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Modified Network Simplex Method to Solve a Sheltering Network Planning and Management Problem

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MODIFIED NETWORK SIMPLEX METHOD TO SOLVE A SHELTERING NETWORK PLANNING AND MANAGEMENT PROBLEM

By

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NETWORK PLANNING AND MANAGEMENT PROBLEM

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This dissertation considers sheltering network planning and operations for natural disaster preparedness and responses with a two-stage stochastic program. The first phase of the network design decides the locations, capacities and held resources of new permanent shelters. Both fixed costs for building a new permanent shelter and variable costs based on capacity are considered. Under each disaster scenario featured by the evacuee demand and transportation network condition, the flows of evacuees and resources to shelters, including permanent and temporary ones, are determined in the second stage to minimize the transportation and shortage/surplus costs. Typically, a large number of scenarios are involved in the problem and cause a huge computational burden. The L-shaped algorithm is applied to decompose the problem into the scenario level with each sub-problem as a linear program. The Sheltering Network Planning and Operation Problem considered in this dissertation also has a special structure in the second-stage sub-problem that is a minimum cost network flow problem with equal flow side constraints. Therefore, the dissertation also takes advantages of the network simplex method to solve the response part of the problem in order to solve the problem more efficiently. This dissertation investigates the extending application of special minimum cost equal flow problem. A case study for preparedness and response to hurricanes in the Gulf Coast region of the United States is conducted to demonstrate the usage of the model including how to define scenarios and cost structures. The numerical experiment results also verify the fast convergence of the L-shaped algorithm for the model.

DEDICATION

I would like to dedicate this research to my parents, Jun Li and Lijun Ren; my grandparents, Xisheng Li, Suzhen Hu, Zhong Ren, and Sumei Ni.

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CHAPTER I

INTRODUCTION

There are four milestones in this research work. Firstly, a two-stage stochastic model is developed to minimize the total evacuation cost. Secondly, in order to tackle large evacuation problems, L-shaped method is introduced to solve the Sheltering Network Planning and Operation Problem (SNPOP). Thirdly, the second-stage sub-problem of the SNPOP is a minimum cost network flow problem with side constraints of equal flows. Therefore, a modified network simplex method is implemented to solve SNPOP more efficiently. Lastly, the problem with two echelons is further studied, and we demonstrate that the structure of the minimum cost network flow problem with side constraints of equal flows could be extended into many other domains. Therefore, in this chapter, introductions related to these four milestones will be stated.

Introduction to sheltering network

In May 2009, the U.S. Department of Homeland Security (DHS) announced a new national shelter system to help victims of natural disasters, especially evacuees of hurricanes (Gibson, 2009). The system would have a database of thousands of places for evacuees to go in an emergency. Currently, the Red Cross National Shelter System (NSS) keeps information regarding over 54,000 potential sheltering facilities, and the information can be accessed by the Federal Emergency Management Agency (FEMA) (American Red Cross, 2009). However, most shelters in NSS are not specialized for evacuation. They have other functions during regular hours, such as churches, convention centers, stadiums, schools, etc. FEMA provides funding to upgrade some potential shelters to meet FEMA standards to provide quality mass sheltering. For example, FEMA's Hazard Mitigation Grant Program (HMGP) provided funding to have a community shelter built to meet FEMA standards at D'Iberville High School in Biloxi, Mississippi to protect approximately 3,000 occupants during a possible disaster (FEMA, 2009). In this dissertation, shelters that have already met FEMA standards are defined as Existing Permanent Shelters. All other shelters in NSS are defined as Temporary Shelters. New Permanent Shelters could be built from scratch or through an updating of current Temporary Shelters. The candidate sites of new Permanent Shelters are defined as Potential Permanent Shelter locations. FEMA has to decide how many new Permanent Shelters should be built during preparedness. Usually, this decision process is complicated mainly because of two reasons: the stochastic manner of disasters and the tradeoff between evacuation requirements and evacuation budgets. Once a natural disaster is imminent, FEMA and the Red Cross need to decide which shelters are open and provide the relevant information to the public. Obviously, those shelters meeting FEMA standards (Permanent Shelters) have higher priority and are more appealing to evacuees.

In this dissertation, a two-stage stochastic programming model is established in order to address both the sheltering network planning issue in the preparedness stage and the sheltering network management problem in the response stage. The first stage is also called the preparedness stage in terms of evacuation management or the master problem in terms of problem solving. Similarly, the second stage is also called the response stage or the recourse problem. The first stage decides locations, capacities, and holding resources of new Permanent Shelters. The solution can help FEMA decide how to allocate funding to build new Permanent Shelters. The first stage also decides how much of each resource should be held in each new Permanent Shelter during long-term preparedness. The second stage allocates evacuees to shelters and transports resources to shelters. Permanent Shelters are given higher priority in this process and are assigned with a lower evacuation cost per evacuee compared to Temporary Shelters.

The advantages of L-shaped method

SNPOP is a mixed integer problem that can be solved using ordinary algorithms like Branch and Bound Algorithm. However, these Mixed Integer algorithms are only efficient when the problem-size is restricted to a certain scope. Unfortunately, for most evacuation problems caused by serious disasters, they usually involve thousands of evacuees, hundreds of shelters, a large amount of evacuation activities and different kinds of evacuation resources, which cause a large calculation burden. Therefore, we have to consider another method which can solve the SNPOP more efficiently. By observing the structure of SNPOP, we find that the SNPOP consists of two stages, preparedness and response. In the preparedness stage, we decide how many new FEMA shelters should be established and their corresponding capacity and number of inventories in each new FEMA shelter. Then, after evacuees suffer a disaster, we try to transfer evacuees from disaster affected areas to shelters by satisfying capacity constraints, demand constraints, supply constraints, and resource balance constraints so that the total evacuation cost is minimized. It is obvious that data generated from the variables of the first stage will be applied in the second stage problem as parameters, in which the objective function of both problems, first stage problem and recourse problem, will be affected. Therefore, L-shaped method is applied to minimize the total objective value by adding cuts to the first stage problem to make the upper bound and the lower bound of the problem converge together. In addition, the problem itself has stochastic behavior, which means that we cannot predict the future precisely. For instance, in a case of a hurricane, we cannot forecast the exact hurricane landing point and the category of the hurricane. Another example: so far, we still do not have ability to accurately predict the center point of an earthquake and the category of the earthquake. If we want to predict the future, the most effective approach is to use the statistic method which gives us only probabilities. In other words, by using statistic method, we can know how many possible disaster scenarios and their probabilities, and the sum of all scenario probabilities is equal to one. L-shaped method is powerful for solving such scenario-based problems, because we can treat each scenario as a sub-problem in the second stage and solve each sub-problem individually, which we divide a large-size problem into many relatively small sub-problems. Therefore, we know that L-shaped method is more efficient to solve such scenario-based problems, and we will demonstrate the efficiency of the L-shaped method in the next chapter.

Minimum cost network flow problem with equal flows

The second stage sub-problem of the Sheltering Network Planning and Operation Problem is actually a minimum cost network flow problem to which equal flow side constraints are added. Therefore, we can take advantage of network structure and the existing network algorithm to solve the response part of Sheltering Network Planning and Operation Problem more efficiently. However, one challenge is that traditional network algorithms like Network Simplex Algorithm cannot be directly applied to the minimum cost network flow structure with equal flow side constraints for a reason that will be explained in Chapter 3. Therefore, modifications are needed to make the Network Simplex Method adapt to the Sheltering Network Planning and Operation Problem. In Chapter 3, we will introduce how to obtain an appropriate model structure to apply the modified Network Simplex Method. We will also describe how to obtain an initial feasible solution, perform pivoting and value update procedure. Finally, numerical experiment is tested to demonstrate the efficiency of the modified network simplex algorithm.

CHAPTER II

SHELTERING NETWORK PLANNING AND MANAGEMENT

A literature review of relevant models and algorithms for evacuation is given in Section 2. The problem statement and the two-stage stochastic programming model for the Sheltering Network Planning and Operation Problem (SNPOP) are given in Section 3. Section 4 discusses the L-shaped algorithm for attacking the computational complexity. A case study is conducted in Section 5 for hurricane preparedness and response in the Gulf Coast region in the United States. Section 6 concludes the chapter with discussions and provides future research directions.

Literature review

Numerous mathematical programming, queuing, and simulation models for studying evacuation have been presented in the literature. Yamada (1996) studied a city emergency evacuation planning problem with two network flow models. The first model, which sought the shortest paths on an undirected graph, assigned each evacuee to a corresponding shelter. Then, the shortest path network was transformed to a minimum cost flow problem by adding capacities in each shelter. Choi et al. (1988) proposed a network flow model for an evacuation problem considering arc capacity constraints. Liu et al. (2006) presented a two-level integrated optimization system for optimal evacuation plans. The high-level optimization maximized the throughput during a given evacuation duration. The low-level optimization minimized the total time of the whole operation, including transportation time and waiting time. Multiple objectives, such as total traveling time and total overload at safe areas, may be involved in transferring evacuees to safe areas (Saadatseresht et al., 2009). Li et al. (2008) proposed a two-stage stochastic evacuation model in the perspective of traffic allocation planning to identify evacuation routes based on total travel times, environmental influences, and economic factors. Bakuli and Smith (1996) used a state-dependent queuing network to attack the resource allocation problem faced in various emergency situations. Integer programs may be combined with state-dependent models to decide the routes in emergency evacuation planning (Stepanov and Smith, 2009). Because of the complexity of the evacuation problem, intelligent simulation models are widely used as an alternative method to construct the evacuation model. Weinroth (1989) developed a simulation model called MOBILIXE for a complex and large scale building evacuation problem. Drager et al. (1992) provided a model called EVACSIM that could be used to study escape and rescue activities on vessels. REMS is another simulation and optimization module to calculate the estimated evacuation time and traffic flow during a hurricane evacuation (Tufekci, 1995).

Shelters play a critical role in response to massive natural disasters, and several papers in the literature consider the locations of shelters in an evacuation process. The shelter locations could influence the total congestion-related evacuation time in hurricane response (Sheral et al., 1991). A Stakelberg game could be used to build a shelter location-allocation model for flood evacuation (Kongsomsaksakul et al., 2005), in which the authority decides the location of shelters to minimize the total evacuation time and the evacuees, as followers, choose shelters and routes. During the response stage, the authority needs to decide which shelter should be opened and how many evacuees should be assigned to each shelter (Altay and Green, 2006). Logistics management in emergencies includes dispatching resources and transporting commodities and personnel to shelters. Yi and Özdamar (2007) used a mixed integer multi-commodity network flow model to decide the routes of shipping both resources and evacuees. Rawls and Turnquist (2010) used a stochastic program to determine the location and quantities of emergency supplies during the preparedness stage. In this chapter, we will consider both the location and capacity issues of shelters in the preparedness stage and the evacuee and resource allocation issues in the response stage. This chapter considers the stochastic natures of natural disasters and incorporates various scenarios with probabilities into the model. For emergency management problems, stochastic models provide a more accurate evacuation model that could take all possible scenarios into consideration. In addition to the travel time (travel cost) of evacuees, this chapter also considers the transportation of resources in the response stage.

Stochastic programming models have been well applied in transportation planning and operations for disaster response. The types and impacts of disasters can be modeled as various scenarios with associated probabilities. The impacts may include transportation demand of evacuees and resources and reduced transportation capacity. Two-stage stochastic programming models could be used to plan the transportation of commodities to disaster-affected areas during response (Barbarosogcaronlu and Arda, 2004), to manage evacuation (Li et al., 2008), or to locate distribution centers of supplies in the preparedness (Rawls and Turnquist 2010). Two-stage stochastic programs are used in long-term transportation planning to minimize a mean-risk objective of the system loss while considering the interdependencies of individual facilities (Liu et al., 2009). This chapter will consider the management of both evacuation and resources.

