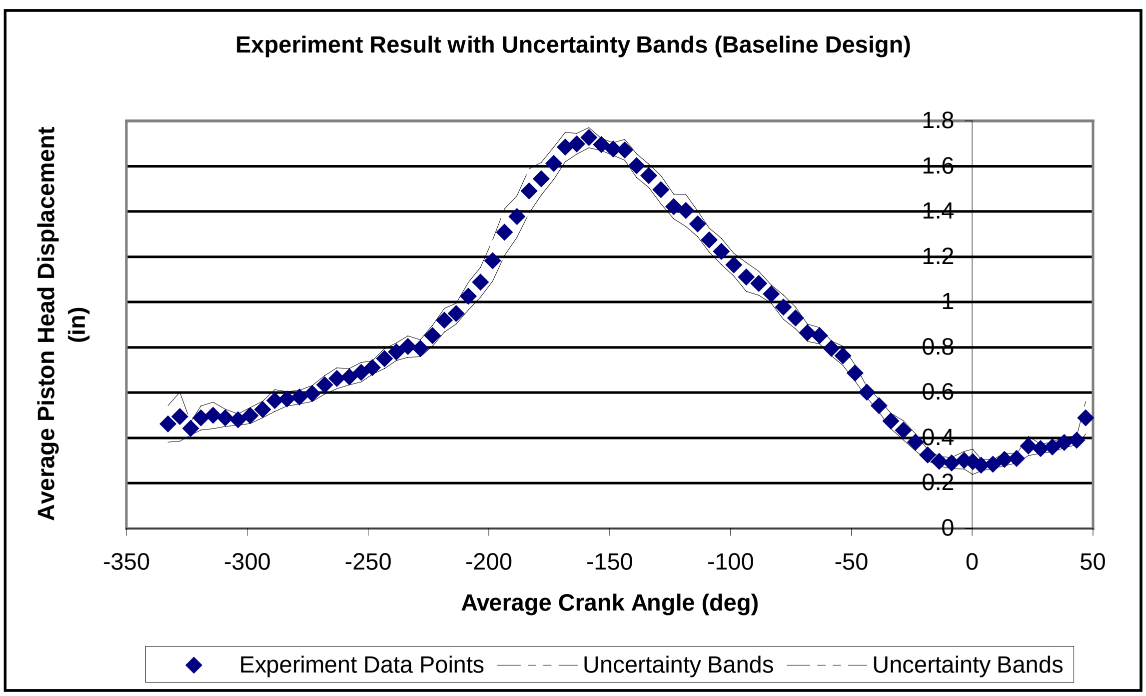


Figure 3.9: Experiment Result of Baseline Design with Uncertainty Bands

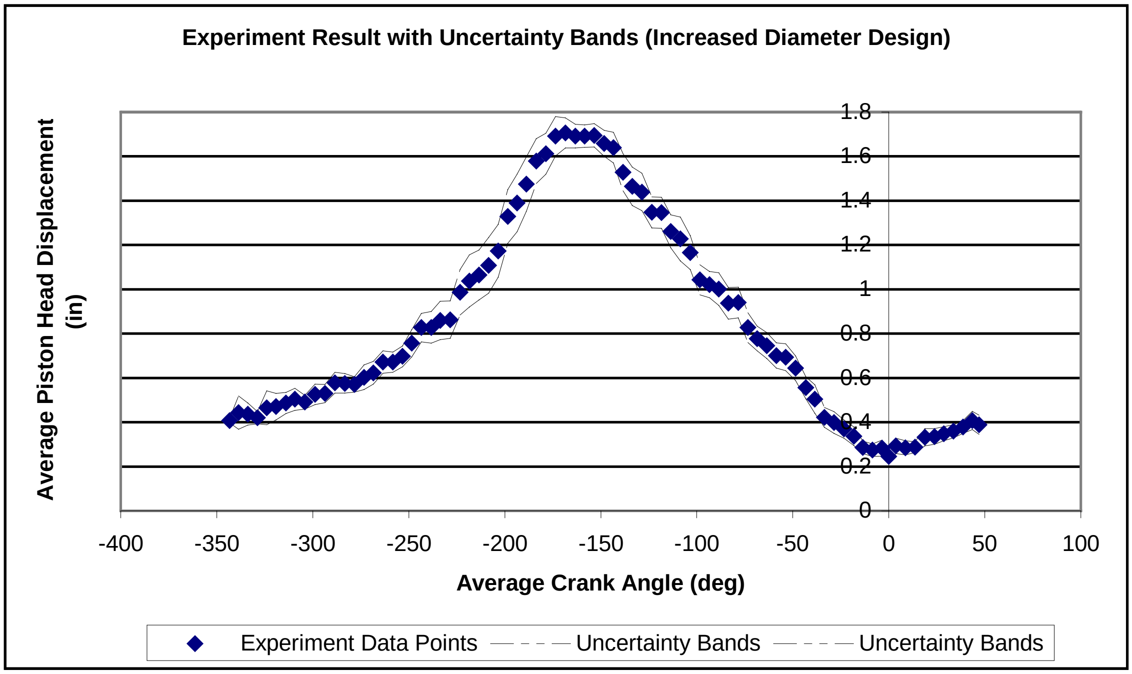


Figure 3.10: Experiment Result of Increased Diameter Design with Uncertainty Bands

Crank angle was found by applying Equation (3.12), and there were uncertainties associated with the variables in Equation (3.12). As mentioned in the earlier paragraph, the engine speed was assumed to be constant when Equation (3.12) was formulated. However, during the execution of the experiment, it was found that the engine speed could not be held constant. There was an additional uncertainty source due to the inconsistency of the engine speed during a cycle. This source was believed to be the dominant effect for the uncertainty associated with the crank angle, $\theta$. That uncertainty source could not be physically characterized or quantified due to insufficient information. An estimate of the crank angle uncertainty was assigned to complete the computing of the crank angle uncertainty. Three values, 1 degree, 5 degrees, and 10 degrees, were assigned for the crank angle uncertainty in the uncertainty analysis. The effects of varying the crank angle uncertainty from 1 degree to 10 degrees will be discussed in Chapter IV.

# CHAPTER IV

# PILOT PROJECT RESULTS COMPARISONS

Comparisons are important in any product development process. Comparisons lead to the validation of a model. Also, the effects of the manufacture on both the model and the experiment can be clearly observed and conclusions can be drawn. In this pilot project, the main idea of the comparisons was to observe the effects of manufacture on the model and to make direct comparisons between the model and the experiment.

## Manufacture Effects on Model

In the initial model step, all the uncertainty sources were purely determined through the measurement of the components such as the connecting rod, the crankshaft, and the piston. However, the connecting rod was customized and manufactured; therefore, the manufacturing tolerances were treated as additional uncertainty sources for the uncertainty analysis. Equation (3.10) and Equation (3.11) showed that the inner and outer lengths of the connecting rod were the terms that incorporated the manufacturing tolerances into the model uncertainty analysis. Now, the question is how do the manufacturing tolerances affect the model uncertainty analysis? Figure 4.1 shows the comparison between the initial model uncertainty bands and the uncertainty with the manufacture effects.

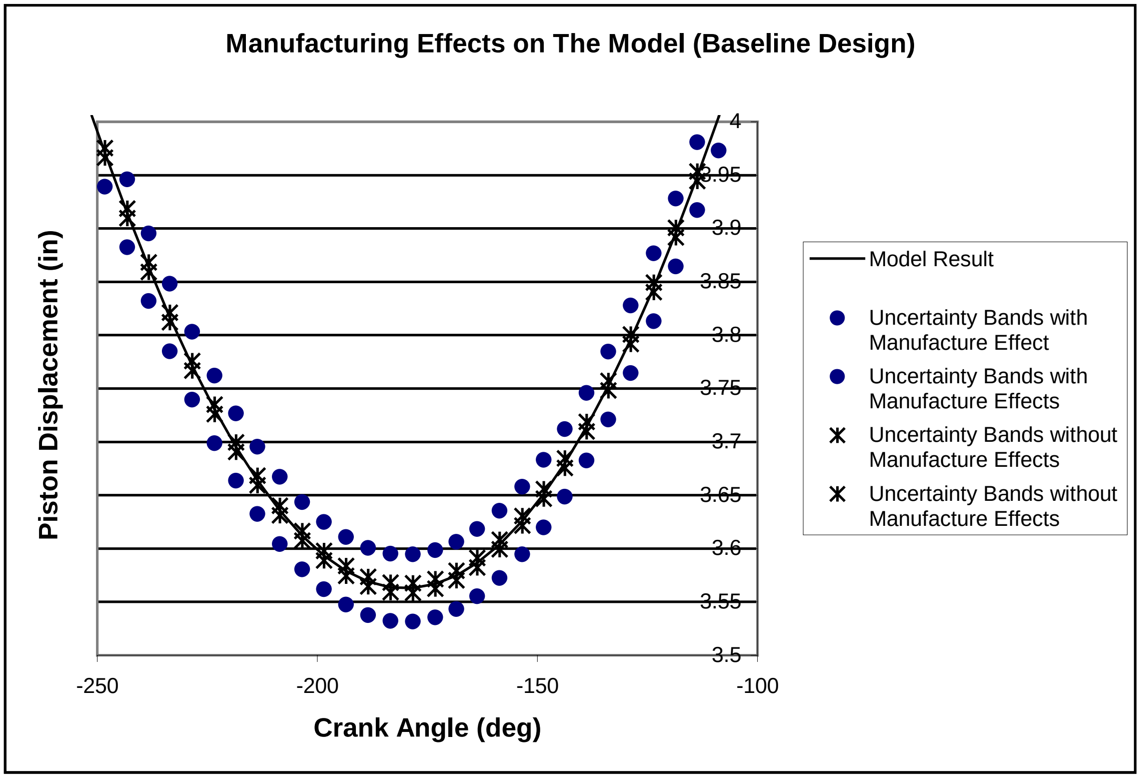


Figure 4.1: Manufacture Effects on Baseline Design Model Uncertainty Analysis

The uncertainties of the model result increase with manufacture effects because of the contributions of the systematic uncertainties of the inner and the outer lengths. In the modeling step, the systematic uncertainties of the inner and outer length were determined through measurements as discussed in Chapter III. However, these systematic uncertainties were small compared to the given manufacture tolerances. Figure 4.1 shows only a part of the original plot because the original model uncertainty bands were not observable compared to the model uncertainty with manufacture effects in a full-scale plot. Figure 4.2 shows that the initial model uncertainty for the increased diameter design was also smaller than the model uncertainty with manufacture effects as expected.

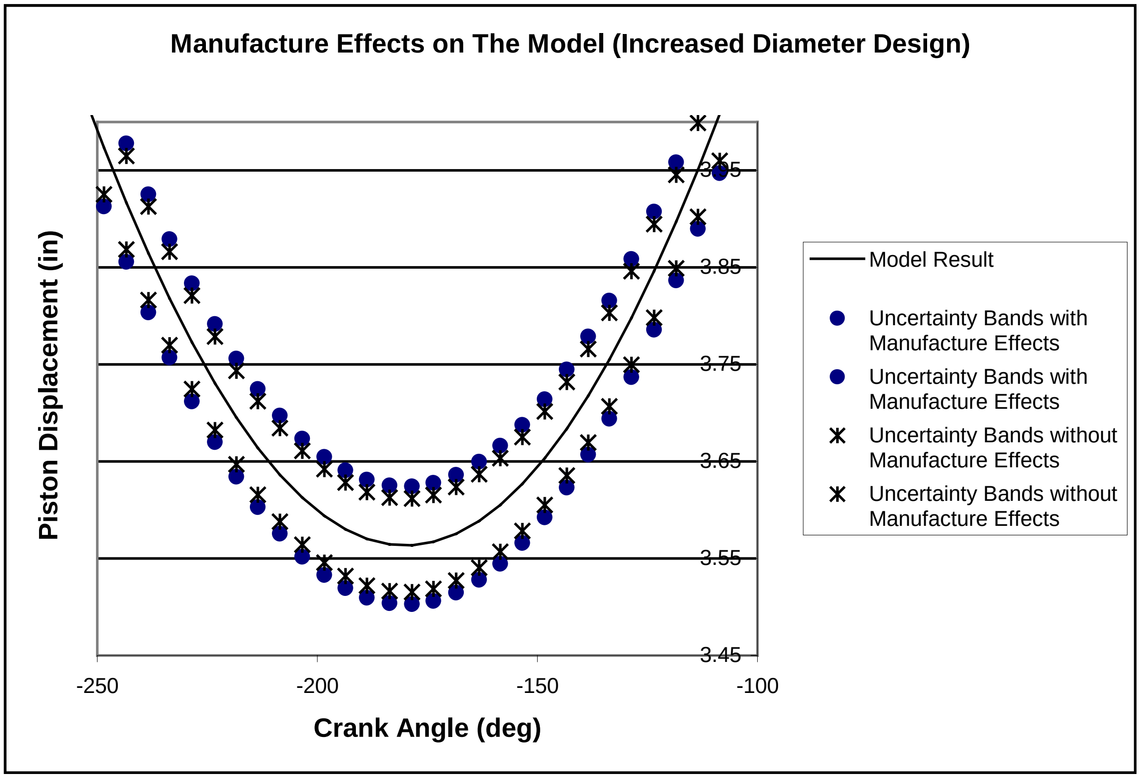


Figure 4.2: Manufacture Effects on Increased Diameter Design Model Uncertainty Analysis

The explanation was that the inner and the outer lengths were no longer a single measured uncertainty term as described in the Chapter III, but were determined by Equation (3.10) and Equation (3.11). As shown by Equation (3.10) and Equation (3.11), both the inner and outer lengths were functions and not direct measurements since the rod was manufactured to a defined center-to-center distance. Therefore, the combined systematic uncertainties with respect to these two terms increased which increased the uncertainty bands around the model result. The manufacturing tolerances are valid as uncertainty estimates in the common situation where many parts are manufactured and a few parts are chosen to be measured to see if they meet the manufacturing specifications within the set tolerances for quality control.

## Total Displacement Determination

As mentioned in Chapter III, the experiment measured the piston head displacement, which was the opposite direction of the model prediction due to the experiment setup (Figure 4.3). From Figure 4.3, the model prediction determined the piston displacement from the crankshaft to the top of the piston head. In contrast, the experiment measured the piston displacement from the cylinder top to the piston head. The total displacement measurement should have been a one-time measurement. However, a mistake was made, and the total displacement was not measured. Therefore, an equation was formulated to calculate the total displacement, and the equation was applied to both the baseline design and the increased diameter design.

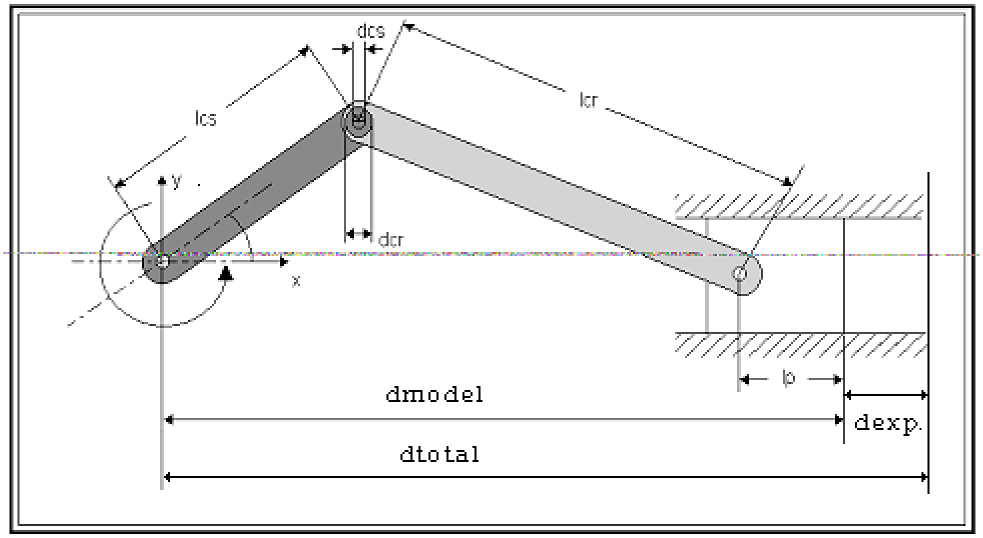


Figure 4.3: Total Displacement Measurement

$$d_{total} = \frac{\left(\max\left(d_{model}\right) + \min\left(d_{experiment}\right)\right) + \left(\min\left(d_{model}\right) + \max\left(d_{experiment}\right)\right)}{2} \tag{4.1}$$

In Equation (4.1), the *max*($d_{model}$) and *min*($d_{model}$) were the maximum and the minimum piston displacements of the model prediction, and the corresponding crank angles were 0 degrees and 180 degrees. The *max*($d_{experiment}$) and *min*($d_{experiment}$) were the maximum and minimum piston displacements measured from the experiment. The total displacement for both the baseline design and the increased diameter design should be the same because the total length from the crankshaft to the cylinder top was fixed. Therefore, Equation (4.1) was applied to both designs, and an average of the two designs was used in the analysis.

$$d_{total} = \frac{d_{total}(Baseline) + d_{total}(IncreasedDiameter)}{2} \tag{4.2}$$

The average total displacement found through Equation (4.2) was then treated as a one-time measurement.

With Equation (4.2) as the data reduction equation to determine the average total displacement, the uncertainty associated with the average total displacement was determined by using the experimental uncertainty analysis techniques discussed in Chapter I. The overall uncertainty of the average total displacement was

$$Ud_{total} = \sqrt{\left[\frac{Ud_{total}(Baseline)}{2}\right]^2 + \left[\frac{Ud_{total}(IncreasedDiameter)}{2}\right]^2} \tag{4.3}$$

where

$$Ud_{total}(Baseline) = \sqrt{\left(\frac{\max(Ud_{model})}{2}\right)^2 + \left(\frac{\min(Ud_{\exp})}{2}\right)^2 + \ldots \left(\frac{\min(Ud_{model})}{2}\right)^2 + \left(\frac{\max(Ud_{\exp})}{2}\right)^2} \tag{4.4}$$

and

$$Ud_{total}(IncreasedDiameter) = \sqrt{\left(\frac{\max(Ud_{model})}{2}\right)^2 + \left(\frac{\min(Ud_{\exp})}{2}\right)^2 + \ldots \left(\frac{\min(Ud_{model})}{2}\right)^2 + \left(\frac{\max(Ud_{\exp})}{2}\right)^2} \tag{4.5}$$

The term $Ud_{total}(Baseline)$ is the uncertainty of the total displacement of the baseline design and $Ud_{total}(Increased\ Diameter)$ is the uncertainty of the total displacement of the increased diameter design. The other terms, $max(Ud_{model})$, $min(Ud_{model})$, $max(Ud_{exp})$, and $min(Ud_{exp})$, are the maximums and minimums of the model predictions and experimental data points for each case. The calculated overall uncertainty of the average total displacement was treated as the uncertainty associated with the average total displacement measurement.

## Model and Experiment Comparisons

The calculated total displacement measurement from Equation (4.2) was treated as the total displacement as shown in Figure 4.3. With the model prediction direction as the new frame of reference to keep all measurements in the same direction, the value calculated from Equation (4.2) was used to convert the experimental data points to the model frame of reference. The data reduction equation used was

$$d_{\exp} = d_{total} - d_{av} \tag{4.6}$$

where $d_{av}$, $d_{total}$, and $d_{exp}$ are the actual average experiment data points, the total displacement, and the equivalent experiment data points that fit the model frame of reference, respectively.

