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A Theoretical Analysis of Multiproduct Mergers: Application in the Major Meat Processing Sectors

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By

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A theoretical analysis of multiproduct mergers: Application in the major meat processing sectors.

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The research is motivated by the significant increase in multiproduct mergers in the meat-protein processing sector, whereby the largest firms now process beef, pork, and chicken. This thesis conducts a theoretical merger analysis, accounting for both within- and across-submarket substitution of demand related goods. The model developed is suitable for analyzing markets in which there are identifiable consumer submarkets within a larger market.

The results indicate two primary findings. The first finding is that Bertrand firms have a unilateral incentive to merge. Firms involved in a given merger increase profit, as well as those not included in the merger. Second, it is found that without sufficient realized scope economies by the merged firm, significant anticompetitive price increases are likely. However, as substitutability within and across submarkets tend towards each other in magnitude, the required cost reductions for welfare neutrality increase vastly. Additionally, guidelines for future empirical analysis are discussed.

DEDICATION

This thesis is dedicated to my family and Rachelle Thomason, whose unwavering love, support, and understanding allowed me to complete this challenge. I could not have done it without you.

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I'd like to thank my friends, family, professors, staff, and fellow graduate students that have made this research successful. Without your help, this would have been an impossible endeavor. First, I'd like to thank my major professor, Dr. Coatney. Your support along the way has been invaluable. This work would not have been possible without your mentorship. I'd also like to thank Dr. Tack and Dr. Parman for providing support and valuable insight into this effort. Your help has been greatly appreciated.

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CHAPTER I

INTRODUCTION

> "Tyson Foods has a multi-protein business model. We produce about one out of every five pounds of chicken, beef and pork in the United States along with a broad portfolio of prepared foods…".
>
> Tyson Foods, Inc.'s Fiscal 2013 Fact Book (Tyson Foods, Inc., 2014)

The preceding statement published by a leading processor of beef, pork, and chicken in the United States illustrates the meat protein industry's belief that the relevant consumer product market in the U.S. is that for 'meat-protein'. The objective of producing all meat-proteins is not new, but the realization of these multiproduct firms is relatively new in the history of mergers and acquisitions, within the meat-protein sector.

Beef, pork, and chicken are the primary meat-protein sources for American consumers. The United States Department of Agriculture's Economic Research Service (USDA-ERS) calculated that 2013 per capita availability[1] of beef was 53.6 boneless pounds, pork was 43.4 boneless pounds, and chicken was 57.7 boneless pounds (USDA-

[1] Per capita availability is a commonly used proxy for U.S. food consumption (USDA-ERS, 2015b; Bentley, 2012). Bentley (2012) describes per capita availability, "Per capita estimates are calculated by dividing the total annual supply of a food by the U.S. population for that year. Although these estimates do not directly measure actual quantities eaten, they provide an indication of whether Americans, on average, are consuming more or less of various foods over time."

ERS, 2015a). Figure 1.1 provides per capita availability for beef, pork, and chicken from 1963 to 2013, in boneless weight equivalents. Figure 1.1 is adapted from the work of Bentley (2012), published in USDA's Amber Waves Magazine, using more recent data.

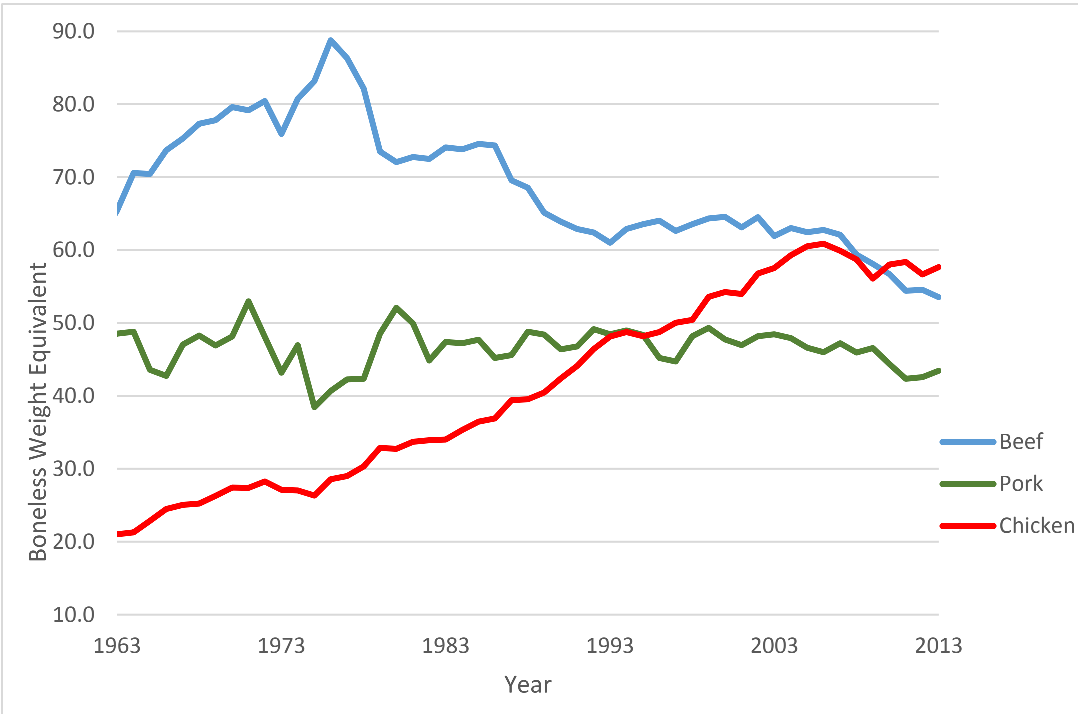

Figure 1.1 1963-2013 U.S. Per Capita Meat Availability: Beef, Pork, and Chicken

Source: USDA-ERS

In recent years, there have been several mergers within and across the three major meat-protein industries; beef, pork, and chicken. Today, two of the three top meat processors in the United States, Tyson Foods, Inc. and JBS USA, maintain significant market shares in beef, poultry and pork processing (Tyson Foods, Inc., 2016; JBS, 2016). Further, Cargill, Inc. remains a significant competitive force in the beef industry, as well as competing in the domestic turkey industry (Cargill, 2016).

On October 30, 2015, JBS SA purchased Cargill's pork division with no comment or restriction from the Department of Justice (Reuters, 2015). Given the high percentage of domestic consumption of beef, pork, and chicken, consumers are at risk of being negatively impacted by increased market concentration and changing market structures in the meat protein processing sector. One reason for these impacts is that demand for meat protein (and by extension the derived demand to processors) is necessarily more inelastic than that for any one sub-product. Hence, there is a potential for market power extension and higher aggregate meat prices.

To understand the potential competitive implications of multiproduct mergers, it is important to first understand how the Department of Justice (DOJ) has historically evaluated the potential for economic harm from multiproduct mergers. In any merger analysis, the definition of the relevant market is a critical step, and frequently the most contentious when courts evaluate the competitive implications of a challenged merger. The relevant market has two dimensions, product space and geographic location[2] (DOJ, 2010). The predicted competitive implications of a merger are necessarily impacted by how broadly or narrowly the relevant market is defined. For instance, if the relevant market is narrowly defined, the number of competitors is necessarily reduced and increases the market concentration and the potential for economic harm from a merger. As a result of this, the firms proposing the merger will contend that the relevant market is much broader and thus the merger has little potential for economic harm.

[2] Refer to the 2010 Horizontal Merger Guidelines (HMG) (DOJ, 2010) pages 7-14 for detailed information regarding the market definition concepts of product space and geographic location.

Problem Statement

Little research has addressed the impacts of any multi-product merger in the meat protein sector. In past meat processing mergers, the DOJ has restricted the definition of the relevant product market to include only beef, pork, or chicken. For example, when the DOJ challenged the JBS-National Beef merger, they considered only the competitive effect in beef processing, primarily on the potential for a lower price paid to local producers and higher beef prices to consumers, as seen in the DOJ statement of the abandonment of the merger (DOJ, 2009). No mention was made towards the effects the merger would have on competition, if allowed, in the other meat protein markets. A limited number of empirical multiproduct merger analyses exist, and still even fewer generalizable models exist for such an analysis.

Economic Research Contributions

The primary objective of this analysis is to develop a theoretical model to analyze the competitive implications of multi-product mergers and their potential welfare impacts. The resulting theoretical model will then be applied to the major meat processing sectors. Further, this model generalizes to k differentiated markets, with j unique submarkets within the larger market. The results of this work are intended to identify conditions in which multiproduct mergers reduce competition and welfare. Additionally, this research is intended to provide guidance for future merger simulations and econometric analysis, first in the meat processing industry but applicable to any differentiated product industry.

CHAPTER II

HISTORICAL AND LEGAL CONTEXT OF THE ISSUE

This chapter provides insight into the historical aspects of the beef, pork, and chicken processing industries. Their past conduct and practices have shaped how these industries are viewed today. Understanding the origins and early history of the meat processing industries is fundamental to understanding how mergers and acquisitions will affect the structure and conduct of those engaged in these industries. Azzam and Anderson (1996) and Azzam (1998) also provide historical perspectives of meat processing firms and a review of the relevant economic methods.

To understand the economic concepts involved is not sufficient for proper analysis. The legal environment must also be considered. This chapter provides background about some the applicable legal concepts. Understanding the legal environment of antitrust is essential because antitrust policy issues and merger analysis are largely legal issues that are heavily informed and guided by sound application of economic principles.

A Short History of the Meat Processing Industries

Early History of the Beef and Pork Processing Industries

The meat sector has an intriguing history in the United States. This sector has long been associated with competitive concerns. Francis Walker (1906) provided a thorough background of the early meat-packing sector. Walker describes that in the early 1900's, four firms (known as the “Beef Trust”) dominated both the beef and pork processing industry, primarily in the Midwestern United States. The Beef Trust consisted of several dominant firms that owned the stockyards where producers sold their livestock, processing facilities that slaughtered both beef and pork, rendering plants, and hide plants. The Beef Trust also controlled cold storage warehousing, refrigerated rail cars, marketing resources, and other distribution systems. This market structure brought concerns by livestock producers, consumers, and federal regulators regarding price-fixing and other anticompetitive actions extending to unfair treatment of workers and unsanitary production conditions. Walker goes on to detail that on March 20, 1905 a grand jury indictment was given for violation of the “Anti-Trust Act”. However, the meat-packers were not prosecuted due to technicalities surrounding immunity and subpoena (Walker, 1906). Walker concludes with the following:

> “In the report on the “Beef Trust” the country has shown that the popular opinions respecting the wholesale prices of beef and the profits in the beef industry were founded on ignorance and error. At the same time it prosecuted the packers for alleged violation of the anti-trust law. The failure of the Government in that undertaking was not due to any lack of zeal and energy, but to a technical legal obstruction which no one could foresee—the opinion of one judge on a new point of law; and if that obstruction had not appeared, the case would have been fought to a finish on its merits.” (Walker, 1906).

Later, in 1918, President Woodrow Wilson received a report from the Federal Trade Commission (FTC) detailing the extent of continued anticompetitive actions by the meat packing companies (FTC, 1918). The FTC found significant abuses of market power; unethical, anticompetitive, and often illegal business practices; and evidence of collusion among meat packers. The FTC recommended that the government take over much of the railroad sectors utilized or owned by the packers (rolling stock, stockyards, refrigerator car equipment and operations, cold storage facilities, etc.) and establish fair marketing and storage systems(FTC, 1918). In 1920, the largest meatpackers signed a consent decree with the DOJ agreeing to liquidate all activities except meat processing. (Fewster, 1930). Soon after, the Packers and Stockyards Act of 1921 was enacted into law, giving the Secretary of Agriculture regulatory authority over meat packers and live poultry dealers (Folsom, 1980).

Early History of Poultry Processing

The poultry processing industry also has unique beginnings. According to the National Chicken Council (National Chicken Council), it was largely a subsistence-level activity until the 1920's and 1930's. Until that time, broilers were largely a byproduct of egg production. Large broiler production operations began to gain in popularity in the 1940's. At first, these were separated from feed mill operations and processing centers. Quickly, the poultry industry began to show signs of vertical integration (National Chicken Council, 2015). Currently, companies in the poultry industry are vertically integrated; a typical poultry firm consists of hatcheries, feed mills, chicken houses, and processing plants.

Recent Consolidation and Concentration in the Meat Processing Industries

Beginning in the 1970's and 1980's, consolidation and concentration have been on the rise within the beef processing industries, while the pork industry has experienced increased consolidation with moderate concentration increases, as well as a significant shift toward vertical coordination (MacDonald et al., 2000; Azzam, 1998). Ollinger et al. (2000) argues that there has been increasing consolidation in the poultry industry as well, although the four firm concentration ratios are not unacceptably high. The authors argue that the impacts of increased concentration may be limited by large increases in demand, both domestic and abroad. Additionally, in the poultry sector, the need for quality birds and increasing size of poultry plant facilities has led to vertical integration (Ollinger et al., 2000).

Consolidation in these industries has given rise to concerns about market power by some, including those in government. An example of this is the Senate Hearings on Agricultural Market Concentration (2001), at which several notable individuals testified concerns about concentration and market power in agriculture, including the beef packing industry. Interestingly, a vast majority of the literature supports that increased concentration in beef packing has occurred for reasons other than purely gaining and exerting market power. For example, MacDonald et al. (1996), MacDonald et al. (2000), and Ollinger et al. (2000), discuss the increasing economics of scale as a primary reason why these industries have become more concentrated and processing plant sizes have increased.

Four-firm concentration ratios, which measure the percentage of the market slaughtered by the four largest firms, provide a widely used measure of market concentration. GIPSA provides the following information: the largest four firms (in their respective industries) accounted for 85 percent of steer and heifer slaughter, 56 percent of cow and bull slaughter, 64 percent of hog slaughter, and 51 percent of broiler chicken slaughter (GIPSA, 2014).

Legal Context of Merger Analysis

Historically, mergers and acquisitions in meat processing were largely within animal species and allowed to continue unfettered until recently (DOJ, 2009). To evaluate horizontal mergers, The DOJ and FTC utilize the Horizontal Merger Guidelines, herein HMG, (DOJ, 2010). The HMG detail many of the conditions and tests used to determine if a merger is anticompetitive, increases market power unacceptably, or provides increased opportunities for collusion or exclusion (DOJ, 2010). Of these possible anticompetitive outcomes, increases in prices to consumers are often the greatest concern. With the rejection of the JBS/National Beef merger in 2009, the DOJ seems to have halted further concentration in the beef packing industry. The DOJ rejected the merger on the basis that it would lower prices received by producers and result in higher beef prices for consumers (DOJ, 2009).

Over the last 20 years, there have been more multiproduct mergers (i.e., horizontal mergers across species and related meat products). Firms engaged in one of the three meat-protein industries have merged with firms producing a different meat-protein, resulting in a new merged firm that is engaged in multiple major meat-protein industries. Historically, these multiproduct mergers have not been challenged. On October 30, 2015, JBS announced the completion of the acquisition of Cargill's Pork assets (Reuters, 2015). The DOJ did not challenge the merger. The DOJ may have considered the merger a conglomerate, a type of merger very infrequently challenged by the DOJ (Kolasky, 2001). The DOJ may have looked at the merger as simply a "change of ownership" in a separate market. It does appear that to date, beef, pork, and chicken products are not included in the same relevant product market by the DOJ or FTC.

The DOJ has in the past defined "beef" products markets as separable markets, as stated by then Deputy Assistant Attorney General Turetsky (1996):

> "Past analyses of mergers in the meat packing industry suggest that steer/heifer and cow/bull are usually distinct product markets for antitrust purposes, for example. This is because the kind of livestock used in each of these two markets is not readily suitable for use in the other."

Notwithstanding the fact that Mr. Turetsky appeared to be focusing on the input side of the meat packing industry, this statement emphasizes a clear preference for a smaller, more narrowly defined relevant market, when dealing with meat products. This further limits the possibility that beef, pork, and chicken might be included in the same relevant market by the DOJ or the FTC. Given its importance to the current problem, further discussion regarding defining the relevant product market is included in the next section.

Market Definition

The HMG detail many important aspects regarding the analysis of mergers. One of the major focal points for nearly any merger analysis is the 'relevant product market'. The HMG state that a relevant market is composed of two dimensions: product market definition and geographic market definition, utilizing the hypothetical monopolist test to help in determinations (DOJ, 2010). Given that the relevant market plays a critical role in the determination of competitive harm, there is much debate about the adequacy of the HMGs role in defining a relevant product market.

Remer and Warren-Boulton (2014) provide a timely analysis of recent developments of market definition use in merger analyses, including the consideration of differentiated products when the final consumer purchases the goods. The authors provide background about the modes of thought regarding market definition: the traditional structural approach reliant on a market definition and the more recent use of a direct, market effects approach using simulation. In the context of *United States v. H&R Block*, Remer and Warren-Boulton conclude that:

> "The H&R Block trial demonstrates that despite some desire in the antitrust community to move beyond the two-step approach to merger analysis, market definition is still an important part of presenting a case at trial. However, effects analysis, such as merger simulation, can be used as part of the market definition exercise, and therefore market definition and effects analysis can be viewed as complementary. Indeed, merger simulation was used by the DOJ for both the effects analysis and market definition, and the court relied on the results in reaching the conclusion that the merger was anticompetitive."

Additionally, Coate and Simons (2012) provide a thorough background on market definition. Coate (2014), in regards to the *H&R Block* Case (among others), also finds the following:

> "Although these markets are difficult to define, the replacement of fact with theory is problematic. As noted, additional competitive analysis in *H&R Block* could have identified an empirically supportable market or isolated customer niches at risk for price discrimination. Either approach would have allowed standard analysis to be undertaken without the need to apply the problematic

diversion model to effectively assume a narrow market. Further discussion notes that even if a narrow market is used and a monopoly or near monopoly structure is generated, fringe expansion and entry issues must be carefully addressed prior to concluding the merger is likely to substantially lessen competition."

Prior to *H&R Block*, several others were already questioning the traditional view of market definition. Farrell and Shapiro (2010) develop a method for considering differentiated products industries when unilateral effects are the concern. They provide an economics based test that would identify the likely anticompetitive effects resulting from such a merger. Kaplow (2011) strongly favors abandoning the use of the HMG market definition, opting in favor of methods more based in economic theory. He describes the HMG as beginning to show some openness to the idea of alternatives to the traditional market definition approach, although he concedes that the guidelines still tend towards using the traditional approach[3].

Coate and Simons (2012) counter these claims, stating that the traditional market definition practices are applicable, and offer insight into whether a merger may be anticompetitive, despite the arguments of Shapiro and Ferrell (2010) and Kaplow (2011). Zimmer (2016) attempts to define the "new" role for market definition as a combination of sorts, stating,

"The role of market definition is in a state of flux: instead of forming the point of departure for determining market shares, its future will be that of describing the

[3] Kaplow (2011) states, "Nevertheless, some controversy concerning the revised Guidelines questions their increased openness toward more direct, economically based methods of predicting the competitive effects of mergers. By contrast, this article suggest that, as a matter of economic logic, the Guidelines revision can only be criticized for its timidity."

competitive landscape. The application of modern methods of directly determining market power and competitive conditions also often requires the identification of those undertakings that exert competitive pressure on each other. In addition, an analysis of the degree of product differentiation and of the nature of completion that exists on the market is often necessary. All this can in the future be encompassed—in a broad sense of the term—by the definition of the relevant market."

Understanding the market definition concept serves to help understand the issues encountered when identifying a modeling construct in this thesis, as well as framing the problem in its appropriate antitrust context. Some of the ideologies of the noted scholars will be adapted to the current problem, such as the status quo role of DOJ's apparent market definition in the meat industries. This is accomplished by formulating the assumptions and conventions adopted in this thesis. First, it is assumed that the product market includes the three major meat protein sources (beef, pork, and chicken) due to these products being substitutes in retail demand (Capps, Jr., 1989) and the movement of industry toward this multiproduct structure. Previously, it appears the DOJ considers each meat product constitutes its own unique product market. The relevant market for meat-protein is considered the United States, thereby addressing the geographic component of market definition. To incorporate a methodology more in line with the newly embraced effects approach, mergers are simulated in order to see how scenarios generated by the new market definition may affect market power in a differentiated product market.

CHAPTER III

LITERATURE REVIEW

The existing literature relevant to this problem pertains to four primary areas: meat processing specific literature, horizontal merger analysis, multiproduct firm merger analysis, and differentiated products and differentiated product merger analysis. To begin, a select amount of literature focusing on mergers, acquisitions, and more generally, the market structure and major demand considerations relevant to the meat processing sector is discussed. Then, the horizontal merger analysis literature, provides the basic framework and tools utilized in merger analysis, is highlighted.

Finally, the literature regarding modeling of mergers in differentiated products and multiproduct settings are utilized in the development of the model herein. It is important to note that the streams of literature surrounding multiproduct firm competition and differentiated products analysis often work in tandem. That is, often these streams of literature will incorporate aspects of each other in their analysis. An easy example of this, and one discussed below is the work of Xu and Coatney (2015), who allow multiproduct firms to produce two demand related products that are differentiated from each other.

Mergers, Market Structure, Structural Considerations in Meat Processing

Formal justification for including beef, pork, and chicken as substitutes in a meat-protein market comes from Capps, Jr. (1989). Capps Jr. also demonstrates empirically the following using scanner data to analyze retail demand functions: “All other meat products excluding beef (nonbeef) are substitutes for roast beef, ground beef, and steak. Similarly, all other meat products excluding poultry (nonpoultry) are substitutes for chicken. All other meat products excluding pork are substitutes for pork chops, ham, and pork loin" (1989).

Nevo (2001) analyzed the ready-to-eat cereals industry assuming Nash-Bertrand competition for differentiated products. Using scanner data, Nevo estimated that high price-cost margins in the ready-to eat-cereals markets were due in part by firms maintaining several brands and the use of effective advertising. This is an early example of a highly concentrated, product-differentiated market in agricultural products, as well as an example of potential government concern about market concentration. Additionally, Nevo does two things that are relevant to this analysis. First, he obtains elasticities for each brand. This is relevant because the model created herein is suited for empirical estimation if the appropriate substitutability parameters can be estimated. Second, he then utilizes them in a Nash-Bertrand pricing game between the firms to analyze if they are reaching the collusive outcome. This thesis will also utilize Bertrand competition in the analysis.

Additionally, Nguyen and Ollinger (2006) study the meat processing industry to determine how productivity in both meatpacking and poultry plants are affected by mergers and acquisitions. They find that after a merger, both meat packing and poultry slaughter plants generally show an increase in productivity for most plant sizes compared to those who have not merged. They note that the largest meat packing plants and small poultry plants did not show these gains in productivity, as compared to those who had not merged. It is important to note that the authors organized the three types of industries by

SIC code (meat packing, prepared meat products, and poultry slaughter and processing). Nguyen and Ollinger make another important conclusion that plant closures and/or reselling was a distinct possibility after a merger or acquisition, which may have other welfare consequences that were not included in the scope of their analysis. Nguyen and Ollinger conclude that the evidence supports that mergers and acquisitions in these industries are driven by efficiencies and synergies.

Gallet (2010) conducted a meta-analysis on price elasticities of meat. Utilizing and accounting for various functional forms, publication types, and other potential biases to the elasticities, several important results are discovered. Gallet finds that the price elasticity of poultry is consistently less elastic compared to beef, lamb, and fish. Gallet's study also yields important information regarding the price elasticity of meat (he uses a composite variable for meat consisting of several meats). Gallet finds that the price elasticity of meat is susceptible to influence by three things in his meta-analysis: the demand specification employed, the method used for estimation, and characteristics of the journal in which the elasticity value was published. Gallet's work highlights the challenges of estimating consumer substitutability behavior estimates from price elasticity estimates.

Horizontal Merger Analysis

As previously discussed, horizontal mergers are between firms engaged in the 'same' market level, such as two beef-packing firms in the same geographic market. Hay and Werden (1993) provide a basic primer for analyzing horizontal mergers, pointing out some of the major issues surrounding horizontal mergers. They provide overviews about how Cournot, Bertrand, and dominant-firm models have been used in merger analysis.

They also discuss the potential for collusion, both overt and tacit, as being a concern associated with mergers. Additionally, Hay and Werden suggest that a merger policy should not be bound by only using calculable benefits and costs. Their discussion identifies that though the DOJ/FTC led merger investigations are primarily based on economics, other more qualitative aspects are incorporated into the determinations about competitive harm.

Salant et al. (1983) discover that in a Cournot setting, horizontal mergers may result in losses. In their analysis, only when at least eighty percent of firms collude, does the Cournot model result in a profitable merger. Facilitating this result are the following assumptions: identical Cournot-behaving firms, constant marginal costs, and a linear demand system.

Joseph Farrell and Carl Shapiro (1990) study horizontal mergers with homogenous goods using a Cournot oligopoly model. They find that horizontal mergers typically raise prices when no synergies are available. They also indicate that in order for a horizontal merger to lower price, significant realized economies of scale and/or learning must take place. However, they make clear that their results hold only when goods are homogenous and firms behave in the Cournot fashion. They leave for future research how their results will apply if products are differentiated. They also suggest that under the Cournot framework with homogenous goods, that it may be possible that mergers, which reduce output, may actually improve total welfare by removing less efficient firms from the industry.

Nocke and Whinston (2013) also apply a Cournot framework to horizontal mergers in order to discern an optimal merger policy. The authors accomplish this by extending Williamson's (1968) basic merger principle that even if some market power is gained, the merger must create efficiencies, which must have the ultimate effect of improving consumer welfare. Their results indicate that some larger mergers may need to be rejected in favor of smaller mergers that have larger increases on consumer welfare. The efficiencies' mentioned by Nocke and Whinston would come in the form of economies of scale in the Cournot framework.

Multiproduct Firm Competition and Mergers

Early attempts at understanding market power exertion in agricultural markets with multiproduct markets were conducted by Schroeter and Azzam (1990) in the U.S. beef and pork industries and Wann and Sexton (1992) in the California pear industry. Of the extensions Wann and Sexton make to Schroeter and Azzam, perhaps most important is that the output products were considered to be heterogeneous. These authors utilized conjectural variations frameworks. Also, these articles address market power in their specified industries, but do little to create a generalizable competitive model capable of analyzing multiproduct industries from a consumer utility perspective.

Generally, the literature related to multiproduct firms has not addressed the issue of merger analysis. Rather, the multiproduct literature discusses other aspects of merger analysis. Typically, when multiproduct firms are considered in an analysis, a property of their nature is being discussed.

For instance, Zhang and Zhang (1996) analyze the conditions encountered for stability to be achieved in related multiproduct markets of various compositions, assuming Cournot competition. The authors find that the conditions required for a stable equilibrium in one market do not necessarily translate in the overall equilibrium for all considered markets. Others have considered elements of strategy regarding the behavior of multiproduct firms, such as Symeonidis (2002), who analyzed whether or not cartels are sustainable in a multiproduct setting.

Yet others have analyzed other issues associated with multiproduct industries, such as De Fraja (1992) who analyzed how multiproduct structures affect optimal product line choices. De Fraja's results provide some interesting insight into why a meat-protein firm might choose to merge into another meat product. Specifically, De Fraja states, "As long as the economies of scale are non-negligible, a firm will never supply products which are very good substitutes. However, the negative effect on the length of the product line of the substitutability between products can be offset by high economies of scope: a firm may supply products which are very good substitutes if the extra cost involved by the broader product line is small."

An early compilation of the primary concepts applicable to multiproduct industries was achieved in Bailey and Friedlaender (1982). Their work reviews the state of the literature and highlights the contributions from authors associated with multiproduct firms, especially with respect to cost concepts.

Differentiated Product Analyses and Mergers

This literature review will primarily discuss product differentiation as it relates to merger analyses. For a more thorough review of the differentiated products literature, generally, consult Xu and Coatney (2015). Differentiated product mergers, as the name implies, are mergers between firms in the same industry that have some form of product differentiation. Product differentiation includes having branded products within a product group, improved service, quality, customer service or other ways by which firms can differentiate their products to the consumer (Shapiro, 1995). Product differentiation becomes pivotal to the current problem when the relevant market is considered the meat protein market instead of simply beef, pork, and chicken in isolation; the relevant market is now conceptually a differentiated protein market. The core change in perspective of how to define the relevant market is to identify how the consumer views the substitutability of these meat products.

Carl Shapiro (1995) states that products which are 'close' substitutes to each other offered from two potentially merging firms are especially conducive to producing anticompetitive price increases. He goes further to detail how to calculate diversion ratios, a rough measure for calculating price increases, for such a merger when better methods are not available. Closeness is of particular relevance for the meat industry because how 'close' substitutes chicken, pork, and beef are to each other will be shown to alter the impact a multiproduct merger will have on competition. Unfortunately, there is no standard for how 'close' is 'close enough' for inclusion in the relevant product market for merger analysis (Shapiro, 1995).

Baker and Bresnahan (1985) study mergers and/or collusion in an n-firm product differentiated market assuming Bertrand competition. To estimate their derived equations, they find the residual demand curves. Empirically, they find that both options would increase market power for the dominant firm after the merger, using the U.S. beer market as an example. This paper provides an illustrated example and background to the issue, but does not broach the subject of how consumer welfare would be affected.

Deneckere and Davidson (1985) investigate incentives for firms to form coalitions through mergers in a differentiated Bertrand oligopoly setting. They use Shubik's (1980) demand specification as a framework. Of note, they find that gross substitutability allows coalition members to raise price after a merger. Assumptions of their analysis include constant marginal costs and symmetric demand. The authors also limit their analysis to firm profits, and do not consider total welfare. Their analysis characterizes mergers in price setting games. Deneckere and Davidson state:

"Price setting games, on the other hand, seem to capture traditional industrial organization insights rather well. Under certain plausible conditions on the demand system, mergers are always beneficial to existing members and become more profitable as the size of the merger increases. The resulting industrial concentration confers large positive externalities on other industry members, so that coalitions producing a small number of varieties earn more than larger ones. Not surprisingly, short of antitrust policy, the industry would concentrate almost completely towards monopoly."

Mcelroy (1993) also investigated mergers in differentiated products industries. Mcelroy finds that without cost savings being possible, Bertrand duopoly with linear demand and marginal costs results in lower welfare after merger. Similarly, Mcelroy finds that after merger and no cost savings, Cournot duopoly with constant marginal cost results in lower welfare.

Hausman et al. (1994) evaluate differentiated product mergers in the beer industry with specific attention given to multi-product firms. The study identifies the impacts and importance of controlling for own-firm cannibalization, increased efficiencies on pricing of products, use of demand elasticities for each relevant product, competition between different market segments, and how demand for other products sold by the same firm and competing firms may be affected by the merger and resulting price changes. Additionally, the authors do include basic calculations for the effects of mergers on consumer welfare. Additionally, using empirical estimates, they calculate the required reduction in marginal cost to offset any changes in post-merger prices. Interestingly, they find that competition from different market segments in the beer industry is often enough to keep price increases limited after a hypothetical merger, even when no efficiency gains are present. Hausman et al. also show that other brands produced by the same firm (a multi-product firm) affect the price-cost margin markup. They point out that most previous analyses have not identified that some of the products that consumers switch to, with a price increase, may in fact be produced by the same firm.

Werden and Froeb (1994) expand the literature by using a logit model to study the welfare implications of mergers in a differentiated products industry under Bertrand competition using long distance telephone carriers as an example. They find that some mergers lessen welfare, while others have relatively little impact or slightly raise welfare, depending on the specific attributes of the merging parties. Their study includes a welfare analysis. However, the authors assume economies of scale and scope are not incorporated, although they include the ability for cost advantages to be incorporated. This analysis will preclude economies of scale from merger, but allow economies of scale to be obtained in application. Werden and Froeb also incorporate a framework that is easily used; requiring only market shares, prices and demand elasticity parameters.

Davis (2002) added to the literature by incorporating experimental economics to studying the effects of mergers in product-differentiated markets. They analyze both Bertrand and Cournot competition with different amounts of available information (basic amounts and extra information), using the DOJ's ALM (Antitrust Logit Model). They assume that all firms are identical and symmetric in a four firm oligopoly. They find that the ALM does reasonably well at predicting large increases in price, and suggest it may be a decent “screening tool”.

Xu and Coatney (2015) advance the differentiated products literature by easing a prevalent restriction in differentiated products modeling that the firm can only produce one product. By doing so, Xu and Coatney introduce the potential for a differentiated products market composed potentially of multiproduct firms. Although this article did not delve into merger analysis, it did serve as the cornerstone for the multiproduct, differentiated product model created and analyzed in this thesis.

Other authors (Dixit (1979); Singh and Vives (1984); Häckner (2000); others) also contributed to the development of the model and hence, will be discussed in the next chapter.

CHAPTER IV

MODEL DEVELOPMENT

In this chapter, the Bowley differentiated products utility function, Bowley (1924), is altered to facilitate various merger scenarios (cases for comparison). Though the Shubik-Levitan utility function is more appropriate when the analysis must consider the addition or subtraction of products (Martin, 2002), it is assumed that for the instant case (meat protein), no additional major categories can or will be created or eliminated as a result of a merger. Additionally, various attributes of the meat industry are explored to formulate which type of competition, Cournot or Bertrand, best describes competition.

Consumer Utility

The starting point for representing consumer utility in this analysis was first presented in Häckner (2000), which was an n-firm extension of the two product Bowley function (Bowley, 1924; Dixit, 1979; Singh and Vives, 1984). Early analyses include the assumption that the consumer only identifies each product by the firm that produces the product, and each firm produces only one product. Xu and Coatney (2015) relaxed this assumption to allow for firms to produce multiple products and the consumer identifies products by the firm producing each product. The Häckner representation, with minor notational adjustments here, is provided in (4.1).

$$U(\mathbf{q},Z)=\sum_{i=1}^{n}\alpha_i\,\mathrm{q}_i-\frac{1}{2}\left(\sum_{i=1}^{n}\beta_i q_i^2+2\theta\sum_{i\neq j}\mathrm{q}_i\mathrm{q}_j\right)+Z \tag{4.1}$$

In this form of utility, the α_i's represent the representative consumer's reservation prices and the β_i's are independent inverse demand slope parameters. For simplicity, it is assumed the α_i's are symmetric[4]. Though the slope of inverse demands may vary, this analysis follows Häckner's simplifying assumption that the β_i's are symmetric and normalized to one. The parameter $\theta \in [-1,\ 1]$ represents a symmetric product substitutability, where a value of -1 indicates perfect complements, 0 independent, and 1 perfect substitutes. Because the current analysis is applied to the meat sector, only the region of substitutes will be considered. Finally, Z is a composite numeraire good.

The most important modification to the previous model is that utility can be further refined to include the possibility of sub-markets within the aggregate market. A submarket would entail any subset of products whose attributes are considered to be 'relatively close substitutes' within a wider sector of consumer products. For example, the meat protein sector is comprised of several potential submarkets, such as beef, pork, and chicken. Each submarket is in turn comprised of competing firms, each producing aggregate composite of similar products from animal carcasses. Even if the quality characteristics of the composite products produced by each firm are viewed as homogenous to the consumer, some differentiation may be established by well-known branding, such as Tyson Foods, Inc. and Pilgrim's Pride Corporation chicken. Brand differentiation across beef firms is much weaker at the retail level, as these firms have yet to significantly brand their products. However, a small degree of indirect differentiation

[4] Varying the reservation prices not only complicates merger solutions, but will also detracts from the within- and across-submarket substitutability impacts on mergers.

may be attained via product quality and service provided to the retailer, regardless of branding.

To facilitate the analysis of submarkets, product substitutability is further broken down into two major components: within-submarket and across-submarket. The within-submarket substitutability will be denoted by θ, while the across-submarket substitutability will be denoted by δ . The δ parameter measures how substitutable a within-submarket firm's product is with those products not included in the same submarket. Because the consumer views subsets of products to be closer substitutes than other subsets, it is logical to assume that within-submarket substitutability is greater than the across-submarket substitutability. This relationship is formalized as $0 < \delta < \theta < 1$. An analogous interpretation is that the difference between two differentiated beef products is less than difference between beef and chicken products. The resulting general representation of utility is provided in 4.2.

$$U(\mathbf{q}, Z) = \alpha \sum_{i=1}^{n} \mathrm{q}_i - \frac{1}{2}\left(\sum_{i=1}^{n} q_i^2 + 2\theta \sum_{i \neq j} \mathrm{q}_i \mathrm{q}_j + 2\delta \sum_{h \neq k} \mathrm{q}_h \mathrm{q}_k \right) + Z \tag{4.2}$$

The utility identifies the $i \neq j$ firm combinations within-submarket and the $h \neq k$ firm combinations across-submarkets.

In the merger analyses that follow, it is assumed the DOJ has stopped all within-submarket increases in concentration, leaving two firms within each submarket. Additionally, it is assumed there are only three relevant submarkets (beef, pork, and chicken). To begin, each firm is assumed to produce only one product within its respective submarket, therefore, six firms in total comprise sector competition. The resulting set of pairwise within-submarket product combinations identified by firm are

$\{1,2\}$, $\{3,4\}$, and $\{5,6\}$. The resulting pairwise across-submarket combinations identified by firm are $\{1,3\}$, $\{1,4\}$, $\{1,5\}$, $\{1,6\}$, $\{2,3\}$, $\{2,4\}$, $\{2,5\}$, $\{2,6\}$, $\{3,5\}$, $\{3,6\}$, $\{4,5\}$, and $\{4,6\}$. The firm/product specific utility function is provided in (4.3).

$$U = \alpha \sum_{i=1}^{6} q_i - \frac{1}{2}\left(\begin{array}{l} \sum_{i=1}^{6} q_i^2 + 2\theta\left(q_1q_2 + q_3q_4 + q_5q_6\right) \\ +2\delta\left(\begin{array}{l} q_1q_3 + q_1q_4 + q_1q_5 + q_1q_6 + q_2q_3 + q_2q_4 \\ +q_2q_5 + q_2q_6 + q_3q_5 + q_3q_6 + q_4q_5 + q_4q_6 \end{array} \right) \end{array} \right) + Z \tag{4.3}$$

Consumers maximize utility subject to a budget constraint $\sum_{i=1}^{6} p_i q_i + Z \le I$, where I is income and the price for the composite good is normalized to one. The resulting system of firm/product specific linear inverse demands are provided in (4.4).

$$\begin{cases} p_1 = \alpha - q_1 - \theta q_2 - \delta\left(q_3 + q_4 + q_5 + q_6\right) \\ \ldots \\ p_6 = \alpha - q_6 - \theta q_5 - \delta\left(q_1 + q_2 + q_3 + q_4\right) \end{cases} \tag{4.4}$$

The corresponding system of linear demands is in (4.5).

$$\begin{cases} q_1 = \dfrac{\alpha\left(1 - 2\delta + 2\delta\theta - \theta^2\right) - \left(1 + 2\delta - 4\delta^2 + \theta\right)p_1 - \left(4\delta^2 - \theta - 2\delta\theta - \theta^2\right)p_2 - \left(\delta\theta - \delta\right)\left(p_3 + p_4 + p_5 + p_6\right)}{\left(2\delta - \theta - 1\right)\left(\theta - 1\right)\left(1 + 4\delta + \theta\right)} \\ \ldots \\ q_6 = \dfrac{\alpha\left(1 - 2\delta + 2\delta\theta - \theta^2\right) - \left(1 + 2\delta - 4\delta^2 + \theta\right)p_6 - \left(4\delta^2 - \theta - 2\delta\theta - \theta^2\right)p_5 - \left(\delta\theta - \delta\right)\left(p_1 + p_2 + p_3 + p_4\right)}{\left(2\delta - \theta - 1\right)\left(\theta - 1\right)\left(1 + 4\delta + \theta\right)} \end{cases} \tag{4.5}$$

It is now possible to formulate own- and cross-price elasticities as a function of the parameters, primarily θ, δ and α. Comparative statics of these elasticities for the premerger case will yield insight into their properties.

To find the own price elasticity for q_i , $i=[1...6]$ it is necessary to take the partial derivative of q_i with respect to p_i. The representative and symmetric derivative is provided in 4.6

$$\frac{\partial q_i}{\partial p_i}=\frac{-1-2\delta+4\delta^2-\theta}{(-1+2\delta-\theta)(-1+\theta)(1+4\delta+\theta)} \tag{4.6}$$

The resulting own price elasticity for firm one is provided in 4.7 as an example.

$$\begin{aligned}E_{d,q1}&=\frac{\partial q_1}{\partial p_1}*\frac{p_1}{q_1(p_1,...,p_6)}\\&=\frac{\left(-1-2\delta+4\delta^2-\theta\right)p_1}{\left(-1-2\delta+4\delta^2-\theta\right)p_1+\left(-4\delta^2+\theta+2\delta\theta+\theta^2\right)p_2-(-1+\theta)\left(\alpha(1-2\delta+\theta)+\delta\left(p_3+p_4+p_5+p_6\right)\right)}<0\end{aligned} \tag{4.7}$$

From the own-price elasticity in 4.7, several comparative statics are now conducted. However, to facilitate this process and maintain the premerger prices, several restrictions are required. These assumptions include that all prices are positive, all prices are equal to each other, and $\alpha > p_i$. It is found that both $\frac{\partial E_{d,q1}}{\partial \delta}<0$ and $\frac{\partial E_{d,q1}}{\partial \theta}<0$, indicating that increasing substitutability within or across submarkets would cause the own price elasticity to become more relatively elastic. Conversely, $\frac{\partial E_{d,q1}}{\partial \alpha}>0$ which shows that increasing the representative consumer's maximum willingness to pay would force the own price elasticity to become more inelastic.

Additionally, cross-price elasticities were calculated for q_1 with respect to p_1. This will allow for evaluation of the cross-price effects of the other good in the same submarket. Again, given symmetric demand only the derivative q_1 with respect to p_2 will be taken, and is provided in 4.8.

$$\frac{\partial q_1}{\partial p_2} = \frac{-4\delta^2 + \theta + 2\delta\theta + \theta^2}{(-1+2\delta-\theta)(-1+\theta)(1+4\delta+\theta)} \tag{4.8}$$

The cross price elasticity is provided in 4.9.

$$E_{d,q1,p2} = \frac{\partial q_1}{\partial p_2} * \frac{p_2}{q_1(p_1,...,p_6)} =$$
$$= \frac{\left(-4\delta^2 + \theta + 2\delta\theta + \theta^2\right) p_2}{\left(-1-2\delta+4\delta^2-\theta\right) p_1 + \left(-4\delta^2+\theta+2\delta\theta+\theta^2\right) p_2 - (-1+\theta)\left(\alpha(1-2\delta+\theta)+\delta\left(p_3+p_4+p_5+p_6\right)\right)} > 0 \tag{4.9}$$

As was the case for the comparative statics of own price elasticity $\frac{\partial E_{d,q1,p2}}{\partial \theta} > 0$ and $\frac{\partial E_{d,q1,p2}}{\partial \delta} > 0$, indicating that increasing substitutability within or across submarkets positively increases the cross-price elasticity. This is to be expected because increasing the substitutability between the two goods, increases the amount of the good switched when good 1 has a price increase. Alternatively, $\frac{\partial E_{d,q1,p2}}{\partial \alpha} < 0$, which indicates that if the representative consumer's maximum willingness to pay were to increase, then an increase in the price of good 1 would cause a small shift towards good 2. This follows intuitively because if the consumer were to value the good more (higher maximum willingness to pay) then, a marginal price change would cause a much smaller price change than if they did not value the good as much.

Firm Competition

Shapiro (1989) stated, “The choice between a pricing game and a quantity game cannot be made on a priori grounds. Rather, one must fashion theory in a particular industry to reflect the technology of production and exchange in that industry.” Following this logic, competition in the meat processing sector, be it Cournot or Bertrand, is identified in relation to supply chain characteristics within each meat industry. Specifically, the structure of the supply chain directly impacts the level of control the processing firm has over the quantity produced.

To begin, the biological production cycles for the three live animal inputs varies significantly and there are various levels of production sectors from conception to slaughter. According to Ward (1997), the biological production cycle for beef cattle is twenty-four months. The major production sectors before processing are cow-calf producers, stocker operators, and feeders. It is only the cow-calf producers that set quantity in the market. In stark contrast, the biological broiler chicken production cycle takes about five months, and the production system is fully vertically integrated, hence quantity produced is directly controlled by the processor in two major stages: hatching and growing. Pork production exists between these two extremes. The biological production cycle for pork is twelve months and the supply chain is moderately vertically coordinated. The supply chain generally is comprised of farrowing and finishing firms (Ward, 1997).

These production cycle and supply chain structures impact the processor’s ability to control quantity in response to changing prices. It appears that there is no clear identification which best describes competition among all competitors in the meat processing industry. However, due to the long production cycle in one (beef) and the breeding of two (beef and pork) are largely controlled by upstream suppliers (for the exception of chicken), it is assumed the processors are more prone to Bertrand competition. This is in stark contrast to the prevalent assumption of Cournot competition in the Agricultural Economics literature, even for beef processors (Schroeter and Azzam, 1990; Crespi et al., 2010, and others). It is of note that it is possible to allow some firms to be Cournot competitors and others to be Bertrand competitors[5] (Tremblay and Tremblay, 2011).

Given the assumption of competition and q_i is the firm/product specific demand, firms maximize the profit objective function provided in (4.10).

$$\pi_i = (p_i - c_i)q_i - F_i \tag{4.10}$$

Firms maximize the objective function by choosing their optimal output price p_i^*, subject to the reaction of their within- and across submarket rivals. It is also assumed that firms have reached economies of scale within each submarket from previous mergers and thus experience constant marginal costs, c_i, within the relevant region of production. Fixed costs are denoted by F_i and are assumed to equal zero. To ensure a solution exists, $\alpha > c_i$. This assumption is made such that the marginal cost of producing a product is not greater than the maximum the representative consumer would be willing to pay. Finally, α_i was previously assumed to be symmetric, and appears to be inconsistent with market price differentials across meat submarkets at relatively equal quantity demanded

[5] However, this approach was not pursued for the sake of tractability.

(Bentley, 2012). However, to maintain ‘relatively’ symmetric profitability across meat processors, it follows that c_i can also be assumed to be symmetric.

CHAPTER V

MERGER CASE ANALYSIS

This chapter uses the theoretical consumer model developed in Chapter IV to model several merger cases and the resulting impacts, in terms of prices, quantities, profit, and welfare, each merger would have. This chapter is organized in two parts. The first part provides a description of each of the merger cases being analyzed, including each Case's relevance to the analysis. The second part of this chapter reports, discusses, and compares the impacts of each merger case.

Merger Case Descriptions

Case I: Premerger

Case I, the premerger case consists of three submarkets with two firms in each of the submarkets. This will serve as the baseline, from which later calculations can be compared. In the context of this research, each submarket would represent a different meat protein submarket (i.e., beef, pork, or chicken) as a part of the larger meat-protein market. A visual representation of the market depicted in Case I is presented in Figure 5.1.

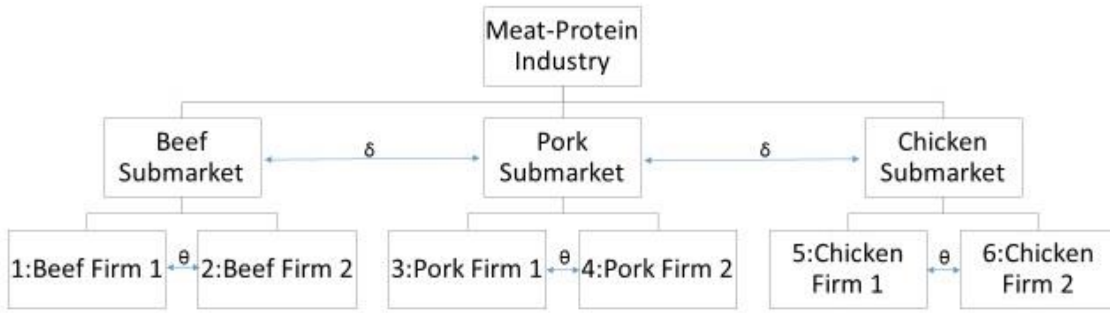

Figure 5.1 Case I: 3 Submarkets, 2 Firms per Submarket

This figure demonstrates the relationships between different actors in Case I. Also provided in the figure are the relevant substitutability relationships.

Case II: A Single Multiproduct Firm across all Three Submarkets

Case II depicts a scenario in which one firm has merged across each submarket, much like we see in these markets today, resulting in a multiproduct firm with contact in all three submarkets. It seems unlikely that the DOJ would challenge such a merger because it would be viewed a change of ownership in a new market. This stems from the traditional view that the each meat product makes up its own unique market. However, a cornerstone of this analysis is entertaining the notion of an alternative market definition encompassing the major meats in the meat-protein sector.

Now, each submarket consists of an entity operated by the multiproduct firm and a fringe firm, where the fringe firm is specific to that submarket. These relationships are provided in Figure 5.2. Mathematically, Case II is different from Case I in that the multiproduct firm is no longer maximizing profit for each product individually. Instead, they are now maximizing the joint profit for all products they produce.

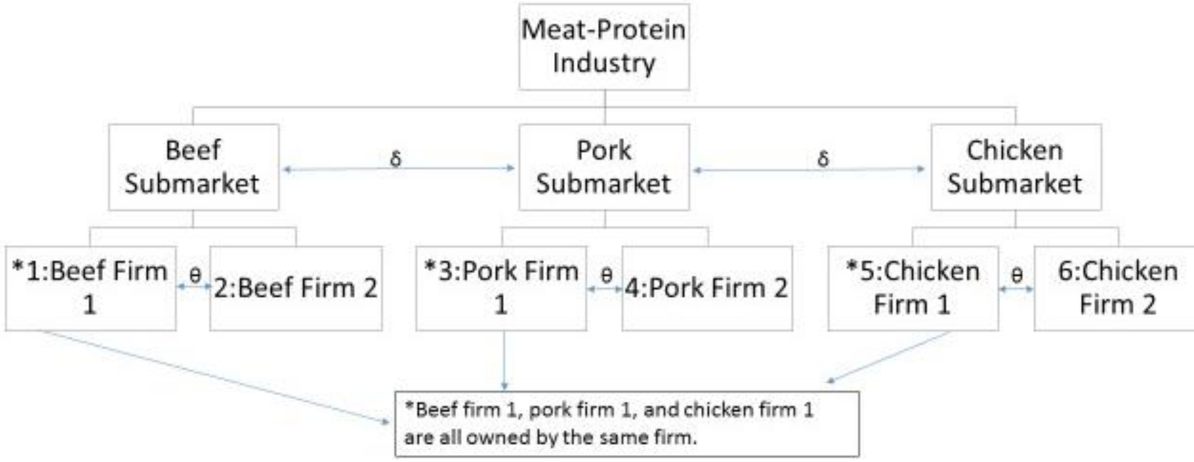

Figure 5.2 Across-submarket Merger by One Firm

Case III: Two Multiproduct Firms across all Three Submarkets

Case III depicts the next logical step from Case II. If one firm is allowed to merge across products unchallenged due to being a change of ownership, then it would be possible for the three fringe firms to merge together and create a matching multiproduct firm. Figure 5.3 depicts the new industry structure. In Case III, both multiproduct firms are now maximizing joint profits across all their products.

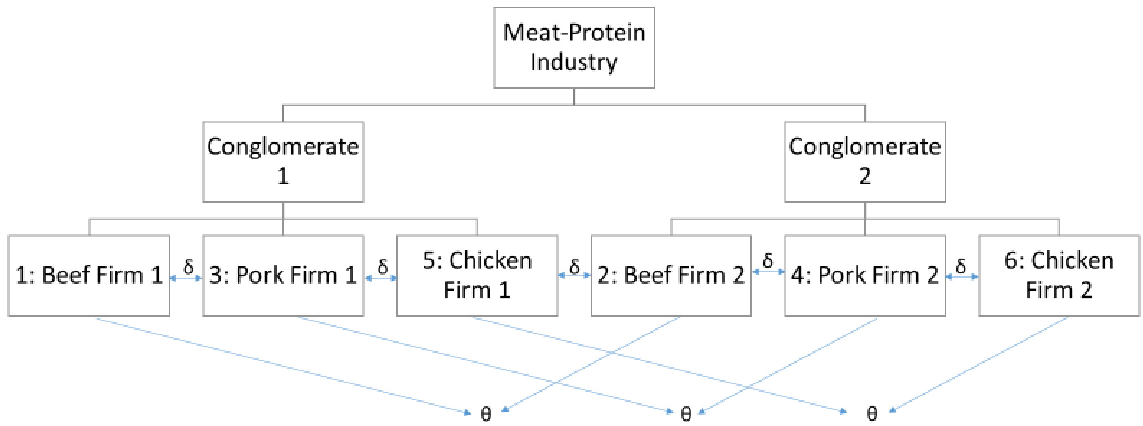

Figure 5.3 Meat-Protein Industry: Two Conglomerates

Note: As before, substitutability between the beef, pork and chicken submarkets is the same (δ) and within-product substitutability is the same (θ).

Case IV: Monopolization of One Submarket

Case IV provides a "what-if" analysis, given the DOJ interpretation of market definition. DOJ would surely challenge any submarket attempting to merge to monopoly. However, this likely constraint is removed in order to compare the impacts of a change in the relevant market. In our analysis, the beef submarket merges to. In order to avoid functional form issues with the Bowley function and to maintain the slight differentiation between the two fresh beef products, the monopolized beef industry would still consist of two products. This coincides with real world conditions because often a merged firm maintains the brand they have purchased, so that they may not risk losing those consumers who display brand loyalty to the acquired brand. A visual of Case IV is provided in Figure 5.4.

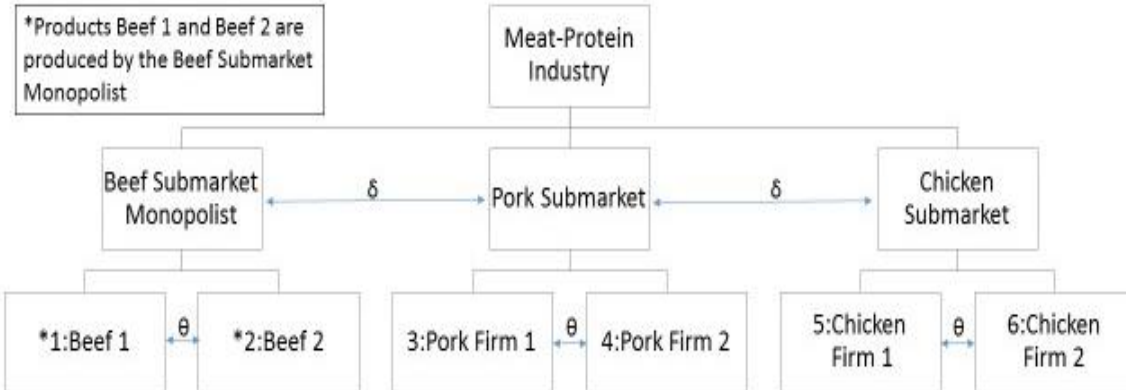

Figure 5.4 Monopolization of the Beef Submarket

Note that beef submarket has merged to monopoly while the pork and chicken submarkets each consist of two individual firms.

Case IV allows for insight into whether a merger to monopoly (Case IV), multiproduct firm (Case II), or conglomerates (Case III) would be more or less competitive under the market definition given for this analysis. In Case IV, the beef submarket monopolist is maximizing joint profits across his two beef products. The other firms are maximizing their profits.

Case V: Hypothetical Meat-Protein Monopolist

Case V depicts a scenario in which a monopolist controls the entire meat-protein market. This is visualized in Figure 5.5.

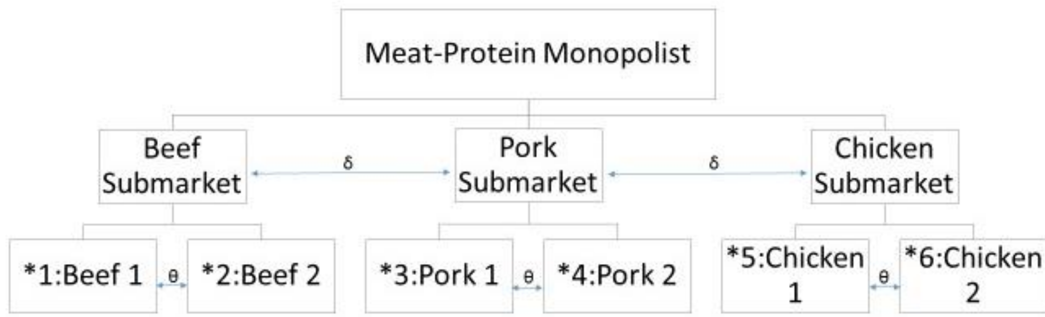

Figure 5.5 Meat-Protein Monopolist

*The meat-protein monopolist controls the entire industry, but maintains separate lines for each product (a necessity with the Bowley function).

Case V is extremely unrealistic and highly unlikely to be allowed by the DOJ. However, this case highlights several important outcomes. Note that the monopolist is still allowed to carry all 6 products, such that he can still capitalize on brand loyalty[6].

Results

The mergers in each case were analyzed using standard profit maximization techniques utilizing Wolfram Mathematica Version 10.2 software, in order to obtain equilibrium price, quantity, profit, and welfare results. These results are discussed below. For further understanding of the methods used to obtain results, see Appendix A, which details the calculations utilized in the Bowley differentiated product duopoly model and how the mathematical system functions. Appendix B provides supplementary information about the price results and comparisons, while Appendix C provides further details for quantity comparisons. Additionally, Appendix D lists the resulting price, quantity, and profit expressions, sorted by each case. Finally, Appendix E details the symmetry relationships established in the various cases. This is included as an aid to the reader.

[6] Additionally, this is required because it satisfies previously mentioned requirements of the Bowley functional form.

Results indicate that the DOJ's stance toward mergers in the meat-protein sector is understandable; mergers with submarket do carry significant potential lessening of competition. Interestingly, results also show that mergers across submarkets, absent sufficient cost reductions, are also capable of anticompetitive harm. Further, this becomes stronger when within submarket substitutability and across submarket substitutability are sufficiently close.

Equilibrium Prices

Table B.1 in Appendix B and its corresponding figures, show all comparisons among prices. Compared to the premerger case (Case I), all prices were higher after merger. Case V, the meat-protein monopolist case, results in higher prices than in any other case presented. The multiproduct firm in Case II charges higher prices than any fringe firm in Case II. Additionally, the price charged by either conglomerate firm in Case III is higher than the price charged by both the multiproduct firm and fringe firm in Case II. The price charged by the submarket monopolist in Case IV is greater than the price charged by any of the fringe firms in Case IV.

An interesting result was found when comparing the prices charged by firms in either Case II or Case III versus the prices charged by firms in Case IV. Clear results were not immediate. Depending on the levels of substitutability within and across submarkets, different results arose. This, itself, is a valuable discovery in that in a differentiated products industry, how consumers view the relationships between substitute goods affects the prices firms can charge, and subsequently the quantities produced, firm profits, and welfare. In order to simplify the depiction of the results, restrictions are placed on the substitutability parameters, $0 < \delta < .75 < \theta < 1$. However arbitrary the break point, the qualitative nature of the results are not affected in any meaningful way. These parameters are likely justifiable for any industry It is logical that consumers view products within a submarket as more highly substitutable than across submarkets. For example, two firm's chicken products (boneless breasts, for example) are likely to be highly substitutable, with only branding or some relatively small magnitude quality differentiation. However, consumers likely view beef products as less substitutable for chicken products.

Appendix B provides further details about the equilibrium price comparisons between Case IV and Case II and Case III. It is important to note that the Case IV yields higher prices so long as the two substitutability parameters are "sufficiently different" from each other and in agreement with the other restrictions made for many of the cases. However, this should not be misconstrued into an absolute meaning regarding Case IV. Refer to Appendix B for specific analysis. The equilibrium prices show that substitutability greatly affects the price a firm is willing to charge for their product or products. Given optimal pricing for each case, equilibrium quantities, profits, and welfare can now be provided.