Problem statement and model formulation for sheltering network planning and operation problem

An effective sheltering management strategy should consider a disaster preparedness stage before knowing the information of a specific disaster and a disaster response stage to optimize the evacuation process for a disaster. The planning stage needs to decide the locations and capacities for new Permanent Shelters, which meet FEMA standards. Evacuation resources may also be planned to store in new Permanent Shelters in preparedness. In the response stage, the authority needs to decide how to assign evacuees from demand points to shelters, including both Permanent Shelters and Temporary Shelters, which do not meet FEMA standards. Note that the Permanent Shelters in the response stage consists of new Permanent Shelters built in the Preparedness Stage and Existing Permanent Shelters. The capacity of shelters restricts the evacuation assignment decisions (Saadatseresht et al., 2009). Typically, Permanent Shelters should have higher priority so that the following model will assign lower occupancy cost per evacuee for Permanent Shelters. At the same time, resources, including commodities and personnel, need to be transported to shelters to support their operations. Therefore, the overall response costs include transportation cost of evacuees, transportation cost of resources, operational cost per evacuee in each shelter, and shortage or surplus costs of resources if any shortage or surplus happens. Figure 1 illustrates the overall sheltering network planning and operation problem (SNPOP). In the middle layer, there are two Existing Permanent Shelters, three Temporary Shelters, and two Potential Permanent Shelter locations that could be selected to be new Permanent Shelters with certain capacities and resource inventory levels in the first stage of SNPOP. After knowing the information of a specific natural disaster, perhaps before the area is overwhelmed by the disaster, transportation network condition and evacuee demands, including locations and volumes, are assumed to be known. The second stage of SNPOP decides the evacuee assignment to shelters and the resource shipment from distribution centers (or other resource origins) to shelters. The first stage and second stage of SNPOP interact with each other so that this chapter proposes a two-stage stochastic program to capture various scenarios of disasters and to consider the total costs in the preparedness and response stages.

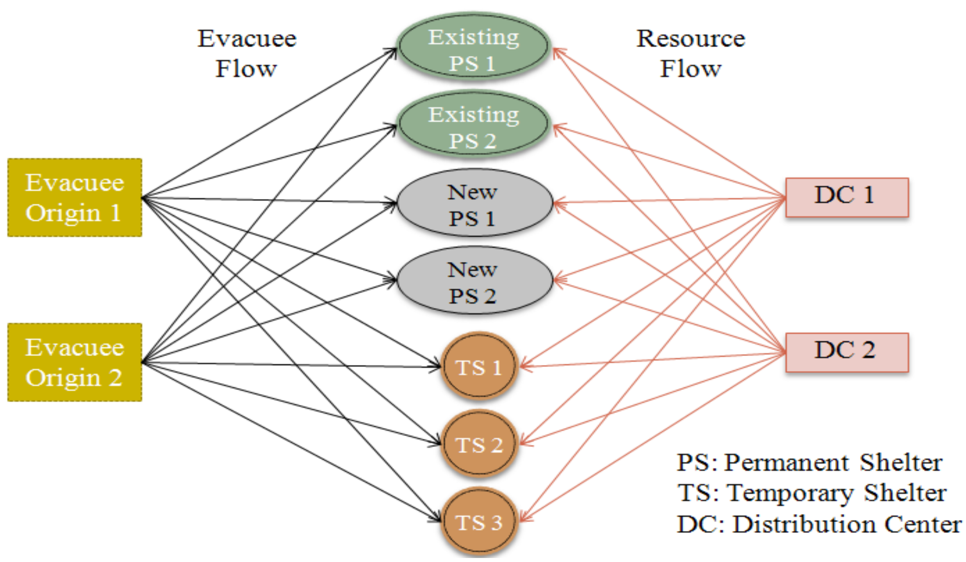

Figure 1 Illustration of SNPOP

The following information is assumed to be known as parameters for the SNPOP model.

ES : Set of Existing Permanent Shelters;

PS : Set of Potential Permanent Shelter locations;

TS : Set of Temporary Shelters;

S: Set of all shelters, $S = ES \cup TS \cup PS$;

K: Set of evacuee origins;

I : Set of distribution centers (or origins of resources), i is its index;

R: Set of resources (commodities or personnel) needed for sheltering, r is its index;

U_j^e: Capacity of Existing Permanent Shelter j in the number of evacuees, $j \in ES$;

U_j^t: Capacity of Temporary Shelter j in the number of evacuees, $j \in TS$;

a_{ir} : Available amount of commodity r at distribution center i;

h_{jr}^e: Available amount of commodity r at Existing Permanent Shelter j, $j \in ES$;

s_j^p : Fixed cost of setting up a new Permanent Shelter at location j divided by the expected number of disasters in the study area during the shelter's expected lifetime, $j \in PS$;

e_r: Unit cost of holding resource r at location j per year divided by the expected number of disasters per year, $j \in PS$;

c^p: Unit cost of having capacity for one evacuee at Permanent Shelters divided by the expected number of disasters in the study area during shelters' expected lifetime, $j \in PS$;

Ω: Set of disaster scenarios, ω is its index;

$Prob(\omega)$: Probability of scenario ω;

$d_k(\omega)$: Total evacuees generated at demand point (affected area) k under scenario ω;

$v_{kj}(\omega)$: Cost of allocating one person from demand point k to shelter j (transportation cost plus operational cost of one evacuee at shelter j) under scenario ω;

$q_{ijr}(\omega)$: Cost of transporting one unit of commodity r from distribution center i to shelter j under scenario ω;

b_r^+ : Unit cost of surplus for commodity r after evacuation;

b_r^- : Unit cost of shortage for commodity r after evacuation.

Note that the units for all kinds of resources are normalized to the required amount for each evacuee during one disaster period.

The SNPOP model includes the following decision variables.

z_j^p: 1: If the Potential Permanent Shelter location j is chosen for setting up a new Permanent Shelter, 0: Otherwise, $j \in PS$;

U_j^p: Capacity of the Permanent Shelter at potential location j, $j \in PS$;

h_{jr}^p: Available amount of resource r at a Potential Permanent Shelter at location j, $j \in PS$;

$x_{kj}(\omega)$: Number of evacuees transported from evacuee origin k to shelter j under scenario $\omega, j \in S$;

$y_{ijr}(\omega)$: Amount of commodity r shipped from distribution center i to shelter j under scenario $\omega, j \in S$;

$s_{jr}^+(\omega)$: Surplus amount for commodity r after evacuation at shelter j under scenario ω;

$s_{jr}^-(\omega)$: Shortage amount for commodity r after evacuation at shelter j under scenario ω.

Here, the first three variables of z_j^p, U_j^p, *and* h_{jr}^p are decisions in the first (preparedness) stage and the remaining variables of $x_{kj}(\omega)$, $y_{ijr}(\omega)$, $s_{jr}^+(\omega)$ and $s_{jr}^-(\omega)$ are decisions in the second (response) stage under a specific scenario ω. With the above definition of variables and parameters, the SNPOP model is given as follows.

Minimize

$$\sum_{j \in PS} s_j^p z_j^p + \sum_{j \in PS} c^p U_j^p + \sum_{j \in PS} \sum_{r \in R} e_r h_{jr}^p +$$

$$\sum_{\omega \in \Omega} Prob(\omega) \Big\{ \sum_{k \in K} \sum_{j \in S} v_{kj}(\omega) x_{kj}(\omega) + \sum_{i \in I} \sum_{j \in S} \sum_{r \in R} q_{ijr}(\omega) y_{ijr}(\omega) +$$

$$\sum_{j \in S} \sum_{r \in R} \left(b_r^+ \; s_{jr}^+(\omega) + b_r^- \; s_{jr}^-(\omega) \right) \Big\} \qquad (1)$$

S.T.

$$U_j^p \le M z_j^p \qquad \forall j \in PS; \qquad (2)$$

$$\sum_{k \in K} x_{kj}(\omega) \le U_j^t \qquad \forall j \in TS, \forall \omega \in \Omega; \qquad (3)$$

$$\sum_{k \in K} x_{kj}(\omega) \le U_j^e \qquad \forall j \in ES, \forall \omega \in \Omega; \qquad (4)$$

$$\sum_{k \in K} x_{kj}(\omega) \le U_j^p \qquad \forall j \in PS, \forall \omega \in \Omega; \qquad (5)$$

$$\sum_{j \in S} x_{kj}(\omega) = d_k(\omega) \qquad \forall k \in K, \forall \omega \in \Omega; \qquad (6)$$

$$\sum_{j \in S} y_{ijr}(\omega) \le a_{ir} \qquad \forall i \in I, \forall r \in R, \forall \omega \in \Omega; \qquad (7)$$

$$\sum_{i \in I} y_{ijr}(\omega) - \sum_{k \in K} x_{kj}(\omega) = s_{jr}^+(\omega) - s_{jr}^-(\omega) \quad \forall j \in TS, \forall r \in R, \forall \omega \in \Omega; \qquad (8)$$

$$\sum_{i \in I} y_{ijr}(\omega) + h_{jr}^e - \sum_{k \in K} x_{kj}(\omega) = s_{jr}^+(\omega) - s_{jr}^-(\omega) \quad \forall j \in ES, \forall r \in R, \forall \omega \in \Omega; \qquad (9)$$

$$\sum_{i \in I} y_{ijr}(\omega) + h_{jr}^p - \sum_{k \in K} x_{kj}(\omega) = s_{jr}^+(\omega) - s_{jr}^-(\omega) \quad \forall j \in PS, \forall r \in R, \forall \omega \in \Omega; \qquad (10)$$

$$z_j^p \in \{0,1\}\,; \; U_j^p, h_{jr}^p, x_{kj}(\omega), y_{ijr}(\omega), s_{jr}^+(\omega), s_{jr}^-(\omega) \in R_+^n.$$

The objective function (1) minimizes the total first stage cost and the expected cost of the second stage over all scenarios. The first stage cost includes the fixed cost to have new Permanent Shelters, the variable cost based on capacity for new Permanent Shelters, and the inventory cost of resources stored at new Permanent Shelters. All costs are normalized for a disaster. The second stage cost includes transportation costs of evacuees, transportation costs of resource distribution, and the surplus and shortage costs for resources after an evacuation. The first constraint set (2), where M is a big number, is the only constraint set in the first stage and guarantees a new Permanent Shelter has to be established before it is used. Constraint sets (3-5) are capacity constraints of all three kinds of shelters. Constraint set (6) ships all evacuees to shelters. Constraint set (7) guarantees that the total shipment of resource r from distribution center i will not exceed the available amount at the center. Constraint sets (8-10) are used to obtain the shortage and surplus of each resource type at each shelter after a disaster. Note that we define the unit of one resource type as the required amount of the resource for each evacuee in (8-10). In practice, the capacity provided by all Temporary Shelters is huge because of their big number. Furthermore, the SNPOP model allows resource shortage and surplus at shelters. Therefore, the feasibility of the SNPOP is guaranteed under each scenario.

The SNPOP model is an integer program with binary variables z_j^p in the first stage, which decides the locations of new Permanent Shelters. The second stage problem under each scenario ω formed by constraint sets (2-10) and the objective function of

$\min \sum_{k\in K, j\in S} v_{kj}(\omega)x_{kj}(\omega) + \sum_{i\in I, j\in S, r\in R} q_{ijr}(\omega)y_{ijr}(\omega) + \sum_{j\in S, r\in R} \left(b_r^+ s_{jr}^+(\omega) + b_r^- s_{jr}^-(\omega)\right)$ is a linear program. Though the second stage problem seems like a network flow problem as shown in Figure 1, the model has additional side constraints called equal flow constraints. Note that each variable $x_{kj}(\omega)$ appears $|R|$ times in constraint sets (8-10) and appears once in constraint set (6), which violates the requirement that one variable (the flow on one arc) can only appear in two constraints (on two nodes). Therefore, optimization solvers specifically developed for the network flow problem cannot be directly used to solve the second stage problem though the Network Simplex method is typically faster than the regular Simplex method for generic linear programs. Based on preliminary numerical experiments, the SNPOP with a real-world size cannot be solved by optimization solvers, such as ILOG CPLEX 9.0, in a reasonable amount of time.

The L-Shaped algorithm for the SNPOP

The computational challenge of solving the SNPOP model (1-10) is mainly from the large number of scenarios that are used to describe future disaster events and the binary variables in the preparedness stage. The SNPOP model (1-10) could be written as follows by separating the two stages of preparedness and response. The first stage problem is as follows.

Minimize

$$\sum_{j \in PS} s_j^p z_j^p + \sum_{j \in PS} c^p U_j^p + \sum_{j \in PS} \sum_{r \in R} e_r h_{jr}^p + E_\omega[Q(U_j^p, h_{jr}^p, \omega)]$$

S.T. (11)

Constrain (2); $z_j^p \in \{0,1\}, U_j^p, h_{jr}^p \in R_+^n$.