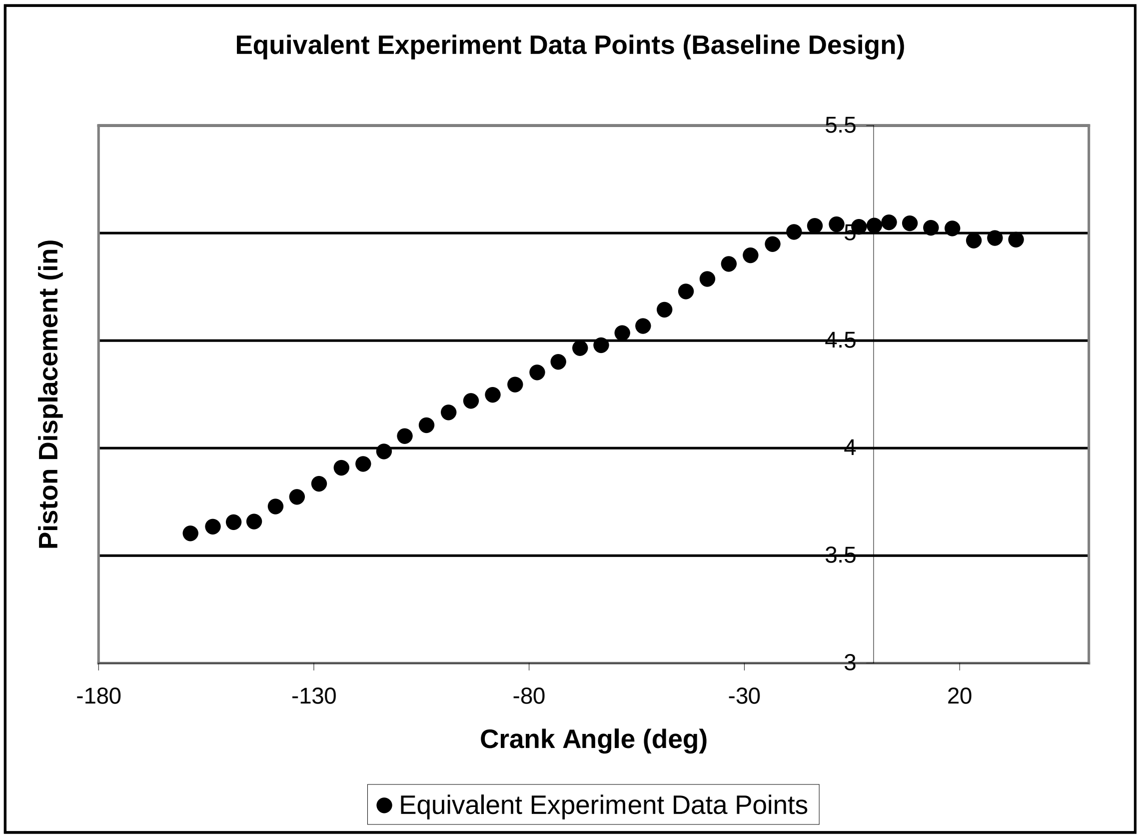


Figure 4.4: Equivalent Experiment Data Points of Baseline Design

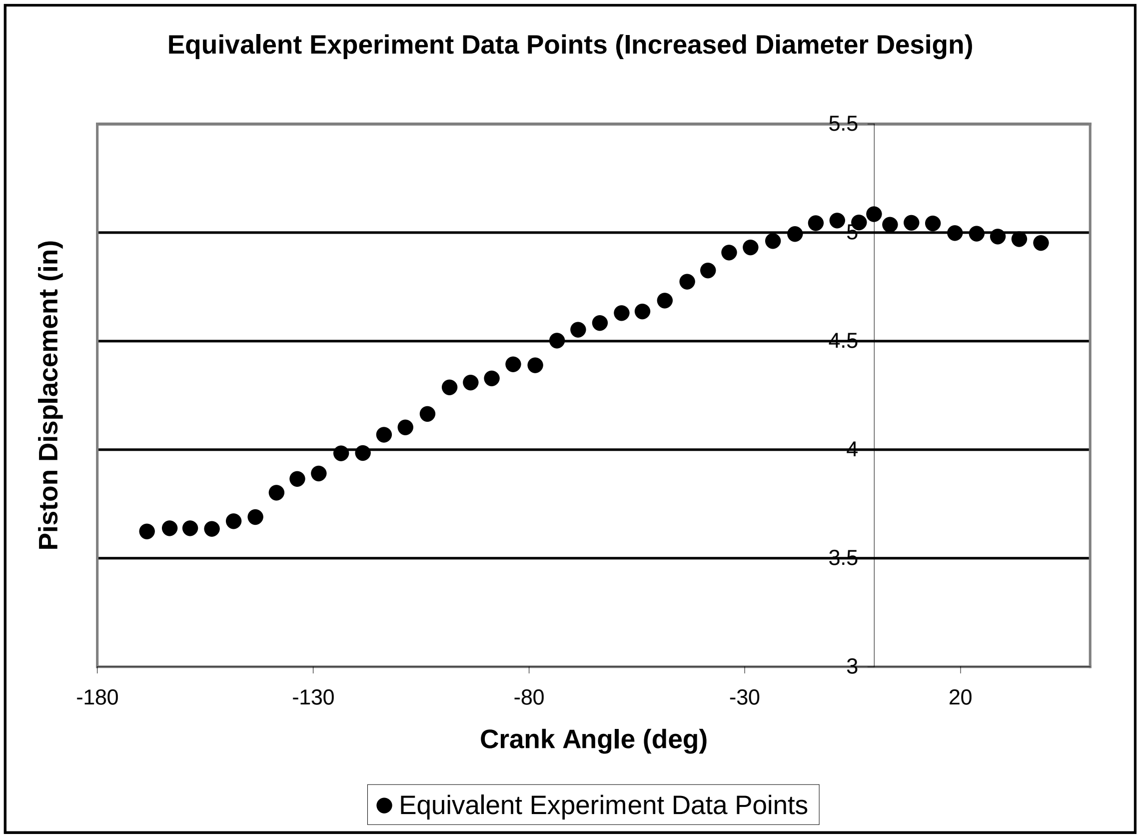


Figure 4.5: Equivalent Experiment Data Points of Increased Diameter Design

Figure 4.4 and Figure 4.5 show the range of equivalent experimental data points that pass through the maximum and the minimum points plotted against the calculated crank angle. These plots do have the maximum and the minimum points as predicted by the model equation. With Equation (4.6) as the data reduction equation, the uncertainty associated with the equivalent experimental data points is

$$Ud_{\exp} = \sqrt{(Ud_{total})^2 + (Ud_{av})^2} \tag{4.7}$$

where $Ud_{total}$ and $Ud_{av}$ are the uncertainties associated with the total displacement and the actual average experiment data points. $Ud_{total}$ was given by Equation (4.3) and $Ud_{av}$ was calculated by Equation (3.9) in Chapter III. Again, the results in the figures only include the displacement uncertainty. The uncertainty of the crank angle is not yet included.

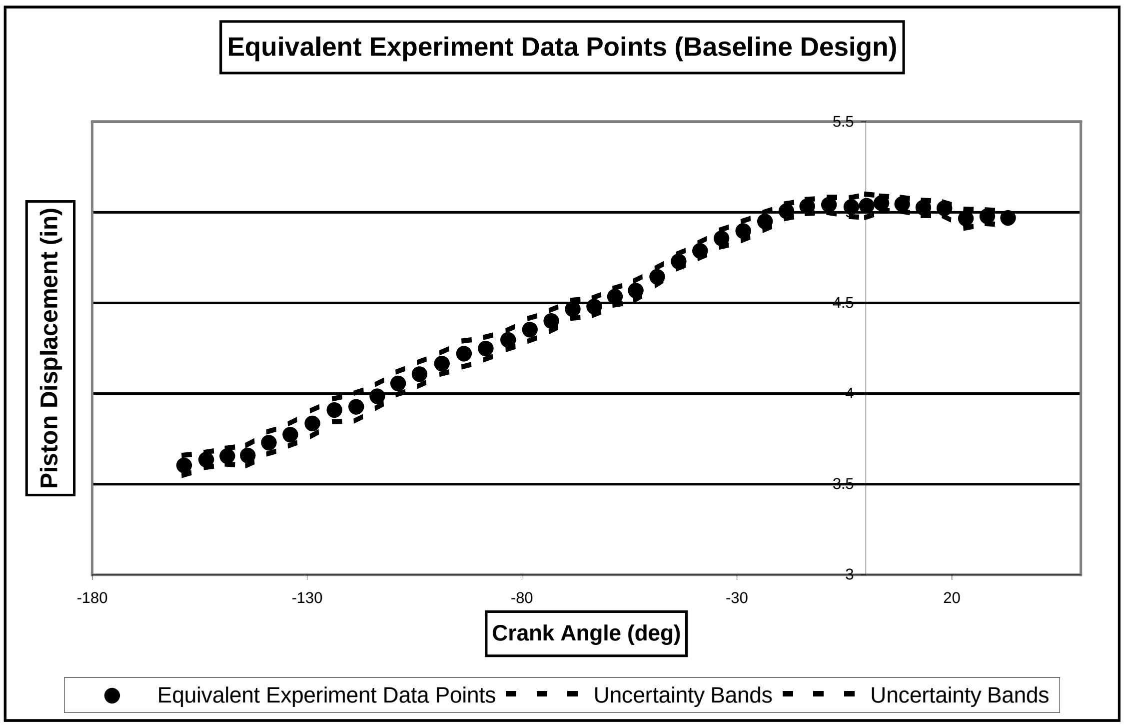


Figure 4.6: Equivalent Experiment Data Points of Baseline Design with Uncertainty Bands

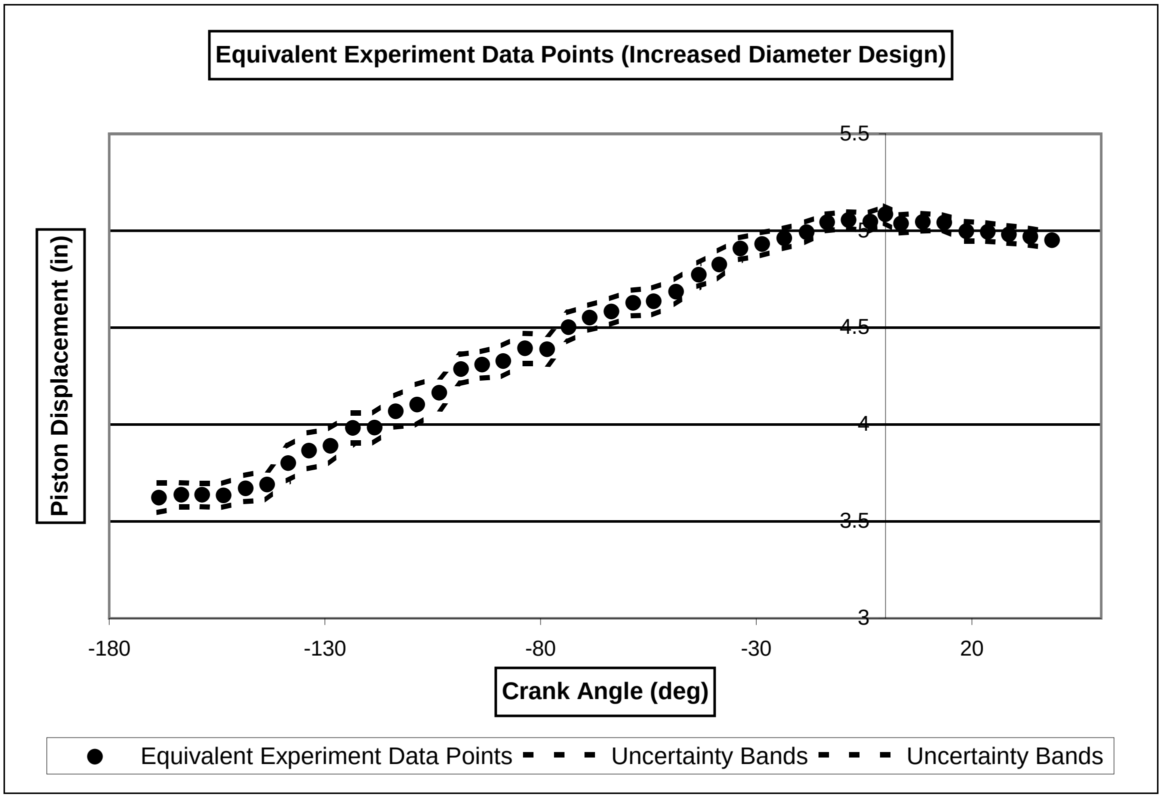


Figure 4.7: Equivalent Experiment Data Points of Increased Diameter Design with Uncertainty Bands

Figure 4.6 and Figure 4.7 show the equivalent experiment data points with the uncertainty bands. The uncertainty bands of the increased diameter design were larger than the uncertainty bands of the baseline design because the slop term increased the randomness of the piston displacement during the experiment.

The next step was a direct comparison between the model prediction and the equivalent experiment data points. Both the model predictions and the equivalent experiment data points were plotted on the same plots as shown below.

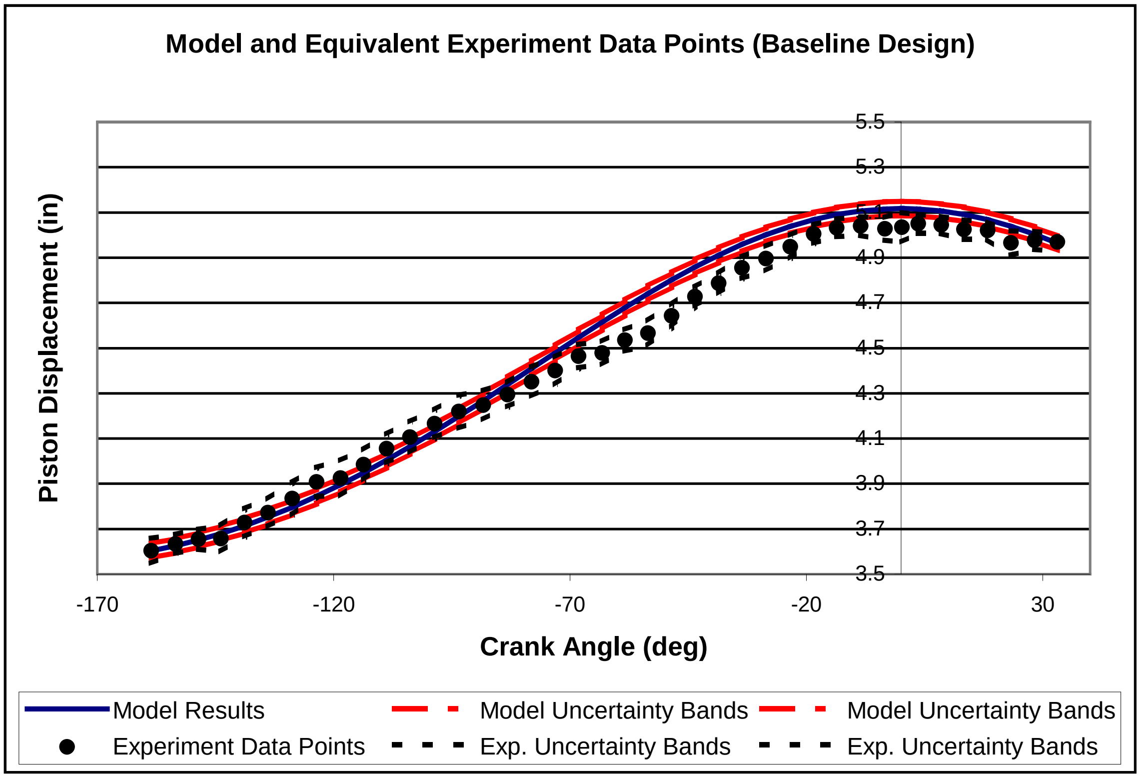


Figure 4.8: Model and Equivalent Experiment Data Points for Baseline Design

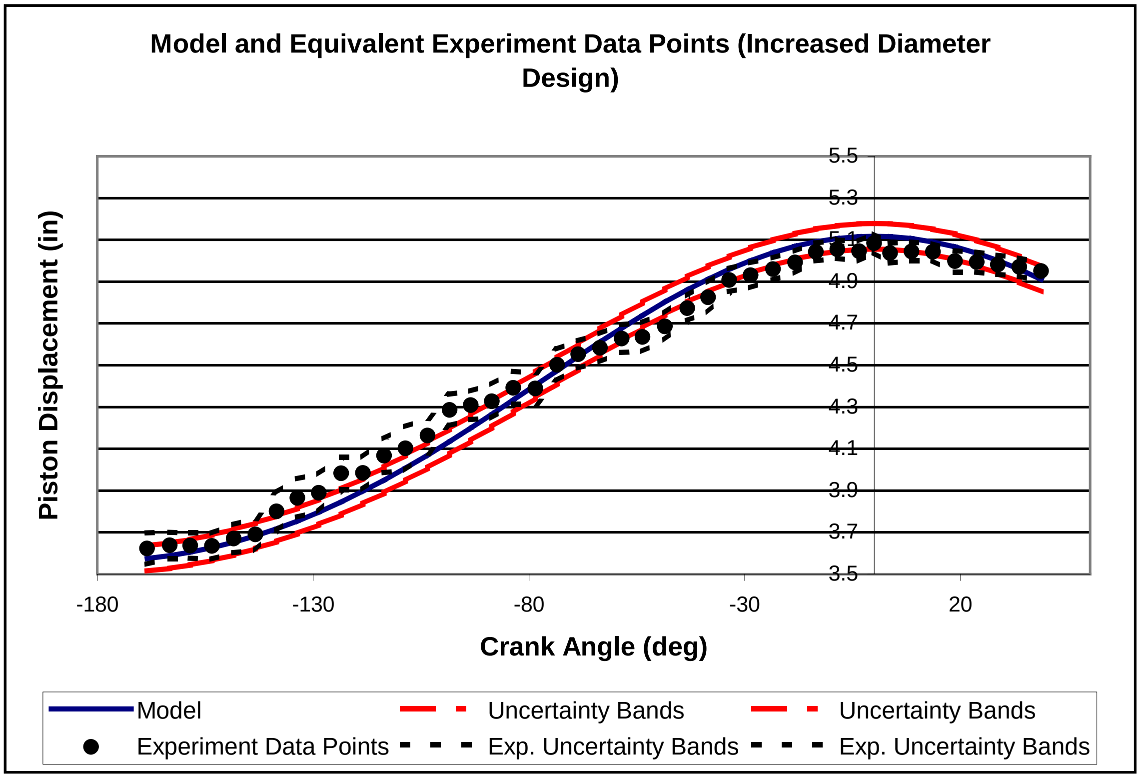


Figure 4.9: Model and Equivalent Experiment Data Points for Increased Diameter Design

Both Figure 4.8 and Figure 4.9 show that the equivalent experimental data points do have the same pattern predicted by the model equation.

As discussed in Chapter III, there was uncertainty associated with the calculated average crank angle, and this uncertainty should not be left out in the uncertainty analysis. The crank angle uncertainty must be included in the uncertainty analysis to determine the effect of the crank angle uncertainty on the overall uncertainty. According to Coleman and Steele,[1] the overall experimental uncertainty should include both the uncertainty of the actual experiment data points and the additional uncertainty from the crank angle measurement. The equation used to calculate the overall experimental uncertainty was

$$Ud_{\exp NEW} = \sqrt{Ud_{\exp}{}^2 + \left(\frac{\partial d_{\exp}}{\partial \theta}\right)^2 (U_\theta)^2} \tag{4.8}$$

where $Ud_{exp}$ was the uncertainty of the experimental data points and was determined by Equation (4.7). $U_\theta$ was the assigned crank angle uncertainty that was discussed in Chapter III. The term that multiplied the crank angle uncertainty was the sensitivity coefficient of the experimental data points with respect to the crank angle and was determined through numerical methods.

Since the model well predicted the experimental results, the partial derivatives of the model equation with respect to the crank angle were used as the sensitivity coefficients of the experimental data points with respect to the crank angle. Therefore, the sensitivity coefficients were expressed as

$$\frac{\partial d_{\exp}}{\partial \theta} = \frac{\partial d(\theta)}{\partial \theta}$$

$$\frac{\partial d_{\exp}}{\partial \theta} = -l_{cs}{}^2 (\sin(\theta)) - \frac{l_{cs}^2 (\sin(\theta)\cos(\theta))}{\sqrt{\left(\frac{l_1 + l_2}{2}\right)^2 - l_{cs}^2 (\sin(\theta))^2}} \tag{4.9}$$

The crank angle uncertainty was included to determine the overall experimental uncertainty. As expected, the calculations showed that the increase in the crank angle uncertainty from 1 degree to 10 degrees increased the overall experimental uncertainty.

Thus the experiment had the largest uncertainty bands when the crank angle uncertainty was 10 degrees as seen in Figure 4.10a, Figure 4.10b, and Figure 4.10c.