Equilibrium Quantities

Comparisons were made among the resulting quantities produced by each firm for each resulting case. Some general findings are presented below, while the complete series of comparisons is available in Appendix C and its associated tables and figures. The first finding is that every merger case results in lower output quantities as compared to the premerger case (Case I). Additionally, Case V results in the lowest quantities produced, as compared to any of the other cases.

In Case II (the case with a single multiproduct firm and fringe competitors), the multiproduct firm produces less of any given product than does the fringe. However, comparing the quantities produced in Case II and Case III some intriguing results occur. For good 1, the conglomerate firm (Case III, product 1) produces more than the single multiproduct firm (Case II, product 1). However, the fringe firm in Case II (Case II, product 2) produces more than the corresponding product in Case III.

Several interesting outcomes are observed regarding Case IV, the submarket monopolist case. To start, any fringe firm in Case IV produces a larger quantity of product than the submarket monopolist in Case IV. The submarket monopolist firm producing product 2 (Case IV), produces less than the fringe firm in Case II, product 2. However, any fringe firm in Case IV produces more per product than the multiproduct firm in Case II, or a product of one of the conglomerate firms in Case III.

Relative to product substitutability, the first comparison is of quantity produced of product 1 for a submarket monopolist (Case IV) against one of the products (product 1) produced by a multiproduct firm in Case II. For product 1, the submarket monopolist only produces a higher quantity when the two substitutability parameters are sufficiently close to one another. As such, the submarket monopolist (Case IV) produces less of product 1 than does a firm in the conglomerate case (Case III). Finally, any fringe firm in Case IV will produce less than any fringe firm in Case II unless the substitutability parameters are sufficiently close.

The equilibrium quantity results, much like the equilibrium price results, indicate that within submarket and across submarket substitutability play a prominent role in the actual quantity produced.

Equilibrium Profits

Next, the profits earned by the firms in the various merger cases will be analyzed and compared. These findings will highlight the incentives firms may have to merge. Table 5.1 below provides the relevant profit comparisons.

Table 5.1 Profit Comparisons by Case (I-V) and Firm (Or Firm Combination in after A Merger)

Comparison	Explanation/Notes (if Necessary)
$\pi_{II,135} > \pi_{I,1}$	
$\pi_{II,2} > \pi_{I,2}$	
$\pi_{III,135} > \pi_{I,1}$	
$\pi_{IV,12} > \pi_{I,1}$	
$\pi_{IV,3} > \pi_{I,3}$	
$\pi_{V} > \pi_{I,1}$	
$\pi_{II,135} > \pi_{II,2}$	
$\pi_{II,135} < \pi_{III,135}$	
$\pi_{II,135} \overset{<}{\underset{>}{=}} \pi_{IV,12}$	See Figure 5.6.
$\pi_{II,135} > \pi_{IV,3}$	
$\pi_{II,135} < \pi_{V}$	
$\pi_{II,2} < \pi_{III,135}$	
$\pi_{II,2} < \pi_{IV,12}$	
$\pi_{II,2} \overset{<}{\underset{>}{=}} \pi_{IV,3}$	See Figure 5.7.
$\pi_{II,2} < \pi_{V}$	

Table 5.1 (Continued)

$<$ $\pi_{III,135} = \pi_{IV,12}$ $>$	See Figure 5.8.
$\pi_{III,135} > \pi_{IV,3}$	
$\pi_{III,135} < \pi_{V}$	
$\pi_{IV,12} < \pi_{V}$	
$\pi_{IV,3} < \pi_{V}$	
$\pi_{IV,3} < \pi_{IV,12}$	

Unless otherwise noted, these comparisons hold for the conditions $0 < \delta < 1$, $\delta < \theta < 1$, $\alpha > 0$, and $0 < c < \alpha$. Notation for this consists of the item being described in regular script. Subscripts directly below allow for each case to be identified (I, II, III, IV, V), as well as the firm/product to be identified (1..6). Refer back to Tables (5.1-5.5) for additional information, if needed. In the context of the problem, firms 1 and 2 produce beef, firms 3 and 4 produce pork, and firms 5 and 6 produce chicken.

Given the equilibrium price and quantities, the profits earned by both the merged parties and remaining fringe firms were higher than any firm in the premerger case. This illustrates that even if a given firm is not involved in a merger, they still benefit from higher profits. The fact that profits for the multiproduct firm (Case II; products 1, 3, and 5) are greater than the premerger profits shows a unilateral incentive for firms to become multiproduct firms.

The multiproduct firm (Case II) achieves higher profits than any fringe firm in Case II. Additionally, profits for both conglomerate firms (Case III) were unilaterally greater than the profits earned by the multiproduct firm in Case II or any fringe firm in Case II. Given that one firm has merged to become a multiproduct firm (as in Case II), it is in the remaining fringe firms' self-interest to achieve higher profits by forming the matching multiproduct firm, resulting in Case III. This again verifies the unilateral incentive to merge.

Comparisons between Case II and Case IV as well as between Case III and Case IV also highlight some highly significant findings, especially as they relate to the DOJ's apparent position regarding product conglomerate mergers. First, the multiproduct firm in Case II earns higher profits than any single fringe firm in Case IV. Also, the submarket monopolist in Case IV earns higher profits than any fringe firm in Case II. Similar to this, a conglomerate firm in Case III earns higher profits than any single fringe firm in Case IV. Additionally, the submarket monopolist in Case IV obtains higher profits than any fringe firm in the same case. Profits for the hypothetical meat-protein monopolist were higher than profits earned by a single firm in any other case. This is to be expected, and is a somewhat rudimentary check to ensure the model has reliable predictive properties.

A pivotal comparison for analysis into whether or not a multiproduct frim is more damaging than a submarket monopolist can now be considered. This comes from comparing the multiproduct firm in Case II against the submarket monopolist in Case IV. Figure 5.6 provides this comparison.

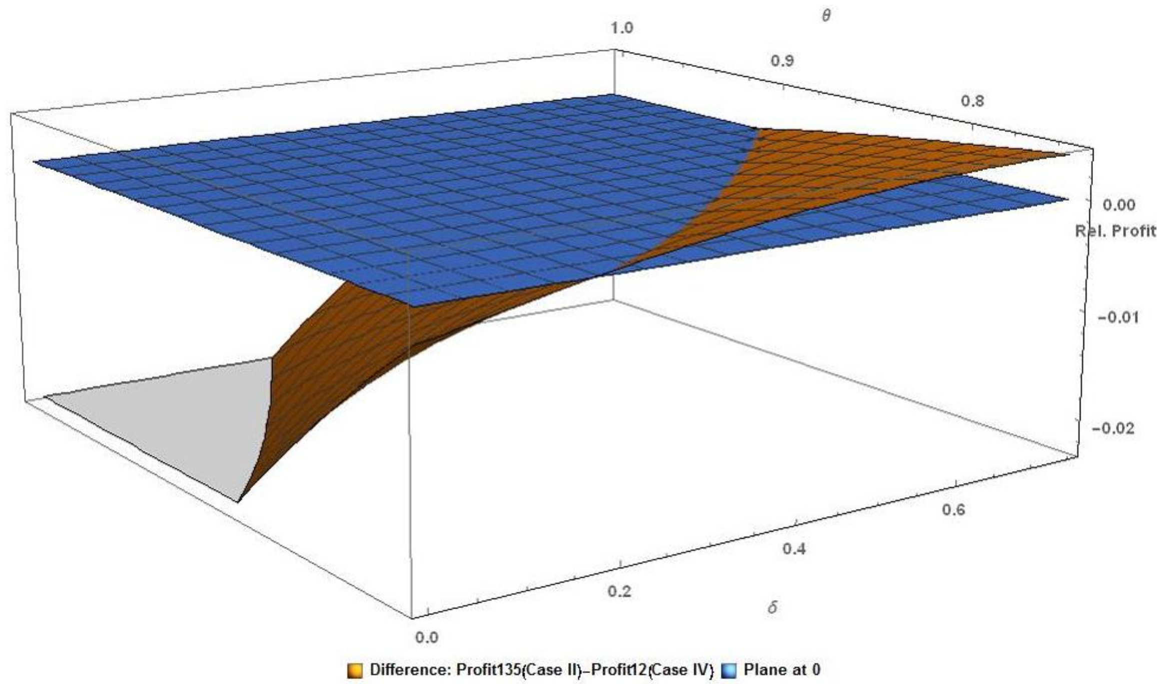

Figure 5.6 Relative Profit Comparison: π_{135} (Case II) vs. π_{12} (Case IV)

Note: To aid in graphing, α=1.5 and c=1. Areas in orange, which are above the horizontal plane at 0, highlight the areas in which the multiproduct firm (Case II) obtains a higher profit than a submarket monopolist (Case IV). This occurs under the restriction $0 < \delta < .75 < \theta < 1$.

From Figure 5.6 it can be seen that there are significant regions of product substitutability in which the submarket monopolist will have higher profits than the multiproduct firm. Also note that for some values of the substitutability parameters, those values tending close to perfect substitutes within submarket and/or independent across submarkets, there exists no possibility for the multiproduct firm from Case II to earn higher profits than the submarket monopolist of Case IV.

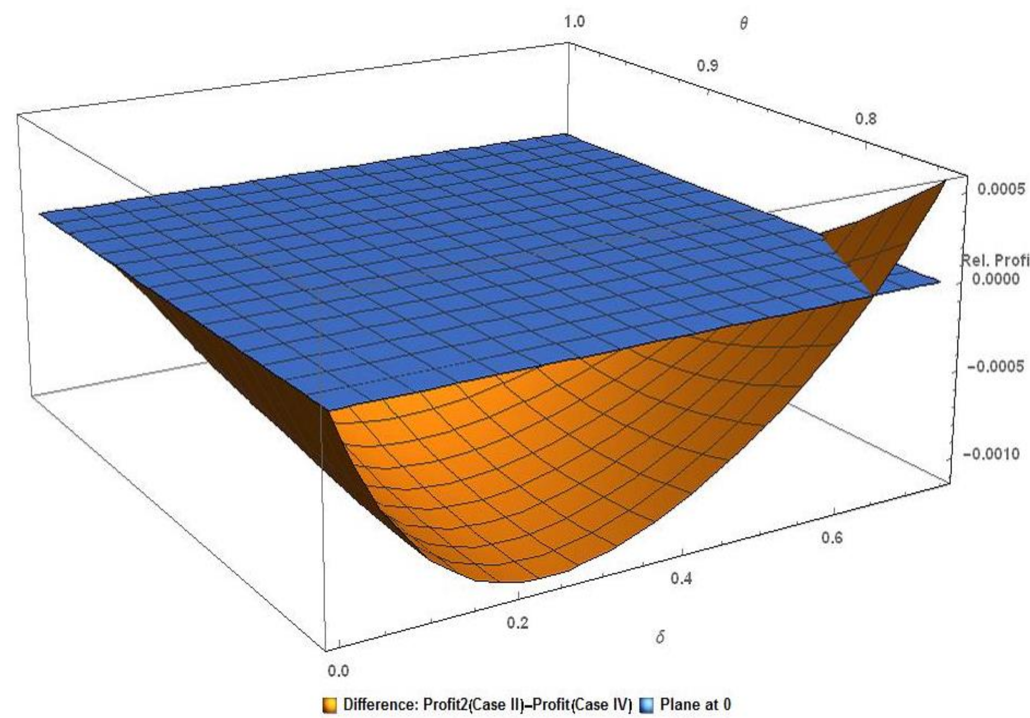

Figure 5.7 Relative Profit Comparison: π_2 (Case II) vs. π_3 (Case IV)

Note: To aid in graphing, α=1.5 and c=1. Areas in orange, which are above the horizontal plane at 0, highlight the areas in which the multiproduct fringe firm (Case II) obtains a higher profit than a fringe firm in Case IV. This occurs under the restriction $0 < \delta < .75 < \theta < 1$.

Similar to Figure 5.7, Figure 5.6 provides the areas in which a fringe firm in Case II will have a higher profit than a corresponding fringe firm in Case IV. Like before, only when the substitutability parameters are sufficiently close, will the fringe firm in Case II have a higher profit than a fringe firm in Case IV. The final profit comparison, provided in Figure 5.8, compares the profits of the submarket monopolist in Case IV against a conglomerate firm in Case III.

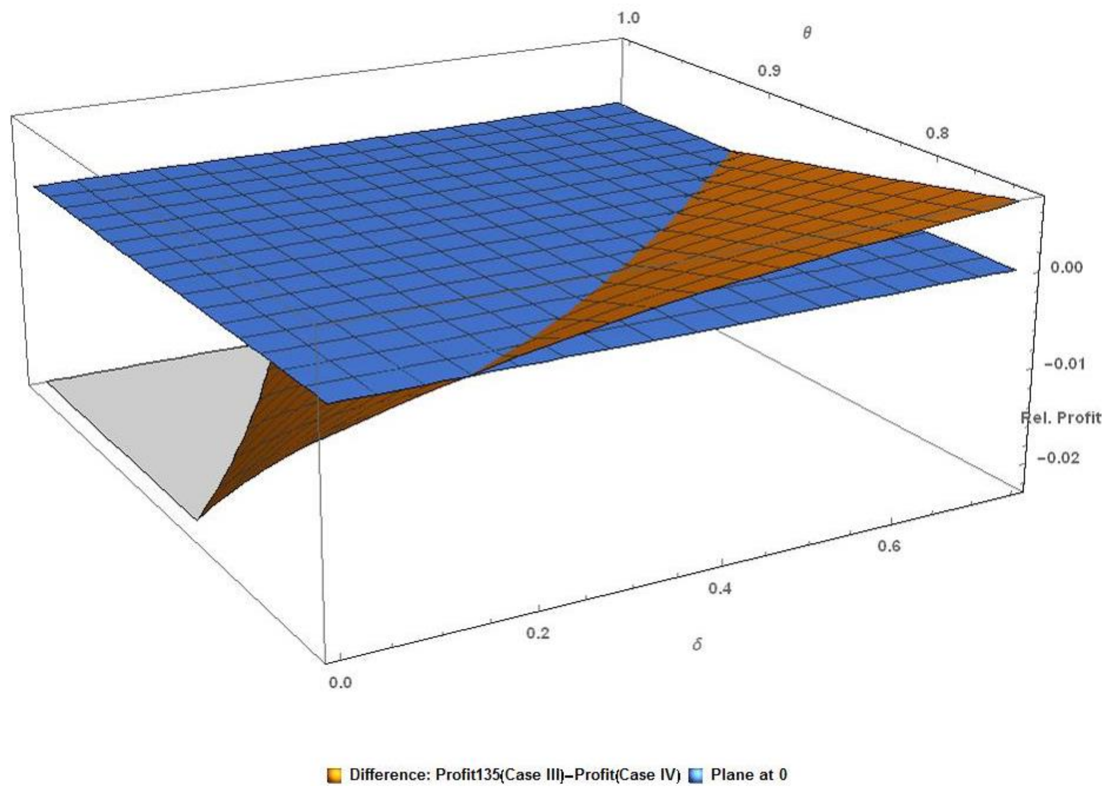

Figure 5.8 Relative Profit Comparison: π_{135} (Case III) vs. π_{12} (Case IV)

Note: To aid in graphing, α=1.5 and c=1. Areas in orange, which are above the horizontal plane at 0, highlight the areas in which the conglomerate firm (Case III) earns a higher profit than a submarket monopolist (Case IV). This occurs under the restriction $0 < \delta < .75 < \theta < 1$.

The comparison between the conglomerate firm's profit in Case III and the submarket monopolist's profits in Case IV, are similar to that of the Case II multiproduct firm and submarket monopolist in Case IV. There are still regions of substitutability where the Case IV submarket monopolist is able to earn higher profits than even a two multiproduct firm of Case III. In its totality, these results indicate that there exists significant credence to the DOJ's views towards not allowing merger to monopoly in any submarket.

Welfare Consequences

Perhaps the most important economic concept for merger analysis is the calculation of welfare (including consumer, producer and total welfare). This allows for the net effects of the merger to be calculated in totality. Practically, this yields a justifiable set of results, in terms of pure economics. However, this misses much in the larger realm of antitrust analysis. Kirkwood and Lande (2008) show that antitrust laws intended, and courts have consistently upheld, that the purpose of these laws is consumer protection, not economic efficiency. Kirkwood and Lande also provide information on the 'traditional' view focused on the efficiency argument, but make clear that the courts have not embraced this view. The courts have opted for the consumer protection argument. Further, Zerbe (2015) finds the following, "This nevertheless means that in at least 1,478 cases, or ninety-eight percent of all federal antitrust cases, consumer protection was the overriding concern."

This thesis will not address the merits of these arguments. Rather, results reflecting both modes of thought will be presented. Total welfare results will be presented, noting both producer and consumer surplus. This will allow for comparisons of total welfare, consumer surplus, and producer surplus. This thesis is merely showing the welfare implications in the traditional economic sense, whilst carefully noting the broader, legal perspective of antitrust analysis.

To calculate consumer and producer surplus, the methods of Chung et al. (2013) are utilized. They performed similar welfare calculations based on a modification of the Bowley functional form, from which their calculations may be modified to fit the specification provided in this paper. Producer surplus (PS) is simply the total industry profits (for a given case). Because the underlying utility function is that of the representative consumer, consumer surplus (CS) can be calculated in the manner provided in expression 5.4.

$$CS = U - \sum_{i=1}^{6} p_i q_i \tag{5.1}$$

Total surplus, or total welfare, is simply $TS = CS + PS$.

Consumer Surplus

Table 5.2 provides the relevant consumer surplus comparisons.

Table 5.2 Consumer Surplus Comparisons for Merger Cases

Comparison	**Notes/Explanation (if Necessary)**
$CS_{II} < CS_I$	
$CS_{III} < CS_I$	
$CS_{IV} < CS_I$	
$CS_V < CS_I$	
$CS_{III} < CS_{II}$	
$CS_{IV} \overset{<}{\underset{>}{=}} CS_{II}$	See Figure 5.9.
$CS_V < CS_{II}$	
$CS_{IV} \overset{<}{\underset{>}{=}} CS_{III}$	See Figure 5.10.
$CS_V < CS_{III}$	
$CS_V < CS_{IV}$	

Unless otherwise noted, these comparisons hold for the conditions $0 < \delta < 1$, $\delta < \theta < 1$, $\alpha > 0$, and $0 < c < \alpha$. See related graphs for additional information.

From Table 5.2 it is seen that consumer surplus in every case is less than Case I. Additionally, Case V unambiguously results in the lowest consumer welfare compared to any other case. Consumer surplus in the two multiproduct firm case (Case III) is lower than in the multiproduct case (Case II). Figure 5.9 visualizes the comparison. Given the restrictions, it is seen that as long as the substitutability parameters are sufficiently different from each other, that consumer welfare is greater in Case II than in Case IV.

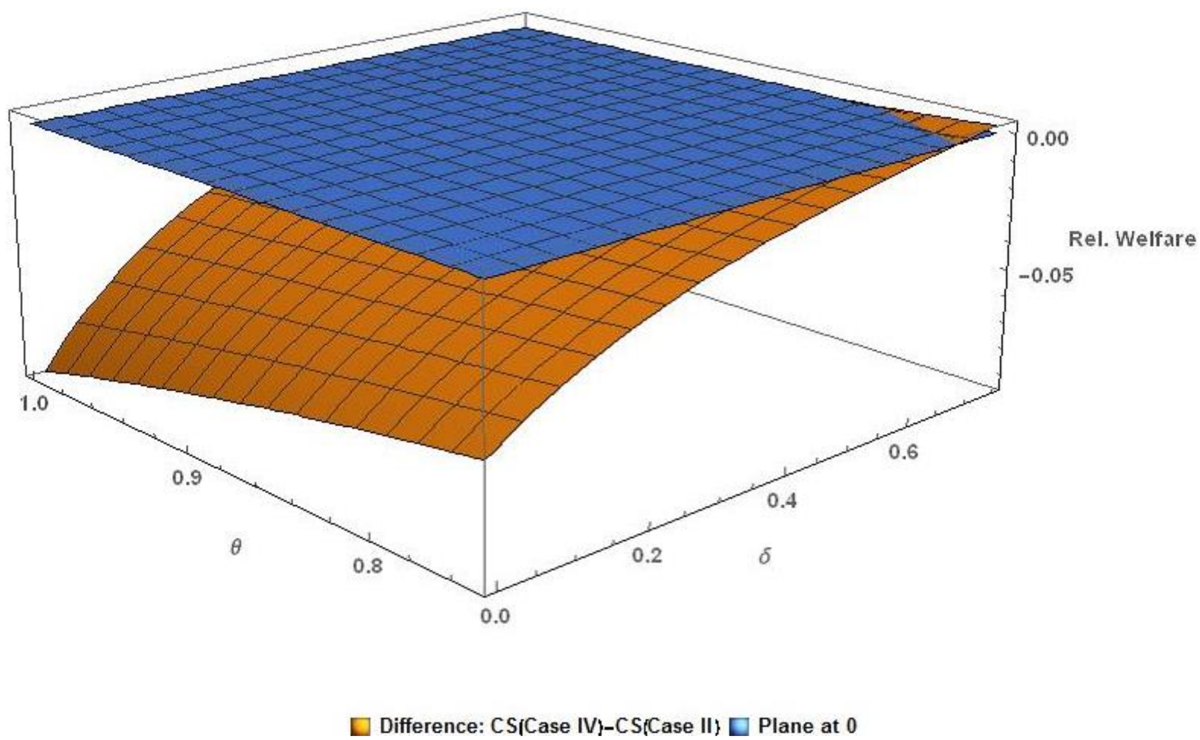

Figure 5.9 Relative Consumer Surplus Comparison: Case IV vs. Case II

Note: To aid in graphing, α=1.5, c=1, and Z=1. Areas in orange, which are above the horizontal plane at 0, visualize the areas in which consumer surplus is greater for Case IV than Case II. Notice that given the additional restrictions, $0 < \delta < .75 < \theta < 1$, which are graphed here, it can be seen that this only occurs when the substitutability parameters are sufficiently close.

As provided in Figure 5.10, Case IV compared against Case III results in very similar findings. When the substitutability parameters are sufficiently close, consumer surplus is greater in Case IV. Otherwise, when the substitutability parameters are sufficiently different, consumer surplus is greater in Case III than in Case IV.

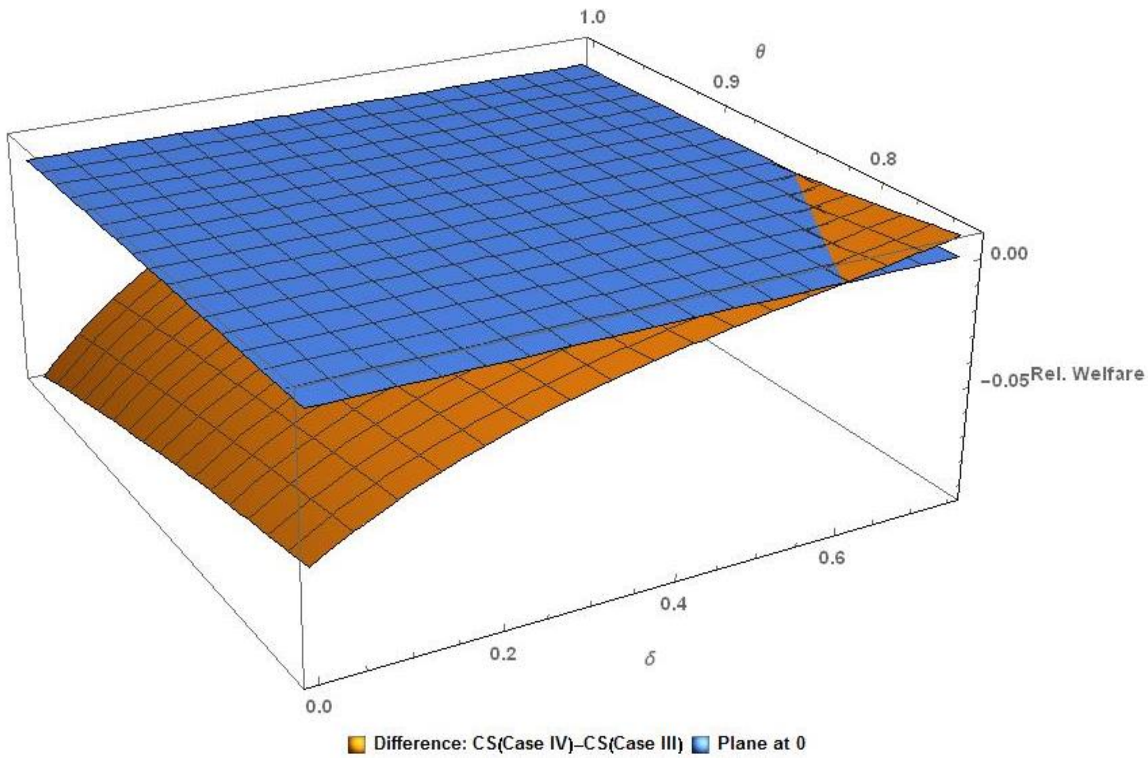

Figure 5.10 Relative Consumer Surplus Comparison: Case IV vs. Case III

Note: To aid in graphing, α=1.5, c=1, and Z=1. Areas in orange, which are above the horizontal plane at 0, visualize the areas in which consumer surplus is greater for Case IV than Case II. Notice that given the additional restrictions, $0 < \delta < .75 < \theta < 1$, which are graphed here, it can be seen that this only occurs when the substitutability parameters are sufficiently close

Producer Surplus

The producer surplus comparisons made here result in nearly the exact opposite outcomes as the consumer surplus comparisons. When compared to the premerger case

(Case I), every subsequent case results in higher producer surplus. Along with this, the meat-protein monopolist case (Case V) results in higher producer surplus than any other case. Additionally, producer surplus is higher in the two multiproduct firm case (Case III) than in the single multiproduct firm case (Case II). These comparisons are provided in Table 5.3.

Table 5.3 Producer Surplus Comparisons for the Merger Cases

Comparison	**Notes/Explanation (if Necessary)**
$PS_{II} > PS_{I}$	
$PS_{III} > PS_{I}$	
$PS_{IV} > PS_{I}$	
$PS_{V} > PS_{I}$	
$PS_{III} > PS_{II}$	
$PS_{IV} \overset{<}{\underset{>}{=}} PS_{II}$	See Figure 5.11.
$PS_{V} > PS_{II}$	
$PS_{IV} \overset{<}{\underset{>}{=}} PS_{III}$	See Figure 5.12.
$PS_{V} > PS_{III}$	
$PS_{V} > PS_{IV}$	

Unless otherwise noted, these comparisons hold for the conditions $0 < \delta < 1$, $\delta < \theta < 1$, $\alpha > 0$, and $0 < c < \alpha$. Notation for this consists of the item being described in regular script. Subscripts directly below allow for each case to be identified (I, II, III, IV, V). Refer back to Tables (5.1-5.5) for additional information.

Comparison between the submarket monopolist case (Case IV) and the single multiproduct firm (Case II) is provided in Figure 5.11. Given the restrictions, producer surplus in Case IV is always greater than that in Case II so long as the substitutability parameters are sufficiently different from one another.

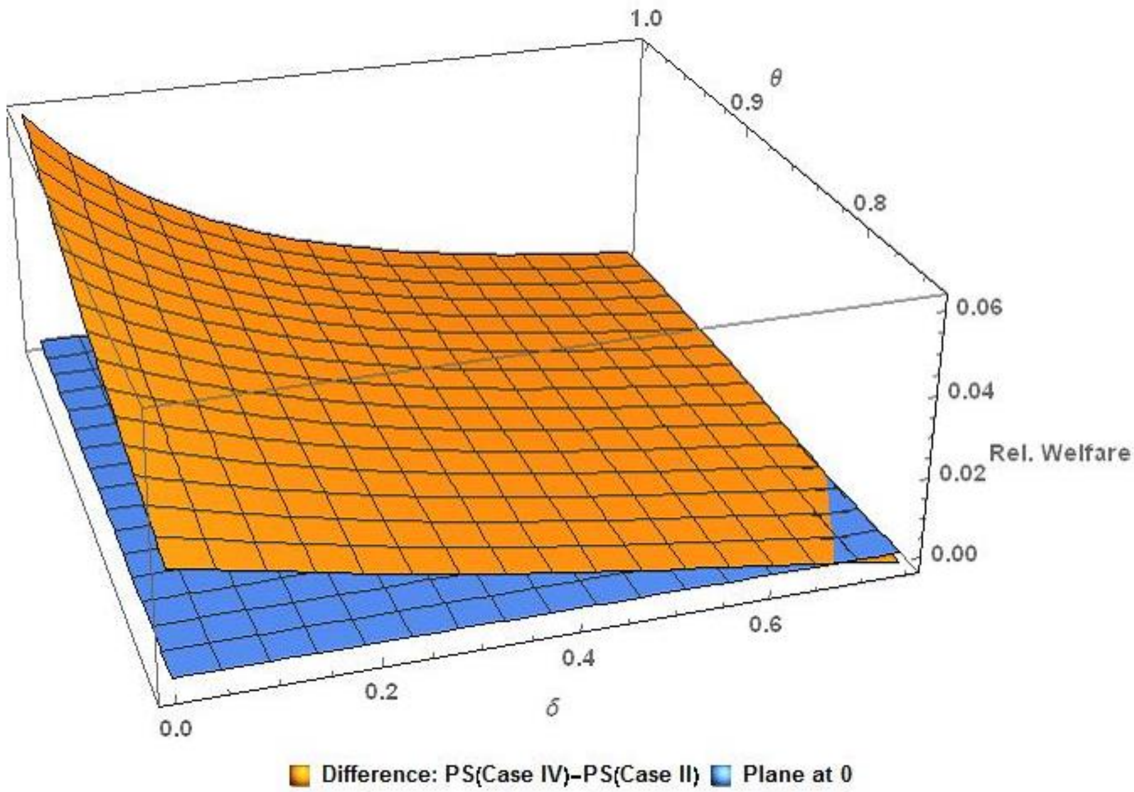

Figure 5.11 Relative Producer Surplus Comparison: Case IV vs. Case II

Note: To aid in graphing, α=1.5, c=1, and Z=1. Areas in orange, which are above the horizontal plane at 0, visualize the areas in which producer surplus is greater for Case IV than Case II. Notice that given the additional restrictions $0<\delta<.75<\theta<1$, which are graphed here, it can be seen that this only occurs when the substitutability parameters are sufficiently different.

Producer surplus in Case IV is also compared to the producer surplus in Case III, as provided in Figure 5.12. Given the restrictions, $0<\delta<.75<\theta<1$, producer surplus in Case IV is greater than Case II in all cases where the substitutability parameters are sufficiently different.

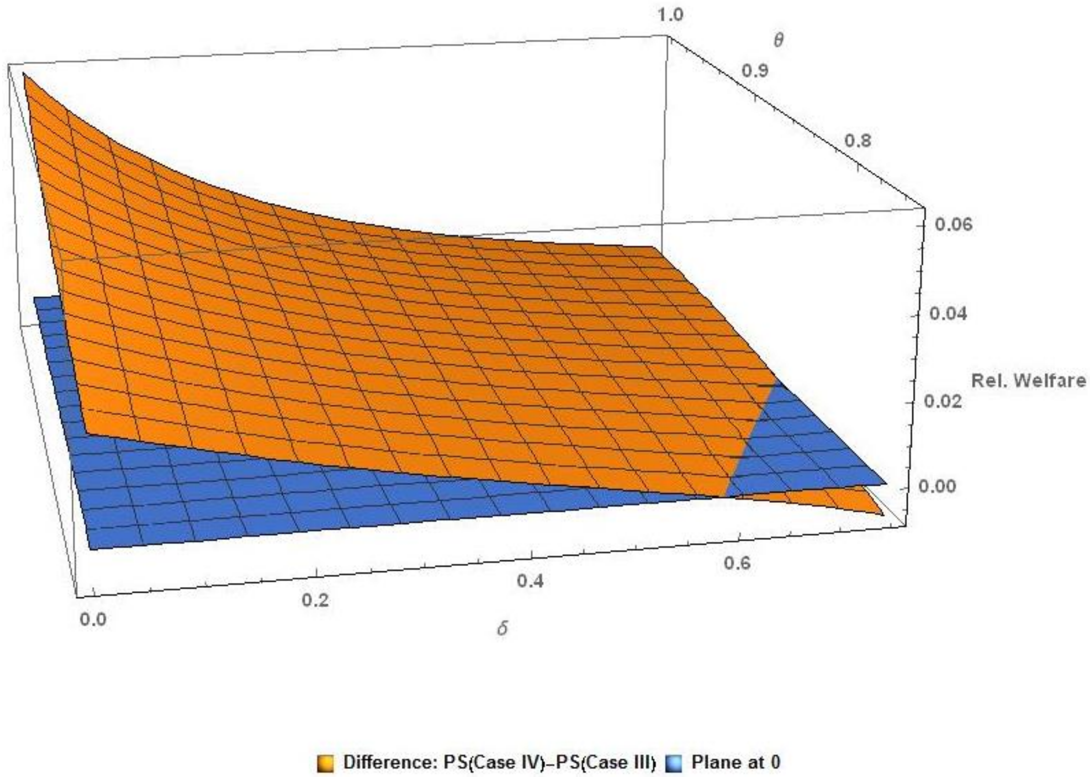

Figure 5.12 Relative Producer Surplus Comparison: Case IV vs. Case III

Note: To aid in graphing, α=1.5, c=1, and Z=1. Areas in orange, which are above the horizontal plane at 0, visualize the areas in which producer surplus is greater for Case IV than Case III. Notice that given the additional restrictions $0 < \delta < .75 < \theta < 1$, which are graphed here, it can be seen that this occurs when the substitutability parameters are sufficiently different.

Total Welfare

Total welfare comparisons were also conducted as a part of this section. Generally speaking, the results for total welfare did not qualitatively differ from the results for consumer surplus. Out of the ten possible comparisons, only Case III compared to Case II was somewhat different. In the consumer surplus case, Case III was unambiguously lower than Case II. However, in terms of total welfare, Case III compared to Case II required further restrictions to reasonably classify. This, along with other total welfare comparisons will be discussed below. Table 5.4 provides the total welfare comparisons.

Table 5.4 Total Welfare Comparisons for the Merger Cases

Comparison	Notes/Explanation (if Necessary)
$TS_{II} < TS_{I}$	
$TS_{III} < TS_{I}$	
$TS_{IV} < TS_{I}$	
$TS_{V} < TS_{I}$	
$TS_{III} \gtreqless TS_{II}$	See Figure 5.13 and 5.14.
$TS_{IV} \gtreqless TS_{II}$	See Figure 5.15.
$TS_{V} < TS_{II}$	
$TS_{IV} \gtreqless TS_{III}$	See Figure 5.16.
$TS_{V} < TS_{III}$	
$TS_{V} < TS_{IV}$	

Unless otherwise noted, these comparisons hold for the conditions $0 < \delta < 1$, $\delta < \theta < 1$, $\alpha > 0$, and $0 < c < \alpha$. Notation for this consists of the item being described in regular script. Subscripts directly below allow for each case to be identified (I, II, III, IV, V). Refer back to Tables (5.1-5.5) for additional information.

Total welfare in the premerger case was greater than that of any other case presented. Additionally, total welfare was unambiguously lower in Case V than any other case. Again, the further restrictions on the two substitutability parameters was required for further analysis. The total welfare comparison that is slightly different from its related case in consumer surplus was the comparison between total welfare in Case II and Case III. Figure 5.13, provides this comparison.

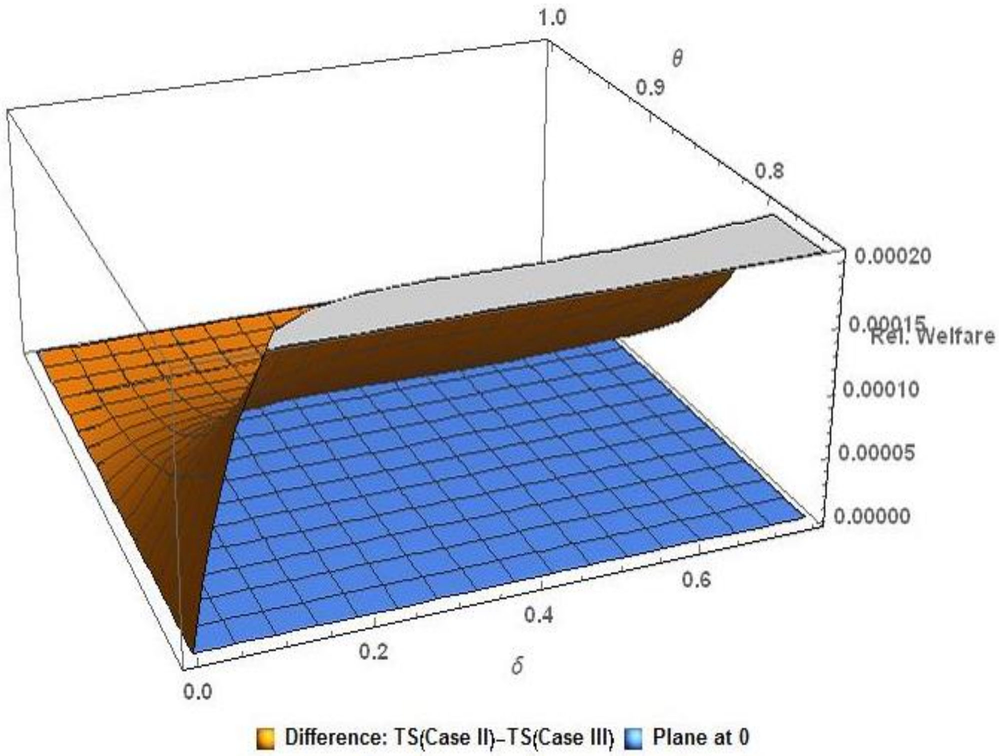

Figure 5.13 View 1:Relative Total Surplus Comparison: Case II vs. Case III

Note: To aid in graphing, α=1.5, c=1, and Z=1. Areas in orange, which are above the horizontal plane at 0, visualize the areas in which total surplus is greater for Case II than Case III. Notice that given the additional restrictions $0 < \delta < .75 < \theta < 1$, which are graphed here, it can be seen that this occurs generally, albeit with some exceptions.

Strictly speaking, this result does not differ from its consumer surplus iteration. Generally, total surplus in Case II is greater than that of Case III. However, due to some strange nonlinearities near the limit of the model where δ tends towards 0 and θ near 1, total surplus can be greater for Case III than Case II. Figure 5.14 provides the extreme example of this. More research should be done to understand this peculiarity.

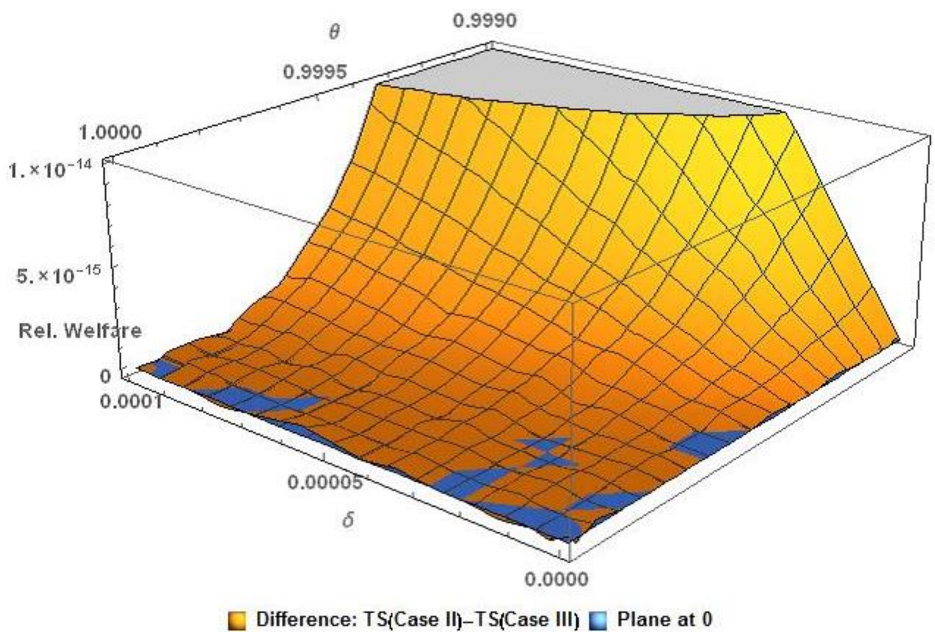

Figure 5.14 View 2:Relative Total Surplus Comparison: Case II vs. Case III

Note: To aid in graphing, α=1.5, c=1, and Z=1. Areas in orange, which are above the horizontal plane at 0, visualize the areas in which total surplus is greater for Case II than Case III.

Two other comparisons warrant further discussion. The first of Case IV and Case II. This is presented in Figure 5.15. With the added restrictions, total welfare in Case IV is only greater than total welfare in Case II when the substitutability parameters are sufficiently close.

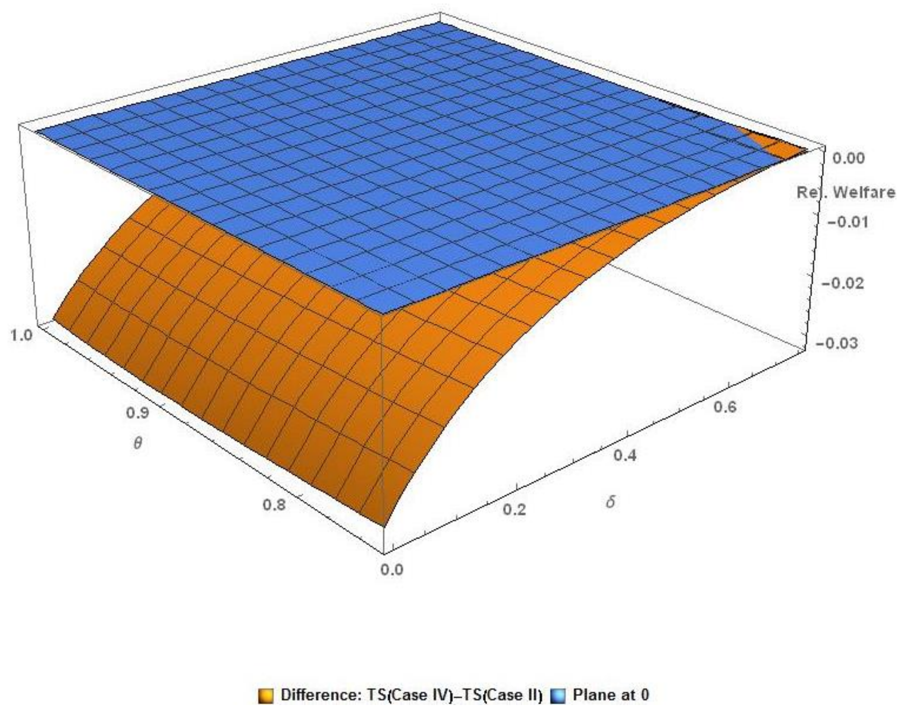

Figure 5.15 Relative Total Surplus Comparison: Case IV vs. Case II

Note: To aid in graphing, α=1.5, c=1, and Z=1. Areas in orange, which are above the horizontal plane at 0, visualize the areas in which total surplus is greater for Case IV than Case II. Notice that given the additional restrictions, $0 < \delta < .75 < \theta < 1$, which are graphed here, Case IV is only greater than Case II when the substitutability parameters are sufficiently close.

The final total welfare comparison is that of total welfare for the submarket monopolist case (Case IV) compared against the two multiproduct firms case (Case III). This comparison is provided in Figure 5.16. With the restrictions $0 < \delta < .75 < \theta < 1$, it can be seen that total surplus for the two multiproduct firm case (Case III) is greater than the total surplus for the submarket monopolist case (Case IV) for all areas except those areas in which the substitutability parameters are sufficiently close to one another.

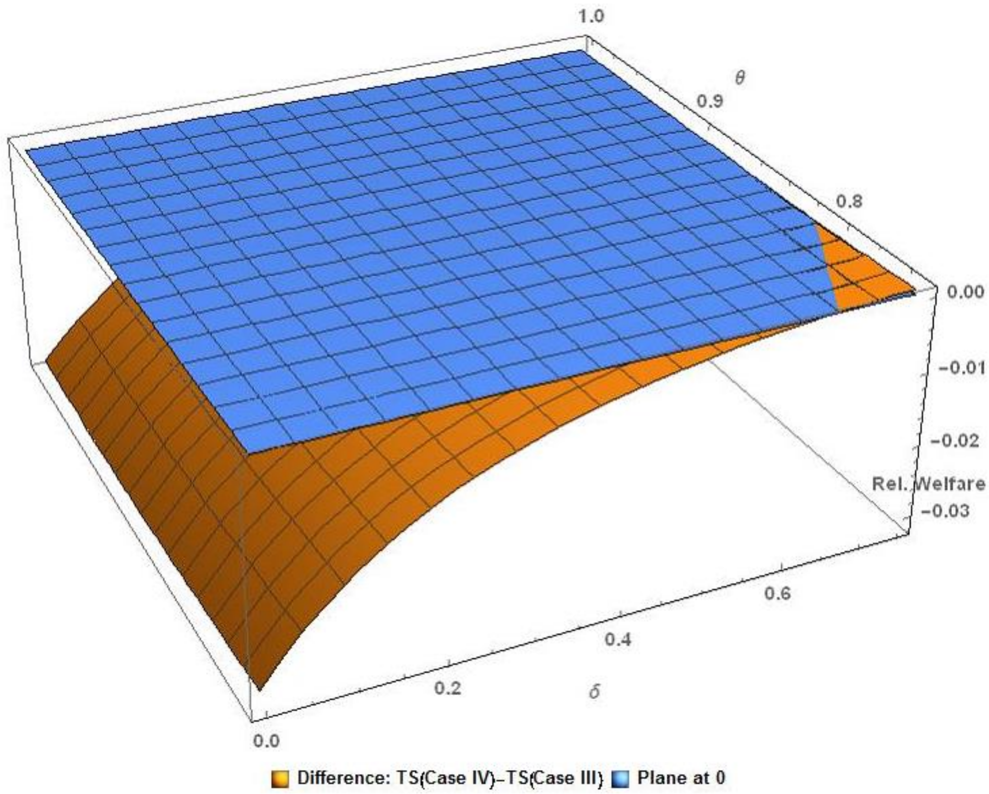

Figure 5.16 Relative Total Surplus Comparison: Case IV vs. Case III

Note: To aid in graphing, α=1.5, c=1, and Z=1. Areas in orange, which are above the horizontal plane at 0, visualize the areas in which total surplus is greater for Case IV than Case II. Notice that given the additional restrictions, $0 < \delta < .75 < \theta < 1$, which are graphed here, Case IV is only greater than Case II when the substitutability parameters are sufficiently close.

Generally speaking, it did not make a significant difference as to whether a consumer surplus standard or total welfare standard was utilized as the criterion for harm.

CHAPTER VI

ECONOMIES OF SCOPE

Case II demonstrated that when a firm merges across all three submarkets, market power increased resulting in harm to both consumers and economic efficiency. These results assumed no economies of scale or scope were gained as a result of the merger. Economies of scope, if present, are a possible source of market power neutralization. Case I and Case II will now be revisited with a focus on incorporating economies of scope into the analysis. This allows for derivation of the required cost reductions to keep prices, and by default welfare, at the premerger levels.

In their landmark text, "Contestable Markets and the Theory of Industry Structure", Baumol, Panzar, and Willig (1982) explain economies of scope as,

> "…there is also the possibility that cost savings may result from simultaneous production of several different outputs in a single enterprise, as contrasted with their production in isolation, each by its own specialized firm. That is, there may exist economies resulting from the scope of the firm's operations."

Economies of scope apply when the merged firm produces a product in three submarkets. To incorporate economies of scope, the constant cost parameter, c, will first be redefined for each of the i pre-merger firms in expression 6.1.

$$c_i = k_i + m_i \qquad (6.1)$$

In expression 6.1, the parameter *k* represents product specific costs. These costs are unique to the product being produced and cannot be reduced or increased due to merger. Reductions in production specific costs is not expected due the heterogeneous production processes for each animal species. For example, if a beef firm merged with a chicken firm, the processing equipment in either plant would not substitute into the other production process.[7] The parameter *m* represents the general business costs common to all products produced, for example "management" costs. Management cost captures corporate level costs primarily in three areas: management, marketing, and distribution. In this model, scope economies (diseconomies) can come from reductions (increases) in management cost.

For instance, the reduction in salary and benefits expenses from only having one CEO and Board of Directors instead of two would be an area in which some management cost savings may be realized. Another probable area targeted for management cost reductions likely would be post-slaughter cold storage and transportation.

After a merger, the merged firm's new cost structure of the *j* across product market firms is provided in expression 6.2.

$$C = \sum_{i=1}^{j} k_i + \lambda \sum_{i=1}^{j} m_i \tag{6.2}$$

The λ parameter is a scalar that measures the realized management cost changes after a multiproduct merger. The cost savings parameter λ can take on values between $0 < \lambda < \infty$.

[7] Sexton and Zhang (2001) state, "For example, although pork, beef, and poultry may substitute for one another in consumers' budgets, they do not substitute at all as inputs into a particular processing plant."

Next, the relationship between λ and economies of scope are addressed. For this analysis, Baumol, Panzar, and Willig's (1982) measure for economies of scope (S). The adapted equation 6.2 with adapted notation is

$$S = \frac{\eta(q_1,0,0)+\eta(0,q_3,0)+\eta(0,0,q_5)-\eta(q_1,q_3,q_5)}{\eta(q_1,q_3,q_5)}, \tag{6.3}$$

where $\eta(\mathbf{q}_i)$ represents the cost as a function of the relevant differentiated product quantities. The presence of economies of scope occurs when it is less expensive to produce all three products than it would be to produce each one separately. This would result in the numerator being positive, along with the denominator.

In regards to one firm merging across the three submarkets as in Case II, substitution of 6.1 and 6.2 into 6.3 the derivation of post-merger economies of scope presented in 6.4.

$$S = \frac{3(k+m)(q_1+q_3+q_5)-3(k+\lambda m)(q_1+q_3+q_5)}{3(k+\lambda m)(q_1+q_3+q_5)} \tag{6.4}$$

As such, when $S>0$ there are economies of scope, $S<0$ indicates that diseconomies of scope, and when $S=0$ there are no scope economies. As can be seen, when $0<\lambda<1$, there are economies of scope and diseconomies when $\lambda>1$. When $\lambda=1$ there are no economies of scope.

Next, the exercise presented in Chapter V, Case I was repeated utilizing the redefined cost function in expression 6.2. Each firm's objective is to maximize profit given their actions and those of their competitors. From this, the optimal prices charged by all firms and the resulting output quantities were obtained. The optimal prices and quantities for each firm are provided in expression 6.5.

$$p^*_{1,\dots,6} = \frac{\alpha(-1+2\delta-\theta)(-1+\theta)+k\left(1+2\delta-4\delta^2+\theta\right)+m\left(1+2\delta-4\delta^2+\theta\right)}{2-4\delta^2+\theta+2\delta\theta-\theta^2}$$

$$q^*_{1,\dots,6} = \frac{(\alpha-k-m)\left(-1-2\delta+4\delta^2-\theta\right)}{(1+4\delta+\theta)\left(4\delta^2-2\delta\theta+(-2+\theta)(1+\theta)\right)} \tag{6.5}$$

Expression 6.6 provides the premerger profits.

$$\pi^*_{1,\dots,6} = \frac{(k+m-\alpha)^2(1-\theta)\left(8\delta^3+(1+\theta)^2-4\delta^2(2+\theta)\right)}{(1+4\delta+\theta)\left(2-4\delta^2+\theta+2\delta\theta-\theta^2\right)^2} \tag{6.6}$$

Now, reiterating Case II with the expanded cost function (6.2), results in equations 6.7 and 6.8 which illustrates the new resulting equilibrium prices and quantities.

$$p^*_{n=1,3,5} = \frac{\begin{pmatrix} \alpha(-1+\theta)\left(-12\delta^2+2\delta(2+\theta)+(1+\theta)(2+\theta)\right) \\ +k\left(24\delta^3-(1+\theta)(2+\theta)-2\delta(4+5\theta)\right) \\ +m\left(8\delta^3(1+2\lambda)-(1+\theta)(\theta+2\lambda)+4\delta^2(-1+\theta+\lambda-\theta\lambda)-2\delta\left(1+2\theta+3(1+\theta)\lambda\right)\right) \end{pmatrix}}{24\delta^3-12\delta^2(-1+\theta)+(-2+\theta)(1+\theta)(2+\theta)+2\delta\left(-6+(-4+\theta)\theta\right)}$$

$$p^*_{n=2,4,6} = \frac{\begin{pmatrix} -\alpha(-1+2\delta-\theta)(-1+\theta)(2+6\delta+\theta) \\ +k(1+2\delta)\left(6\delta(-1+2\delta)-(1+\theta)(2+\theta)\right) \\ +m(1+2\delta)\left(-2\delta(2+\lambda)+4\delta^2(2+\lambda)-(1+\theta)(2+\theta\lambda)\right) \end{pmatrix}}{24\delta^3-12\delta^2(-1+\theta)+(-2+\theta)(1+\theta)(2+\theta)+2\delta\left(-6+(-4+\theta)\theta\right)} \tag{6.7}$$

$$q^*_{n=1,3,5} = \frac{\begin{array}{c}(1+2\delta)(k(-1+\theta)(-12\delta^2+2\delta(2+\theta)+(1+\theta)(2+\theta))-\alpha(-1+\theta)(-12\delta^2+2\delta(2+\theta)+(1+\theta)(2+\theta))\\ +m(8\delta^3(-1+\lambda)-4\delta^2(-1+\theta)(1+2\lambda)+2\delta(1+2\theta+(-3+(-1+\theta)\theta)\lambda)+(1+\theta)(\theta+(-2+\theta^2)\lambda)))\end{array}}{(-1+\theta)(1+4\delta+\theta)(24\delta^3-12\delta^2(-1+\theta)+(-2+\theta)(1+\theta)(2+\theta)+2\delta(-6+(-4+\theta)\theta))}$$

$$q^*_{n=2,4,6} = \frac{(1+2\delta-4\delta^2+\theta)(k(-1+\theta)(2+6\delta+\theta)-\alpha(-1+\theta)(2+6\delta+\theta)+m(-2+4\delta^2(-1+\lambda)+\theta(\theta+\lambda)+2\delta(-4+\lambda+\theta(2+\lambda))))}{(-1+\theta)(1+4\delta+\theta)(24\delta^3-12\delta^2(-1+\theta)+(-2+\theta)(1+\theta)(2+\theta)+2\delta(-6+(-4+\theta)\theta))} \tag{6.8}$$

The required cost savings λ^* needed for a welfare neutral merger can now be derived. Derivation is accomplished by setting $p^*_{n=1,3,5}$ (the multiproduct firm price in Case II) equal to $p^*_{1,\ldots,6}$ (the premerger case price) and solving for λ^*. The result is provided in 6.9.

$$\lambda^* = \frac{m(2+2\delta-\theta)(-1-2\delta+4\delta^2-\theta)-2(k-\alpha)\delta(-1+\theta)^2}{m(1+2\delta)(4\delta^2-2\delta\theta+(-2+\theta)(1+\theta))} \tag{6.9}$$

Thus, when $\lambda^* < \dfrac{m(2+2\delta-\theta)(-1-2\delta+4\delta^2-\theta)+2(k-\alpha)\delta(-1+\theta)^2}{m(1+2\delta)(4\delta^2-2\delta\theta+(-2+\theta)(1+\theta))}$, then the premerger price from Case I is ensured to be greater than the post-merger price in Case II.

Comparative statics also provide insight into the properties of the required λ^*. The partial derivative of λ^* with respect to across-submarket substitutability is $\dfrac{\partial\lambda^*}{\partial\delta} < 0$. This shows that an increase in across-submarket substitutability lowers the required λ^* value for the merger to be welfare neutral, meaning that a larger cost reduction is required for welfare neutrality when across-submarket substitutability rises. The partial derivative of λ^* with respect to within-submarket substitutability is greater than zero, $\dfrac{\partial\lambda^*}{\partial\theta} > 0$.

This comparative static reveals that increasing substitutability within submarkets increases the required λ^*, thus lowering the required cost reduction for welfare neutrality.

Together, these two substitutability parameters create an interesting relationship, whereby as they approach one another in magnitude, increasingly higher cost reductions are required for welfare neutrality, as visualized in Figure 6.1. From Figure 6.1 it is seen that as within submarket and across submarket substitutability tend toward values close to one another, more management cost reductions are necessary to offset any price increase.

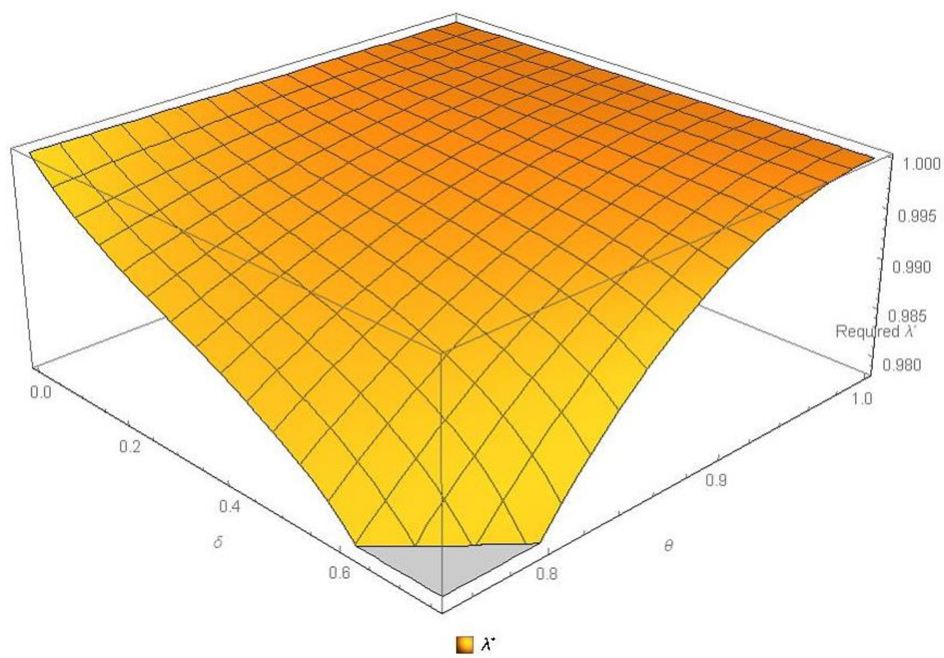

Figure 6.1 Required Cost Reduction to Ensure No Welfare Loss

Note: For simplicity, $\alpha = 3$. The parameters m and k were also normalized to one each, respectively.

This is especially important because the firms engaged in these markets are trying to just that: differentiate their product from other direct competitors in the same subsector and differentiating their products from other possible substitutes in the broader market.

When the firm pursues product differentiation both within and across submarkets, it allows them to increase the price-cost margin for each product in their portfolio of goods, which improves firm profits.

Other comparative statics yield informative results. For instance, $\frac{\partial \lambda^*}{\partial \alpha} < 0$, which means that the higher the existing consumer maximum willingness-to-pay increases the required cost reduction for welfare neutrality. This occurs because by doing so the firm has effectively increased the price-cost margin, which increases the required cost reduction for welfare neutrality. To counter this, greater cost reductions are required by the firm (and passed on to the consumer via pricing) for welfare neutrality. Antitrust agencies would be advised to look into to situations in which the price-cost margin increases in this modeling framework. Also, the comparative static for both subsets of costs results in $\frac{\partial \lambda^*}{\partial m} > 0$ or $\frac{\partial \lambda^*}{\partial k} > 0$. Therefore, if either or both costs are initially high requires lesser cost savings for a welfare neutral merger. Higher costs lower the price-cost margin for a firm's product. The results indicate that with a lower price-cost margin, it requires less scope economies for a welfare neutral merger. It follows that the antitrust regulator might forgo analyzing situations in which the price-cost margin is lower in favor of investigating situations in which the price-cost margins have increased.