Where $Q(U_j^p, h_{jr}^p, \omega)$ is the objective function value of the second stage with given values of U_j^p and h_{jr}^p and under a given scenario ω. The second-stage sheltering network problem under one particular scenario ω can be expressed as:

$Q\left(U_j^p, h_{jr}^p, \omega\right) =$

Minimize

$$\sum_{k \in K} \sum_{j \in S} v_{kj}(\omega) x_{kj}(\omega) + \sum_{i \in I} \sum_{j \in S} \sum_{r \in R} q_{ijr}(\omega) y_{ijr}(\omega) +$$

$$\sum_{j \in S} \sum_{r \in R} \left(b_r^+ \; s_{jr}^+(\omega) + b_r^- \; s_{jr}^-(\omega) \right) \tag{12}$$

S.T.

Constraints (2-10) under scenario ω;

$x_{kj}(\omega), y_{ijr}(\omega), s_{jr}^+(\omega), s_{jr}^-(\omega) \in R_+^n$.

The second-stage sub-problem is feasible under any scenario because it is assumed that the total capacity of all shelters, including a large number of Temporary Shelters, is enough for all evacuees under any scenarios, and shortage and surplus of resources are allowed. The second-stage sub-problem is a linear program with continuous variables, so the recourse function of $E_\omega\left[Q\left(U_j^p, h_{jr}^p, \omega\right)\right]$ is continuous, convex, and piece-wise linear. It is well known that if the number of second-stage scenarios is finite and the second-stage sub-problem for each scenario is a linear problem, the whole stochastic program can be solved by building the combination of outer linearization of the recourse cost function (RCF) representing $E_\omega\left[Q\left(U_j^p, h_{jr}^p, \omega\right)\right]$ and by solving the master cost function (MCF) iteratively using a cutting plane method. This method is called the L-shaped method, which was developed by extending Dantzig-Wolfe decomposition of the dual problem and Bender's decomposition of the primal problem to the stochastic programming domain (Birge and Louveaux, 1997). The key point of the L-shaped algorithm is to represent $E_\omega[Q\left(U_j^p, h_{jr}^p, \omega\right)$ for any $\left(U_j^p, h_{jr}^p\right)$ with a convex hull that is formed iteratively by solving the first-stage problem and the second-stage problem (12). The first-stage problem at iteration *v* is written into (13).

Minimize

$$\sum_{j\in PS} s_j^p z_j^p + \sum_{j\in PS} c^p U_j^p + \sum_{j\in PS}\sum_{r\in R} e_r h_{jr}^p + \theta \tag{13}$$

S.T.

$$U_j^p \le M z_j^p \qquad \forall j \in PS; \tag{14}$$

$$\theta \ge \theta^v + \sum_{j\in PS} \pi_j^{p,v}(U_j^p - U_j^{p,v}) + \sum_{j\in PS}\sum_{r\in R} \rho_{jr}^p (h_{jr}^{p,v} - h_{jr}^p) \quad \forall v \in \{1,2,\dots,V\}; \tag{15}$$

$$z_j^p \in \{0,1\}, U_j^p, h_{jr}^p, \theta \in R_+^n.$$

Here, $\theta^v = E_\omega\left[Q\left(U_j^{p,v}, h_{jr}^{p,v}, \omega\right)\right]$ is the expected value of objective function values of the second stage problems over all scenarios at iteration v by solving model (12) individually for each scenario ω with $U_j^p = U_j^{p,v}$ and $h_{jr}^p = h_{jr}^{p,v}$. Assume the simplex multipliers associated with constraint set (5) when solving model (12) individually for each scenario ω at iteration v are $\pi_j^{p,v}(\omega)$, and the simplex multipliers associated with constraint set (10) are $\rho_{jr}^{p,v}(\omega)$. Therefore, $\pi_j^{p,v} = \sum_\omega Prob(\omega)\pi_j^{p,v}(\omega)$ and $\rho_{jr}^{p,v} = \sum_\omega Prob(\omega)\,\rho_{jr}^{p,v}(\omega)$. The algorithm adds one cut of $\theta \ge \theta^v + \sum_{j\in PS} \pi_j^{p,v}(U_j^p - U_j^{p,v}) + \sum_{j\in PS}\sum_{r\in R} \rho_{jr}^{p,v} (h_{jr}^{p,v} - h_{jr}^p)$ into the master problem at each iteration v. The overall algorithm is as follows.

Step 0. $V = 0$ and $UB^0 = \infty$.

Step 1. Solve the master problem (13) and let $V = V + 1$.

Step 2, Let the objective function value be LB^V (the lower bound of the SNPOP model), obtain the values of U_j^p and h_{jr}^p from the solution in step 1 and set them as $U_j^{p,V}$ and $h_{jr}^{p,V}$, and let $UB^V = \sum_{j\in PS} s_j^p z_j^p + \sum_{j\in PS} c^p U_j^p + \sum_{j\in PS}\sum_{r\in R} e_r h_{jr}^p$ under the current solution from step 1.

Step 3. If $LB^V < min_{v=0,\dots,V-1} UB^v$, continue; otherwise, go to step 6.

Step 4. Solve model (12) individually for each scenario ω with $U_j^p = U_j^{p,V}$ and $h_{jr}^p = h_{jr}^{p,V}$, obtain θ^V, $\pi_j^{p,V}$, and $\rho_{jr}^{p,V}$, and let $UB^V = UB^V + \Theta^V$ be the upper bound of the SNPOP problem.

Step 5. Add the cut of $\theta \geq \theta^V + \sum_{j\in PS} \pi_j^{p,V}\left(U_j^p - U_j^{p,V}\right) + \sum_{j\in PS}\sum_{r\in R} \rho_{jr}^{p,V}\,(h_{jr}^{p,V} - h_{jr}^p)$ into (13) and go to step 1.

Step 6. Stop with the optimal solution.

Please note that the lower bound LB^V is non-decreasing over iterations but the upper bound UB^V are not so that the overall upper bound is $min_{v=0,\dots,V} UB^v$ at iteration V.

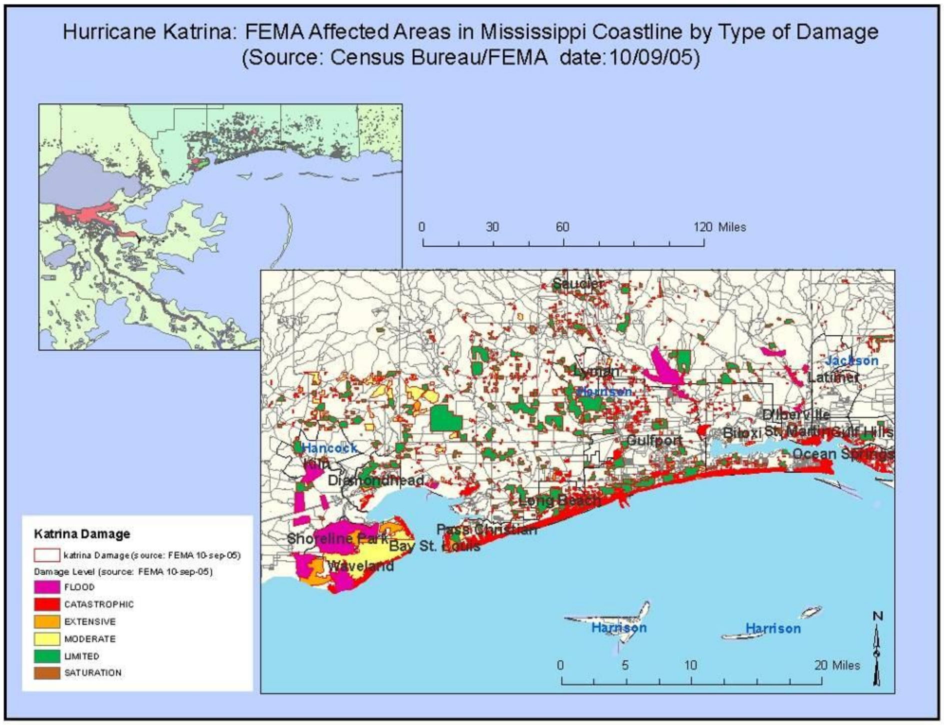

Figure 2 Areas affected by Hurricane Katrina in FEMA map (FEMA, 2005)

Case study

To demonstrate the implementation of the proposed SNPOP model and evaluate the effectiveness of the L-shaped algorithm, a case study is conducted for sheltering network planning and operations against hurricanes in the Gulf Coast region of the United States.

Case description

This case study covers the Gulf Coast region of Louisiana, Mississippi, Alabama, and Florida. For hurricanes, the most overwhelmed areas are along the coast around the landfall. The impact is reduced quickly when a hurricane moves to inland, as shown in Figure 2. The figure presents affected areas categorized by types of damage caused by

Hurricane Katrina in 2005. Therefore, this research assumes that evacuees are generated from the coastal areas near the hurricane landfall.

The number of evacuees generated in each area under each hurricane scenario is mainly decided by two factors: the landfall location and hurricane intensity. Klotzbach et al. (2009) predicted the probabilities of landfalls at the county level for eleven regions from Brownsville, TX to Eastport, ME based on past tropical cyclones, storms and hurricanes occurrences from 1880-2007. There are 205 coastal and near-coastal counties within the eleven regions. The historical data of hurricane information for each region were from the North Atlantic hurricane database (HURDAT) Reanalysis Project conducted by the Hurricane Research Division (HRD) and the Atlantic Oceanographic and Meteorological Laboratory (AOML). In addition to hurricane landfall locations, hurricane intensity is also an important factor to the volume of evacuees. Usually when hurricane intensity increases, the affected area is larger and the number of evacuees increases. The Saffir-Simpson Scale is widely used to represent the hurricane category, which is based on the wind speed of a hurricane. Like the project conducted by Klotzbach et al. (2009), this research classifies hurricanes based on the Saffir-Simpson Scale into three broad categories: Storm (tropical storm), Hurricane (category 1and 2 on the Saffir-Simpson Scale), and Intense Hurricane (category 3, 4 and 5 on the Saffir-Simpson Scale). The number of landfalls in each county *k* under category *t* during 1880-2007, N_t^k, is calculated as $L_t^{l_k} \cdot D_k$, where $L_t^{l_k}$ is the number of category *t* hurricanes that occurred in region l_k in which county *k* is located and D_k is the coastline distance of county *k* divided by region l_k's whole coastline distance. In this research, each scenario ω is characterized by a pair of (*k*, *t*), the landfall county *k* and hurricane category *t*. The probability of scenario ω is $Prob(\omega) = \frac{N_t^k}{\sum_{t,k} N_t^k}$. This case study considers all hurricane scenarios in which the landfall is between New Iberia, LA and Robertsdale, AL. A hurricane may affect neighboring counties beyond the landfall county, but the affected areas are restricted along the coastline at the county level (In this chapter, we refer to parishes in the State of Louisiana as counties). Table 1 provides affected counties with their landfall probabilities under each hurricane category. Note $K' \subset K$, where K' is the set of landfall counties and K is the set of all evacuee origins/affected areas. In Table 1, $K = \{1,2,\dots,19\}$ while $K' = \{3,4,\dots,17\}$. This study does not consider the landfalls at county 1, county 2, county 18, and county 19.

The number of evacuees in each county generated under each scenario depends on three factors, its population, the landfall location, and the hurricane category. Table 1 lists the assumed percentage of population who will be evacuees in each county when a category t hurricane makes landfall in county k.