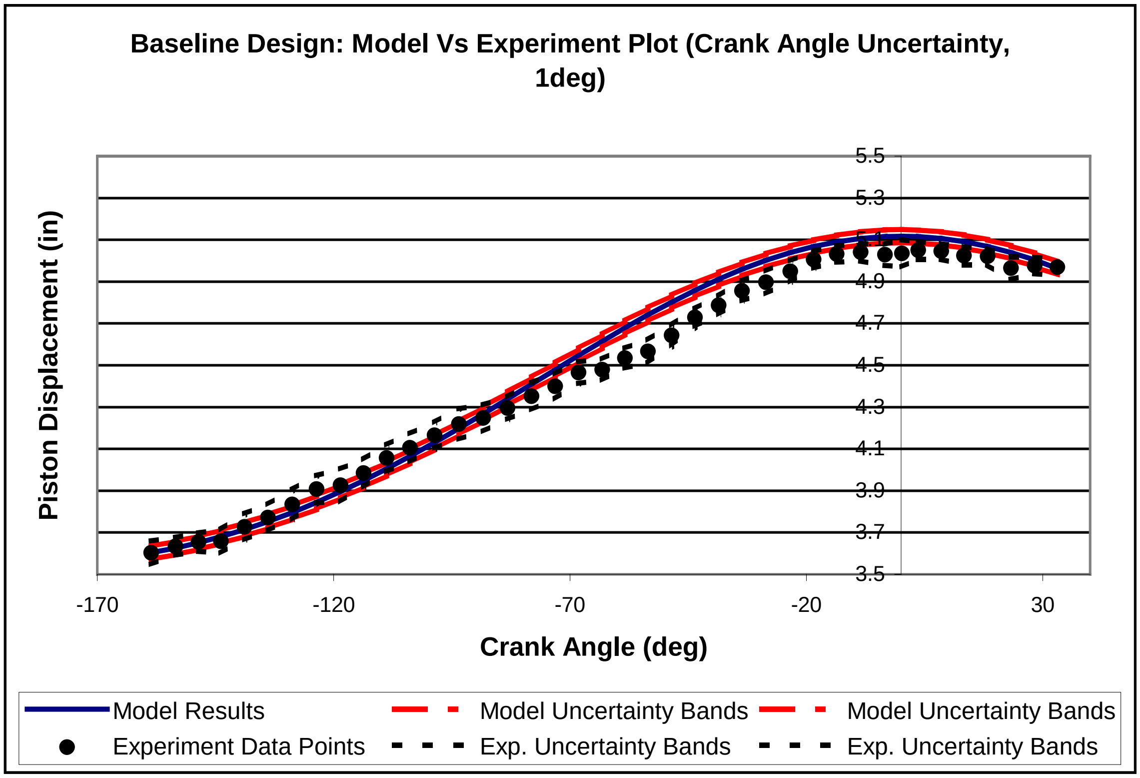


Figure 4.10a: 1-Degree Crank Angle Uncertainty – Baseline Design

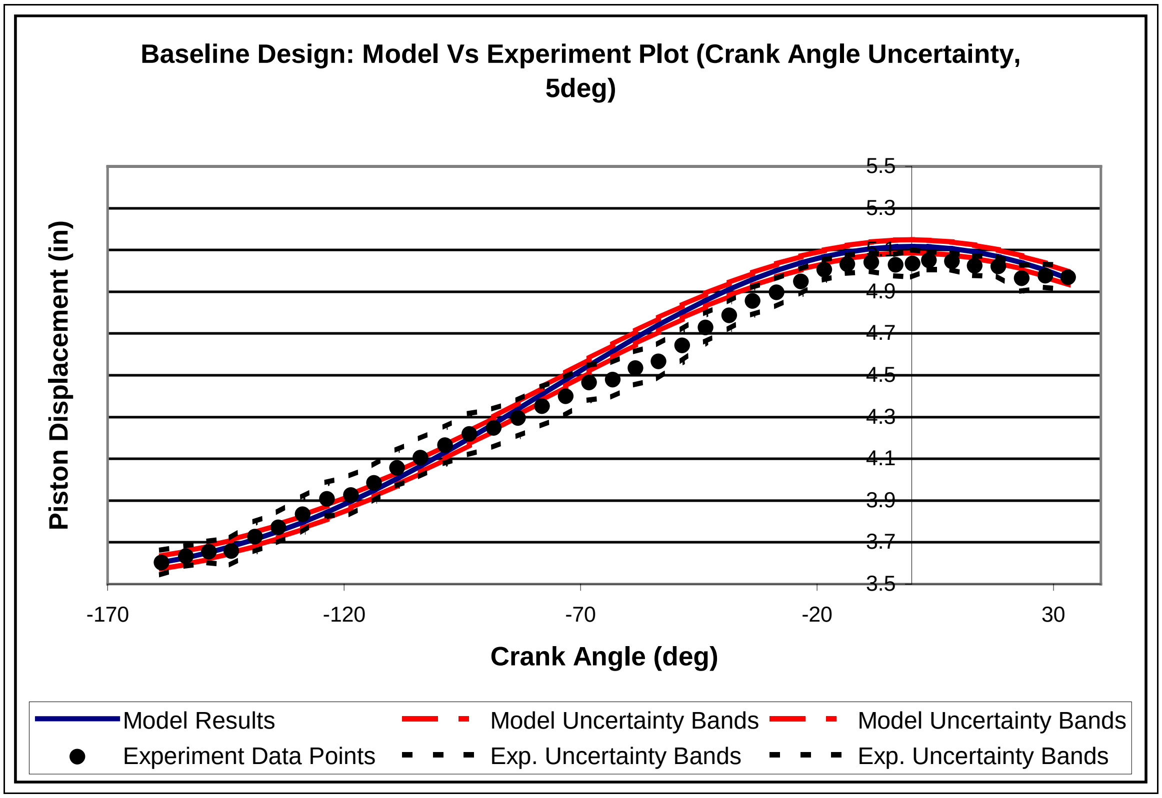


Figure 4.10b: 5-Degree Crank Angle Uncertainty – Baseline Design

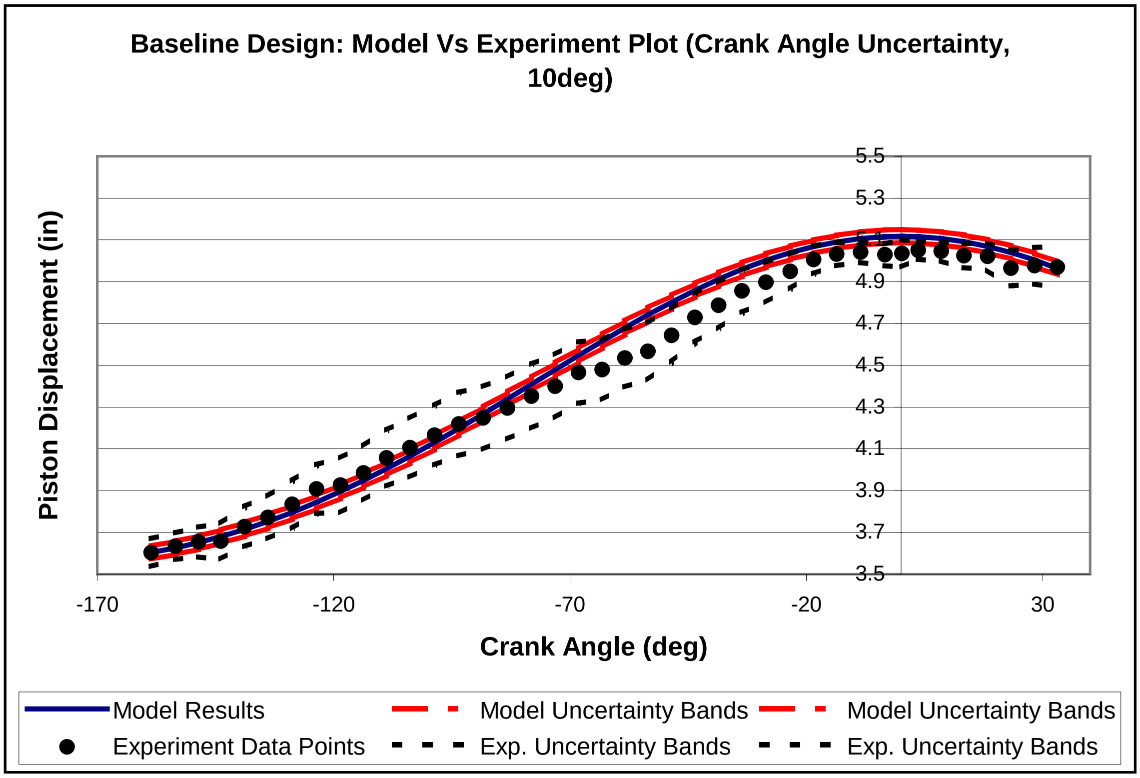


Figure 4.10c: 10-Degree Crank Angle Uncertainty – Baseline Design

The overall experimental uncertainty also increased with the increase in the crank angle uncertainty for the increased diameter design, as expected. The range of the crank angle uncertainty is believed to be the range that best describes or predicts the crank angle uncertainty. Ten degrees is believed to be the maximum allowable crank angle uncertainty. With a crank angle uncertainty greater than 10 degrees, the overall experimental uncertainty will be totally dominated by the crank angle uncertainty, and it is meaningless to have such a huge crank angle uncertainty.

## Validation Analysis

Based on the model and experiment comparisons, the model seemed to predict the experimental results well. But, how valid is that prediction? For model and experiment comparisons, the final step is the validation analysis. This validation analysis was conducted using experimental uncertainty analysis techniques.

According to Coleman and Steele,[1] a comparisons error, *E*, is the resultant of all the errors associated with both the experimental data and the model prediction. The data reduction equation to determine the comparison error between the model prediction and the experimental data was

$$E = d(\theta_{NEW}) - d_{\exp NEW} \tag{4.10}$$

$d(\theta_{NEW})$ is the model prediction that is obtained from Equation (3.2) with respect to the crank angle range covering the maximum and the minimum displacement values, and $d_{expNEW}$ is determined by Equation (4.6) with respect to the crank angle range covering the maximum and the minimum displacement values. With Equation (4.10) as the data reduction equation for the comparison error, the uncertainty of the comparison error, $U_E$, is

$$U_E = \sqrt{\left(\frac{\partial E}{\partial d(\theta_{NEW})}\right)^2 \left(Ud(\theta_{NEW})\right)^2 + \left(\frac{\partial E}{\partial d_{\exp NEW}}\right)^2 \left(Ud_{\exp NEW}\right)^2} \tag{4.11}$$

Ud($\theta_{NEW}$ ) is determined through Equation (3.5) with respect to the crank angle range covering the maximum and the minimum displacement values and $Ud_{expNEW}$ is given by Equation (4.8).

According to Coleman and Steele,[1] if the magnitude of the comparison error, $|E|$, is less than the comparison error uncertainty, $U_E$, then the validation has been achieved at the $U_E$ level. Otherwise, improvement is still needed on either the proposed model or the experimental setup. Figures 4.11 and 4.12 show the validation plots of both the baseline design and the increased diameter design with different crank angle uncertainties.

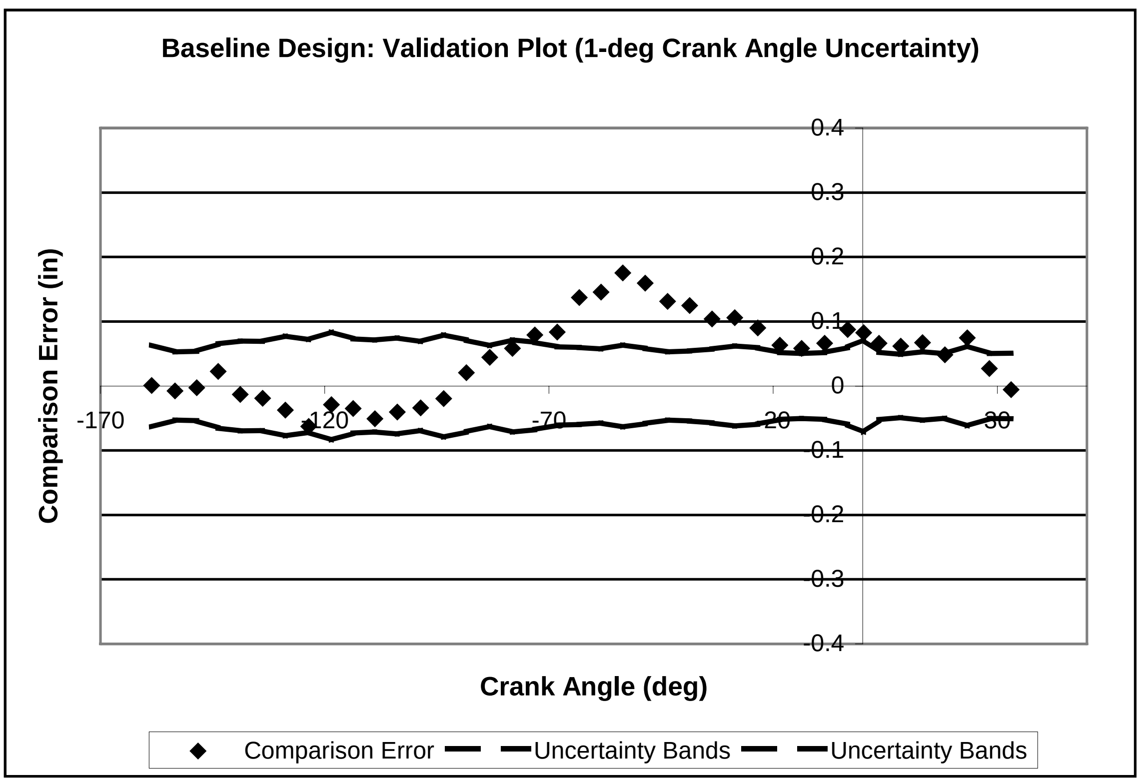


Figure 4.11a: Baseline Design Validation Plot with 1-degree Crank Angle Uncertainty

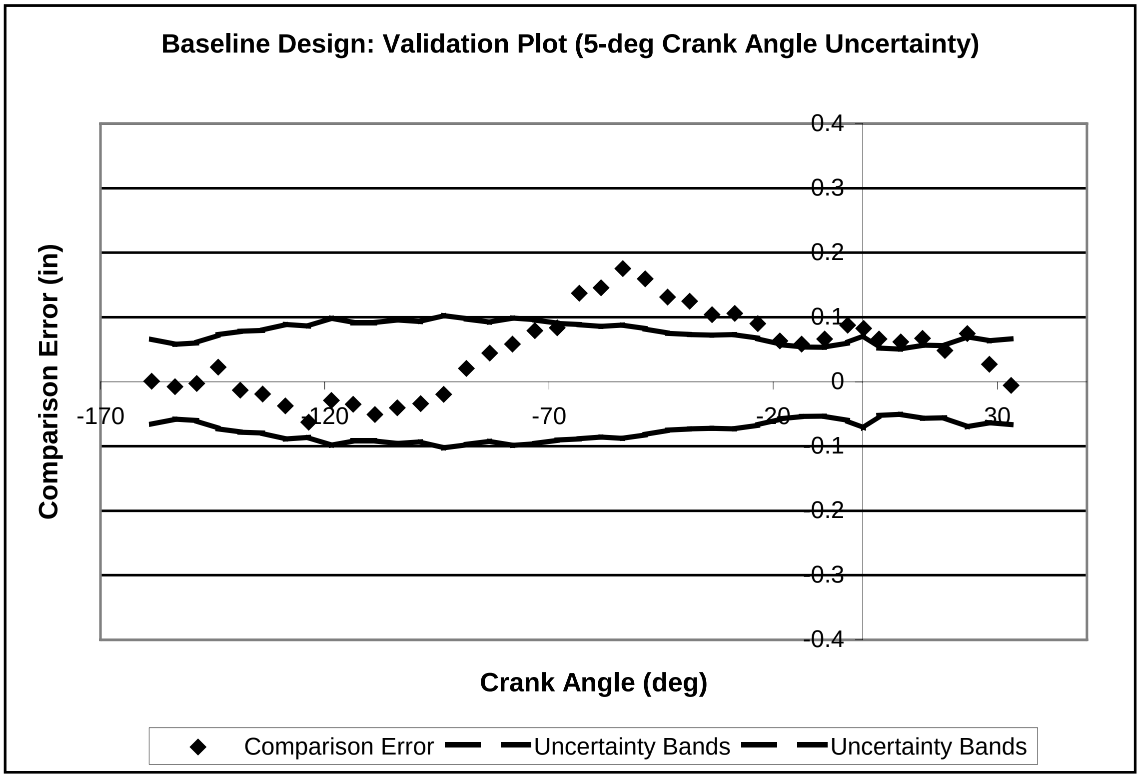


Figure 4.11b: Baseline Design Validation Plot with 5-degree Crank Angle Uncertainty

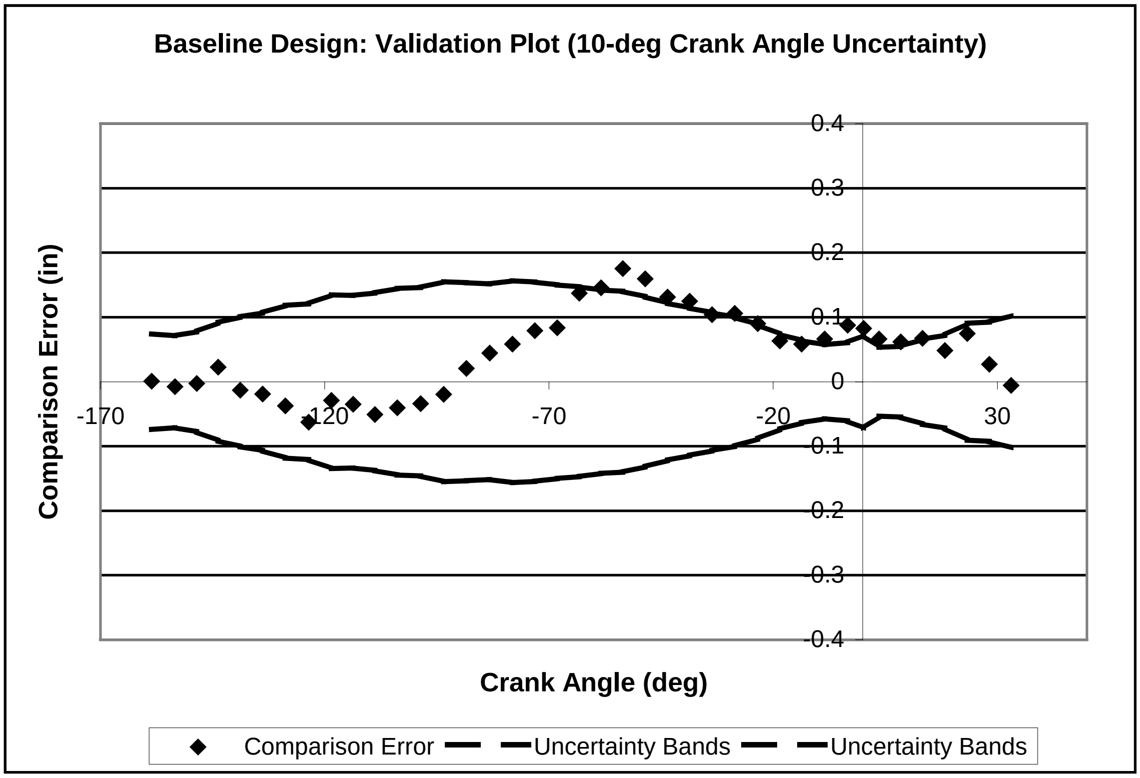


Figure 4.11c: Baseline Design Validation Plot with 10-degree Crank Angle Uncertainty

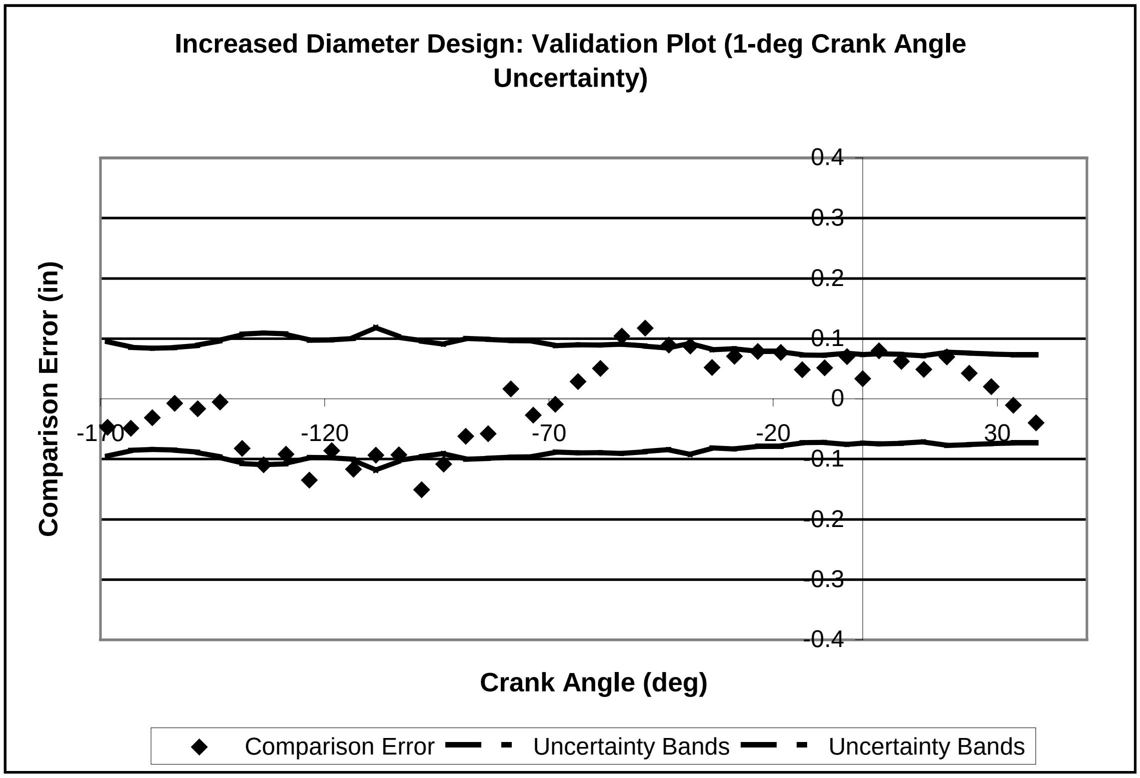


Figure 4.12a: Increased Diameter Design Validation Plot with 1-degree Crank Angle Uncertainty

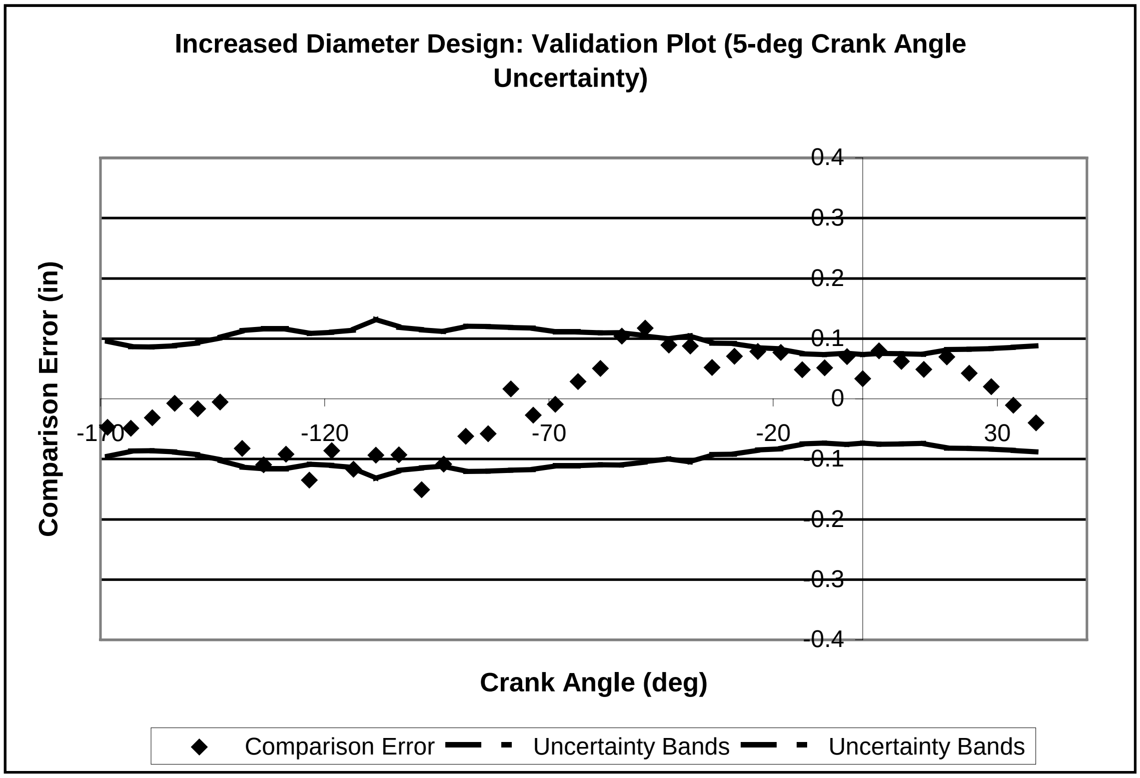


Figure 4.12b: Increased Diameter Design Validation Plot with 5-degree Crank Angle Uncertainty