In all, the expected post-merger economies of scope are an important consideration in differentiated product merger analysis. Without sufficient cost savings, it would not be possible to offset the increases in market power, and depending upon the initial state of the industry, even greater cost savings may be required.

CHAPTER VII

CONCLUSIONS AND GUIDANCE FOR FUTURE RESEARCH

The analysis is relevant and timely due to the prevalence of this merger pattern in meat protein markets. The product differentiation model developed in this study is a mathematically tractable and informative. Although this analysis focuses on the meat-protein industry as a guiding example, it is easily extended and adapted to other markets in which mergers and acquisitions of a similar nature are occurring.

This thesis provides several contributions to the literature that are relevant to academicians, antitrust regulators, and firm decision makers. The modeling framework and findings presented contribute to a better understanding of the potential effects of multi-product mergers, considers the required cost savings to offset market power, identifies factors which inhibit adequate cost savings to be realized by the merger, and has further characterized the role that within- and across-submarket substitutability play in affecting the welfare neutrality of the merger.

The modeling approach also considers alternative market definitions and provides several important findings. The primary finding of this thesis is that there appears to be merit in the DOJ's view that mergers within submarkets are more likely to cause harm and must be thoroughly scrutinized. However, the results also indicate the importance of recognizing that mergers across submarkets may also have considerable anticompetitive effects as well; especially if sufficient economies of scope cannot be realized.

Additionally, the results also indicate that firms should not blindly attempt to define the relevant more broadly. This is because that even in a broader market definition, significant anticompetitive harm may still occur, especially when the likelihood that sufficient management cost reductions are small.

Finally, the results of the analysis also provide a plausible explanation for the emergence of multiproduct firms and increased consolidation, especially as has been observed in the meat processing sectors. The emergence of multiproduct firms and increased consolidation is much in agreeance with Deneckere and Davidson (1985), who demonstrated that in a Bertrand game with differentiated products, firms have an incentive to merge. However, there may be non-market power driven motives firms have for merging beyond strict profit maximization. Motivations for mergers and acquisitions are not always transparent, and there often are several motivations for merger (Nguyen et al., 2012). Given the demand relationships of the industry described in this thesis, it is left for future research whether portfolio diversification may be a reason for these mergers in the meat-protein sector. Additionally, this analysis ignores the fact that mergers are not always profitable endeavors. In fact, Ravenscraft and Scherer (1989) find that average profitability decreases after merger, with very few exceptions. The authors also suggest 'control loss' of the merged firm as a possible explanation for this.

Limitations and Extensions

The model developed in this thesis does not come without limitations. Assumptions and simplifications are made as a necessary step for clarity of exposition. As such, the major shortcoming of the current work is that the modeling framework has not been fully generalized so as to be empirically testable in in a realistic market setting.

Additionally, this model is specified using the Bowley function, limiting its application to market settings which exhibit similar traits, making the Bowley specification suitable. If the Bowley model does not accurately describe the market under consideration, a more suitable functional form must be used. However, it is unknown if the model and results presented in this thesis is robust to changes in functional form.

Assuming the broader market has a finite set of possible products as in the meat protein industry, the generalized consumer utility was presented in expression 4.2[8]. As described in Chapter IV, several other generalizations are possible: allowing the parameters α_i and β_i to vary for each product in the market, and allowing the number of firms in each submarket to vary. To allow for a wider range of product competition, another set of substitutability parameters may be added to account for each pairing of products. For instance, it is likely the case that differentiation of products within submarkets are asymmetric. In turn, every product, both within and across submarkets, would have a unique substitutability parameter. This would result in a sizable matrix of differentiation parameters. Additional extensions include allowing for costs to vary across products being produced. This step would only improve the current modeling framework if it is reasonable to assume the relative cost to maximum willingness-to-pay differentials are significantly different.

Additionally, opportunities for extension of this model to include various strategic concepts may present emerging research opportunities. For example, it is unknown how

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[8] For clarity, the fully generalized model is shown here:

$$U(\mathbf{q}, Z) = \alpha_i \sum_{i=1}^{n} q_i - \frac{1}{2}\left(\sum_{i=1}^{n} \beta_i q_i^2 + 2\theta_{i,j} \sum_{i \neq j} q_i q_j + 2\delta_{h,k} \sum_{h \neq k} q_h q_k \right) + Z .$$

For detailed parameter descriptions and relationships, see Chapter IV.

the outcomes might change if exclusionary bundling or tie-in sales were incorporated as a second phase strategy of the merged firm.

As an intermediate step for empirical analysis of an industry, or if relevant firm data (costs) are not readily available, the modeling framework is well suited for use with simulation methods. This type of testing has a distinct advantage for further understanding the power and implications of the modeling framework. The advantage of using simulation methods to test this model is that the relevant parameters can be easily adjusted to "realistically relevant" levels, thus allowing for a wider set of predictions and scenarios.

Finally, steps for future empirical estimation will be discussed. First, data on market prices and quantities consumed are readily available (e.g. scanner data and USDA reports). Once the data are acquired, econometric estimation of the relevant consumer demand parameters is possible by utilizing the system of linear inverse demands shown in 4.4. From this system, direct estimates of α_i , β_i, , and $\delta_{h,\mathrm{k}}$ can be obtained. In turn, realistic estimates for the required cost reductions for welfare neutrality can be either simulated by obtaining estimated for production specific and management costs from the firm and inserted into the generalized expression 6.9. The value of this simple approach is that a rough measure of the required cost reductions for welfare neutrality can be obtained as an initial merger screening or "quick-look" tool, utilizing a limited amount of data and low time commitment.

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APPENDIX A

TWO PRODUCT BOWLEY MODEL CALCULATIONS

This appendix illustrates the mathematics for the two product duopoly model with the Bowley function, notably published by Dixit (1979) in the Bell Journal of Economics. The Bowley function portrays the utility of a representative consumer for two goods using a quadratic function, shown in (A.1).

$$U(q_1, q_2) = a_1 q_1 + a_2 q_2 - \frac{1}{2}(2\theta q_1 q_2 + \beta_1 q_1^2 + \beta_2 q_2^2) + z \tag{A.1}$$

In (A.1), *a* is a reservation price, *b* is a slope parameter, and z is a representative good for all other goods and has been normalized to 1. θ is the substitutability parameter between the two goods. At $\theta = 0$, the goods are independent of each other. When $0 < \theta < 1$, the goods are imperfect substitutes. At $\theta = 1$, the goods are perfect substitutes. Next, Equation (A.2) shows the inverse linear demand equations implied by the Bowley function.

$$\begin{aligned} p_1 &= a_1 - \theta q_2 - q_1 \beta_1 \\ p_2 &= a_2 - \theta q_1 - q_2 \beta_2 \end{aligned} \tag{A.2}$$

These can be solved for quantity to find the Bertrand-Bowley demands for each quantity, as in (A.3).

$$\begin{aligned} q_1 &= \frac{\theta a_2 - \theta p_2 + (-a_1 + p_1)\beta_2}{\theta^2 - \beta_1 \beta_2} \\ q_2 &= \frac{\theta a_1 - \theta p_1 + (-a_2 + p_2)\beta_1}{\theta^2 - \beta_1 \beta_2} \end{aligned} \tag{A.3}$$

Note that it is possible for *a* and *b* to vary across products, making them no longer symmetric, but this is not done here.

Next, the quantities will be added together in order to find aggregate demand. This is done because $q_1 + q_2 = Q$. After simplification, (A.4) will result.

$$Q = (q_1 + q_2) = \frac{-\theta p_1 - \theta p_2 + a_2(\theta - \beta_1) + p_2\beta_1 + a_1(\theta - \beta_2) + p_1\beta_2}{\theta^2 - \beta_1\beta_2} \tag{A.4}$$

The differentiated aggregate elasticity for q_1 will now be calculated, which will also work for q_2 because the firms are assumed to be symmetric. The first step is to take the partial derivative of q_1 with respect to p_1, resulting in (A.5).

$$\frac{\partial q_1}{\partial p_1} = \frac{\beta_2}{\theta^2 - \beta_1\beta_2} \tag{A.5}$$

Then, $\frac{\partial q_1}{\partial p_1}$ can be multiplied by $\frac{p_1}{\frac{\theta a_2 - \theta p_2 + (-a_1 + p_1)\beta_2}{\theta^2 - \beta_1\beta_2}}$, in order to find the differentiated elasticity for q_1, shown in equation (A.6).

$$E_{d,q_1} = -\frac{1}{b(1-\theta^2)} * \frac{p_1}{\frac{\theta a_2 - \theta p_2 + (-a_1 + p_1)\beta_2}{\theta^2 - \beta_1\beta_2}} = \frac{p_1\beta_2}{\theta a_2 - \theta p_2 + (-a_1 + p_1)\beta_2} \tag{A.6}$$

For this comparison, the elasticity of composite aggregate demand will be calculated. This requires making a composite price variable, p. This is done in (A.7). Note: Prices are normalized such that $p_1 = p_2 = 1$. This can be done because the firms are symmetric.

$$p = .5(p_1 + p_2) \tag{A.7}$$

Given this, $p = 1$. With these changes, expression (A.4) can be rephrased as expression (A.8).

$$\frac{a_2(\theta - \beta_1) + a_1(\theta - \beta_2) + p(-2\theta + \beta_1 + \beta_2)}{\theta^2 - \beta_1\beta_2} \tag{A.8}$$

Taking the first derivative of (A.8) with respect to p allows (A.9) to be obtained, which is the composite partial derivative.

$$\frac{\partial Q}{\partial p} = \frac{-2\theta + \beta_1 + \beta_2}{\theta^2 - \beta_1\beta_2} \tag{A.9}$$

From (A.9), the composite elasticity of demand for the composite aggregate demand can be found, as in (A.10).

$$\begin{aligned} E_{d,c} &= -\frac{-2\theta + \beta_1 + \beta_2}{\theta^2 - \beta_1\beta_2} * \frac{p}{\frac{-\theta p_1 - \theta p_2 + a_2(\theta - \beta_1) + p_2\beta_1 + a_1(\theta - \beta_2) + p_1\beta_2}{\theta^2 - \beta_1\beta_2}} \\ &= \frac{p(-2\theta + \beta_1 + \beta_2)}{a_2(\theta - \beta_1) + a_1(\theta - \beta_2) + p(-2\theta + \beta_1 + \beta_2)} \end{aligned} \tag{A.10}$$

Now, it is possible to check to see if composite aggregate demand is more inelastic than for each individual aggregate demand. The elasticity of demand for q_1 should be more negative than elasticity of demand for the composite aggregate demand. From (A.6) and (A.10), (A.11) can be obtained.

$$\frac{p(-2\theta + \beta_1 + \beta_2)}{a_2(\theta - \beta_1) + a_1(\theta - \beta_2) + p(-2\theta + \beta_1 + \beta_2)} > \frac{p_1\beta_2}{\theta a_2 - \theta p_2 + (-a_1 + p_1)\beta_2} \tag{A.11}$$

It is known that p , p_1 and p_2 are all equal to 1. Plugging these values in, (A.12) results.

$$\frac{-2\theta + \beta_1 + \beta_2}{-2\theta + a_2(\theta - \beta_1) + \beta_1 + a_1(\theta - \beta_2) + \beta_2} > -\frac{\beta_2}{\theta - \theta a_2 + (-1 + a_1)\beta_2} \tag{A.12}$$

This is the required expression for which the Bowley function will need to satisfy in order to be said to have the properties desired. It is found that for values of $-1 < \theta < 1$ and $a > 1$ this condition is satisfied. Happily, this means that under the assumptions made by this exercise, the full range of θ possibilities from $\theta = -1$ (perfect complements) to $\theta = 1$ (perfect substitutes) is possible so long as $a > 1$.

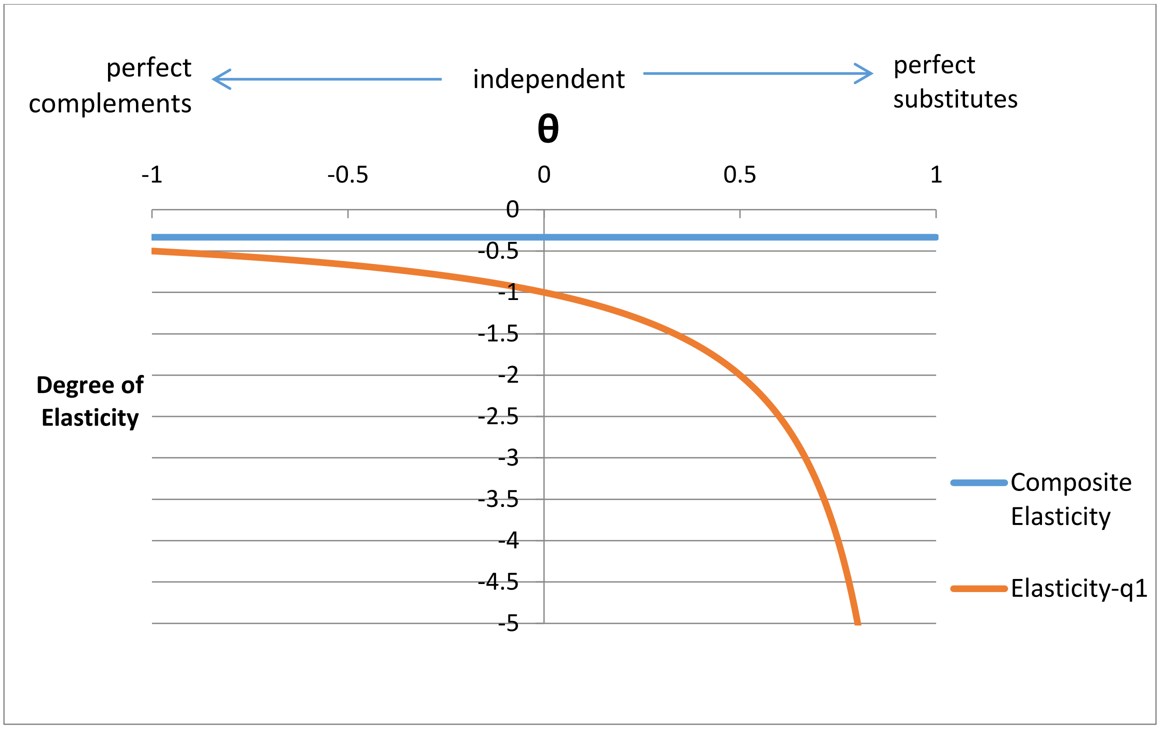


Figure A.1 Comparison of Composite Own-price Elasticity of Demand and Own-price Elasticity of Demand for Good 1

APPENDIX B

PRICE COMPARISON TABLES FOR MERGER CASES

Table B.1 Price Comparisons for Merger Cases

Comparison	Notes/explanation (if necessary)
$P_{II,1} > P_{I,1}$	
$P_{II,2} > P_{I,2}$	
$P_{III,1} > P_{I,1}$	
$P_{IV,1} > P_{I,1}$	
$P_{IV,6} > P_{I,6}$	
$P_{V,1} > P_{\mathrm{I},1}$	
$P_{III,1} > P_{II,1}$	
$P_{III,2} > P_{II,2}$	
$P_{IV,1} \overset{<}{\underset{>}{=}} P_{II,1}$	See Figure B.1 and discussion.
$P_{IV,2} \overset{<}{\underset{>}{=}} P_{II,2}$	See Figure B.2. $P_{IV,2} > P_{II,2}$ when the condition $0 < \delta < .75 < \theta < 1$ is satisfied.
$P_{IV,3} \overset{<}{\underset{>}{=}} P_{II,3}$	See Figure B.3 and discussion.
$P_{V,1} > P_{\mathrm{II},1}$	
$P_{V,2} > P_{\mathrm{II},2}$	
$P_{IV,1} \overset{<}{\underset{>}{=}} P_{III,1}$	See Figure B.4 and discussion.
$P_{IV,3} \overset{<}{\underset{>}{=}} P_{III,3}$	See Figure B.5 and discussion.
$P_{V,1} > P_{\mathrm{III},1}$	
$P_{V,1} > P_{\mathrm{IV},1}$	
$P_{V,3} > P_{\mathrm{IV},3}$	
$P_{II,1} > P_{II,2}$	

Table B.1 (continued)

Comparison	Notes/explanation (if necessary)
$<$ $P_{\mathrm{II},2} = P_{IV,3}$ $>$	See Figure B6 and discussion.
$P_{IV,3} < P_{IV,1}$	

Unless otherwise noted, these comparisons hold for the conditions $0 < \delta < 1$, $\delta < \theta < 1$, $\alpha > 0$, and $0 < c < \alpha$. Notation for this consists of the item being described in regular script. Subscripts directly below allow for each case to be identified (I, II, III, IV, V), as well as the firm/product to be identified (1..6). Refer back to Tables (5.1-5.5) for additional information, if needed. In the context of the problem, firms 1 and 2 produce beef, firms 3 and 4 produce pork, and firms 5 and 6 produce chicken.

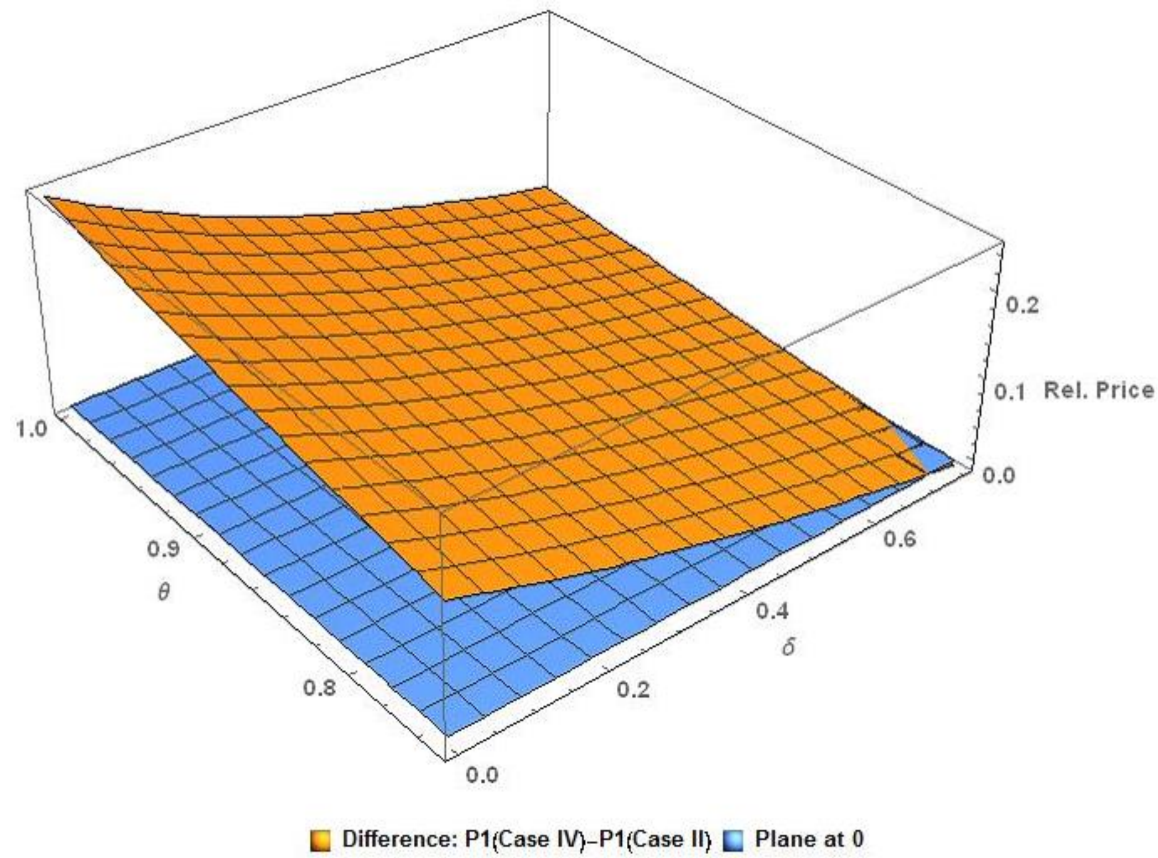


Figure B.1 Relative Price Comparison: P_1 (Case IV) vs. P_1 (Case II)

Note: To aid in graphing, α=1.5 and c=1. Areas in orange, which are above the horizontal plane at 0, detail the areas in which the price for the submarket monopolist in Case IV is greater than the price for the multiproduct firm in Case II. Conditions required for $P_{IV,1} > P_{II,1}$, as reported by Wolfram Mathematica are

$$0<\theta<1,\ 0<\delta<\left(\begin{array}{l} Root[-4\theta-12\theta^2-11\theta^3-\theta^4+3\theta^5+\theta^6 \\ +\left(8-16\theta-54\theta^2-24\theta^3+10\theta^4+4\theta^5\right)\#1 \\ +\left(48+20\theta-8\theta^2+16\theta^3-4\theta^4\right)\#1^2 \\ +\left(24+144\theta+112\theta^2-16\theta^3\right)\#1^3 \\ +\left(-208+112\theta-48\theta^2\right)\#1^4 \\ +\left(-96-192\theta\right)\#1^5+192\#1^6\&,2] \end{array}\right),\ \alpha>0,\ 0<c<\alpha.$$

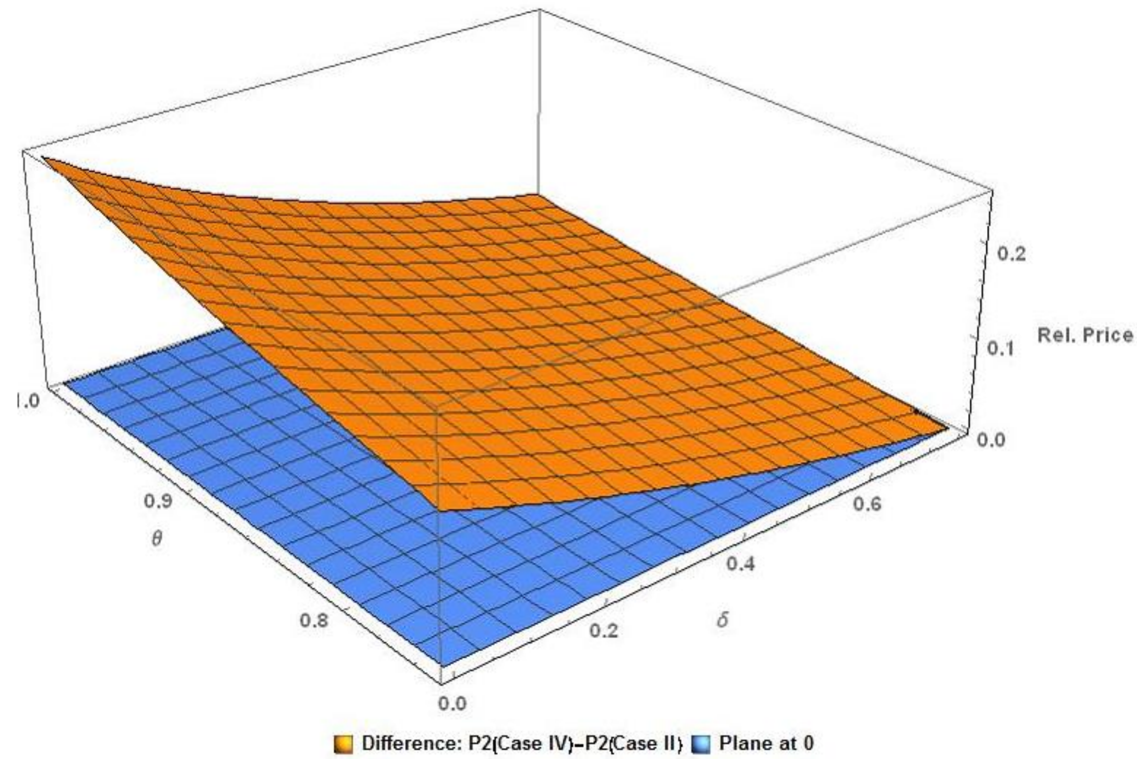


Figure B.2 Relative Price Comparison: P_2 (Case IV) vs. P_2 (Case II)

Note: To aid in graphing, α=1.5 and c=1. Areas in orange, which are above the horizontal plane at 0, detail the areas in which the price for the submarket monopolist in Case IV is greater than the price for the multiproduct fringe firm in Case II. Two possible conditions required for $P_{IV,2} > P_{II,2}$, as reported by Wolfram Mathematica are:

(1) $$\begin{pmatrix} 0 < \theta \leq \frac{2}{19}\left(3+2\sqrt{7}\right), \\ 0 < \delta < \theta, \\ \alpha > 0, \\ 0 < c < \alpha \end{pmatrix}$$

or

(2) $$\left(\begin{array}{l} \frac{2}{19}\left(3+2\sqrt{7}\right) < \theta < 1, \\ 0 < \delta < \left(Root\left[\begin{array}{l} 4\theta + 8\theta^2 + 3\theta^3 - 2\theta^4 - \theta^5 + \left(20\theta + 34\theta^2 + 4\theta^3 - 10\theta^4\right)\#1 \\ + \left(-24 + 12\theta + 48\theta^2 - 24\theta^3\right)\#1^2 \\ + \left(-88 - 32\theta^2\right)\#1^3 - 48\theta\#1^4 + 96\#1^5 \&, 4 \end{array}\right]\right) \\ \alpha > 0, \\ 0 < c < \alpha \end{array}\right).$$

This implies that for the restrictions $0 < \delta < .75 < \theta < 1$, then $P_{IV,2} > P_{II,2}$.

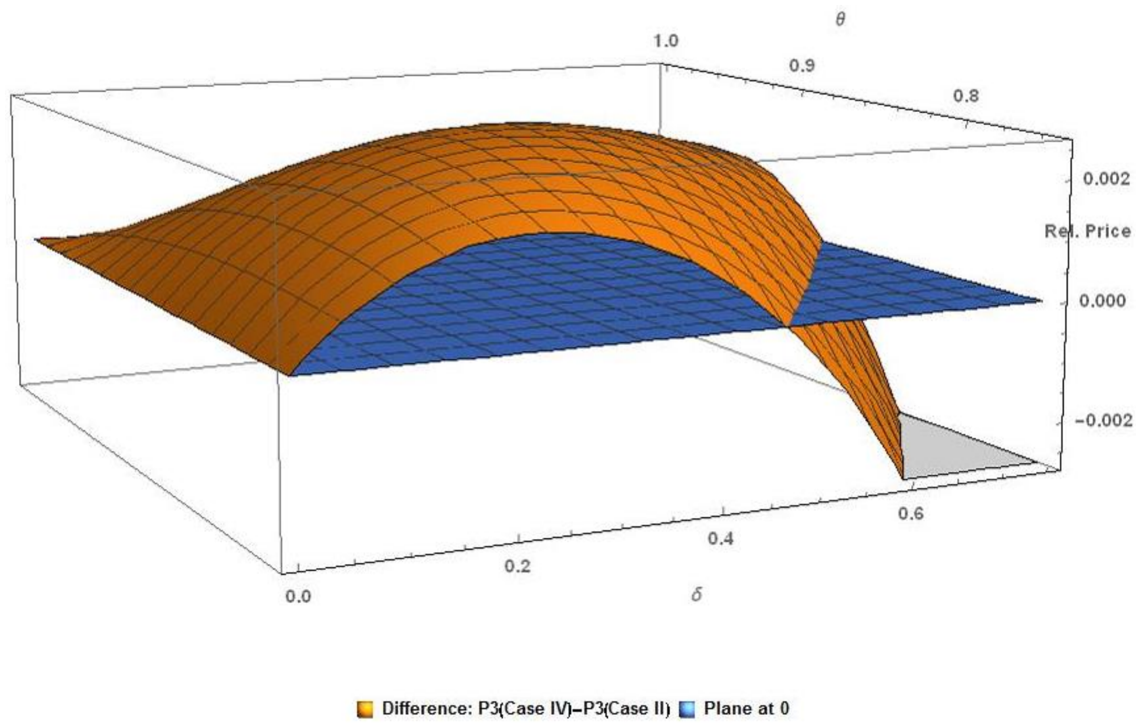


Figure B.3 Relative Price Comparison: P_3 (Case IV) vs. P_3 (Case II)

Note: To aid in graphing, α=1.5 and c=1. Areas in orange, which are above the horizontal plane at 0, detail the areas in which the price for the Case IV fringe firm is greater than the price for the multiproduct firm in Case II. The conditions required for $P_{IV,2} > P_{II,2}$, as reported by Wolfram Mathematica are

$$-3+\sqrt{13} < \theta < 1,$$

$$0 < \delta < \left(Root \begin{bmatrix} -4-2\theta+9\theta^2+8\theta^3+\theta^4+\left(-16+4\theta+28\theta^2+8\theta^3\right)\#1 \\ +\left(-4-16\theta-16\theta^2\right)\#1^2+\left(32-80\theta\right)\#1^3+48\#1^4\&,3 \end{bmatrix} \right),$$

$$\alpha > 0,$$

$$0 < c < \alpha$$

Within the realm of $0 < \delta < .75 < \theta < 1$, $P_{IV,3} > P_{II,3}$ so long as the substitutability parameters are 'sufficiently different.'

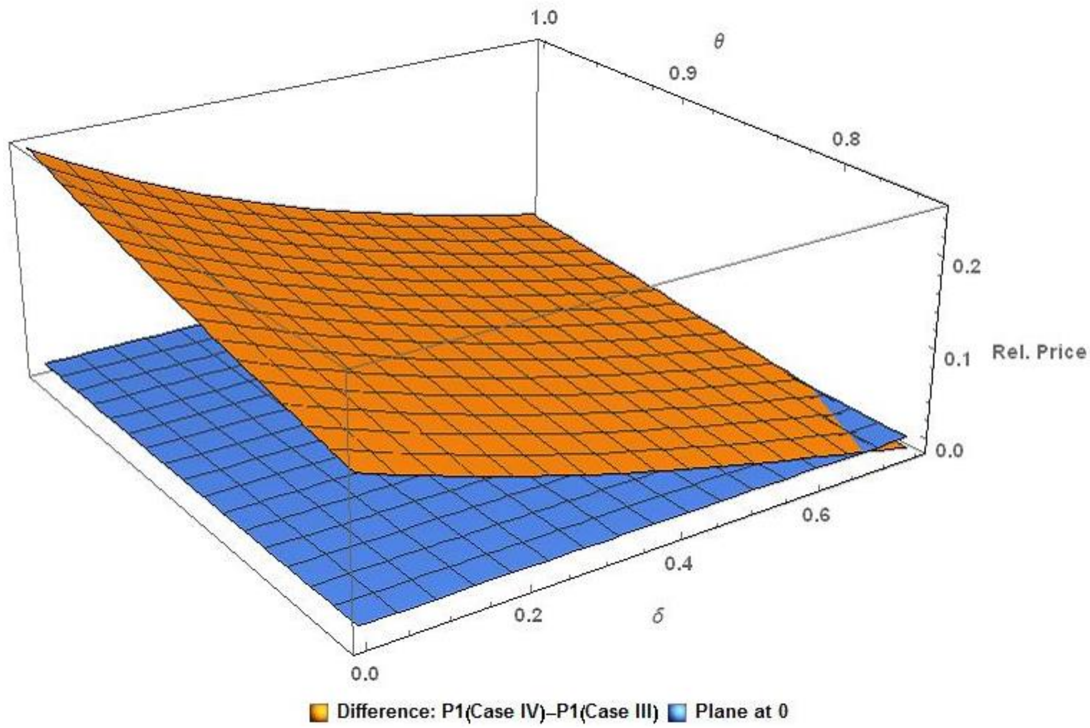


Figure B.4 Relative Price Comparison: P_1 (Case IV) vs. P_1 (Case III)

Note: To aid in graphing, α=1.5 and c=1. Areas in orange, which are above the horizontal plane at 0, detail the areas in which the price for the submarket monopolist in Case IV is greater than the a price for a firm in the two multiproduct firm Case III. The condition required for $P_{IV,1} > P_{III,1}$, as reported by Wolfram Mathematica is:

$$0 < \theta < 1$$

$$0 < \delta < \left(Root \begin{bmatrix} 2\theta + 3\theta^2 - \theta^4 + \left(-4 + 8\theta + 8\theta^2 - 4\theta^3\right)\#1 \\ + \left(-20 + 16\theta - 8\theta^2\right)\#1^2 - 16\theta\#1^3 + 16\#1^4 \&, 3 \end{bmatrix} \right).$$

$$\alpha > 0$$

$$0 < c < \alpha$$

It suffices to say that under the restrictions $0 < \delta < .75 < \theta < 1$, $P_{IV,1} > P_{III,1}$ when the within and across submarket substitutability's are 'sufficiently different'.

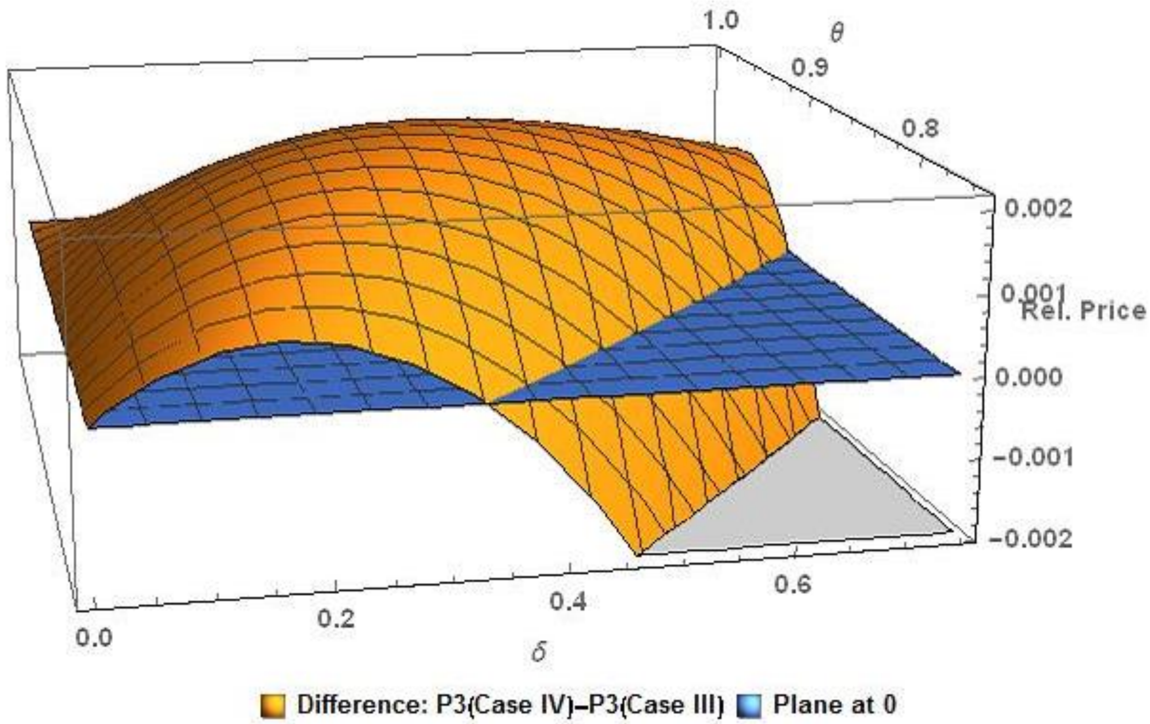


Figure B.5 Relative Price Comparison: P_3 (Case IV) vs. P_3 (Case III)

Note: To aid in graphing, α=1.5 and c=1. Areas in orange, which are above the horizontal plane at 0, detail the areas in which the price for the submarket monopolist in Case IV is greater than the a price for a firm in the two multiproduct firm (Case III). The condition required for $P_{IV,3} > P_{III,3}$, as reported by Wolfram Mathematica is

$$\frac{2}{3} < \theta < 1$$

$$0 < \delta < \frac{1}{4}(-3+4\theta)+\frac{1}{4}\sqrt{1-20\theta+28\theta^2}\ .$$

$$\alpha > 0$$

$$0 < c < \alpha$$

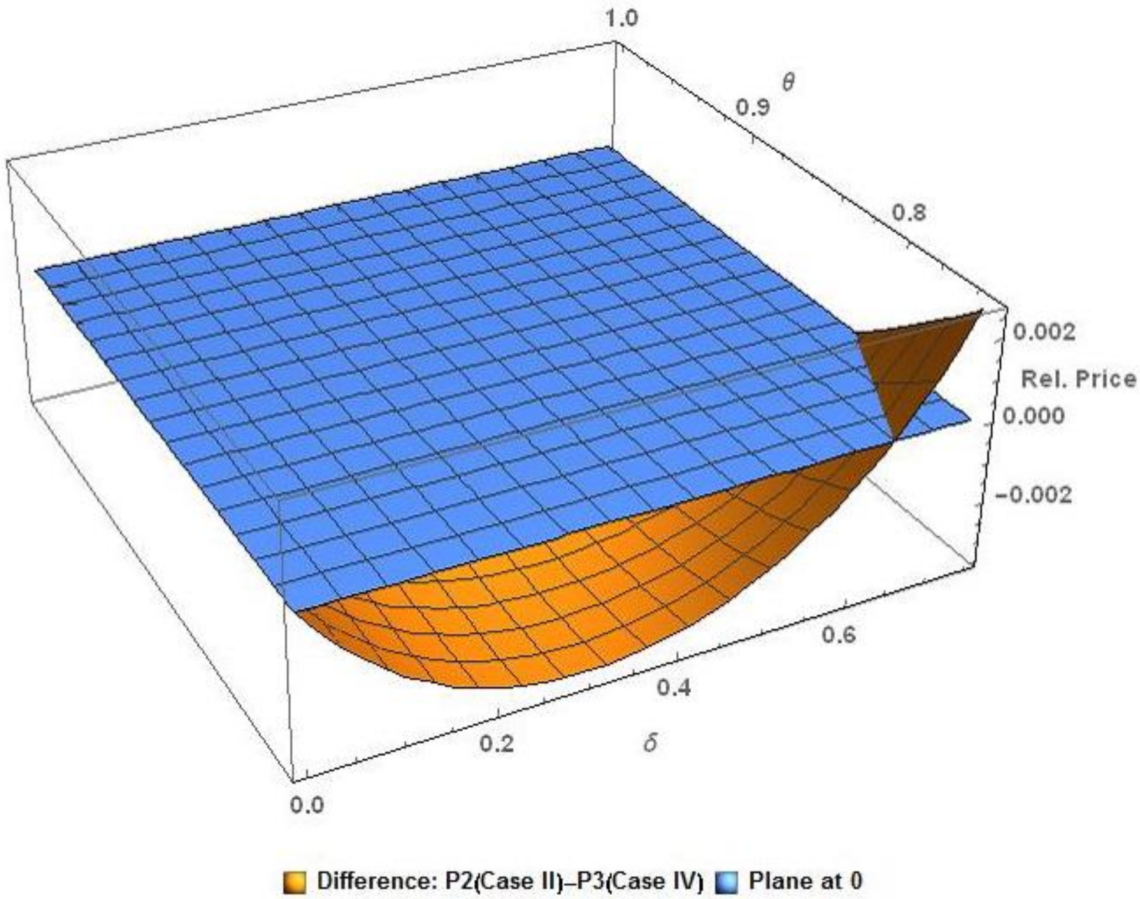


Figure B.6 Relative Price Comparison: P_2 (Case II) vs. P_3 (Case IV)

Note: To aid in graphing, α=1.5 and c=1. Areas in orange, which are above the horizontal plane at 0, show the areas in which the price for the fringe firm in Case II is greater than the a price for a fringe firm in Case IV. The two possible conditions required for $P_{II,2} > P_{IV,3}$, as reported by Wolfram Mathematica are:

$$(1)\ \begin{pmatrix} 0 < \theta \leq Root\begin{bmatrix} -1 + 26\#1 - 94\#1^2 \\ +180\#1^3 + 51\#1^4 \&, 2 \end{bmatrix} \\ Root\begin{bmatrix} -3\theta^2 - 3\theta^3 + \left(4 - 4\theta - 18\theta^2\right)\#1 \\ +\left(20 - 20\theta\right)\#1^2 + 24\#1^3 \&, 3 \end{bmatrix} < \delta < \theta \\ \alpha > 0 \\ 0 < c < \alpha \end{pmatrix} \text{ or}$$

$$(2)\ \begin{pmatrix} Root\begin{bmatrix} -1+26\#1-94\#1^2+180\#1^3 \\ +51\#1^4\&,2 \end{bmatrix} < \theta < 1 \\ Root\begin{bmatrix} -3\theta^2-3\theta^3+\left(4-4\theta-18\theta^2\right)\#1 \\ +\left(20-20\theta\right)\#1^2+24\#1^3\&,1 \end{bmatrix} < \delta < \theta \\ \alpha > 0 \\ 0 < c < \alpha \end{pmatrix}.$$

As opposed to previous results, for the restriction $0<\delta<.75<\theta<1$, $P_{II,2} > P_{IV,3}$ when the substitutability parameters are sufficiently close. Or, conversely, the price for a Case IV fringe firm is greater than the price for a Case II fringe firm only when the substitutability parameters are sufficiently different from one another.

APPENDIX C

QUANTITY COMPARISON TABLES FOR MERGER CASES

Table C.1 Quantity Comparisons for Merger Cases

Comparison	Notes/explanation (if necessary)
$q_{II,1} < q_{I,1}$	
$q_{III,1} < q_{I,1}$	
$q_{IV,1} < q_{I,1}$	
$q_{IV,6} > q_{I,6}$	
$q_{V,1} < q_{I,1}$	
$q_{III,1} > q_{II,1}$	
$q_{III,2} < q_{II,2}$	
$<$ $q_{IV,1} = q_{II,1}$ $>$	See Figure C.1 and discussion.
$q_{IV,2} < q_{II,2}$	
$q_{IV,3} > q_{II,3}$	
$q_{V,1} < q_{II,1}$	
$q_{V,2} < q_{II,2}$	
$<$ $q_{IV,1} = q_{III,1}$ $>$	See Figure C.2 and discussion. $q_{IV,1} < q_{III,1}$ under restrictions $0 < \delta < .75 < \theta < 1$.
$q_{IV,3} > q_{III,3}$	
$q_{V,1} < q_{III,1}$	
$q_{V,1} < q_{IV,1}$	
$q_{V,3} < q_{IV,3}$	
$q_{II,2} > q_{I,2}$	
$q_{II,1} < q_{II,2}$	
$<$ $q_{II,2} = q_{IV,3}$ $>$	See Figure C.3 and discussion.
$q_{IV,3} > q_{IV,1}$	

Unless otherwise noted, these comparisons hold for the conditions $0 < \delta < 1$, $\delta < \theta < 1$, $\alpha > 0$, and $0 < c < \alpha$. Notation for this consists of the item being described in regular script. Subscripts directly below allow for each case to be identified (I, II, III, IV, V), as well as the firm/product to be identified (1..6). Refer back to Tables (5.1-5.5) for additional information, if needed. In the context of the problem, firms 1 and 2 produce beef, firms 3 and 4 produce pork, and firms 5 and 6 produce chicken.

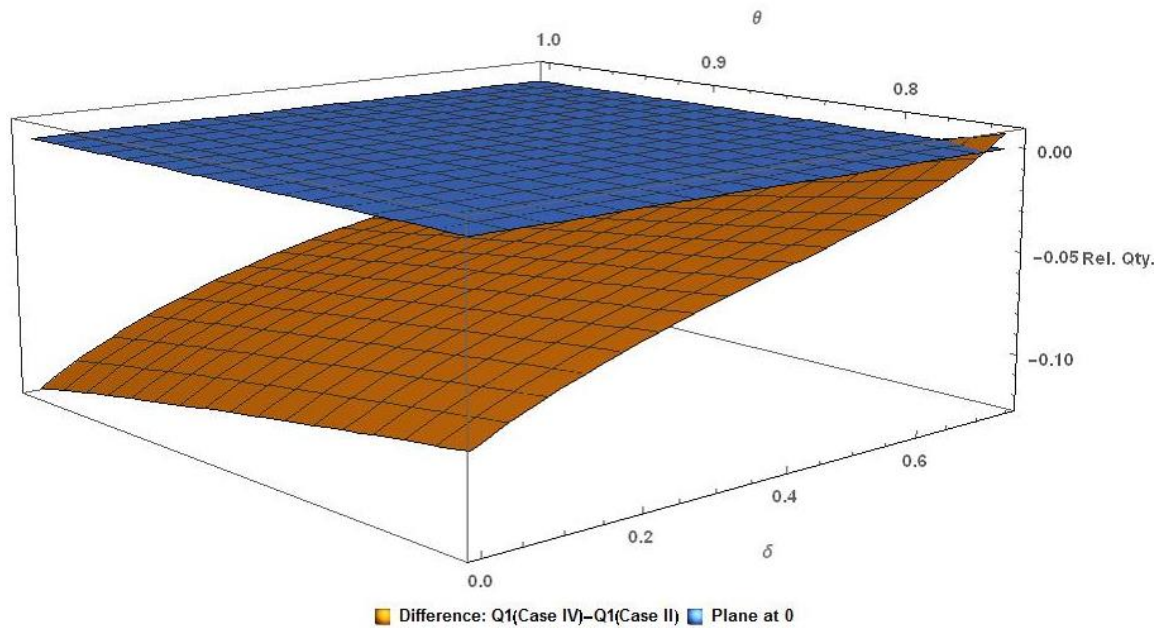


Figure C.1 Relative Quantity Comparison: : Q_1 (Case IV) vs. Q_1 (Case II)

Note: To aid in graphing, α=1.5 and c=1. Areas in orange, which are above the horizontal plane at 0, detail the areas in which the quantity for product 1 of the submarket monopolist in Case IV is less than the quantity for product 1 for the multiproduct firm in Case II. The actual conditions required for $q_{IV,1} > q_{II,1}$ are laborious to present and detract from the analysis. However, it is appropriate to state that given the additional restrictions, $0 < \delta < .75 < \theta < 1$, only when the substitutability parameters are sufficiently close will the submarket monopolist produce a higher quantity of good 1 than a multiproduct firm.

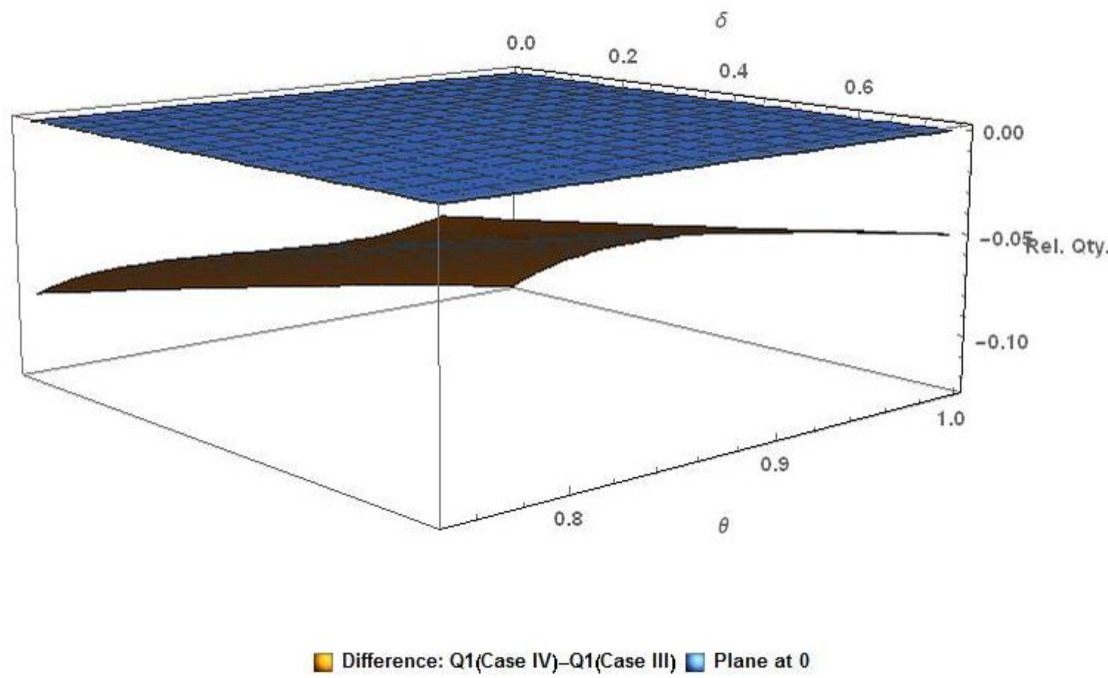


Figure C.2 Relative Quantity Comparison: : Q_1 (Case IV) vs. Q_1 (Case I)

Note: To aid in graphing, α=1.5 and c=1. Under the restrictions $0 < \delta < .75 < \theta < 1$, the area in orange is never above the zero plane indicating that for product 1, a firm in the two multiproduct case always produces more than the submarket monopolist. Without these restrictions, the actual conditions in which $q_{IV,1} > q_{III,1}$ are

$$\begin{pmatrix} 0 < \theta < \frac{1}{34}\left(3+\sqrt{145}\right) \\ Root\begin{bmatrix} -2\theta - 3\theta^2 + \theta^4 + \left(4 - 16\theta - 12\theta^2 + 8\theta^3\right)\#1 \\ +\left(20 - 40\theta + 8\theta^2\right)\#1^2 + \left(32 - 16\theta\right)\#1^3 + 16\#1^4 \&, 2 \end{bmatrix} < \delta < \theta \\ \alpha > 0 \\ 0 < c < \alpha \end{pmatrix}.$$

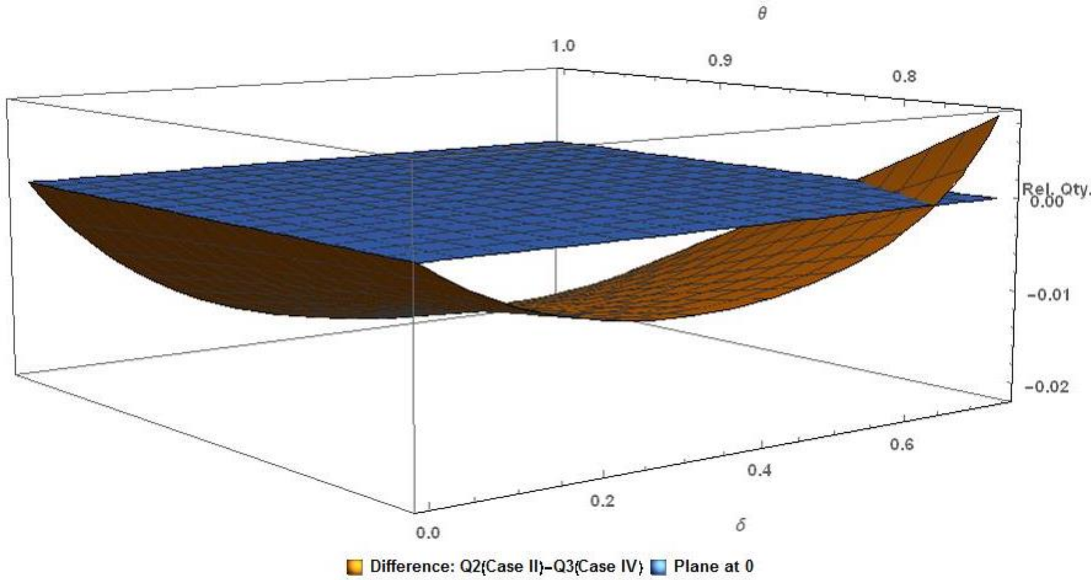


Figure C.3 Relative Quantity Comparison: : Q_2 (Case II) vs. Q_3 (Case IV))

Note: To aid in graphing, α=1.5 and c=1. Areas in orange, which are above the horizontal plane at 0, detail the areas in which the quantity for Case II, product 2 (a fringe firm) is greater than the fringe firm in Case IV. Given the restrictions, $0 < \delta < .75 < \theta < 1$, only when the substitutability parameters are sufficiently close will the Case II fringe firm produce a higher quantity than a submarket monopolist in Case IV. Without the restrictions presented, the actual conditions in which $q_{II,2} > q_{IV,3}$ are

$$\begin{pmatrix} 0 < \delta < 1 \\ \delta < \theta < Root\begin{bmatrix} -4\delta - 20\delta^2 - 24\delta^3 + \left(4\delta + 20\delta^2\right)\#1 \\ +\left(3 + 18\delta\right)\#1^2 + 3\#1^3 \&, 3 \end{bmatrix} \\ \alpha > 0 \\ 0 < c < \alpha \end{pmatrix}.$$

APPENDIX D

CALCULATIONS BY CASE & WELFARE CALCULATIONS BY CASE

Table D.1 Case I: Prices, Quantities, and Profits

Price	$p^*_{i=1,\dots,6} = \dfrac{\alpha(-1+2\delta-\theta)(-1+\theta)+c(1+2\delta-4\delta^2+\theta)}{2-4\delta^2+\theta+2\delta\theta-\theta^2}$
Quantity	$q^*_{i \atop i=1,\dots,6} = \dfrac{(\alpha-c)(-1-2\delta+4\delta^2-\theta)}{(1+4\delta+\theta)(4\delta^2-2\delta\theta+(-2+\theta)(1+\theta))}$
Profit	$\pi^*_{i \atop i=1,\dots,6} = \dfrac{(c-\alpha)^2(-1+2\delta-\theta)(-1-2\delta+4\delta^2-\theta)(1-\theta)}{(1+4\delta+\theta)(2-4\delta^2+\theta+2\delta\theta-\theta^2)^2}$

Table D.2 Case II: Prices, Quantities, and Profits

Price	$p^*_{mp \atop i=1,3,5} = \dfrac{\alpha(-1+\theta)(-12\delta^2+2\delta(2+\theta)+(1+\theta)(2+\theta))+c(24\delta^3-(1+\theta)(2+\theta)-2\delta(4+5\theta))}{24\delta^3-12\delta^2(-1+\theta)+(-2+\theta)(1+\theta)(2+\theta)+2\delta(-6+(-4+\theta)\theta)}$ $p^*_{f \atop i=2,4,6} \dfrac{-\alpha(-1+2\delta-\theta)(-1+\theta)(2+6\delta+\theta)+c(1+2\delta)(6\delta(-1+2\delta)-(1+\theta)(2+\theta))}{24\delta^3-12\delta^2(-1+\theta)+(-2+\theta)(1+\theta)(2+\theta)+2\delta(-6+(-4+\theta)\theta)}$
Quantity	$q^*_{mp \atop i=1,3,5} = \dfrac{(\alpha-c)(1+2\delta)(12\delta^2-2\delta(2+\theta)-(1+\theta)(2+\theta))}{(1+4\delta+\theta)(24\delta^3-12\delta^2(-1+\theta)+(-2+\theta)(1+\theta)(2+\theta)+2\delta(-6+(-4+\theta)\theta))}$ $q^*_{f \atop i=2,4,6} = \dfrac{(\alpha-c)(-1-2\delta+4\delta^2-\theta)(2+6\delta+\theta)}{(1+4\delta+\theta)(24\delta^3-12\delta^2(-1+\theta)+(-2+\theta)(1+\theta)(2+\theta)+2\delta(-6+(-4+\theta)\theta))}$
Profit	$\pi^*_{mp \atop i=1} = \dfrac{3(c-\alpha)^2(1+2\delta)(1-\theta)(-12\delta^2+2\delta(2+\theta)+(1+\theta)(2+\theta))^2}{(1+4\delta+\theta)(24\delta^3-12\delta^2(-1+\theta)+(-2+\theta)(1+\theta)(2+\theta)+2\delta(-6+(-4+\theta)\theta))^2}$ $\pi^*_{f \atop i=2} = \dfrac{(c-\alpha)^2(-1+2\delta-\theta)(-1-2\delta+4\delta^2-\theta)(1-\theta)(2+6\delta+\theta)^2}{(1+4\delta+\theta)(24\delta^3-12\delta^2(-1+\theta)+(-2+\theta)(1+\theta)(2+\theta)+2\delta(-6+(-4+\theta)\theta))^2}$

Table D.3 Case III: Prices, Quantities, and Profits

Price	$\underset{i=1,\ldots,6}{p^*_{i,c}} = \dfrac{c+\alpha+2c\delta-\alpha\theta}{2+2\delta-\theta}$
Quantity	$\underset{i=1,\ldots,6}{q^*_{i,c}} = \dfrac{(\alpha-c)(1+2\delta)}{(2+2\delta-\theta)(1+4\delta+\theta)}$
Profit	$\underset{i=1,\ldots,6}{\pi^*_{i,c}} = \dfrac{3(c-\alpha)^2(1+2\delta)(1-\theta)}{(-2-2\delta+\theta)^2(1+4\delta+\theta)}$

Table D.4 Case IV: Prices, Quantities and Profits

Price	$\underset{i=1,2}{p^*_{i,bm}} = \dfrac{1}{2}\left(c+\alpha+\dfrac{4(c-\alpha)(-1+\delta)\delta(1+4\delta+\theta)}{-2+4\delta^2+8\delta^3-3\theta+\theta^3+2\delta(-3+\theta)(1+\theta)}\right)$ $\underset{i=3,4,5,6}{p^*_{i,nm}} = \dfrac{-\alpha(-1+2\delta-\theta)(-1+\theta)(1+3\delta+\theta)+c(-1+\delta)(1+2\delta+\theta)(1+4\delta+\theta)}{-2+4\delta^2+8\delta^3-3\theta+\theta^3+2\delta(-3+\theta)(1+\theta)}$
Quantity	$\underset{i=1,2}{q^*_{i,\mathrm{bm}}} = \dfrac{(\alpha-c)(1+2\delta+\theta)(4\delta^2+(-2+\theta)(1+\theta)+\delta(-6+4\theta))}{2(1+4\delta+\theta)(-2+4\delta^2+8\delta^3-3\theta+\theta^3+2\delta(-3+\theta)(1+\theta))}$ $\underset{i=3,4,5,6}{q^*_{i,\mathrm{nm}}} = \dfrac{(\alpha-c)(-1-2\delta+4\delta^2-\theta)(1+3\delta+\theta)}{(1+4\delta+\theta)(-2+4\delta^2+8\delta^3-3\theta+\theta^3+2\delta(-3+\theta)(1+\theta))}$
Profit	$\underset{i=1,2}{\pi^*_{bm}} = \dfrac{(c-\alpha)^2(1-2\delta+\theta)(1+2\delta+\theta)(4\delta^2+(-2+\theta)(1+\theta)+\delta(-6+4\theta))^2}{2(1+4\delta+\theta)(-2+4\delta^2+8\delta^3-3\theta+\theta^3+2\delta(-3+\theta)(1+\theta))^2}$ $\underset{i=3,4,5,6}{\pi^*_{i,\mathrm{nm}}} = \dfrac{(c-\alpha)^2(1-2\delta+\theta)(-1-2\delta+4\delta^2-\theta)(-1+\theta)(1+3\delta+\theta)^2}{(1+4\delta+\theta)(-2+4\delta^2+8\delta^3-3\theta+\theta^3+2\delta(-3+\theta)(1+\theta))^2}$

Table D.5 Case V: Prices, Quantities, and Profits

Price	$\underset{i=1,\ldots,6}{p^*_{v,i}} = \dfrac{c+\alpha}{2}$
Quantity	$\underset{i=1,\ldots,6}{q^*_{v,i}} = \dfrac{\alpha-c}{2(1+4\delta+\theta)}$
Profit	$\underset{i=1,\ldots,6}{\pi^*_{v,i}} = \dfrac{(c-\alpha)^2}{4(1+4\delta+\theta)}$

Table D.6 Case I Welfare Calculations

Consumer Surplus	$\dfrac{\left(\begin{array}{l}3c^2(1+2\delta-4\delta^2+\theta)^2-6c\alpha(1+2\delta-4\delta^2+\theta)^2\\+3\alpha^2(1+2\delta-4\delta^2+\theta)^2+z(1+4\delta+\theta)(2-4\delta^2+\theta+2\delta\theta-\theta^2)^2\end{array}\right)}{(1+4\delta+\theta)(2-4\delta^2+\theta+2\delta\theta-\theta^2)^2}$
Producer Surplus	$\dfrac{6(c-\alpha)^2(-1+2\delta-\theta)(-1-2\delta+4\delta^2-\theta)(1-\theta)}{(1+4\delta+\theta)(2-4\delta^2+\theta+2\delta\theta-\theta^2)^2}$
Total Surplus	$\dfrac{\left(\begin{array}{l}(-6(c-\alpha)^2(-1+2\delta-\theta)(-1-2\delta+4\delta^2-\theta)(-1+\theta)\\+3c^2(1+2\delta-4\delta^2+\theta)^2-6c\alpha(1+2\delta-4\delta^2+\theta)^2+3\alpha^2(1+2\delta-4\delta^2+\theta)^2\\+z(1+4\delta+\theta)(2-4\delta^2+\theta+2\delta\theta-\theta^2)^2)\end{array}\right)}{(1+4\delta+\theta)(2-4\delta^2+\theta+2\delta\theta-\theta^2)^2}$

Table D.7 Case II Welfare Calculations

Consumer Surplus	$z+\frac{3(c-\alpha)^2(1+2\delta)\begin{pmatrix}-144\delta^4+288\delta^5+2\delta(1+\theta)(2+\theta)(7+5\theta)+\left(2+3\theta+\theta^2\right)^2\\-24\delta^3(6+\theta(8+\theta))-2\delta^2(-13+\theta(-9+\theta(3+\theta)))\end{pmatrix}}{(1+4\delta+\theta)\left(24\delta^3-12\delta^2(-1+\theta)+(-2+\theta)(1+\theta)(2+\theta)+2\delta(-6+(-4+\theta)\theta)\right)^2}$
Producer Surplus	$\frac{3(c-\alpha)^2(-1+\theta)\begin{pmatrix}-(-1+2\delta-\theta)\left(-1-2\delta+4\delta^2-\theta\right)(2+6\delta+\theta)^2\\-(1+2\delta)\left(-12\delta^2+2\delta(2+\theta)+(1+\theta)(2+\theta)\right)^2\end{pmatrix}}{(1+4\delta+\theta)\left(24\delta^3-12\delta^2(-1+\theta)+(-2+\theta)(1+\theta)(2+\theta)+2\delta(-6+(-4+\theta)\theta)\right)^2}$
Total Surplus	$z+\frac{\begin{pmatrix}3(c-\alpha)^2(576\delta^6-576\delta^5(-1+\theta)-(1+\theta)^2(2+\theta)^2(-3+2\theta)\\+96\delta^4(-6+(-4+\theta)\theta)-2\delta(1+\theta)(2+\theta)(-21+\theta(-7+\theta(9+\theta)))\\+4\delta^3(-103+\theta(-39+\theta(51+19\theta)))+2\delta^2(43+\theta(85+\theta(41-\theta(5+2\theta)))))\end{pmatrix}}{(1+4\delta+\theta)\left(24\delta^3-12\delta^2(-1+\theta)+(-2+\theta)(1+\theta)(2+\theta)+2\delta(-6+(-4+\theta)\theta)\right)^2}$

Table D.8 Case III Welfare Calculations

Consumer Surplus	$z+\frac{-6c\alpha(1+2\delta)^2+3(c+2c\delta)^2+3(\alpha+2\alpha\delta)^2}{(-2-2\delta+\theta)^2(1+4\delta+\theta)}$
Producer Surplus	$\frac{6(c-\alpha)^2(1+2\delta)(1-\theta)}{(-2-2\delta+\theta)^2(1+4\delta+\theta)}$
Total Surplus	$z+\frac{3(c-\alpha)^2(1+2\delta)(3+2\delta-2\theta)}{(-2-2\delta+\theta)^2(1+4\delta+\theta)}$

Table D.9 Case IV Welfare Calculations

Consumer Surplus	$z+\frac{-6c\alpha(1+2\delta)^2+3(c+2c\delta)^2+3(\alpha+2\alpha\delta)^2}{(-2-2\delta+\theta)^2(1+4\delta+\theta)}$
Producer Surplus	$\frac{6(c-\alpha)^2(1+2\delta)(1-\theta)}{(-2-2\delta+\theta)^2(1+4\delta+\theta)}$
Total Surplus	$z+\frac{3(c-\alpha)^2(1+2\delta)(3+2\delta-2\theta)}{(-2-2\delta+\theta)^2(1+4\delta+\theta)}$

Table D.10 Case V Welfare Calculations

Consumer Surplus	$z+\frac{3(c-\alpha)^2}{4(1+4\delta+\theta)}$
Producer Surplus	$\frac{3(\alpha-c)^2}{2(1+4\delta+\theta)}$
Total Surplus	$z+\frac{9(c-\alpha)^2}{4(1+4\delta+\theta)}$

APPENDIX E

SYMMETRY RELATIONSHIPS IN MERGER CASES

Appendix E provides an explicit listing of the symmetry relationships exhibited by the model. Table E.1 shows the relationships for all five cases presented. This reference will aid readers looking at the comparisons.