Table 1 Locations and population of affected areas and landfall/scenario probabilities

	County Index	County	Population (2007)	Storm Probability	Hurricane Probability	Intense Hurricane Probability	
Possible Affected Areas *K*	1	Cameron, LA	7,238				
	2	Abbeville, LA	56,096				
	3	New Iberia, LA	74,965	0.03162	0.01551	0.00699	Possible Landfall Areas *K'*
	4	Franklin, LA	51,311	0.04278	0.02098	0.00946	
	5	Houma, LA	108,424	0.08835	0.04333	0.01954	
	6	Thibodaux, LA	92,713	0.03162	0.01551	0.00699	
	7	Hahnville, LA	52,044	0.02581	0.01266	0.00571	
	8	Gretna, LA	423,520	0.01674	0.00821	0.00370	
	9	Pointe a la Hache, LA	21,540	0.04836	0.02372	0.01069	
	10	Chalmette, LA	19,826	0.04371	0.02143	0.00967	
	11	New Orleans, LA	239,124	0.03139	0.01539	0.00694	
	12	Covington, LA	226,625	0.04255	0.02086	0.00941	
	13	Woodville, MS	40,421	0.02604	0.01277	0.00576	
	14	Bay St. Louis, MS	171,875	0.03441	0.01687	0.00761	
	15	Pascagoula, MS	130,577	0.03813	0.01870	0.00843	
	16	Mobile, AL	406,309	0.03441	0.01687	0.00761	
	17	Robertsdale, AL	174,439	0.04836	0.02372	0.01069	
	18	Pensacola, FL	37,600				
	19	Milton, FL	147,044				

Table 2 Percentage of evacuees when a category *t* hurricane makes landfall in county *k*

Hurricane	County Index				
Category	*k*-2	*k*-1	*k*	*k*+1	*k*+2
t=1	0	5%	10%	5%	0%
t=2	0	10%	20%	10%	0%
t=3	20	50%	70%	50%	20%

This case study considers a total of 57 Existing Permanent Shelters and 26 Potential Permanent Shelter locations. Locations and capacities of Permanent Shelters are provided in Figure 4. The locations and capacities of the Existing Permanent Shelters are based on the published information from the state government of Louisiana and the American Red Cross (2009). The Potential Permanent Shelter locations are randomly selected in highly populated areas. As mentioned in Section 1, there are thousands of Temporary Shelters in the study region. It is not possible or necessary to consider individual Temporary Shelters separately. Under each scenario, the authority needs to decide how much Temporary Shelter capacity should be used in each area and open Temporary Shelters based on a priority table decided in the preparedness stage. This case study consolidates Temporary Shelters into 31 regions (see Table 3). The capacities are randomly created based on a uniform distribution U [10,000, 20,000].

Table 3 Locations and capacities of temporary shelters

New Orleans, LA 19,859	Baton Rouge, LA 16,043	Shreveport, LA 12,428	Metairie, LA 18,028
Lafayette, LA 14,200	Lake Charles, LA 15,920	Kenner, LA 18,350	Bossier City, LA 16,365
Monroe, LA 17,305	Alexandria, LA 12,783	Jackson, MS 19,791	Gulfport, MS 17,498
Biloxi, MS 19,719	Hattiesburg, MS ,19,874	Greenville, MS 11,080	Meridian, MS 15,217
Tupelo, MS 11,017	Birmingham, AL 13,896	Montgomery, AL 10,085	Mobile, AL 14,738
Huntsville, AL 14,415	Tuscaloosa, AL 15,264	Hoover, AL 11,572	Dothan, AL 15,459
Decatur, AL 10,405	Auburn, AL 13,619	Gadsden, AL 10,265	Houston, TX 14,804
Austin, TX 15,978	Dallas, TX 18,103	San Antonio, TX 15,414	

Seven distribution centers and five resource types are considered in this case (i.e., $|I| = 7$ and $|R| = 5$). The assumed locations of the distribution centers and their available resource amounts, a_{ir}, are listed in Table 4. Please note the unit of each resource type is defined as the required amount for each evacuee. The values of a_{ir} are randomly created based on a uniform distribution U [200,000, 250,000]. Some resources are assumed to be held already at Existing Permanent Shelters, and the amount of resource r at shelter j, h^{e}_{jr}, is randomly drawn from a uniform distribution U [300, 700]. A recent paper by Rawls and Turnquist (2010) discussed how to determine the location of distribution centers and corresponding quantities of emergency supplies during the preparedness stage.

Table 4 Available resources at distribution centers, a_{ir}

Distribution Center	Resource 1	Resource 2	Resource 3	Resource 4	Resource 5
Shreveport,LA	225,256	228,134	217,613	215,397	228,707
Baton Rouge,LA	207,045	217,146	204,282	222,609	214,199
Jackson,MS	224,312	209,614	213,999	216,399	227,476
Hattiesburg,MS	223,112	209,816	230,161	212,570	229,716
Birmingham,AL	214,038	219,692	211,080	215,177	209,016
Montgomery,AL	206,149	205,763	207,242	232,612	220,534
Dallas, TX	210,505	220,394	220,399	207,767	201,767

The transportation costs include two parts: the costs of transporting evacuees from affected areas to shelters and the costs of shipping resources from distribution centers to shelters. The cost of allocating one evacuee from affected area k to shelter j in scenario ω is calculated based on the formula of $v_{kj}(\omega) = LH_k(\omega) \cdot vb \cdot d_{kj}$, in which d_{kj} is the distance (in miles) from evacuee demand origin k to shelter j, vb is the unit transportation cost (in dollars per mile per evacuee), and $LH_k(\omega)$ is a scenario-based weight capturing the increased transportation costs of the affected areas because of possible infrastructure damages and traffic congestion. The values of $LH_k(\omega)$ are decided by uniform distributions in Table 5 when the landfall county is *m* and the hurricane category is *t*. For other counties, $LH_k(\omega) = 1$. This case study sets $vb = \$\ 0.5$ per mile per person.

Table 5 $LH_k(\omega)$ values when a category *t* hurricane makes landfall in county *m*

Hurricane Category	County Index *k*				
	m-2	*m*-1	*m*	*m*+1	*m*+2
t=1	1	*U*(1.00, 1.10)	*U*(1.20, 1.25)	*U*(1.00, 1.10)	1
t=2	1	*U*(1.05, 1.15)	*U*(1.25, 1.35)	*U*(1.05, 1.15)	1
t=3	*U*(1.05, 1.15)	*U*(1.30, 1.35)	*U*(1.40, 1.45)	*U*(1.30, 1.35)	*U*(1.05, 1.15)

With better facilities and management, Permanent Shelters are usually preferred over Temporary Shelters. The additional cost, randomly drawn from a uniform distribution *U* [100,110], is considered for evacuees going to a Temporary Shelter and is added into the transportation cost $v_{kj}(\omega)$ from evacuee demand point k to Temporary Shelter $j \in TS$. The cost of shipping one unit of resource *r* from distribution center *i* to shelter *j* is calculated as $q_{ijr}(\omega) = LH(\omega) \cdot rvb_r \cdot d_{ij}$, in the same fashion as $v_{kj}(\omega)$. The transportation cost per unit per mile, rvb_r, is assumed to be different for resources, and their values are given in Table 6. The table also lists the unit surplus cost and shortage cost at shelters after an evacuation process and the unit holding cost at Permanent Shelters for each resource type. Here, the values of rvb_r, v_r^+, v_r^-, and e^r are randomly created based on uniform distributions of *U* [0.1, 0.2], *U* [40, 70], *U* [50, 80], and *U* [20,70] respectively.

Table 6 Unit transportation cost, surplus cost, and shortage cost for resources

	Resource Type				
	1	2	3	4	5
Unit Transportation Cost (rvb_r) ($ per mile per unit)	0.11	0.12	0.15	0.1	0.14
Unit Surplus Cost (v_r^+) ($ per unit)	40	66	63	58	70
Unit Shortage Cost (v_r^-) ($ per unit)	57	70	63	53	57
Unit Holding Cost at Permanent Shelters (e^r) ($ per unit)	40	38	48	28	31

Results and analysis

The case described in 5.1, including 57 Existing Permanent Shelters, 26 Potential Permanent Shelter locations, 31 Temporary Shelters, 7 distribution centers, 5 types of emergency resources, 19 hurricane affected areas, and 45 evacuation scenarios, is solved with the L-shaped algorithm described in Section 4. The algorithm is coded with Microsoft C++ on a Dell desktop with Intel® Core (TM) 2 CPU, 6600 @ 2.40 GHz and 2.00 GB of RAM by calling the optimization solver of CPLEX 9.0 for solving the master and sub-problems. The algorithm reaches the optimal solution of $21,824,600 after 206 iterations and 3,509 seconds. Figure 3 illustrates the convergence of the upper bound and lower bound of the SNPOP over iterations.

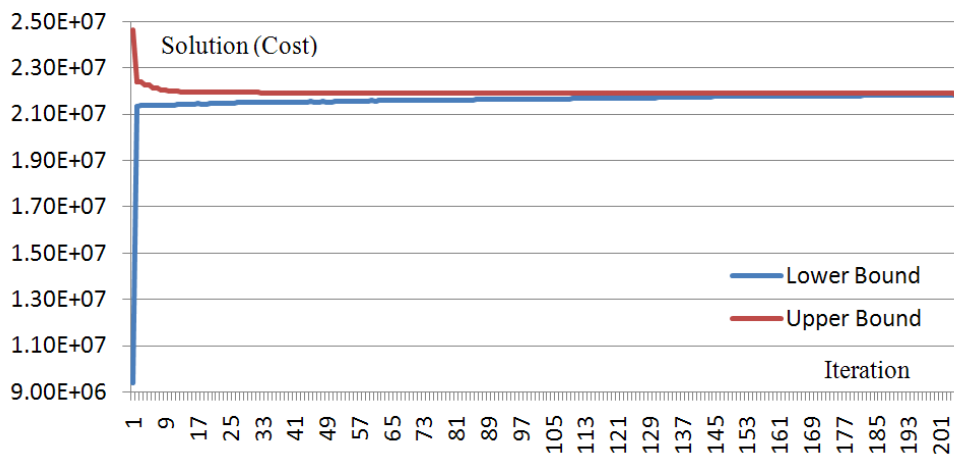

Figure 3 Convergence of the L-shaped algorithm in solving SNPOP

The result is illustrated in Figure 4, a map created by Google Earth©. The map displays the distribution of Existing Permanent Shelters, Potential Permanent Shelter locations, Temporary Shelters, distribution centers and affected areas. The information about Existing Permanent Shelters including shelter name, location, and capacity is sourced from American Red Cross National Shelter System. Figure 4, for example, shows that Faulkner State Community College Shelter is an Existing Permanent Shelter located in the city of Bay Minette, and it has a capacity of 746. There are a total of 26 potential locations for new Permanent Shelters. The solution selects 6 of them, marked as pink in Figure 4, to build new Permanent Shelters. All selected potential shelter locations are close to affected areas to reduce the second-stage evacuation cost.

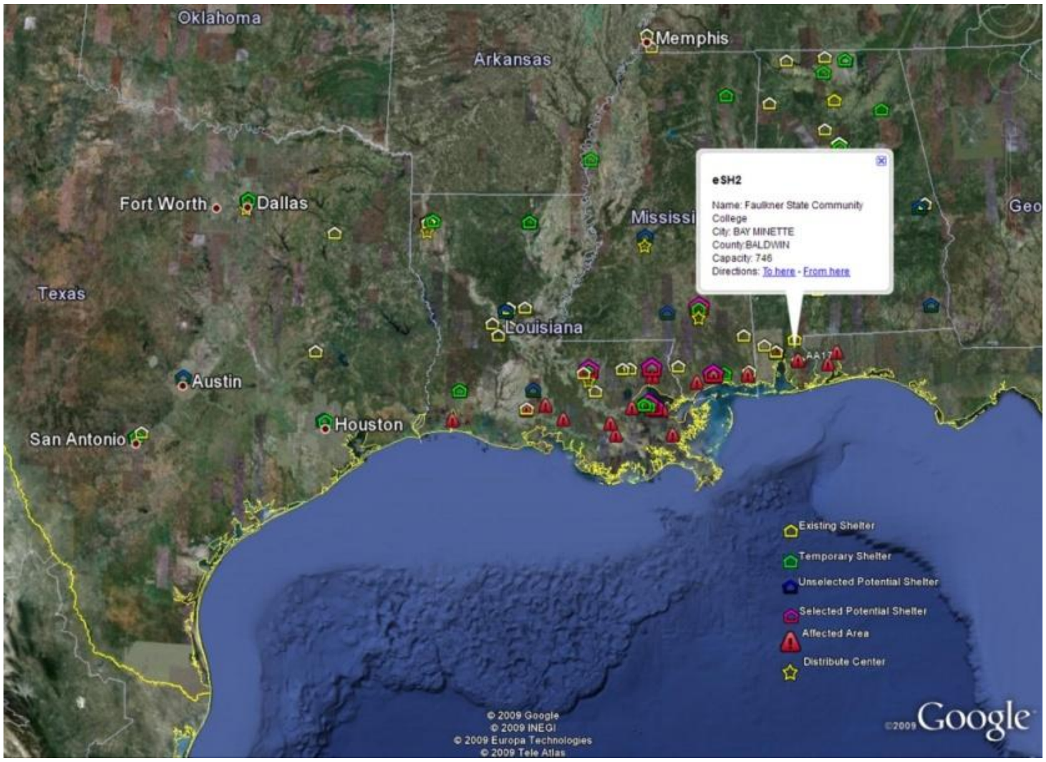

Figure 4 Distribution of shelters, affected areas, and distribution centers

In addition to evacuation preparedness, emergency managers may also be interested in the evacuation operations under each scenario, including evacuee transportation, resource shipment, and how much Temporary Shelter capacity should be used. Figure 5, also created by Google Earth©, illustrates the recommended operations under scenario 45, in which an intense (Category 3) hurricane makes landfall in county 17 and affects counties 15 through 19. The map includes the information of evacuee flows from affected areas to various shelters and resource flows from distribution centers to shelters. There are six route networks with different colors in the map, five for evacuee flows from different affected areas and one for the resource flow. Because of this interactive map, a user can click the shelter or route to find out relevant information. Figure 5 shows that shelter 53 will use all its capacity to host 500 evacuees while shelter 54 will hold 716 evacuees, below its 800 capacity. All the evacuees at these two shelters will be from affected area 15. Figure 5 also shows that, in general, evacuees should be transported or guided to nearby Permanent Shelters. If the nearby existing and newly built Permanent Shelters do not have enough capacity, Temporary Shelters or far away Permanent Shelters may be used.