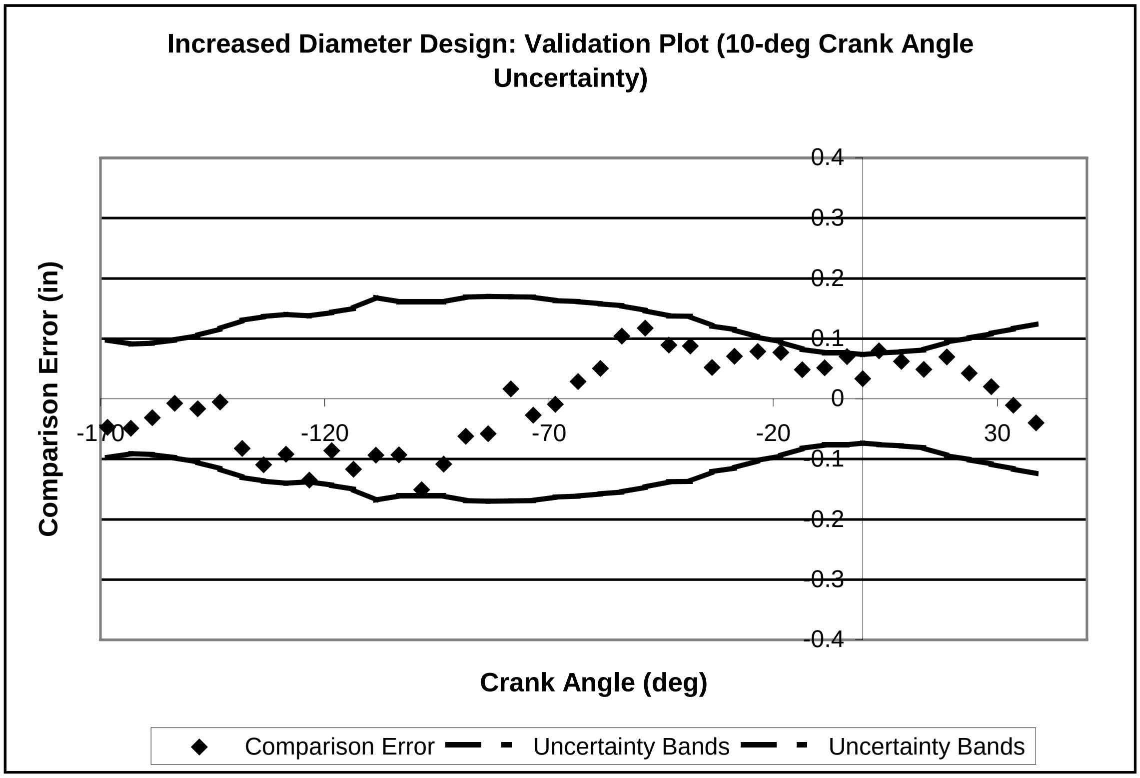


Figure 4.12c: Increased Diameter Design Validation Plot with 10-degree Crank Angle Uncertainty

From the figures, as the crank angle uncertainty increases, the uncertainty bands for the comparison error also become larger and more towards a sinusoidal curve rather than a straight line. The sensitivity coefficient of the experimental data points with respect to the crank angle is given by Equation (4.9). This shows the sine function effect on the overall expression. At a crank angle of zero degrees, the uncertainty bands have a fixed gap for both the baseline design and the increased diameter design since the sine function at zero degrees is approaching a nominal value of zero. Also, from the figures above, there are a few data points that exceed the uncertainty bands. This means that an improvement is needed on either the proposed model or the experimental setup because the validation was not achieved at the calculated $U_E$ level. Therefore, how should either the proposed model or the experiment be refined for improvement? Uncertainty Percentage Contribution (UPC) terms will be used to answer the question.

## Uncertainty Percentage Contribution Analysis

As mentioned in Chapter I, Uncertainty Percentage Contribution (UPC) is a nondimensionlized indicator of the percentage contribution of each variable to the final overall uncertainty. The UPC values are calculated according to Equation (1.7), and the total of all the UPC values is 100% as shown in Equation (1.6). UPC analysis highlights the variables with the highest contributions to the total uncertainty allowing the researcher to focus on those variables for improvement.

$$1 = \frac{\left(\frac{\partial r}{\partial X_1}\right)^2 U_{X1}^2 + \left(\frac{\partial r}{\partial X_2}\right)^2 U_{X2}^2 + \ldots + \left(\frac{\partial r}{\partial X_J}\right)^2 U_{XJ}^2}{U_r^2} \tag{1.6}$$

$$UPC_i = \frac{\left(\frac{\partial r}{\partial X_i}\right)^2 (U_{Xi})^2}{U_r^2} \tag{1.7}$$

Since there are many data points in the determined crank angle range, only one point was chosen to be evaluated with the UPC analysis. For the baseline design, a crank angle of –78.1 degrees was chosen for the UPC evaluation. For the increased diameter design, the chosen crank angle was –83.6 degrees. These values were chosen because they corresponded to the highest $U_E$ value in each case. The UPC evaluations are summarized in the Table 4.1 and Table 4.2.

Table 4.1: Summary of UPC Evaluation for Baseline Design

| **Baseline Design** | | | |
|---|---|---|---|
| **Crank Angle (deg)** | -78.1 | -78.1 | -78.1 |
| **Maximum $U_E$ Value (in)** | 0.072 | 0.099 | 0.156 |
| **Theta Uncertainty (deg)** | 1 | 5 | 10 |
| | | | |
| **Elemental Sources** | **UPC %** | **UPC %** | **UPC %** |
| Unc. of Total Displacement | 20.433 | 10.739 | 4.326 |
| Unc. of Actual Experiment Data Pts. | 56.205 | 29.54 | 11.899 |
| Unc. of Crank Angle | 3.761 | 49.418 | 79.625 |
| Random Unc. of the Slop | 0.085 | 0.045 | 0.018 |
| Systematic Unc. of the Slop | 12.087 | 6.353 | 2.559 |
| Combined Systematic Crankshaft Length Unc. | 1.07E-05 | 5.63E-06 | 2.27E-06 |
| Combined Systematic Piston Length Unc. | 1.40E-02 | 7.23E-03 | 2.91E-03 |
| Systematic Unc. of Inner Length, L1 | 3.708 | 1.949 | 0.785 |
| Systematic Unc. of Outer Length, L2 | 3.708 | 1.949 | 0.785 |

Table 4.2: Summary of UPC Evaluation for Increased Diameter Design

| **Increased Diameter Design** | | | |
|---|---|---|---|
| **Crank Angle (deg)** | -83.567 | -83.567 | -83.567 |
| **Maximum $U_E$ Value (in)** | 0.101 | 0.121 | 0.171 |
| **Theta Uncertainty (deg)** | 1 | 5 | 10 |
| | | | |
| **Elemental Sources** | **UPC %** | **UPC %** | **UPC %** |
| Unc. of Total Displacement | 10.196 | 7.125 | 3.671 |
| Unc. of Actual Experiment Data Pts. | 52.088 | 36.401 | 18.753 |
| Unc. of Crank Angle | 1.796 | 31.371 | 64.644 |
| Random Unc. of the Slop | 26.174 | 18.291 | 9.423 |
| Systematic Unc. of the Slop | 6.031 | 4.215 | 2.171 |
| Combined Systematic Crankshaft Length Unc. | 2.90E-04 | 2.03E-04 | 1.04E-04 |
| Combined Systematic Piston Length Unc. | 6.87E-03 | 4.80E-03 | 2.47E-03 |
| Systematic Unc. of Inner Length, L1 | 1.855 | 1.296 | 0.668 |
| Systematic Unc. of Outer Length, L2 | 1.855 | 1.296 | 0.668 |

From the tables above, the UPC of the crank angle uncertainty increased as the crank angle uncertainty increased for both designs. At 10 degrees crank angle uncertainty, the effect of this crank angle uncertainty dominated the overall uncertainty contributions. Therefore, this implies that the experiment needs improvement. The crank angle should be measured directly in the experiment to improve the overall $U_E$ value.

Several other terms also had significant UPC values in the various cases. First, the randomness of the actual experiment data points had a relatively high UPC value for all cases. This should be the correct measurement because the experiment was measuring the piston displacement directly. However, increasing the number of repeat data runs could decrease this uncertainty. Next, the total displacement uncertainty was a fairly significant contributor to the overall uncertainty, particularly when the crank angle uncertainty was lower. This indicates that the total displacement should be measured directly during the experiment setup, as known. The manufacture tolerances for the connecting rod diameter and connecting rod length also had a measurable effect on $U_E$ even though the percentage contributions were small compared to contributions from the experiment. Finally, the slop term was a significant factor. The random uncertainty of the slop term was calculated by Equation (3.3), which was totally dependent on the diameters of the connecting rod and the crankshaft.

$$R_{Sx} = \frac{(d_{cr} - d_{cs})}{2} \qquad (3.3)$$

The random uncertainty of the slop term in the baseline design was relatively small because the diameter of the connecting rod was supposed to be equivalent to the diameter of the crankshaft creating a tight fit. On the other hand, the random uncertainty of the slop term in the increased diameter design had a larger contribution because the diameter of the connecting rod was customized to be 0.1 inches larger than the baseline design. The effects of the slop were clearly shown in the increased diameter design; this was the reason that the uncertainty of the increased diameter design was much larger than the uncertainty of the baseline design.

Both Table 4.1 and Table 4.2 showed the UPC of each measured variable, but what how would the four general steps (model, experiment, manufacture, and comparisons) contribute to the overall uncertainty? Placing the particular uncertainty values under one of the 4 general steps in the product development process is fairly subjective and could vary depending on the situation and information needed. As an example in this case, the uncertainty values were placed under the 4 steps as given in Tables 4.3 and 4.4.

Table 4.3: UPC of 4-Product Development Steps for the Baseline Design

| **Baseline Design** | | | |
|---|---|---|---|
| **Crank Angle (deg)** | -78.1 | -78.1 | -78.1 |
| **Maximum $U_E$ Value (in)** | 0.072 | 0.099 | 0.156 |
| **Theta Uncertainty (deg)** | 1 | 5 | 10 |
| | | | |
| **Model** | **UPC %** | **UPC %** | **UPC %** |
| Combined Systematic Crankshaft Length Unc. | 1.07E-05 | 5.63E-06 | 2.27E-06 |
| Combined Systematic Piston Length Unc. | 1.40E-02 | 7.23E-03 | 2.91E-03 |
| **Total** | **1.40E-02** | **7.24E-03** | **2.92E-03** |
| | | | |
| **Experiment** | **UPC %** | **UPC %** | **UPC %** |
| Unc. of Actual Experiment Data Pts. | 56.205 | 29.54 | 11.899 |
| Unc. of Crank Angle | 3.761 | 49.418 | 79.625 |
| **Total** | **59.966** | **78.958** | **91.524** |
| | | | |
| **Manufacture** | **UPC %** | **UPC %** | **UPC %** |
| Random Unc. of the Slop | 0.085 | 0.045 | 0.018 |
| Systematic Unc. of the Slop | 12.087 | 6.353 | 2.559 |
| Systematic Unc. of Inner Length, L1 | 3.708 | 1.949 | 0.785 |
| Systematic Unc. of Outer Length, L2 | 3.708 | 1.949 | 0.785 |
| **Total** | **19.588** | **10.296** | **4.147** |
| | | | |
| **Comparisons** | **UPC %** | **UPC %** | **UPC %** |
| Unc. of Total Displacement | 20.433 | 10.739 | 4.326 |
| **Total** | **20.433** | **10.739** | **4.326** |

Table 4.4: UPC of 4-Product Development Steps for Increased Diameter Design

| | | | |
|---|---|---|---|
| **Increased Diameter Design** | | | |
| **Crank Angle (deg)** | -83.567 | -83.567 | -83.567 |
| **Maximum $U_E$ Value (in)** | 0.101 | 0.121 | 0.171 |
| **Theta Uncertainty (deg)** | 1 | 5 | 10 |
| | | | |
| **Model** | **UPC %** | **UPC %** | **UPC %** |
| Combined Systematic Crankshaft Length Unc. | 2.90E-04 | 2.03E-04 | 1.04E-04 |
| Combined Systematic Piston Length Unc. | 6.87E-03 | 4.80E-03 | 2.47E-03 |
| **Total** | **7.16E-03** | **5.00E-03** | **2.58E-03** |
| | | | |
| **Experiment** | **UPC %** | **UPC %** | **UPC %** |
| Unc. of Actual Experiment Data Pts. | 52.088 | 36.401 | 18.753 |
| Unc. of Crank Angle | 1.796 | 31.371 | 64.644 |
| **Total** | **53.884** | **67.772** | **83.397** |
| | | | |
| **Manufacture** | **UPC %** | **UPC %** | **UPC %** |
| Random Unc. of the Slop | 26.174 | 18.291 | 9.423 |
| Systematic Unc. of the Slop | 6.031 | 4.215 | 2.171 |
| Systematic Unc. of Inner Length, L1 | 1.855 | 1.296 | 0.668 |
| Systematic Unc. of Outer Length, L2 | 1.855 | 1.296 | 0.668 |
| **Total** | **35.915** | **25.098** | **12.93** |
| | | | |
| **Comparisons** | **UPC %** | **UPC %** | **UPC %** |
| Unc. of Total Displacement | 10.196 | 7.125 | 3.671 |
| **Total** | **10.196** | **7.125** | **3.671** |

With this division of the uncertainty values, from Table 4.3 and Table 4.4, the uncertainty contributed by the experiment step had the highest contribution due to the randomness of the actual experiment data points and/or the crank angle uncertainty. This clearly shows that the experiment in this product development process needed improvement in order to get a better experiment result. The manufacture step also made a significant contribution to the overall uncertainty due to the slop term and the manufacture tolerances. The uncertainty contributed by the comparisons step was solely due to the uncertainty of the total displacement. The model uncertainty contributed the least to the overall UPC because the uncertainty was calculated based on the measurements of the variables, which was too small to make a significant impact to the overall uncertainty. From the overall view, the experimentation needs improvement because the UPC analysis showed that the experiment had a relatively high uncertainty contribution to the overall $U_E$.

Realize that the model did not account for slop. It could be argued that the uncertainty of the slop should be considered in the model step to account for uncertainty related to the basic assumption that slop was not important. If this were done, then the model would appear to be a significant contributor to the overall uncertainty. Also, the crank angle uncertainty could be considered part of the comparisons step since the conversion of the time measured in the experiment to crank angle was only needed to compare to the model results. This would reduce the contribution of the experiment step and increase the contribution of the comparisons step. Many other examples could be given; however, the point is that the division of the uncertainty terms under the 4 general steps is subjective and depends on the situation and information needed. This shows that uncertainty is an implicit part of the product development process using the methodology developed in this research. There are no longer clear divisions for uncertainty for the particular steps. All steps are an integral part of the process.

Finally, the UPC analysis results are strictly valid only when the validation has been achieved. Until the validation is achieved, refinements must be made in the steps and uncertainty estimates. The previous example was used to show how the data could be interpreted and used for improvements.

# CHAPTER V

# CONCLUSIONS

Evaluation of the overall product development process through uncertainty analysis techniques was a new approach taken to combine the four general steps of a product development process (model, experiment, manufacture, and comparisons). As stated in Chapter I, the formula that determined the final product was given by Equation (1.16) and the uncertainty of the final product was given by Equation (1.17). The uncertainty of each step could be determined, and the proposed idea stated that the uncertainty of the final product was a function of the uncertainties within each individual step. Therefore, a total understanding was needed to define the relationships between the steps and determine the uncertainty of the final product.

A literature survey was conducted with the hope that other methodologies could aid in defining the relationships between the steps and determining the uncertainty of the final product. The key areas of the literature survey were modeling, multidisciplinary design optimization (MDO), robust design, and design of experiments (DoE). In modeling, the important criterion is the verification and validation (V&V) of a model, and uncertainty analysis techniques can be used as tools in the V&V process.[1] Uncertainty analysis techniques can be very beneficial in modeling because uncertainty analysis can highlight controlling parameters thus saving both time and cost. MDO is an optimization tool that is very useful in model simulation because through MDO the best design parameter can be determined. MDO that incorporates uncertainty analysis into the design process would accelerate the optimization process. Robust design is another tool used to search for the best design parameter that would give a robust result. The links between the subsystems in the robust design could be helpful in defining the relationships of the steps of the product development process. DoE is another common tool that is used to improve the quality of the experiment result. DoE is capable of determining the controlling parameter(s) and reorganizing the data entry to decrease the variability of the parameter(s). All of these areas of research could contribute to the long-term goals of the work described in this thesis.

A pilot project, a four-bar-slider-mechanism, was initiated to show the methodology of the product development process uncertainty analysis. The model uncertainty analysis was completed, and the connecting rod was customized and manufactured according to the requirements of this research. The manufacture tolerances were incorporated into the uncertainty analysis, and the model prediction result was obtained. An experiment was conducted to measure the piston displacement directly, and the crank angle was also determined. Direct comparisons between the model prediction and the experiment data were made. The comparison showed that the model equation did predict the experiment data. A validation analysis was performed, and the UPC of each step was calculated. Both the validation analysis and the UPC calculations showed that the experimentation of this research needed improvement since the experiment uncertainty dominated the overall uncertainty of the final product.

Several suggestions can be made to improve the experiment result and the experiment uncertainty. First, the motor used to drive the experiment needs to be replaced by a more powerful motor to give the least variation in engine speed. Second, the total displacement of the piston must be measured directly and not calculated through an equation. Finally, the crank angle of this mechanism should be measured directly and not determined by an equation. Following these suggestions would give a better experiment result and thus will directly improve the credibility of the model equation for this research.

The listed objectives of this pilot project in Table 3.1 have been met. The ultimate goals of this research to determine the uncertainty of the final product and the relative contributions of each step to the overall final product uncertainty have been achieved. The methodology developed for combining the uncertainties in the individual steps and the calculation of the relative contributions of each step has not been done in the past. Table 4.3 and Table 4.4 in Chapter IV showed the percentage contributions of each step to the overall final product uncertainty. The UPC analysis proved the uncertainty of each step has been built into the product development process in determining the uncertainty of the final product. This product development process uncertainty methodology is unique and different from the traditional approach. The uncertainty analysis is no longer a simple comparisons tool but rather an important tool that brings together the computational work, experimental work, and manufacturing. For future work, this methodology needs to be extended to other product development process. The detail analysis and result of this pilot project can be used to assist in the analysis of other product development process. With repetition of this product development process uncertainty analysis, a more general model of this technique will be developed.

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# APPENDIX

## MATHCAD WORKSHEETS

## Model Results:

Establish origin in MathCad: ORIGIN $\equiv$ 1

10 measurements were made for each measured variable in the DRE of the model.