Table E.1 Symmetry Relationships in Merger Cases: Prices, Quantities, and Profits

Case	**Prices**	**Quantities**	**Profits**
I	$P_1 = P_2 = P_3 = P_4 = P_5 = P_6$	$q_1 = q_2 = q_3 = q_4 = q_5 = q_6$	$\pi_1 = \pi_2 = \pi_3 = \pi_4 = \pi_5 = \pi_6$
II	$P_1 = P_3 = P_5$ $P_2 = P_4 = P_6$	$q_1 = q_3 = q_5$ $q_2 = q_4 = q_6$	$\pi_1 = \pi_3 = \pi_5$ $\pi_2 = \pi_4 = \pi_6$
III	$P_1 = P_2 = P_3 = P_4 = P_5 = P_6$	$q_1 = q_2 = q_3 = q_4 = q_5 = q_6$	$\pi_1 = \pi_2 = \pi_3 = \pi_4 = \pi_5 = \pi_6$
IV	$P_1 = P_2$ $P_3 = P_4 = P_5 = P_6$	$q_1 = q_2$ $q_3 = q_4 = q_5 = q_6$	$\pi_1 = \pi_2$ $\pi_3 = \pi_4 = \pi_5 = \pi_6$
V	$P_1 = P_2 = P_3 = P_4 = P_5 = P_6$	$q_1 = q_2 = q_3 = q_4 = q_5 = q_6$	$\pi_1 = \pi_2 = \pi_3 = \pi_4 = \pi_5 = \pi_6$

APPENDIX F

MATHEMATICA CODE

```
(*Modified Bowley: 3 product submarkets and 2 firms per submarket,
substitutability within submarkets equal
 (theta) and across submarkets equal (delta)*)

FullSimplify[m - p1 q1 - p2 q2 - p3 q3 - p4 q4 - p5 q5 -
   p6 q6 + α (q1 + q2 + q3 + q4 + q5 + q6) + 1/2 (-q1^2 - q2^2 - q3^2 - q4^2 - q5^2 - q6^2 -
      2 * δ (q1 q3 + q2 q3 + q1 q4 + q2 q4 + q1 q5 + q2 q5 + q3 q5 + q4 q5 + q1 q6 + q2 q6 + q3 q6 + q4 q6) -
      2 * θ (q1 q2 + q3 q4 + q5 q6))]

m - p1 q1 - p2 q2 - p3 q3 - p4 q4 - p5 q5 - p6 q6 + α (q1 + q2 + q3 + q4 + q5 + q6) +
 1/2 (-q1^2 - q2^2 - q3^2 - 2 θ q3 q4 - q4^2 - 2 δ q3 q5 - 2 δ q4 q5 - q5^2 - 2 (δ (q3 + q4) + θ q5) q6 -
     q6^2 - 2 δ q2 (q3 + q4 + q5 + q6) - 2 q1 (θ q2 + δ (q3 + q4 + q5 + q6)))

FullSimplify[D[m - p1 q1 - p2 q2 - p3 q3 - p4 q4 - p5 q5 -
    p6 q6 + α (q1 + q2 + q3 + q4 + q5 + q6) + 1/2 (-q1^2 - q2^2 - q3^2 - q4^2 - q5^2 - q6^2 -
       2 * δ (q1 q3 + q2 q3 + q1 q4 + q2 q4 + q1 q5 + q2 q5 + q3 q5 + q4 q5 + q1 q6 + q2 q6 + q3 q6 + q4 q6) -
       2 * θ (q1 q2 + q3 q4 + q5 q6)), {{q1, q2, q3, q4, q5, q6}}]]

{α - p1 - q1 - θ q2 - δ (q3 + q4 + q5 + q6), α - p2 - θ q1 - q2 - δ (q3 + q4 + q5 + q6),
 α - p3 - q3 - θ q4 - δ (q1 + q2 + q5 + q6), α - p4 - θ q3 - q4 - δ (q1 + q2 + q5 + q6),
 α - p5 - δ (q1 + q2 + q3 + q4) - q5 - θ q6, α - p6 - δ (q1 + q2 + q3 + q4) - θ q5 - q6}

(*Demands*)

FullSimplify[Solve[α - p1 - q1 - θ q2 - δ (q3 + q4 + q5 + q6) == 0 &&
   α - p2 - θ q1 - q2 - δ (q3 + q4 + q5 + q6) == 0 && α - p3 - q3 - θ q4 - δ (q1 + q2 + q5 + q6) == 0 &&
   α - p4 - θ q3 - q4 - δ (q1 + q2 + q5 + q6) == 0 && α - p5 - δ (q1 + q2 + q3 + q4) - q5 - θ q6 == 0 &&
   α - p6 - δ (q1 + q2 + q3 + q4) - θ q5 - q6 == 0, {q1, q2, q3, q4, q5, q6}]]

{{q1 → ((-1 - 2 δ + 4 δ^2 - θ) p1 + (-4 δ^2 + θ + 2 δ θ + θ^2) p2 -
       (-1 + θ) (α (1 - 2 δ + θ) + δ (p3 + p4 + p5 + p6))) / ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ)),
  q2 → ((4 δ^2 - 2 δ θ - θ (1 + θ)) p1 + (1 + 2 δ - 4 δ^2 + θ) p2 +
       (-1 + θ) (α (1 - 2 δ + θ) + δ (p3 + p4 + p5 + p6))) / ((-1 + θ) (1 - 2 δ + θ) (1 + 4 δ + θ)),
  q3 → (α (-1 + 2 δ - θ) (-1 + θ) + (δ - δ θ) p1 + (δ - δ θ) p2 - p3 - 2 δ p3 + 4 δ^2 p3 - θ p3 - 4 δ^2 p4 +
       θ p4 + 2 δ θ p4 + θ^2 p4 + δ p5 - δ θ p5 - δ (-1 + θ) p6) / ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ)),
  q4 → (α (-1 + 2 δ - θ) (-1 + θ) + (δ - δ θ) p1 + (δ - δ θ) p2 - 4 δ^2 p3 + θ p3 + 2 δ θ p3 + θ^2 p3 - p4 -
       2 δ p4 + 4 δ^2 p4 - θ p4 + δ p5 - δ θ p5 - δ (-1 + θ) p6) / ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ)),
  q5 → (α (-1 + 2 δ - θ) (-1 + θ) + (δ - δ θ) p1 + (δ - δ θ) p2 + δ p3 - δ θ p3 + δ p4 - δ θ p4 - p5 -
       2 δ p5 + 4 δ^2 p5 - θ p5 + (-4 δ^2 + θ + 2 δ θ + θ^2) p6) / ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ)),
  q6 → (α - 2 α δ + 2 α δ θ - α θ^2 + (δ - δ θ) p1 + (δ - δ θ) p2 + δ p3 - δ θ p3 + δ p4 - δ θ p4 - 4 δ^2 p5 +
       θ p5 + 2 δ θ p5 + θ^2 p5 + (-1 - 2 δ + 4 δ^2 - θ) p6) / ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ))}}

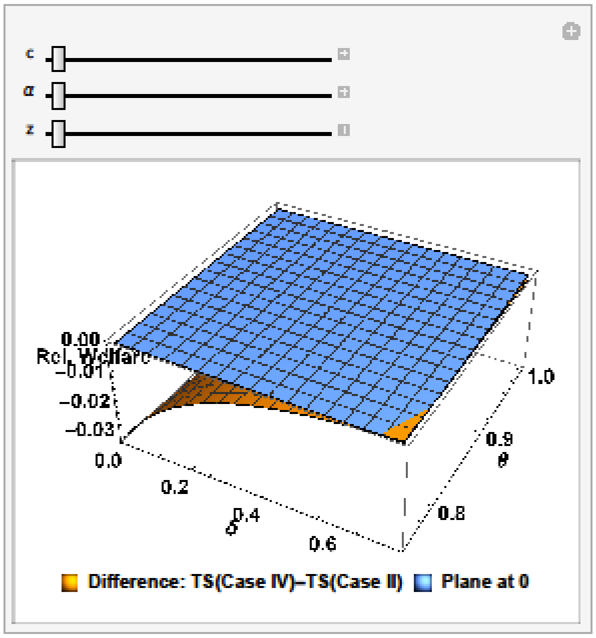
(*Inverse demands*)
```

```
Reduce[(z + ((c - α)^2 (832 δ^6 + 64 δ^5 (15 - 4 θ) + (1 + θ)^4 (36 + θ (-28 + 3 θ)) +
          4 δ (1 + θ)^3 (66 + θ (-49 + 6 θ)) + 16 δ^4 (-96 + θ (-4 + 9 θ)) +
          4 δ^2 (1 + θ)^2 (97 + 3 θ (-34 + 11 θ)) + 16 δ^3 (1 + θ) (-52 + θ (-16 + 23 θ))))/
      (4 (1 + 4 δ + θ) (-2 + 4 δ^2 + 8 δ^3 - 3 θ + θ^3 + 2 δ (-3 + θ) (1 + θ))^2)) >
   (z + (3 (c - α)^2 (576 δ^6 - 576 δ^5 (-1 + θ) - (1 + θ)^2 (2 + θ)^2 (-3 + 2 θ) +
          96 δ^4 (-6 + (-4 + θ) θ) - 2 δ (1 + θ) (2 + θ) (-21 + θ (-7 + θ (9 + θ))) +
          4 δ^3 (-103 + θ (-39 + θ (51 + 19 θ))) + 2 δ^2 (43 + θ (85 + θ (41 - θ (5 + 2 θ))))))/
      ((1 + 4 δ + θ) (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))^2)) &&
  0 < δ < 1 && δ < θ < 1 && α > 0 && 0 <
   c <
   α]
z ∈ Reals && ((0 < θ ≤ Root[
        -12 370 864 909 569 909 577 184 662 732 800 + 27 835 235 449 030 503 065 997 793 198 080
           #1 + 846 931 092 083 635 988 682 417 761 538 048 #1^2 -
          281 560 285 044 301 781 098 780 498 907 136 #1^3 -
          9 971 606 065 780 409 651 386 170 338 442 240 #1^4 +
          20 122 390 909 238 787 675 874 080 591 796 224 #1^5 +
          18 022 561 443 916 094 241 436 181 112 137 472 #1^6 -
          145 419 317 350 936 304 847 465 531 288 548 608 #1^7 +
          67 192 460 922 889 883 255 010 190 249 620 480 #1^8 +
          548 162 676 342 916 059 152 203 678 288 149 888 #1^9 -
          37 560 299 765 779 596 282 123 814 542 092 352 #1^10 -
          1 971 663 539 850 980 889 311 248 843 572 948 480 #1^11 -
          1 667 028 431 180 980 654 971 869 380 829 247 216 #1^12 +
          7 591 724 825 013 922 076 163 328 512 951 276 528 #1^13 +
          6 842 615 293 048 105 386 094 597 080 189 351 864 #1^14 -
          23 157 188 699 722 601 472 443 322 606 237 281 520 #1^15 -
          12 452 216 166 254 911 218 359 979 987 181 959 200 #1^16 +
          49 308 416 781 940 996 339 695 888 384 757 670 448 #1^17 +
          10 329 374 064 954 573 559 512 353 529 055 404 420 #1^18 -
          73 362 388 861 083 861 226 505 665 149 961 096 272 #1^19 +
          2 083 624 521 320 468 567 110 244 933 409 311 628 #1^20 +
          78 214 224 529 453 695 189 703 996 228 298 360 388 #1^21 -
          15 626 863 679 016 596 320 989 056 120 795 972 800 #1^22 -
          61 152 804 706 528 341 490 269 504 171 495 233 748 #1^23 +
          20 156 715 959 215 533 710 252 881 092 793 953 975 #1^24 +
          35 606 919 305 527 694 835 641 341 102 989 963 288 #1^25 -
          15 561 274 367 277 878 373 812 870 800 143 262 212 #1^26 -
          15 475 268 793 350 055 323 251 778 788 517 689 224 #1^27 +
          8 346 332 453 447 085 101 459 385 692 564 337 536 #1^28 +
          4 912 031 674 599 676 282 348 716 509 088 348 048 #1^29 -
```

```
        3 239 431 097 497 717 410 163 526 311 076 830 884 #1^30 -
        1 060 488 561 432 041 244 746 273 963 827 508 396 #1^31 +
        901 607 412 256 406 417 841 523 816 356 689 298 #1^32 +
        125 229 840 618 772 606 502 609 905 094 046 576 #1^33 -
        170 387 616 471 556 637 783 551 877 644 351 800 #1^34 +
        620 321 000 170 194 690 538 107 468 386 928 #1^35 +
        19 408 264 839 356 822 543 058 006 965 863 428 #1^36 -
        2 166 148 263 566 858 401 905 608 020 074 980 #1^37 -
        998 813 893 018 808 926 996 220 598 485 544 #1^38 +
        190 740 516 564 108 460 442 592 160 769 400 #1^39 +
        916 331 128 635 909 864 072 350 486 331 #1^40 &, 10] &&
   Root[-64 θ - 336 θ^2 - 640 θ^3 - 392 θ^4 + 332 θ^5 + 595 θ^6 + 182 θ^7 - 155 θ^8 - 112 θ^9 -
        3 θ^10 + 14 θ^11 + 3 θ^12 + (192 - 160 θ - 3120 θ^2 - 5832 θ^3 - 1792 θ^4 +
            4620 θ^5 + 3840 θ^6 - 740 θ^7 - 1632 θ^8 - 252 θ^9 + 208 θ^10 + 60 θ^11) #1 +
        (2080 + 2752 θ - 8232 θ^2 - 15 416 θ^3 + 980 θ^4 + 13 088 θ^5 + 2056 θ^6 - 6056 θ^7 -
            2092 θ^8 + 736 θ^9 + 312 θ^10) #1^2 + (5696 + 14 976 θ + 11 376 θ^2 - 304 θ^3 - 6752 θ^4 -
            8816 θ^5 - 4816 θ^6 + 112 θ^7 + 256 θ^8 - 208 θ^9) #1^3 + (-10 272 + 21 088 θ +
            87 024 θ^2 + 24 832 θ^3 - 69 568 θ^4 - 19 616 θ^5 + 27 584 θ^6 + 3936 θ^7 - 4528 θ^8) #1^4 +
        (-60 480 - 32 512 θ + 72 896 θ^2 + 9472 θ^3 - 21 632 θ^4 + 32 768 θ^5 + 16 128 θ^6 - 2816 θ^7)
         #1^5 + (-5696 - 176 128 θ - 119 168 θ^2 + 150 400 θ^3 + 32 256 θ^4 - 41 216 θ^5 + 19 584 θ^6)
         #1^6 + (215 040 - 179 456 θ - 92 928 θ^2 + 76 032 θ^3 - 83 200 θ^4 - 11 520 θ^5) #1^7 +
        (99 072 + 327 168 θ - 262 400 θ^2 + 77 824 θ^3 - 68 864 θ^4) #1^8 +
        (-313 344 + 463 872 θ - 178 176 θ^2 + 119 808 θ^3) #1^9 +
        (-138 240 - 116 736 θ + 135 168 θ^2) #1^10 + (147 456 - 184 320 θ) #1^11 +
        36 864 #1^12 &, 6] < δ < θ && α > 0 && 0 < c < α) ||
 (Root[-12 370 864 909 569 909 577 184 662 732 800 +
        27 835 235 449 030 503 065 997 793 198 080 #1 +
        846 931 092 083 635 988 682 417 761 538 048 #1^2 -
        281 560 285 044 301 781 098 780 498 907 136 #1^3 -
        9 971 606 065 780 409 651 386 170 338 442 240 #1^4 +
        20 122 390 909 238 787 675 874 080 591 796 224 #1^5 +
        18 022 561 443 916 094 241 436 181 112 137 472 #1^6 -
        145 419 317 350 936 304 847 465 531 288 548 608 #1^7 +
        67 192 460 922 889 883 255 010 190 249 620 480 #1^8 +
        548 162 676 342 916 059 152 203 678 288 149 888 #1^9 -
        37 560 299 765 779 596 282 123 814 542 092 352 #1^10 -
        1 971 663 539 850 980 889 311 248 843 572 948 480 #1^11 -
        1 667 028 431 180 980 654 971 869 380 829 247 216 #1^12 +
        7 591 724 825 013 922 076 163 328 512 951 276 528 #1^13 +
        6 842 615 293 048 105 386 094 597 080 189 351 864 #1^14 -
        23 157 188 699 722 601 472 443 322 606 237 281 520 #1^15 -
        12 452 216 166 254 911 218 359 979 987 181 959 200 #1^16 +
        49 308 416 781 940 996 339 695 888 384 757 670 448 #1^17 +
```

```
10 329 374 064 954 573 559 512 353 529 055 404 420 #1^18 -
73 362 388 861 083 861 226 505 665 149 961 096 272 #1^19 +
2 083 624 521 320 468 567 110 244 933 409 311 628 #1^20 +
78 214 224 529 453 695 189 703 996 228 298 360 388 #1^21 -
15 626 863 679 016 596 320 989 056 120 795 972 800 #1^22 -
61 152 804 706 528 341 490 269 504 171 495 233 748 #1^23 +
20 156 715 959 215 533 710 252 881 092 793 953 975 #1^24 +
35 606 919 305 527 694 835 641 341 102 989 963 288 #1^25 -
15 561 274 367 277 878 373 812 870 800 143 262 212 #1^26 -
15 475 268 793 350 055 323 251 778 788 517 689 224 #1^27 +
8 346 332 453 447 085 101 459 385 692 564 337 536 #1^28 +
4 912 031 674 599 676 282 348 716 509 088 348 048 #1^29 -
3 239 431 097 497 717 410 163 526 311 076 830 884 #1^30 -
1 060 488 561 432 041 244 746 273 963 827 508 396 #1^31 +
901 607 412 256 406 417 841 523 816 356 689 298 #1^32 +
125 229 840 618 772 606 502 609 905 094 046 576 #1^33 -
170 387 616 471 556 637 783 551 877 644 351 800 #1^34 +
620 321 000 170 194 690 538 107 468 386 928 #1^35 +
19 408 264 839 356 822 543 058 006 965 863 428 #1^36 -
2 166 148 263 566 858 401 905 608 020 074 980 #1^37 -
998 813 893 018 808 926 996 220 598 485 544 #1^38 +
190 740 516 564 108 460 442 592 160 769 400 #1^39 +
916 331 128 635 909 864 072 350 486 331 #1^40 &, 10] < θ < 1 &&
Root[-64 θ - 336 θ^2 - 640 θ^3 - 392 θ^4 + 332 θ^5 + 595 θ^6 + 182 θ^7 - 155 θ^8 - 112 θ^9 -
  3 θ^10 + 14 θ^11 + 3 θ^12 + (192 - 160 θ - 3120 θ^2 - 5832 θ^3 - 1792 θ^4 +
     4620 θ^5 + 3840 θ^6 - 740 θ^7 - 1632 θ^8 - 252 θ^9 + 208 θ^10 + 60 θ^11) #1 +
  (2080 + 2752 θ - 8232 θ^2 - 15 416 θ^3 + 980 θ^4 + 13 088 θ^5 + 2056 θ^6 - 6056 θ^7 -
     2092 θ^8 + 736 θ^9 + 312 θ^10) #1^2 + (5696 + 14 976 θ + 11 376 θ^2 - 304 θ^3 - 6752 θ^4 -
     8816 θ^5 - 4816 θ^6 + 112 θ^7 + 256 θ^8 - 208 θ^9) #1^3 + (-10 272 + 21 088 θ +
     87 024 θ^2 + 24 832 θ^3 - 69 568 θ^4 - 19 616 θ^5 + 27 584 θ^6 + 3936 θ^7 - 4528 θ^8) #1^4 +
  (-60 480 - 32 512 θ + 72 896 θ^2 + 9472 θ^3 - 21 632 θ^4 + 32 768 θ^5 + 16 128 θ^6 - 2816 θ^7)
   #1^5 + (-5696 - 176 128 θ - 119 168 θ^2 + 150 400 θ^3 + 32 256 θ^4 - 41 216 θ^5 + 19 584 θ^6)
   #1^6 + (215 040 - 179 456 θ - 92 928 θ^2 + 76 032 θ^3 - 83 200 θ^4 - 11 520 θ^5) #1^7 +
  (99 072 + 327 168 θ - 262 400 θ^2 + 77 824 θ^3 - 68 864 θ^4) #1^8 +
  (-313 344 + 463 872 θ - 178 176 θ^2 + 119 808 θ^3) #1^9 +
  (-138 240 - 116 736 θ + 135 168 θ^2) #1^10 + (147 456 - 184 320 θ) #1^11 +
  36 864 #1^12 &, 4] < δ < θ && α > 0 && 0 < c < α))
```

```
Manipulate[
 Plot3D[{((((z + ((c - α)^2 (832 δ^6 + 64 δ^5 (15 - 4 θ) + (1 + θ)^4 (36 + θ (-28 + 3 θ)) + 4 δ
              (1 + θ)^3 (66 + θ (-49 + 6 θ)) + 16 δ^4 (-96 + θ (-4 + 9 θ)) +
             4 δ^2 (1 + θ)^2 (97 + 3 θ (-34 + 11 θ)) + 16 δ^3 (1 + θ) (-52 + θ (-16 + 23 θ))))/
        (4 (1 + 4 δ + θ) (-2 + 4 δ^2 + 8 δ^3 - 3 θ + θ^3 + 2 δ (-3 + θ) (1 + θ))^2))) -
     ((z + (3 (c - α)^2 (576 δ^6 - 576 δ^5 (-1 + θ) - (1 + θ)^2 (2 + θ)^2 (-3 + 2 θ) +
             96 δ^4 (-6 + (-4 + θ) θ) - 2 δ (1 + θ) (2 + θ) (-21 + θ (-7 + θ (9 + θ))) +
             4 δ^3 (-103 + θ (-39 + θ (51 + 19 θ))) +
             2 δ^2 (43 + θ (85 + θ (41 - θ (5 + 2 θ)))))))/ ((1 + 4 δ + θ)
          (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))^2)))),
   0}, {δ, 0, .75}, {θ, .75, 1}, PlotLabel → "", AxesLabel →
   {"δ",
    "θ",
    "Rel. Welfare"},
  AxesStyle → Larger, LabelStyle →
   Bold,
  PlotLegends →
   Placed[
    {"Difference: TS(Case IV)-TS(Case II)", "Plane at 0"},
    Below]],
 {c, 1, 1}, {α, 1.5, 1}, {z, 1,
  1}]
```



(*case 5 vs case 2*)

Reduce[

$$\left(z+\frac{9\,(c-\alpha)^2}{4\,(1+4\,\delta+\theta)}\right) > \Big(z+\Big(3\,(c-\alpha)^2\,\big(576\,\delta^6-576\,\delta^5\,(-1+\theta)-(1+\theta)^2\,(2+\theta)^2\,(-3+2\,\theta)+96\,\delta^4\,(-6+(-4+\theta)\,\theta)-2\,\delta\,(1+\theta)\,(2+\theta)\,(-21+\theta\,(-7+\theta\,(9+\theta)))+4\,\delta^3\,(-103+\theta\,(-39+\theta\,(51+19\,\theta)))+2\,\delta^2\,(43+\theta\,(85+\theta\,(41-\theta\,(5+2\,\theta))))\big)\Big)\Big/\Big((1+4\,\delta+\theta)\,\big(24\,\delta^3-12\,\delta^2\,(-1+\theta)+(-2+\theta)\,(1+\theta)\,(2+\theta)+2\,\delta\,(-6+(-4+\theta)\,\theta)\big)^2\Big)\Big)\ \&\&$$

$$0<\delta<1\ \&\&\ \delta<\theta<1\ \&\&\ \alpha>0\ \&\&\ 0<c<\alpha\,]$$

False

(*case 4 vs case 3*)

$$\text{Reduce}\Big[\Big(z+\Big((c-\alpha)^2\Big(832\,\delta^6+64\,\delta^5\,(15-4\,\theta)+(1+\theta)^4\,(36+\theta\,(-28+3\,\theta))+4\,\delta\,(1+\theta)^3\,(66+\theta\,(-49+6\,\theta))+16\,\delta^4\,(-96+\theta\,(-4+9\,\theta))+4\,\delta^2\,(1+\theta)^2\,(97+3\,\theta\,(-34+11\,\theta))+16\,\delta^3\,(1+\theta)\,(-52+\theta\,(-16+23\,\theta))\Big)\Big)\Big/\Big(4\,(1+4\,\delta+\theta)\,\big(-2+4\,\delta^2+8\,\delta^3-3\,\theta+\theta^3+2\,\delta\,(-3+\theta)\,(1+\theta)\big)^2\Big)\Big) >$$

$$\left(z+\frac{3\,(c-\alpha)^2\,(1+2\,\delta)\,(3+2\,\delta-2\,\theta)}{(-2-2\,\delta+\theta)^2\,(1+4\,\delta+\theta)}\right) \&\& \; 0 <$$

$$\delta <$$

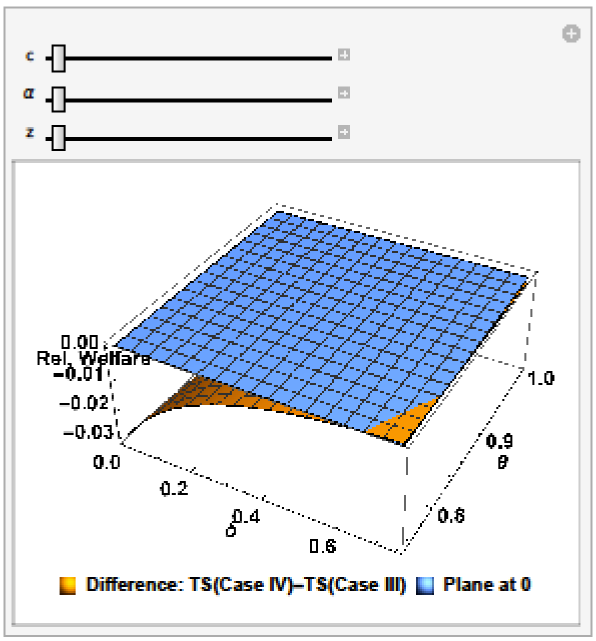
$$1 \&\& \; \delta <$$

$$\theta < 1 \&\& \; \alpha >$$

$$0 \&\& \; 0 < c < \alpha\Big]$$

```
z ∈ Reals &&
 ((0 < θ ≤ Root[123 511 232 462 688 - 2 152 108 622 855 296 #1 + 17 217 688 820 943 872 #1^2 -
          84 276 043 604 725 696 #1^3 + 282 230 444 306 700 768 #1^4 - 682 810 149 323 378 720 #1^5 +
          1 224 909 225 577 170 532 #1^6 - 1 638 852 075 838 399 868 #1^7 +
          1 605 846 340 585 401 858 #1^8 - 1 073 342 467 733 190 444 #1^9 +
          354 355 253 552 470 205 #1^10 + 154 067 200 931 017 175 #1^11 -
          305 626 043 847 094 996 #1^12 + 228 302 656 885 454 616 #1^13 -
          108 543 254 967 467 414 #1^14 + 35 357 419 832 511 406 #1^15 - 7 838 379 740 750 990 #1^16 +
          1 117 927 930 236 804 #1^17 - 90 079 181 444 907 #1^18 + 2 933 192 621 223 #1^19 &, 2] &&
     Root[-16 θ - 36 θ^2 + 43 θ^4 + 12 θ^5 - 18 θ^6 - 4 θ^7 + 3 θ^8 +
          (96 - 144 θ - 432 θ^2 + 216 θ^3 + 420 θ^4 - 180 θ^5 - 132 θ^6 + 60 θ^7) #1 +
          (592 - 704 θ - 1008 θ^2 + 1360 θ^3 + 160 θ^4 - 720 θ^5 + 192 θ^6) #1^2 +
          (640 - 128 θ - 304 θ^2 + 304 θ^3 - 224 θ^4 - 64 θ^5) #1^3 +
          (-1648 + 5056 θ - 4320 θ^2 + 1664 θ^3 - 224 θ^4) #1^4 +
          (-2560 + 4096 θ - 3072 θ^2 + 1408 θ^3) #1^5 + (256 - 1792 θ + 896 θ^2) #1^6 +
          (1280 - 1280 θ) #1^7 + 256 #1^8 &, 6] < δ < θ && α > 0 && 0 < c < α) ||
   (Root[123 511 232 462 688 - 2 152 108 622 855 296 #1 + 17 217 688 820 943 872 #1^2 -
          84 276 043 604 725 696 #1^3 + 282 230 444 306 700 768 #1^4 - 682 810 149 323 378 720 #1^5 +
          1 224 909 225 577 170 532 #1^6 - 1 638 852 075 838 399 868 #1^7 +
          1 605 846 340 585 401 858 #1^8 - 1 073 342 467 733 190 444 #1^9 +
          354 355 253 552 470 205 #1^10 + 154 067 200 931 017 175 #1^11 -
          305 626 043 847 094 996 #1^12 + 228 302 656 885 454 616 #1^13 -
          108 543 254 967 467 414 #1^14 + 35 357 419 832 511 406 #1^15 - 7 838 379 740 750 990 #1^16 +
          1 117 927 930 236 804 #1^17 - 90 079 181 444 907 #1^18 + 2 933 192 621 223 #1^19 &, 2] <
      θ < Root[-80 - 652 #1 - 1220 #1^2 + 741 #1^3 + 1247 #1^4 &, 4] &&
     Root[-16 θ - 36 θ^2 + 43 θ^4 + 12 θ^5 - 18 θ^6 - 4 θ^7 + 3 θ^8 +
          (96 - 144 θ - 432 θ^2 + 216 θ^3 + 420 θ^4 - 180 θ^5 - 132 θ^6 + 60 θ^7) #1 +
          (592 - 704 θ - 1008 θ^2 + 1360 θ^3 + 160 θ^4 - 720 θ^5 + 192 θ^6) #1^2 +
          (640 - 128 θ - 304 θ^2 + 304 θ^3 - 224 θ^4 - 64 θ^5) #1^3 +
          (-1648 + 5056 θ - 4320 θ^2 + 1664 θ^3 - 224 θ^4) #1^4 +
          (-2560 + 4096 θ - 3072 θ^2 + 1408 θ^3) #1^5 + (256 - 1792 θ + 896 θ^2) #1^6 +
          (1280 - 1280 θ) #1^7 + 256 #1^8 &, 4] < δ < θ && α > 0 && 0 < c < α))
```

```
Manipulate[
 Plot3D[{(((z + ((c - α)^2 (832 δ^6 + 64 δ^5 (15 - 4 θ) + (1 + θ)^4 (36 + θ (-28 + 3 θ)) + 4 δ
          (1 + θ)^3 (66 + θ (-49 + 6 θ)) + 16 δ^4 (-96 + θ (-4 + 9 θ)) +
         4 δ^2 (1 + θ)^2 (97 + 3 θ (-34 + 11 θ)) + 16 δ^3 (1 + θ) (-52 + θ (-16 + 23 θ))))/
       (4 (1 + 4 δ + θ) (-2 + 4 δ^2 + 8 δ^3 - 3 θ + θ^3 + 2 δ (-3 + θ) (1 + θ))^2))) -
    ((z + (3 (c - α)^2 (1 + 2 δ) (3 + 2 δ - 2 θ))/((-2 - 2 δ + θ)^2 (1 + 4 δ + θ))))), 0}, {δ, 0, .75},
  {θ, .75, 1}, PlotLabel → "", AxesLabel →
   {"δ", "θ", "Rel. Welfare"},
  AxesStyle → Larger, LabelStyle →
   Bold, PlotLegends →
   Placed[{"Difference: TS(Case IV)-TS(Case III)", "Plane at 0"}, Below]],
 {c, 1, 1}, {α, 1.5, 1}, {z,
  1,
  1}]
```




```
(*case 5 vs case 3*)
```

Reduce$\left[\left(z+\frac{9\,(c-\alpha)^2}{4\,(1+4\,\delta+\theta)}\right)>\left(z+\frac{3\,(c-\alpha)^2\,(1+2\,\delta)\,(3+2\,\delta-2\,\theta)}{(-2-2\,\delta+\theta)^2\,(1+4\,\delta+\theta)}\right)\,\&\&\right.$

$0<\delta<1\,\&\&\,\delta<\theta<1\,\&\&\,\alpha>0\,\&\&\,0<c<\alpha\big]$

False

(*case 5 vs case 4*)

Reduce[

$\left(z+\frac{9\,(c-\alpha)^2}{4\,(1+4\,\delta+\theta)}\right)>\Big(z+\Big((c-\alpha)^2\,\big(832\,\delta^6+64\,\delta^5\,(15-4\,\theta)+(1+\theta)^4\,(36+\theta\,(-28+3\,\theta))+$

$4\,\delta\,(1+\theta)^3\,(66+\theta\,(-49+6\,\theta))+16\,\delta^4\,(-96+\theta\,(-4+9\,\theta))+$

$4\,\delta^2\,(1+\theta)^2\,(97+3\,\theta\,(-34+11\,\theta))+16\,\delta^3\,(1+\theta)\,(-52+\theta\,(-16+23\,\theta))\big)\Big)\Big/$

$\left(4\,(1+4\,\delta+\theta)\,(-2+4\,\delta^2+8\,\delta^3-3\,\theta+\theta^3+2\,\delta\,(-3+\theta)\,(1+\theta))^2\right)\Big)\,\&\&$

$0<\delta<1\,\&\&\,\delta<\theta<1\,\&\&\,\alpha>0\,\&\&\,0<c<\alpha\big]$

False

```
FullSimplify[Solve[{q1 == ((-1 - 2 δ + 4 δ^2 - θ) p1 + (-4 δ^2 + θ + 2 δ θ + θ^2) p2 -
        (-1 + θ) (α (1 - 2 δ + θ) + δ (p3 + p4 + p5 + p6))) / ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ)),
    q2 == ((4 δ^2 - 2 δ θ - θ (1 + θ)) p1 + (1 + 2 δ - 4 δ^2 + θ) p2 +
        (-1 + θ) (α (1 - 2 δ + θ) + δ (p3 + p4 + p5 + p6))) / ((-1 + θ) (1 - 2 δ + θ) (1 + 4 δ + θ)),
    q3 == (α (-1 + 2 δ - θ) (-1 + θ) + (δ - δ θ) p1 + (δ - δ θ) p2 - p3 - 2 δ p3 + 4 δ^2 p3 - θ p3 - 4 δ^2 p4 +
        θ p4 + 2 δ θ p4 + θ^2 p4 + δ p5 - δ θ p5 - δ (-1 + θ) p6) / ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ)),
    q4 == (α (-1 + 2 δ - θ) (-1 + θ) + (δ - δ θ) p1 + (δ - δ θ) p2 - 4 δ^2 p3 + θ p3 + 2 δ θ p3 + θ^2 p3 - p4 -
        2 δ p4 + 4 δ^2 p4 - θ p4 + δ p5 - δ θ p5 - δ (-1 + θ) p6) / ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ)),
    q5 == (α (-1 + 2 δ - θ) (-1 + θ) + (δ - δ θ) p1 + (δ - δ θ) p2 + δ p3 - δ θ p3 + δ p4 - δ θ p4 - p5 -
        2 δ p5 + 4 δ^2 p5 - θ p5 + (-4 δ^2 + θ + 2 δ θ + θ^2) p6) / ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ)),
    q6 == (α - 2 α δ + 2 α δ θ - α θ^2 + (δ - δ θ) p1 + (δ - δ θ) p2 + δ p3 - δ θ p3 + δ p4 -
        δ θ p4 - 4 δ^2 p5 + θ p5 + 2 δ θ p5 + θ^2 p5 + (-1 - 2 δ + 4 δ^2 - θ) p6) /
      ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ))}, {p1, p2, p3, p4, p5, p6}]]
{{p1 → α - q1 - θ q2 - δ (q3 + q4 + q5 + q6), p2 → α - θ q1 - q2 - δ (q3 + q4 + q5 + q6),
  p3 → α - q3 - θ q4 - δ (q1 + q2 + q5 + q6), p4 → α - θ q3 - q4 - δ (q1 + q2 + q5 + q6),
  p5 → α - δ (q1 + q2 + q3 + q4) - q5 - θ q6, p6 → α - δ (q1 + q2 + q3 + q4) - θ q5 - q6}}

(*profit firm 1*)

(p1 - c) * (((-1 - 2 δ + 4 δ^2 - θ) p1 + (-4 δ^2 + θ + 2 δ θ + θ^2) p2 -
      (-1 + θ) (α (1 - 2 δ + θ) + δ (p3 + p4 + p5 + p6))) / ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ)))
((-c + p1) ((-1 - 2 δ + 4 δ^2 - θ) p1 + (-4 δ^2 + θ + 2 δ θ + θ^2) p2 -
      (-1 + θ) (α (1 - 2 δ + θ) + δ (p3 + p4 + p5 + p6)))) / ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ))

(*Case I: Premerger (Or "Base Case")*)

(*profit max firm 1*)

FullSimplify[
 D[((-c + p1) ((-1 - 2 δ + 4 δ^2 - θ) p1 + (-4 δ^2 + θ + 2 δ θ + θ^2) p2 - (-1 + θ) (α (1 - 2 δ + θ) +
            δ (p3 + p4 + p5 + p6)))) / ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ)), p1]]
(α (-1 + 2 δ - θ) (-1 + θ) + c (1 + 2 δ - 4 δ^2 + θ) + (-4 δ + 8 δ^2 - 2 (1 + θ)) p1 +
    (-4 δ^2 + θ + 2 δ θ + θ^2) p2 - δ (-1 + θ) (p3 + p4 + p5 + p6)) / ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ))

(*profit firm 2*)

(p2 - c) * (((4 δ^2 - 2 δ θ - θ (1 + θ)) p1 + (1 + 2 δ - 4 δ^2 + θ) p2 +
      (-1 + θ) (α (1 - 2 δ + θ) + δ (p3 + p4 + p5 + p6))) / ((-1 + θ) (1 - 2 δ + θ) (1 + 4 δ + θ)))
((-c + p2) ((4 δ^2 - 2 δ θ - θ (1 + θ)) p1 + (1 + 2 δ - 4 δ^2 + θ) p2 +
      (-1 + θ) (α (1 - 2 δ + θ) + δ (p3 + p4 + p5 + p6)))) / ((-1 + θ) (1 - 2 δ + θ) (1 + 4 δ + θ))

(*profit max firm 2*)
```

```
FullSimplify[
 D[((-c + p2) ((4 δ² - 2 δ θ - θ (1 + θ)) p1 + (1 + 2 δ - 4 δ² + θ) p2 + (-1 + θ) (α (1 - 2 δ + θ) +
          δ (p3 + p4 + p5 + p6)))) / ((-1 + θ) (1 - 2 δ + θ) (1 + 4 δ + θ)), p2]]
((4 δ² - 2 δ θ - θ (1 + θ)) p1 + (1 + 2 δ - 4 δ² + θ) p2 + (1 + 2 δ - 4 δ² + θ) (-c + p2) +
   (-1 + θ) (α (1 - 2 δ + θ) + δ (p3 + p4 + p5 + p6))) / ((-1 + θ) (1 - 2 δ + θ) (1 + 4 δ + θ))

(*profit firm 3*)

FullSimplify[(p3 - c) *
  ((α (-1 + 2 δ - θ) (-1 + θ) + (δ - δ θ) p1 + (δ - δ θ) p2 - p3 - 2 δ p3 + 4 δ² p3 - θ p3 - 4 δ² p4 + θ p4 +
      2 δ θ p4 + θ² p4 + δ p5 - δ θ p5 - δ (-1 + θ) p6) / ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ)))]
((-c + p3)
   (α (-1 + 2 δ - θ) (-1 + θ) + (δ - δ θ) p1 + (δ - δ θ) p2 - p3 - 2 δ p3 + 4 δ² p3 - θ p3 - 4 δ² p4 + θ p4 +
     2 δ θ p4 + θ² p4 + δ p5 - δ θ p5 - δ (-1 + θ) p6)) / ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ))

(*profit max firm 3*)

FullSimplify[
 D[((-c + p3) (α (-1 + 2 δ - θ) (-1 + θ) + (δ - δ θ) p1 + (δ - δ θ) p2 - p3 - 2 δ p3 + 4 δ² p3 -
        θ p3 - 4 δ² p4 + θ p4 + 2 δ θ p4 + θ² p4 + δ p5 - δ θ p5 - δ (-1 + θ) p6)) /
   ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ)), p3]]
(α (-1 + 2 δ - θ) (-1 + θ) + c (1 + 2 δ - 4 δ² + θ) + (δ - δ θ) p1 + (δ - δ θ) p2 - 2 p3 -
   4 δ p3 + 8 δ² p3 - 2 θ p3 - 4 δ² p4 + θ p4 + 2 δ θ p4 + θ² p4 + δ p5 - δ θ p5 - δ (-1 + θ) p6) /
 ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ))

(*profit firm 4*)

(p4 - c) *
 ((α (-1 + 2 δ - θ) (-1 + θ) + (δ - δ θ) p1 + (δ - δ θ) p2 - 4 δ² p3 + θ p3 + 2 δ θ p3 + θ² p3 - p4 - 2 δ p4 +
     4 δ² p4 - θ p4 + δ p5 - δ θ p5 - δ (-1 + θ) p6) / ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ)))
((-c + p4)
   (α (-1 + 2 δ - θ) (-1 + θ) + (δ - δ θ) p1 + (δ - δ θ) p2 - 4 δ² p3 + θ p3 + 2 δ θ p3 + θ² p3 - p4 -
     2 δ p4 + 4 δ² p4 - θ p4 + δ p5 - δ θ p5 - δ (-1 + θ) p6)) / ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ))

(*profit max firm 4*)

FullSimplify[
 D[((-c + p4) (α (-1 + 2 δ - θ) (-1 + θ) + (δ - δ θ) p1 + (δ - δ θ) p2 - 4 δ² p3 + θ p3 + 2 δ θ p3 +
        θ² p3 - p4 - 2 δ p4 + 4 δ² p4 - θ p4 + δ p5 - δ θ p5 - δ (-1 + θ) p6)) /
   ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ)), p4]]
(α (-1 + 2 δ - θ) (-1 + θ) + c (1 + 2 δ - 4 δ² + θ) + (δ - δ θ) p1 + (δ - δ θ) p2 - 4 δ² p3 +
   θ p3 + 2 δ θ p3 + θ² p3 - 2 p4 - 4 δ p4 + 8 δ² p4 - 2 θ p4 + δ p5 - δ θ p5 - δ (-1 + θ) p6) /
 ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ))

(*profit firm 5*)
```

```
(p5 - c) *
 ((α (-1 + 2 δ - θ) (-1 + θ) + (δ - δ θ) p1 + (δ - δ θ) p2 + δ p3 - δ θ p3 + δ p4 - δ θ p4 - p5 - 2 δ p5 +
     4 δ^2 p5 - θ p5 + (-4 δ^2 + θ + 2 δ θ + θ^2) p6) / ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ)))
((-c + p5) (α (-1 + 2 δ - θ) (-1 + θ) + (δ - δ θ) p1 + (δ - δ θ) p2 + δ p3 - δ θ p3 + δ p4 - δ θ p4 - p5 -
     2 δ p5 + 4 δ^2 p5 - θ p5 + (-4 δ^2 + θ + 2 δ θ + θ^2) p6)) / ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ))

(*profit max firm 5*)

FullSimplify[
 D[((-c + p5) (α (-1 + 2 δ - θ) (-1 + θ) + (δ - δ θ) p1 + (δ - δ θ) p2 + δ p3 - δ θ p3 + δ p4 -
        δ θ p4 - p5 - 2 δ p5 + 4 δ^2 p5 - θ p5 + (-4 δ^2 + θ + 2 δ θ + θ^2) p6)) /
   ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ)), p5]]
(α (-1 + 2 δ - θ) (-1 + θ) + c (1 + 2 δ - 4 δ^2 + θ) + (δ - δ θ) p1 + (δ - δ θ) p2 + δ p3 -
   δ θ p3 + δ p4 - δ θ p4 - 2 p5 - 4 δ p5 + 8 δ^2 p5 - 2 θ p5 + (-4 δ^2 + θ + 2 δ θ + θ^2) p6) /
 ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ))

(*profit firm 6*)

(p6 - c) *
 ((α - 2 α δ + 2 α δ θ - α θ^2 + (δ - δ θ) p1 + (δ - δ θ) p2 + δ p3 - δ θ p3 + δ p4 - δ θ p4 - 4 δ^2 p5 + θ p5 +
     2 δ θ p5 + θ^2 p5 + (-1 - 2 δ + 4 δ^2 - θ) p6) / ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ)))
((-c + p6) (α - 2 α δ + 2 α δ θ - α θ^2 + (δ - δ θ) p1 + (δ - δ θ) p2 + δ p3 - δ θ p3 + δ p4 - δ θ p4 - 4 δ^2 p5 +
     θ p5 + 2 δ θ p5 + θ^2 p5 + (-1 - 2 δ + 4 δ^2 - θ) p6)) / ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ))

FullSimplify[
 D[((-c + p6) (α - 2 α δ + 2 α δ θ - α θ^2 + (δ - δ θ) p1 + (δ - δ θ) p2 + δ p3 - δ θ p3 + δ p4 -
        δ θ p4 - 4 δ^2 p5 + θ p5 + 2 δ θ p5 + θ^2 p5 + (-1 - 2 δ + 4 δ^2 - θ) p6)) /
   ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ)), p6]]
(c + α + 2 c δ - 2 α δ - 4 c δ^2 + c θ + 2 α δ θ - α θ^2 + (δ - δ θ) p1 + (δ - δ θ) p2 + δ p3 -
   δ θ p3 + δ p4 - δ θ p4 - 4 δ^2 p5 + θ p5 + 2 δ θ p5 + θ^2 p5 + 2 (-1 - 2 δ + 4 δ^2 - θ) p6) /
 ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ))

(*premerger prices*)
```

```
FullSimplify[
 Solve[{(α (-1 + 2 δ - θ) (-1 + θ) + c (1 + 2 δ - 4 δ^2 + θ) + (-4 δ + 8 δ^2 - 2 (1 + θ)) p1 +
       (-4 δ^2 + θ + 2 δ θ + θ^2) p2 - δ (-1 + θ) (p3 + p4 + p5 + p6)) /
     ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ)) == 0,
   ((4 δ^2 - 2 δ θ - θ (1 + θ)) p1 + (1 + 2 δ - 4 δ^2 + θ) p2 + (1 + 2 δ - 4 δ^2 + θ) (-c + p2) +
       (-1 + θ) (α (1 - 2 δ + θ) + δ (p3 + p4 + p5 + p6))) / ((-1 + θ) (1 - 2 δ + θ) (1 + 4 δ + θ)) == 0,
   (α (-1 + 2 δ - θ) (-1 + θ) + c (1 + 2 δ - 4 δ^2 + θ) + (δ - δ θ) p1 + (δ - δ θ) p2 - 2 p3 -
       4 δ p3 + 8 δ^2 p3 - 2 θ p3 - 4 δ^2 p4 + θ p4 + 2 δ θ p4 + θ^2 p4 + δ p5 - δ θ p5 - δ (-1 + θ) p6) /
     ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ)) == 0,
   (α (-1 + 2 δ - θ) (-1 + θ) + c (1 + 2 δ - 4 δ^2 + θ) + (δ - δ θ) p1 + (δ - δ θ) p2 - 4 δ^2 p3 +
       θ p3 + 2 δ θ p3 + θ^2 p3 - 2 p4 - 4 δ p4 + 8 δ^2 p4 - 2 θ p4 + δ p5 - δ θ p5 - δ (-1 + θ) p6) /
     ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ)) == 0,
   (α (-1 + 2 δ - θ) (-1 + θ) + c (1 + 2 δ - 4 δ^2 + θ) + (δ - δ θ) p1 + (δ - δ θ) p2 + δ p3 -
       δ θ p3 + δ p4 - δ θ p4 - 2 p5 - 4 δ p5 + 8 δ^2 p5 - 2 θ p5 + (-4 δ^2 + θ + 2 δ θ + θ^2) p6) /
     ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ)) == 0,
   (c + α + 2 c δ - 2 α δ - 4 c δ^2 + c θ + 2 α δ θ - α θ^2 + (δ - δ θ) p1 + (δ - δ θ) p2 + δ p3 -
       δ θ p3 + δ p4 - δ θ p4 - 4 δ^2 p5 + θ p5 + 2 δ θ p5 + θ^2 p5 + 2 (-1 - 2 δ + 4 δ^2 - θ) p6) /
     ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ)) == 0}, {p1, p2, p3, p4, p5, p6}]]
```

$$\left\{\left\{p_1 \to \frac{\alpha\,(-1+2\delta-\theta)\,(-1+\theta)+c\,\left(1+2\delta-4\delta^2+\theta\right)}{2-4\delta^2+\theta+2\delta\theta-\theta^2},\right.\right.$$
$$p_2 \to \frac{\alpha\,(-1+2\delta-\theta)\,(-1+\theta)+c\,\left(1+2\delta-4\delta^2+\theta\right)}{2-4\delta^2+\theta+2\delta\theta-\theta^2},$$
$$p_3 \to \frac{\alpha\,(-1+2\delta-\theta)\,(-1+\theta)+c\,\left(1+2\delta-4\delta^2+\theta\right)}{2-4\delta^2+\theta+2\delta\theta-\theta^2},$$
$$p_4 \to \frac{\alpha\,(-1+2\delta-\theta)\,(-1+\theta)+c\,\left(1+2\delta-4\delta^2+\theta\right)}{2-4\delta^2+\theta+2\delta\theta-\theta^2},$$
$$p_5 \to \frac{\alpha\,(-1+2\delta-\theta)\,(-1+\theta)+c\,\left(1+2\delta-4\delta^2+\theta\right)}{2-4\delta^2+\theta+2\delta\theta-\theta^2},$$
$$\left.\left. p_6 \to \frac{\alpha\,(-1+2\delta-\theta)\,(-1+\theta)+c\,\left(1+2\delta-4\delta^2+\theta\right)}{2-4\delta^2+\theta+2\delta\theta-\theta^2}\right\}\right\}$$

```
(*Solving for Qty's*)

(* Solving for q1*)
```

```
FullSimplify[1/((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ))
  ((-1 - 2 δ + 4 δ^2 - θ) ((α (-1 + 2 δ - θ) (-1 + θ) + c (1 + 2 δ - 4 δ^2 + θ))/(2 - 4 δ^2 + θ + 2 δ θ - θ^2)) +
   (-4 δ^2 + θ + 2 δ θ + θ^2) ((α (-1 + 2 δ - θ) (-1 + θ) + c (1 + 2 δ - 4 δ^2 + θ))/(2 - 4 δ^2 + θ + 2 δ θ - θ^2)) -
   (-1 + θ) (α (1 - 2 δ + θ) + δ (((α (-1 + 2 δ - θ) (-1 + θ) + c (1 + 2 δ - 4 δ^2 + θ))/(2 - 4 δ^2 + θ + 2 δ θ - θ^2)) +
        ((α (-1 + 2 δ - θ) (-1 + θ) + c (1 + 2 δ - 4 δ^2 + θ))/(2 - 4 δ^2 + θ + 2 δ θ - θ^2)) +
        ((α (-1 + 2 δ - θ) (-1 + θ) + c (1 + 2 δ - 4 δ^2 + θ))/(2 - 4 δ^2 + θ + 2 δ θ - θ^2)) +
        ((α (-1 + 2 δ - θ) (-1 + θ) + c (1 + 2 δ - 4 δ^2 + θ))/(2 - 4 δ^2 + θ + 2 δ θ - θ^2)))))]
```

$$-\frac{(c-\alpha)\left(-1-2\delta+4\delta^2-\theta\right)}{(1+4\delta+\theta)\left(4\delta^2-2\delta\theta+(-2+\theta)(1+\theta)\right)}$$

```
(*q6 to check*)
```

```
FullSimplify[1/((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ))
  (α - 2 α δ + 2 α δ θ - α θ² + (δ - δ θ) ((α (-1 + 2 δ - θ) (-1 + θ) + c (1 + 2 δ - 4 δ² + θ))/(2 - 4 δ² + θ + 2 δ θ - θ²)) +
    (δ - δ θ) ((α (-1 + 2 δ - θ) (-1 + θ) + c (1 + 2 δ - 4 δ² + θ))/(2 - 4 δ² + θ + 2 δ θ - θ²)) +
    δ ((α (-1 + 2 δ - θ) (-1 + θ) + c (1 + 2 δ - 4 δ² + θ))/(2 - 4 δ² + θ + 2 δ θ - θ²)) -
    δ θ ((α (-1 + 2 δ - θ) (-1 + θ) + c (1 + 2 δ - 4 δ² + θ))/(2 - 4 δ² + θ + 2 δ θ - θ²)) +
    δ ((α (-1 + 2 δ - θ) (-1 + θ) + c (1 + 2 δ - 4 δ² + θ))/(2 - 4 δ² + θ + 2 δ θ - θ²)) -
    δ θ ((α (-1 + 2 δ - θ) (-1 + θ) + c (1 + 2 δ - 4 δ² + θ))/(2 - 4 δ² + θ + 2 δ θ - θ²)) -
    4 δ² ((α (-1 + 2 δ - θ) (-1 + θ) + c (1 + 2 δ - 4 δ² + θ))/(2 - 4 δ² + θ + 2 δ θ - θ²)) +
    θ ((α (-1 + 2 δ - θ) (-1 + θ) + c (1 + 2 δ - 4 δ² + θ))/(2 - 4 δ² + θ + 2 δ θ - θ²)) +
    2 δ θ ((α (-1 + 2 δ - θ) (-1 + θ) + c (1 + 2 δ - 4 δ² + θ))/(2 - 4 δ² + θ + 2 δ θ - θ²)) +
    θ² ((α (-1 + 2 δ - θ) (-1 + θ) + c (1 + 2 δ - 4 δ² + θ))/(2 - 4 δ² + θ + 2 δ θ - θ²)) +
    (-1 - 2 δ + 4 δ² - θ) ((α (-1 + 2 δ - θ) (-1 + θ) + c (1 + 2 δ - 4 δ² + θ))/(2 - 4 δ² + θ + 2 δ θ - θ²)))]
```

$$-\frac{(c-\alpha)\left(-1-2\delta+4\delta^2-\theta\right)}{(1+4\delta+\theta)\left(4\delta^2-2\delta\theta+(-2+\theta)(1+\theta)\right)}$$

```
Reduce[
 ((α - c) (-1 - 2 δ + 4 δ² - θ))/((1 + 4 δ + θ) (4 δ² - 2 δ θ + (-2 + θ) (1 + θ))) == -((c - α) (-1 - 2 δ + 4 δ² - θ))/((1 + 4 δ + θ) (4 δ² - 2 δ θ + (-2 + θ) (1 + θ)))]
True

(*Checking properties*)

Reduce[((α - c) (-1 - 2 δ + 4 δ² - θ))/((1 + 4 δ + θ) (4 δ² - 2 δ θ + (-2 + θ) (1 + θ))) > 0 && 0 < δ < θ < 1]
α ∈ Reals && 0 < δ < 1 && δ < θ < 1 && c < α

(*profit*)
```

```
FullSimplify[((α (-1+2δ-θ) (-1+θ) + c (1+2δ-4δ^2+θ))/(2-4δ^2+θ+2δθ-θ^2) - c) *
  (-((c-α) (-1-2δ+4δ^2-θ))/((1+4δ+θ) (4δ^2-2δθ+(-2+θ) (1+θ))))]
```

$$-\frac{(c-\alpha)^2\,(-1+2\delta-\theta)\,(-1-2\delta+4\delta^2-\theta)\,(-1+\theta)}{(1+4\delta+\theta)\,(2-4\delta^2+\theta+2\delta\theta-\theta^2)^2}$$

```
Reduce[-((c-α)^2 (-1+2δ-θ) (-1-2δ+4δ^2-θ) (-1+θ))/((1+4δ+θ) (2-4δ^2+θ+2δθ-θ^2)^2) ==
  ((c-α)^2 (-1+2δ-θ) (-1-2δ+4δ^2-θ) (1-θ))/((1+4δ+θ) (2-4δ^2+θ+2δθ-θ^2)^2)]
```