Figure 5 One scenario of evacuation process

Chapter conclusion

This chapter considers sheltering network planning in the preparedness stage and operations in the response stage for national disasters, especially for hurricanes. The locations and capacities of new Permanent Shelters are decided in the planning phase. Building a new Permanent Shelter involves both fixed costs and variable costs based on capacity. Once information from one specific disaster is known, the operational issues are addressed, including the transportation of evacuees from affected areas and the shipment of resources from distribution centers. The evacuee flows also decide how many Temporary Shelters should be opened in each area. The Sheltering Network Planning and Operation Problem (SNPOP) is modeled as a two-stage stochastic programming model with integer variables in the first stage. The first-stage master problem captures the planning problem while the second-stage sub-problems deal with the response problem under all possible scenarios. Because of the large size, the stochastic programming model cannot be solved directly with existing optimization solvers. Therefore, this dissertation adopts the L-shaped algorithm to separate the two stages and solve the second-stage sub-problems individually with iterations.

A comprehensive case study for hurricane preparedness and response in the Gulf Coast region of the United States is presented to demonstrate the method of data collection and verify the SNPOP model and the L-shaped algorithm. Each hurricane scenario is characterized by its landfall and intensity. The data collection includes the definition of scenarios with probabilities, the location and capacity information of Existing Permanent Shelters, and various cost components. The numerical experiment results show that the L-shaped algorithm converges well, and a real-world problem could be solved in a reasonable amount of time.

Though the case study could be solved within thousands of seconds, the computational burden could still be an issue if we increase the number of scenarios further to capture more stochastic features of disasters or increase the resolution of the problem from counties to smaller areas. A future direction is to develop a more efficient algorithm to solve each sub-problem. As mentioned before, each sub-problem is not exactly a network flow problem because of the side constraints of equal flows. Specific algorithms could be developed to solve the minimum-cost network flow problem with equal flow constraints after taking advantage of the special location of the equal flow constraints in the network for the SNPOP sub-problems.

CHAPTER III

EQUAL FLOW STRUCTURE

In Chapter 2, we use ILOG CPLEX solver to directly solve the second-stage sub-problem of SNPOP as a part of the L-Shaped algorithm. With a larger network, the computational time for solving each sub-problem is dramatic long. In this chapter, the second-stage sub-problem is reconsidered as a network flow problem with equal flow constraints, and we use a Revised Network Simplex algorithm (RNS) to solve it. The network consists of two parts, the first part for evacuee flows and the second part for resource flows. In the first part of the network, evacuees are transferred from disaster affected areas to shelters. Each evacuee requires a certain amount of evacuation resources. In the second part of the network, distribution centers supply resources to shelters. In order to consist with SNPOP, the second network also has a surplus node and a shortage node for each type of resource. Dummy nodes are added to make sure the whole network problem is always feasible. These two sub-networks are connected by resource equal flow arcs, which cause the whole network different from the minimum-cost network flow problem and make the traditional network Simplex algorithm inappropriate. To address the computational challenge, we will revise the traditional network Simplex to incorporate the equal flow constraints. The motivation to have an algorithm based on network Simplex is that network Simplex algorithms can typically solve LP problems more efficiently compared with the Simplex algorithm in terms of solution time.

Minimum cost network flow problem with equal flow constraints

Similar to Calvete (2003), let's consider a network of $[N, A]$. $N = \{1, ..., n\}$ is the set of all nodes while $A = \{(i, j) : i, j \in N\}$ is the set of all arcs. b_i is the supply volume at node *i*. Negative b_i means that node *i* is a demand node. c_{ij} represents unit flow cost on arc (*i*, *j*) and u_{ij} denotes the capacity of arc (*i*, *j*). Assume there are *p* equal flow requirements. Let A_s denote the s^{th} set of arcs that must have equal flows, *s* = {1, 2,…, *p*}. The minimum cost network flow problem with equal flow constraints can be modeled as (16 - 21), in which f_{ij} is the decision variable representing the flow over arc (*i*, *j*).

Minimize

$$\sum_{(i,j)\in A} c_{ij} f_{ij} \tag{16}$$

S.T.

$$\sum_{\{j:(i,j)\in A\}} f_{ij} - \sum_{\{j:(j,i)\in A\}} f_{ji} = b_i \qquad \forall i \in N \tag{17}$$

$$f_{ij} = f_1 \qquad \forall (i,j) \in A_1 \tag{18}$$

$$f_{ij} = f_2 \qquad \forall (i,j) \in A_2 \tag{19}$$

$$f_{ij} = f_p \qquad \forall (i,j) \in A_p \tag{20}$$

$$0 \le f_{ij} \le u_{ij} \qquad \forall (i,j) \in A \tag{21}$$

In model (16 - 21), the objective function (16) is to minimize flow costs over all arcs. The first constraint set (17) is for flow conservation at nodes. Each of other constraint sets (e.g., 18 - 20) is to guarantee the same flows over arcs belonging to the same equal flow set, where f_s is the flow amount for the s^{th} set of equal flow arcs. Finally, additional capacity restriction is imposed on each arc.

In order to solve model (16 - 21) by taking advantage of the high computational speed of the network Simplex method, we will reformulate the model and introduce some properties. Let $\overline{\Phi}$ denote the network $[N, \overline{A}]$ and $\overline{A} = A - \bigcup_{s=1}^{p} A_s$. Then, model (16 - 21) can be rewritten into (22 - 24).

Minimize

$$\sum_{(i,j)\in\bar{A}} c_{ij} f_{ij} + \sum_{s=1}^{p} c_s f_s \tag{22}$$

S.T.

$$\sum_{\{j:(i,j)\in\bar{A}\}} f_{ij} - \sum_{\{j:(i,j)\in\bar{A}\}} f_{ji} + \sum_{s=1}^{p} a_s^i f_s = b_i \qquad \forall i \in N \tag{23}$$

$$0 \le f_{ij} \le u_{ij}, \forall (i,j) \in \bar{A};\ 0 \le f_s \le u_s, s = 1, \ldots, p. \tag{24}$$

Here, a_s^i is the number of arcs in the equal flow arc set s flowing out of node i minus the number of arcs in the equal flow arc set s flowing into node i. Its value could be positive, negative, or zero. The revised network simplex algorithm discussed below will be based on model (22 - 24). To address the difference of equal flow constraints, Calvete (2003) revised the network simplex algorithm for the general equal flow problem. Therefore, we apply Calvete's algorithm to solve the second-stage sub-problem of SNPOP. However, since the second-stage sub-problem of SNPOP has its own specific structure, we plan to develop a more efficient algorithm based on Calvete's network simplex algorithm to solve the second-stage sub-problem of SNPOP.

An illustration of the equal flow problem with the second-stage sub-problem of SNPOP

The second-stage sub-problem of SNPOP can be represented as a minimum-cost network flow problem with equal flow constraints. The whole network is comprised by two sub-networks, and the two sub-networks are connected by the equal flow arcs. Based on the structure of the second-stage sub-problem of SNPOP in chapter 2, a small example that has 2 affected areas, 2 shelters, 2 types of resources, and 2 distribution centers is presented in Figure 6. The sub-network for evacuee-flow, which is at the left side and has Nodes 0 through 3 in Figure 6, has two sets of nodes, the set of affected areas (Nodes 0 and 1 in Figure 6) denoted by *K* and the set of shelters (Nodes 2 and 3) denoted by *S*. Evacuees from any affected area could be transferred to any shelter, such as from affected area 0 to shelter 2 or shelter 3. The other sub-network is for resources, which is in the right-hand side of Figure 6 and includes Nodes 4 through 17. To have consistent flow directions with the evacuee flows, the resource flows are defined from shelters to distribution centers. In other words, the resource flows in Figure 6 represent demand flows rather than physical flows of resources. In this sub-network there are five sets of nodes, the set of shelter-resource nodes denoted by *SR* (Nodes 4 through 7), the set of distribution-center-resource nodes denoted by *DR* (Nodes 14 through 17), the set of shortage nodes for each resource type denoted by *ST* (Nodes 8 and 9), the set of surplus nodes for each resource type denoted by *SP* (Nodes 10 and 11), and the set of dummy nodes for overall surplus or shortage for all resource types (Nodes 12 and 13). In Figure 6, all even-numbered nodes are for resource type 1 while others are for resource type 2 in the resource flow sub-network. Arcs connecting the shelter-resource nodes and distribution-center-resource nodes represent the demand amount of one resource type at one shelter that is satisfied by the shipment from some distribution centers. In Figure 6, for example, type 1 resource demand is flown from node 4 (a shelter-resource node) to node 14 (a distribution-center-node). If demands of one type of resource at a shelter are less than the total available amount, including the amount from distribution centers and the existing amount at the shelter, the additional demand will be flown from a surplus node that is for this resource type. For example the flow on arc (4, 10) in Figure 6 represents the surplus of resource type 1 at shelter 1 in operations. Otherwise, if the total available amount of one resource type is not enough to meet demands in a shelter, there will be a shortage flow from the shelter to the shortage node for that resource type. For instance, if the demand for type 1 resource from shelter 1 in node 4 cannot be satisfied, there will be a shortage flow form node 4 to shortage node 8. In order to guarantee that the total supply is equal to the total demand for the whole network, two dummy nodes of 12 and 13 are introduced into Figure 6 to make the whole structure always feasible. Table 7 is given to summarize the definition of all node sets with the example shown in Figure 6.

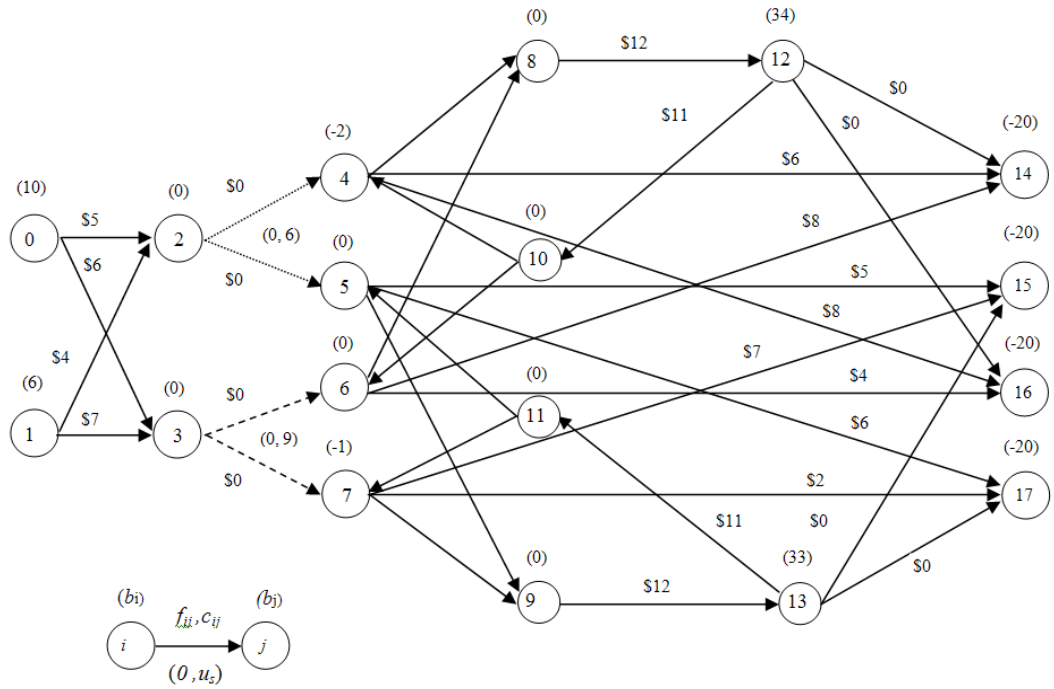

Figure 6 An example of the second stage sub-problem of SNPOP

Table 7 Summary of node set definition

Set	Definition	Example in Fig 3.1
K:	the set of affected areas	{0,1}
S:	the set of shelters	{2,3}
SR:	the set of shelter-resource nodes	{4,5,7,8}
ST:	the set of shortage nodes for resources	{8,9}
SP:	the set of surplus nodes for resources	{10,11}
DM:	the set of dummy nodes	{12,13}
DR:	the set of distribution-center-resource nodes	{14,15,16,17}

The two sub-networks are connected by equal flow arcs. For evacuees in each shelter, since they require all types of resources (two kinds of resources in Figure 6) and the demand quantities for each resource (measured as the required amount for each evacuee) are the same, there are two arcs with the same flow from each shelter (e.g. arc (2, 4) and arc (2, 5)) in the example illustrated in Figure 6. The number of equal flow arc sets is $|S|$ and each set has $\frac{|SR|}{|S|}$ arcs. Since the flows on equal flow arcs are the same, they should enter or exit the basis simultaneously over Simplex iterations. This feature causes the failure of implementing the traditional network simplex algorithm, which introduces only one new variable/arc into the basis in an iteration and finds a pivoting cycle after adding the entering arc. The new value of each arc in a pivoting cycle depends on the maximum value that can be flown in the cycle. However, it is difficult to carry out the pivoting process by using the traditional network simplex algorithm if there are multiple arcs entering the basis together with the same amount of flow.