Crank Shaft Length:

$$ml_{cs} := \begin{pmatrix} .777 \\ .778 \\ .777 \\ .776 \\ .777 \\ .778 \\ .778 \\ .776 \\ .779 \\ .777 \end{pmatrix} \cdot in$$

L1 Length:

$$ml_{1} := \begin{pmatrix} 2.575 \\ 2.578 \\ 2.576 \\ 2.578 \\ 2.577 \\ 2.575 \\ 2.574 \\ 2.578 \\ 2.579 \\ 2.575 \end{pmatrix} \cdot in$$

L2 Length:

$$ml_{2} := \begin{pmatrix} 3.899 \\ 3.898 \\ 3.904 \\ 3.901 \\ 3.901 \\ 3.904 \\ 3.907 \\ 3.902 \\ 3.904 \\ 3.903 \end{pmatrix} \cdot in$$

Crank Shaft Diameter:

$$md_{cs} := \begin{pmatrix} .746 \\ .745 \\ .749 \\ .748 \\ .745 \\ .747 \\ .746 \\ .744 \\ .742 \\ .746 \end{pmatrix} \cdot in$$

Connecting Rod Diameter:

$$md_{cr} := \begin{pmatrix} .743 \\ .752 \\ .754 \\ .757 \\ .759 \\ .758 \\ .751 \\ .752 \\ .758 \\ .752 \end{pmatrix} \cdot in$$

Piston Length

$$ml_{p} := \begin{pmatrix} 1.101 \\ 1.099 \\ 1.102 \\ 1.100 \\ 1.102 \\ 1.101 \\ 1.099 \\ 1.100 \\ 1.101 \\ 1.101 \end{pmatrix} \cdot in$$

The mean value will be used for each set of 10 measurements:

**Crank Shaft Length:**

$l_{cs} := \text{mean}\left(ml_{cs}\right)$

$l_{cs} = 0.777\ in$

**L1 Length:**

$l_{1} := \text{mean}\left(ml_{1}\right)$

$l_{1} = 2.577\ in$

**L2 Length:**

$l_{2} := \text{mean}\left(ml_{2}\right)$

$l_{2} = 3.902\ in$

**Crank Shaft Diameter:**

$d_{cs} := \text{mean}\left(md_{cs}\right)$

$d_{cs} = 0.746\ in$

**Connecting Rod Diameter:**

$d_{cr} := \text{mean}\left(md_{cr}\right)$

$d_{cr} = 0.754\ in$

**Piston Length:**

$l_{p} := \text{mean}\left(ml_{p}\right)$

$l_{p} = 1.101\ in$

Assume **perfect fit;** therefore, **slop** is **negligible** : $s_x := 0 \cdot in$

The **model equation** :

$$d(\theta) := l_{cs} \cdot \cos(\theta) + \sqrt{\left(\frac{l_1 + l_2}{2}\right)^2 - l_{cs}^{\,2} \cdot (\sin(\theta))^2} + l_p + s_x$$

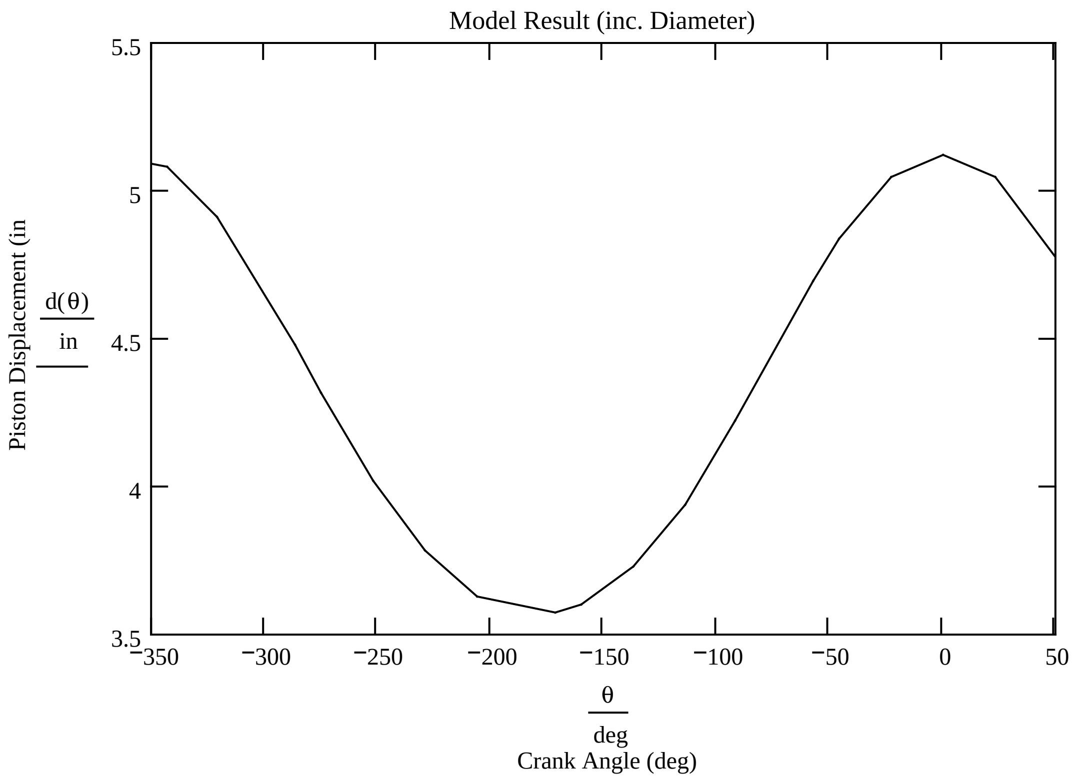


Figure A.1: Model Result

## MODEL UNCERTAINTY

**Partial derivatives** of piston displacement with respect to (w.r.t.) each variable:

$$pl_{cs}(\theta) := \cos(\theta) + \frac{-l_{cs}\cdot(\sin(\theta))^2}{\sqrt{\left(\frac{l_1 + l_2}{2}\right)^2 - l_{cs}{}^2\cdot(\sin(\theta))^2}}$$

w.r.t. **crankshaft length**

$$pl_1(\theta) := \frac{1}{2}\cdot\frac{\frac{l_1+l_2}{2}}{\sqrt{\left(\frac{l_1 + l_2}{2}\right)^2 - l_{cs}{}^2\cdot(\sin(\theta))^2}}$$

w.r.t. **L1 Length**

$$pl_2(\theta) := \frac{1}{2}\cdot\frac{\frac{l_1+l_2}{2}}{\sqrt{\left(\frac{l_1 + l_2}{2}\right)^2 - l_{cs}{}^2\cdot(\sin(\theta))^2}}$$

w.r.t. **L2 Length**

$pl_p := 1$ w.r.t. **Piston Length**

$ps_x := 1$ w.r.t. **Slop**

The **standard deviations** from the ten measurements:

$Sl_{cs} := \text{Stdev}(ml_{cs})$ $Sl_{cs} = 9.487\times 10^{-4}\,\text{in}$

$Sl_1 := \text{Stdev}(ml_1)$ $Sl_1 = 1.716\times 10^{-3}\,\text{in}$

$Sl_2 := \text{Stdev}(ml_2)$ $Sl_2 = 2.669\times 10^{-3}\,\text{in}$

$Sl_p := \text{Stdev}(ml_p)$ $Sl_p = 1.075\times 10^{-3}\,\text{in}$

$Sd_{cr} := \text{Stdev}(md_{cr})$ $Sd_{cr} = 4.789\times 10^{-3}\,\text{in}$

$Sd_{cs} := \text{Stdev}(md_{cs})$ $Sd_{cs} = 1.989\times 10^{-3}\,\text{in}$

The **random uncertainty** values based on the standard deviation of the 10 samples:

The large sample assumption, if N > 10, the *t* value is $t := 2$ $N := 10$

$$rl_{cs} := \frac{t \cdot Sl_{cs}}{\sqrt{N}} \quad rl_1 := \frac{t \cdot Sl_1}{\sqrt{N}} \quad rl_p := \frac{t \cdot Sl_p}{\sqrt{N}} \quad rl_2 := \frac{t \cdot Sl_2}{\sqrt{N}} \quad rd_{cs} := \frac{t \cdot Sd_{cs}}{\sqrt{N}} \quad rd_{cr} := \frac{t \cdot Sd_{cr}}{\sqrt{N}}$$

A micrometer was used to determine these measurements and the **least count** (LC) of the micrometer is

$$LC := .001 \cdot in$$

The **systematic uncertainty** in these measurements:

$$sl_{cs1} := \frac{1}{2} \cdot LC \quad sl_{p1} := \frac{1}{2} \cdot LC \quad sl1_1 := \frac{1}{2} \cdot LC \quad sl2_1 := \frac{1}{2} \cdot LC \quad sd_{cs1} := \frac{1}{2} \cdot LC \quad sd_{cr1} := \frac{1}{2} \cdot LC$$

The **second elemental source** was the baseline design:

$$sl_{cs2} := rl_{cs} \quad sd_{cs2} := rd_{cs} \quad sl_{p2} := rl_p \quad sl1_2 := rl_1 \quad sl2_2 := rl_2 \quad sd_{cr2} := rd_{cr}$$

The **combined systematic uncertainty** for the pre-designed pieces:

$$sd_{cs} := \sqrt{sd_{cs1}^2 + sd_{cs2}^2} \quad sl_{cs} := \sqrt{sl_{cs1}^2 + sl_{cs2}^2} \quad sl_p := \sqrt{sl_{p1}^2 + sl_{p2}^2}$$

$$sl_1 := \sqrt{sl1_1^2 + sl1_2^2} \quad sl_2 := \sqrt{sl2_1^2 + sl2_2^2} \quad sd_{cr} := \sqrt{sd_{cr1}^2 + sd_{cr2}^2}$$

The **total uncertainty** for the **diameters** :

$$ud_{cs} := sd_{cs} \quad ud_{cr} := sd_{cr}$$

Slop uncertainty equations:

$$rs_x := \frac{d_{cr} - d_{cs}}{2} \quad ss_x := \sqrt{\frac{1}{4} \cdot ud_{cr}^2 + \frac{1}{4} \cdot ud_{cs}^2}$$

The model uncertainty was obtained from the experimental uncertainty equation:

$$rd(\theta) := \left(ps_x^2 \cdot rs_x^2\right)^{\frac{1}{2}}$$

$$sd(\theta) := \left(pl_{cs}(\theta)^2 \cdot sl_{cs}^2 + pl_1(\theta)^2 \cdot sl_1^2 + pl_2(\theta)^2 \cdot sl_2^2 + pl_p^2 \cdot sl_p^2 + ps_x^2 \cdot ss_x^2\right)^{\frac{1}{2}}$$

Finally, the **combined model uncertainty** : $ud(\theta) := \sqrt{rd(\theta)^2 + sd(\theta)^2}$

The model equation:

$$d(\theta) := l_{cs} \cdot \cos(\theta) + \sqrt{\left(\frac{l_1 + l_2}{2}\right)^2 - l_{cs}{}^2 \cdot (\sin(\theta))^2} + l_p + s_x$$

## Prepare for comparisons:

$dm(\theta) := d(\theta)$ **Model Equation** $Udm(\theta) := ud(\theta)$ **Model Uncertainty**

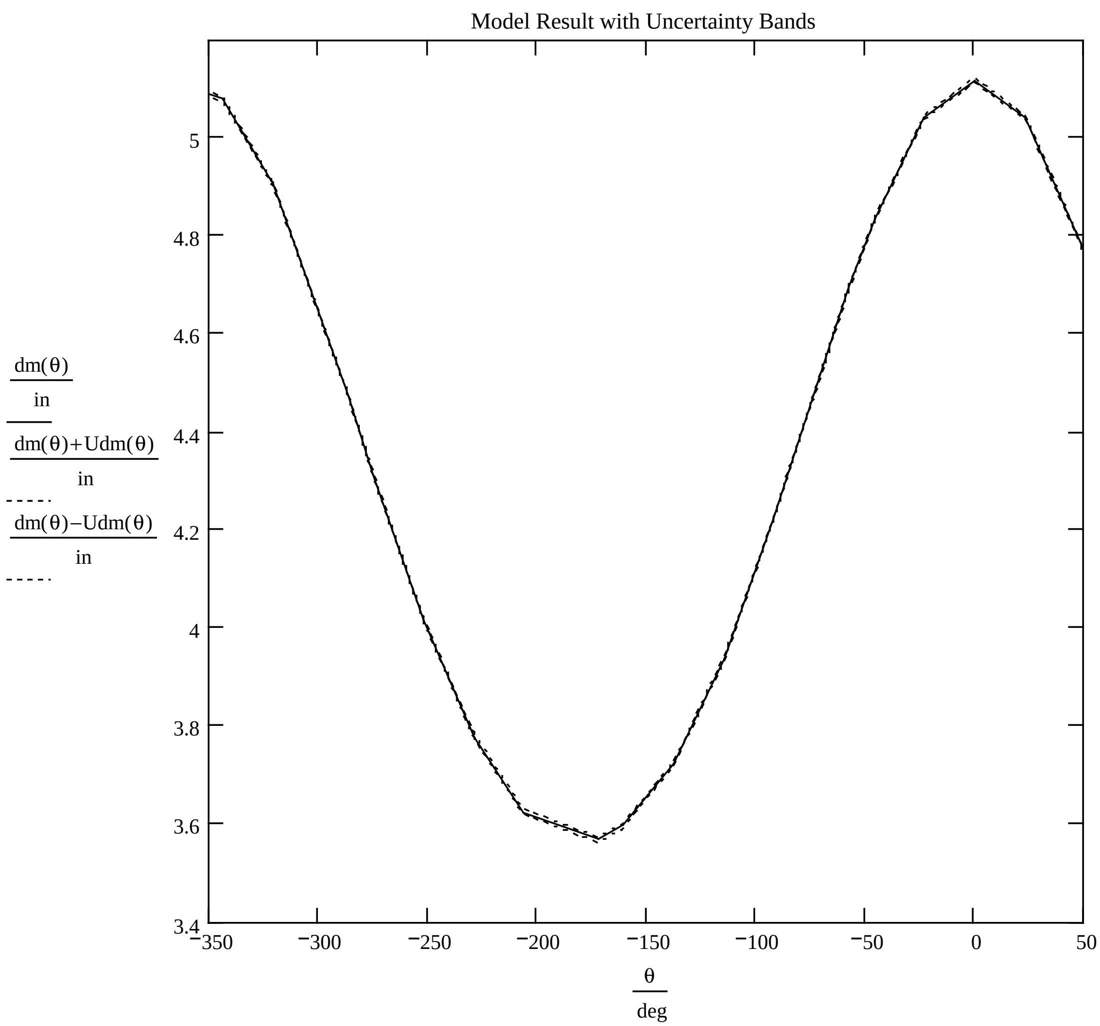


Figure A.2: Model Result with Uncertainty Bands

# MANUFACTURE RESULTS

The uncertainty calculation including manufacturing tolerances :

The measured piston diameter, $md_p := \begin{pmatrix} 0.489 \\ 0.488 \\ 0.49 \\ 0.488 \\ 0.49 \\ 0.489 \\ 0.488 \\ 0.49 \\ 0.488 \\ 0.489 \end{pmatrix} \cdot in$

**Piston Diameter:** $d_p := mean(md_p)$ $d_p = 0.489in$

The standard deviation of the piston diameter, $Stdev(md_p) = 8.756 \times 10^{-4} in$

The **random uncertainty** values based on the standard deviation of the 10 samples:

The large sample assumption, if N > 10, the *t* value is $t := 2$ $N := 10$

$$rd_p := \frac{t \cdot Stdev(md_p)}{\sqrt{N}} \qquad rd_p = 5.538 \times 10^{-4} in$$

A micrometer was used to determine these measurements and the **least count** (LC) of the micrometer is

$LC := .001 \cdot in$

The **systematic uncertainty** piston diameter:

$$sd_p := \frac{1}{2} \cdot LC \qquad sd_p = 5 \times 10^{-4} in$$

The overall uncertainty of the piston diameter will be the **rss** of the random and systematic components of the piston diameter.

$$Ud_p := \sqrt{sd_p^2 + rd_p^2} \qquad Ud_p = 7.461 \times 10^{-4} in$$

**Specified Values for Manufacture** :

**Specified Connecting Rod Diameter:** $d_{cr} := .75 \cdot in$

**Specified Connecting Rod Length:** $l_{cr} := 3.25 \cdot in$

Manufacturing tolerances specified for new connecting rod:

**Connecting Rod Length Tolerance** $tl_{cr} := .01 \cdot in$

Exaggerated uncertainty in the connecting rod diameter:

**Connecting Rod Diameter Tolerance** $td_{cr} := .05 \cdot in$

**Redefine Systematic Uncertainty of the Connecting Rod Diameter** $sd_{cr} := td_{cr}$

The data reduction equations for manufacture:

$$l1 = l_{cr} - \frac{d_{cr} + d_p}{2} \qquad l2 = l_{cr} + \frac{d_{cr} + d_p}{2}$$

## MANUFACTURING UNCERTAINTY

The **systematic uncertainty** based on manufacturing tolerances:

**L1 Length** $sl_1 := \sqrt{tl_{cr}^2 + \left(\frac{1}{4}\right) \cdot td_{cr}^2 + \left(\frac{1}{4}\right) \cdot Ud_p^2}$ $sl_1 = 0.027in$

**L2 Length** $sl_2 := \sqrt{tl_{cr}^2 + \left(\frac{1}{4}\right) \cdot td_{cr}^2 + \left(\frac{1}{4}\right) \cdot Ud_p^2}$ $sl_2 = 0.027in$

# EXPERIMENTAL RESULTS

Due to the fact that there were repeated cycles for each experiment, an average piston displacement and average crank angle were used. The average crank angle was calculated based on 5 degree increments. The average piston displacement was sorted first according to the crank angle and then only followed by calculating the average of the piston displacement.

**Average piston displacement:**

$d_{av} :=$

| | 1 |
|---|---|
| 1 | 0.4619 |
| 2 | 0.494103 |
| 3 | 0.441736 |
| 4 | 0.488343 |
| 5 | 0.499784 |
| 6 | 0.488515 |
| 7 | 0.48107 |
| 8 | 0.49874 |
| 9 | 0.525857 |
| 10 | 0.564552 |
| 11 | 0.572576 |
| 12 | 0.58012 |
| 13 | 0.596519 |
| 14 | 0.633897 |
| 15 | 0.662999 |
| 16 | 0.669909 |
| 17 | 0.690147 |
| 18 | 0.710236 |
| 19 | 0.749645 |
| 20 | 0.78037 |
| 21 | 0.803341 |
| 22 | 0.795794 |
| 23 | 0.851448 |
| 24 | 0.919653 |
| 25 | 0.948764 |
| 26 | 1.025803 |
| 27 | 1.087376 |
| 28 | 1.18298 |
| 29 | 1.308156 |
| 30 | 1.377472 |
| 31 | 1.490046 |
| 32 | 1.544272 |
| 33 | 1.612412 |
| 34 | 1.684202 |

**Standard deviation of the average piston displacement:**

$Std_{dav} :=$

| | 1 |
|---|---|
| 1 | 0.076499 |
| 2 | 0.087292 |
| 3 | 0.027378 |
| 4 | 0.073728 |
| 5 | 0.097446 |
| 6 | 0.073758 |
| 7 | 0.05365 |
| 8 | 0.079114 |
| 9 | 0.082318 |
| 10 | 0.10832 |
| 11 | 0.074506 |
| 12 | 0.076118 |
| 13 | 0.081558 |
| 14 | 0.101506 |
| 15 | 0.103934 |
| 16 | 0.090882 |
| 17 | 0.088936 |
| 18 | 0.070226 |
| 19 | 0.103471 |
| 20 | 0.086689 |
| 21 | 0.113317 |
| 22 | 0.088924 |
| 23 | 0.108931 |
| 24 | 0.117626 |
| 25 | 0.108968 |
| 26 | 0.143272 |
| 27 | 0.156475 |
| 28 | 0.211476 |
| 29 | 0.243191 |
| 30 | 0.21815 |
| 31 | 0.231083 |
| 32 | 0.171838 |
| 33 | 0.168974 |
| 34 | 0.144168 |

The equation used to calculate the crank angle, theta, from the engine speed and the elapsed time with an assumption that the engine speed ( $\omega$) is constant.

$$\theta = \theta_0 + \omega \cdot (\Delta t)$$

**Average crank angle:**

$\theta_{av} :=$ deg

| | 1 |
|---|---|
| 1 | -332.8161 |
| 2 | -327.8667 |
| 3 | -323.3906 |
| 4 | -319.1057 |
| 5 | -314.179 |
| 6 | -309.084 |
| 7 | -303.821 |
| 8 | -298.7423 |
| 9 | -293.637 |
| 10 | -288.5633 |
| 11 | -283.6013 |
| 12 | -278.3261 |
| 13 | -273.1152 |
| 14 | -267.9455 |
| 15 | -262.8727 |
| 16 | -257.686 |
| 17 | -252.8011 |
| 18 | -248.2747 |
| 19 | -243.1921 |
| 20 | -238.3299 |
| 21 | -233.4699 |
| 22 | -228.3996 |
| 23 | -223.3307 |
| 24 | -218.4638 |
| 25 | -213.5974 |
| 26 | -208.5149 |
| 27 | -203.4323 |
| 28 | -198.4638 |
| 29 | -193.4945 |
| 30 | -188.412 |
| 31 | -183.333 |
| 32 | -178.2627 |
| 33 | -173.1802 |
| 34 | -168.4196 |
| 35 | -163.6726 |
| 36 | -158.5901 |

**Standard deviation of the a verage crank angle:**

$Std_{\theta av} :=$ deg

| | 1 |
|---|---|
| 1 | 1.5414 |
| 2 | 1.4023 |
| 3 | 1.5414 |
| 4 | 1.3091 |
| 5 | 1.3276 |
| 6 | 1.3249 |
| 7 | 1.4736 |
| 8 | 1.4304 |
| 9 | 1.4484 |
| 10 | 1.4299 |
| 11 | 1.4063 |
| 12 | 1.5504 |
| 13 | 1.3407 |
| 14 | 1.5753 |
| 15 | 1.3036 |
| 16 | 1.636 |
| 17 | 1.1642 |
| 18 | 1.5138 |
| 19 | 1.5275 |
| 20 | 1.4389 |
| 21 | 1.5597 |
| 22 | 1.5518 |
| 23 | 1.5188 |
| 24 | 1.3489 |
| 25 | 1.4362 |
| 26 | 1.412 |
| 27 | 1.4108 |
| 28 | 1.349 |
| 29 | 1.4142 |
| 30 | 1.4203 |
| 31 | 1.443 |
| 32 | 1.4613 |
| 33 | 1.4791 |
| 34 | 1.334 |
| 35 | 1.5179 |
| 36 | 1.5176 |

The **uncertainty of the reference angle, $\theta_0$** becomes a **fossilized systematic uncertainty** in determining the uncertainty of the average crank angle.
Through experiment it was comfirmed that there was an additional uncertainty source due to the engine speed.
However, due to the fact that there's no sufficient information to quantify the uncertainty due to the engine speed and we also believe that this uncertainty due to engine speed will be the dominant source in computing the uncertainty of the crank angle, theta, a range of estimated crank angle uncertainties will be assigned to complete the uncertainty analysis.
Crank Angle Uncertainty: 1 deg, 5 deg, 10 deg.