True

```
(*Checking*)
Reduce[
 ((c-α)^2 (-1+2δ-θ) (-1-2δ+4δ^2-θ) (1-θ))/((1+4δ+θ) (2-4δ^2+θ+2δθ-θ^2)^2) > 0 && 0 < δ < 1 && δ < θ < 1 && α > 1 && c < α]
```

0 < δ < 1 && δ < θ < 1 && α > 1 && c < α

```
(*total industry profit  (Producer surplus)*)
FullSimplify[-((c-α)^2 (-1+2δ-θ) (-1-2δ+4δ^2-θ) (-1+θ))/((1+4δ+θ) (2-4δ^2+θ+2δθ-θ^2)^2) +
  -((c-α)^2 (-1+2δ-θ) (-1-2δ+4δ^2-θ) (-1+θ))/((1+4δ+θ) (2-4δ^2+θ+2δθ-θ^2)^2) +
  -((c-α)^2 (-1+2δ-θ) (-1-2δ+4δ^2-θ) (-1+θ))/((1+4δ+θ) (2-4δ^2+θ+2δθ-θ^2)^2) +
  -((c-α)^2 (-1+2δ-θ) (-1-2δ+4δ^2-θ) (-1+θ))/((1+4δ+θ) (2-4δ^2+θ+2δθ-θ^2)^2) +
  -((c-α)^2 (-1+2δ-θ) (-1-2δ+4δ^2-θ) (-1+θ))/((1+4δ+θ) (2-4δ^2+θ+2δθ-θ^2)^2) +
  -((c-α)^2 (-1+2δ-θ) (-1-2δ+4δ^2-θ) (-1+θ))/((1+4δ+θ) (2-4δ^2+θ+2δθ-θ^2)^2)]
```

$$-\frac{6\,(c-\alpha)^2\,(-1+2\delta-\theta)\,(-1-2\delta+4\delta^2-\theta)\,(-1+\theta)}{(1+4\delta+\theta)\,(2-4\delta^2+\theta+2\delta\theta-\theta^2)^2}$$

```
(*Consumer surplus*)
FullSimplify[s - ((α (-1+2δ-θ) (-1+θ) + c (1+2δ-4δ^2+θ))/(2-4δ^2+θ+2δθ-θ^2))
```

$$\left(\frac{(\alpha-c)\left(-1-2\delta+4\delta^2-\theta\right)}{\left(1+4\delta+\theta\right)\left(4\delta^2-2\delta\theta+\left(-2+\theta\right)\left(1+\theta\right)\right)}\right)-$$
$$\left(\frac{\alpha\left(-1+2\delta-\theta\right)\left(-1+\theta\right)+c\left(1+2\delta-4\delta^2+\theta\right)}{2-4\delta^2+\theta+2\delta\theta-\theta^2}\right)$$
$$\left(\frac{(\alpha-c)\left(-1-2\delta+4\delta^2-\theta\right)}{\left(1+4\delta+\theta\right)\left(4\delta^2-2\delta\theta+\left(-2+\theta\right)\left(1+\theta\right)\right)}\right)-$$
$$\left(\frac{\alpha\left(-1+2\delta-\theta\right)\left(-1+\theta\right)+c\left(1+2\delta-4\delta^2+\theta\right)}{2-4\delta^2+\theta+2\delta\theta-\theta^2}\right)$$
$$\left(\frac{(\alpha-c)\left(-1-2\delta+4\delta^2-\theta\right)}{\left(1+4\delta+\theta\right)\left(4\delta^2-2\delta\theta+\left(-2+\theta\right)\left(1+\theta\right)\right)}\right)-$$
$$\left(\frac{\alpha\left(-1+2\delta-\theta\right)\left(-1+\theta\right)+c\left(1+2\delta-4\delta^2+\theta\right)}{2-4\delta^2+\theta+2\delta\theta-\theta^2}\right)$$
$$\left(\frac{(\alpha-c)\left(-1-2\delta+4\delta^2-\theta\right)}{\left(1+4\delta+\theta\right)\left(4\delta^2-2\delta\theta+\left(-2+\theta\right)\left(1+\theta\right)\right)}\right)-$$
$$\left(\frac{\alpha\left(-1+2\delta-\theta\right)\left(-1+\theta\right)+c\left(1+2\delta-4\delta^2+\theta\right)}{2-4\delta^2+\theta+2\delta\theta-\theta^2}\right)$$
$$\left(\frac{(\alpha-c)\left(-1-2\delta+4\delta^2-\theta\right)}{\left(1+4\delta+\theta\right)\left(4\delta^2-2\delta\theta+\left(-2+\theta\right)\left(1+\theta\right)\right)}\right)-$$
$$\left(\frac{\alpha\left(-1+2\delta-\theta\right)\left(-1+\theta\right)+c\left(1+2\delta-4\delta^2+\theta\right)}{2-4\delta^2+\theta+2\delta\theta-\theta^2}\right)$$
$$\left(\frac{(\alpha-c)\left(-1-2\delta+4\delta^2-\theta\right)}{\left(1+4\delta+\theta\right)\left(4\delta^2-2\delta\theta+\left(-2+\theta\right)\left(1+\theta\right)\right)}\right)+$$
$$\alpha\left(\left(\frac{(\alpha-c)\left(-1-2\delta+4\delta^2-\theta\right)}{\left(1+4\delta+\theta\right)\left(4\delta^2-2\delta\theta+\left(-2+\theta\right)\left(1+\theta\right)\right)}\right)+\right.$$
$$\left(\frac{(\alpha-c)\left(-1-2\delta+4\delta^2-\theta\right)}{\left(1+4\delta+\theta\right)\left(4\delta^2-2\delta\theta+\left(-2+\theta\right)\left(1+\theta\right)\right)}\right)+$$
$$\left(\frac{(\alpha-c)\left(-1-2\delta+4\delta^2-\theta\right)}{\left(1+4\delta+\theta\right)\left(4\delta^2-2\delta\theta+\left(-2+\theta\right)\left(1+\theta\right)\right)}\right)+$$
$$\left(\frac{(\alpha-c)\left(-1-2\delta+4\delta^2-\theta\right)}{\left(1+4\delta+\theta\right)\left(4\delta^2-2\delta\theta+\left(-2+\theta\right)\left(1+\theta\right)\right)}\right)+$$
$$\left(\frac{(\alpha-c)\left(-1-2\delta+4\delta^2-\theta\right)}{\left(1+4\delta+\theta\right)\left(4\delta^2-2\delta\theta+\left(-2+\theta\right)\left(1+\theta\right)\right)}\right)+$$
$$\left.\left(\frac{(\alpha-c)\left(-1-2\delta+4\delta^2-\theta\right)}{\left(1+4\delta+\theta\right)\left(4\delta^2-2\delta\theta+\left(-2+\theta\right)\left(1+\theta\right)\right)}\right)\right)+$$
$$\frac{1}{2}\left(-\left(\frac{(\alpha-c)\left(-1-2\delta+4\delta^2-\theta\right)}{\left(1+4\delta+\theta\right)\left(4\delta^2-2\delta\theta+\left(-2+\theta\right)\left(1+\theta\right)\right)}\right)^2-\right.$$
$$\left(\frac{(\alpha-c)\left(-1-2\delta+4\delta^2-\theta\right)}{\left(1+4\delta+\theta\right)\left(4\delta^2-2\delta\theta+\left(-2+\theta\right)\left(1+\theta\right)\right)}\right)^2-$$

$$\left(\frac{(\alpha - c)\left(-1 - 2\,\delta + 4\,\delta^2 - \theta\right)}{(1 + 4\,\delta + \theta)\left(4\,\delta^2 - 2\,\delta\,\theta + (-2 + \theta)\,(1 + \theta)\right)}\right)^2 -$$

$$2\,\theta\left(\frac{(\alpha - c)\left(-1 - 2\,\delta + 4\,\delta^2 - \theta\right)}{(1 + 4\,\delta + \theta)\left(4\,\delta^2 - 2\,\delta\,\theta + (-2 + \theta)\,(1 + \theta)\right)}\right)$$

$$\left(\frac{(\alpha - c)\left(-1 - 2\,\delta + 4\,\delta^2 - \theta\right)}{(1 + 4\,\delta + \theta)\left(4\,\delta^2 - 2\,\delta\,\theta + (-2 + \theta)\,(1 + \theta)\right)}\right) -$$

$$\left(\frac{(\alpha - c)\left(-1 - 2\,\delta + 4\,\delta^2 - \theta\right)}{(1 + 4\,\delta + \theta)\left(4\,\delta^2 - 2\,\delta\,\theta + (-2 + \theta)\,(1 + \theta)\right)}\right)^2 -$$

$$2\,\delta\left(\frac{(\alpha - c)\left(-1 - 2\,\delta + 4\,\delta^2 - \theta\right)}{(1 + 4\,\delta + \theta)\left(4\,\delta^2 - 2\,\delta\,\theta + (-2 + \theta)\,(1 + \theta)\right)}\right)$$

$$\left(\frac{(\alpha - c)\left(-1 - 2\,\delta + 4\,\delta^2 - \theta\right)}{(1 + 4\,\delta + \theta)\left(4\,\delta^2 - 2\,\delta\,\theta + (-2 + \theta)\,(1 + \theta)\right)}\right) -$$

$$2\,\delta\left(\frac{(\alpha - c)\left(-1 - 2\,\delta + 4\,\delta^2 - \theta\right)}{(1 + 4\,\delta + \theta)\left(4\,\delta^2 - 2\,\delta\,\theta + (-2 + \theta)\,(1 + \theta)\right)}\right)$$

$$\left(\frac{(\alpha - c)\left(-1 - 2\,\delta + 4\,\delta^2 - \theta\right)}{(1 + 4\,\delta + \theta)\left(4\,\delta^2 - 2\,\delta\,\theta + (-2 + \theta)\,(1 + \theta)\right)}\right) -$$

$$\left(\frac{(\alpha - c)\left(-1 - 2\,\delta + 4\,\delta^2 - \theta\right)}{(1 + 4\,\delta + \theta)\left(4\,\delta^2 - 2\,\delta\,\theta + (-2 + \theta)\,(1 + \theta)\right)}\right)^2 -$$

$$2\left(\delta\left(\left(\frac{(\alpha - c)\left(-1 - 2\,\delta + 4\,\delta^2 - \theta\right)}{(1 + 4\,\delta + \theta)\left(4\,\delta^2 - 2\,\delta\,\theta + (-2 + \theta)\,(1 + \theta)\right)}\right) +\right.\right.$$

$$\left.\left(\frac{(\alpha - c)\left(-1 - 2\,\delta + 4\,\delta^2 - \theta\right)}{(1 + 4\,\delta + \theta)\left(4\,\delta^2 - 2\,\delta\,\theta + (-2 + \theta)\,(1 + \theta)\right)}\right)\right) +$$

$$\left.\theta\left(\frac{(\alpha - c)\left(-1 - 2\,\delta + 4\,\delta^2 - \theta\right)}{(1 + 4\,\delta + \theta)\left(4\,\delta^2 - 2\,\delta\,\theta + (-2 + \theta)\,(1 + \theta)\right)}\right)\right)$$

$$\left(\frac{(\alpha - c)\left(-1 - 2\,\delta + 4\,\delta^2 - \theta\right)}{(1 + 4\,\delta + \theta)\left(4\,\delta^2 - 2\,\delta\,\theta + (-2 + \theta)\,(1 + \theta)\right)}\right) -$$

$$\left(\frac{(\alpha - c)\left(-1 - 2\,\delta + 4\,\delta^2 - \theta\right)}{(1 + 4\,\delta + \theta)\left(4\,\delta^2 - 2\,\delta\,\theta + (-2 + \theta)\,(1 + \theta)\right)}\right)^2 -$$

$$2\,\delta\left(\frac{(\alpha - c)\left(-1 - 2\,\delta + 4\,\delta^2 - \theta\right)}{(1 + 4\,\delta + \theta)\left(4\,\delta^2 - 2\,\delta\,\theta + (-2 + \theta)\,(1 + \theta)\right)}\right)$$

$$\left(\left(\frac{(\alpha - c)\left(-1 - 2\,\delta + 4\,\delta^2 - \theta\right)}{(1 + 4\,\delta + \theta)\left(4\,\delta^2 - 2\,\delta\,\theta + (-2 + \theta)\,(1 + \theta)\right)}\right) +\right.$$

$$\left(\frac{(\alpha - c)\left(-1 - 2\,\delta + 4\,\delta^2 - \theta\right)}{(1 + 4\,\delta + \theta)\left(4\,\delta^2 - 2\,\delta\,\theta + (-2 + \theta)\,(1 + \theta)\right)}\right) +$$

$$\left(\frac{(\alpha - c)\left(-1 - 2\,\delta + 4\,\delta^2 - \theta\right)}{(1 + 4\,\delta + \theta)\left(4\,\delta^2 - 2\,\delta\,\theta + (-2 + \theta)\,(1 + \theta)\right)}\right) +$$

$$\left(\frac{(\alpha-c)\left(-1-2\delta+4\delta^2-\theta\right)}{(1+4\delta+\theta)\left(4\delta^2-2\delta\theta+(-2+\theta)(1+\theta)\right)}\right)\Bigg]\Bigg)-$$

$$2\left(\frac{(\alpha-c)\left(-1-2\delta+4\delta^2-\theta\right)}{(1+4\delta+\theta)\left(4\delta^2-2\delta\theta+(-2+\theta)(1+\theta)\right)}\right)$$

$$\left(\theta\left(\frac{(\alpha-c)\left(-1-2\delta+4\delta^2-\theta\right)}{(1+4\delta+\theta)\left(4\delta^2-2\delta\theta+(-2+\theta)(1+\theta)\right)}\right)+\right.$$

$$\delta\left(\left(\frac{(\alpha-c)\left(-1-2\delta+4\delta^2-\theta\right)}{(1+4\delta+\theta)\left(4\delta^2-2\delta\theta+(-2+\theta)(1+\theta)\right)}\right)+\right.$$

$$\left(\frac{(\alpha-c)\left(-1-2\delta+4\delta^2-\theta\right)}{(1+4\delta+\theta)\left(4\delta^2-2\delta\theta+(-2+\theta)(1+\theta)\right)}\right)+$$

$$\left(\frac{(\alpha-c)\left(-1-2\delta+4\delta^2-\theta\right)}{(1+4\delta+\theta)\left(4\delta^2-2\delta\theta+(-2+\theta)(1+\theta)\right)}\right)+$$

$$\left.\left.\left(\frac{(\alpha-c)\left(-1-2\delta+4\delta^2-\theta\right)}{(1+4\delta+\theta)\left(4\delta^2-2\delta\theta+(-2+\theta)(1+\theta)\right)}\right)\right)\right)\Bigg)\Bigg]$$

$$\left(3c^2\left(1+2\delta-4\delta^2+\theta\right)^2-6c\alpha\left(1+2\delta-4\delta^2+\theta\right)^2+3\alpha^2\left(1+2\delta-4\delta^2+\theta\right)^2+z(1+4\delta+\theta)\left(2-4\delta^2+\theta+2\delta\theta-\theta^2\right)^2\right)\Big/\left((1+4\delta+\theta)\left(2-4\delta^2+\theta+2\delta\theta-\theta^2\right)^2\right)$$

(*Total surplus*)

$$\mathbf{FullSimplify}\Big[-\frac{6(c-\alpha)^2\left(-1+2\delta-\theta\right)\left(-1-2\delta+4\delta^2-\theta\right)\left(-1+\theta\right)}{\left(1+4\delta+\theta\right)\left(2-4\delta^2+\theta+2\delta\theta-\theta^2\right)^2}+\left(3c^2\left(1+2\delta-4\delta^2+\theta\right)^2-6c\alpha\left(1+2\delta-4\delta^2+\theta\right)^2+3\alpha^2\left(1+2\delta-4\delta^2+\theta\right)^2+z\left(1+4\delta+\theta\right)\left(2-4\delta^2+\theta+2\delta\theta-\theta^2\right)^2\right)\Big/\left(\left(1+4\delta+\theta\right)\left(2-4\delta^2+\theta+2\delta\theta-\theta^2\right)^2\right)\Big]$$

$$\frac{1}{(1+4\delta+\theta)\left(2-4\delta^2+\theta+2\delta\theta-\theta^2\right)^2}\left(-6(c-\alpha)^2(-1+2\delta-\theta)\left(-1-2\delta+4\delta^2-\theta\right)(-1+\theta)+3c^2\left(1+2\delta-4\delta^2+\theta\right)^2-6c\alpha\left(1+2\delta-4\delta^2+\theta\right)^2+3\alpha^2\left(1+2\delta-4\delta^2+\theta\right)^2+z(1+4\delta+\theta)\left(2-4\delta^2+\theta+2\delta\theta-\theta^2\right)^2\right)$$

$$\mathbf{FullSimplify}\Big[\frac{1}{\left(1+4\delta+\theta\right)\left(2-4\delta^2+\theta+2\delta\theta-\theta^2\right)^2}\left(-6(c-\alpha)^2\left(-1+2\delta-\theta\right)\left(-1-2\delta+4\delta^2-\theta\right)\left(-1+\theta\right)+3c^2\left(1+2\delta-4\delta^2+\theta\right)^2-6c\alpha\left(1+2\delta-4\delta^2+\theta\right)^2+3\alpha^2\left(1+2\delta-4\delta^2+\theta\right)^2+z\left(1+4\delta+\theta\right)\left(2-4\delta^2+\theta+2\delta\theta-\theta^2\right)^2\right)\Big]$$

$$\frac{1}{(1+4\delta+\theta)\left(2-4\delta^2+\theta+2\delta\theta-\theta^2\right)^2}\left(-6(c-\alpha)^2(-1+2\delta-\theta)\left(-1-2\delta+4\delta^2-\theta\right)(-1+\theta)+3c^2\left(1+2\delta-4\delta^2+\theta\right)^2-6c\alpha\left(1+2\delta-4\delta^2+\theta\right)^2+3\alpha^2\left(1+2\delta-4\delta^2+\theta\right)^2+z(1+4\delta+\theta)\left(2-4\delta^2+\theta+2\delta\theta-\theta^2\right)^2\right)$$

(*Case II: Single Multiproduct Firm with fringe firm in each submarket*)

(*profit firm 1*)

```
(p1 - c) * (((-1 - 2 δ + 4 δ^2 - θ) p1 + (-4 δ^2 + θ + 2 δ θ + θ^2) p2 -
      (-1 + θ) (α (1 - 2 δ + θ) + δ (p3 + p4 + p5 + p6))) / ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ)))

((-c + p1) ((-1 - 2 δ + 4 δ^2 - θ) p1 + (-4 δ^2 + θ + 2 δ θ + θ^2) p2 -
      (-1 + θ) (α (1 - 2 δ + θ) + δ (p3 + p4 + p5 + p6)))) / ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ))

(*profit firm 2*)

(p2 - c) * (((4 δ^2 - 2 δ θ - θ (1 + θ)) p1 + (1 + 2 δ - 4 δ^2 + θ) p2 +
      (-1 + θ) (α (1 - 2 δ + θ) + δ (p3 + p4 + p5 + p6))) / ((-1 + θ) (1 - 2 δ + θ) (1 + 4 δ + θ)))

((-c + p2) ((4 δ^2 - 2 δ θ - θ (1 + θ)) p1 + (1 + 2 δ - 4 δ^2 + θ) p2 +
      (-1 + θ) (α (1 - 2 δ + θ) + δ (p3 + p4 + p5 + p6)))) / ((-1 + θ) (1 - 2 δ + θ) (1 + 4 δ + θ))

(*profit max firm 2*)

FullSimplify[
 D[((-c + p2) ((4 δ^2 - 2 δ θ - θ (1 + θ)) p1 + (1 + 2 δ - 4 δ^2 + θ) p2 + (-1 + θ) (α (1 - 2 δ + θ) +
            δ (p3 + p4 + p5 + p6)))) / ((-1 + θ) (1 - 2 δ + θ) (1 + 4 δ + θ)), p2]]

((4 δ^2 - 2 δ θ - θ (1 + θ)) p1 + (1 + 2 δ - 4 δ^2 + θ) p2 + (1 + 2 δ - 4 δ^2 + θ) (-c + p2) +
    (-1 + θ) (α (1 - 2 δ + θ) + δ (p3 + p4 + p5 + p6))) / ((-1 + θ) (1 - 2 δ + θ) (1 + 4 δ + θ))

(*profit firm 3*)

FullSimplify[(p3 - c) *
   ((α (-1 + 2 δ - θ) (-1 + θ) + (δ - δ θ) p1 + (δ - δ θ) p2 - p3 - 2 δ p3 + 4 δ^2 p3 - θ p3 - 4 δ^2 p4 + θ p4 +
       2 δ θ p4 + θ^2 p4 + δ p5 - δ θ p5 - δ (-1 + θ) p6) / ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ)))]

((-c + p3)
    (α (-1 + 2 δ - θ) (-1 + θ) + (δ - δ θ) p1 + (δ - δ θ) p2 - p3 - 2 δ p3 + 4 δ^2 p3 - θ p3 - 4 δ^2 p4 + θ p4 +
       2 δ θ p4 + θ^2 p4 + δ p5 - δ θ p5 - δ (-1 + θ) p6)) / ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ))

(*profit firm 4*)

(p4 - c) *
  ((α (-1 + 2 δ - θ) (-1 + θ) + (δ - δ θ) p1 + (δ - δ θ) p2 - 4 δ^2 p3 + θ p3 + 2 δ θ p3 + θ^2 p3 - p4 - 2 δ p4 +
      4 δ^2 p4 - θ p4 + δ p5 - δ θ p5 - δ (-1 + θ) p6) / ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ)))

((-c + p4)
    (α (-1 + 2 δ - θ) (-1 + θ) + (δ - δ θ) p1 + (δ - δ θ) p2 - 4 δ^2 p3 + θ p3 + 2 δ θ p3 + θ^2 p3 - p4 -
       2 δ p4 + 4 δ^2 p4 - θ p4 + δ p5 - δ θ p5 - δ (-1 + θ) p6)) / ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ))

(*profit max firm 4*)
```

```
FullSimplify[
 D[((-c + p4) (α (-1 + 2 δ - θ) (-1 + θ) + (δ - δ θ) p1 + (δ - δ θ) p2 - 4 δ^2 p3 + θ p3 + 2 δ θ p3 +
        θ^2 p3 - p4 - 2 δ p4 + 4 δ^2 p4 - θ p4 + δ p5 - δ θ p5 - δ (-1 + θ) p6)) /
   ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ)), p4]]
(α (-1 + 2 δ - θ) (-1 + θ) + c (1 + 2 δ - 4 δ^2 + θ) + (δ - δ θ) p1 + (δ - δ θ) p2 - 4 δ^2 p3 +
   θ p3 + 2 δ θ p3 + θ^2 p3 - 2 p4 - 4 δ p4 + 8 δ^2 p4 - 2 θ p4 + δ p5 - δ θ p5 - δ (-1 + θ) p6) /
 ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ))

(*profit firm 5*)

(p5 - c) *
 ((α (-1 + 2 δ - θ) (-1 + θ) + (δ - δ θ) p1 + (δ - δ θ) p2 + δ p3 - δ θ p3 + δ p4 - δ θ p4 - p5 - 2 δ p5 +
     4 δ^2 p5 - θ p5 + (-4 δ^2 + θ + 2 δ θ + θ^2) p6) / ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ)))
((-c + p5) (α (-1 + 2 δ - θ) (-1 + θ) + (δ - δ θ) p1 + (δ - δ θ) p2 + δ p3 - δ θ p3 + δ p4 - δ θ p4 - p5 -
     2 δ p5 + 4 δ^2 p5 - θ p5 + (-4 δ^2 + θ + 2 δ θ + θ^2) p6)) / ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ))

(*profit firm 6*)

(p6 - c) *
 ((α - 2 α δ + 2 α δ θ - α θ^2 + (δ - δ θ) p1 + (δ - δ θ) p2 + δ p3 - δ θ p3 + δ p4 - δ θ p4 - 4 δ^2 p5 + θ p5 +
     2 δ θ p5 + θ^2 p5 + (-1 - 2 δ + 4 δ^2 - θ) p6) / ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ)))
((-c + p6) (α - 2 α δ + 2 α δ θ - α θ^2 + (δ - δ θ) p1 + (δ - δ θ) p2 + δ p3 - δ θ p3 + δ p4 - δ θ p4 - 4 δ^2 p5 +
     θ p5 + 2 δ θ p5 + θ^2 p5 + (-1 - 2 δ + 4 δ^2 - θ) p6)) / ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ))

FullSimplify[
 D[((-c + p6) (α - 2 α δ + 2 α δ θ - α θ^2 + (δ - δ θ) p1 + (δ - δ θ) p2 + δ p3 - δ θ p3 + δ p4 -
        δ θ p4 - 4 δ^2 p5 + θ p5 + 2 δ θ p5 + θ^2 p5 + (-1 - 2 δ + 4 δ^2 - θ) p6)) /
   ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ)), p6]]
(c + α + 2 c δ - 2 α δ - 4 c δ^2 + c θ + 2 α δ θ - α θ^2 + (δ - δ θ) p1 + (δ - δ θ) p2 + δ p3 -
   δ θ p3 + δ p4 - δ θ p4 - 4 δ^2 p5 + θ p5 + 2 δ θ p5 + θ^2 p5 + 2 (-1 - 2 δ + 4 δ^2 - θ) p6) /
 ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ))

(*profit multiproduct f1 f3 f5*)
```

```
FullSimplify[
 ((p1 - c) * (((-1 - 2 δ + 4 δ^2 - θ) p1 + (-4 δ^2 + θ + 2 δ θ + θ^2) p2 - (-1 + θ) (α (1 - 2 δ + θ) +
          δ (p3 + p4 + p5 + p6))) / ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ)))) +
  ((p3 - c) * ((α (-1 + 2 δ - θ) (-1 + θ) + (δ - δ θ) p1 + (δ - δ θ) p2 - p3 - 2 δ p3 +
        4 δ^2 p3 - θ p3 - 4 δ^2 p4 + θ p4 + 2 δ θ p4 + θ^2 p4 + δ p5 - δ θ p5 - δ (-1 + θ) p6) /
      ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ)))) + ((p5 - c) *
    ((α (-1 + 2 δ - θ) (-1 + θ) + (δ - δ θ) p1 + (δ - δ θ) p2 + δ p3 - δ θ p3 + δ p4 - δ θ p4 - p5 - 2 δ p5 +
        4 δ^2 p5 - θ p5 + (-4 δ^2 + θ + 2 δ θ + θ^2) p6) / ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ))))]
```

```
                  1
-----------------------------------
(-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ)
 ((-c + p3) (α (-1 + 2 δ - θ) (-1 + θ) + (δ - δ θ) p1 + (δ - δ θ) p2 - p3 - 2 δ p3 +
       4 δ^2 p3 - θ p3 - 4 δ^2 p4 + θ p4 + 2 δ θ p4 + θ^2 p4 + δ p5 - δ θ p5 - δ (-1 + θ) p6) +
    (-c + p5) (α (-1 + 2 δ - θ) (-1 + θ) + (δ - δ θ) p1 + (δ - δ θ) p2 + δ p3 - δ θ p3 +
       δ p4 - δ θ p4 - p5 - 2 δ p5 + 4 δ^2 p5 - θ p5 + (-4 δ^2 + θ + 2 δ θ + θ^2) p6) +
    (-c + p1) ((-1 - 2 δ + 4 δ^2 - θ) p1 + (-4 δ^2 + θ + 2 δ θ + θ^2) p2 -
       (-1 + θ) (α (1 - 2 δ + θ) + δ (p3 + p4 + p5 + p6))))
```

```
(*joint profit max multiproduct f1 f3 f5*)
FullSimplify[D[                  1
                -----------------------------------
                (-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ)
   ((-c + p3) (α (-1 + 2 δ - θ) (-1 + θ) + (δ - δ θ) p1 + (δ - δ θ) p2 - p3 - 2 δ p3 +
         4 δ^2 p3 - θ p3 - 4 δ^2 p4 + θ p4 + 2 δ θ p4 + θ^2 p4 + δ p5 - δ θ p5 - δ (-1 + θ) p6) +
      (-c + p5) (α (-1 + 2 δ - θ) (-1 + θ) + (δ - δ θ) p1 + (δ - δ θ) p2 + δ p3 - δ θ p3 +
         δ p4 - δ θ p4 - p5 - 2 δ p5 + 4 δ^2 p5 - θ p5 + (-4 δ^2 + θ + 2 δ θ + θ^2) p6) +
      (-c + p1) ((-1 - 2 δ + 4 δ^2 - θ) p1 + (-4 δ^2 + θ + 2 δ θ + θ^2) p2 -
         (-1 + θ) (α (1 - 2 δ + θ) + δ (p3 + p4 + p5 + p6)))), p1]]
```

```
(- (-1 + 2 δ - θ) (c + α + 2 c δ - α θ) + (-4 δ + 8 δ^2 - 2 (1 + θ)) p1 + (-4 δ^2 + θ + 2 δ θ + θ^2) p2 -
   δ (-1 + θ) (2 p3 + p4 + 2 p5 + p6)) / ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ))
```

```
FullSimplify[D[                  1
                -----------------------------------
                (-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ)
   ((-c + p3) (α (-1 + 2 δ - θ) (-1 + θ) + (δ - δ θ) p1 + (δ - δ θ) p2 - p3 - 2 δ p3 +
         4 δ^2 p3 - θ p3 - 4 δ^2 p4 + θ p4 + 2 δ θ p4 + θ^2 p4 + δ p5 - δ θ p5 - δ (-1 + θ) p6) +
      (-c + p5) (α (-1 + 2 δ - θ) (-1 + θ) + (δ - δ θ) p1 + (δ - δ θ) p2 + δ p3 - δ θ p3 +
         δ p4 - δ θ p4 - p5 - 2 δ p5 + 4 δ^2 p5 - θ p5 + (-4 δ^2 + θ + 2 δ θ + θ^2) p6) +
      (-c + p1) ((-1 - 2 δ + 4 δ^2 - θ) p1 + (-4 δ^2 + θ + 2 δ θ + θ^2) p2 -
         (-1 + θ) (α (1 - 2 δ + θ) + δ (p3 + p4 + p5 + p6)))), p3]]
```

```
(- (-1 + 2 δ - θ) (c + α + 2 c δ - α θ) - 2 δ (-1 + θ) p1 + (δ - δ θ) p2 - 2 p3 - 4 δ p3 +
    8 δ^2 p3 - 2 θ p3 - 4 δ^2 p4 + θ p4 + 2 δ θ p4 + θ^2 p4 + 2 δ p5 - 2 δ θ p5 - δ (-1 + θ) p6) /
  ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ))
```

```
FullSimplify[D[1/((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ))
    ((-c + p3) (α (-1 + 2 δ - θ) (-1 + θ) + (δ - δ θ) p1 + (δ - δ θ) p2 - p3 - 2 δ p3 +
          4 δ^2 p3 - θ p3 - 4 δ^2 p4 + θ p4 + 2 δ θ p4 + θ^2 p4 + δ p5 - δ θ p5 - δ (-1 + θ) p6) +
      (-c + p5) (α (-1 + 2 δ - θ) (-1 + θ) + (δ - δ θ) p1 + (δ - δ θ) p2 + δ p3 - δ θ p3 +
          δ p4 - δ θ p4 - p5 - 2 δ p5 + 4 δ^2 p5 - θ p5 + (-4 δ^2 + θ + 2 δ θ + θ^2) p6) +
      (-c + p1) ((-1 - 2 δ + 4 δ^2 - θ) p1 + (-4 δ^2 + θ + 2 δ θ + θ^2) p2 -
          (-1 + θ) (α (1 - 2 δ + θ) + δ (p3 + p4 + p5 + p6)))), p5]]
(- (-1 + 2 δ - θ) (c + α + 2 c δ - α θ) - 2 δ (-1 + θ) p1 + (δ - δ θ) p2 + 2 δ p3 - 2 δ θ p3 + δ p4 - δ θ p4 -
    2 p5 - 4 δ p5 + 8 δ^2 p5 - 2 θ p5 + (-4 δ^2 + θ + 2 δ θ + θ^2) p6) / ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ))

(*Postmerger prices: f135, f2, f4, f6*)

FullSimplify[Solve[
  {((- (-1 + 2 δ - θ) (c + α + 2 c δ - α θ) + (-4 δ + 8 δ^2 - 2 (1 + θ)) p1 + (-4 δ^2 + θ + 2 δ θ + θ^2) p2 -
          δ (-1 + θ) (2 p3 + p4 + 2 p5 + p6)) / ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ))) == 0,
    ((((4 δ^2 - 2 δ θ - θ (1 + θ)) p1 + (1 + 2 δ - 4 δ^2 + θ) p2 + (1 + 2 δ - 4 δ^2 + θ) (-c + p2) +
          (-1 + θ) (α (1 - 2 δ + θ) + δ (p3 + p4 + p5 + p6))) / ((-1 + θ) (1 - 2 δ + θ) (1 + 4 δ + θ))) ==
     0, ((- (-1 + 2 δ - θ) (c + α + 2 c δ - α θ) - 2 δ (-1 + θ) p1 + (δ - δ θ) p2 - 2 p3 - 4 δ p3 +
          8 δ^2 p3 - 2 θ p3 - 4 δ^2 p4 + θ p4 + 2 δ θ p4 + θ^2 p4 + 2 δ p5 - 2 δ θ p5 - δ (-1 + θ) p6) /
        ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ))) == 0,
    ((α (-1 + 2 δ - θ) (-1 + θ) + c (1 + 2 δ - 4 δ^2 + θ) + (δ - δ θ) p1 + (δ - δ θ) p2 - 4 δ^2 p3 +
          θ p3 + 2 δ θ p3 + θ^2 p3 - 2 p4 - 4 δ p4 + 8 δ^2 p4 - 2 θ p4 + δ p5 - δ θ p5 - δ (-1 + θ) p6) /
        ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ))) == 0,
    ((- (-1 + 2 δ - θ) (c + α + 2 c δ - α θ) - 2 δ (-1 + θ) p1 + (δ - δ θ) p2 + 2 δ p3 - 2 δ θ p3 +
          δ p4 - δ θ p4 - 2 p5 - 4 δ p5 + 8 δ^2 p5 - 2 θ p5 + (-4 δ^2 + θ + 2 δ θ + θ^2) p6) /
        ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ))) == 0,
    ((c + α + 2 c δ - 2 α δ - 4 c δ^2 + c θ + 2 α δ θ - α θ^2 + (δ - δ θ) p1 + (δ - δ θ) p2 + δ p3 -
          δ θ p3 + δ p4 - δ θ p4 - 4 δ^2 p5 + θ p5 + 2 δ θ p5 + θ^2 p5 + 2 (-1 - 2 δ + 4 δ^2 - θ) p6) /
        ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ))) == 0}, {p1, p2, p3, p4, p5, p6}]]
{{p1 →
    (α (-1 + θ) (-12 δ^2 + 2 δ (2 + θ) + (1 + θ) (2 + θ)) + c (24 δ^3 - (1 + θ) (2 + θ) - 2 δ (4 + 5 θ))) /
     (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ)),
   p2 → (-α (-1 + 2 δ - θ) (-1 + θ) (2 + 6 δ + θ) + c (1 + 2 δ) (6 δ (-1 + 2 δ) - (1 + θ) (2 + θ))) /
     (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ)), p3 →
    (α (-1 + θ) (-12 δ^2 + 2 δ (2 + θ) + (1 + θ) (2 + θ)) + c (24 δ^3 - (1 + θ) (2 + θ) - 2 δ (4 + 5 θ))) /
     (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ)),
   p4 → (-α (-1 + 2 δ - θ) (-1 + θ) (2 + 6 δ + θ) + c (1 + 2 δ) (6 δ (-1 + 2 δ) - (1 + θ) (2 + θ))) /
     (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ)), p5 →
    (α (-1 + θ) (-12 δ^2 + 2 δ (2 + θ) + (1 + θ) (2 + θ)) + c (24 δ^3 - (1 + θ) (2 + θ) - 2 δ (4 + 5 θ))) /
     (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ)),
   p6 → (-α (-1 + 2 δ - θ) (-1 + θ) (2 + 6 δ + θ) + c (1 + 2 δ) (6 δ (-1 + 2 δ) - (1 + θ) (2 + θ))) /
     (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))}}
```

```
FullSimplify[
 (-α (-1 + 2 δ - θ) (-1 + θ) (2 + 6 δ + θ) + c (1 + 2 δ) (6 δ (-1 + 2 δ) - (1 + θ) (2 + θ))) /
  (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))]
(-α (-1 + 2 δ - θ) (-1 + θ) (2 + 6 δ + θ) + c (1 + 2 δ) (6 δ (-1 + 2 δ) - (1 + θ) (2 + θ))) /
 (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))

Reduce [
 (α (-1 + θ) (-12 δ^2 + 2 δ (2 + θ) + (1 + θ) (2 + θ)) + c (24 δ^3 - (1 + θ) (2 + θ) - 2 δ (4 + 5 θ))) /
     (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ)) -
    (-α (-1 + 2 δ - θ) (-1 + θ) (2 + 6 δ + θ) + c (1 + 2 δ) (6 δ (-1 + 2 δ) - (1 + θ) (2 + θ))) /
     (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ)) >
   0 && 0 < δ < θ < 1 && α > c > 0]
0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < c < α

(*Solving for quantities*)

(*solving for q1*)

FullSimplify[1 / ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ))
  ((-1 - 2 δ + 4 δ^2 - θ) ((α (-1 + θ) (-12 δ^2 + 2 δ (2 + θ) + (1 + θ) (2 + θ)) +
          c (24 δ^3 - (1 + θ) (2 + θ) - 2 δ (4 + 5 θ))) / (24 δ^3 - 12 δ^2 (-1 + θ) +
          (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))) + (-4 δ^2 + θ + 2 δ θ + θ^2)
     ((-α (-1 + 2 δ - θ) (-1 + θ) (2 + 6 δ + θ) + c (1 + 2 δ) (6 δ (-1 + 2 δ) - (1 + θ) (2 + θ))) /
      (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))) -
    (-1 + θ) (α (1 - 2 δ + θ) + δ (((α (-1 + θ) (-12 δ^2 + 2 δ (2 + θ) + (1 + θ) (2 + θ)) +
              c (24 δ^3 - (1 + θ) (2 + θ) - 2 δ (4 + 5 θ))) / (24 δ^3 - 12 δ^2 (-1 + θ) +
              (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))) + ((-α (-1 + 2 δ - θ)
               (-1 + θ) (2 + 6 δ + θ) + c (1 + 2 δ) (6 δ (-1 + 2 δ) - (1 + θ) (2 + θ))) /
            (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))) +
          ((α (-1 + θ) (-12 δ^2 + 2 δ (2 + θ) + (1 + θ) (2 + θ)) +
              c (24 δ^3 - (1 + θ) (2 + θ) - 2 δ (4 + 5 θ))) / (24 δ^3 - 12 δ^2 (-1 + θ) +
              (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))) + ((-α (-1 + 2 δ - θ)
               (-1 + θ) (2 + 6 δ + θ) + c (1 + 2 δ) (6 δ (-1 + 2 δ) - (1 + θ) (2 + θ))) /
            (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))))))]
-(((c - α) (1 + 2 δ) (12 δ^2 - 2 δ (2 + θ) - (1 + θ) (2 + θ))) /
   ((1 + 4 δ + θ) (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))))

Reduce[((α - c) (1 + 2 δ) (12 δ^2 - 2 δ (2 + θ) - (1 + θ) (2 + θ))) /
   ((1 + 4 δ + θ) (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))) ==
  -(((c - α) (1 + 2 δ) (12 δ^2 - 2 δ (2 + θ) - (1 + θ) (2 + θ))) /
     ((1 + 4 δ + θ) (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))))]
True
```

```
(*Solving for q2*)

FullSimplify[1/((-1 + θ) (1 - 2 δ + θ) (1 + 4 δ + θ))
  ((4 δ^2 - 2 δ θ - θ (1 + θ)) ((α (-1 + θ) (-12 δ^2 + 2 δ (2 + θ) + (1 + θ) (2 + θ)) +
          c (24 δ^3 - (1 + θ) (2 + θ) - 2 δ (4 + 5 θ))) / (24 δ^3 - 12 δ^2 (-1 + θ) +
          (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))) + (1 + 2 δ - 4 δ^2 + θ)
      ((-α (-1 + 2 δ - θ) (-1 + θ) (2 + 6 δ + θ) + c (1 + 2 δ) (6 δ (-1 + 2 δ) - (1 + θ) (2 + θ))) /
        (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))) +
    (-1 + θ) (α (1 - 2 δ + θ) + δ (((α (-1 + θ) (-12 δ^2 + 2 δ (2 + θ) + (1 + θ) (2 + θ)) +
                c (24 δ^3 - (1 + θ) (2 + θ) - 2 δ (4 + 5 θ))) / (24 δ^3 - 12 δ^2 (-1 + θ) +
                (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))) + ((-α (-1 + 2 δ - θ)
                 (-1 + θ) (2 + 6 δ + θ) + c (1 + 2 δ) (6 δ (-1 + 2 δ) - (1 + θ) (2 + θ))) /
              (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))) +
            ((α (-1 + θ) (-12 δ^2 + 2 δ (2 + θ) + (1 + θ) (2 + θ)) +
                c (24 δ^3 - (1 + θ) (2 + θ) - 2 δ (4 + 5 θ))) / (24 δ^3 - 12 δ^2 (-1 + θ) +
                (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))) + ((-α (-1 + 2 δ - θ)
                 (-1 + θ) (2 + 6 δ + θ) + c (1 + 2 δ) (6 δ (-1 + 2 δ) - (1 + θ) (2 + θ))) /
              (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))))))]
-(((c - α) (-1 - 2 δ + 4 δ^2 - θ) (2 + 6 δ + θ)) /
   ((1 + 4 δ + θ) (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))))

Reduce[((α - c) (-1 - 2 δ + 4 δ^2 - θ) (2 + 6 δ + θ)) /
   ((1 + 4 δ + θ) (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))) ==
  -(((c - α) (-1 - 2 δ + 4 δ^2 - θ) (2 + 6 δ + θ)) /
     ((1 + 4 δ + θ) (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))))]
True

(*PROFIT multiproduct firm 135*)
```

```
FullSimplify[
 ((((α (-1 + θ) (-12 δ^2 + 2 δ (2 + θ) + (1 + θ) (2 + θ)) + c (24 δ^3 - (1 + θ) (2 + θ) - 2 δ (4 + 5 θ))) /
       (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))) - c) *
    (- (((c - α) (1 + 2 δ) (12 δ^2 - 2 δ (2 + θ) - (1 + θ) (2 + θ))) / ((1 + 4 δ + θ)
          (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ)))))) +
  ((((α (-1 + θ) (-12 δ^2 + 2 δ (2 + θ) + (1 + θ) (2 + θ)) +
          c (24 δ^3 - (1 + θ) (2 + θ) - 2 δ (4 + 5 θ))) /
       (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))) - c) *
    (- (((c - α) (1 + 2 δ) (12 δ^2 - 2 δ (2 + θ) - (1 + θ) (2 + θ))) / ((1 + 4 δ + θ)
          (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ)))))) +
  ((((α (-1 + θ) (-12 δ^2 + 2 δ (2 + θ) + (1 + θ) (2 + θ)) +
          c (24 δ^3 - (1 + θ) (2 + θ) - 2 δ (4 + 5 θ))) /
       (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))) - c) *
    (- (((c - α) (1 + 2 δ) (12 δ^2 - 2 δ (2 + θ) - (1 + θ) (2 + θ))) / ((1 + 4 δ + θ)
          (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))))))]

-((3 (c - α)^2 (1 + 2 δ) (-1 + θ) (-12 δ^2 + 2 δ (2 + θ) + (1 + θ) (2 + θ))^2) /
   ((1 + 4 δ + θ) (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))^2))

Reduce[(3 (c - α)^2 (1 + 2 δ) (1 - θ) (-12 δ^2 + 2 δ (2 + θ) + (1 + θ) (2 + θ))^2) /
   ((1 + 4 δ + θ) (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))^2) ==
  -((3 (c - α)^2 (1 + 2 δ) (-1 + θ) (-12 δ^2 + 2 δ (2 + θ) + (1 + θ) (2 + θ))^2) /
     ((1 + 4 δ + θ) (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))^2))]

True

(*PROFIT 2, 4, or 6*)

FullSimplify[
 (((-α (-1 + 2 δ - θ) (-1 + θ) (2 + 6 δ + θ) + c (1 + 2 δ) (6 δ (-1 + 2 δ) - (1 + θ) (2 + θ))) /
      (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))) - c) *
   (- (((c - α) (-1 - 2 δ + 4 δ^2 - θ) (2 + 6 δ + θ)) /
       ((1 + 4 δ + θ) (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ)))))]

-(((c - α)^2 (-1 + 2 δ - θ) (-1 - 2 δ + 4 δ^2 - θ) (-1 + θ) (2 + 6 δ + θ)^2) /
   ((1 + 4 δ + θ) (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))^2))

Reduce[((c - α)^2 (-1 + 2 δ - θ) (-1 - 2 δ + 4 δ^2 - θ) (1 - θ) (2 + 6 δ + θ)^2) /
   ((1 + 4 δ + θ) (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))^2) ==
  -(((c - α)^2 (-1 + 2 δ - θ) (-1 - 2 δ + 4 δ^2 - θ) (-1 + θ) (2 + 6 δ + θ)^2) /
     ((1 + 4 δ + θ) (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))^2))]

True

(*Industry Profit (Producer Surplus)*)
```

```
FullSimplify[-((3 (c - α)^2 (1 + 2 δ) (-1 + θ) (-12 δ^2 + 2 δ (2 + θ) + (1 + θ) (2 + θ))^2) /
      ((1 + 4 δ + θ) (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))^2)) +
  -(((c - α)^2 (-1 + 2 δ - θ) (-1 - 2 δ + 4 δ^2 - θ) (-1 + θ) (2 + 6 δ + θ)^2) /
      ((1 + 4 δ + θ) (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))^2)) +
  -(((c - α)^2 (-1 + 2 δ - θ) (-1 - 2 δ + 4 δ^2 - θ) (-1 + θ) (2 + 6 δ + θ)^2) /
      ((1 + 4 δ + θ) (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))^2)) +
  -(((c - α)^2 (-1 + 2 δ - θ) (-1 - 2 δ + 4 δ^2 - θ) (-1 + θ) (2 + 6 δ + θ)^2) /
      ((1 + 4 δ + θ) (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))^2))]
(3 (c - α)^2 (-1 + θ) (-(-1 + 2 δ - θ) (-1 - 2 δ + 4 δ^2 - θ) (2 + 6 δ + θ)^2 -
      (1 + 2 δ) (-12 δ^2 + 2 δ (2 + θ) + (1 + θ) (2 + θ))^2)) /
 ((1 + 4 δ + θ) (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))^2)

(*Consumer surplus*)

FullSimplify[s -
   ((α (-1 + θ) (-12 δ^2 + 2 δ (2 + θ) + (1 + θ) (2 + θ)) + c (24 δ^3 - (1 + θ) (2 + θ) - 2 δ (4 + 5 θ))) /
      (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ)))
    (((α - c) (1 + 2 δ) (12 δ^2 - 2 δ (2 + θ) - (1 + θ) (2 + θ))) /
      ((1 + 4 δ + θ) (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ)))) -
   ((-α (-1 + 2 δ - θ) (-1 + θ) (2 + 6 δ + θ) + c (1 + 2 δ) (6 δ (-1 + 2 δ) - (1 + θ) (2 + θ))) /
      (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ)))
    (((α - c) (-1 - 2 δ + 4 δ^2 - θ) (2 + 6 δ + θ)) /
      ((1 + 4 δ + θ) (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ)))) -
   ((α (-1 + θ) (-12 δ^2 + 2 δ (2 + θ) + (1 + θ) (2 + θ)) + c (24 δ^3 - (1 + θ) (2 + θ) - 2 δ (4 + 5 θ))) /
      (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ)))
    (((α - c) (1 + 2 δ) (12 δ^2 - 2 δ (2 + θ) - (1 + θ) (2 + θ))) /
      ((1 + 4 δ + θ) (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ)))) -
   ((-α (-1 + 2 δ - θ) (-1 + θ) (2 + 6 δ + θ) + c (1 + 2 δ) (6 δ (-1 + 2 δ) - (1 + θ) (2 + θ))) /
      (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ)))
    (((α - c) (-1 - 2 δ + 4 δ^2 - θ) (2 + 6 δ + θ)) /
      ((1 + 4 δ + θ) (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ)))) -
   ((α (-1 + θ) (-12 δ^2 + 2 δ (2 + θ) + (1 + θ) (2 + θ)) + c (24 δ^3 - (1 + θ) (2 + θ) - 2 δ (4 + 5 θ))) /
      (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ)))
    (((α - c) (1 + 2 δ) (12 δ^2 - 2 δ (2 + θ) - (1 + θ) (2 + θ))) /
      ((1 + 4 δ + θ) (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ)))) -
   ((-α (-1 + 2 δ - θ) (-1 + θ) (2 + 6 δ + θ) + c (1 + 2 δ) (6 δ (-1 + 2 δ) - (1 + θ) (2 + θ))) /
      (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ)))
    (((α - c) (-1 - 2 δ + 4 δ^2 - θ) (2 + 6 δ + θ)) /
      ((1 + 4 δ + θ) (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ)))) +
  α ((((α - c) (1 + 2 δ) (12 δ^2 - 2 δ (2 + θ) - (1 + θ) (2 + θ))) /
        ((1 + 4 δ + θ) (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ)))) +
```

$$(((\alpha - c)\ (-1 - 2\delta + 4\delta^2 - \theta)\ (2 + 6\delta + \theta))\ /$$
$$((1 + 4\delta + \theta)\ (24\delta^3 - 12\delta^2\ (-1 + \theta) + (-2 + \theta)\ (1 + \theta)\ (2 + \theta) + 2\delta\ (-6 + (-4 + \theta)\ \theta)))) +$$
$$(((\alpha - c)\ (1 + 2\delta)\ (12\delta^2 - 2\delta\ (2 + \theta) - (1 + \theta)\ (2 + \theta)))\ /$$
$$((1 + 4\delta + \theta)\ (24\delta^3 - 12\delta^2\ (-1 + \theta) + (-2 + \theta)\ (1 + \theta)\ (2 + \theta) + 2\delta\ (-6 + (-4 + \theta)\ \theta)))) +$$
$$(((\alpha - c)\ (-1 - 2\delta + 4\delta^2 - \theta)\ (2 + 6\delta + \theta))\ /$$
$$((1 + 4\delta + \theta)\ (24\delta^3 - 12\delta^2\ (-1 + \theta) + (-2 + \theta)\ (1 + \theta)\ (2 + \theta) + 2\delta\ (-6 + (-4 + \theta)\ \theta)))) +$$
$$(((\alpha - c)\ (1 + 2\delta)\ (12\delta^2 - 2\delta\ (2 + \theta) - (1 + \theta)\ (2 + \theta)))\ /$$
$$((1 + 4\delta + \theta)\ (24\delta^3 - 12\delta^2\ (-1 + \theta) + (-2 + \theta)\ (1 + \theta)\ (2 + \theta) + 2\delta\ (-6 + (-4 + \theta)\ \theta)))) +$$
$$(((\alpha - c)\ (-1 - 2\delta + 4\delta^2 - \theta)\ (2 + 6\delta + \theta))\ /\ ((1 + 4\delta + \theta)$$
$$(24\delta^3 - 12\delta^2\ (-1 + \theta) + (-2 + \theta)\ (1 + \theta)\ (2 + \theta) + 2\delta\ (-6 + (-4 + \theta)\ \theta))))) +$$
$$\frac{1}{2}\ (-(((\alpha - c)\ (1 + 2\delta)\ (12\delta^2 - 2\delta\ (2 + \theta) - (1 + \theta)\ (2 + \theta)))\ /\ ((1 + 4\delta + \theta)$$
$$(24\delta^3 - 12\delta^2\ (-1 + \theta) + (-2 + \theta)\ (1 + \theta)\ (2 + \theta) + 2\delta\ (-6 + (-4 + \theta)\ \theta))))^2 -$$
$$(((\alpha - c)\ (-1 - 2\delta + 4\delta^2 - \theta)\ (2 + 6\delta + \theta))\ /\ ((1 + 4\delta + \theta)$$
$$(24\delta^3 - 12\delta^2\ (-1 + \theta) + (-2 + \theta)\ (1 + \theta)\ (2 + \theta) + 2\delta\ (-6 + (-4 + \theta)\ \theta))))^2 -$$
$$(((\alpha - c)\ (1 + 2\delta)\ (12\delta^2 - 2\delta\ (2 + \theta) - (1 + \theta)\ (2 + \theta)))\ /\ ((1 + 4\delta + \theta)$$
$$(24\delta^3 - 12\delta^2\ (-1 + \theta) + (-2 + \theta)\ (1 + \theta)\ (2 + \theta) + 2\delta\ (-6 + (-4 + \theta)\ \theta))))^2 -$$
$$2\theta\ (((\alpha - c)\ (1 + 2\delta)\ (12\delta^2 - 2\delta\ (2 + \theta) - (1 + \theta)\ (2 + \theta)))\ /$$
$$((1 + 4\delta + \theta)\ (24\delta^3 - 12\delta^2\ (-1 + \theta) + (-2 + \theta)\ (1 + \theta)\ (2 + \theta) + 2\delta\ (-6 + (-4 + \theta)\ \theta))))$$
$$(((\alpha - c)\ (-1 - 2\delta + 4\delta^2 - \theta)\ (2 + 6\delta + \theta))\ /\ ((1 + 4\delta + \theta)$$
$$(24\delta^3 - 12\delta^2\ (-1 + \theta) + (-2 + \theta)\ (1 + \theta)\ (2 + \theta) + 2\delta\ (-6 + (-4 + \theta)\ \theta)))) -$$
$$(((\alpha - c)\ (-1 - 2\delta + 4\delta^2 - \theta)\ (2 + 6\delta + \theta))\ /\ ((1 + 4\delta + \theta)$$
$$(24\delta^3 - 12\delta^2\ (-1 + \theta) + (-2 + \theta)\ (1 + \theta)\ (2 + \theta) + 2\delta\ (-6 + (-4 + \theta)\ \theta))))^2 -$$
$$2\delta\ (((\alpha - c)\ (1 + 2\delta)\ (12\delta^2 - 2\delta\ (2 + \theta) - (1 + \theta)\ (2 + \theta)))\ /$$
$$((1 + 4\delta + \theta)\ (24\delta^3 - 12\delta^2\ (-1 + \theta) + (-2 + \theta)\ (1 + \theta)\ (2 + \theta) + 2\delta\ (-6 + (-4 + \theta)\ \theta))))$$
$$(((\alpha - c)\ (1 + 2\delta)\ (12\delta^2 - 2\delta\ (2 + \theta) - (1 + \theta)\ (2 + \theta)))\ /\ ((1 + 4\delta + \theta)$$
$$(24\delta^3 - 12\delta^2\ (-1 + \theta) + (-2 + \theta)\ (1 + \theta)\ (2 + \theta) + 2\delta\ (-6 + (-4 + \theta)\ \theta)))) -$$
$$2\delta\ (((\alpha - c)\ (-1 - 2\delta + 4\delta^2 - \theta)\ (2 + 6\delta + \theta))\ /\ ((1 + 4\delta + \theta)$$
$$(24\delta^3 - 12\delta^2\ (-1 + \theta) + (-2 + \theta)\ (1 + \theta)\ (2 + \theta) + 2\delta\ (-6 + (-4 + \theta)\ \theta))))$$
$$(((\alpha - c)\ (1 + 2\delta)\ (12\delta^2 - 2\delta\ (2 + \theta) - (1 + \theta)\ (2 + \theta)))\ /\ ((1 + 4\delta + \theta)$$
$$(24\delta^3 - 12\delta^2\ (-1 + \theta) + (-2 + \theta)\ (1 + \theta)\ (2 + \theta) + 2\delta\ (-6 + (-4 + \theta)\ \theta)))) -$$
$$(((\alpha - c)\ (1 + 2\delta)\ (12\delta^2 - 2\delta\ (2 + \theta) - (1 + \theta)\ (2 + \theta)))\ /\ ((1 + 4\delta + \theta)$$
$$(24\delta^3 - 12\delta^2\ (-1 + \theta) + (-2 + \theta)\ (1 + \theta)\ (2 + \theta) + 2\delta\ (-6 + (-4 + \theta)\ \theta))))^2 -$$
$$2\ (\delta\ ((((\alpha - c)\ (1 + 2\delta)\ (12\delta^2 - 2\delta\ (2 + \theta) - (1 + \theta)\ (2 + \theta)))\ /\ ((1 + 4\delta + \theta)$$
$$(24\delta^3 - 12\delta^2\ (-1 + \theta) + (-2 + \theta)\ (1 + \theta)\ (2 + \theta) + 2\delta\ (-6 + (-4 + \theta)\ \theta)))) +$$
$$(((\alpha - c)\ (-1 - 2\delta + 4\delta^2 - \theta)\ (2 + 6\delta + \theta))\ /\ ((1 + 4\delta + \theta)\ (24\delta^3 -$$
$$12\delta^2\ (-1 + \theta) + (-2 + \theta)\ (1 + \theta)\ (2 + \theta) + 2\delta\ (-6 + (-4 + \theta)\ \theta))))) +$$
$$\theta\ (((\alpha - c)\ (1 + 2\delta)\ (12\delta^2 - 2\delta\ (2 + \theta) - (1 + \theta)\ (2 + \theta)))\ /\ ((1 + 4\delta + \theta)$$
$$(24\delta^3 - 12\delta^2\ (-1 + \theta) + (-2 + \theta)\ (1 + \theta)\ (2 + \theta) + 2\delta\ (-6 + (-4 + \theta)\ \theta)))))$$
$$(((\alpha - c)\ (-1 - 2\delta + 4\delta^2 - \theta)\ (2 + 6\delta + \theta))\ /\ ((1 + 4\delta + \theta)$$
$$(24\delta^3 - 12\delta^2\ (-1 + \theta) + (-2 + \theta)\ (1 + \theta)\ (2 + \theta) + 2\delta\ (-6 + (-4 + \theta)\ \theta)))) -$$
$$(((\alpha - c)\ (-1 - 2\delta + 4\delta^2 - \theta)\ (2 + 6\delta + \theta))\ /\ ((1 + 4\delta + \theta)$$
$$(24\delta^3 - 12\delta^2\ (-1 + \theta) + (-2 + \theta)\ (1 + \theta)\ (2 + \theta) + 2\delta\ (-6 + (-4 + \theta)\ \theta))))^2 -$$

```
2 δ (((α - c) (-1 - 2 δ + 4 δ^2 - θ) (2 + 6 δ + θ)) / ((1 + 4 δ + θ)
      (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))))
   ((((α - c) (1 + 2 δ) (12 δ^2 - 2 δ (2 + θ) - (1 + θ) (2 + θ))) / ((1 + 4 δ + θ)
        (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ)))) +
    (((α - c) (-1 - 2 δ + 4 δ^2 - θ) (2 + 6 δ + θ)) / ((1 + 4 δ + θ)
        (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ)))) +
    (((α - c) (1 + 2 δ) (12 δ^2 - 2 δ (2 + θ) - (1 + θ) (2 + θ))) / ((1 + 4 δ + θ)
        (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ)))) +
    (((α - c) (-1 - 2 δ + 4 δ^2 - θ) (2 + 6 δ + θ)) / ((1 + 4 δ + θ)
        (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))))) -
 2 (((α - c) (1 + 2 δ) (12 δ^2 - 2 δ (2 + θ) - (1 + θ) (2 + θ))) /
    ((1 + 4 δ + θ) (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))))
   (θ (((α - c) (-1 - 2 δ + 4 δ^2 - θ) (2 + 6 δ + θ)) / ((1 + 4 δ + θ)
         (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ)))) +
    δ ((((α - c) (1 + 2 δ) (12 δ^2 - 2 δ (2 + θ) - (1 + θ) (2 + θ))) / ((1 + 4 δ + θ)
           (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ)))) +
       (((α - c) (-1 - 2 δ + 4 δ^2 - θ) (2 + 6 δ + θ)) / ((1 + 4 δ + θ) (24 δ^3 -
             12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ)))) +
       (((α - c) (1 + 2 δ) (12 δ^2 - 2 δ (2 + θ) - (1 + θ) (2 + θ))) / ((1 + 4 δ + θ)
           (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ)))) +
       (((α - c) (-1 - 2 δ + 4 δ^2 - θ) (2 + 6 δ + θ)) / ((1 + 4 δ + θ) (24 δ^3 -
             12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ)))))))]

(z (1 + 4 δ + θ) (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))^2 +
   3 c^2 (1 + 2 δ) (-144 δ^4 + 288 δ^5 + 2 δ (1 + θ) (2 + θ) (7 + 5 θ) +
      (2 + 3 θ + θ^2)^2 - 24 δ^3 (6 + θ (8 + θ)) - 2 δ^2 (-13 + θ (-9 + θ (3 + θ)))) -
   6 c α (1 + 2 δ) (-144 δ^4 + 288 δ^5 + 2 δ (1 + θ) (2 + θ) (7 + 5 θ) + (2 + 3 θ + θ^2)^2 -
      24 δ^3 (6 + θ (8 + θ)) - 2 δ^2 (-13 + θ (-9 + θ (3 + θ)))) +
   3 α^2 (1 + 2 δ) (-144 δ^4 + 288 δ^5 + 2 δ (1 + θ) (2 + θ) (7 + 5 θ) + (2 + 3 θ + θ^2)^2 -
      24 δ^3 (6 + θ (8 + θ)) - 2 δ^2 (-13 + θ (-9 + θ (3 + θ))))) /
 ((1 + 4 δ + θ) (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))^2)

FullSimplify[z -
  ((α (-1 + θ) (-12 δ^2 + 2 δ (2 + θ) + (1 + θ) (2 + θ)) + c (24 δ^3 - (1 + θ) (2 + θ) - 2 δ (4 + 5 θ))) /
     (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ)))
   (-(((c - α) (1 + 2 δ) (12 δ^2 - 2 δ (2 + θ) - (1 + θ) (2 + θ))) / ((1 + 4 δ + θ)
          (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))))) -
  ((-α (-1 + 2 δ - θ) (-1 + θ) (2 + 6 δ + θ) + c (1 + 2 δ) (6 δ (-1 + 2 δ) - (1 + θ) (2 + θ))) /
     (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ)))
   (-(((c - α) (-1 - 2 δ + 4 δ^2 - θ) (2 + 6 δ + θ)) / ((1 + 4 δ + θ)
          (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))))) -
  ((α (-1 + θ) (-12 δ^2 + 2 δ (2 + θ) + (1 + θ) (2 + θ)) + c (24 δ^3 - (1 + θ) (2 + θ) - 2 δ (4 + 5 θ))) /
```