In this example, c_s corresponds to the cost for equal flow arc s. The second-stage sub-problem of SNPOP has all $c_s = 0$ for s=1,…, p and c_{ij} corresponds to the cost in (2.1) in Chapter 2 as follows:

$$c_{ij} = \begin{cases} \text{Unit evacuee transportation from node i to node } j & \forall i \in AA, \forall j \in S \\ \text{Unit resource transportation from node i to node } j & \forall i \in SR, \forall j \in DR \\ \text{Unit resource shortage cost for shortage node } j & \forall (i,j) \in \bar{A} | i \in SR, j \in ST \\ \text{Unit resource surplus cost for surplus node } j & \forall (i,j) \in \bar{A} | i \in SP, j \in SR \\ 0 & \text{any other } (i,j) \in \bar{A} \end{cases} \quad (25)$$

Regarding arc capacity of this example, u_s is the capacity of s^{th} shelter, which is from the first stage solution of SNPOP. For any other arcs, $u_{ij} = \infty$, $\forall (i,j) \in \bar{A}$. Also showed in Figure 6, π_i is the node potential of node i at one iteration and will be used in the modified Simplex method later.

Revised network simplex algorithm for the minimum-cost network flow problem with equal flow constraints

The network Simplex algorithm for the minimum-cost network flow problem roughly follows six steps.

Step 1. Find the first basic feasible solution.

Step 2. Calculate the node potentials and reduced costs for all non-basic arcs.

Step 3. Check the optimality conditions. If it is optimal already, stop.

Step 4. Decide the entering non-basic variable (arc).

Step 5. Decide the exiting basic variable (arc).

Step 6. Update the values of all basic variables (arcs) and go to step 2.

In this Section, for each step, we first introduce the methods of the network Simplex algorithm for regular minimum-cost network problem and discuss the necessary changes of the algorithms for the network with equal flow constrains.

Basic feasible solutions

As described in previous section, for any basic feasible solution in the network simplex method (Ahuja, 1999), a network $[N, A]$ with n nodes has n-1 basic arcs, and all other arcs have flows either on their lower bounds or on their upper bounds. Let (T, L, U) denote a basic feasible solution. T is the set of basic arcs whose flows could be between their lower bounds and upper bounds. L is the set of non-basic arcs whose flows are on their lower bounds. U is the set of non-basic arcs whose flows are on their upper bounds. For the network with equal flow constraints, if there are r equal flow arc sets in the basis, where r is between 1 and p, the remaining arcs, which all belong to $\bar{A}$, form a forest with $r+1$ spanning trees. The forest is denoted as F. In F, each spanning tree is denoted as $T_1, T_2, \ldots, T_k$, where T_k represents the k^{th} spanning tree. Let B denote node-arc incidence matrix of basic arcs. It basically consists of two parts: the first part is constructed by basic equal flow arc sets and represented by column matrices of $A_1, A_2, \ldots, A_r$; The second part is constructed by all other basic arcs and their node-arc incidence matrix is denoted as $\bar{B}$. Therefore, $B = [\bar{B} \quad A_1 \quad A_2 \quad \ldots \quad A_r]$ and in order to obtain a feasible solution to model (22 - 24), B has to have a rank of n-1. The rank of matrix B is equal to n-1 if and only if the matrix D has full rank (Calvete, 2003).

Any basic solution of a network $[N, A]$ with r equal flow arc sets in the basis, where r could be between 0 and p, consists of (r+1) spanning trees in $\overline{\Phi}$ and requires that $\text{rank}(D) = r$. Based on Calvete (2003), an (r+1) spanning forest F in $\overline{\Phi}$ is a 'good $(r+1)$ forest' with respect to the variables $\{f_s\}_{s \in S}, S \subseteq \{1, \ldots, p\}, |S| = r$, if $\text{rank}(D) = r$. For the network structure for the second-stage sub-problem of SNPOP illustrated in Fig. 5, there are at least $|ST|+1$ trees in $\bar{\Phi}$, where $|ST|$ is the number of resource types involved in the original SNPOP problem. In other words, when $r < |ST|$, there is no way to construct an $(r+1)$ forest. For example, if there is no equal flow set in the basis for the example in Fig. 5, there will be at least three trees formed by basic arcs in $\overline{\Phi}$ and we cannot find any tree in $\bar{\Phi}$.

Initial basic feasible solution

There are several methods to obtain an initial basic feasible solution for minimum-cost network flow problems. In Ahujia (2003), one dummy node denoted by "-1" is simply added and is connected to each of all nodes with an arc having infinite upper bounds. If node *i* has a nonnegative demand, an artificial arc of (*i*, -1) is added; otherwise, an artificial arc (-1, *i*) is added. The first basic feasible solution is established by letting the flow of the artificial arc connected to node *i* be the demand (or supply) of node *i*. Since the second-stage sub-problem of the SNPOP is always feasible, there is at least one basic feasible solution in which all artificial variables are zeros. Therefore, a very big number is assigned to artificial arcs/variables $c_{i,-1}$ or $c_{-1,i}$ as unit flow costs. The advantage of this initial solution generating method is that it can be easily applied to general minimum-cost network Simplex algorithm. However, for the second-stage sub-problem of SNPOP, this method is time-consuming because of two major reasons. First, the second-stage sub-problem of SNPOP usually involves a large number of nodes and arcs. If we introduce the node -1 to construct the initial basic solution, *n* number of additional arcs will be added and it will take at least *n* iterations to move these artificial arcs out of the basis. Second, the pivoting procedure of the second-stage sub-problem of SNPOP is much more complicated than the general minimum-cost network problem and causes more computational burden. Thus, we try to save pivoting iterations to reduce the overall computational time. In order to achieve this purpose, another initial solution generating method is introduced to get the first basic feasible solution for the second-state sub-problem of SNPOP under scenario ω as follows.

BEGIN

Step 1: Let *k* be the first affected area in the set of *K*, whose evacuee demand is $d_k(\omega)$.

Step 2: Among shelters that have any positive available resources and have not reached their capacities, select the one with the least evacuee transportation cost $v_{kj}(\omega)$.

In other words, $j^* = argmin_{j \in S} v_{kj}(\omega)$, where $S = \{j | \sum_r h_{jr} > 0 \ and \ U_j > 0\}$. Here, h_{jr} is the amount of resource *r* at shelter *j* and U_j is the upper bound of shelter *j*.

Step 3: Flow the amount of $x_{kj^*} = min\{d_k(\omega), U_{j^*}\}$ from affected area *k* to shelter j^*. Flow necessary resources from distribution centers to shelter j^* to meet evacuees' demand with the least cost while considering the available amount of each resource at the distribution centers. Update $d_k(\omega) = d_k(\omega) - x_{kj^*}$, $U_{j^*} = U_{j^*} - x_{kj^*}$, and $h_{j^*r} = [h_{j^*r} - x_{kj^*}/|R|]$. Here, R is the set of resource types.

Step 4: If $d_k(\omega) > 0$, go to Step 2.

Step 5: If $d_k(\omega) = 0$ and there is no remaining affected area in *K*, go to END.

Step 6: If U_{j^*}> 0, remove be the next affected area in *K* and go to Step 3. If $U_{j^*} = d_k(\omega) = 0$, let *k* be the next affected area in *K* and go to Step 2.

END

The procedure to obtain the first basic feasible solution is very myopic and does not lead to a quality solution. The procedure is based on the assumption that utilizing the available resources at a shelter first has potential of avoiding the shipping cost for resources from distribution centers to shelters. This procedure will force the flows on equal flow arcs reach upper bounds one by one. Therefore, there is at most one equal flow arc set in the basis so that the initial feasible solution is simple and has a good structure (the initial forest will have one equal arc set and two trees and we do not have to develop techniques to satisfy a pre-pivoting condition (Calvete, 2003) that a forest must have a good tree structure). Please see the example in Figure 7.

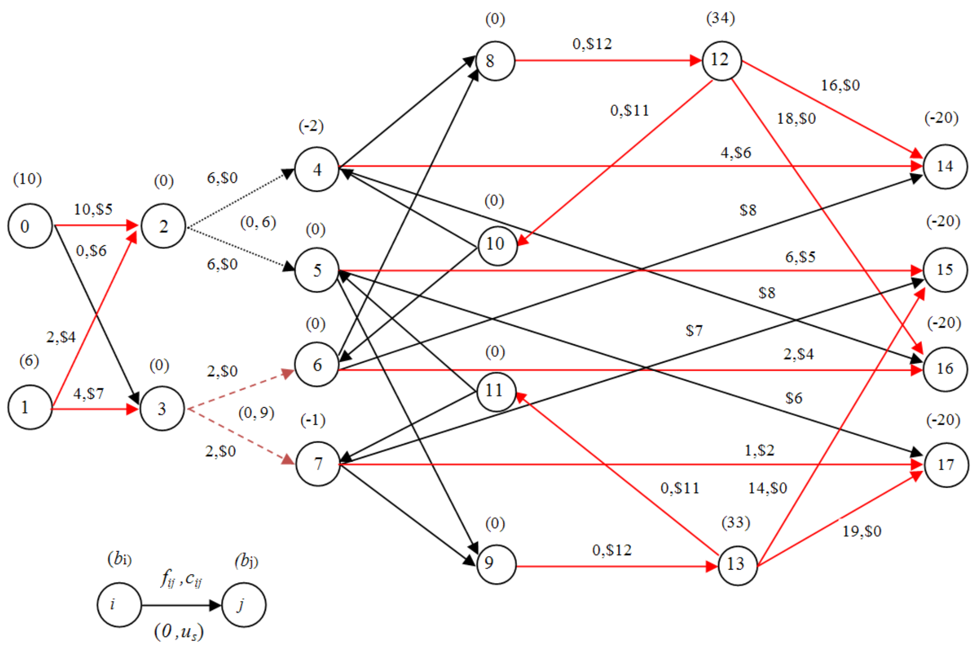

Figure 7 An example of the initial solution of SNPOP

Calculating the value of equal flow arcs

A basic equal flow set connects trees in a forest corresponding to a basic feasible solution. In order to obtain the flow of an equal flow arc set, we need to know the supply or demand for the trees that are connected by the equal flow set. The total flow of all basic equal flow arcs out of a tree is equal to the total supply minus total demand of all nodes in the tree minus the net flow amount of non-basic equal flow arcs and non-basic arcs (the flows on their upper bounds) connected to the tree. According to Calvete (2003), the values of the basic variables $f_1, \dots, f_r$ are determined by solving the following linear system:

$$D\begin{pmatrix} f_1 \\ \vdots \\ f_r \end{pmatrix} = \begin{pmatrix} b(T_1) \\ \vdots \\ b(T_r) \end{pmatrix} \tag{26}$$

$$b(T_h) = \sum_{k \in T_h} b_k - \sum_{k \in T_h} \sum_{(i,j) \in V_h^1} u_{ij} + \sum_{k \in T_h} \sum_{(i,j) \in V_h^2} u_{ij} - \sum_{k \in T_h} \sum_{s \in \bar{U}} a_s^k u_s, \; h = 1, \dots, r \quad (27)$$

$$\text{where, } V_h^1 = \{(i,j): i \in T_h, j \notin T_h, (i,j) \in U\}, \quad (28)$$

$$V_h^2 = \{(i,j): i \notin T_h, j \in T_h, (i,j) \in U\}, \quad (29)$$

$$U = \{(i,j): f_{ij} = u_{ij,} \, (i,j) \in \bar{A}\}, \quad (30)$$

$$\text{and } \bar{U} = \{s: f_s = u_{s,} \; s \in \{1, \dots, p\}\}. \quad (31)$$

Optimality condition

In the Network Simplex algorithm for a regular minimum-cost network problem, node potentials π_i of a basic feasible solution, which is a tree connecting all nodes, can be calculated based on the relationship of $c_{ij} - \pi_i + \pi_j = 0, \forall (i,j) \in T$ *b*y arbitrarily setting the node potential of a node equal to zero. Then, reduced cost for each non-basic arc is calculated as $c_{ij}^{\pi} = c_{ij} - \pi_i + \pi_j$. The current basic feasible solution is an optimal solution if and only if the following two conditions are satisfied:

a. $c_{ij}^{\pi} \geq 0$ for every arc $(i,j) \in L$, and

b. $c_{ij}^{\pi} \leq 0$ for every arc $(i,j) \in U$.