$U_\theta := 1\cdot deg$

$U_\theta = 0.017rad$

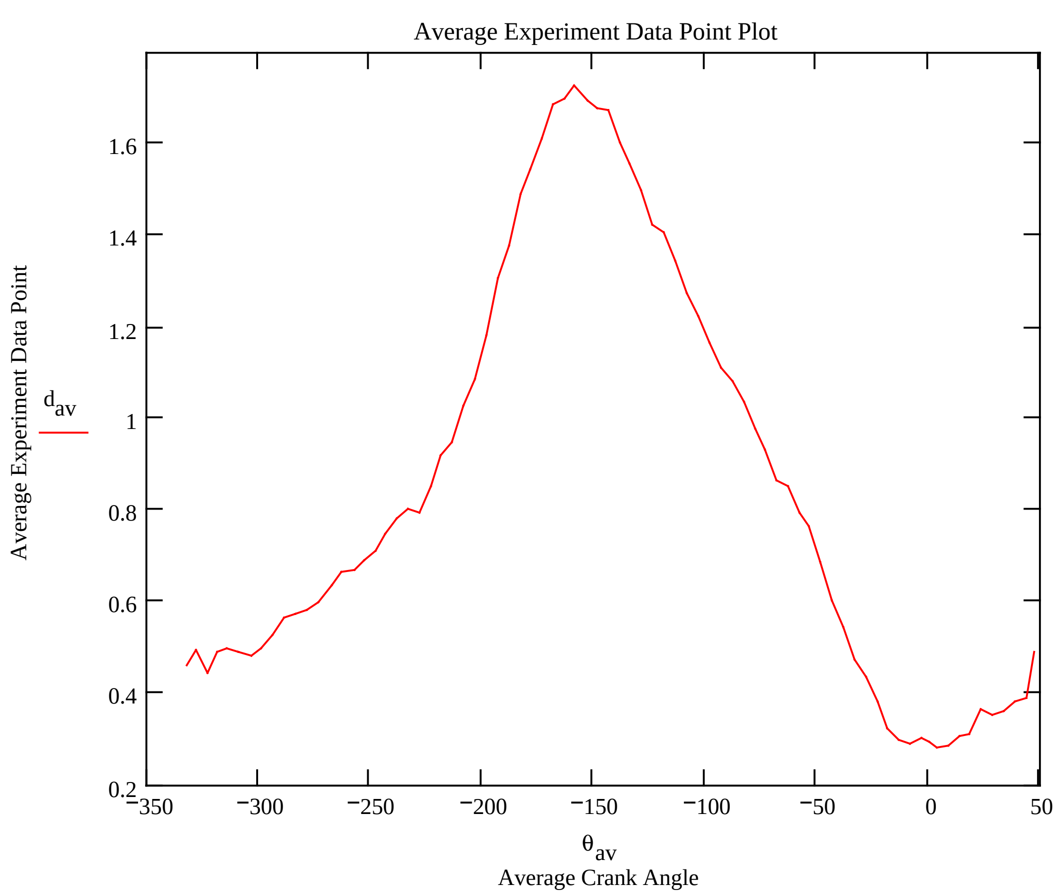


Figure A.3: Actual Experiment Data Points with Calculated Crank Angle

## EXPERIMENTAL UNCERTAINTY

Since the repeated experiment N > 10, therefore t-value is 2

The **random uncertainty** associated with the **experiment data point** is found to be

$$R_{exp} = \frac{t \cdot Std_{dav}}{\sqrt{N}} \qquad R_{exp} :=$$

| | 1 |
|---|---|
| 1 | 0.080294 |
| 2 | 0.10837 |
| 3 | 0.028736 |
| 4 | 0.052738 |
| 5 | 0.058891 |
| 6 | 0.037925 |
| 7 | 0.025108 |
| 8 | 0.035084 |
| 9 | 0.036505 |
| 10 | 0.048035 |
| 11 | 0.031467 |
| 12 | 0.030752 |
| 13 | 0.035271 |
| 14 | 0.041008 |
| 15 | 0.04609 |
| 16 | 0.03596 |
| 17 | 0.042867 |
| 18 | 0.029659 |
| 19 | 0.043699 |
| 20 | 0.038443 |
| 21 | 0.047858 |
| 22 | 0.037555 |
| 23 | 0.046005 |
| 24 | 0.052162 |
| 25 | 0.046021 |
| 26 | 0.060508 |
| 27 | 0.066085 |
| 28 | 0.091455 |
| 29 | 0.102707 |
| 30 | 0.092132 |
| 31 | 0.097594 |
| 32 | 0.072573 |
| 33 | 0.071363 |
| 34 | 0.065626 |
| 35 | 0.046251 |
| 36 | 0.044655 |
| 37 | 0.026715 |

The manufacture uncertainty implicitly included in randomness of **d** measurements during experiment. Therefore, **NO** additional uncertainty terms for manufacture.

The total random uncertainty of the experiment is

$$R_{dexp} := R_{exp}$$

The **total experimental uncertainty** is random components above because the random uncertainty is so big that it dominants the effect of the systematic uncertainty. Therefore, compared to the random uncertainty in the experiment data, the systematic uncertainty is negligible.

$$U_{dav} := R_{dexp}$$

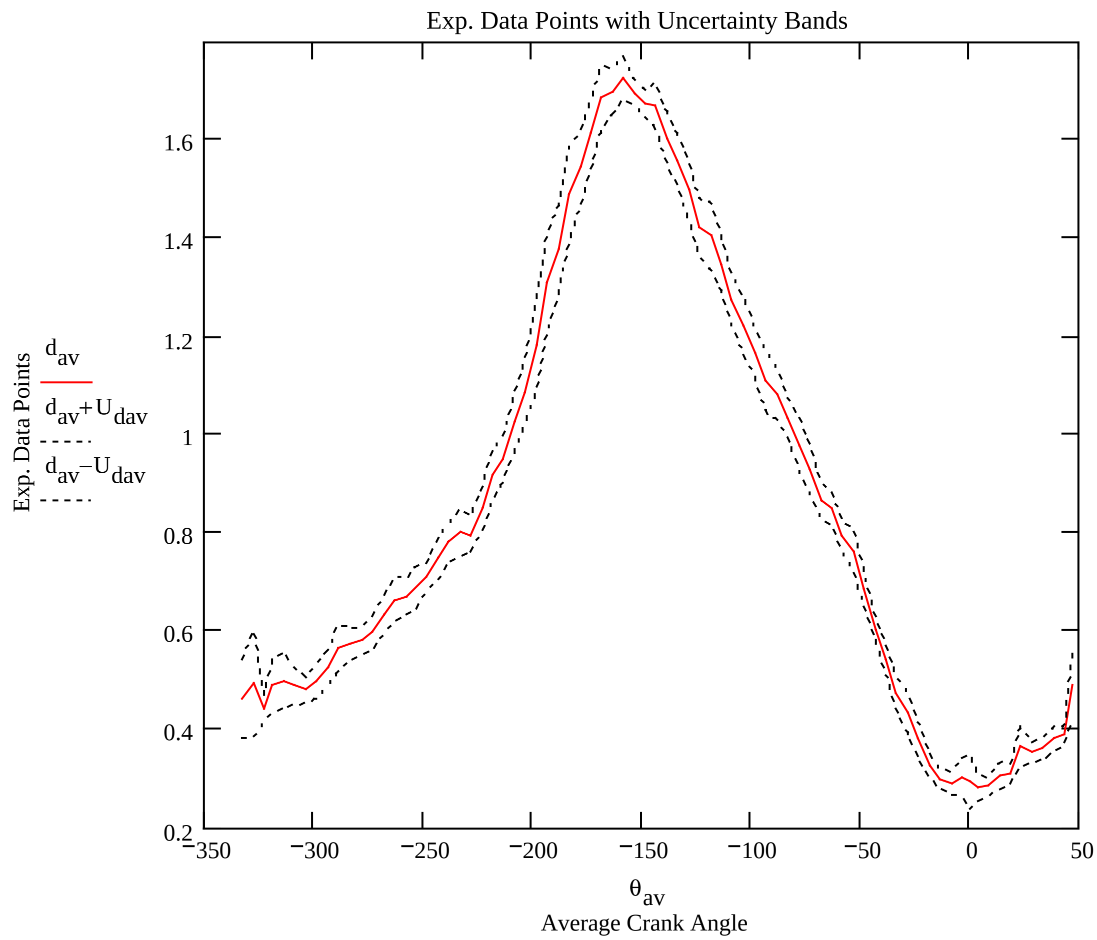


Figure A.4: Actual Experiment Data Points with Uncertainty Bands

# COMPARISONS

**Manufacturing Effects on the Model:**

The uncertainty for the connecting rod diameter:

$ud_{cr} := sd_{cr}$ **the manufacture tolerance**

Repeat slop uncertainty equations:

$$rs_x := \frac{d_{cr} - d_{cs}}{2} \qquad ss_x := \sqrt{\frac{1}{4} \cdot ud_{cr}^2 + \frac{1}{4} \cdot ud_{cs}^2}$$

The model uncertainty that incorporated the manufacture effects:

$$rd(\theta) := \left(ps_x^2 \cdot rs_x^2\right)^{\frac{1}{2}}$$

$$sd(\theta) := \left(pl_{cs}(\theta)^2 \cdot sl_{cs}^2 + pl_1(\theta)^2 \cdot sl_1^2 + pl_2(\theta)^2 \cdot sl_2^2 + pl_p^2 \cdot sl_p^2 + ps_x^2 \cdot ss_x^2\right)^{\frac{1}{2}}$$

Finally, the **combined uncertainty with manufacture effects** :

$$Ud(\theta) := \sqrt{(rd(\theta))^2 + (sd(\theta))^2}$$

The **model equation with manufacture effects** :

$$d(\theta) := l_{cs} \cdot \cos(\theta) + \sqrt{\left(\frac{l_1 + l_2}{2}\right)^2 - l_{cs}^2 \cdot (\sin(\theta))^2} + l_p + s_x \qquad i := 1..\, rows(\theta_{av})$$

**Model Displacement:** $d\left(\theta_{av_i} \cdot deg\right) =$

| |
|---|
| 5.01192029 |
| 4.97173637 |
| 4.93061611 |
| 4.88735744 |
| 4.83337721 |
| 4.77336992 |
| 4.70762674 |
| 4.64127617 |
| 4.57241569 |
| 4.50253589 |
| 4.43346094 |
| 4.35993226 |
| 4.28788597 |
| 4.21761702 |
| 4.15039542 |
| 4.08395263 |
| 4.02390853 |
| 3.97077171 |
| 3.9142768 |
| 3.86362625 |
| 3.81653111 |
| 3.77135015 |
| 3.73037331 |
| 3.69508779 |
| 3.66386851 |
| 3.63566843 |
| 3.61202233 |
| 3.59334479 |
| 3.57907737 |
| 3.569066 |
| 3.56369958 |
| 3.56297158 |
| 3.56688449 |
| 3.57476247 |
| 3.58666904 |
| 3.60388875 |
| 3.6261652 |
| 3.65133517 |
| 3.68015857 |
| 3.71420591 |
| 3.75267043 |
| 3.79612105 |

**Uncertainty of Model Displacement:** $Ud\left(\theta_{av_i} \cdot deg\right) =$

| |
|---|
| 0.03159097 |
| 0.03161524 |
| 0.03163925 |
| 0.03166358 |
| 0.03169249 |
| 0.03172259 |
| 0.03175289 |
| 0.0317804 |
| 0.03180538 |
| 0.03182671 |
| 0.03184345 |
| 0.03185615 |
| 0.03186305 |
| 0.03186408 |
| 0.0318594 |
| 0.03184898 |
| 0.03183429 |
| 0.03181688 |
| 0.03179361 |
| 0.03176838 |
| 0.03174107 |
| 0.03171124 |
| 0.03168101 |
| 0.03165249 |
| 0.03162528 |
| 0.03159909 |
| 0.03157592 |
| 0.03155682 |
| 0.03154176 |
| 0.03153094 |
| 0.03152506 |
| 0.03152426 |
| 0.03152856 |
| 0.03153712 |
| 0.03154983 |
| 0.03156769 |
| 0.03158991 |
| 0.03161383 |
| 0.03163971 |
| 0.03166823 |
| 0.03169784 |
| 0.03172804 |

in

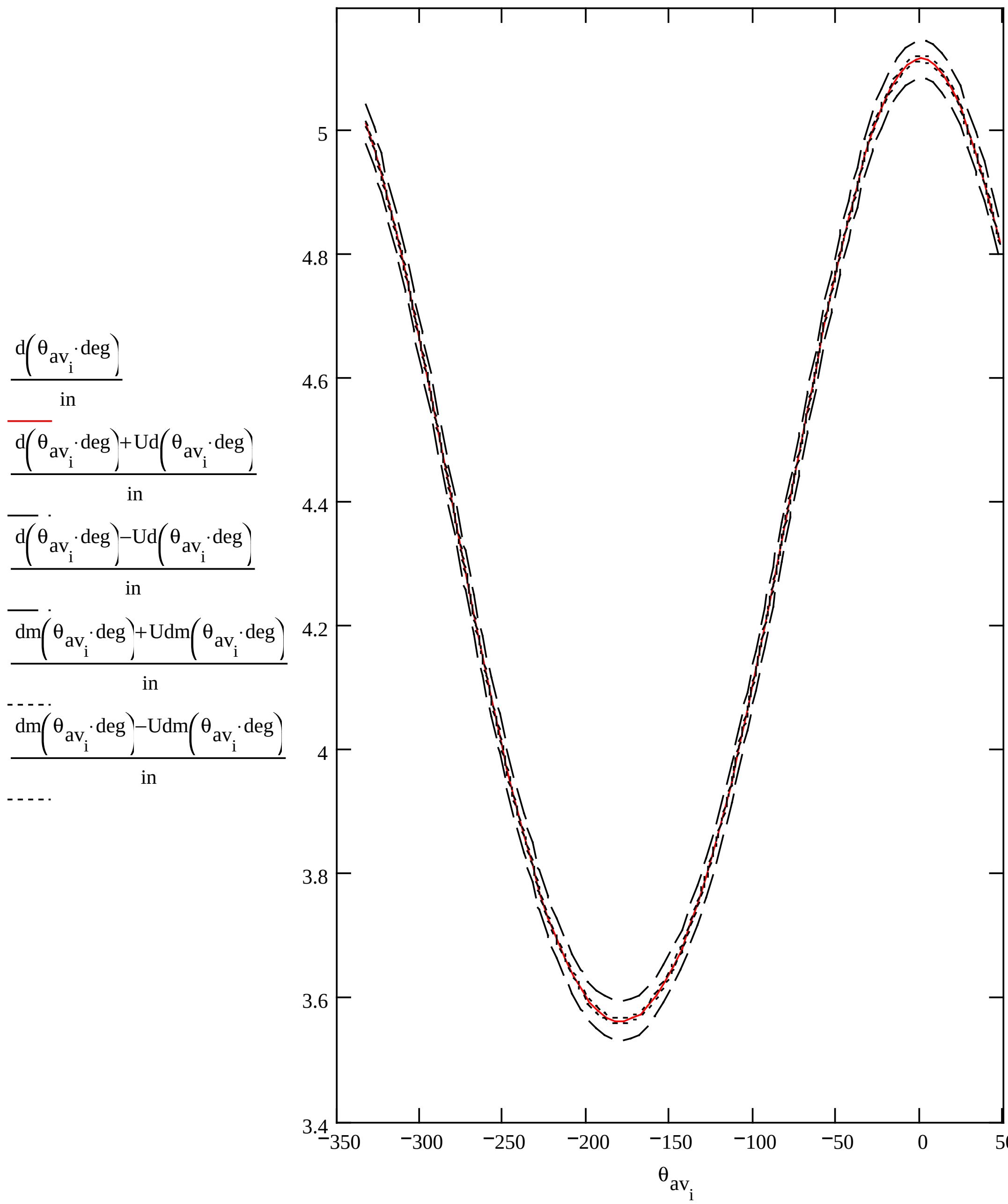


Figure A.5: Model with Manufacturing Effects and Original Model Comparison

# Total Displacement Measurement

Since the total displacement was not measured, an equation was used to calculate the total displacement. For both cases (baseline design & the increased diameter case), the total displacement should be the same and

The **total displacement** : $d_{total} = \frac{(dm(0) + d_{min}) + (dm(\pi) + d_{max})}{2}$

From the **Baseline Design** Case:

**Max from Model Equation:**

$d(0 \cdot deg) = 5.1173 in$

$d_{BaselineModelMax} := 5.1173 \cdot in$

**Max from Exp. Data Points:**

$max(d_{av}) \cdot in = 1.7261 in$

$d_{BaselineExpMax} := 1.7261 \cdot in$

**Min from Model Equation:**

$d(180 \cdot deg) = 3.5627 in$

$d_{BaselineModelMin} := 3.5627 \cdot in$

**Min from Exp. Data Points:**

$min(d_{av}) \cdot in = 0.2802 in$

$d_{BaselineExpMin} := 0.2802 \cdot in$

$$d_{totalBaseline} := \frac{(d_{BaselineModelMax} + d_{BaselineExpMin}) + (d_{BaselineModelMin} + d_{BaselineExpMax})}{2}$$

$d_{totalBaseline} = 5.343 in$

From the **Increased Diameter** Case:

**Max from Model Equation:**

$d_{IncreasedModelMax} := 5.1173 \cdot in$

**Max from Exp. Data Points:**

$d_{IncreasedExpMax} := 1.7065 \cdot in$

**Min from Model Equation:**

$d_{IncreasedModelMin} := 3.5627 \cdot in$

**Min from Exp. Data Points:**

$d_{IncreasedExpMin} := 0.2449 \cdot in$

$$d_{totalIncreased} := \frac{(d_{IncreasedModelMax} + d_{IncreasedExpMin}) + (d_{IncreasedModelMin} + d_{IncreasedExpMax})}{2}$$

$d_{totalIncreased} = 5.316 in$

## Overall Total Displacement

$$d_{total} := \frac{d_{totalBaseline} + d_{totalIncreased}}{2}$$

$d_{total} = 5.329in$

## Uncertainty of the Total Displacement

$$Ud_{total} = \sqrt{\left(\frac{Ud_{totalBaseline}}{2}\right)^2 + \left(\frac{Ud_{totalIncreased}}{2}\right)^2}$$

Let's break down the two components into basic variables

**PS:** Due to the fact that the systematic uncertainty of the experiment data is negligible compared to the random uncertainty of the experiment data points, the systematic uncertainty will not be included in the following calculations.

## Baseline Design Case:

$$Ud_{totalBaseline} = \sqrt{\left(\frac{Ud_{BaselineModelMax}}{2}\right)^2 + \left(\frac{Ud_{BaselineExpMin}}{2}\right)^2 \cdots + \left(\frac{Ud_{BaselineModelMin}}{2}\right)^2 + \left(\frac{Ud_{BaselineExpMax}}{2}\right)^2}$$

**Uncertainty** of the **maximum point** given by the **model equation** prediction [d model (0 deg) ] is

$Udmodel_{max} := Ud(0 \cdot deg)$ $Udmodel_{max} = 0.031524in$

$Ud_{BaselineModelMax} := 0.031524$

**Uncertainty** of the **minimum point** given by the **model equation** prediction [d model (180deg)] is

$Udmodel_{min} := Ud(180 \cdot deg)$ $Udmodel_{min} = 0.031524in$

$Ud_{BaselineModelMin} := 0.031524$

$Ud_{max} := R_{exp_{39}}$  $Ud_{max} = 0.046445$

$Ud_{BaselineExpMax} := 0.046445$

**Uncertainty** associated with the **minimum point** of the **experiment data points** (d min) is

$Ud_{min} := R_{exp_{72}}$  $Ud_{min} = 0.019631$

$Ud_{BaselineExpMin} := 0.019631$

$$Ud_{totalBaseline} := \sqrt{\left(\frac{Ud_{BaselineModelMax}}{2}\right)^2 + \left(\frac{Ud_{BaselineExpMin}}{2}\right)^2 \ldots + \left(\frac{Ud_{BaselineModelMin}}{2}\right)^2 + \left(\frac{Ud_{BaselineExpMax}}{2}\right)^2}$$

$Ud_{totalBaseline} = 0.03365$

## Increased Diameter Case:

$$Ud_{totalIncreased} = \sqrt{\left(\frac{Ud_{IncreasedModelMax}}{2}\right)^2 + \left(\frac{Ud_{IncreasedExpMin}}{2}\right)^2 \ldots + \left(\frac{Ud_{IncreasedModelMin}}{2}\right)^2 + \left(\frac{Ud_{IncreasedExpMax}}{2}\right)^2}$$

**Uncertainty** of the **maximum point** given by the **model equation** prediction [d model (0 deg) ] is

$Ud_{IncreasedModelMax} := 0.058234$

**Uncertainty** of the **minimum point** given by the **model equation** prediction [d model (180deg)] is

$Ud_{IncreasedModelMin} := 0.058234$

**Uncertainty** associated with the **maximum point** of the **experiment data points** (d max) is

$Ud_{IncreasedExpMax} := 0.068193$

**Uncertainty** associated with the **minimum point** of the **experiment data points** (d min) is

$Ud_{IncreasedExpMin} := 0.030899$

$$Ud_{totalIncreased} := \sqrt{\left(\frac{Ud_{IncreasedModelMax}}{2}\right)^2 + \left(\frac{Ud_{IncreasedExpMin}}{2}\right)^2 \cdots + \left(\frac{Ud_{IncreasedModelMin}}{2}\right)^2 + \left(\frac{Ud_{IncreasedExpMax}}{2}\right)^2}$$

$Ud_{totalIncreased} = 0.055649$

## Overall Total Displacement Uncertainty

$$Ud_{total} := \sqrt{\left(\frac{Ud_{totalBaseline}}{2}\right)^2 + \left(\frac{Ud_{totalIncreased}}{2}\right)^2}$$

$Ud_{total} = 0.032517$

With the model as the new frame of reference, the experiment data points will be converted to fit the model frame of reference.

The data reduction equation ( **DRE**) of converting the experiment data points to fit the model frame of reference is

$$dexp_{av} := \frac{d_{total}}{in} - d_{av}$$

$dexp_{av} =$

| | 1 |
|---|---|
| 1 | 4.8675249 |
| 2 | 4.83532169 |
| 3 | 4.88768862 |
| 4 | 4.84108178 |
| 5 | 4.82964144 |
| 6 | 4.84091006 |
| 7 | 4.84835543 |
| 8 | 4.83068542 |
| 9 | 4.80356789 |
| 10 | 4.76487314 |
| 11 | 4.75684885 |
| 12 | 4.74930548 |
| 13 | 4.73290596 |
| 14 | 4.69552844 |
| 15 | 4.66642638 |
| 16 | 4.65951645 |
| 17 | 4.63927817 |
| 18 | 4.61918858 |
| 19 | 4.57977951 |
| 20 | 4.54905525 |
| 21 | 4.52608441 |
| 22 | 4.5336315 |
| 23 | 4.47797717 |
| 24 | 4.40977188 |
| 25 | 4.38066064 |
| 26 | 4.30362218 |
| 27 | 4.24204935 |
| 28 | 4.14644501 |
| 29 | 4.02126874 |

Using the crank angle obtained from the experiment and substituting into the model equation to determine the model displacement with the same crank angle.