$$(24\,\delta^3-12\,\delta^2\,(-1+\theta)+(-2+\theta)\,(1+\theta)\,(2+\theta)+2\,\delta\,(-6+(-4+\theta)\,\theta)))$$
$$(-(((c-\alpha)\,(1+2\,\delta)\,(12\,\delta^2-2\,\delta\,(2+\theta)-(1+\theta)\,(2+\theta)))\,/\,((1+4\,\delta+\theta)$$
$$(24\,\delta^3-12\,\delta^2\,(-1+\theta)+(-2+\theta)\,(1+\theta)\,(2+\theta)+2\,\delta\,(-6+(-4+\theta)\,\theta)))))\,-$$
$$((-\alpha\,(-1+2\,\delta-\theta)\,(-1+\theta)\,(2+6\,\delta+\theta)+c\,(1+2\,\delta)\,(6\,\delta\,(-1+2\,\delta)-(1+\theta)\,(2+\theta)))\,/$$
$$(24\,\delta^3-12\,\delta^2\,(-1+\theta)+(-2+\theta)\,(1+\theta)\,(2+\theta)+2\,\delta\,(-6+(-4+\theta)\,\theta)))$$
$$(-(((c-\alpha)\,(-1-2\,\delta+4\,\delta^2-\theta)\,(2+6\,\delta+\theta))\,/\,((1+4\,\delta+\theta)$$
$$(24\,\delta^3-12\,\delta^2\,(-1+\theta)+(-2+\theta)\,(1+\theta)\,(2+\theta)+2\,\delta\,(-6+(-4+\theta)\,\theta)))))\,-$$
$$((\alpha\,(-1+\theta)\,(-12\,\delta^2+2\,\delta\,(2+\theta)+(1+\theta)\,(2+\theta))+c\,(24\,\delta^3-(1+\theta)\,(2+\theta)-2\,\delta\,(4+5\,\theta)))\,/$$
$$(24\,\delta^3-12\,\delta^2\,(-1+\theta)+(-2+\theta)\,(1+\theta)\,(2+\theta)+2\,\delta\,(-6+(-4+\theta)\,\theta)))$$
$$(-(((c-\alpha)\,(1+2\,\delta)\,(12\,\delta^2-2\,\delta\,(2+\theta)-(1+\theta)\,(2+\theta)))\,/\,((1+4\,\delta+\theta)$$
$$(24\,\delta^3-12\,\delta^2\,(-1+\theta)+(-2+\theta)\,(1+\theta)\,(2+\theta)+2\,\delta\,(-6+(-4+\theta)\,\theta)))))\,-$$
$$((-\alpha\,(-1+2\,\delta-\theta)\,(-1+\theta)\,(2+6\,\delta+\theta)+c\,(1+2\,\delta)\,(6\,\delta\,(-1+2\,\delta)-(1+\theta)\,(2+\theta)))\,/$$
$$(24\,\delta^3-12\,\delta^2\,(-1+\theta)+(-2+\theta)\,(1+\theta)\,(2+\theta)+2\,\delta\,(-6+(-4+\theta)\,\theta)))$$
$$(-(((c-\alpha)\,(-1-2\,\delta+4\,\delta^2-\theta)\,(2+6\,\delta+\theta))\,/\,((1+4\,\delta+\theta)$$
$$(24\,\delta^3-12\,\delta^2\,(-1+\theta)+(-2+\theta)\,(1+\theta)\,(2+\theta)+2\,\delta\,(-6+(-4+\theta)\,\theta)))))\,+$$
$$\alpha\,((-(((c-\alpha)\,(1+2\,\delta)\,(12\,\delta^2-2\,\delta\,(2+\theta)-(1+\theta)\,(2+\theta)))\,/\,((1+4\,\delta+\theta)$$
$$(24\,\delta^3-12\,\delta^2\,(-1+\theta)+(-2+\theta)\,(1+\theta)\,(2+\theta)+2\,\delta\,(-6+(-4+\theta)\,\theta)))))\,+$$
$$(-(((c-\alpha)\,(-1-2\,\delta+4\,\delta^2-\theta)\,(2+6\,\delta+\theta))\,/\,((1+4\,\delta+\theta)$$
$$(24\,\delta^3-12\,\delta^2\,(-1+\theta)+(-2+\theta)\,(1+\theta)\,(2+\theta)+2\,\delta\,(-6+(-4+\theta)\,\theta)))))\,+$$
$$(-(((c-\alpha)\,(1+2\,\delta)\,(12\,\delta^2-2\,\delta\,(2+\theta)-(1+\theta)\,(2+\theta)))\,/\,((1+4\,\delta+\theta)$$
$$(24\,\delta^3-12\,\delta^2\,(-1+\theta)+(-2+\theta)\,(1+\theta)\,(2+\theta)+2\,\delta\,(-6+(-4+\theta)\,\theta)))))\,+$$
$$(-(((c-\alpha)\,(-1-2\,\delta+4\,\delta^2-\theta)\,(2+6\,\delta+\theta))\,/\,((1+4\,\delta+\theta)$$
$$(24\,\delta^3-12\,\delta^2\,(-1+\theta)+(-2+\theta)\,(1+\theta)\,(2+\theta)+2\,\delta\,(-6+(-4+\theta)\,\theta)))))\,+$$
$$(-(((c-\alpha)\,(1+2\,\delta)\,(12\,\delta^2-2\,\delta\,(2+\theta)-(1+\theta)\,(2+\theta)))\,/\,((1+4\,\delta+\theta)$$
$$(24\,\delta^3-12\,\delta^2\,(-1+\theta)+(-2+\theta)\,(1+\theta)\,(2+\theta)+2\,\delta\,(-6+(-4+\theta)\,\theta)))))\,+$$
$$(-(((c-\alpha)\,(-1-2\,\delta+4\,\delta^2-\theta)\,(2+6\,\delta+\theta))\,/\,((1+4\,\delta+\theta)$$
$$(24\,\delta^3-12\,\delta^2\,(-1+\theta)+(-2+\theta)\,(1+\theta)\,(2+\theta)+2\,\delta\,(-6+(-4+\theta)\,\theta))))))\,+$$
$$\frac{1}{2}\,(-(-(((c-\alpha)\,(1+2\,\delta)\,(12\,\delta^2-2\,\delta\,(2+\theta)-(1+\theta)\,(2+\theta)))\,/\,((1+4\,\delta+\theta)$$
$$(24\,\delta^3-12\,\delta^2\,(-1+\theta)+(-2+\theta)\,(1+\theta)\,(2+\theta)+2\,\delta\,(-6+(-4+\theta)\,\theta)))))^2\,-$$
$$(-(((c-\alpha)\,(-1-2\,\delta+4\,\delta^2-\theta)\,(2+6\,\delta+\theta))\,/\,((1+4\,\delta+\theta)\,(24\,\delta^3-12\,\delta^2$$
$$(-1+\theta)+(-2+\theta)\,(1+\theta)\,(2+\theta)+2\,\delta\,(-6+(-4+\theta)\,\theta)))))^2\,-$$
$$(-(((c-\alpha)\,(1+2\,\delta)\,(12\,\delta^2-2\,\delta\,(2+\theta)-(1+\theta)\,(2+\theta)))\,/\,((1+4\,\delta+\theta)$$
$$(24\,\delta^3-12\,\delta^2\,(-1+\theta)+(-2+\theta)\,(1+\theta)\,(2+\theta)+2\,\delta\,(-6+(-4+\theta)\,\theta)))))^2\,-$$
$$(-(((c-\alpha)\,(-1-2\,\delta+4\,\delta^2-\theta)\,(2+6\,\delta+\theta))\,/\,((1+4\,\delta+\theta)$$
$$(24\,\delta^3-12\,\delta^2\,(-1+\theta)+(-2+\theta)\,(1+\theta)\,(2+\theta)+2\,\delta\,(-6+(-4+\theta)\,\theta)))))^2\,-$$
$$(-(((c-\alpha)\,(1+2\,\delta)\,(12\,\delta^2-2\,\delta\,(2+\theta)-(1+\theta)\,(2+\theta)))\,/\,((1+4\,\delta+\theta)$$
$$(24\,\delta^3-12\,\delta^2\,(-1+\theta)+(-2+\theta)\,(1+\theta)\,(2+\theta)+2\,\delta\,(-6+(-4+\theta)\,\theta)))))^2\,-$$
$$(-(((c-\alpha)\,(-1-2\,\delta+4\,\delta^2-\theta)\,(2+6\,\delta+\theta))\,/\,((1+4\,\delta+\theta)$$
$$(24\,\delta^3-12\,\delta^2\,(-1+\theta)+(-2+\theta)\,(1+\theta)\,(2+\theta)+2\,\delta\,(-6+(-4+\theta)\,\theta)))))^2\,-$$
$$2*\delta\,((-(((c-\alpha)\,(1+2\,\delta)\,(12\,\delta^2-2\,\delta\,(2+\theta)-(1+\theta)\,(2+\theta)))\,/\,((1+4\,\delta+\theta)$$
$$(24\,\delta^3-12\,\delta^2\,(-1+\theta)+(-2+\theta)\,(1+\theta)\,(2+\theta)+2\,\delta\,(-6+(-4+\theta)\,\theta)))))$$
$$(-(((c-\alpha)\,(1+2\,\delta)\,(12\,\delta^2-2\,\delta\,(2+\theta)-(1+\theta)\,(2+\theta)))\,/\,((1+4\,\delta+\theta)$$

$$
\begin{array}{l}
(24 \delta^3 - 12 \delta^2 (-1 + \theta) + (-2 + \theta) (1 + \theta) (2 + \theta) + 2 \delta (-6 + (-4 + \theta) \theta))))) + \\
(-(((c - \alpha) (-1 - 2 \delta + 4 \delta^2 - \theta) (2 + 6 \delta + \theta)) / ((1 + 4 \delta + \theta) (24 \delta^3 - \\
12 \delta^2 (-1 + \theta) + (-2 + \theta) (1 + \theta) (2 + \theta) + 2 \delta (-6 + (-4 + \theta) \theta))))) \\
(-(((c - \alpha) (1 + 2 \delta) (12 \delta^2 - 2 \delta (2 + \theta) - (1 + \theta) (2 + \theta))) / ((1 + 4 \delta + \theta) \\
(24 \delta^3 - 12 \delta^2 (-1 + \theta) + (-2 + \theta) (1 + \theta) (2 + \theta) + 2 \delta (-6 + (-4 + \theta) \theta))))) + \\
(-(((c - \alpha) (1 + 2 \delta) (12 \delta^2 - 2 \delta (2 + \theta) - (1 + \theta) (2 + \theta))) / ((1 + 4 \delta + \theta) \\
(24 \delta^3 - 12 \delta^2 (-1 + \theta) + (-2 + \theta) (1 + \theta) (2 + \theta) + 2 \delta (-6 + (-4 + \theta) \theta))))) \\
(-(((c - \alpha) (-1 - 2 \delta + 4 \delta^2 - \theta) (2 + 6 \delta + \theta)) / ((1 + 4 \delta + \theta) (24 \delta^3 - \\
12 \delta^2 (-1 + \theta) + (-2 + \theta) (1 + \theta) (2 + \theta) + 2 \delta (-6 + (-4 + \theta) \theta))))) + \\
(-(((c - \alpha) (-1 - 2 \delta + 4 \delta^2 - \theta) (2 + 6 \delta + \theta)) / ((1 + 4 \delta + \theta) (24 \delta^3 - \\
12 \delta^2 (-1 + \theta) + (-2 + \theta) (1 + \theta) (2 + \theta) + 2 \delta (-6 + (-4 + \theta) \theta))))) \\
(-(((c - \alpha) (-1 - 2 \delta + 4 \delta^2 - \theta) (2 + 6 \delta + \theta)) / ((1 + 4 \delta + \theta) (24 \delta^3 - \\
12 \delta^2 (-1 + \theta) + (-2 + \theta) (1 + \theta) (2 + \theta) + 2 \delta (-6 + (-4 + \theta) \theta))))) + \\
(-(((c - \alpha) (1 + 2 \delta) (12 \delta^2 - 2 \delta (2 + \theta) - (1 + \theta) (2 + \theta))) / ((1 + 4 \delta + \theta) \\
(24 \delta^3 - 12 \delta^2 (-1 + \theta) + (-2 + \theta) (1 + \theta) (2 + \theta) + 2 \delta (-6 + (-4 + \theta) \theta))))) \\
(-(((c - \alpha) (1 + 2 \delta) (12 \delta^2 - 2 \delta (2 + \theta) - (1 + \theta) (2 + \theta))) / ((1 + 4 \delta + \theta) \\
(24 \delta^3 - 12 \delta^2 (-1 + \theta) + (-2 + \theta) (1 + \theta) (2 + \theta) + 2 \delta (-6 + (-4 + \theta) \theta))))) + \\
(-(((c - \alpha) (-1 - 2 \delta + 4 \delta^2 - \theta) (2 + 6 \delta + \theta)) / ((1 + 4 \delta + \theta) (24 \delta^3 - \\
12 \delta^2 (-1 + \theta) + (-2 + \theta) (1 + \theta) (2 + \theta) + 2 \delta (-6 + (-4 + \theta) \theta))))) \\
(-(((c - \alpha) (1 + 2 \delta) (12 \delta^2 - 2 \delta (2 + \theta) - (1 + \theta) (2 + \theta))) / ((1 + 4 \delta + \theta) \\
(24 \delta^3 - 12 \delta^2 (-1 + \theta) + (-2 + \theta) (1 + \theta) (2 + \theta) + 2 \delta (-6 + (-4 + \theta) \theta))))) + \\
(-(((c - \alpha) (1 + 2 \delta) (12 \delta^2 - 2 \delta (2 + \theta) - (1 + \theta) (2 + \theta))) / ((1 + 4 \delta + \theta) \\
(24 \delta^3 - 12 \delta^2 (-1 + \theta) + (-2 + \theta) (1 + \theta) (2 + \theta) + 2 \delta (-6 + (-4 + \theta) \theta))))) \\
(-(((c - \alpha) (1 + 2 \delta) (12 \delta^2 - 2 \delta (2 + \theta) - (1 + \theta) (2 + \theta))) / ((1 + 4 \delta + \theta) \\
(24 \delta^3 - 12 \delta^2 (-1 + \theta) + (-2 + \theta) (1 + \theta) (2 + \theta) + 2 \delta (-6 + (-4 + \theta) \theta))))) + \\
(-(((c - \alpha) (-1 - 2 \delta + 4 \delta^2 - \theta) (2 + 6 \delta + \theta)) / ((1 + 4 \delta + \theta) (24 \delta^3 - \\
12 \delta^2 (-1 + \theta) + (-2 + \theta) (1 + \theta) (2 + \theta) + 2 \delta (-6 + (-4 + \theta) \theta))))) \\
(-(((c - \alpha) (1 + 2 \delta) (12 \delta^2 - 2 \delta (2 + \theta) - (1 + \theta) (2 + \theta))) / ((1 + 4 \delta + \theta) \\
(24 \delta^3 - 12 \delta^2 (-1 + \theta) + (-2 + \theta) (1 + \theta) (2 + \theta) + 2 \delta (-6 + (-4 + \theta) \theta))))) + \\
(-(((c - \alpha) (1 + 2 \delta) (12 \delta^2 - 2 \delta (2 + \theta) - (1 + \theta) (2 + \theta))) / ((1 + 4 \delta + \theta) \\
(24 \delta^3 - 12 \delta^2 (-1 + \theta) + (-2 + \theta) (1 + \theta) (2 + \theta) + 2 \delta (-6 + (-4 + \theta) \theta))))) \\
(-(((c - \alpha) (-1 - 2 \delta + 4 \delta^2 - \theta) (2 + 6 \delta + \theta)) / ((1 + 4 \delta + \theta) (24 \delta^3 - \\
12 \delta^2 (-1 + \theta) + (-2 + \theta) (1 + \theta) (2 + \theta) + 2 \delta (-6 + (-4 + \theta) \theta))))) + \\
(-(((c - \alpha) (-1 - 2 \delta + 4 \delta^2 - \theta) (2 + 6 \delta + \theta)) / ((1 + 4 \delta + \theta) (24 \delta^3 - \\
12 \delta^2 (-1 + \theta) + (-2 + \theta) (1 + \theta) (2 + \theta) + 2 \delta (-6 + (-4 + \theta) \theta))))) \\
(-(((c - \alpha) (-1 - 2 \delta + 4 \delta^2 - \theta) (2 + 6 \delta + \theta)) / ((1 + 4 \delta + \theta) (24 \delta^3 - \\
12 \delta^2 (-1 + \theta) + (-2 + \theta) (1 + \theta) (2 + \theta) + 2 \delta (-6 + (-4 + \theta) \theta))))) + \\
(-(((c - \alpha) (1 + 2 \delta) (12 \delta^2 - 2 \delta (2 + \theta) - (1 + \theta) (2 + \theta))) / ((1 + 4 \delta + \theta) \\
(24 \delta^3 - 12 \delta^2 (-1 + \theta) + (-2 + \theta) (1 + \theta) (2 + \theta) + 2 \delta (-6 + (-4 + \theta) \theta))))) \\
(-(((c - \alpha) (-1 - 2 \delta + 4 \delta^2 - \theta) (2 + 6 \delta + \theta)) / ((1 + 4 \delta + \theta) (24 \delta^3 - \\
12 \delta^2 (-1 + \theta) + (-2 + \theta) (1 + \theta) (2 + \theta) + 2 \delta (-6 + (-4 + \theta) \theta))))) + \\
(-(((c - \alpha) (-1 - 2 \delta + 4 \delta^2 - \theta) (2 + 6 \delta + \theta)) / ((1 + 4 \delta + \theta) (24 \delta^3 - \\
12 \delta^2 (-1 + \theta) + (-2 + \theta) (1 + \theta) (2 + \theta) + 2 \delta (-6 + (-4 + \theta) \theta)))))
\end{array}
$$

```
      (- (((c - α) (-1 - 2 δ + 4 δ^2 - θ) (2 + 6 δ + θ)) / ((1 + 4 δ + θ) (24 δ^3 -
            12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ)))))) -
   2 * θ ((- (((c - α) (1 + 2 δ) (12 δ^2 - 2 δ (2 + θ) - (1 + θ) (2 + θ))) / ((1 + 4 δ + θ)
            (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ)))))
      (- (((c - α) (-1 - 2 δ + 4 δ^2 - θ) (2 + 6 δ + θ)) / ((1 + 4 δ + θ) (24 δ^3 -
            12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))))) +
     (- (((c - α) (1 + 2 δ) (12 δ^2 - 2 δ (2 + θ) - (1 + θ) (2 + θ))) / ((1 + 4 δ + θ)
            (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ)))))
      (- (((c - α) (-1 - 2 δ + 4 δ^2 - θ) (2 + 6 δ + θ)) / ((1 + 4 δ + θ) (24 δ^3 -
            12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))))) +
     (- (((c - α) (1 + 2 δ) (12 δ^2 - 2 δ (2 + θ) - (1 + θ) (2 + θ))) / ((1 + 4 δ + θ)
            (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ)))))
      (- (((c - α) (-1 - 2 δ + 4 δ^2 - θ) (2 + 6 δ + θ)) / ((1 + 4 δ + θ) (24 δ^3 -
            12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))))))]

(z (1 + 4 δ + θ) (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))^2 +
   3 c^2 (1 + 2 δ) (-144 δ^4 + 288 δ^5 + 2 δ (1 + θ) (2 + θ) (7 + 5 θ) +
       (2 + 3 θ + θ^2)^2 - 24 δ^3 (6 + θ (8 + θ)) - 2 δ^2 (-13 + θ (-9 + θ (3 + θ)))) -
   6 c α (1 + 2 δ) (-144 δ^4 + 288 δ^5 + 2 δ (1 + θ) (2 + θ) (7 + 5 θ) + (2 + 3 θ + θ^2)^2 -
       24 δ^3 (6 + θ (8 + θ)) - 2 δ^2 (-13 + θ (-9 + θ (3 + θ)))) +
   3 α^2 (1 + 2 δ) (-144 δ^4 + 288 δ^5 + 2 δ (1 + θ) (2 + θ) (7 + 5 θ) + (2 + 3 θ + θ^2)^2 -
       24 δ^3 (6 + θ (8 + θ)) - 2 δ^2 (-13 + θ (-9 + θ (3 + θ))))) /
  ((1 + 4 δ + θ) (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))^2)

Reduce[(z (1 + 4 δ + θ) (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))^2 +
     3 c^2 (1 + 2 δ) (-144 δ^4 + 288 δ^5 + 2 δ (1 + θ) (2 + θ) (7 + 5 θ) +
         (2 + 3 θ + θ^2)^2 - 24 δ^3 (6 + θ (8 + θ)) - 2 δ^2 (-13 + θ (-9 + θ (3 + θ)))) -
     6 c α (1 + 2 δ) (-144 δ^4 + 288 δ^5 + 2 δ (1 + θ) (2 + θ) (7 + 5 θ) + (2 + 3 θ + θ^2)^2 -
         24 δ^3 (6 + θ (8 + θ)) - 2 δ^2 (-13 + θ (-9 + θ (3 + θ)))) +
     3 α^2 (1 + 2 δ) (-144 δ^4 + 288 δ^5 + 2 δ (1 + θ) (2 + θ) (7 + 5 θ) + (2 + 3 θ + θ^2)^2 -
         24 δ^3 (6 + θ (8 + θ)) - 2 δ^2 (-13 + θ (-9 + θ (3 + θ))))) /
    ((1 + 4 δ + θ) (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))^2) ==
   z + (3 (c - α)^2 (1 + 2 δ) (-144 δ^4 + 288 δ^5 + 2 δ (1 + θ) (2 + θ) (7 + 5 θ) +
         (2 + 3 θ + θ^2)^2 - 24 δ^3 (6 + θ (8 + θ)) - 2 δ^2 (-13 + θ (-9 + θ (3 + θ))))) /
    ((1 + 4 δ + θ) (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))^2)]

True
```

```
FullSimplify[Collect[
  (z (1 + 4 δ + θ) (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))^2 +
      3 c^2 (1 + 2 δ) (-144 δ^4 + 288 δ^5 + 2 δ (1 + θ) (2 + θ) (7 + 5 θ) +
         (2 + 3 θ + θ^2)^2 - 24 δ^3 (6 + θ (8 + θ)) - 2 δ^2 (-13 + θ (-9 + θ (3 + θ)))) -
      6 c α (1 + 2 δ) (-144 δ^4 + 288 δ^5 + 2 δ (1 + θ) (2 + θ) (7 + 5 θ) + (2 + 3 θ + θ^2)^2 -
         24 δ^3 (6 + θ (8 + θ)) - 2 δ^2 (-13 + θ (-9 + θ (3 + θ)))) +
      3 α^2 (1 + 2 δ) (-144 δ^4 + 288 δ^5 + 2 δ (1 + θ) (2 + θ) (7 + 5 θ) + (2 + 3 θ + θ^2)^2 -
         24 δ^3 (6 + θ (8 + θ)) - 2 δ^2 (-13 + θ (-9 + θ (3 + θ))))) /
   ((1 + 4 δ + θ) (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))^2),
  (1 + 4 δ + θ)]]
z + (3 (c - α)^2 (1 + 2 δ) (-144 δ^4 + 288 δ^5 + 2 δ (1 + θ) (2 + θ) (7 + 5 θ) +
       (2 + 3 θ + θ^2)^2 - 24 δ^3 (6 + θ (8 + θ)) - 2 δ^2 (-13 + θ (-9 + θ (3 + θ))))) /
  ((1 + 4 δ + θ) (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))^2)
```

```
(*Total Surplus*)
```

```
FullSimplify[(3 (c - α)^2 (-1 + θ) (-(-1 + 2 δ - θ) (-1 - 2 δ + 4 δ^2 - θ) (2 + 6 δ + θ)^2 -
         (1 + 2 δ) (-12 δ^2 + 2 δ (2 + θ) + (1 + θ) (2 + θ))^2)) /
    ((1 + 4 δ + θ) (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))^2) +
  (z + (3 (c - α)^2 (1 + 2 δ) (-144 δ^4 + 288 δ^5 + 2 δ (1 + θ) (2 + θ) (7 + 5 θ) +
           (2 + 3 θ + θ^2)^2 - 24 δ^3 (6 + θ (8 + θ)) - 2 δ^2 (-13 + θ (-9 + θ (3 + θ))))) /
      ((1 + 4 δ + θ) (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))^2))]
z + (3 (c - α)^2 (576 δ^6 - 576 δ^5 (-1 + θ) - (1 + θ)^2 (2 + θ)^2 (-3 + 2 θ) +
        96 δ^4 (-6 + (-4 + θ) θ) - 2 δ (1 + θ) (2 + θ) (-21 + θ (-7 + θ (9 + θ))) +
        4 δ^3 (-103 + θ (-39 + θ (51 + 19 θ))) + 2 δ^2 (43 + θ (85 + θ (41 - θ (5 + 2 θ)))))) /
  ((1 + 4 δ + θ) (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))^2)
```

```
(*Case III: Two Multiproduct Firms Firm135 and Firm 246*)

(*profit Firm 135*)
```

```
FullSimplify[
 ((p1 - c) * (((-1 - 2 δ + 4 δ^2 - θ) p1 + (-4 δ^2 + θ + 2 δ θ + θ^2) p2 - (-1 + θ) (α (1 - 2 δ + θ) +
          δ (p3 + p4 + p5 + p6))) / ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ)))) +
  ((p3 - c) * ((α (-1 + 2 δ - θ) (-1 + θ) + (δ - δ θ) p1 + (δ - δ θ) p2 - p3 - 2 δ p3 +
        4 δ^2 p3 - θ p3 - 4 δ^2 p4 + θ p4 + 2 δ θ p4 + θ^2 p4 + δ p5 - δ θ p5 - δ (-1 + θ) p6) /
      ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ)))) + ((p5 - c) *
    ((α (-1 + 2 δ - θ) (-1 + θ) + (δ - δ θ) p1 + (δ - δ θ) p2 + δ p3 - δ θ p3 + δ p4 - δ θ p4 - p5 - 2 δ p5 +
        4 δ^2 p5 - θ p5 + (-4 δ^2 + θ + 2 δ θ + θ^2) p6) / ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ))))]

             1
─────────────────────────────
(-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ)
 ((-c + p3) (α (-1 + 2 δ - θ) (-1 + θ) + (δ - δ θ) p1 + (δ - δ θ) p2 - p3 - 2 δ p3 +
       4 δ^2 p3 - θ p3 - 4 δ^2 p4 + θ p4 + 2 δ θ p4 + θ^2 p4 + δ p5 - δ θ p5 - δ (-1 + θ) p6) +
    (-c + p5) (α (-1 + 2 δ - θ) (-1 + θ) + (δ - δ θ) p1 + (δ - δ θ) p2 + δ p3 - δ θ p3 +
       δ p4 - δ θ p4 - p5 - 2 δ p5 + 4 δ^2 p5 - θ p5 + (-4 δ^2 + θ + 2 δ θ + θ^2) p6) +
    (-c + p1) ((-1 - 2 δ + 4 δ^2 - θ) p1 + (-4 δ^2 + θ + 2 δ θ + θ^2) p2 -
       (-1 + θ) (α (1 - 2 δ + θ) + δ (p3 + p4 + p5 + p6))))

FullSimplify[D[                 1
               ─────────────────────────────
               (-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ)
   ((-c + p3) (α (-1 + 2 δ - θ) (-1 + θ) + (δ - δ θ) p1 + (δ - δ θ) p2 - p3 - 2 δ p3 +
         4 δ^2 p3 - θ p3 - 4 δ^2 p4 + θ p4 + 2 δ θ p4 + θ^2 p4 + δ p5 - δ θ p5 - δ (-1 + θ) p6) +
      (-c + p5) (α (-1 + 2 δ - θ) (-1 + θ) + (δ - δ θ) p1 + (δ - δ θ) p2 + δ p3 - δ θ p3 +
         δ p4 - δ θ p4 - p5 - 2 δ p5 + 4 δ^2 p5 - θ p5 + (-4 δ^2 + θ + 2 δ θ + θ^2) p6) +
      (-c + p1) ((-1 - 2 δ + 4 δ^2 - θ) p1 + (-4 δ^2 + θ + 2 δ θ + θ^2) p2 -
         (-1 + θ) (α (1 - 2 δ + θ) + δ (p3 + p4 + p5 + p6)))), p1]]

(- (-1 + 2 δ - θ) (c + α + 2 c δ - α θ) + (-4 δ + 8 δ^2 - 2 (1 + θ)) p1 + (-4 δ^2 + θ + 2 δ θ + θ^2) p2 -
   δ (-1 + θ) (2 p3 + p4 + 2 p5 + p6)) / ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ))

FullSimplify[D[                 1
               ─────────────────────────────
               (-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ)
   ((-c + p3) (α (-1 + 2 δ - θ) (-1 + θ) + (δ - δ θ) p1 + (δ - δ θ) p2 - p3 - 2 δ p3 +
         4 δ^2 p3 - θ p3 - 4 δ^2 p4 + θ p4 + 2 δ θ p4 + θ^2 p4 + δ p5 - δ θ p5 - δ (-1 + θ) p6) +
      (-c + p5) (α (-1 + 2 δ - θ) (-1 + θ) + (δ - δ θ) p1 + (δ - δ θ) p2 + δ p3 - δ θ p3 +
         δ p4 - δ θ p4 - p5 - 2 δ p5 + 4 δ^2 p5 - θ p5 + (-4 δ^2 + θ + 2 δ θ + θ^2) p6) +
      (-c + p1) ((-1 - 2 δ + 4 δ^2 - θ) p1 + (-4 δ^2 + θ + 2 δ θ + θ^2) p2 -
         (-1 + θ) (α (1 - 2 δ + θ) + δ (p3 + p4 + p5 + p6)))), p3]]

(- (-1 + 2 δ - θ) (c + α + 2 c δ - α θ) - 2 δ (-1 + θ) p1 + (δ - δ θ) p2 - 2 p3 - 4 δ p3 +
   8 δ^2 p3 - 2 θ p3 - 4 δ^2 p4 + θ p4 + 2 δ θ p4 + θ^2 p4 + 2 δ p5 - 2 δ θ p5 - δ (-1 + θ) p6) /
 ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ))
```

```
FullSimplify[D[1/((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ))
    ((-c + p3) (α (-1 + 2 δ - θ) (-1 + θ) + (δ - δ θ) p1 + (δ - δ θ) p2 - p3 - 2 δ p3 +
          4 δ^2 p3 - θ p3 - 4 δ^2 p4 + θ p4 + 2 δ θ p4 + θ^2 p4 + δ p5 - δ θ p5 - δ (-1 + θ) p6) +
      (-c + p5) (α (-1 + 2 δ - θ) (-1 + θ) + (δ - δ θ) p1 + (δ - δ θ) p2 + δ p3 - δ θ p3 +
          δ p4 - δ θ p4 - p5 - 2 δ p5 + 4 δ^2 p5 - θ p5 + (-4 δ^2 + θ + 2 δ θ + θ^2) p6) +
      (-c + p1) ((-1 - 2 δ + 4 δ^2 - θ) p1 + (-4 δ^2 + θ + 2 δ θ + θ^2) p2 -
          (-1 + θ) (α (1 - 2 δ + θ) + δ (p3 + p4 + p5 + p6)))), p5]]
```

$$\Big(-(-1+2\delta-\theta)\,(c+\alpha+2c\delta-\alpha\theta)-2\delta\,(-1+\theta)\,p_1+(\delta-\delta\theta)\,p_2+2\delta p_3-2\delta\theta p_3+\delta p_4-\delta\theta p_4-2p_5-4\delta p_5+8\delta^2 p_5-2\theta p_5+\left(-4\delta^2+\theta+2\delta\theta+\theta^2\right)p_6\Big)\Big/\left((-1+2\delta-\theta)\,(-1+\theta)\,(1+4\delta+\theta)\right)$$

```
(*profit Firm 246*)
```

```
FullSimplify[((p2 - c) * (((4 δ^2 - 2 δ θ - θ (1 + θ)) p1 + (1 + 2 δ - 4 δ^2 + θ) p2 +
          (-1 + θ) (α (1 - 2 δ + θ) + δ (p3 + p4 + p5 + p6))) / ((-1 + θ) (1 - 2 δ + θ) (1 + 4 δ + θ)))) +
   ((p4 - c) * ((α (-1 + 2 δ - θ) (-1 + θ) + (δ - δ θ) p1 + (δ - δ θ) p2 - 4 δ^2 p3 + θ p3 +
          2 δ θ p3 + θ^2 p3 - p4 - 2 δ p4 + 4 δ^2 p4 - θ p4 + δ p5 - δ θ p5 - δ (-1 + θ) p6) /
        ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ)))) + ((p6 - c) *
      ((α - 2 α δ + 2 α δ θ - α θ^2 + (δ - δ θ) p1 + (δ - δ θ) p2 + δ p3 - δ θ p3 + δ p4 - δ θ p4 - 4 δ^2 p5 +
          θ p5 + 2 δ θ p5 + θ^2 p5 + (-1 - 2 δ + 4 δ^2 - θ) p6) / ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ))))]
```

$$\frac{1}{(-1+\theta)\,(1+4\delta+\theta)}\left(\frac{1}{-1+2\delta-\theta}(-c+p_6)\left(\alpha-2\alpha\delta+2\alpha\delta\theta-\alpha\theta^2+(\delta-\delta\theta)\,p_1+(\delta-\delta\theta)\,p_2+\delta p_3-\delta\theta p_3+\delta p_4-\delta\theta p_4-4\delta^2 p_5+\theta p_5+2\delta\theta p_5+\theta^2 p_5+\left(-1-2\delta+4\delta^2-\theta\right)p_6\right)+\frac{1}{-1+2\delta-\theta}(-c+p_4)\left(\alpha\,(-1+2\delta-\theta)\,(-1+\theta)+(\delta-\delta\theta)\,p_1+(\delta-\delta\theta)\,p_2-4\delta^2 p_3+\theta p_3+2\delta\theta p_3+\theta^2 p_3-p_4-2\delta p_4+4\delta^2 p_4-\theta p_4+\delta p_5-\delta\theta p_5-\delta\,(-1+\theta)\,p_6\right)+\frac{1}{1-2\delta+\theta}(-c+p_2)\left(\left(4\delta^2-2\delta\theta-\theta\,(1+\theta)\right)p_1+\left(1+2\delta-4\delta^2+\theta\right)p_2+(-1+\theta)\,(\alpha\,(1-2\delta+\theta)+\delta\,(p_3+p_4+p_5+p_6))\right)\right)$$

$$\text{FullSimplify}\Big[\text{D}\Big[\frac{1}{(-1+\theta)\,(1+4\,\delta+\theta)}\Big(\frac{1}{-1+2\,\delta-\theta}(-c+p_6)\,\big(\alpha-2\,\alpha\,\delta+2\,\alpha\,\delta\,\theta-\alpha\,\theta^2+(\delta-\delta\,\theta)\,p_1+(\delta-\delta\,\theta)\,p_2+\delta\,p_3-\delta\,\theta\,p_3+\delta\,p_4-\delta\,\theta\,p_4-4\,\delta^2\,p_5+\theta\,p_5+2\,\delta\,\theta\,p_5+\theta^2\,p_5+\big(-1-2\,\delta+4\,\delta^2-\theta\big)\,p_6\big)+\frac{1}{-1+2\,\delta-\theta}(-c+p_4)\,\big(\alpha\,\big(-1+2\,\delta-\theta\big)\,\big(-1+\theta\big)+(\delta-\delta\,\theta)\,p_1+(\delta-\delta\,\theta)\,p_2-4\,\delta^2\,p_3+\theta\,p_3+2\,\delta\,\theta\,p_3+\theta^2\,p_3-p_4-2\,\delta\,p_4+4\,\delta^2\,p_4-\theta\,p_4+\delta\,p_5-\delta\,\theta\,p_5-\delta\,\big(-1+\theta\big)\,p_6\big)+\frac{1}{1-2\,\delta+\theta}(-c+p_2)\,\Big(\big(4\,\delta^2-2\,\delta\,\theta-\theta\,\big(1+\theta\big)\big)\,p_1+\big(1+2\,\delta-4\,\delta^2+\theta\big)\,p_2+\big(-1+\theta\big)\,\big(\alpha\,\big(1-2\,\delta+\theta\big)+\delta\,(p_3+p_4+p_5+p_6)\big)\Big)\Big],\ p_2\Big]\Big]$$

$$\Big((-1+2\,\delta-\theta)\,(c+\alpha+2\,c\,\delta-\alpha\,\theta)+\big(4\,\delta^2-2\,\delta\,\theta-\theta\,(1+\theta)\big)\,p_1+2\,\big(1+2\,\delta-4\,\delta^2+\theta\big)\,p_2+\delta\,(-1+\theta)\,(p_3+2\,p_4+p_5+2\,p_6)\Big)\Big/\big((-1+\theta)\,(1-2\,\delta+\theta)\,(1+4\,\delta+\theta)\big)$$

$$\text{FullSimplify}\Big[\text{D}\Big[\frac{1}{(-1+\theta)\,(1+4\,\delta+\theta)}\Big(\frac{1}{-1+2\,\delta-\theta}(-c+p_6)\,\big(\alpha-2\,\alpha\,\delta+2\,\alpha\,\delta\,\theta-\alpha\,\theta^2+(\delta-\delta\,\theta)\,p_1+(\delta-\delta\,\theta)\,p_2+\delta\,p_3-\delta\,\theta\,p_3+\delta\,p_4-\delta\,\theta\,p_4-4\,\delta^2\,p_5+\theta\,p_5+2\,\delta\,\theta\,p_5+\theta^2\,p_5+\big(-1-2\,\delta+4\,\delta^2-\theta\big)\,p_6\big)+\frac{1}{-1+2\,\delta-\theta}(-c+p_4)\,\big(\alpha\,\big(-1+2\,\delta-\theta\big)\,\big(-1+\theta\big)+(\delta-\delta\,\theta)\,p_1+(\delta-\delta\,\theta)\,p_2-4\,\delta^2\,p_3+\theta\,p_3+2\,\delta\,\theta\,p_3+\theta^2\,p_3-p_4-2\,\delta\,p_4+4\,\delta^2\,p_4-\theta\,p_4+\delta\,p_5-\delta\,\theta\,p_5-\delta\,\big(-1+\theta\big)\,p_6\big)+\frac{1}{1-2\,\delta+\theta}(-c+p_2)\,\Big(\big(4\,\delta^2-2\,\delta\,\theta-\theta\,\big(1+\theta\big)\big)\,p_1+\big(1+2\,\delta-4\,\delta^2+\theta\big)\,p_2+\big(-1+\theta\big)\,\big(\alpha\,\big(1-2\,\delta+\theta\big)+\delta\,(p_3+p_4+p_5+p_6)\big)\Big)\Big],\ p_4\Big]\Big]$$

$$\Big((-1+2\,\delta-\theta)\,(c+\alpha+2\,c\,\delta-\alpha\,\theta)+\delta\,(-1+\theta)\,p_1+2\,\delta\,(-1+\theta)\,p_2+4\,\delta^2\,p_3-\theta\,p_3-2\,\delta\,\theta\,p_3-\theta^2\,p_3+2\,p_4+4\,\delta\,p_4-8\,\delta^2\,p_4+2\,\theta\,p_4-\delta\,p_5+\delta\,\theta\,p_5+2\,\delta\,(-1+\theta)\,p_6\Big)\Big/\big((-1+\theta)\,(1-2\,\delta+\theta)\,(1+4\,\delta+\theta)\big)$$

$$\text{FullSimplify}\Big[\text{D}\Big[\frac{1}{(-1+\theta)\,(1+4\,\delta+\theta)}\Big(\frac{1}{-1+2\,\delta-\theta}(-c+p_6)\,\big(\alpha-2\,\alpha\,\delta+2\,\alpha\,\delta\,\theta-\alpha\,\theta^2+(\delta-\delta\,\theta)\,p_1+(\delta-\delta\,\theta)\,p_2+\delta\,p_3-\delta\,\theta\,p_3+\delta\,p_4-\delta\,\theta\,p_4-4\,\delta^2\,p_5+\theta\,p_5+2\,\delta\,\theta\,p_5+\theta^2\,p_5+\big(-1-2\,\delta+4\,\delta^2-\theta\big)\,p_6\big)+\frac{1}{-1+2\,\delta-\theta}(-c+p_4)\,\big(\alpha\,\big(-1+2\,\delta-\theta\big)\,\big(-1+\theta\big)+(\delta-\delta\,\theta)\,p_1+(\delta-\delta\,\theta)\,p_2-4\,\delta^2\,p_3+\theta\,p_3+2\,\delta\,\theta\,p_3+\theta^2\,p_3-p_4-2\,\delta\,p_4+4\,\delta^2\,p_4-\theta\,p_4+\delta\,p_5-\delta\,\theta\,p_5-\delta\,\big(-1+\theta\big)\,p_6\big)+\frac{1}{1-2\,\delta+\theta}(-c+p_2)\,\Big(\big(4\,\delta^2-2\,\delta\,\theta-\theta\,\big(1+\theta\big)\big)\,p_1+\big(1+2\,\delta-4\,\delta^2+\theta\big)\,p_2+\big(-1+\theta\big)\,\big(\alpha\,\big(1-2\,\delta+\theta\big)+\delta\,(p_3+p_4+p_5+p_6)\big)\Big)\Big],\ p_6\Big]\Big]$$

$$\Big((-1+2\,\delta-\theta)\,(c+\alpha+2\,c\,\delta-\alpha\,\theta)+\delta\,(-1+\theta)\,p_1+2\,\delta\,(-1+\theta)\,p_2-\delta\,p_3+\delta\,\theta\,p_3-2\,\delta\,p_4+2\,\delta\,\theta\,p_4+4\,\delta^2\,p_5-\theta\,p_5-2\,\delta\,\theta\,p_5-\theta^2\,p_5+2\,p_6+2\,\big(2\,\delta-4\,\delta^2+\theta\big)\,p_6\Big)\Big/\big((-1+\theta)\,(1-2\,\delta+\theta)\,(1+4\,\delta+\theta)\big)$$

```
FullSimplify[Solve[
  {(-(-1 + 2 δ - θ) (c + α + 2 c δ - α θ) + (-4 δ + 8 δ^2 - 2 (1 + θ)) p1 + (-4 δ^2 + θ + 2 δ θ + θ^2) p2 -
        δ (-1 + θ) (2 p3 + p4 + 2 p5 + p6)) / ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ)) == 0,
   ((-1 + 2 δ - θ) (c + α + 2 c δ - α θ) + (4 δ^2 - 2 δ θ - θ (1 + θ)) p1 + 2 (1 + 2 δ - 4 δ^2 + θ) p2 +
        δ (-1 + θ) (p3 + 2 p4 + p5 + 2 p6)) / ((-1 + θ) (1 - 2 δ + θ) (1 + 4 δ + θ)) == 0,
   (-(-1 + 2 δ - θ) (c + α + 2 c δ - α θ) - 2 δ (-1 + θ) p1 + (δ - δ θ) p2 - 2 p3 - 4 δ p3 +
        8 δ^2 p3 - 2 θ p3 - 4 δ^2 p4 + θ p4 + 2 δ θ p4 + θ^2 p4 + 2 δ p5 - 2 δ θ p5 - δ (-1 + θ) p6) /
      ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ)) == 0,
   ((-1 + 2 δ - θ) (c + α + 2 c δ - α θ) + δ (-1 + θ) p1 + 2 δ (-1 + θ) p2 + 4 δ^2 p3 - θ p3 -
        2 δ θ p3 - θ^2 p3 + 2 p4 + 4 δ p4 - 8 δ^2 p4 + 2 θ p4 - δ p5 + δ θ p5 + 2 δ (-1 + θ) p6) /
      ((-1 + θ) (1 - 2 δ + θ) (1 + 4 δ + θ)) == 0, (-(-1 + 2 δ - θ) (c + α + 2 c δ - α θ) -
        2 δ (-1 + θ) p1 + (δ - δ θ) p2 + 2 δ p3 - 2 δ θ p3 + δ p4 - δ θ p4 - 2 p5 - 4 δ p5 + 8 δ^2 p5 -
        2 θ p5 + (-4 δ^2 + θ + 2 δ θ + θ^2) p6) / ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ)) == 0,
   ((-1 + 2 δ - θ) (c + α + 2 c δ - α θ) + δ (-1 + θ) p1 + 2 δ (-1 + θ) p2 - δ p3 + δ θ p3 -
        2 δ p4 + 2 δ θ p4 + 4 δ^2 p5 - θ p5 - 2 δ θ p5 - θ^2 p5 + 2 p6 + 2 (2 δ - 4 δ^2 + θ) p6) /
      ((-1 + θ) (1 - 2 δ + θ) (1 + 4 δ + θ)) == 0}, {p1, p2, p3, p4, p5, p6}]]
```

$$\left\{\left\{p_1 \to \frac{c+\alpha+2 c \delta-\alpha \theta}{2+2 \delta-\theta},\ p_2 \to \frac{c+\alpha+2 c \delta-\alpha \theta}{2+2 \delta-\theta},\ p_3 \to \frac{c+\alpha+2 c \delta-\alpha \theta}{2+2 \delta-\theta},\ p_4 \to \frac{c+\alpha+2 c \delta-\alpha \theta}{2+2 \delta-\theta},\ p_5 \to \frac{c+\alpha+2 c \delta-\alpha \theta}{2+2 \delta-\theta},\ p_6 \to \frac{c+\alpha+2 c \delta-\alpha \theta}{2+2 \delta-\theta}\right\}\right\}$$

```
(*solving for q1*)
```

FullSimplify[

$$\left(\left(-1-2 \delta+4 \delta^2-\theta\right)\left(\frac{c+\alpha+2 c \delta-\alpha \theta}{2+2 \delta-\theta}\right)+\left(-4 \delta^2+\theta+2 \delta \theta+\theta^2\right)\left(\frac{c+\alpha+2 c \delta-\alpha \theta}{2+2 \delta-\theta}\right)-(-1+\theta)\left(\alpha(1-2 \delta+\theta)+\delta\left(\left(\frac{c+\alpha+2 c \delta-\alpha \theta}{2+2 \delta-\theta}\right)+\left(\frac{c+\alpha+2 c \delta-\alpha \theta}{2+2 \delta-\theta}\right)+\left(\frac{c+\alpha+2 c \delta-\alpha \theta}{2+2 \delta-\theta}\right)+\left(\frac{c+\alpha+2 c \delta-\alpha \theta}{2+2 \delta-\theta}\right)\right)\right)\right) \Big/ \left((-1+2 \delta-\theta)(-1+\theta)(1+4 \delta+\theta)\right)$$

]

$$-\frac{(c-\alpha)(1+2 \delta)}{(2+2 \delta-\theta)(1+4 \delta+\theta)}$$

$$\text{Reduce}\left[\frac{(\alpha-c)(1+2 \delta)}{(2+2 \delta-\theta)(1+4 \delta+\theta)} == -\frac{(c-\alpha)(1+2 \delta)}{(2+2 \delta-\theta)(1+4 \delta+\theta)}\right]$$

True

```
(*PROFIT 1*)
```

$$\left(\left(\frac{c+\alpha+2 c \delta-\alpha \theta}{2+2 \delta-\theta}\right)-c\right) * \left(-\frac{(c-\alpha)(1+2 \delta)}{(2+2 \delta-\theta)(1+4 \delta+\theta)}\right)$$

$$-\frac{(c-\alpha)(1+2 \delta)\left(-c+\frac{c+\alpha+2 c \delta-\alpha \theta}{2+2 \delta-\theta}\right)}{(2+2 \delta-\theta)(1+4 \delta+\theta)}$$

```
(*Profit 135, by symmetry 246*)
```

$$\text{FullSimplify}\left[\left(-\frac{(c-\alpha)\,(1+2\,\delta)\,\left(-c+\frac{c+\alpha+2\,c\,\delta-\alpha\,\theta}{2+2\,\delta-\theta}\right)}{(2+2\,\delta-\theta)\,(1+4\,\delta+\theta)}\right)+\left(-\frac{(c-\alpha)\,(1+2\,\delta)\,\left(-c+\frac{c+\alpha+2\,c\,\delta-\alpha\,\theta}{2+2\,\delta-\theta}\right)}{(2+2\,\delta-\theta)\,(1+4\,\delta+\theta)}\right)+\left(-\frac{(c-\alpha)\,(1+2\,\delta)\,\left(-c+\frac{c+\alpha+2\,c\,\delta-\alpha\,\theta}{2+2\,\delta-\theta}\right)}{(2+2\,\delta-\theta)\,(1+4\,\delta+\theta)}\right)\right]$$

$$-\frac{3\,(c-\alpha)^2\,(1+2\,\delta)\,(-1+\theta)}{(-2-2\,\delta+\theta)^2\,(1+4\,\delta+\theta)}$$

$$\text{Reduce}\left[\frac{3\,(c-\alpha)^2\,(1+2\,\delta)\,(1-\theta)}{(-2-2\,\delta+\theta)^2\,(1+4\,\delta+\theta)}==-\frac{3\,(c-\alpha)^2\,(1+2\,\delta)\,(-1+\theta)}{(-2-2\,\delta+\theta)^2\,(1+4\,\delta+\theta)}\right]$$

True

(*Industry (*profit*)*)

$$\text{FullSimplify}\left[-\frac{3\,(c-\alpha)^2\,(1+2\,\delta)\,(-1+\theta)}{(-2-2\,\delta+\theta)^2\,(1+4\,\delta+\theta)}+-\frac{3\,(c-\alpha)^2\,(1+2\,\delta)\,(-1+\theta)}{(-2-2\,\delta+\theta)^2\,(1+4\,\delta+\theta)}\right]$$

$$-\frac{6\,(c-\alpha)^2\,(1+2\,\delta)\,(-1+\theta)}{(-2-2\,\delta+\theta)^2\,(1+4\,\delta+\theta)}$$

$$\text{Reduce}\left[\frac{6\,(c-\alpha)^2\,(1+2\,\delta)\,(1-\theta)}{(-2-2\,\delta+\theta)^2\,(1+4\,\delta+\theta)}==-\frac{6\,(c-\alpha)^2\,(1+2\,\delta)\,(-1+\theta)}{(-2-2\,\delta+\theta)^2\,(1+4\,\delta+\theta)}\right]$$