However, this procedure needs to be slightly modified for the equal flow network simplex method. A given basic feasible solution is a "good (r+1) forest" rather a single tree. If we calculate the node potentials separately for each tree by arbitrarily setting one node potential in each tree to be zero, the problem that $c_s^{\pi} \neq 0$ for $s \in T, s = 1,..,r$ may occur. Therefore, node potential normalization procedure across trees is must be developed. After getting node potentials π_i in a separate way for each tree, calculate the reduced cost for each basic equal flow set as

$$c_s^\pi = c_s - \sum_{i \in N} a_s^i \pi_i, s = 1, \ldots, r. \tag{32}$$

If all $c_s^\pi = 0, \forall s = 1, \ldots, r$, then π_i, $i \in N$are the right node potentials and no adjustment is necessary. Otherwise, calculate the adjustment amount σ_h for each tree T_h as

$$D \begin{pmatrix} \sigma_1 \\ \vdots \\ \sigma_r \end{pmatrix} = \begin{pmatrix} c_1^\pi \\ \vdots \\ c_r^\pi \end{pmatrix} \tag{33}$$

Then, the node potentials are adjusted as follows.

$$\pi_i' = \begin{cases} \pi_i + \sigma_1 & i \in T_1 \\ \cdots & \cdots \\ \pi_i + \sigma_r & i \in T_r \\ \pi_i & i \in T_{r+1}. \end{cases} \tag{34}$$

In the Network Simplex algorithm for a regular minimum-cost network problem [*N*, *A*] that has n nodes, a basic feasible solution has *n*-1 basic variables. When the optimality condition is not satisfied, the pivoting procedure decides which non-basic arc will enter the basis and which basic arc will leave the basis. The number of basic variables is always equal to *n*-1 over iterations. The entering non-basic arc is usually selected based on its reduced cost c_{ij}^π and the most common way is to choose the arc with the largest $|c_{ij}^\pi|$ among all admissible arcs. A non-basic arc (i, j) is called admissible if

$$c_{ij}^\pi < 0 \text{ and } (i, j) \in L \tag{35}$$

$$\text{or } c_{ij}^{\pi} > 0 \text{ and } (i,j) \in U. \tag{36}$$

The entering arc, together with some basic arcs, forms a unique cycle because basic arcs form a tree. If the entering arc is on its lower bound in the current basic solution, to determine the leaving arc, we keep increasing the flow of the entering arc along its direction by θ units until the value of an arc in the cycle reaches its bounds, which will leave the basis. Then, the flows on all arcs in the cycle are increased by θ units in the direction of the entering arc to complete one pivoting with one arc entering the basis and one arc leaving the basis. The leaving arc could be the entering arc itself if θ is equal to the upper bound of the entering arc. It is Vice Versa if the flow of the entering arc is on its upper bound.

When the network involves equal flow constraints, the pivoting procedure is modified and the following three different cases are considered. In the following discussion, we assume the entering non-basic variable is on its lower bound and therefore we consider an increase of its flow by θ. If the entering non-basic variable is on its upper bound, we just modify all steps by considering a decrease of its flow by θ.

Case 1. The entering arc is $f_{ij}, i \in T_h, j \in T_h$ for some h.

In case 1, the entering arc has both node *i* and node *j* belonging to the same tree T_h . In this case, the pivoting procedure will be performed in tree T_h only, and all other trees will not be affected. The whole process including cycle identification and leaving arc determination is the same as the Simplex method for the regular minimum-cost network problem.

Case 2. The entering arc is $f_{ij}, i \in T_h, j \in T_q$ *and* $h \neq q$.

If the entering arc $(i,j) \in \bar{A}$ and node i and node j belong to different trees, increasing the flow over arc (i, j) by θ (for the situation that $f_{ij} = 0$) will affect both tree h and tree q. Thus, after introducing the new arc (i, j) into the basis, $b(T_h)$ will decrease by θ and $b(T_q)$ will increase by θ. Based on equation (25), the values on equal flow arc sets will change as follows.

$$D\begin{pmatrix} f_1 \\ \vdots \\ f_r \end{pmatrix} = \begin{pmatrix} b(T_1) \\ \cdots \\ b(T_h) - \theta \\ \cdots \\ b(T_q) + \theta \\ \cdots \\ b(T_r) \end{pmatrix} \tag{37}$$

After obtaining the new flows on the equal flow arcs, we need to update the flows on all trees after updating the supply and demand on the nodes connecting the trees and equal flow arcs. In algorithm development, we first set $\theta = 1$ and calculate its impact on all basic variables, including both $f_{ij}, (i,j) \in T_h, h = 1, \dots, r+1$ and $f_s, s = 1, \dots, r$. A ratio test is then conducted to see which basic variable will reach its bound first when we increase the value of θ and call this basic variable the exiting variable. All basic variables' values are updated with the determined θ and finish a pivoting.

Case 3. The entering variable is f_s, for one $s = r+1, \dots, p$.

When f_s is increased by θ, the net supply of each tree T_h, $b(T_h)$, will decrease by $\theta \sum_{m \in T_h} a_s^m$. Therefore, the flows of all basic equal flow sets should be updated as

$$D\begin{pmatrix} f_1 \\ \vdots \\ f_r \end{pmatrix} = \begin{pmatrix} b(T_1) - \theta \sum_{m \in T_1} a_s^m \\ \vdots \\ b(T_r) - \theta \sum_{m \in T_r} a_s^m \end{pmatrix} \tag{38}$$

With the changes of equal flow arc flows, including the entering variable and current basic variable, the flows of other basic arcs, $f_{ij}, (i,j) \in T_h, h = 1, \ldots, r+1$ are updated. Following the same procedure in Case 2, the exiting variable could be determined and all basic variables' values could be updated.

Computational results

The numerical experiments are conducted on the second-stage sub-problem of SNPOP introduced in the case study in Chapter 2, which is a network flow problem with equal flow side constraints. In Chapter 2, the problem is solved directly by CPLEX. Here, we convert the second-stage sub-problem of SNPOP into a network structure and solve it with a RNS Simplex algorithm (RNS) for equal flow constraints described in this chapter. The RNS algorithm was programmed by Visual C++. Both the CPLEX solver and the revised network Simplex algorithm are tested on a PC that has Intel(R) Core (TM)2 CPU with 2.40 GHz and 2.39 Ghz, 2.00GB of RAM under Windows XP professional operation system.

Seven instances with different sizes are tested to compare the efficiency of the two algorithms. The network is increased over instance by increasing the number of temporary shelters (ts), the number of potential permanent shelters (ps) and the number of existing permanent shelters (es), the number of distribution centers (n), types of resources (l) and the number of affected areas (t) in Table 8. Among these seven networks, the smallest network consists of 55 nodes and 285 arcs and the largest network is constructed by 352 nodes and 1,596 arcs.

Table 8 Test problems

	# Nodes	# Arcs	*ts*	*ps*	*es*	*n*	*l*	*t*
Problem 1	55	285	5	5	5	4	2	2
Problem 2	100	420	10	10	10	4	2	2
Problem 3	190	840	20	20	20	4	2	2
Problem 4	235	1050	25	25	25	4	2	2
Problem 5	283	1274	35	26	30	4	2	2
Problem 6	316	1482	45	26	31	4	2	2
Problem 7	352	1596	57	26	31	4	2	2

Both the RNS and CPLEX solver provide the same optimal solution but the RNS algorithm is more efficient in terms of the average CPU time. For instance, the RNS algorithm takes 0.122 seconds to solve the largest instance, instance 7, a nearly 91.87% improvement compared with CPLEX solver.

Table 9 Computational results

Instance	Optimal Objective Value		Average CPU Time (Sec.)	
	RNS	CPLEX	RNS	CPLEX
1	$ 942,664.00	$ 942,664.00	0.000	1.360
2	$ 1,252,880.00	$ 1,252,880.00	0.011	1.375
3	$ 1,540,800.00	$ 1,540,800.00	0.041	1.375
4	$ 1,803,020.00	$ 1,803,020.00	0.065	1.390
5	$ 2,325,250.00	$ 2,325,250.00	0.081	1.406
6	$ 2,946,690.00	$ 2,946,690.00	0.098	1.406
7	$ 3,697,570.00	$ 3,697,570.00	0.122	1.500

Please note that the second-stage problem has to be solved for many times when using the L-shaped method to solve the SNPOP. The time saving from the RNS method could improve the overall computational efficiency for the L-shaped method solving the SNPOP. The improvement will allow us to attack SNPOP instances with larger network and more scenarios.

CHAPTER IV

PROBLEM EXTENSION AND APPLICATIONS

In the previous chapter, the network Simplex is modified to solve the minimum-cost network flow problems with equal flow side constraints. The numerical results demonstrate that the revised network Simplex outperforms the CPLEX solver when solving the second-stage sub-problem of SNPOP in terms of computational time. The revised network Simplex algorithm is mainly based on Calvete (2003)'s work, which is for general minimum-cost network flow problems. We believe the second-stage sub-problem of SNPOP has some special properties that can be used to further improve the algorithm so that a larger SNPOP with more areas and more scenarios can be solved. In this chapter, the properties will be identified for future improvement. In addition to the sheltering network planning and operation problem, the general structure of the network with equal flow side constraints can be extended to many other fields in which equal amount of entities are required. In the chapter, the minimum cost network flow problem with echelon equal flow constraints is defined and studied.

Minimum cost network flow problem with echelon equal flow constraints

Consider a directed network $G = [N, A]$, where $N = \{1, \ldots, n\}$ is the set of nodes and $A = \{(i,j): i,j \in N\}$ is the set of directed arcs. u_{ij} and c_{ij} are the upper bound and the unit flowing cost from node i to node j for $\forall (i,j) \in A$. N is comprised by two exclusive sets N^L and N^R such that $N^L \cup N^R = N$ and $N^L \cap N^R = \emptyset$. Let $A^L = \{(i,j) \in A: i,j \in N^L\}$ and $A^R = \{(i,j) \in A: i,j \in N^R\}$. N^R is further comprised by

q exclusive sets $N^{R,1}, \cdots, N^{R,q}$, where $N^{R,1} \cup N^{R,2} \cup \cdots \cup N^{R,q} = N^R$ and $N^{R,t} \cap N^{R,u} = \emptyset, \forall t, u = 1, \ldots, q$. Let $A^{R,t} = \{(i,j) \in A^R : i,j \in N^{R,t}\}$, where $t = 1, \ldots, q$. We assume that $A^R = \cup_{t=1,\ldots,q} A^{R,t}$. Let $G^L = [N^L, A^L]$ and $G^{R,t} = [N^{R,t}, A^{R,t}]$, where $t = 1, \ldots, q$. Here, G^L, $G^{R,1}$, …, and $G^{R,q}$ do not communicate with each other in G. The network is illustrated in Figure 8. To consider the equal flow constraints, let $A_1, \ldots, A_p$ be subsets of A. The equal flow constraints require that $f_{ij} = f_s, \forall (i,j) \in A_s, s = 1, \ldots, p$, where $f_{ij} = f_s$ is the flow along arc (i,j). Each subset A_s connects one node in N^L (denoted by n_s^L) and q nodes in N^R, one from each of $N^{R,1}, \cdots, N^{R,q}$ and is denoted by $n_s^{R,t}$, where $t = 1, \ldots, q$. Please note that in the network under study, two arcs from different sets of $A_1, \ldots, A_p$ do not share any common nodes. In order to facilitate the analysis and create the first basic feasible solution, we add (q - 1) artificial arcs to connect q nodes of $n_1^{R,1}, \cdots, n_1^{R,q}$ and call this set of arcs A'. Please note that $u_{ij} = 0$ and $c_{ij} = 0$ for $\forall (i,j) \in A'$.The new network is denoted by $G' = [N, A \cup A']$ and will be used in the following analysis.