Displacement by **model equation** : $d(\theta_{av} \cdot deg)$

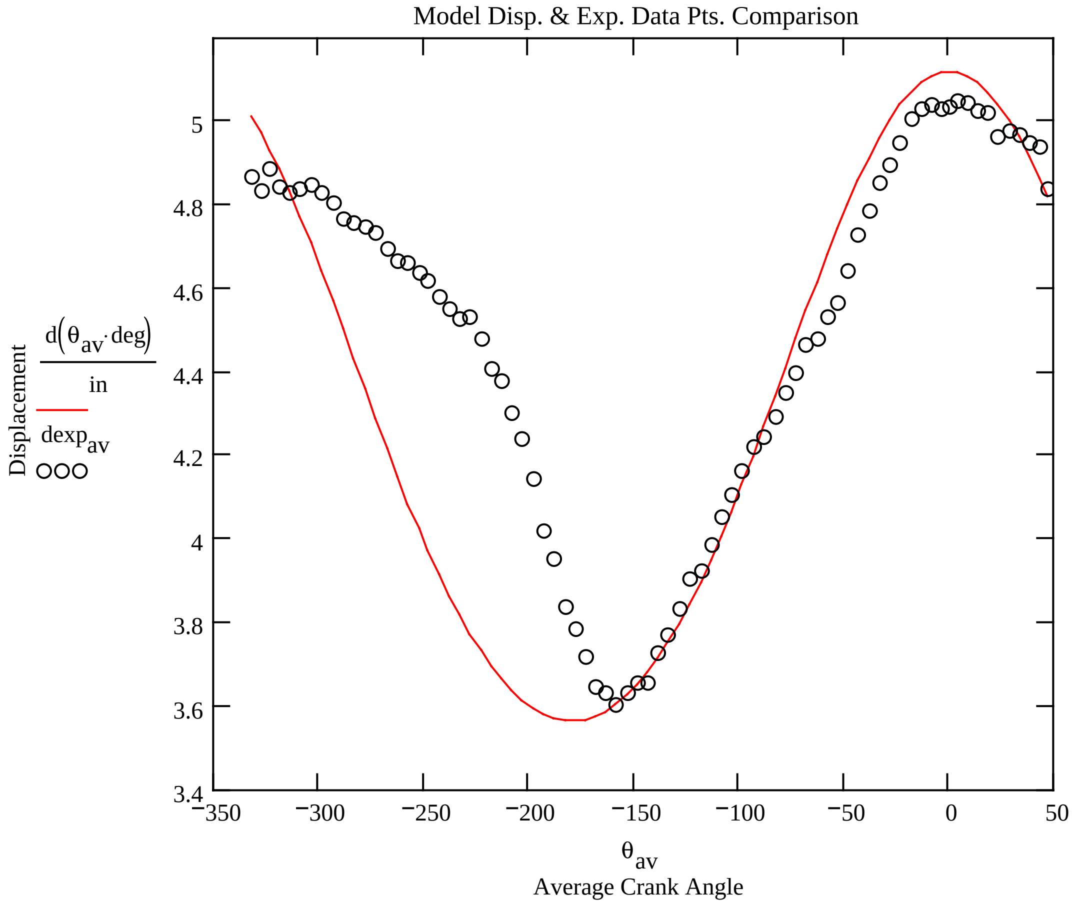


Figure A.6: Model and Experiment Comparison

The components of the systematic uncertainty of the converted experiment data points are

Uncertainty of all the experiment data points $U_{dav}$

$$Udexp_{av} := \sqrt{\left(Ud_{total}\right)^2 + \left(U_{dav}\right)^2}$$

$Udexp_{av} =$

| | 1 |
|---|---|
| 1 | 0.08662849 |
| 2 | 0.11314313 |
| 3 | 0.04339444 |
| 4 | 0.06195678 |
| 5 | 0.0672717 |
| 6 | 0.04995623 |
| 7 | 0.04108256 |
| 8 | 0.04783511 |
| 9 | 0.04888683 |
| 10 | 0.0580063 |
| 11 | 0.04524914 |
| 12 | 0.04475501 |
| 13 | 0.0479724 |
| 14 | 0.0523357 |
| 15 | 0.05640623 |
| 16 | 0.04848157 |
| 17 | 0.05380479 |
| 18 | 0.04401127 |
| 19 | 0.05446976 |
| 20 | 0.05035081 |
| 21 | 0.05785922 |
| 22 | 0.04967643 |
| 23 | 0.05633647 |
| 24 | 0.0614672 |
| 25 | 0.05634954 |
| 26 | 0.06869206 |
| 27 | 0.07365135 |
| 28 | 0.09706354 |
| 29 | 0.10773192 |
| 30 | 0.09770194 |
| 31 | 0.10286851 |
| 32 | 0.07952457 |
| 33 | 0.07842216 |
| 34 | 0.07323982 |
| 35 | 0.05653755 |
| 36 | 0.05523936 |
| 37 | 0.0420836 |
| 38 | 0.0431332 |
| 39 | 0.05669634 |
| 40 | 0.06142111 |
| 41 | 0.06101555 |

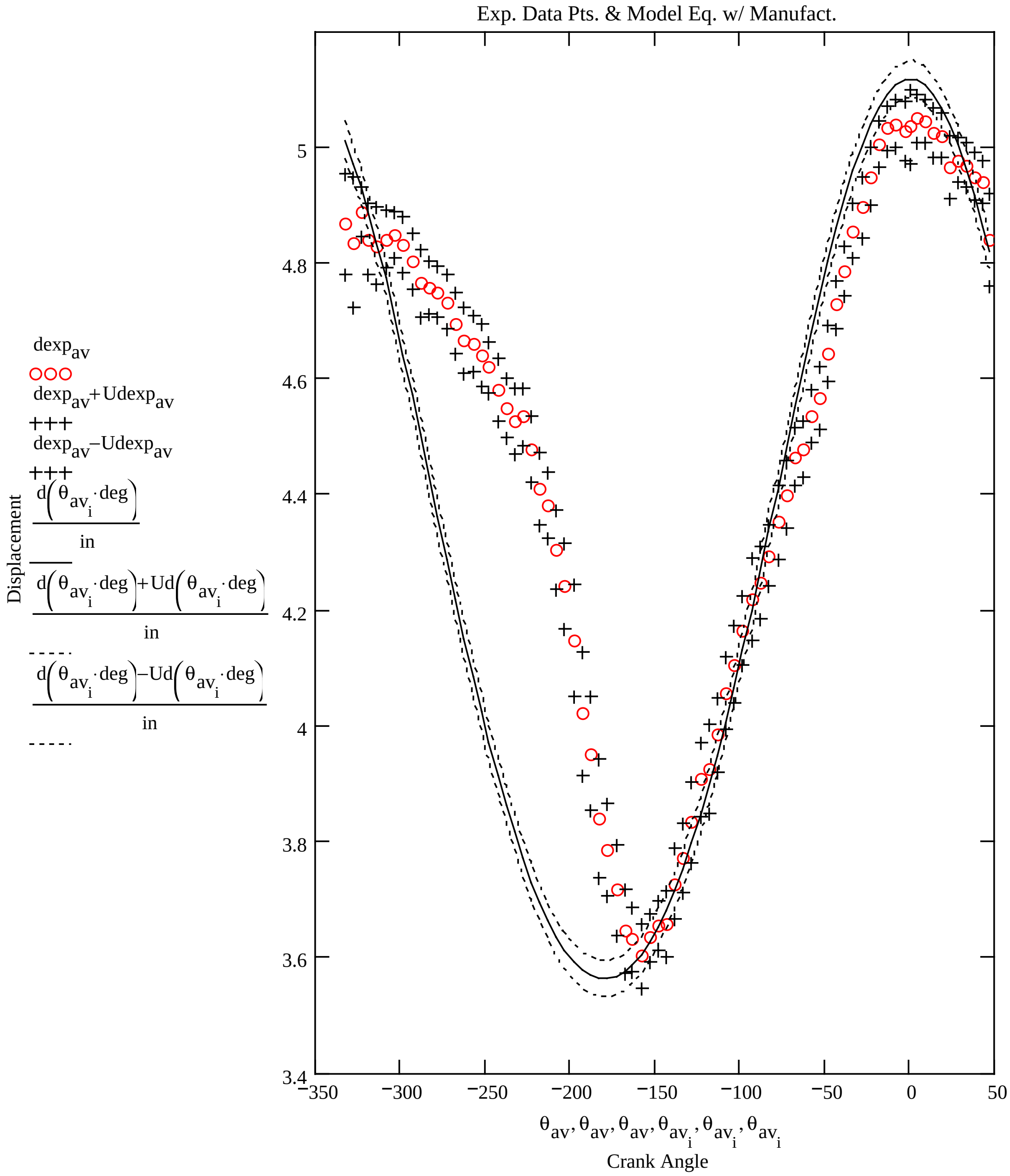


Figure A.7: Model with Manufacture Effects and Experiment Comparison

From the previous model-experiment comparison plot, it showed that the left hand side of the plot (approximately from -350 deg to -160 deg) was not suitable for comparison. The gap between the experiment data points and the model equation was caused by the variations of engine speed during the experimental phase. Therefore, comparison of the experiment data points with the model equation are only valid on the right hand side of the plot will be more valid because the curves were like overlapping each others.

From the minimum displacement to the far right of the plot, the range of angles is

$\theta_{av_{36}} = -158.59$ deg The crank angle associated with the minimum point

$\theta_{av_{75}} = 33.127$ deg The crank angle of the right-most of the plot

The new range of crank angle used for comparisons **-158.59 deg < $\theta$ < 33.13 deg** .

$\theta av_{NEW} :=$ deg

| | 1 |
|---|---|
| 1 | -158.5901 |
| 2 | -153.4121 |
| 3 | -148.5617 |
| 4 | -143.784 |
| 5 | -138.8307 |
| 6 | -133.8462 |
| 7 | -128.7636 |
| 8 | -123.5738 |
| 9 | -118.5063 |
| 10 | -113.6458 |
| 11 | -108.7962 |
| 12 | -103.813 |
| 13 | -98.6232 |
| 14 | -93.4497 |
| 15 | -88.3672 |
| 16 | -83.1954 |
| 17 | -78.0996 |
| 18 | -73.1455 |
| 19 | -68.153 |
| 20 | -63.1994 |
| 21 | -58.3379 |
| 22 | -53.4772 |
| 23 | -48.5106 |
| 24 | -43.5394 |
| 25 | -38.569 |
| 26 | -33.5901 |
| 27 | -28.5076 |
| 28 | -23.425 |

$\alpha := 1 .. \text{rows}\left(\theta av_{NEW}\right)$

The experiment displacement uncertainty with respect to new crank angle range:

$Udexp_{avNEW} :=$ in

| | 1 |
|---|---|
| 1 | 0.055239 |
| 2 | 0.042084 |
| 3 | 0.043133 |
| 4 | 0.056696 |
| 5 | 0.061421 |
| 6 | 0.061016 |
| 7 | 0.070353 |
| 8 | 0.063354 |
| 9 | 0.077353 |
| 10 | 0.065124 |
| 11 | 0.06235 |
| 12 | 0.066511 |
| 13 | 0.059411 |
| 14 | 0.071785 |
| 15 | 0.062232 |
| 16 | 0.051864 |
| 17 | 0.062974 |
| 18 | 0.058256 |
| 19 | 0.04983 |
| 20 | 0.048738 |
| 21 | 0.045874 |
| 22 | 0.053698 |
| 23 | 0.04771 |
| 24 | 0.040583 |
| 25 | 0.043094 |
| 26 | 0.046888 |
| 27 | 0.053287 |
| 28 | 0.049516 |
| 29 | 0.04079 |
| 30 | 0.038776 |
| 31 | 0.04105 |
| 32 | 0.050854 |
| 33 | 0.06508 |
| 34 | 0.041573 |
| 35 | 0.037469 |
| 36 | 0.042827 |
| 37 | 0.037983 |
| 38 | 0.053255 |
| 39 | 0.038463 |
| 40 | 0.038588 |

The theta uncertainty must be included in the total uncertainty of the experiment. The **partial derivative** of the model equation w.r.t. the crank angle is

$$d'_{\theta}(\theta) := -l_{cs}\cdot\sin(\theta) - \frac{1}{\left[\left(\frac{1}{2}\cdot l_1 + \frac{1}{2}\cdot l_2\right)^2 - l_{cs}^{\,2}\cdot\sin(\theta)^2\right]^{\frac{1}{2}}}\cdot l_{cs}^{\,2}\cdot\sin(\theta)\cdot\cos(\theta)$$

$$d'_{\theta}\left(\theta av_{NEW_{\alpha}}\cdot deg\right) =$$

| | 1 |
|---|---|
| 1 | 0.22011299 |
| 2 | 0.27281203 |
| 3 | 0.32176445 |
| 4 | 0.36943836 |
| 5 | 0.41808565 |
| 6 | 0.46598154 |
| 7 | 0.51339519 |
| 8 | 0.55991796 |
| 9 | 0.60304093 |
| 10 | 0.64179482 |
| 11 | 0.67742959 |
| 12 | 0.71035336 |
| 13 | 0.74005157 |
| 14 | 0.76435292 |
| 15 | 0.78245655 |
| 16 | 0.79441864 |
| 17 | 0.79931098 |
| 18 | 0.79708706 |
| 19 | 0.78755747 |
| 20 | 0.77064993 |
| 21 | 0.74673071 |
| 22 | 0.71556406 |
| 23 | 0.67634888 |
| 24 | 0.6298803 |
| 25 | 0.57656475 |
| 26 | 0.51676528 |
| 27 | 0.44972934 |
| 28 | 0.37736339 |
| 29 | 0.30224921 |
| 30 | 0.22553015 |
| 31 | 0.14258704 |
| 32 | 0.05613159 |
| 33 | $301193\cdot 10^{-3}$ |
| 34 | -0.06136488 |
| 35 | -0.14229995 |

in

Using **Eq. 5.87 (Coleman & Steele, 2nd Edition)** to incorporate the **uncertainty in crank angle** to the **uncertainty of the experiment displacement**, to give the **final total uncertainty in the experiment data points**.

$$Udexp_{NEW_{\alpha}} := \sqrt{U_{\theta}^{2} \cdot d'_{\theta}\left(\theta av_{NEW_{\alpha}} \cdot deg\right)^{2} + \left(Udexp_{avNEW_{\alpha}} \cdot in\right)^{2}}$$

$Udexp_{NEW} =$ (in)

| | 1 |
|---|---|
| 1 | 0.05537279 |
| 2 | 0.04235211 |
| 3 | 0.04349725 |
| 4 | 0.05706181 |
| 5 | 0.06185304 |
| 6 | 0.06155519 |
| 7 | 0.0709218 |
| 8 | 0.0641034 |
| 9 | 0.07806563 |
| 10 | 0.06608025 |
| 11 | 0.06346152 |
| 12 | 0.06765673 |
| 13 | 0.06079873 |
| 14 | 0.07301447 |
| 15 | 0.06371297 |
| 16 | 0.05368498 |
| 17 | 0.0645008 |
| 18 | 0.05989387 |
| 19 | 0.05169152 |
| 20 | 0.05056034 |
| 21 | 0.0476897 |
| 22 | 0.05513158 |
| 23 | 0.04914833 |
| 24 | 0.04204519 |
| 25 | 0.04425335 |
| 26 | 0.04774753 |
| 27 | 0.05386173 |
| 28 | 0.04995209 |
| 29 | 0.04113006 |
| 30 | 0.03897517 |
| 31 | 0.04112565 |
| 32 | 0.05086383 |
| 33 | 0.06508021 |
| 34 | 0.04158633 |
| 35 | 0.03755158 |
| 36 | 0.04300117 |
| 37 | 0.03834282 |

The **new converted experiment data points with manufacture effects** that **fit the model frame of reference**

$dexp_{avNEW} :=$ in

| | 1 |
|---|---|
| 1 | 3.60333 |
| 2 | 3.63371 |
| 3 | 3.65417 |
| 4 | 3.65744 |
| 5 | 3.72724 |
| 6 | 3.77172 |
| 7 | 3.83339 |
| 8 | 3.90737 |
| 9 | 3.92515 |
| 10 | 3.98413 |
| 11 | 4.05546 |
| 12 | 4.10544 |
| 13 | 4.16472 |
| 14 | 4.21865 |
| 15 | 4.24695 |
| 16 | 4.29427 |
| 17 | 4.35133 |
| 18 | 4.39958 |
| 19 | 4.46455 |
| 20 | 4.47809 |
| 21 | 4.5341 |
| 22 | 4.56641 |
| 23 | 4.64291 |
| 24 | 4.72784 |
| 25 | 4.78642 |
| 26 | 4.85499 |
| 27 | 4.8959 |
| 28 | 4.94857 |
| 29 | 5.00475 |
| 30 | 5.03216 |
| 31 | 5.04034 |
| 32 | 5.02795 |
| 33 | 5.03482 |
| 34 | 5.04919 |
| 35 | 5.04483 |
| 36 | 5.02434 |
| 37 | 5.02049 |
| 38 | 4.96459 |
| 39 | 4.97651 |

The **new converted experiment data points with manufacture effects and the total combined converted experimental uncertainty**

$$\left(dexp_{avNEW} \cdot in\right) + Udexp_{NEW} =$$

| | 1 |
|---|---|
| 1 | 3.65870279 |
| 2 | 3.67606211 |
| 3 | 3.69766725 |
| 4 | 3.71450181 |
| 5 | 3.78909304 |
| 6 | 3.83327519 |
| 7 | 3.9043118 |
| 8 | 3.9714734 |
| 9 | 4.00321563 |
| 10 | 4.05021025 |
| 11 | 4.11892152 |
| 12 | 4.17309673 |
| 13 | 4.22551873 |
| 14 | 4.29166447 |
| 15 | 4.31066297 |
| 16 | 4.34795498 |
| 17 | 4.4158308 |
| 18 | 4.45947387 |
| 19 | 4.51624152 |
| 20 | 4.52865034 |
| 21 | 4.5817897 |
| 22 | 4.62154158 |
| 23 | 4.69205833 |
| 24 | 4.76988519 |
| 25 | 4.83067335 |
| 26 | 4.90273753 |
| 27 | 4.94976173 |
| 28 | 4.99852209 |
| 29 | 5.04588006 |
| 30 | 5.07113517 |
| 31 | 5.08146565 |
| 32 | 5.07881383 |
| 33 | 5.09990021 |
| 34 | 5.09077633 |
| 35 | 5.08238158 |
| 36 | 5.06734117 |
| 37 | 5.05883282 |
| 38 | 5.01824646 |
| 39 | 5.01575408 |
| 40 | 5.00853501 |

in

$$\left(\text{dexp}_{\text{avNEW}} \cdot \text{in}\right) - \text{Udexp}_{\text{NEW}} =$$

| | 1 |
|---|---|
| 1 | 3.54795721 |
| 2 | 3.59135789 |
| 3 | 3.61067275 |
| 4 | 3.60037819 |
| 5 | 3.66538696 |
| 6 | 3.71016481 |
| 7 | 3.7624682 |
| 8 | 3.8432666 |
| 9 | 3.84708437 |
| 10 | 3.91804975 |
| 11 | 3.99199848 |
| 12 | 4.03778327 |
| 13 | 4.10392127 |
| 14 | 4.14563553 |
| 15 | 4.18323703 |
| 16 | 4.24058502 |
| 17 | 4.2868292 |
| 18 | 4.33968613 |
| 19 | 4.41285848 |
| 20 | 4.42752966 |
| 21 | 4.4864103 |
| 22 | 4.51127842 |
| 23 | 4.59376167 |
| 24 | 4.68579481 |
| 25 | 4.74216665 |
| 26 | 4.80724247 |
| 27 | 4.84203827 |
| 28 | 4.89861791 |
| 29 | 4.96361994 |
| 30 | 4.99318483 |
| 31 | 4.99921435 |
| 32 | 4.97708617 |
| 33 | 4.96973979 |
| 34 | 5.00760367 |
| 35 | 5.00727842 |
| 36 | 4.98133883 |
| 37 | 4.98214718 |
| 38 | 4.91093354 |
| 39 | 4.93726592 |
| 40 | 4.92932499 |

in

The **new model equation with manufacture effects** data

$d(\theta av_{NEW} \cdot deg) =$

| | 1 |
|---|---|
| 1 | 3.60388875 |
| 2 | 3.6261652 |
| 3 | 3.65133517 |
| 4 | 3.68015857 |
| 5 | 3.71420591 |
| 6 | 3.75267043 |
| 7 | 3.79612105 |
| 8 | 3.84474709 |
| 9 | 3.89619439 |
| 10 | 3.9490151 |
| 11 | 4.00486955 |
| 12 | 4.06524817 |
| 13 | 4.13097402 |
| 14 | 4.19893586 |
| 15 | 4.26758678 |
| 16 | 4.33880583 |
| 17 | 4.40973024 |
| 18 | 4.4787976 |
| 19 | 4.54789184 |
| 20 | 4.61530389 |
| 21 | 4.6797302 |
| 22 | 4.74180795 |
| 23 | 4.80218889 |
| 24 | 4.85890707 |
| 25 | 4.91128432 |
| 26 | 4.9588324 |
| 27 | 5.00174225 |
| 28 | 5.03846337 |
| 29 | 5.06796034 |
| 30 | 5.09033052 |
| 31 | 5.10666205 |
| 32 | 5.11566333 |
| 33 | 5.11729152 |
| 34 | 5.11534341 |
| 35 | 5.10670521 |
| 36 | 5.09130077 |
| 37 | 5.06866737 |
| 38 | 5.03925725 |
| 39 | 5.00365513 |
| 40 | 4.96298773 |

in

$Ud(\theta av_{NEW_{\alpha}} \cdot deg) =$

| |
|---|
| 0.03156769 |
| 0.03158991 |
| 0.03161383 |
| 0.03163971 |
| 0.03166823 |
| 0.03169784 |
| 0.03172804 |
| 0.03175789 |
| 0.03178509 |
| 0.03180852 |
| 0.03182854 |
| 0.03184496 |
| 0.03185697 |
| 0.03186335 |
| 0.03186394 |
| 0.03185876 |
| 0.03184815 |
| 0.03183296 |
| 0.03181334 |
| 0.03179026 |
| 0.03176485 |
| 0.0317375 |
| 0.03170841 |
| 0.03167902 |
| 0.03165025 |
| 0.03162286 |
| 0.03159719 |
| 0.03157454 |
| 0.03155591 |
| 0.03154154 |
| 0.03153093 |
| 0.03152503 |
| 0.03152396 |
| 0.03152524 |
| 0.0315309 |
| 0.03154092 |
| 0.03155546 |
| 0.03157404 |
| 0.03159602 |
| 0.03162042 |

ir

The **model equation with manufacture effects data and the total combined converted experimental uncertainty**

$$d\left(\theta av_{NEW_{\alpha}} \cdot deg\right) + Ud\left(\theta av_{NEW_{\alpha}} \cdot deg\right) = \text{ in}$$

| |
|---|
| 3.63545643 |
| 3.65775511 |
| 3.68294901 |
| 3.71179828 |
| 3.74587414 |
| 3.78436826 |
| 3.82784909 |
| 3.87650498 |
| 3.92797948 |
| 3.98082362 |
| 4.03669809 |
| 4.09709312 |
| 4.16283099 |
| 4.23079921 |
| 4.29945073 |
| 4.37066459 |
| 4.44157838 |
| 4.51063056 |
| 4.57970518 |
| 4.64709415 |
| 4.71149505 |
| 4.77354545 |
| 4.83389729 |
| 4.89058609 |
| 4.94293457 |
| 4.99045526 |
| 5.03333944 |
| 5.0700379 |
| 5.09951625 |
| 5.12187206 |
| 5.13819298 |
| 5.14718836 |
| 5.14881548 |
| 5.14686866 |
| 5.13823611 |
| 5.12284168 |
| 5.10022283 |
| 5.07083129 |
| 5.03525115 |
| 4.99460815 |

$$d\left(\theta av_{NEW_{\alpha}} \cdot deg\right) - Ud\left(\theta av_{NEW_{\alpha}} \cdot deg\right) =$$

| |
|---|
| 3.57232106 |
| 3.59457529 |
| 3.61972134 |
| 3.64851886 |
| 3.68253768 |
| 3.72097259 |
| 3.76439301 |
| 3.8129892 |
| 3.8644093 |
| 3.91720659 |
| 3.97304101 |
| 4.03340321 |
| 4.09911705 |
| 4.16707251 |
| 4.23572284 |
| 4.30694707 |
| 4.37788209 |
| 4.44696464 |
| 4.51607849 |
| 4.58351362 |
| 4.64796536 |
| 4.71007046 |
| 4.77048048 |
| 4.82722805 |
| 4.87963407 |
| 4.92720954 |
| 4.97014506 |
| 5.00688883 |
| 5.03640442 |
| 5.05878897 |
| 5.07513112 |
| 5.08413829 |
| 5.08576756 |
| 5.08381817 |
| 5.07517431 |
| 5.05975985 |
| 5.03711191 |
| 5.00768321 |
| 4.97205911 |
| 4.93136731 |

in

**Solid line** represents the **model displacement** , **dotted line** represents the **uncertainty bands of the model displacement** , **circle s** represent the **experiment data points** and the **dashed line** represents the **uncertainty bands of the experiment data points** .