True

(*Consumer surplus*)

$$\text{FullSimplify}\Big[z-p_1\,q_1-p_2\,q_2-p_3\,q_3-p_4\,q_4-p_5\,q_5-p_6\,q_6+\alpha\,(q_1+q_2+q_3+q_4+q_5+q_6)+\frac{1}{2}\Big(-q_1^2-q_2^2-q_3^2-q_4^2-q_5^2-q_6^2-2*\delta\,(q_1\,q_3+q_2\,q_3+q_1\,q_4+q_2\,q_4+q_1\,q_5+q_2\,q_5+q_3\,q_5+q_4\,q_5+q_1\,q_6+q_2\,q_6+q_3\,q_6+q_4\,q_6)-2*\theta\,(q_1\,q_2+q_3\,q_4+q_5\,q_6)\Big)\Big]$$

$$z-p_1\,q_1-p_2\,q_2-p_3\,q_3-p_4\,q_4-p_5\,q_5-p_6\,q_6+\alpha\,(q_1+q_2+q_3+q_4+q_5+q_6)+\frac{1}{2}\Big(-q_1^2-q_2^2-q_3^2-2\,\theta\,q_3\,q_4-q_4^2-2\,\delta\,q_3\,q_5-2\,\delta\,q_4\,q_5-q_5^2-2\,(\delta\,(q_3+q_4)+\theta\,q_5)\,q_6-q_6^2-2\,\delta\,q_2\,(q_3+q_4+q_5+q_6)-2\,q_1\,(\theta\,q_2+\delta\,(q_3+q_4+q_5+q_6))\Big)$$

$$\text{FullSimplify}\Big[z-\left(\frac{c+\alpha+2\,c\,\delta-\alpha\,\theta}{2+2\,\delta-\theta}\right)\left(\frac{(\alpha-c)\,(1+2\,\delta)}{(2+2\,\delta-\theta)\,(1+4\,\delta+\theta)}\right)-\left(\frac{c+\alpha+2\,c\,\delta-\alpha\,\theta}{2+2\,\delta-\theta}\right)\left(\frac{(\alpha-c)\,(1+2\,\delta)}{(2+2\,\delta-\theta)\,(1+4\,\delta+\theta)}\right)-\left(\frac{c+\alpha+2\,c\,\delta-\alpha\,\theta}{2+2\,\delta-\theta}\right)\left(\frac{(\alpha-c)\,(1+2\,\delta)}{(2+2\,\delta-\theta)\,(1+4\,\delta+\theta)}\right)-\left(\frac{c+\alpha+2\,c\,\delta-\alpha\,\theta}{2+2\,\delta-\theta}\right)\left(\frac{(\alpha-c)\,(1+2\,\delta)}{(2+2\,\delta-\theta)\,(1+4\,\delta+\theta)}\right)-\left(\frac{c+\alpha+2\,c\,\delta-\alpha\,\theta}{2+2\,\delta-\theta}\right)$$

$$\left(\frac{(\alpha-c)\,(1+2\delta)}{(2+2\delta-\theta)\,(1+4\delta+\theta)}\right)-\left(\frac{c+\alpha+2c\delta-\alpha\theta}{2+2\delta-\theta}\right)\left(\frac{(\alpha-c)\,(1+2\delta)}{(2+2\delta-\theta)\,(1+4\delta+\theta)}\right)+$$

$$\alpha\Bigg(\Bigg(\left(\frac{(\alpha-c)\,(1+2\delta)}{(2+2\delta-\theta)\,(1+4\delta+\theta)}\right)+\left(\frac{(\alpha-c)\,(1+2\delta)}{(2+2\delta-\theta)\,(1+4\delta+\theta)}\right)+\left(\frac{(\alpha-c)\,(1+2\delta)}{(2+2\delta-\theta)\,(1+4\delta+\theta)}\right)+$$

$$\left(\frac{(\alpha-c)\,(1+2\delta)}{(2+2\delta-\theta)\,(1+4\delta+\theta)}\right)+\left(\frac{(\alpha-c)\,(1+2\delta)}{(2+2\delta-\theta)\,(1+4\delta+\theta)}\right)+\left(\frac{(\alpha-c)\,(1+2\delta)}{(2+2\delta-\theta)\,(1+4\delta+\theta)}\right)\Bigg)\Bigg)+$$

$$\frac{1}{2}\Bigg(-\left(\frac{(\alpha-c)\,(1+2\delta)}{(2+2\delta-\theta)\,(1+4\delta+\theta)}\right)^2-\left(\frac{(\alpha-c)\,(1+2\delta)}{(2+2\delta-\theta)\,(1+4\delta+\theta)}\right)^2-\left(\frac{(\alpha-c)\,(1+2\delta)}{(2+2\delta-\theta)\,(1+4\delta+\theta)}\right)^2-$$

$$2\theta\left(\frac{(\alpha-c)\,(1+2\delta)}{(2+2\delta-\theta)\,(1+4\delta+\theta)}\right)\left(\frac{(\alpha-c)\,(1+2\delta)}{(2+2\delta-\theta)\,(1+4\delta+\theta)}\right)-$$

$$\left(\frac{(\alpha-c)\,(1+2\delta)}{(2+2\delta-\theta)\,(1+4\delta+\theta)}\right)^2-2\delta\left(\frac{(\alpha-c)\,(1+2\delta)}{(2+2\delta-\theta)\,(1+4\delta+\theta)}\right)\left(\frac{(\alpha-c)\,(1+2\delta)}{(2+2\delta-\theta)\,(1+4\delta+\theta)}\right)-$$

$$2\delta\left(\frac{(\alpha-c)\,(1+2\delta)}{(2+2\delta-\theta)\,(1+4\delta+\theta)}\right)\left(\frac{(\alpha-c)\,(1+2\delta)}{(2+2\delta-\theta)\,(1+4\delta+\theta)}\right)-\left(\frac{(\alpha-c)\,(1+2\delta)}{(2+2\delta-\theta)\,(1+4\delta+\theta)}\right)^2-$$

$$2\Bigg(\delta\Bigg(\left(\frac{(\alpha-c)\,(1+2\delta)}{(2+2\delta-\theta)\,(1+4\delta+\theta)}\right)+\left(\frac{(\alpha-c)\,(1+2\delta)}{(2+2\delta-\theta)\,(1+4\delta+\theta)}\right)\Bigg)+$$

$$\theta\left(\frac{(\alpha-c)\,(1+2\delta)}{(2+2\delta-\theta)\,(1+4\delta+\theta)}\right)\Bigg)\left(\frac{(\alpha-c)\,(1+2\delta)}{(2+2\delta-\theta)\,(1+4\delta+\theta)}\right)-$$

$$\left(\frac{(\alpha-c)\,(1+2\delta)}{(2+2\delta-\theta)\,(1+4\delta+\theta)}\right)^2-2\delta\left(\frac{(\alpha-c)\,(1+2\delta)}{(2+2\delta-\theta)\,(1+4\delta+\theta)}\right)$$

$$\Bigg(\left(\frac{(\alpha-c)\,(1+2\delta)}{(2+2\delta-\theta)\,(1+4\delta+\theta)}\right)+\left(\frac{(\alpha-c)\,(1+2\delta)}{(2+2\delta-\theta)\,(1+4\delta+\theta)}\right)+$$

$$\left(\frac{(\alpha-c)\,(1+2\delta)}{(2+2\delta-\theta)\,(1+4\delta+\theta)}\right)+\left(\frac{(\alpha-c)\,(1+2\delta)}{(2+2\delta-\theta)\,(1+4\delta+\theta)}\right)\Bigg)-$$

$$2\left(\frac{(\alpha-c)\,(1+2\delta)}{(2+2\delta-\theta)\,(1+4\delta+\theta)}\right)\Bigg(\theta\left(\frac{(\alpha-c)\,(1+2\delta)}{(2+2\delta-\theta)\,(1+4\delta+\theta)}\right)+$$

$$\delta\Bigg(\left(\frac{(\alpha-c)\,(1+2\delta)}{(2+2\delta-\theta)\,(1+4\delta+\theta)}\right)+\left(\frac{(\alpha-c)\,(1+2\delta)}{(2+2\delta-\theta)\,(1+4\delta+\theta)}\right)+$$

$$\left(\frac{(\alpha-c)\,(1+2\delta)}{(2+2\delta-\theta)\,(1+4\delta+\theta)}\right)+\left(\frac{(\alpha-c)\,(1+2\delta)}{(2+2\delta-\theta)\,(1+4\delta+\theta)}\right)\Bigg)\Bigg)\Bigg)\Bigg)\Bigg]$$

$$\left(-6c\alpha\,(1+2\delta)^2+3\,(c+2c\delta)^2+3\,(\alpha+2\alpha\delta)^2+z\,(-2-2\delta+\theta)^2\,(1+4\delta+\theta)\right)\Big/\left((-2-2\delta+\theta)^2\,(1+4\delta+\theta)\right)$$

```
Collect[(-6 c α (1+2 δ)^2 + 3 (c+2 c δ)^2 + 3 (α+2 α δ)^2 + z (-2-2 δ+θ)^2 (1+4 δ+θ)) /
  ((-2-2 δ+θ)^2 (1+4 δ+θ)), (-2-2 δ+θ)^2 (1+4 δ+θ)]
```

$$z+\frac{-6c\alpha\,(1+2\delta)^2+3\,(c+2c\delta)^2+3\,(\alpha+2\alpha\delta)^2}{(-2-2\delta+\theta)^2\,(1+4\delta+\theta)}$$

```
Reduce[z + (-6 c α (1 + 2 δ)^2 + 3 (c + 2 c δ)^2 + 3 (α + 2 α δ)^2)/((-2 - 2 δ + θ)^2 (1 + 4 δ + θ)) ==
  (-6 c α (1 + 2 δ)^2 + 3 (c + 2 c δ)^2 + 3 (α + 2 α δ)^2 + z (-2 - 2 δ + θ)^2 (1 + 4 δ + θ)) /
   ((-2 - 2 δ + θ)^2 (1 + 4 δ + θ))]
True

(*total surplus*)

FullSimplify[(6 (c - α)^2 (1 + 2 δ) (1 - θ))/((-2 - 2 δ + θ)^2 (1 + 4 δ + θ)) + (z + (-6 c α (1 + 2 δ)^2 + 3 (c + 2 c δ)^2 + 3 (α + 2 α δ)^2)/((-2 - 2 δ + θ)^2 (1 + 4 δ + θ)))]
z + (3 (c - α)^2 (1 + 2 δ) (3 + 2 δ - 2 θ))/((-2 - 2 δ + θ)^2 (1 + 4 δ + θ))

(*Case IV: Submarket Monopolist*)

(* monopoly firm 12, 3-6 individual firms*)

(*Profit monopoly 1-2*)

FullSimplify[(p1 - c) * (((-1 - 2 δ + 4 δ^2 - θ) p1 + (-4 δ^2 + θ + 2 δ θ + θ^2) p2 -
       (-1 + θ) (α (1 - 2 δ + θ) + δ (p3 + p4 + p5 + p6))) / ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ))) +
   (p2 - c) * (((4 δ^2 - 2 δ θ - θ (1 + θ)) p1 + (1 + 2 δ - 4 δ^2 + θ) p2 +
       (-1 + θ) (α (1 - 2 δ + θ) + δ (p3 + p4 + p5 + p6))) / ((-1 + θ) (1 - 2 δ + θ) (1 + 4 δ + θ)))]
(1/(-1 + 2 δ - θ) (-c + p1) ((-1 - 2 δ + 4 δ^2 - θ) p1 +
      (-4 δ^2 + θ + 2 δ θ + θ^2) p2 - (-1 + θ) (α (1 - 2 δ + θ) + δ (p3 + p4 + p5 + p6))) +
   1/(1 - 2 δ + θ) (-c + p2) ((4 δ^2 - 2 δ θ - θ (1 + θ)) p1 + (1 + 2 δ - 4 δ^2 + θ) p2 +
      (-1 + θ) (α (1 - 2 δ + θ) + δ (p3 + p4 + p5 + p6)))) / ((-1 + θ) (1 + 4 δ + θ))

(*Profit max monopoly 1-2*)

FullSimplify[
 D[(1/(-1 + 2 δ - θ) (-c + p1) ((-1 - 2 δ + 4 δ^2 - θ) p1 + (-4 δ^2 + θ + 2 δ θ + θ^2) p2 - (-1 + θ)
         (α (1 - 2 δ + θ) + δ (p3 + p4 + p5 + p6))) +
      1/(1 - 2 δ + θ) (-c + p2) ((4 δ^2 - 2 δ θ - θ (1 + θ)) p1 + (1 + 2 δ - 4 δ^2 + θ) p2 +
         (-1 + θ) (α (1 - 2 δ + θ) + δ (p3 + p4 + p5 + p6)))) / ((-1 + θ) (1 + 4 δ + θ)), p1]]
(2 (1 + 2 δ - 4 δ^2 + θ) p1 + (8 δ^2 - 4 δ θ - 2 θ (1 + θ)) p2 +
   (-1 + θ) (c + α + 2 c δ - 2 α δ + (c + α) θ + δ (p3 + p4 + p5 + p6))) /
 ((-1 + θ) (1 - 2 δ + θ) (1 + 4 δ + θ))
```

```
FullSimplify[
 D[(1/(-1 + 2 δ - θ) (-c + p1) ((-1 - 2 δ + 4 δ^2 - θ) p1 + (-4 δ^2 + θ + 2 δ θ + θ^2) p2 - (-1 + θ)
        (α (1 - 2 δ + θ) + δ (p3 + p4 + p5 + p6))) +
     1/(1 - 2 δ + θ) (-c + p2) ((4 δ^2 - 2 δ θ - θ (1 + θ)) p1 + (1 + 2 δ - 4 δ^2 + θ) p2 +
        (-1 + θ) (α (1 - 2 δ + θ) + δ (p3 + p4 + p5 + p6)))) / ((-1 + θ) (1 + 4 δ + θ)), p2]]

((8 δ^2 - 4 δ θ - 2 θ (1 + θ)) p1 + 2 (1 + 2 δ - 4 δ^2 + θ) p2 +
   (-1 + θ) (c + α + 2 c δ - 2 α δ + (c + α) θ + δ (p3 + p4 + p5 + p6))) /
 ((-1 + θ) (1 - 2 δ + θ) (1 + 4 δ + θ))

(*Postmerger prices*)

FullSimplify[
 Solve[{(2 (1 + 2 δ - 4 δ^2 + θ) p1 + (8 δ^2 - 4 δ θ - 2 θ (1 + θ)) p2 + (-1 + θ) (c + α + 2 c δ - 2 α δ +
          (c + α) θ + δ (p3 + p4 + p5 + p6))) / ((-1 + θ) (1 - 2 δ + θ) (1 + 4 δ + θ)) == 0,
   ((8 δ^2 - 4 δ θ - 2 θ (1 + θ)) p1 + 2 (1 + 2 δ - 4 δ^2 + θ) p2 + (-1 + θ) (c + α + 2 c δ - 2 α δ +
          (c + α) θ + δ (p3 + p4 + p5 + p6))) / ((-1 + θ) (1 - 2 δ + θ) (1 + 4 δ + θ)) == 0,
   (α (-1 + 2 δ - θ) (-1 + θ) + c (1 + 2 δ - 4 δ^2 + θ) + (δ - δ θ) p1 + (δ - δ θ) p2 - 2 p3 -
       4 δ p3 + 8 δ^2 p3 - 2 θ p3 - 4 δ^2 p4 + θ p4 + 2 δ θ p4 + θ^2 p4 + δ p5 - δ θ p5 - δ (-1 + θ) p6) /
     ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ)) == 0,
   (α (-1 + 2 δ - θ) (-1 + θ) + c (1 + 2 δ - 4 δ^2 + θ) + (δ - δ θ) p1 + (δ - δ θ) p2 - 4 δ^2 p3 +
       θ p3 + 2 δ θ p3 + θ^2 p3 - 2 p4 - 4 δ p4 + 8 δ^2 p4 - 2 θ p4 + δ p5 - δ θ p5 - δ (-1 + θ) p6) /
     ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ)) == 0,
   (α (-1 + 2 δ - θ) (-1 + θ) + c (1 + 2 δ - 4 δ^2 + θ) + (δ - δ θ) p1 + (δ - δ θ) p2 + δ p3 -
       δ θ p3 + δ p4 - δ θ p4 - 2 p5 - 4 δ p5 + 8 δ^2 p5 - 2 θ p5 + (-4 δ^2 + θ + 2 δ θ + θ^2) p6) /
     ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ)) == 0,
   (c + α + 2 c δ - 2 α δ - 4 c δ^2 + c θ + 2 α δ θ - α θ^2 + (δ - δ θ) p1 + (δ - δ θ) p2 + δ p3 -
       δ θ p3 + δ p4 - δ θ p4 - 4 δ^2 p5 + θ p5 + 2 δ θ p5 + θ^2 p5 + 2 (-1 - 2 δ + 4 δ^2 - θ) p6) /
     ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ)) == 0}, {p1, p2, p3, p4, p5, p6}]]

{{p1 -> 1/2 (c + α + (4 (c - α) (-1 + δ) δ (1 + 4 δ + θ))/(-2 + 4 δ^2 + 8 δ^3 - 3 θ + θ^3 + 2 δ (-3 + θ) (1 + θ))),
  p2 -> 1/2 (c + α + (4 (c - α) (-1 + δ) δ (1 + 4 δ + θ))/(-2 + 4 δ^2 + 8 δ^3 - 3 θ + θ^3 + 2 δ (-3 + θ) (1 + θ))),
  p3 -> (-α (-1 + 2 δ - θ) (-1 + θ) (1 + 3 δ + θ) + c (-1 + δ) (1 + 2 δ + θ) (1 + 4 δ + θ)) /
    (-2 + 4 δ^2 + 8 δ^3 - 3 θ + θ^3 + 2 δ (-3 + θ) (1 + θ)),
  p4 -> (-α (-1 + 2 δ - θ) (-1 + θ) (1 + 3 δ + θ) + c (-1 + δ) (1 + 2 δ + θ) (1 + 4 δ + θ)) /
    (-2 + 4 δ^2 + 8 δ^3 - 3 θ + θ^3 + 2 δ (-3 + θ) (1 + θ)),
  p5 -> (-α (-1 + 2 δ - θ) (-1 + θ) (1 + 3 δ + θ) + c (-1 + δ) (1 + 2 δ + θ) (1 + 4 δ + θ)) /
    (-2 + 4 δ^2 + 8 δ^3 - 3 θ + θ^3 + 2 δ (-3 + θ) (1 + θ)),
  p6 -> (-α (-1 + 2 δ - θ) (-1 + θ) (1 + 3 δ + θ) + c (-1 + δ) (1 + 2 δ + θ) (1 + 4 δ + θ)) /
    (-2 + 4 δ^2 + 8 δ^3 - 3 θ + θ^3 + 2 δ (-3 + θ) (1 + θ))}}
```

$$\text{Reduce}\Big[\big(-\alpha\,(-1+2\,\delta-\theta)\,(-1+\theta)\,(1+3\,\delta+\theta)+c\,(-1+\delta)\,(1+2\,\delta+\theta)\,(1+4\,\delta+\theta)\big)\,/\\ \big(-2+4\,\delta^2+8\,\delta^3-3\,\theta+\theta^3+2\,\delta\,(-3+\theta)\,(1+\theta)\big)==\\ \big(-\alpha\,(-1+2\,\delta-\theta)\,(-1+\theta)\,(1+3\,\delta+\theta)+c\,(-1+\delta)\,(1+2\,\delta+\theta)\,(1+4\,\delta+\theta)\big)\,/\\ \big(-2+4\,\delta^2+8\,\delta^3-3\,\theta+\theta^3+2\,\delta\,(-3+\theta)\,(1+\theta)\big)==\\ \big(-\alpha\,(-1+2\,\delta-\theta)\,(-1+\theta)\,(1+3\,\delta+\theta)+c\,(-1+\delta)\,(1+2\,\delta+\theta)\,(1+4\,\delta+\theta)\big)\,/\\ \big(-2+4\,\delta^2+8\,\delta^3-3\,\theta+\theta^3+2\,\delta\,(-3+\theta)\,(1+\theta)\big)==\\ \big(-\alpha\,(-1+2\,\delta-\theta)\,(-1+\theta)\,(1+3\,\delta+\theta)+c\,(-1+\delta)\,(1+2\,\delta+\theta)\,(1+4\,\delta+\theta)\big)\,/\\ \big(-2+4\,\delta^2+8\,\delta^3-3\,\theta+\theta^3+2\,\delta\,(-3+\theta)\,(1+\theta)\big)\Big]$$

True

(*Solving for q1*)

$$\text{FullSimplify}\Bigg[\frac{1}{(-1+2\,\delta-\theta)\,(-1+\theta)\,(1+4\,\delta+\theta)}\\ \Bigg(\left(-1-2\,\delta+4\,\delta^2-\theta\right)\left(\frac{1}{2}\left(c+\alpha+\frac{4\,(c-\alpha)\,(-1+\delta)\,\delta\,(1+4\,\delta+\theta)}{-2+4\,\delta^2+8\,\delta^3-3\,\theta+\theta^3+2\,\delta\,(-3+\theta)\,(1+\theta)}\right)\right)+\\ \left(-4\,\delta^2+\theta+2\,\delta\,\theta+\theta^2\right)\left(\frac{1}{2}\left(c+\alpha+\frac{4\,(c-\alpha)\,(-1+\delta)\,\delta\,(1+4\,\delta+\theta)}{-2+4\,\delta^2+8\,\delta^3-3\,\theta+\theta^3+2\,\delta\,(-3+\theta)\,(1+\theta)}\right)\right)-\\ (-1+\theta)\,\Big(\alpha\,(1-2\,\delta+\theta)+\delta\,\Big(\big(\big(-\alpha\,(-1+2\,\delta-\theta)\,(-1+\theta)\,(1+3\,\delta+\theta)+c\,(-1+\delta)\,(1+2\,\delta+\theta)\\ (1+4\,\delta+\theta)\big)\,/\,\big(-2+4\,\delta^2+8\,\delta^3-3\,\theta+\theta^3+2\,\delta\,(-3+\theta)\,(1+\theta)\big)\big)+\\ \big(\big(-\alpha\,(-1+2\,\delta-\theta)\,(-1+\theta)\,(1+3\,\delta+\theta)+c\,(-1+\delta)\,(1+2\,\delta+\theta)\,(1+4\,\delta+\theta)\big)\,/\\ \big(-2+4\,\delta^2+8\,\delta^3-3\,\theta+\theta^3+2\,\delta\,(-3+\theta)\,(1+\theta)\big)\big)+\\ \big(\big(-\alpha\,(-1+2\,\delta-\theta)\,(-1+\theta)\,(1+3\,\delta+\theta)+c\,(-1+\delta)\,(1+2\,\delta+\theta)\,(1+4\,\delta+\theta)\big)\,/\\ \big(-2+4\,\delta^2+8\,\delta^3-3\,\theta+\theta^3+2\,\delta\,(-3+\theta)\,(1+\theta)\big)\big)+\\ \big(\big(-\alpha\,(-1+2\,\delta-\theta)\,(-1+\theta)\,(1+3\,\delta+\theta)+c\,(-1+\delta)\,(1+2\,\delta+\theta)\,(1+4\,\delta+\theta)\big)\,/\\ \big(-2+4\,\delta^2+8\,\delta^3-3\,\theta+\theta^3+2\,\delta\,(-3+\theta)\,(1+\theta)\big)\big)\Big)\Big)\Bigg)\Bigg]$$

$$-\frac{(c-\alpha)\,(1+2\,\delta+\theta)\,\left(4\,\delta^2+(-2+\theta)\,(1+\theta)+\delta\,(-6+4\,\theta)\right)}{2\,(1+4\,\delta+\theta)\,\left(-2+4\,\delta^2+8\,\delta^3-3\,\theta+\theta^3+2\,\delta\,(-3+\theta)\,(1+\theta)\right)}$$

$$\text{Reduce}\Big[\frac{(\alpha-c)\,(1+2\,\delta+\theta)\,\left(4\,\delta^2+(-2+\theta)\,(1+\theta)+\delta\,(-6+4\,\theta)\right)}{2\,(1+4\,\delta+\theta)\,\left(-2+4\,\delta^2+8\,\delta^3-3\,\theta+\theta^3+2\,\delta\,(-3+\theta)\,(1+\theta)\right)}==\\ -\frac{(c-\alpha)\,(1+2\,\delta+\theta)\,\left(4\,\delta^2+(-2+\theta)\,(1+\theta)+\delta\,(-6+4\,\theta)\right)}{2\,(1+4\,\delta+\theta)\,\left(-2+4\,\delta^2+8\,\delta^3-3\,\theta+\theta^3+2\,\delta\,(-3+\theta)\,(1+\theta)\right)}\Big]$$

True

(*solving for q3*)

```
FullSimplify[1/((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ))
  (α (-1 + 2 δ - θ) (-1 + θ) + (δ - δ θ) (1/2 (c + α + (4 (c - α) (-1 + δ) δ (1 + 4 δ + θ))/(-2 + 4 δ² + 8 δ³ - 3 θ + θ³ + 2 δ (-3 + θ) (1 + θ)))) +
    (δ - δ θ) (1/2 (c + α + (4 (c - α) (-1 + δ) δ (1 + 4 δ + θ))/(-2 + 4 δ² + 8 δ³ - 3 θ + θ³ + 2 δ (-3 + θ) (1 + θ)))) -
    ((-α (-1 + 2 δ - θ) (-1 + θ) (1 + 3 δ + θ) + c (-1 + δ) (1 + 2 δ + θ) (1 + 4 δ + θ)) /
      (-2 + 4 δ² + 8 δ³ - 3 θ + θ³ + 2 δ (-3 + θ) (1 + θ))) -
    2 δ ((-α (-1 + 2 δ - θ) (-1 + θ) (1 + 3 δ + θ) + c (-1 + δ) (1 + 2 δ + θ) (1 + 4 δ + θ)) /
       (-2 + 4 δ² + 8 δ³ - 3 θ + θ³ + 2 δ (-3 + θ) (1 + θ))) +
    4 δ² ((-α (-1 + 2 δ - θ) (-1 + θ) (1 + 3 δ + θ) + c (-1 + δ) (1 + 2 δ + θ) (1 + 4 δ + θ)) /
       (-2 + 4 δ² + 8 δ³ - 3 θ + θ³ + 2 δ (-3 + θ) (1 + θ))) -
    θ ((-α (-1 + 2 δ - θ) (-1 + θ) (1 + 3 δ + θ) + c (-1 + δ) (1 + 2 δ + θ) (1 + 4 δ + θ)) /
       (-2 + 4 δ² + 8 δ³ - 3 θ + θ³ + 2 δ (-3 + θ) (1 + θ))) -
    4 δ² ((-α (-1 + 2 δ - θ) (-1 + θ) (1 + 3 δ + θ) + c (-1 + δ) (1 + 2 δ + θ) (1 + 4 δ + θ)) /
       (-2 + 4 δ² + 8 δ³ - 3 θ + θ³ + 2 δ (-3 + θ) (1 + θ))) +
    θ ((-α (-1 + 2 δ - θ) (-1 + θ) (1 + 3 δ + θ) + c (-1 + δ) (1 + 2 δ + θ) (1 + 4 δ + θ)) /
       (-2 + 4 δ² + 8 δ³ - 3 θ + θ³ + 2 δ (-3 + θ) (1 + θ))) +
    2 δ θ ((-α (-1 + 2 δ - θ) (-1 + θ) (1 + 3 δ + θ) + c (-1 + δ) (1 + 2 δ + θ) (1 + 4 δ + θ)) /
       (-2 + 4 δ² + 8 δ³ - 3 θ + θ³ + 2 δ (-3 + θ) (1 + θ))) +
    θ² ((-α (-1 + 2 δ - θ) (-1 + θ) (1 + 3 δ + θ) + c (-1 + δ) (1 + 2 δ + θ) (1 + 4 δ + θ)) /
       (-2 + 4 δ² + 8 δ³ - 3 θ + θ³ + 2 δ (-3 + θ) (1 + θ))) +
    δ ((-α (-1 + 2 δ - θ) (-1 + θ) (1 + 3 δ + θ) + c (-1 + δ) (1 + 2 δ + θ) (1 + 4 δ + θ)) /
       (-2 + 4 δ² + 8 δ³ - 3 θ + θ³ + 2 δ (-3 + θ) (1 + θ))) -
    δ θ ((-α (-1 + 2 δ - θ) (-1 + θ) (1 + 3 δ + θ) + c (-1 + δ) (1 + 2 δ + θ) (1 + 4 δ + θ)) /
       (-2 + 4 δ² + 8 δ³ - 3 θ + θ³ + 2 δ (-3 + θ) (1 + θ))) -
    δ (-1 + θ) ((-α (-1 + 2 δ - θ) (-1 + θ) (1 + 3 δ + θ) + c (-1 + δ) (1 + 2 δ + θ) (1 + 4 δ + θ)) /
       (-2 + 4 δ² + 8 δ³ - 3 θ + θ³ + 2 δ (-3 + θ) (1 + θ))))]

-((c - α) (-1 - 2 δ + 4 δ² - θ) (1 + 3 δ + θ))/((1 + 4 δ + θ) (-2 + 4 δ² + 8 δ³ - 3 θ + θ³ + 2 δ (-3 + θ) (1 + θ)))

Reduce[((α - c) (-1 - 2 δ + 4 δ² - θ) (1 + 3 δ + θ))/((1 + 4 δ + θ) (-2 + 4 δ² + 8 δ³ - 3 θ + θ³ + 2 δ (-3 + θ) (1 + θ))) ==
  -((c - α) (-1 - 2 δ + 4 δ² - θ) (1 + 3 δ + θ))/((1 + 4 δ + θ) (-2 + 4 δ² + 8 δ³ - 3 θ + θ³ + 2 δ (-3 + θ) (1 + θ)))]

True

(*Profit monopolist 12*)
```

```
FullSimplify[((((1/2 (c + α + (4 (c - α) (-1 + δ) δ (1 + 4 δ + θ))/(-2 + 4 δ^2 + 8 δ^3 - 3 θ + θ^3 + 2 δ (-3 + θ) (1 + θ)))) - c) *
    (-(((c - α) (1 + 2 δ + θ) (4 δ^2 + (-2 + θ) (1 + θ) + δ (-6 + 4 θ))) /
        (2 (1 + 4 δ + θ) (-2 + 4 δ^2 + 8 δ^3 - 3 θ + θ^3 + 2 δ (-3 + θ) (1 + θ)))))) +
  (((1/2 (c + α + (4 (c - α) (-1 + δ) δ (1 + 4 δ + θ))/(-2 + 4 δ^2 + 8 δ^3 - 3 θ + θ^3 + 2 δ (-3 + θ) (1 + θ)))) - c) *
    (-(((c - α) (1 + 2 δ + θ) (4 δ^2 + (-2 + θ) (1 + θ) + δ (-6 + 4 θ))) /
        (2 (1 + 4 δ + θ) (-2 + 4 δ^2 + 8 δ^3 - 3 θ + θ^3 + 2 δ (-3 + θ) (1 + θ))))))]

-(((c - α)^2 (-1 + 2 δ - θ) (1 + 2 δ + θ) (4 δ^2 + (-2 + θ) (1 + θ) + δ (-6 + 4 θ))^2) /
   (2 (1 + 4 δ + θ) (-2 + 4 δ^2 + 8 δ^3 - 3 θ + θ^3 + 2 δ (-3 + θ) (1 + θ))^2))

Reduce[((c - α)^2 (1 - 2 δ + θ) (1 + 2 δ + θ) (4 δ^2 + (-2 + θ) (1 + θ) + δ (-6 + 4 θ))^2) /
    (2 (1 + 4 δ + θ) (-2 + 4 δ^2 + 8 δ^3 - 3 θ + θ^3 + 2 δ (-3 + θ) (1 + θ))^2) ==
  -(((c - α)^2 (-1 + 2 δ - θ) (1 + 2 δ + θ) (4 δ^2 + (-2 + θ) (1 + θ) + δ (-6 + 4 θ))^2) /
    (2 (1 + 4 δ + θ) (-2 + 4 δ^2 + 8 δ^3 - 3 θ + θ^3 + 2 δ (-3 + θ) (1 + θ))^2))]

True

(*PROFIT 3,4,5, or 6*)

FullSimplify[(((-α (-1 + 2 δ - θ) (-1 + θ) (1 + 3 δ + θ) + c (-1 + δ) (1 + 2 δ + θ) (1 + 4 δ + θ)) /
        (-2 + 4 δ^2 + 8 δ^3 - 3 θ + θ^3 + 2 δ (-3 + θ) (1 + θ))) - c) *
  (-((c - α) (-1 - 2 δ + 4 δ^2 - θ) (1 + 3 δ + θ))/((1 + 4 δ + θ) (-2 + 4 δ^2 + 8 δ^3 - 3 θ + θ^3 + 2 δ (-3 + θ) (1 + θ))))]

-((c - α)^2 (-1 + 2 δ - θ) (-1 - 2 δ + 4 δ^2 - θ) (-1 + θ) (1 + 3 δ + θ)^2)/((1 + 4 δ + θ) (-2 + 4 δ^2 + 8 δ^3 - 3 θ + θ^3 + 2 δ (-3 + θ) (1 + θ))^2)

(*Industry profit (Producer Surplus)*)
```

```
FullSimplify[-(((c-α)^2 (-1+2δ-θ) (1+2δ+θ) (4δ^2+(-2+θ) (1+θ)+δ (-6+4θ))^2)/
     (2 (1+4δ+θ) (-2+4δ^2+8δ^3-3θ+θ^3+2δ (-3+θ) (1+θ))^2)) +
   -(((c-α)^2 (-1+2δ-θ) (-1-2δ+4δ^2-θ) (-1+θ) (1+3δ+θ)^2)/
     ((1+4δ+θ) (-2+4δ^2+8δ^3-3θ+θ^3+2δ (-3+θ) (1+θ))^2)) +
   -(((c-α)^2 (-1+2δ-θ) (-1-2δ+4δ^2-θ) (-1+θ) (1+3δ+θ)^2)/
     ((1+4δ+θ) (-2+4δ^2+8δ^3-3θ+θ^3+2δ (-3+θ) (1+θ))^2)) +
   -(((c-α)^2 (-1+2δ-θ) (-1-2δ+4δ^2-θ) (-1+θ) (1+3δ+θ)^2)/
     ((1+4δ+θ) (-2+4δ^2+8δ^3-3θ+θ^3+2δ (-3+θ) (1+θ))^2)) +
   -(((c-α)^2 (-1+2δ-θ) (-1-2δ+4δ^2-θ) (-1+θ) (1+3δ+θ)^2)/
     ((1+4δ+θ) (-2+4δ^2+8δ^3-3θ+θ^3+2δ (-3+θ) (1+θ))^2))]

-(((c-α)^2 (-1+2δ-θ)
     (32δ^5+368δ^4 (-1+θ)+(1+θ)^3 (12+(-12+θ) θ)+2δ (1+θ)^2 (48+5 (-10+θ) θ)+
       8δ^3 (-7+34 (-1+θ) θ)+4δ^2 (1+θ) (51+2θ (-35+9θ))))/
   (2 (1+4δ+θ) (-2+4δ^2+8δ^3-3θ+θ^3+2δ (-3+θ) (1+θ))^2))

(*Consumer Surplus*)

FullSimplify[z-(1/2 (c+α+(4 (c-α) (-1+δ) δ (1+4δ+θ))/(-2+4δ^2+8δ^3-3θ+θ^3+2δ (-3+θ) (1+θ))))
    (((α-c) (1+2δ+θ) (4δ^2+(-2+θ) (1+θ)+δ (-6+4θ)))/
     (2 (1+4δ+θ) (-2+4δ^2+8δ^3-3θ+θ^3+2δ (-3+θ) (1+θ)))) -
  (1/2 (c+α+(4 (c-α) (-1+δ) δ (1+4δ+θ))/(-2+4δ^2+8δ^3-3θ+θ^3+2δ (-3+θ) (1+θ))))
    (((α-c) (1+2δ+θ) (4δ^2+(-2+θ) (1+θ)+δ (-6+4θ)))/
     (2 (1+4δ+θ) (-2+4δ^2+8δ^3-3θ+θ^3+2δ (-3+θ) (1+θ)))) -
  ((-α (-1+2δ-θ) (-1+θ) (1+3δ+θ)+c (-1+δ) (1+2δ+θ) (1+4δ+θ))/
    (-2+4δ^2+8δ^3-3θ+θ^3+2δ (-3+θ) (1+θ)))
   ((α-c) (-1-2δ+4δ^2-θ) (1+3δ+θ))/((1+4δ+θ) (-2+4δ^2+8δ^3-3θ+θ^3+2δ (-3+θ) (1+θ))) -
  ((-α (-1+2δ-θ) (-1+θ) (1+3δ+θ)+c (-1+δ) (1+2δ+θ) (1+4δ+θ))/
    (-2+4δ^2+8δ^3-3θ+θ^3+2δ (-3+θ) (1+θ)))
   ((α-c) (-1-2δ+4δ^2-θ) (1+3δ+θ))/((1+4δ+θ) (-2+4δ^2+8δ^3-3θ+θ^3+2δ (-3+θ) (1+θ))) -
  ((-α (-1+2δ-θ) (-1+θ) (1+3δ+θ)+c (-1+δ) (1+2δ+θ) (1+4δ+θ))/
    (-2+4δ^2+8δ^3-3θ+θ^3+2δ (-3+θ) (1+θ)))
   ((α-c) (-1-2δ+4δ^2-θ) (1+3δ+θ))/((1+4δ+θ) (-2+4δ^2+8δ^3-3θ+θ^3+2δ (-3+θ) (1+θ))) -
  ((-α (-1+2δ-θ) (-1+θ) (1+3δ+θ)+c (-1+δ) (1+2δ+θ) (1+4δ+θ))/
    (-2+4δ^2+8δ^3-3θ+θ^3+2δ (-3+θ) (1+θ)))
```

$$\left(\frac{(\alpha-c)\,(-1-2\,\delta+4\,\delta^2-\theta)\,(1+3\,\delta+\theta)}{(1+4\,\delta+\theta)\,(-2+4\,\delta^2+8\,\delta^3-3\,\theta+\theta^3+2\,\delta\,(-3+\theta)\,(1+\theta))}\right)+$$

$$\alpha\,((((\alpha-c)\,(1+2\,\delta+\theta)\,(4\,\delta^2+(-2+\theta)\,(1+\theta)+\delta\,(-6+4\,\theta)))\,/$$
$$(2\,(1+4\,\delta+\theta)\,(-2+4\,\delta^2+8\,\delta^3-3\,\theta+\theta^3+2\,\delta\,(-3+\theta)\,(1+\theta))))+$$
$$(((\alpha-c)\,(1+2\,\delta+\theta)\,(4\,\delta^2+(-2+\theta)\,(1+\theta)+\delta\,(-6+4\,\theta)))\,/$$
$$(2\,(1+4\,\delta+\theta)\,(-2+4\,\delta^2+8\,\delta^3-3\,\theta+\theta^3+2\,\delta\,(-3+\theta)\,(1+\theta))))+$$
$$(((\alpha-c)\,(-1-2\,\delta+4\,\delta^2-\theta)\,(1+3\,\delta+\theta))\,/\,((1+4\,\delta+\theta)\,(-2+4\,\delta^2+8\,\delta^3-3\,\theta+$$
$$\theta^3+2\,\delta\,(-3+\theta)\,(1+\theta))))+(((\alpha-c)\,(-1-2\,\delta+4\,\delta^2-\theta)\,(1+3\,\delta+\theta))\,/$$
$$((1+4\,\delta+\theta)\,(-2+4\,\delta^2+8\,\delta^3-3\,\theta+\theta^3+2\,\delta\,(-3+\theta)\,(1+\theta))))+$$
$$(((\alpha-c)\,(-1-2\,\delta+4\,\delta^2-\theta)\,(1+3\,\delta+\theta))\,/\,((1+4\,\delta+\theta)\,(-2+4\,\delta^2+8\,\delta^3-3\,\theta+$$
$$\theta^3+2\,\delta\,(-3+\theta)\,(1+\theta))))+(((\alpha-c)\,(-1-2\,\delta+4\,\delta^2-\theta)\,(1+3\,\delta+\theta))\,/$$
$$((1+4\,\delta+\theta)\,(-2+4\,\delta^2+8\,\delta^3-3\,\theta+\theta^3+2\,\delta\,(-3+\theta)\,(1+\theta)))))+$$

$$\frac{1}{2}\,(-(((\alpha-c)\,(1+2\,\delta+\theta)\,(4\,\delta^2+(-2+\theta)\,(1+\theta)+\delta\,(-6+4\,\theta)))\,/$$
$$(2\,(1+4\,\delta+\theta)\,(-2+4\,\delta^2+8\,\delta^3-3\,\theta+\theta^3+2\,\delta\,(-3+\theta)\,(1+\theta))))^2-$$
$$(((\alpha-c)\,(1+2\,\delta+\theta)\,(4\,\delta^2+(-2+\theta)\,(1+\theta)+\delta\,(-6+4\,\theta)))\,/$$
$$(2\,(1+4\,\delta+\theta)\,(-2+4\,\delta^2+8\,\delta^3-3\,\theta+\theta^3+2\,\delta\,(-3+\theta)\,(1+\theta))))^2-$$
$$(((\alpha-c)\,(-1-2\,\delta+4\,\delta^2-\theta)\,(1+3\,\delta+\theta))\,/\,((1+4\,\delta+\theta)\,(-2+4\,\delta^2+8\,\delta^3-3\,\theta+$$
$$\theta^3+2\,\delta\,(-3+\theta)\,(1+\theta))))^2-2\,\theta\,(((\alpha-c)\,(-1-2\,\delta+4\,\delta^2-\theta)\,(1+3\,\delta+\theta))\,/$$
$$((1+4\,\delta+\theta)\,(-2+4\,\delta^2+8\,\delta^3-3\,\theta+\theta^3+2\,\delta\,(-3+\theta)\,(1+\theta))))$$
$$(((\alpha-c)\,(-1-2\,\delta+4\,\delta^2-\theta)\,(1+3\,\delta+\theta))\,/\,((1+4\,\delta+\theta)\,(-2+4\,\delta^2+8\,\delta^3-3\,\theta+$$
$$\theta^3+2\,\delta\,(-3+\theta)\,(1+\theta))))-(((\alpha-c)\,(-1-2\,\delta+4\,\delta^2-\theta)\,(1+3\,\delta+\theta))\,/$$
$$((1+4\,\delta+\theta)\,(-2+4\,\delta^2+8\,\delta^3-3\,\theta+\theta^3+2\,\delta\,(-3+\theta)\,(1+\theta))))^2-$$
$$2\,\delta\,(((\alpha-c)\,(-1-2\,\delta+4\,\delta^2-\theta)\,(1+3\,\delta+\theta))\,/\,((1+4\,\delta+\theta)\,(-2+4\,\delta^2+8\,\delta^3-$$
$$3\,\theta+\theta^3+2\,\delta\,(-3+\theta)\,(1+\theta))))\,(((\alpha-c)\,(-1-2\,\delta+4\,\delta^2-\theta)\,(1+3\,\delta+\theta))\,/$$
$$((1+4\,\delta+\theta)\,(-2+4\,\delta^2+8\,\delta^3-3\,\theta+\theta^3+2\,\delta\,(-3+\theta)\,(1+\theta))))-$$
$$2\,\delta\,(((\alpha-c)\,(-1-2\,\delta+4\,\delta^2-\theta)\,(1+3\,\delta+\theta))\,/\,((1+4\,\delta+\theta)\,(-2+4\,\delta^2+8\,\delta^3-$$
$$3\,\theta+\theta^3+2\,\delta\,(-3+\theta)\,(1+\theta))))\,(((\alpha-c)\,(-1-2\,\delta+4\,\delta^2-\theta)\,(1+3\,\delta+\theta))\,/$$
$$((1+4\,\delta+\theta)\,(-2+4\,\delta^2+8\,\delta^3-3\,\theta+\theta^3+2\,\delta\,(-3+\theta)\,(1+\theta))))-$$
$$(((\alpha-c)\,(-1-2\,\delta+4\,\delta^2-\theta)\,(1+3\,\delta+\theta))\,/$$
$$((1+4\,\delta+\theta)\,(-2+4\,\delta^2+8\,\delta^3-3\,\theta+\theta^3+2\,\delta\,(-3+\theta)\,(1+\theta))))^2-$$
$$2\,(\delta\,((((\alpha-c)\,(-1-2\,\delta+4\,\delta^2-\theta)\,(1+3\,\delta+\theta))\,/\,((1+4\,\delta+\theta)\,(-2+4\,\delta^2+8\,\delta^3-3\,\theta+$$
$$\theta^3+2\,\delta\,(-3+\theta)\,(1+\theta))))+(((\alpha-c)\,(-1-2\,\delta+4\,\delta^2-\theta)\,(1+3\,\delta+\theta))\,/$$
$$((1+4\,\delta+\theta)\,(-2+4\,\delta^2+8\,\delta^3-3\,\theta+\theta^3+2\,\delta\,(-3+\theta)\,(1+\theta)))))+$$
$$\theta\,(((\alpha-c)\,(-1-2\,\delta+4\,\delta^2-\theta)\,(1+3\,\delta+\theta))\,/\,((1+4\,\delta+\theta)\,(-2+4\,\delta^2+8\,\delta^3-$$
$$3\,\theta+\theta^3+2\,\delta\,(-3+\theta)\,(1+\theta)))))\,(((\alpha-c)\,(-1-2\,\delta+4\,\delta^2-\theta)\,(1+3\,\delta+\theta))\,/$$
$$((1+4\,\delta+\theta)\,(-2+4\,\delta^2+8\,\delta^3-3\,\theta+\theta^3+2\,\delta\,(-3+\theta)\,(1+\theta))))-$$
$$(((\alpha-c)\,(-1-2\,\delta+4\,\delta^2-\theta)\,(1+3\,\delta+\theta))\,/$$
$$((1+4\,\delta+\theta)\,(-2+4\,\delta^2+8\,\delta^3-3\,\theta+\theta^3+2\,\delta\,(-3+\theta)\,(1+\theta))))^2-$$
$$2\,\delta\,(((\alpha-c)\,(1+2\,\delta+\theta)\,(4\,\delta^2+(-2+\theta)\,(1+\theta)+\delta\,(-6+4\,\theta)))\,/$$
$$(2\,(1+4\,\delta+\theta)\,(-2+4\,\delta^2+8\,\delta^3-3\,\theta+\theta^3+2\,\delta\,(-3+\theta)\,(1+\theta))))$$
$$((((\alpha-c)\,(-1-2\,\delta+4\,\delta^2-\theta)\,(1+3\,\delta+\theta))\,/\,((1+4\,\delta+\theta)\,(-2+4\,\delta^2+8\,\delta^3-3\,\theta+$$
$$\theta^3+2\,\delta\,(-3+\theta)\,(1+\theta))))+(((\alpha-c)\,(-1-2\,\delta+4\,\delta^2-\theta)\,(1+3\,\delta+\theta))\,/$$

```
((1+4δ+θ) (-2+4δ^2+8δ^3-3θ+θ^3+2δ (-3+θ) (1+θ)))) +
  (((α-c) (-1-2δ+4δ^2-θ) (1+3δ+θ)) / ((1+4δ+θ) (-2+4δ^2+8δ^3-
      3θ+θ^3+2δ (-3+θ) (1+θ)))) + (((α-c) (-1-2δ+4δ^2-θ) (1+3δ+θ)) /
   ((1+4δ+θ) (-2+4δ^2+8δ^3-3θ+θ^3+2δ (-3+θ) (1+θ))))) -
 2 ((((α-c) (1+2δ+θ) (4δ^2+(-2+θ) (1+θ)+δ (-6+4θ))) /
   (2 (1+4δ+θ) (-2+4δ^2+8δ^3-3θ+θ^3+2δ (-3+θ) (1+θ))))
  (θ (((α-c) (1+2δ+θ) (4δ^2+(-2+θ) (1+θ)+δ (-6+4θ))) /
      (2 (1+4δ+θ) (-2+4δ^2+8δ^3-3θ+θ^3+2δ (-3+θ) (1+θ)))) +
   δ ((((α-c) (-1-2δ+4δ^2-θ) (1+3δ+θ)) / ((1+4δ+θ) (-2+4δ^2+8δ^3-3θ+
          θ^3+2δ (-3+θ) (1+θ)))) + (((α-c) (-1-2δ+4δ^2-θ) (1+3δ+θ)) /
       ((1+4δ+θ) (-2+4δ^2+8δ^3-3θ+θ^3+2δ (-3+θ) (1+θ)))) +
     (((α-c) (-1-2δ+4δ^2-θ) (1+3δ+θ)) / ((1+4δ+θ) (-2+4δ^2+8δ^3-3θ+
          θ^3+2δ (-3+θ) (1+θ)))) + (((α-c) (-1-2δ+4δ^2-θ) (1+3δ+θ)) /
       ((1+4δ+θ) (-2+4δ^2+8δ^3-3θ+θ^3+2δ (-3+θ) (1+θ))))))))]

(4 z (1+4δ+θ) (-2+4δ^2+8δ^3-3θ+θ^3+2δ (-3+θ) (1+θ))^2 +
  c^2 (1+2δ+θ) (480δ^5+48δ^4 (-11+7θ)+4δ^2 (1+θ) (43+4 (-11+θ) θ)+
     6δ (1+θ)^2 (16+(-6+θ) θ)+8δ^3 (-31+10 (-6+θ) θ)+(1+θ)^3 (12+(-4+θ) θ)) -
  2 c α (1+2δ+θ) (480δ^5+48δ^4 (-11+7θ)+4δ^2 (1+θ) (43+4 (-11+θ) θ)+
     6δ (1+θ)^2 (16+(-6+θ) θ)+8δ^3 (-31+10 (-6+θ) θ)+(1+θ)^3 (12+(-4+θ) θ)) +
  α^2 (1+2δ+θ) (480δ^5+48δ^4 (-11+7θ)+4δ^2 (1+θ) (43+4 (-11+θ) θ)+
     6δ (1+θ)^2 (16+(-6+θ) θ)+8δ^3 (-31+10 (-6+θ) θ)+(1+θ)^3 (12+(-4+θ) θ))) /
 (4 (1+4δ+θ) (-2+4δ^2+8δ^3-3θ+θ^3+2δ (-3+θ) (1+θ))^2)

FullSimplify[Collect[1 / (4 (1+4δ+θ) (-2+4δ^2+8δ^3-3θ+θ^3+2δ (-3+θ) (1+θ))^2)
   (4 z (1+4δ+θ) (-2+4δ^2+8δ^3-3θ+θ^3+2δ (-3+θ) (1+θ))^2 +
    c^2 (1+2δ+θ) (480δ^5+48δ^4 (-11+7θ)+4δ^2 (1+θ) (43+4 (-11+θ) θ)+
       6δ (1+θ)^2 (16+(-6+θ) θ)+8δ^3 (-31+10 (-6+θ) θ)+(1+θ)^3 (12+(-4+θ) θ)) -
    2 c α (1+2δ+θ) (480δ^5+48δ^4 (-11+7θ)+4δ^2 (1+θ) (43+4 (-11+θ) θ)+
       6δ (1+θ)^2 (16+(-6+θ) θ)+8δ^3 (-31+10 (-6+θ) θ)+(1+θ)^3 (12+(-4+θ) θ)) +
    α^2 (1+2δ+θ) (480δ^5+48δ^4 (-11+7θ)+4δ^2 (1+θ) (43+4 (-11+θ) θ)+
       6δ (1+θ)^2 (16+(-6+θ) θ)+8δ^3 (-31+10 (-6+θ) θ)+
       (1+θ)^3 (12+(-4+θ) θ))), (1+4δ+θ)]]

z + ((c-α)^2 (1+2δ+θ) (480δ^5+48δ^4 (-11+7θ)+4δ^2 (1+θ) (43+4 (-11+θ) θ)+
     6δ (1+θ)^2 (16+(-6+θ) θ)+8δ^3 (-31+10 (-6+θ) θ)+(1+θ)^3 (12+(-4+θ) θ))) /
  (4 (1+4δ+θ) (-2+4δ^2+8δ^3-3θ+θ^3+2δ (-3+θ) (1+θ))^2)
```


```
z + ((c-α)^2 (1+2δ+θ) (480δ^5+48δ^4 (-11+7θ)+4δ^2 (1+θ) (43+4 (-11+θ) θ)+
     6δ (1+θ)^2 (16+(-6+θ) θ)+8δ^3 (-31+10 (-6+θ) θ)+(1+θ)^3 (12+(-4+θ) θ))) /
  (4 (1+4δ+θ) (-2+4δ^2+8δ^3-3θ+θ^3+2δ (-3+θ) (1+θ))^2)
```

```
Reduce[
 z + ((c - α)^2 (1 + 2 δ + θ) (480 δ^5 + 48 δ^4 (-11 + 7 θ) + 4 δ^2 (1 + θ) (43 + 4 (-11 + θ) θ) + 6 δ
          (1 + θ)^2 (16 + (-6 + θ) θ) + 8 δ^3 (-31 + 10 (-6 + θ) θ) + (1 + θ)^3 (12 + (-4 + θ) θ))) /
    (4 (1 + 4 δ + θ) (-2 + 4 δ^2 + 8 δ^3 - 3 θ + θ^3 + 2 δ (-3 + θ) (1 + θ))^2) ==
  1 / (4 (1 + 4 δ + θ) (-2 + 4 δ^2 + 8 δ^3 - 3 θ + θ^3 + 2 δ (-3 + θ) (1 + θ))^2)
   (4 z (1 + 4 δ + θ) (-2 + 4 δ^2 + 8 δ^3 - 3 θ + θ^3 + 2 δ (-3 + θ) (1 + θ))^2 +
     c^2 (1 + 2 δ + θ) (480 δ^5 + 48 δ^4 (-11 + 7 θ) + 4 δ^2 (1 + θ) (43 + 4 (-11 + θ) θ) +
        6 δ (1 + θ)^2 (16 + (-6 + θ) θ) + 8 δ^3 (-31 + 10 (-6 + θ) θ) + (1 + θ)^3 (12 + (-4 + θ) θ)) -
     2 c α (1 + 2 δ + θ) (480 δ^5 + 48 δ^4 (-11 + 7 θ) + 4 δ^2 (1 + θ) (43 + 4 (-11 + θ) θ) +
        6 δ (1 + θ)^2 (16 + (-6 + θ) θ) + 8 δ^3 (-31 + 10 (-6 + θ) θ) + (1 + θ)^3 (12 + (-4 + θ) θ)) +
     α^2 (1 + 2 δ + θ) (480 δ^5 + 48 δ^4 (-11 + 7 θ) + 4 δ^2 (1 + θ) (43 + 4 (-11 + θ) θ) +
        6 δ (1 + θ)^2 (16 + (-6 + θ) θ) + 8 δ^3 (-31 + 10 (-6 + θ) θ) + (1 + θ)^3 (12 + (-4 + θ) θ)))]

True

(*Total Surplus*)

FullSimplify[
 (((c - α)^2 (1 - 2 δ + θ) (32 δ^5 + 368 δ^4 (-1 + θ) + (1 + θ)^3 (12 + (-12 + θ) θ) + 2 δ (1 + θ)^2
          (48 + 5 (-10 + θ) θ) + 8 δ^3 (-7 + 34 (-1 + θ) θ) + 4 δ^2 (1 + θ) (51 + 2 θ (-35 + 9 θ)))) /
    (2 (1 + 4 δ + θ) (-2 + 4 δ^2 + 8 δ^3 - 3 θ + θ^3 + 2 δ (-3 + θ) (1 + θ))^2)) +
  (z + ((c - α)^2 (1 + 2 δ + θ) (480 δ^5 + 48 δ^4 (-11 + 7 θ) + 4 δ^2 (1 + θ) (43 + 4 (-11 + θ) θ) + 6 δ
           (1 + θ)^2 (16 + (-6 + θ) θ) + 8 δ^3 (-31 + 10 (-6 + θ) θ) + (1 + θ)^3 (12 + (-4 + θ) θ))) /
     (4 (1 + 4 δ + θ) (-2 + 4 δ^2 + 8 δ^3 - 3 θ + θ^3 + 2 δ (-3 + θ) (1 + θ))^2))]

z + ((c - α)^2 (832 δ^6 + 64 δ^5 (15 - 4 θ) + (1 + θ)^4 (36 + θ (-28 + 3 θ)) +
      4 δ (1 + θ)^3 (66 + θ (-49 + 6 θ)) + 16 δ^4 (-96 + θ (-4 + 9 θ)) +
      4 δ^2 (1 + θ)^2 (97 + 3 θ (-34 + 11 θ)) + 16 δ^3 (1 + θ) (-52 + θ (-16 + 23 θ)))) /
  (4 (1 + 4 δ + θ) (-2 + 4 δ^2 + 8 δ^3 - 3 θ + θ^3 + 2 δ (-3 + θ) (1 + θ))^2)

FullSimplify[
 Collect[z + ((c - α)^2 (832 δ^6 + 64 δ^5 (15 - 4 θ) + (1 + θ)^4 (36 + θ (-28 + 3 θ)) + 4 δ (1 + θ)^3
          (66 + θ (-49 + 6 θ)) + 16 δ^4 (-96 + θ (-4 + 9 θ)) +
         4 δ^2 (1 + θ)^2 (97 + 3 θ (-34 + 11 θ)) + 16 δ^3 (1 + θ) (-52 + θ (-16 + 23 θ)))) /
    (4 (1 + 4 δ + θ) (-2 + 4 δ^2 + 8 δ^3 - 3 θ + θ^3 + 2 δ (-3 + θ) (1 + θ))^2), (1 + 4 δ + θ)]]

z + ((c - α)^2 (832 δ^6 + 64 δ^5 (15 - 4 θ) + (1 + θ)^4 (36 + θ (-28 + 3 θ)) +
      4 δ (1 + θ)^3 (66 + θ (-49 + 6 θ)) + 16 δ^4 (-96 + θ (-4 + 9 θ)) +
      4 δ^2 (1 + θ)^2 (97 + 3 θ (-34 + 11 θ)) + 16 δ^3 (1 + θ) (-52 + θ (-16 + 23 θ)))) /
  (4 (1 + 4 δ + θ) (-2 + 4 δ^2 + 8 δ^3 - 3 θ + θ^3 + 2 δ (-3 + θ) (1 + θ))^2)
```

```
(*Case V: Meat Protein Monopolist*)

(*Profit max meat protein monoplist*)
FullSimplify[
 ((p1 - c) * (((-1 - 2 δ + 4 δ^2 - θ) p1 + (-4 δ^2 + θ + 2 δ θ + θ^2) p2 - (-1 + θ) (α (1 - 2 δ + θ) +
          δ (p3 + p4 + p5 + p6))) / ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ)))) +
  ((p2 - c) * (((4 δ^2 - 2 δ θ - θ (1 + θ)) p1 + (1 + 2 δ - 4 δ^2 + θ) p2 +
        (-1 + θ) (α (1 - 2 δ + θ) + δ (p3 + p4 + p5 + p6))) / ((-1 + θ) (1 - 2 δ + θ) (1 + 4 δ + θ)))) +
  ((p3 - c) * ((α (-1 + 2 δ - θ) (-1 + θ) + (δ - δ θ) p1 + (δ - δ θ) p2 - p3 - 2 δ p3 +
        4 δ^2 p3 - θ p3 - 4 δ^2 p4 + θ p4 + 2 δ θ p4 + θ^2 p4 + δ p5 - δ θ p5 - δ (-1 + θ) p6) /
      ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ)))) +
  ((p4 - c) * ((α (-1 + 2 δ - θ) (-1 + θ) + (δ - δ θ) p1 + (δ - δ θ) p2 - 4 δ^2 p3 + θ p3 +
        2 δ θ p3 + θ^2 p3 - p4 - 2 δ p4 + 4 δ^2 p4 - θ p4 + δ p5 - δ θ p5 - δ (-1 + θ) p6) /
      ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ)))) +
  ((p5 - c) * ((α (-1 + 2 δ - θ) (-1 + θ) + (δ - δ θ) p1 + (δ - δ θ) p2 + δ p3 - δ θ p3 +
        δ p4 - δ θ p4 - p5 - 2 δ p5 + 4 δ^2 p5 - θ p5 + (-4 δ^2 + θ + 2 δ θ + θ^2) p6) /
      ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ)))) + ((p6 - c) *
    ((α - 2 α δ + 2 α δ θ - α θ^2 + (δ - δ θ) p1 + (δ - δ θ) p2 + δ p3 - δ θ p3 + δ p4 - δ θ p4 - 4 δ^2 p5 +
        θ p5 + 2 δ θ p5 + θ^2 p5 + (-1 - 2 δ + 4 δ^2 - θ) p6) / ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ))))]
```

$$
-\frac{1}{(-1+2\delta-\theta)\ (-1+\theta)\ (1+4\delta+\theta)}
\Big(6\,c\,\alpha\,(-1+2\delta-\theta)\,(-1+\theta)+\left(1+2\delta-4\delta^2+\theta\right)p_1^2+\left(1+2\delta-4\delta^2+\theta\right)p_2^2-c\,p_3-\alpha\,p_3+2\,c\,\delta\,p_3+
2\,\alpha\,\delta\,p_3-2\,c\,\delta\,\theta\,p_3-2\,\alpha\,\delta\,\theta\,p_3+c\,\theta^2\,p_3+\alpha\,\theta^2\,p_3+p_3^2+2\,\delta\,p_3^2-4\,\delta^2\,p_3^2+\theta\,p_3^2-c\,p_4-\alpha\,p_4+
2\,c\,\delta\,p_4+2\,\alpha\,\delta\,p_4-2\,c\,\delta\,\theta\,p_4-2\,\alpha\,\delta\,\theta\,p_4+c\,\theta^2\,p_4+\alpha\,\theta^2\,p_4+8\,\delta^2\,p_3\,p_4-2\,\theta\,p_3\,p_4-4\,\delta\,\theta\,p_3\,p_4-
2\,\theta^2\,p_3\,p_4+p_4^2+2\,\delta\,p_4^2-4\,\delta^2\,p_4^2+\theta\,p_4^2-c\,p_5-\alpha\,p_5+2\,c\,\delta\,p_5+2\,\alpha\,\delta\,p_5-2\,c\,\delta\,\theta\,p_5-2\,\alpha\,\delta\,\theta\,p_5+
c\,\theta^2\,p_5+\alpha\,\theta^2\,p_5-2\,\delta\,p_3\,p_5+2\,\delta\,\theta\,p_3\,p_5-2\,\delta\,p_4\,p_5+2\,\delta\,\theta\,p_4\,p_5+p_5^2+2\,\delta\,p_5^2-4\,\delta^2\,p_5^2+\theta\,p_5^2-
\left((c+\alpha)\,(-1+2\delta-\theta)\,(-1+\theta)-2\,\delta\,(-1+\theta)\,(p_3+p_4)+2\left(-4\delta^2+\theta+2\delta\theta+\theta^2\right)p_5\right)p_6+
\left(1+2\delta-4\delta^2+\theta\right)p_6^2+(-1+\theta)\,p_2\,\left(-(c+\alpha)\,(-1+2\delta-\theta)+2\,\delta\,(p_3+p_4+p_5+p_6)\right)-
p_1\left(2\left(-4\delta^2+\theta+2\delta\theta+\theta^2\right)p_2-(-1+\theta)\,\left(-(c+\alpha)\,(-1+2\delta-\theta)+2\,\delta\,(p_3+p_4+p_5+p_6)\right)\right)\Big)
$$

$$
\begin{aligned}
&D\Big[-\frac{1}{(-1+2\,\delta-\theta)\,(-1+\theta)\,(1+4\,\delta+\theta)} \\
&\quad \Big(6\,c\,\alpha\,(-1+2\,\delta-\theta)\,(-1+\theta)+(1+2\,\delta-4\,\delta^2+\theta)\,p_1^2+(1+2\,\delta-4\,\delta^2+\theta)\,p_2^2-c\,p_3-\alpha\,p_3+2\,c\,\delta\,p_3+ \\
&\quad\quad 2\,\alpha\,\delta\,p_3-2\,c\,\delta\,\theta\,p_3-2\,\alpha\,\delta\,\theta\,p_3+c\,\theta^2\,p_3+\alpha\,\theta^2\,p_3+p_3^2+2\,\delta\,p_3^2-4\,\delta^2\,p_3^2+\theta\,p_3^2-c\,p_4-\alpha\,p_4+ \\
&\quad\quad 2\,c\,\delta\,p_4+2\,\alpha\,\delta\,p_4-2\,c\,\delta\,\theta\,p_4-2\,\alpha\,\delta\,\theta\,p_4+c\,\theta^2\,p_4+\alpha\,\theta^2\,p_4+8\,\delta^2\,p_3\,p_4-2\,\theta\,p_3\,p_4-4\,\delta\,\theta\,p_3\,p_4- \\
&\quad\quad 2\,\theta^2\,p_3\,p_4+p_4^2+2\,\delta\,p_4^2-4\,\delta^2\,p_4^2+\theta\,p_4^2-c\,p_5-\alpha\,p_5+2\,c\,\delta\,p_5+2\,\alpha\,\delta\,p_5-2\,c\,\delta\,\theta\,p_5-2\,\alpha\,\delta\,\theta\,p_5+ \\
&\quad\quad c\,\theta^2\,p_5+\alpha\,\theta^2\,p_5-2\,\delta\,p_3\,p_5+2\,\delta\,\theta\,p_3\,p_5-2\,\delta\,p_4\,p_5+2\,\delta\,\theta\,p_4\,p_5+p_5^2+2\,\delta\,p_5^2-4\,\delta^2\,p_5^2+\theta\,p_5^2- \\
&\quad\quad \big((c+\alpha)\,(-1+2\,\delta-\theta)\,(-1+\theta)-2\,\delta\,(-1+\theta)\,(p_3+p_4)+2\,(-4\,\delta^2+\theta+2\,\delta\,\theta+\theta^2)\,p_5\big)\,p_6+ \\
&\quad\quad (1+2\,\delta-4\,\delta^2+\theta)\,p_6^2+(-1+\theta)\,p_2\,\big(-(c+\alpha)\,(-1+2\,\delta-\theta)+2\,\delta\,(p_3+p_4+p_5+p_6)\big)- \\
&\quad\quad p_1\,\big(2\,(-4\,\delta^2+\theta+2\,\delta\,\theta+\theta^2)\,p_2-(-1+\theta)\,\big(-(c+\alpha)\,(-1+2\,\delta-\theta)+2\,\delta\,(p_3+p_4+p_5+p_6)\big)\big)\Big), \\
&\quad \{\{p_1,\,p_2,\,p_3,\,p_4,\,p_5,\,p_6\}\}\Big]
\end{aligned}
$$

$$
\begin{aligned}
&\Big\{-\Big(\big(2\,(1+2\,\delta-4\,\delta^2+\theta)\,p_1-2\,(-4\,\delta^2+\theta+2\,\delta\,\theta+\theta^2)\,p_2+(-1+\theta) \\
&\quad\quad ((-c-\alpha)\,(-1+2\,\delta-\theta)+2\,\delta\,(p_3+p_4+p_5+p_6))\big)\big/((-1+2\,\delta-\theta)\,(-1+\theta)\,(1+4\,\delta+\theta))\Big), \\
&\ -\Big(\big(-2\,(-4\,\delta^2+\theta+2\,\delta\,\theta+\theta^2)\,p_1+2\,(1+2\,\delta-4\,\delta^2+\theta)\,p_2+(-1+\theta) \\
&\quad\quad ((-c-\alpha)\,(-1+2\,\delta-\theta)+2\,\delta\,(p_3+p_4+p_5+p_6))\big)\big/((-1+2\,\delta-\theta)\,(-1+\theta)\,(1+4\,\delta+\theta))\Big), \\
&\ -\Big(\big(-c-\alpha+2\,c\,\delta+2\,\alpha\,\delta-2\,c\,\delta\,\theta-2\,\alpha\,\delta\,\theta+c\,\theta^2+\alpha\,\theta^2+2\,\delta\,(-1+\theta)\,p_1+2\,\delta\,(-1+\theta)\,p_2+ \\
&\quad\quad 2\,p_3+4\,\delta\,p_3-8\,\delta^2\,p_3+2\,\theta\,p_3+8\,\delta^2\,p_4-2\,\theta\,p_4-4\,\delta\,\theta\,p_4-2\,\theta^2\,p_4-2\,\delta\,p_5+ \\
&\quad\quad 2\,\delta\,\theta\,p_5+2\,\delta\,(-1+\theta)\,p_6\big)\big/((-1+2\,\delta-\theta)\,(-1+\theta)\,(1+4\,\delta+\theta))\Big), \\
&\ -\Big(\big(-c-\alpha+2\,c\,\delta+2\,\alpha\,\delta-2\,c\,\delta\,\theta-2\,\alpha\,\delta\,\theta+c\,\theta^2+\alpha\,\theta^2+2\,\delta\,(-1+\theta)\,p_1+2\,\delta\,(-1+\theta)\,p_2+ \\
&\quad\quad 8\,\delta^2\,p_3-2\,\theta\,p_3-4\,\delta\,\theta\,p_3-2\,\theta^2\,p_3+2\,p_4+4\,\delta\,p_4-8\,\delta^2\,p_4+2\,\theta\,p_4-2\,\delta\,p_5+ \\
&\quad\quad 2\,\delta\,\theta\,p_5+2\,\delta\,(-1+\theta)\,p_6\big)\big/((-1+2\,\delta-\theta)\,(-1+\theta)\,(1+4\,\delta+\theta))\Big), \\
&\ -\Big(\big(-c-\alpha+2\,c\,\delta+2\,\alpha\,\delta-2\,c\,\delta\,\theta-2\,\alpha\,\delta\,\theta+c\,\theta^2+\alpha\,\theta^2+2\,\delta\,(-1+\theta)\,p_1+2\,\delta\,(-1+\theta)\,p_2-2\,\delta\,p_3+ \\
&\quad\quad 2\,\delta\,\theta\,p_3-2\,\delta\,p_4+2\,\delta\,\theta\,p_4+2\,p_5+4\,\delta\,p_5-8\,\delta^2\,p_5+2\,\theta\,p_5-2\,(-4\,\delta^2+\theta+2\,\delta\,\theta+\theta^2)\,p_6\big)\big/ \\
&\quad ((-1+2\,\delta-\theta)\,(-1+\theta)\,(1+4\,\delta+\theta))\Big), \\
&\ -\Big(\big(-(c+\alpha)\,(-1+2\,\delta-\theta)\,(-1+\theta)+2\,\delta\,(-1+\theta)\,p_1+2\,\delta\,(-1+\theta)\,p_2+ \\
&\quad\quad 2\,\delta\,(-1+\theta)\,(p_3+p_4)-2\,(-4\,\delta^2+\theta+2\,\delta\,\theta+\theta^2)\,p_5+2\,(1+2\,\delta-4\,\delta^2+\theta)\,p_6\big)\big/ \\
&\quad ((-1+2\,\delta-\theta)\,(-1+\theta)\,(1+4\,\delta+\theta))\Big)\Big\}
\end{aligned}
$$

```
FullSimplify[Solve[
  {(-((2 (1 + 2 δ - 4 δ^2 + θ) p1 - 2 (-4 δ^2 + θ + 2 δ θ + θ^2) p2 + (-1 + θ) ((-c - α) (-1 + 2 δ - θ) +
          2 δ (p3 + p4 + p5 + p6)))/((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ)))) == 0,
   (-((-2 (-4 δ^2 + θ + 2 δ θ + θ^2) p1 + 2 (1 + 2 δ - 4 δ^2 + θ) p2 + (-1 + θ) ((-c - α) (-1 + 2 δ - θ) +
          2 δ (p3 + p4 + p5 + p6)))/((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ)))) == 0,
   (-((-c - α + 2 c δ + 2 α δ - 2 c δ θ - 2 α δ θ + c θ^2 + α θ^2 + 2 δ (-1 + θ) p1 + 2 δ (-1 + θ) p2 +
        2 p3 + 4 δ p3 - 8 δ^2 p3 + 2 θ p3 + 8 δ^2 p4 - 2 θ p4 - 4 δ θ p4 - 2 θ^2 p4 - 2 δ p5 +
        2 δ θ p5 + 2 δ (-1 + θ) p6)/((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ)))) == 0,
   (-((-c - α + 2 c δ + 2 α δ - 2 c δ θ - 2 α δ θ + c θ^2 + α θ^2 + 2 δ (-1 + θ) p1 + 2 δ (-1 + θ) p2 +
        8 δ^2 p3 - 2 θ p3 - 4 δ θ p3 - 2 θ^2 p3 + 2 p4 + 4 δ p4 - 8 δ^2 p4 + 2 θ p4 - 2 δ p5 +
        2 δ θ p5 + 2 δ (-1 + θ) p6)/((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ)))) == 0,
   (-((-c - α + 2 c δ + 2 α δ - 2 c δ θ - 2 α δ θ + c θ^2 + α θ^2 + 2 δ (-1 + θ) p1 + 2 δ (-1 + θ) p2 -
        2 δ p3 + 2 δ θ p3 - 2 δ p4 + 2 δ θ p4 + 2 p5 + 4 δ p5 - 8 δ^2 p5 + 2 θ p5 -
        2 (-4 δ^2 + θ + 2 δ θ + θ^2) p6)/((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ)))) == 0,
   (-((-(c + α) (-1 + 2 δ - θ) (-1 + θ) + 2 δ (-1 + θ) p1 + 2 δ (-1 + θ) p2 +
        2 δ (-1 + θ) (p3 + p4) - 2 (-4 δ^2 + θ + 2 δ θ + θ^2) p5 + 2 (1 + 2 δ - 4 δ^2 + θ) p6)/
      ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ)))) == 0}, {p1, p2, p3, p4, p5, p6}]]
```

$$\left\{\left\{p_1 \to \frac{c+\alpha}{2},\ p_2 \to \frac{c+\alpha}{2},\ p_3 \to \frac{c+\alpha}{2},\ p_4 \to \frac{c+\alpha}{2},\ p_5 \to \frac{c+\alpha}{2},\ p_6 \to \frac{c+\alpha}{2}\right\}\right\}$$

```
(*solving for q1*)

FullSimplify[((-1 - 2 δ + 4 δ^2 - θ) ((c + α)/2) + (-4 δ^2 + θ + 2 δ θ + θ^2) ((c + α)/2) -
    (-1 + θ) (α (1 - 2 δ + θ) + δ (((c + α)/2) + ((c + α)/2) + ((c + α)/2) + ((c + α)/2))))/
  ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ))]
```

$$\frac{-c+\alpha}{2\,(1+4\,\delta+\theta)}$$

```
Reduce[(α - c)/(2 (1 + 4 δ + θ)) == (-c + α)/(2 (1 + 4 δ + θ))]
```