Let $c_s = \sum_{(i,j) \in A_s} c_{ij}$ and $u_s = \min_{(i,j) \in A_s} u_{ij}$, $s = 1, \ldots, p$. Define a_s^i the number of arcs in A_s outgoing from node i minus the number of arcs incoming to node i. Therefore,

$$a_s^i = \begin{cases} q & i = n_s^L ; \\ -1 & i \in \{n_s^{R,1}, \ldots, n_s^{R,q}\}; \\ 0 & \text{otherwise.} \end{cases} \tag{39}$$

After defining $\tilde{A} = A' - \cup_{s=1,\ldots,p} A_s$, we can represent the two-echelon equal flow problem with the following linear program (39 - 42) (Calvete, 2003).

Minimize

$$\sum_{(i,j)\in\tilde{A}} c_{ij} f_{ij} + \sum_{s=1}^{p} c_s f_s \qquad (40)$$

S.T.

$$\sum_{\{j:(i,j)\in\tilde{A}\}} f_{ij} - \sum_{\{j:(j,i)\in\tilde{A}\}} f_{ji} + \sum_{s=1}^{p} a_s^i f_s = b_i \qquad \forall i \in N; \qquad (41)$$

$$0 \leq f_s \leq u_s \qquad s = 1, \dots, p; \text{ and} \qquad (42)$$

$$0 \leq f_{ij} \leq u_{ij} \quad \forall (i,j) \in \tilde{A}. \qquad (43)$$

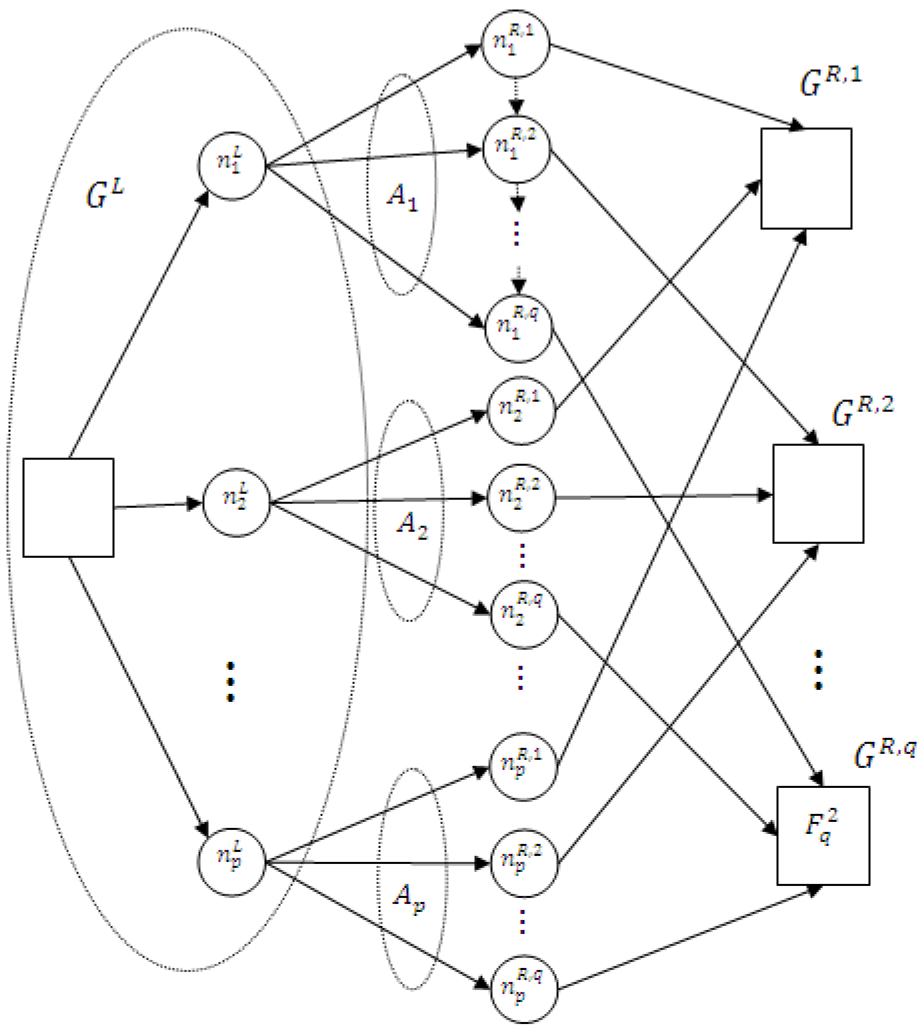

Figure 8 Equal flow network structure

The model (39 - 42) has n constraints and the rank of the technical coefficients of (40) is $n-1$. According to Calvete (2003), when r of the p equal flow variables f_s are in the basis, the remaining basic arcs in $\tilde{A}$ form r+1 spanning trees (i.e., there are $n-r-1$ basic variables are from f_{ij}). Because N^L and N^R do not communicate in the network of $G = [N, \tilde{A}\ ]$, at least of one of equal flow variables f_s is in the basis (i.e., basic arcs in $\tilde{A}$ form at least 2 spanning trees).

Thereon: For any basic feasible solution to the minimum cost network flow problem with echelon equal flow constraints defined by (39 - 42), all f_{ij}, $(i,j) \in A'$ are basic variables.

Proof: When r=1, there are two spanning trees, one belonging to G^L and the other belonging to $[N^R, A^R \cup A']$. Because $G^{R,t} = [N^{R,t}, A^{R,t}]$, where $t = 1, \dots, q$ are connected to each other only through arcs in A', all f_{ij}, $(i,j) \in A'$ must be the basic variable in order to have a single tree in the graph of $[N^R, A^R \cup A']$. During one pivoting procedure, no matter which non-basic variable is selected to enter the basis, when we increase (or decrease) this non-basic variable by θ, all f_{ij}, $(i,j) \in A'$ will stay the same so that all f_{ij}, $(i,j) \in A'$ will not leave the basis. Note that only equal flows and arcs in A' connect each sub-network $G^{R,t}$ with the remaining nodes in N. Because any flows on the equal flows have the same amount of flow-in (or flow-out) for all sub-networks $G^{R,t}$ where $t = 1, \dots, q$, any change of f_{ij}, $(i,j) \in A'$ from 0 will fail the flow balance of sub-networks $G^{R,t}$, which is true before a pivoting.

In order to study the characteristics of a basic feasible solution in which r of $\{f_1, \dots, f_p\}$ are basic, we define the following matrix D, whose dimension is $(r+1) \times r$. Each row corresponds to one spanning tree of basic arcs in $\tilde{A}$, and each column corresponds to one equal flow variable. Without any loss of generality, we assume $f_1, \dots, f_r$ are basic and the first k spanning trees belong to G^L, called $T_1^L, \dots, T_k^L$. We assume T_1^L contains n_1^L. Furthermore, let T^a denote the tree containing the arc set A'. The remaining trees are called $T_1^R, \dots, T_{r-k}^R$. Obviously, $T^a, T_1^R, \dots,$ and T_{r-k}^R belong to the network of $[N^R, A^R \cup A']$.

$$D = \begin{pmatrix} \sum_{i \in T_1^L} a_1^i & \cdots & \sum_{i \in T_1^L} a_s^i & \cdots & \sum_{i \in T_1^L} a_r^i \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \sum_{i \in T_k^L} a_1^i & \cdots & \sum_{i \in T_k^L} a_s^i & \cdots & \sum_{i \in T_k^L} a_r^i \\ \sum_{i \in T^a} a_1^i & \cdots & \sum_{i \in T^a} a_s^i & \cdots & \sum_{i \in T^a} a_r^i \\ \sum_{i \in T_1^R} a_1^i & \cdots & \sum_{i \in T_1^R} a_s^i & \cdots & \sum_{i \in T_1^R} a_r^i \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \sum_{i \in T_{r-k}^R} a_1^i & \cdots & \sum_{i \in T_{r-k}^R} a_s^i & \cdots & \sum_{i \in T_{r-k}^R} a_r^i \end{pmatrix} \tag{44}$$

Additionally, the D matrix has the following characteristics:

1. $D_{ms} \in \{0, q\}\ \forall s = 1, \ldots, r$ and $\forall m = 1, \ldots, k$.
2. $\sum_{m=1}^{k} D_{ms} = q, \forall s = 1, \ldots, r$.
3. $D_{ms} \in \{0, -1\}, \forall s = 1, \ldots, r$ and $\forall m = k + 2, \ldots, r + 1$.
4. $\sum_{m=k+1}^{p+1} D_{ms} = -q, \forall s = 1, \ldots, r$.
5. $D_{k+1,1} = -q$.

When r=1, there are two spanning trees. One belongs to G^L, and the other belongs to $[N^R, A^R \cup A']$. So D is a 2×1 matrix as

$$D = \begin{pmatrix} q \\ -q \end{pmatrix} \tag{44}$$

Construct D' by removing the $(k+1)^{\text{th}}$ row from $\boldsymbol{D}$. Note that the $(k+1)^{\text{th}}$ row corresponds to the tree including all f_{ij}, $(i, j) \in A'$. Based on Theorem 2 in Calvete (2003), $\text{rank}(D') = r$. Note the first column of D' is $(q \quad 0 \quad \ldots \quad 0)^T$. The above prorperties could be used to develop efficient algorithms to solve the minimum cost network flow problem with echelon equal flow constraints.

Potential applications

The minimum-cost network flow problem with equal flow constraints can be applied to many fields. In urban water demand problems, a simple equal flow constraint can be added in different time horizons to determine the maximum dimension of water demand center and then to minimize the total water allocation cost (Manca et al 2008). Another example is the irrigation problem. Usually, the irrigation problem requires proportional water demands during subsequent time periods. We can treat the time-dependent proportional water flows in the same way as equal flows to address the dimensions of water demand centers (Manca et al 2008).

The application of equal flow constraints can also be implemented to solve water scarcity problems. In critical cases, like drought and chemical pollution, insufficient water supply will happen in reservoirs. To satisfy the demands in demand areas and to avoid infeasibility of the problem, dummy nodes are added, and corresponding arcs connecting demand nodes and dummy nodes are established to allow water shortage (Sechi and Zuddas, 2008). To minimize the shortage, heavy costs are associated with those arcs. However, by adding the dummy arcs, we can only guarantee the dummy flow can only be used during the water scarcity case. We still could not manage the water shortage. In order to manage shortages based on different water supply priorities and to apply certain deficit rules instead of just adding heavy cost on shortage arcs, equal flow or proportional flow constraints in different time horizons can be added to the optimization model to manage flow assignments during an extreme case.

Another important application of the minimum-cost network flow problem with equal flow constraints is in the airplane industry. Verweij et al (1997) investigated a problem that airplane production plants and assembly plants are located separately. From the production plants to the assembly plants, airplane parts need to be transported by a special transport aircraft. Because of the restrictions of the transport aircraft, the parts have to be organized and transported as a pre-specified combination. Therefore, equal flow constraints are applied to reflect the part flows. The objective of this problem is to investigate an optimal transportation strategy such that the total flying time of the transport aircraft is minimized.

We believe the minimum-cost network flow problem with equal flow constraints can also be implemented in many other fields, such as supply chain network management and assembly line scheduling. For example, an automotive assembly line needs to manage its part supply chains simultaneously with multiple part suppliers. The orders of different types of parts are also dependent on production plans. For example, producing a car may require four wheels, one engine, one transmission, etc. The flows of all parts, with normalized units, should be equal to guarantee the assembly requirement. In other words, we can manage the whole supply network as a minimum-cost network flow problem with equal flows that connect sub-networks for each part (or assembly) type. The whole network also has the above two properties discussed in this chapter. This dissertation study is expected to identify the application areas of the minimum-cost network problem with equal flow constraints and test the revised network Simplex in those applications.

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