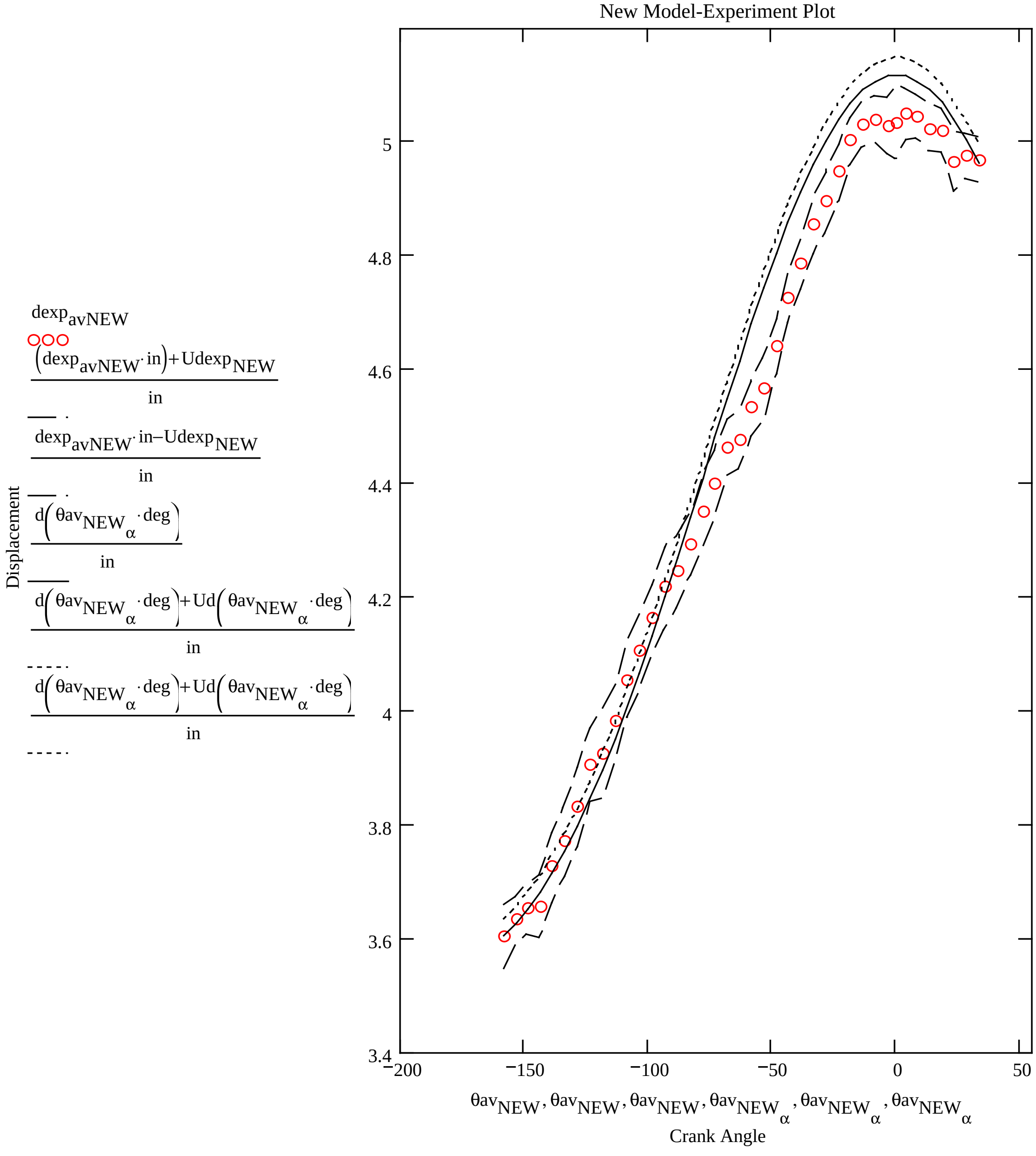


Figure A.8: Maximum and Minimum Dispalcement

## Validation

Result predicted by the Model equation $d_{NEW}$

Result obtained through Experimentation $dexp_{avNEW}$

From Coleman & Steele, 2nd Edition

Equation 5.82

Comparison error, E = Model - Experiment Data Points

$$E := d\left(\theta av_{NEW} \cdot deg\right) - dexp_{avNEW} \cdot in$$

E = (in)

| | 1 |
|---|---|
| 1 | $5.5874612 \cdot 10^{-4}$ |
| 2 | $-7.54479982 \cdot 10^{-3}$ |
| 3 | $-2.83482723 \cdot 10^{-3}$ |
| 4 | 0.02271857 |
| 5 | -0.01303409 |
| 6 | -0.01904957 |
| 7 | -0.03726895 |
| 8 | -0.06262291 |
| 9 | -0.02895561 |
| 10 | -0.0351149 |
| 11 | -0.05059045 |
| 12 | -0.04019183 |
| 13 | -0.03374598 |
| 14 | -0.01971414 |
| 15 | 0.02063678 |
| 16 | 0.04453583 |
| 17 | 0.05840024 |
| 18 | 0.0792176 |
| 19 | 0.08334184 |
| 20 | 0.13721389 |
| 21 | 0.1456302 |
| 22 | 0.17539795 |
| 23 | 0.15927889 |
| 24 | 0.13106707 |
| 25 | 0.12486432 |
| 26 | 0.1038424 |
| 27 | 0.10584225 |
| 28 | 0.08989337 |
| 29 | 0.06321034 |
| 30 | 0.05817052 |
| 31 | 0.06632205 |

The uncertainty of the comparison error $U_{E_\alpha} := \sqrt{\left(Udexp_{NEW_\alpha}\right)^2 + Ud\left(\theta av_{NEW_\alpha} \cdot deg\right)^2}$

$U_E =$ (in)

| | 1 |
|---|---|
| 1 | 0.06373903 |
| 2 | 0.05283582 |
| 3 | 0.05377216 |
| 4 | 0.06524662 |
| 5 | 0.06948867 |
| 6 | 0.06923723 |
| 7 | 0.07769537 |
| 8 | 0.07153887 |
| 9 | 0.0842884 |
| 10 | 0.07333745 |
| 11 | 0.07099592 |
| 12 | 0.07477656 |
| 13 | 0.06863929 |
| 14 | 0.07966421 |
| 15 | 0.0712366 |
| 16 | 0.06242642 |
| 17 | 0.07193509 |
| 18 | 0.06782782 |
| 19 | 0.06069681 |
| 20 | 0.05972411 |
| 21 | 0.0573002 |
| 22 | 0.06361415 |
| 23 | 0.05848916 |
| 24 | 0.05264369 |
| 25 | 0.05440678 |
| 26 | 0.05726982 |
| 27 | 0.06244573 |
| 28 | 0.05909453 |
| 29 | 0.05184069 |
| 30 | 0.05013913 |
| 31 | 0.05182198 |
| 32 | 0.05984109 |
| 33 | 0.07231316 |
| 34 | 0.0521849 |
| 35 | 0.04903385 |
| 36 | 0.05332851 |
| 37 | 0.04965802 |
| 38 | 0.06225701 |
| 39 | 0.0503826 |
| 40 | 0.05067946 |

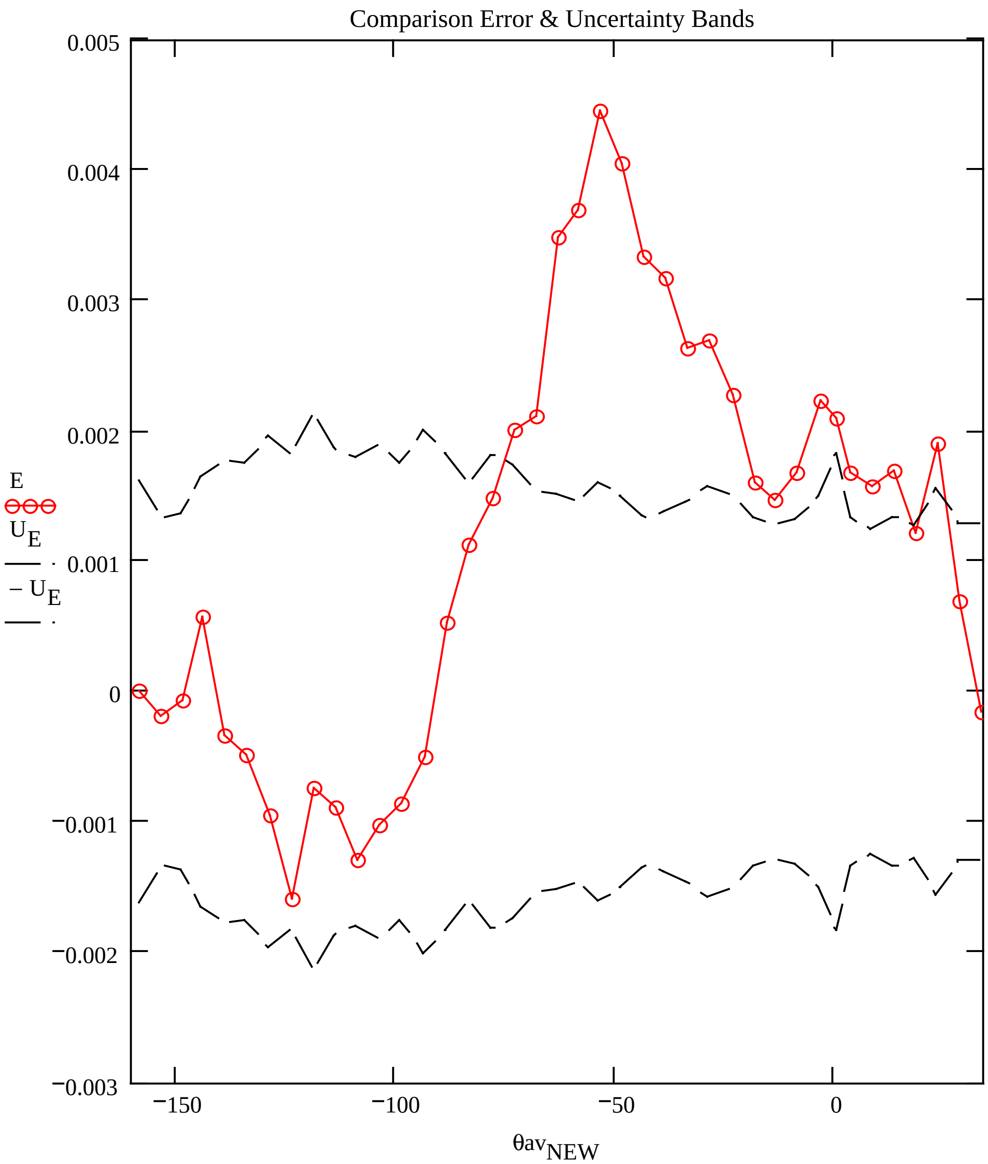


Figure A.9: Validation Plot

At certain range, there are experiment data points that exceed the comparison uncertainty bands, and it is believe to be caused by the variation of the engine speed, $\omega$. With all the information obtained there, I can't characterize the crank angle equation. Therefore, there must be some extra information needed to refine this crank angle equation.

## Uncertainties Percentage Contributions (UPC)

From the **model equation** ;

$$d(\theta) := l_{cs}\cdot\cos(\theta) + \sqrt{\left(\frac{l_1 + l_2}{2}\right)^2 - l_{cs}^{\,2}\cdot(\sin(\theta))^2} + l_p + s_x$$

The partial derivatives of the model equation w.r.t. each variable are:

$$pl_{cs}(\theta) := \cos(\theta) + \frac{-l_{cs}\cdot(\sin(\theta))^2}{\sqrt{\left(\frac{l_1 + l_2}{2}\right)^2 - l_{cs}^{\,2}\cdot(\sin(\theta))^2}}$$

w.r.t. **crankshaft length**

$$pl_1(\theta) := \frac{1}{2}\cdot\frac{\frac{l_1+l_2}{2}}{\sqrt{\left(\frac{l_1 + l_2}{2}\right)^2 - l_{cs}^{\,2}\cdot(\sin(\theta))^2}}$$

w.r.t. **L1 Length**

$$pl_2(\theta) := \frac{1}{2}\cdot\frac{\frac{l_1+l_2}{2}}{\sqrt{\left(\frac{l_1 + l_2}{2}\right)^2 - l_{cs}^{\,2}\cdot(\sin(\theta))^2}}$$

w.r.t. **L2 Length**

$$pl_p := 1$$

w.r.t. **Piston Length**

$$ps_x := 1$$

w.r.t. **Slop**

$$d'_\theta(\theta) := -l_{cs}\cdot\sin(\theta) - \frac{1}{\left[\left(\frac{1}{2}\cdot l_1 + \frac{1}{2}\cdot l_2\right)^2 - l_{cs}^{\,2}\cdot\sin(\theta)^2\right]^{\frac{1}{2}}}\cdot l_{cs}^{\,2}\cdot\sin(\theta)\cdot\cos(\theta)$$

w.r.t. **Crank Angle**

At the **maximum uncertainty of E**, (when the crank angle uncertainty is assumed to be 10 deg)

$U_{E_{17}} = 0.072\text{in}$

The **corresponding crank angle** is $\theta av_{NEW_{17}} = -78.1 \quad \text{deg}$

UPC w.r.t. **total piston displacement measurement**

$$UPC_{dtotal} := \frac{\left(Ud_{total} \cdot in\right)^2}{\left(U_{E_{17}}\right)^2} \qquad UPC_{dtotal} = 20.433\%$$

**COMPARISON**

UPC w.r.t. **total experiment uncertainty (randomness of the experiment data points)**

$$UPC_{dav} := \frac{\left(U_{dav_{52}} \cdot in\right)^2}{\left(U_{E_{17}}\right)^2} \qquad UPC_{dav} = 56.205\%$$

**EXPERIMENT**

UPC w.r.t. **crank angle uncertainty (a given value)**

$$UPC_{\theta} := \frac{U_{\theta}^{2} \cdot d'_{\theta}\left(\theta av_{NEW_{17}} \cdot deg\right)^2}{\left(U_{E_{17}}\right)^2} \qquad UPC_{\theta} = 3.761\%$$

**EXPERIMENT**

UPC w.r.t. **random uncertainty of the slop term**

$$UPC_{rsx} := \frac{ps_x^{\,2} \cdot rs_x^{\,2}}{\left(U_{E_{17}}\right)^2} \qquad UPC_{rsx} = 0.085\%$$

**MEASUREMENTS**

UPC w.r.t. **systematic uncertainty of the slop term (consist of tolerance of the connecting rod diameter & the systematic uncertainty of the crankshaft from measurement)**

$$UPC_{ssx} := \frac{ps_x{}^2 \cdot ss_x{}^2}{\left(U_{E_{17}}\right)^2} \qquad UPC_{ssx} = 12.087\%$$

**TOLERANCE**

$$UPC_{udcr} := \frac{ps_x{}^2 \cdot \left(\frac{1}{4} \cdot ud_{cr}{}^2\right)}{\left(U_{E_{17}}\right)^2} \qquad UPC_{udcr} = 12.078\%$$

**MEASUREMENTS**

$$UPC_{udcs} := \frac{ps_x{}^2 \cdot \left(\frac{1}{4} \cdot ud_{cs}{}^2\right)}{\left(U_{E_{17}}\right)^2} \qquad UPC_{udcs} = 8.852 \times 10^{-3}\%$$

UPC w.r.t. **combined systematic uncertainty of the crankshaft length from measurement**

$$UPC_{Lcs} := \frac{pl_{cs}\left(\theta av_{NEW_{17}} \cdot deg\right)^2 \cdot sl_{cs}{}^2}{\left(U_{E_{17}}\right)^2} \qquad UPC_{Lcs} = 1.071 \times 10^{-5}\%$$

**MEASUREMENTS**

UPC w.r.t. **combined systematic uncertainty of the piston length from measurement**

$$UPC_{Lp} := \frac{pl_p{}^2 \cdot sl_p{}^2}{\left(U_{E_{17}}\right)^2} \qquad UPC_{Lp} = 0.014\%$$

**MEASUREMENTS**

UPC w.r.t. **the inner and outer lengths of the connecting rod (consist of tolerance of the connecting rod length, tolerance of the connecting rod diameter, both systematic and random uncertainties of the piston diameter)**

$$sl_1 = \sqrt{tl_{cr}^2 + \left(\frac{1}{4}\right)\cdot td_{cr}^2 + \left(\frac{1}{4}\right)\cdot Ud_p^2} \qquad sl_2 = \sqrt{tl_{cr}^2 + \left(\frac{1}{4}\right)\cdot td_{cr}^2 + \left(\frac{1}{4}\right)\cdot Ud_p^2}$$

$$UPC_{L1} := \frac{pl_1\left(\theta av_{NEW_{17}}\cdot deg\right)^2\cdot sl_1^2}{\left(U_{E_{17}}\right)^2} \qquad UPC_{L1} = 3.708\% \qquad UPC_{L2} := \frac{pl_2\left(\theta av_{NEW_{17}}\cdot deg\right)^2\cdot sl_2^2}{\left(U_{E_{17}}\right)^2} \qquad UPC_{L2} = 3.708\%$$

**TOLERANCE**

$$UPC_{tlcr} := \frac{pl_1\left(\theta av_{NEW_{17}}\cdot deg\right)^2\cdot tl_{cr}^2}{\left(U_{E_{17}}\right)^2} + \frac{pl_2\left(\theta av_{NEW_{17}}\cdot deg\right)^2\cdot tl_{cr}^2}{\left(U_{E_{17}}\right)^2}$$

$UPC_{tlcr} = 1.023\%$

$$UPC_{tdcr} := \frac{pl_1\left(\theta av_{NEW_{17}}\cdot deg\right)^2\cdot\left[\left(\frac{1}{4}\right)\cdot td_{cr}^2\right]}{\left(U_{E_{17}}\right)^2} + \frac{pl_2\left(\theta av_{NEW_{17}}\cdot deg\right)^2\cdot\left[\left(\frac{1}{4}\right)\cdot td_{cr}^2\right]}{\left(U_{E_{17}}\right)^2}$$

$UPC_{tdcr} = 6.391\%$

**MEASUREMENTS**

$$UPC_{udp} := \frac{pl_1\left(\theta av_{NEW_{17}}\cdot deg\right)^2\cdot\left[\left(\frac{1}{4}\right)\cdot Ud_p^2\right]}{\left(U_{E_{17}}\right)^2} + \frac{pl_2\left(\theta av_{NEW_{17}}\cdot deg\right)^2\cdot\left[\left(\frac{1}{4}\right)\cdot Ud_p^2\right]}{\left(U_{E_{17}}\right)^2}$$

$UPC_{udp} = 1.423\times 10^{-3}\%$

$$UPC_{rsx} + UPC_{udcr} + UPC_{udcs} + UPC_{Lcs} \cdots + UPC_{tlcr} + UPC_{tdcr} + UPC_{udp} + UPC_{Lp} + UPC_{dtotal} + UPC_{dav} + UPC_{\theta} = 100\%$$

Total % due to the **tolerance of the connecting rod diameter** is

$UPC_{udcr} + UPC_{tdcr} = 18.469\%$