True

```
(*profit 1*)

FullSimplify[(((c + α)/2) - c) * ((-c + α)/(2 (1 + 4 δ + θ)))]
```

$$\frac{(c-\alpha)^2}{4\,(1+4\,\delta+\theta)}$$

```
(*profit 5*)
```

```
FullSimplify[
 (((c + α)/2 - c) * ((α (-1 + 2 δ - θ) (-1 + θ) + (δ - δ θ) (c + α)/2 + (δ - δ θ) (c + α)/2 + δ (c + α)/2 -
      δ θ (c + α)/2 + δ (c + α)/2 - δ θ (c + α)/2 - (c + α)/2 - 2 δ (c + α)/2 + 4 δ^2 (c + α)/2 - θ (c + α)/2 +
      (-4 δ^2 + θ + 2 δ θ + θ^2) (c + α)/2) / ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ))))]
```

$$\frac{(c-\alpha)^2}{4\,(1+4\,\delta+\theta)}$$

```
(*PROTEIN MONOPOLIST PROFIT (Producer Surplus)*)

FullSimplify[((c - α)^2/(4 (1 + 4 δ + θ))) + ((c - α)^2/(4 (1 + 4 δ + θ))) +
  ((c - α)^2/(4 (1 + 4 δ + θ))) + ((c - α)^2/(4 (1 + 4 δ + θ))) + ((c - α)^2/(4 (1 + 4 δ + θ))) + ((c - α)^2/(4 (1 + 4 δ + θ)))]
```

$$\frac{3\,(c-\alpha)^2}{2\,(1+4\,\delta+\theta)}$$

```
Reduce[(3 (α - c)^2)/(2 (1 + 4 δ + θ)) == (3 (c - α)^2)/(2 (1 + 4 δ + θ))]
```

True

```
(*Consumer Surplus*)

FullSimplify[z - p1 q1 - p2 q2 - p3 q3 - p4 q4 - p5 q5 -
  p6 q6 + α (q1 + q2 + q3 + q4 + q5 + q6) + 1/2 (-q1^2 - q2^2 - q3^2 - q4^2 - q5^2 - q6^2 -
     2 * δ (q1 q3 + q2 q3 + q1 q4 + q2 q4 + q1 q5 + q2 q5 + q3 q5 + q4 q5 + q1 q6 + q2 q6 + q3 q6 + q4 q6) -
     2 * θ (q1 q2 + q3 q4 + q5 q6))]
```

$$z - p_1 q_1 - p_2 q_2 - p_3 q_3 - p_4 q_4 - p_5 q_5 - p_6 q_6 + \alpha\,(q_1 + q_2 + q_3 + q_4 + q_5 + q_6) + \frac{1}{2}\left(-q_1^2 - q_2^2 - q_3^2 - 2\,\theta\,q_3 q_4 - q_4^2 - 2\,\delta\,q_3 q_5 - 2\,\delta\,q_4 q_5 - q_5^2 - 2\,(\delta\,(q_3 + q_4) + \theta\,q_5)\,q_6 - q_6^2 - 2\,\delta\,q_2\,(q_3 + q_4 + q_5 + q_6) - 2\,q_1\,(\theta\,q_2 + \delta\,(q_3 + q_4 + q_5 + q_6))\right)$$

FullSimplify[

$$\begin{aligned}
&z-\left(\frac{c+\alpha}{2}\right)\left(\frac{\alpha-c}{2\,(1+4\,\delta+\theta)}\right)-\left(\frac{c+\alpha}{2}\right)\left(\frac{\alpha-c}{2\,(1+4\,\delta+\theta)}\right)-\left(\frac{c+\alpha}{2}\right)\left(\frac{\alpha-c}{2\,(1+4\,\delta+\theta)}\right)-\\
&\left(\frac{c+\alpha}{2}\right)\left(\frac{\alpha-c}{2\,(1+4\,\delta+\theta)}\right)-\left(\frac{c+\alpha}{2}\right)\left(\frac{\alpha-c}{2\,(1+4\,\delta+\theta)}\right)-\left(\frac{c+\alpha}{2}\right)\left(\frac{\alpha-c}{2\,(1+4\,\delta+\theta)}\right)+\\
&\alpha\left(\left(\frac{\alpha-c}{2\,(1+4\,\delta+\theta)}\right)+\left(\frac{\alpha-c}{2\,(1+4\,\delta+\theta)}\right)+\left(\frac{\alpha-c}{2\,(1+4\,\delta+\theta)}\right)+\left(\frac{\alpha-c}{2\,(1+4\,\delta+\theta)}\right)+\left(\frac{\alpha-c}{2\,(1+4\,\delta+\theta)}\right)+\right.\\
&\left.\left(\frac{\alpha-c}{2\,(1+4\,\delta+\theta)}\right)\right)+\frac{1}{2}\left(-\left(\frac{\alpha-c}{2\,(1+4\,\delta+\theta)}\right)^2-\left(\frac{\alpha-c}{2\,(1+4\,\delta+\theta)}\right)^2-\left(\frac{\alpha-c}{2\,(1+4\,\delta+\theta)}\right)^2-\right.\\
&2\,\theta\left(\frac{\alpha-c}{2\,(1+4\,\delta+\theta)}\right)\left(\frac{\alpha-c}{2\,(1+4\,\delta+\theta)}\right)-\left(\frac{\alpha-c}{2\,(1+4\,\delta+\theta)}\right)^2-2\,\delta\left(\frac{\alpha-c}{2\,(1+4\,\delta+\theta)}\right)\\
&\left(\frac{\alpha-c}{2\,(1+4\,\delta+\theta)}\right)-2\,\delta\left(\frac{\alpha-c}{2\,(1+4\,\delta+\theta)}\right)\left(\frac{\alpha-c}{2\,(1+4\,\delta+\theta)}\right)-\left(\frac{\alpha-c}{2\,(1+4\,\delta+\theta)}\right)^2-\\
&2\left(\delta\left(\left(\frac{\alpha-c}{2\,(1+4\,\delta+\theta)}\right)+\left(\frac{\alpha-c}{2\,(1+4\,\delta+\theta)}\right)\right)+\theta\left(\frac{\alpha-c}{2\,(1+4\,\delta+\theta)}\right)\right)\left(\frac{\alpha-c}{2\,(1+4\,\delta+\theta)}\right)-\\
&\left(\frac{\alpha-c}{2\,(1+4\,\delta+\theta)}\right)^2-2\,\delta\left(\frac{\alpha-c}{2\,(1+4\,\delta+\theta)}\right)\left(\left(\frac{\alpha-c}{2\,(1+4\,\delta+\theta)}\right)+\left(\frac{\alpha-c}{2\,(1+4\,\delta+\theta)}\right)+\right.\\
&\left.\left(\frac{\alpha-c}{2\,(1+4\,\delta+\theta)}\right)+\left(\frac{\alpha-c}{2\,(1+4\,\delta+\theta)}\right)\right)-2\left(\frac{\alpha-c}{2\,(1+4\,\delta+\theta)}\right)\left(\theta\left(\frac{\alpha-c}{2\,(1+4\,\delta+\theta)}\right)+\right.\\
&\left.\left.\delta\left(\left(\frac{\alpha-c}{2\,(1+4\,\delta+\theta)}\right)+\left(\frac{\alpha-c}{2\,(1+4\,\delta+\theta)}\right)+\left(\frac{\alpha-c}{2\,(1+4\,\delta+\theta)}\right)+\left(\frac{\alpha-c}{2\,(1+4\,\delta+\theta)}\right)\right)\right)\right)]
\end{aligned}$$

$$z+\frac{3\,(c-\alpha)^2}{4\,(1+4\,\delta+\theta)}$$

(*Total Surplus*)

FullSimplify$\left[\left(\frac{3\,(c-\alpha)^2}{2\,(1+4\,\delta+\theta)}\right)+\left(z+\frac{3\,(c-\alpha)^2}{4\,(1+4\,\delta+\theta)}\right)\right]$

$$z+\frac{9\,(c-\alpha)^2}{4\,(1+4\,\delta+\theta)}$$

(*ECONOMIES OF SCOPE*)

(*profit firm 1*)

$$\begin{aligned}
&(p_1-k-m)*\left(\left(\left(-1-2\,\delta+4\,\delta^2-\theta\right)p_1+\left(-4\,\delta^2+\theta+2\,\delta\,\theta+\theta^2\right)p_2-\right.\right.\\
&\left.\left.(-1+\theta)\,\left(\alpha\,(1-2\,\delta+\theta)+\delta\,(p_3+p_4+p_5+p_6)\right)\right)\Big/\left((-1+2\,\delta-\theta)\,(-1+\theta)\,(1+4\,\delta+\theta)\right)\right)
\end{aligned}$$

$$\begin{aligned}
&\left((-k-m+p_1)\,\left(\left(-1-2\,\delta+4\,\delta^2-\theta\right)p_1+\left(-4\,\delta^2+\theta+2\,\delta\,\theta+\theta^2\right)p_2-\right.\right.\\
&\left.\left.(-1+\theta)\,\left(\alpha\,(1-2\,\delta+\theta)+\delta\,(p_3+p_4+p_5+p_6)\right)\right)\right)\Big/\left((-1+2\,\delta-\theta)\,(-1+\theta)\,(1+4\,\delta+\theta)\right)
\end{aligned}$$

(*profit max firm 1*)

```
FullSimplify[
 D[((-k - m + p1) ((-1 - 2 δ + 4 δ^2 - θ) p1 + (-4 δ^2 + θ + 2 δ θ + θ^2) p2 - (-1 + θ) (α (1 - 2 δ + θ) +
        δ (p3 + p4 + p5 + p6)))) / ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ)), p1]]
(-(-1 - 2 δ + 4 δ^2 - θ) (k + m - p1) + (-1 - 2 δ + 4 δ^2 - θ) p1 + (-4 δ^2 + θ + 2 δ θ + θ^2) p2 -
   (-1 + θ) (α (1 - 2 δ + θ) + δ (p3 + p4 + p5 + p6))) / ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ))

(*profit firm 2*)

(p2 - k - m) * (((4 δ^2 - 2 δ θ - θ (1 + θ)) p1 + (1 + 2 δ - 4 δ^2 + θ) p2 +
      (-1 + θ) (α (1 - 2 δ + θ) + δ (p3 + p4 + p5 + p6))) / ((-1 + θ) (1 - 2 δ + θ) (1 + 4 δ + θ)))
((-k - m + p2) ((4 δ^2 - 2 δ θ - θ (1 + θ)) p1 + (1 + 2 δ - 4 δ^2 + θ) p2 +
      (-1 + θ) (α (1 - 2 δ + θ) + δ (p3 + p4 + p5 + p6)))) / ((-1 + θ) (1 - 2 δ + θ) (1 + 4 δ + θ))

(*profit max firm 2*)

FullSimplify[
 D[((-k - m + p2) ((4 δ^2 - 2 δ θ - θ (1 + θ)) p1 + (1 + 2 δ - 4 δ^2 + θ) p2 + (-1 + θ) (α (1 - 2 δ + θ) +
        δ (p3 + p4 + p5 + p6)))) / ((-1 + θ) (1 - 2 δ + θ) (1 + 4 δ + θ)), p2]]
((4 δ^2 - 2 δ θ - θ (1 + θ)) p1 + (1 + 2 δ - 4 δ^2 + θ) p2 + (1 + 2 δ - 4 δ^2 + θ) (-k - m + p2) +
   (-1 + θ) (α (1 - 2 δ + θ) + δ (p3 + p4 + p5 + p6))) / ((-1 + θ) (1 - 2 δ + θ) (1 + 4 δ + θ))

(*profit firm 3*)

FullSimplify[(p3 - k - m ) *
  ((α (-1 + 2 δ - θ) (-1 + θ) + (δ - δ θ) p1 + (δ - δ θ) p2 - p3 - 2 δ p3 + 4 δ^2 p3 - θ p3 - 4 δ^2 p4 + θ p4 +
      2 δ θ p4 + θ^2 p4 + δ p5 - δ θ p5 - δ (-1 + θ) p6) / ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ)))]
-(((k + m - p3) (α (-1 + 2 δ - θ) (-1 + θ) + (δ - δ θ) p1 + (δ - δ θ) p2 - p3 - 2 δ p3 +
       4 δ^2 p3 - θ p3 - 4 δ^2 p4 + θ p4 + 2 δ θ p4 + θ^2 p4 + δ p5 - δ θ p5 - δ (-1 + θ) p6)) /
   ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ)))

(*profit max firm 3*)

FullSimplify[
 D[((-k - m + p3) (α (-1 + 2 δ - θ) (-1 + θ) + (δ - δ θ) p1 + (δ - δ θ) p2 - p3 - 2 δ p3 + 4 δ^2 p3 -
        θ p3 - 4 δ^2 p4 + θ p4 + 2 δ θ p4 + θ^2 p4 + δ p5 - δ θ p5 - δ (-1 + θ) p6)) /
   ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ)), p3]]
(α (-1 + 2 δ - θ) (-1 + θ) + k (1 + 2 δ - 4 δ^2 + θ) + m (1 + 2 δ - 4 δ^2 + θ) + (δ - δ θ) p1 + (δ - δ θ) p2 -
   2 p3 - 4 δ p3 + 8 δ^2 p3 - 2 θ p3 - 4 δ^2 p4 + θ p4 + 2 δ θ p4 + θ^2 p4 + δ p5 - δ θ p5 - δ (-1 + θ) p6) /
 ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ))

(*profit firm 4*)
```

```
(p4 - k - m) *
 ((α (-1 + 2 δ - θ) (-1 + θ) + (δ - δ θ) p1 + (δ - δ θ) p2 - 4 δ^2 p3 + θ p3 + 2 δ θ p3 + θ^2 p3 - p4 - 2 δ p4 +
     4 δ^2 p4 - θ p4 + δ p5 - δ θ p5 - δ (-1 + θ) p6) / ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ)))
((-k - m + p4)
   (α (-1 + 2 δ - θ) (-1 + θ) + (δ - δ θ) p1 + (δ - δ θ) p2 - 4 δ^2 p3 + θ p3 + 2 δ θ p3 + θ^2 p3 - p4 -
     2 δ p4 + 4 δ^2 p4 - θ p4 + δ p5 - δ θ p5 - δ (-1 + θ) p6)) / ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ))

(*profit max firm 4*)

FullSimplify[
 D[((-k - m + p4) (α (-1 + 2 δ - θ) (-1 + θ) + (δ - δ θ) p1 + (δ - δ θ) p2 - 4 δ^2 p3 + θ p3 +
        2 δ θ p3 + θ^2 p3 - p4 - 2 δ p4 + 4 δ^2 p4 - θ p4 + δ p5 - δ θ p5 - δ (-1 + θ) p6)) /
    ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ)), p4]]
(α (-1 + 2 δ - θ) (-1 + θ) + k (1 + 2 δ - 4 δ^2 + θ) + m (1 + 2 δ - 4 δ^2 + θ) + (δ - δ θ) p1 + (δ - δ θ) p2 -
   4 δ^2 p3 + θ p3 + 2 δ θ p3 + θ^2 p3 - 2 p4 - 4 δ p4 + 8 δ^2 p4 - 2 θ p4 + δ p5 - δ θ p5 - δ (-1 + θ) p6) /
 ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ))

(*profit firm 5*)

(p5 - k - m) *
 ((α (-1 + 2 δ - θ) (-1 + θ) + (δ - δ θ) p1 + (δ - δ θ) p2 + δ p3 - δ θ p3 + δ p4 - δ θ p4 - p5 - 2 δ p5 +
     4 δ^2 p5 - θ p5 + (-4 δ^2 + θ + 2 δ θ + θ^2) p6) / ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ)))
((-k - m + p5) (α (-1 + 2 δ - θ) (-1 + θ) + (δ - δ θ) p1 + (δ - δ θ) p2 + δ p3 - δ θ p3 + δ p4 - δ θ p4 - p5 -
     2 δ p5 + 4 δ^2 p5 - θ p5 + (-4 δ^2 + θ + 2 δ θ + θ^2) p6)) / ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ))

(*profit max firm 5*)

FullSimplify[
 D[((-k - m + p5) (α (-1 + 2 δ - θ) (-1 + θ) + (δ - δ θ) p1 + (δ - δ θ) p2 + δ p3 - δ θ p3 +
        δ p4 - δ θ p4 - p5 - 2 δ p5 + 4 δ^2 p5 - θ p5 + (-4 δ^2 + θ + 2 δ θ + θ^2) p6)) /
    ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ)), p5]]
(α (-1 + 2 δ - θ) (-1 + θ) + k (1 + 2 δ - 4 δ^2 + θ) + m (1 + 2 δ - 4 δ^2 + θ) + (δ - δ θ) p1 + (δ - δ θ) p2 +
   δ p3 - δ θ p3 + δ p4 - δ θ p4 - 2 p5 - 4 δ p5 + 8 δ^2 p5 - 2 θ p5 + (-4 δ^2 + θ + 2 δ θ + θ^2) p6) /
 ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ))

(*profit firm 6*)

FullSimplify[(p6 - k - m) *
  ((α - 2 α δ + 2 α δ θ - α θ^2 + (δ - δ θ) p1 + (δ - δ θ) p2 + δ p3 - δ θ p3 + δ p4 - δ θ p4 - 4 δ^2 p5 + θ p5 +
      2 δ θ p5 + θ^2 p5 + (-1 - 2 δ + 4 δ^2 - θ) p6) / ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ)))]
-(((k + m - p6)
     (α - 2 α δ + 2 α δ θ - α θ^2 + (δ - δ θ) p1 + (δ - δ θ) p2 + δ p3 - δ θ p3 + δ p4 - δ θ p4 - 4 δ^2 p5 +
       θ p5 + 2 δ θ p5 + θ^2 p5 + (-1 - 2 δ + 4 δ^2 - θ) p6)) / ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ)))
```

```
FullSimplify[
 D[-(((k + m - p6) (α - 2 α δ + 2 α δ θ - α θ^2 + (δ - δ θ) p1 + (δ - δ θ) p2 + δ p3 - δ θ p3 + δ p4 -
        δ θ p4 - 4 δ^2 p5 + θ p5 + 2 δ θ p5 + θ^2 p5 + (-1 - 2 δ + 4 δ^2 - θ) p6)) /
     ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ))), p6]]
(k + m + α + 2 k δ + 2 m δ - 2 α δ - 4 k δ^2 - 4 m δ^2 + k θ + m θ + 2 α δ θ - α θ^2 + (δ - δ θ) p1 + (δ - δ θ) p2 +
   δ p3 - δ θ p3 + δ p4 - δ θ p4 - 4 δ^2 p5 + θ p5 + 2 δ θ p5 + θ^2 p5 + 2 (-1 - 2 δ + 4 δ^2 - θ) p6) /
 ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ))

(*premerger prices*)

FullSimplify[
 Solve[{(-(-1 - 2 δ + 4 δ^2 - θ) (k + m - p1) + (-1 - 2 δ + 4 δ^2 - θ) p1 + (-4 δ^2 + θ + 2 δ θ + θ^2) p2 -
        (-1 + θ) (α (1 - 2 δ + θ) + δ (p3 + p4 + p5 + p6))) / ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ)) ==
     0, ((4 δ^2 - 2 δ θ - θ (1 + θ)) p1 + (1 + 2 δ - 4 δ^2 + θ) p2 + (1 + 2 δ - 4 δ^2 + θ) (-k - m + p2) +
        (-1 + θ) (α (1 - 2 δ + θ) + δ (p3 + p4 + p5 + p6))) / ((-1 + θ) (1 - 2 δ + θ) (1 + 4 δ + θ)) == 0,
    (α (-1 + 2 δ - θ) (-1 + θ) + k (1 + 2 δ - 4 δ^2 + θ) + m (1 + 2 δ - 4 δ^2 + θ) + (δ - δ θ) p1 +
        (δ - δ θ) p2 - 2 p3 - 4 δ p3 + 8 δ^2 p3 - 2 θ p3 - 4 δ^2 p4 + θ p4 + 2 δ θ p4 + θ^2 p4 +
        δ p5 - δ θ p5 - δ (-1 + θ) p6) / ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ)) == 0,
    (α (-1 + 2 δ - θ) (-1 + θ) + k (1 + 2 δ - 4 δ^2 + θ) + m (1 + 2 δ - 4 δ^2 + θ) + (δ - δ θ) p1 +
        (δ - δ θ) p2 - 4 δ^2 p3 + θ p3 + 2 δ θ p3 + θ^2 p3 - 2 p4 - 4 δ p4 + 8 δ^2 p4 - 2 θ p4 +
        δ p5 - δ θ p5 - δ (-1 + θ) p6) / ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ)) == 0,
    (α (-1 + 2 δ - θ) (-1 + θ) + k (1 + 2 δ - 4 δ^2 + θ) + m (1 + 2 δ - 4 δ^2 + θ) + (δ - δ θ) p1 +
        (δ - δ θ) p2 + δ p3 - δ θ p3 + δ p4 - δ θ p4 - 2 p5 - 4 δ p5 + 8 δ^2 p5 - 2 θ p5 +
        (-4 δ^2 + θ + 2 δ θ + θ^2) p6) / ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ)) == 0,
    (k + m + α + 2 k δ + 2 m δ - 2 α δ - 4 k δ^2 - 4 m δ^2 + k θ + m θ + 2 α δ θ - α θ^2 + (δ - δ θ) p1 + (δ - δ θ)
         p2 + δ p3 - δ θ p3 + δ p4 - δ θ p4 - 4 δ^2 p5 + θ p5 + 2 δ θ p5 + θ^2 p5 + 2 (-1 - 2 δ + 4 δ^2 - θ) p6) /
      ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ)) == 0}, {p1, p2, p3, p4, p5, p6}]]
{{p1 → (α (-1 + 2 δ - θ) (-1 + θ) + k (1 + 2 δ - 4 δ^2 + θ) + m (1 + 2 δ - 4 δ^2 + θ)) /
     (2 - 4 δ^2 + θ + 2 δ θ - θ^2),
  p2 → -((α (-1 + 2 δ - θ) (-1 + θ) + k (1 + 2 δ - 4 δ^2 + θ) + m (1 + 2 δ - 4 δ^2 + θ)) /
      (4 δ^2 - 2 δ θ + (-2 + θ) (1 + θ))),
  p3 → -((α (-1 + 2 δ - θ) (-1 + θ) + k (1 + 2 δ - 4 δ^2 + θ) + m (1 + 2 δ - 4 δ^2 + θ)) /
      (4 δ^2 - 2 δ θ + (-2 + θ) (1 + θ))),
  p4 → -((α (-1 + 2 δ - θ) (-1 + θ) + k (1 + 2 δ - 4 δ^2 + θ) + m (1 + 2 δ - 4 δ^2 + θ)) /
      (4 δ^2 - 2 δ θ + (-2 + θ) (1 + θ))),
  p5 → -((α (-1 + 2 δ - θ) (-1 + θ) + k (1 + 2 δ - 4 δ^2 + θ) + m (1 + 2 δ - 4 δ^2 + θ)) /
      (4 δ^2 - 2 δ θ + (-2 + θ) (1 + θ))),
  p6 → -((α (-1 + 2 δ - θ) (-1 + θ) + k (1 + 2 δ - 4 δ^2 + θ) + m (1 + 2 δ - 4 δ^2 + θ)) /
      (4 δ^2 - 2 δ θ + (-2 + θ) (1 + θ)))}}

(*premerger q1*)
```

```
FullSimplify[
 ((-1 - 2 δ + 4 δ^2 - θ) ((α (-1 + 2 δ - θ) (-1 + θ) + k (1 + 2 δ - 4 δ^2 + θ) + m (1 + 2 δ - 4 δ^2 + θ)) /
       (2 - 4 δ^2 + θ + 2 δ θ - θ^2)) + (-4 δ^2 + θ + 2 δ θ + θ^2)
     ((α (-1 + 2 δ - θ) (-1 + θ) + k (1 + 2 δ - 4 δ^2 + θ) + m (1 + 2 δ - 4 δ^2 + θ)) /
       (2 - 4 δ^2 + θ + 2 δ θ - θ^2)) - (-1 + θ)
     (α (1 - 2 δ + θ) + δ (((α (-1 + 2 δ - θ) (-1 + θ) + k (1 + 2 δ - 4 δ^2 + θ) + m (1 + 2 δ - 4 δ^2 + θ)) /
            (2 - 4 δ^2 + θ + 2 δ θ - θ^2)) + ((α (-1 + 2 δ - θ) (-1 + θ) + k (1 + 2 δ - 4 δ^2 + θ) +
              m (1 + 2 δ - 4 δ^2 + θ)) / (2 - 4 δ^2 + θ + 2 δ θ - θ^2)) + ((α (-1 + 2 δ - θ) (-1 + θ) +
              k (1 + 2 δ - 4 δ^2 + θ) + m (1 + 2 δ - 4 δ^2 + θ)) / (2 - 4 δ^2 + θ + 2 δ θ - θ^2)) +
          ((α (-1 + 2 δ - θ) (-1 + θ) + k (1 + 2 δ - 4 δ^2 + θ) + m (1 + 2 δ - 4 δ^2 + θ)) /
            (2 - 4 δ^2 + θ + 2 δ θ - θ^2))))) / ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ))]
```

$$-\frac{(k+m-\alpha)\left(-1-2\delta+4\delta^2-\theta\right)}{(1+4\delta+\theta)\left(4\delta^2-2\delta\theta+(-2+\theta)(1+\theta)\right)}$$

```
Reduce[
 (α - k - m) (-1 - 2 δ + 4 δ^2 - θ) / ((1 + 4 δ + θ) (4 δ^2 - 2 δ θ + (-2 + θ) (1 + θ))) ==
  -(k + m - α) (-1 - 2 δ + 4 δ^2 - θ) / ((1 + 4 δ + θ) (4 δ^2 - 2 δ θ + (-2 + θ) (1 + θ)))]
```

True

```
(* ********************************************************************************
 *******************)

(*profit multiproduct f1 f3 f5*)

FullSimplify[
 ((p1 - k - λ*m) * (((-1 - 2 δ + 4 δ^2 - θ) p1 + (-4 δ^2 + θ + 2 δ θ + θ^2) p2 - (-1 + θ) (α (1 - 2 δ + θ) +
            δ (p3 + p4 + p5 + p6))) / ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ)))) +
  ((p3 - k - λ*m) * ((α (-1 + 2 δ - θ) (-1 + θ) + (δ - δ θ) p1 + (δ - δ θ) p2 - p3 - 2 δ p3 +
          4 δ^2 p3 - θ p3 - 4 δ^2 p4 + θ p4 + 2 δ θ p4 + θ^2 p4 + δ p5 - δ θ p5 - δ (-1 + θ) p6) /
        ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ)))) + ((p5 - k - λ*m) *
     ((α (-1 + 2 δ - θ) (-1 + θ) + (δ - δ θ) p1 + (δ - δ θ) p2 + δ p3 - δ θ p3 + δ p4 - δ θ p4 - p5 - 2 δ p5 +
          4 δ^2 p5 - θ p5 + (-4 δ^2 + θ + 2 δ θ + θ^2) p6) / ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ))))]
```

$$\frac{1}{(-1+2\delta-\theta)(-1+\theta)(1+4\delta+\theta)}\Big((-k-m\lambda+p_3)\big(\alpha(-1+2\delta-\theta)(-1+\theta)+(\delta-\delta\theta)p_1+(\delta-\delta\theta)p_2-p_3-2\delta p_3+4\delta^2 p_3-\theta p_3-4\delta^2 p_4+\theta p_4+2\delta\theta p_4+\theta^2 p_4+\delta p_5-\delta\theta p_5-\delta(-1+\theta)p_6\big)+(-k-m\lambda+p_5)\big(\alpha(-1+2\delta-\theta)(-1+\theta)+(\delta-\delta\theta)p_1+(\delta-\delta\theta)p_2+\delta p_3-\delta\theta p_3+\delta p_4-\delta\theta p_4-p_5-2\delta p_5+4\delta^2 p_5-\theta p_5+\left(-4\delta^2+\theta+2\delta\theta+\theta^2\right)p_6\big)+(-k-m\lambda+p_1)\Big(\left(-1-2\delta+4\delta^2-\theta\right)p_1+\left(-4\delta^2+\theta+2\delta\theta+\theta^2\right)p_2-(-1+\theta)\left(\alpha(1-2\delta+\theta)+\delta(p_3+p_4+p_5+p_6)\right)\Big)\Big)$$

```
(*joint profit max multiproduct f1 f3 f5*)
FullSimplify[D[1/((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ))
    ((-k - m λ + p3) (α (-1 + 2 δ - θ) (-1 + θ) + (δ - δ θ) p1 + (δ - δ θ) p2 - p3 - 2 δ p3 +
          4 δ^2 p3 - θ p3 - 4 δ^2 p4 + θ p4 + 2 δ θ p4 + θ^2 p4 + δ p5 - δ θ p5 - δ (-1 + θ) p6) +
      (-k - m λ + p5) (α (-1 + 2 δ - θ) (-1 + θ) + (δ - δ θ) p1 + (δ - δ θ) p2 + δ p3 -
          δ θ p3 + δ p4 - δ θ p4 - p5 - 2 δ p5 + 4 δ^2 p5 - θ p5 + (-4 δ^2 + θ + 2 δ θ + θ^2) p6) +
      (-k - m λ + p1) ((-1 - 2 δ + 4 δ^2 - θ) p1 + (-4 δ^2 + θ + 2 δ θ + θ^2) p2 -
          (-1 + θ) (α (1 - 2 δ + θ) + δ (p3 + p4 + p5 + p6)))), p1]]
(-(-1 + 2 δ - θ) (k + α + 2 k δ - α θ + m λ + 2 m δ λ) + (-4 δ + 8 δ^2 - 2 (1 + θ)) p1 +
    (-4 δ^2 + θ + 2 δ θ + θ^2) p2 - δ (-1 + θ) (2 p3 + p4 + 2 p5 + p6)) /
  ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ))
```

```
FullSimplify[D[1/((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ))
    ((-k - m λ + p3) (α (-1 + 2 δ - θ) (-1 + θ) + (δ - δ θ) p1 + (δ - δ θ) p2 - p3 - 2 δ p3 +
          4 δ^2 p3 - θ p3 - 4 δ^2 p4 + θ p4 + 2 δ θ p4 + θ^2 p4 + δ p5 - δ θ p5 - δ (-1 + θ) p6) +
      (-k - m λ + p5) (α (-1 + 2 δ - θ) (-1 + θ) + (δ - δ θ) p1 + (δ - δ θ) p2 + δ p3 -
          δ θ p3 + δ p4 - δ θ p4 - p5 - 2 δ p5 + 4 δ^2 p5 - θ p5 + (-4 δ^2 + θ + 2 δ θ + θ^2) p6) +
      (-k - m λ + p1) ((-1 - 2 δ + 4 δ^2 - θ) p1 + (-4 δ^2 + θ + 2 δ θ + θ^2) p2 -
          (-1 + θ) (α (1 - 2 δ + θ) + δ (p3 + p4 + p5 + p6)))), p3]]
(-(-1 + 2 δ - θ) (k + α + 2 k δ - α θ + m λ + 2 m δ λ) - 2 δ (-1 + θ) p1 + (δ - δ θ) p2 - 2 p3 - 4 δ p3 +
    8 δ^2 p3 - 2 θ p3 - 4 δ^2 p4 + θ p4 + 2 δ θ p4 + θ^2 p4 + 2 δ p5 - 2 δ θ p5 - δ (-1 + θ) p6) /
  ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ))
```

```
FullSimplify[D[1/((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ))
    ((-k - m λ + p3) (α (-1 + 2 δ - θ) (-1 + θ) + (δ - δ θ) p1 + (δ - δ θ) p2 - p3 - 2 δ p3 +
          4 δ^2 p3 - θ p3 - 4 δ^2 p4 + θ p4 + 2 δ θ p4 + θ^2 p4 + δ p5 - δ θ p5 - δ (-1 + θ) p6) +
      (-k - m λ + p5) (α (-1 + 2 δ - θ) (-1 + θ) + (δ - δ θ) p1 + (δ - δ θ) p2 + δ p3 -
          δ θ p3 + δ p4 - δ θ p4 - p5 - 2 δ p5 + 4 δ^2 p5 - θ p5 + (-4 δ^2 + θ + 2 δ θ + θ^2) p6) +
      (-k - m λ + p1) ((-1 - 2 δ + 4 δ^2 - θ) p1 + (-4 δ^2 + θ + 2 δ θ + θ^2) p2 -
          (-1 + θ) (α (1 - 2 δ + θ) + δ (p3 + p4 + p5 + p6)))), p5]]
(-(-1 + 2 δ - θ) (k + α + 2 k δ - α θ + m λ + 2 m δ λ) - 2 δ (-1 + θ) p1 + (δ - δ θ) p2 + 2 δ p3 -
    2 δ θ p3 + δ p4 - δ θ p4 - 2 p5 - 4 δ p5 + 8 δ^2 p5 - 2 θ p5 + (-4 δ^2 + θ + 2 δ θ + θ^2) p6) /
  ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ))
```

```
(*Postmerger prices: f135, f2, f4, f6*)
```

```
FullSimplify[
 Solve[{(-(-1 + 2 δ - θ) (k + α + 2 k δ - α θ + m λ + 2 m δ λ) + (-4 δ + 8 δ^2 - 2 (1 + θ)) p1 +
       (-4 δ^2 + θ + 2 δ θ + θ^2) p2 - δ (-1 + θ) (2 p3 + p4 + 2 p5 + p6)) /
     ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ)) == 0,
   ((4 δ^2 - 2 δ θ - θ (1 + θ)) p1 + (1 + 2 δ - 4 δ^2 + θ) p2 + (1 + 2 δ - 4 δ^2 + θ) (-k - m + p2) +
       (-1 + θ) (α (1 - 2 δ + θ) + δ (p3 + p4 + p5 + p6))) / ((-1 + θ) (1 - 2 δ + θ) (1 + 4 δ + θ)) == 0,
   (-(-1 + 2 δ - θ) (k + α + 2 k δ - α θ + m λ + 2 m δ λ) - 2 δ (-1 + θ) p1 + (δ - δ θ) p2 -
       2 p3 - 4 δ p3 + 8 δ^2 p3 - 2 θ p3 - 4 δ^2 p4 + θ p4 + 2 δ θ p4 + θ^2 p4 + 2 δ p5 -
       2 δ θ p5 - δ (-1 + θ) p6) / ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ)) == 0,
   (α (-1 + 2 δ - θ) (-1 + θ) + k (1 + 2 δ - 4 δ^2 + θ) + m (1 + 2 δ - 4 δ^2 + θ) + (δ - δ θ) p1 +
       (δ - δ θ) p2 - 4 δ^2 p3 + θ p3 + 2 δ θ p3 + θ^2 p3 - 2 p4 - 4 δ p4 + 8 δ^2 p4 - 2 θ p4 +
       δ p5 - δ θ p5 - δ (-1 + θ) p6) / ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ)) == 0,
   (-(-1 + 2 δ - θ) (k + α + 2 k δ - α θ + m λ + 2 m δ λ) - 2 δ (-1 + θ) p1 + (δ - δ θ) p2 +
       2 δ p3 - 2 δ θ p3 + δ p4 - δ θ p4 - 2 p5 - 4 δ p5 + 8 δ^2 p5 - 2 θ p5 + (-4 δ^2 + θ + 2 δ θ + θ^2) p6) /
     ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ)) == 0,
   (k + m + α + 2 k δ + 2 m δ - 2 α δ - 4 k δ^2 - 4 m δ^2 + k θ + m θ + 2 α δ θ - α θ^2 + (δ - δ θ) p1 + (δ - δ θ)
        p2 + δ p3 - δ θ p3 + δ p4 - δ θ p4 - 4 δ^2 p5 + θ p5 + 2 δ θ p5 + θ^2 p5 + 2 (-1 - 2 δ + 4 δ^2 - θ) p6) /
     ((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ)) == 0}, {p1, p2, p3, p4, p5, p6}]]

{{p1 →
   (α (-1 + θ) (-12 δ^2 + 2 δ (2 + θ) + (1 + θ) (2 + θ)) + k (24 δ^3 - (1 + θ) (2 + θ) - 2 δ (4 + 5 θ)) +
      m (8 δ^3 (1 + 2 λ) - (1 + θ) (θ + 2 λ) + 4 δ^2 (-1 + θ + λ - θ λ) - 2 δ (1 + 2 θ + 3 (1 + θ) λ))) /
    (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ)),
  p2 → (-α (-1 + 2 δ - θ) (-1 + θ) (2 + 6 δ + θ) + k (1 + 2 δ) (6 δ (-1 + 2 δ) - (1 + θ) (2 + θ)) +
      m (1 + 2 δ) (-2 δ (2 + λ) + 4 δ^2 (2 + λ) - (1 + θ) (2 + θ λ))) /
    (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ)), p3 →
   (α (-1 + θ) (-12 δ^2 + 2 δ (2 + θ) + (1 + θ) (2 + θ)) + k (24 δ^3 - (1 + θ) (2 + θ) - 2 δ (4 + 5 θ)) +
      m (8 δ^3 (1 + 2 λ) - (1 + θ) (θ + 2 λ) + 4 δ^2 (-1 + θ + λ - θ λ) - 2 δ (1 + 2 θ + 3 (1 + θ) λ))) /
    (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ)),
  p4 → (-α (-1 + 2 δ - θ) (-1 + θ) (2 + 6 δ + θ) + k (1 + 2 δ) (6 δ (-1 + 2 δ) - (1 + θ) (2 + θ)) +
      m (1 + 2 δ) (-2 δ (2 + λ) + 4 δ^2 (2 + λ) - (1 + θ) (2 + θ λ))) /
    (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ)), p5 →
   (α (-1 + θ) (-12 δ^2 + 2 δ (2 + θ) + (1 + θ) (2 + θ)) + k (24 δ^3 - (1 + θ) (2 + θ) - 2 δ (4 + 5 θ)) +
      m (8 δ^3 (1 + 2 λ) - (1 + θ) (θ + 2 λ) + 4 δ^2 (-1 + θ + λ - θ λ) - 2 δ (1 + 2 θ + 3 (1 + θ) λ))) /
    (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ)),
  p6 → (-α (-1 + 2 δ - θ) (-1 + θ) (2 + 6 δ + θ) + k (1 + 2 δ) (6 δ (-1 + 2 δ) - (1 + θ) (2 + θ)) +
      m (1 + 2 δ) (-2 δ (2 + λ) + 4 δ^2 (2 + λ) - (1 + θ) (2 + θ λ))) /
    (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))}}

(*solving for q1*)
```

```
Simplify[        1
         ─────────────────────────────────────
         (-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ)
  ((-1 - 2 δ + 4 δ² - θ) ((α (-1 + θ) (-12 δ² + 2 δ (2 + θ) + (1 + θ) (2 + θ)) +
          k (24 δ³ - (1 + θ) (2 + θ) - 2 δ (4 + 5 θ)) +
          m (8 δ³ (1 + 2 λ) - (1 + θ) (θ + 2 λ) + 4 δ² (-1 + θ + λ - θ λ) - 2 δ (1 + 2 θ + 3 (1 + θ) λ))) /
       (24 δ³ - 12 δ² (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))) +
    (-4 δ² + θ + 2 δ θ + θ²) ((-α (-1 + 2 δ - θ) (-1 + θ) (2 + 6 δ + θ) + k (1 + 2 δ) (6 δ (-1 + 2 δ) -
            (1 + θ) (2 + θ)) + m (1 + 2 δ) (-2 δ (2 + λ) + 4 δ² (2 + λ) - (1 + θ) (2 + θ λ))) /
       (24 δ³ - 12 δ² (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))) -
    (-1 + θ) (α (1 - 2 δ + θ) + δ (((α (-1 + θ) (-12 δ² + 2 δ (2 + θ) + (1 + θ) (2 + θ)) +
              k (24 δ³ - (1 + θ) (2 + θ) - 2 δ (4 + 5 θ)) + m (8 δ³ (1 + 2 λ) -
                 (1 + θ) (θ + 2 λ) + 4 δ² (-1 + θ + λ - θ λ) - 2 δ (1 + 2 θ + 3 (1 + θ) λ))) /
            (24 δ³ - 12 δ² (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))) +
          ((-α (-1 + 2 δ - θ) (-1 + θ) (2 + 6 δ + θ) + k (1 + 2 δ) (6 δ (-1 + 2 δ) - (1 + θ) (2 + θ)) +
              m (1 + 2 δ) (-2 δ (2 + λ) + 4 δ² (2 + λ) - (1 + θ) (2 + θ λ))) /
            (24 δ³ - 12 δ² (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))) +
          ((α (-1 + θ) (-12 δ² + 2 δ (2 + θ) + (1 + θ) (2 + θ)) +
              k (24 δ³ - (1 + θ) (2 + θ) - 2 δ (4 + 5 θ)) + m (8 δ³ (1 + 2 λ) -
                 (1 + θ) (θ + 2 λ) + 4 δ² (-1 + θ + λ - θ λ) - 2 δ (1 + 2 θ + 3 (1 + θ) λ))) /
            (24 δ³ - 12 δ² (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))) +
          ((-α (-1 + 2 δ - θ) (-1 + θ) (2 + 6 δ + θ) + k (1 + 2 δ) (6 δ (-1 + 2 δ) - (1 + θ) (2 + θ)) +
              m (1 + 2 δ) (-2 δ (2 + λ) + 4 δ² (2 + λ) - (1 + θ) (2 + θ λ))) /
            (24 δ³ - 12 δ² (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))))))]

((1 + 2 δ)
    (k (-1 + θ) (2 - 12 δ² + 3 θ + θ² + 2 δ (2 + θ)) - α (-1 + θ) (2 - 12 δ² + 3 θ + θ² + 2 δ (2 + θ)) +
      m (8 δ³ (-1 + λ) - 4 δ² (-1 + θ) (1 + 2 λ) + (1 + θ) (θ - 2 λ + θ² λ) +
          2 δ (1 + 2 θ - 3 λ - θ λ + θ² λ)))) /
 ((-1 + θ) (1 + 4 δ + θ) (-4 + 24 δ³ - 12 δ² (-1 + θ) - 4 θ + θ² + θ³ + 2 δ (-6 - 4 θ + θ²)))

(*solving for q2*)
```

```
FullSimplify[1/((-1 + θ) (1 - 2 δ + θ) (1 + 4 δ + θ))
  ((4 δ^2 - 2 δ θ - θ (1 + θ)) ((α (-1 + θ) (-12 δ^2 + 2 δ (2 + θ) + (1 + θ) (2 + θ)) +
          k (24 δ^3 - (1 + θ) (2 + θ) - 2 δ (4 + 5 θ)) + m
            (8 δ^3 (1 + 2 λ) - (1 + θ) (θ + 2 λ) + 4 δ^2 (-1 + θ + λ - θ λ) - 2 δ (1 + 2 θ + 3 (1 + θ) λ))) /
        (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))) +
    (1 + 2 δ - 4 δ^2 + θ) ((-α (-1 + 2 δ - θ) (-1 + θ) (2 + 6 δ + θ) + k (1 + 2 δ) (6 δ (-1 + 2 δ) -
              (1 + θ) (2 + θ)) + m (1 + 2 δ) (-2 δ (2 + λ) + 4 δ^2 (2 + λ) - (1 + θ) (2 + θ λ))) /
        (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))) +
    (-1 + θ) (α (1 - 2 δ + θ) + δ (((α (-1 + θ) (-12 δ^2 + 2 δ (2 + θ) + (1 + θ) (2 + θ)) +
                k (24 δ^3 - (1 + θ) (2 + θ) - 2 δ (4 + 5 θ)) + m (8 δ^3 (1 + 2 λ) -
                    (1 + θ) (θ + 2 λ) + 4 δ^2 (-1 + θ + λ - θ λ) - 2 δ (1 + 2 θ + 3 (1 + θ) λ))) /
              (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))) +
            ((-α (-1 + 2 δ - θ) (-1 + θ) (2 + 6 δ + θ) + k (1 + 2 δ) (6 δ (-1 + 2 δ) -
                    (1 + θ) (2 + θ)) + m (1 + 2 δ) (-2 δ (2 + λ) + 4 δ^2 (2 + λ) - (1 + θ) (2 + θ λ))) /
              (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))) +
            ((α (-1 + θ) (-12 δ^2 + 2 δ (2 + θ) + (1 + θ) (2 + θ)) +
                k (24 δ^3 - (1 + θ) (2 + θ) - 2 δ (4 + 5 θ)) + m (8 δ^3 (1 + 2 λ) -
                    (1 + θ) (θ + 2 λ) + 4 δ^2 (-1 + θ + λ - θ λ) - 2 δ (1 + 2 θ + 3 (1 + θ) λ))) /
              (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))) +
            ((-α (-1 + 2 δ - θ) (-1 + θ) (2 + 6 δ + θ) + k (1 + 2 δ) (6 δ (-1 + 2 δ) -
                    (1 + θ) (2 + θ)) + m (1 + 2 δ) (-2 δ (2 + λ) + 4 δ^2 (2 + λ) - (1 + θ) (2 + θ λ))) /
              (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))))))]

-(((-1 - 2 δ + 4 δ^2 - θ) (k (-1 + θ) (2 + 6 δ + θ) - α (-1 + θ) (2 + 6 δ + θ) +
        m (-2 + 4 δ^2 (-1 + λ) + θ (θ + λ) + 2 δ (-4 + λ + θ (2 + λ))))) / ((-1 + θ) (1 + 4 δ + θ)
      (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))))
```

```
FullSimplify[
 (((α (-1 + θ) (-12 δ^2 + 2 δ (2 + θ) + (1 + θ) (2 + θ)) + k (24 δ^3 - (1 + θ) (2 + θ) - 2 δ (4 + 5 θ)) + m
          (8 δ^3 (1 + 2 λ) - (1 + θ) (θ + 2 λ) + 4 δ^2 (-1 + θ + λ - θ λ) - 2 δ (1 + 2 θ + 3 (1 + θ) λ))) /
       (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ)) - k - m λ) *
    (((1 + 2 δ) (k (-1 + θ) (2 - 12 δ^2 + 3 θ + θ^2 + 2 δ (2 + θ)) -
          α (-1 + θ) (2 - 12 δ^2 + 3 θ + θ^2 + 2 δ (2 + θ)) + m (8 δ^3 (-1 + λ) -
             4 δ^2 (-1 + θ) (1 + 2 λ) + (1 + θ) (θ - 2 λ + θ^2 λ) + 2 δ (1 + 2 θ - 3 λ - θ λ + θ^2 λ)))) /
      ((-1 + θ) (1 + 4 δ + θ) (-4 + 24 δ^3 - 12 δ^2 (-1 + θ) - 4 θ + θ^2 + θ^3 + 2 δ (-6 - 4 θ + θ^2))))) +
  (((α (-1 + θ) (-12 δ^2 + 2 δ (2 + θ) + (1 + θ) (2 + θ)) +
          k (24 δ^3 - (1 + θ) (2 + θ) - 2 δ (4 + 5 θ)) + m
          (8 δ^3 (1 + 2 λ) - (1 + θ) (θ + 2 λ) + 4 δ^2 (-1 + θ + λ - θ λ) - 2 δ (1 + 2 θ + 3 (1 + θ) λ))) /
       (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ)) - k - m λ) *
    (((1 + 2 δ) (k (-1 + θ) (2 - 12 δ^2 + 3 θ + θ^2 + 2 δ (2 + θ)) -
          α (-1 + θ) (2 - 12 δ^2 + 3 θ + θ^2 + 2 δ (2 + θ)) + m (8 δ^3 (-1 + λ) -
             4 δ^2 (-1 + θ) (1 + 2 λ) + (1 + θ) (θ - 2 λ + θ^2 λ) + 2 δ (1 + 2 θ - 3 λ - θ λ + θ^2 λ)))) /
      ((-1 + θ) (1 + 4 δ + θ) (-4 + 24 δ^3 - 12 δ^2 (-1 + θ) - 4 θ + θ^2 + θ^3 + 2 δ (-6 - 4 θ + θ^2))))) +
  (((α (-1 + θ) (-12 δ^2 + 2 δ (2 + θ) + (1 + θ) (2 + θ)) +
          k (24 δ^3 - (1 + θ) (2 + θ) - 2 δ (4 + 5 θ)) + m
          (8 δ^3 (1 + 2 λ) - (1 + θ) (θ + 2 λ) + 4 δ^2 (-1 + θ + λ - θ λ) - 2 δ (1 + 2 θ + 3 (1 + θ) λ))) /
       (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ)) - k - m λ) *
    (((1 + 2 δ) (k (-1 + θ) (2 - 12 δ^2 + 3 θ + θ^2 + 2 δ (2 + θ)) -
          α (-1 + θ) (2 - 12 δ^2 + 3 θ + θ^2 + 2 δ (2 + θ)) + m (8 δ^3 (-1 + λ) -
             4 δ^2 (-1 + θ) (1 + 2 λ) + (1 + θ) (θ - 2 λ + θ^2 λ) + 2 δ (1 + 2 θ - 3 λ - θ λ + θ^2 λ)))) /
      ((-1 + θ) (1 + 4 δ + θ) (-4 + 24 δ^3 - 12 δ^2 (-1 + θ) - 4 θ + θ^2 + θ^3 + 2 δ (-6 - 4 θ + θ^2)))))]
-((3 (1 + 2 δ) (k (-1 + θ) (-12 δ^2 + 2 δ (2 + θ) + (1 + θ) (2 + θ)) - α (-1 + θ)
         (-12 δ^2 + 2 δ (2 + θ) + (1 + θ) (2 + θ)) + m (8 δ^3 (-1 + λ) - 4 δ^2 (-1 + θ) (1 + 2 λ) +
          2 δ (1 + 2 θ + (-3 + (-1 + θ) θ) λ) + (1 + θ) (θ + (-2 + θ^2) λ)))^2) / ((-1 + θ)
     (1 + 4 δ + θ) (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))^2))

(*premerger profit*)
```

```
FullSimplify[(((α (-1 + 2 δ - θ) (-1 + θ) + k (1 + 2 δ - 4 δ^2 + θ) + m (1 + 2 δ - 4 δ^2 + θ)) /
      (2 - 4 δ^2 + θ + 2 δ θ - θ^2)) - k - m) + 1/((-1 + 2 δ - θ) (-1 + θ) (1 + 4 δ + θ))
   ((-1 - 2 δ + 4 δ^2 - θ) ((α (-1 + 2 δ - θ) (-1 + θ) + k (1 + 2 δ - 4 δ^2 + θ) + m (1 + 2 δ - 4 δ^2 + θ)) /
        (2 - 4 δ^2 + θ + 2 δ θ - θ^2)) + (-4 δ^2 + θ + 2 δ θ + θ^2)
      ((α (-1 + 2 δ - θ) (-1 + θ) + k (1 + 2 δ - 4 δ^2 + θ) + m (1 + 2 δ - 4 δ^2 + θ)) /
        (2 - 4 δ^2 + θ + 2 δ θ - θ^2)) - (-1 + θ)
      (α (1 - 2 δ + θ) + δ (((α (-1 + 2 δ - θ) (-1 + θ) + k (1 + 2 δ - 4 δ^2 + θ) + m (1 + 2 δ - 4 δ^2 + θ)) /
              (2 - 4 δ^2 + θ + 2 δ θ - θ^2)) + ((α (-1 + 2 δ - θ) (-1 + θ) +
                 k (1 + 2 δ - 4 δ^2 + θ) + m (1 + 2 δ - 4 δ^2 + θ)) / (2 - 4 δ^2 + θ + 2 δ θ - θ^2)) +
            ((α (-1 + 2 δ - θ) (-1 + θ) + k (1 + 2 δ - 4 δ^2 + θ) + m (1 + 2 δ - 4 δ^2 + θ)) /
              (2 - 4 δ^2 + θ + 2 δ θ - θ^2)) + ((α (-1 + 2 δ - θ) (-1 + θ) + k (1 + 2 δ - 4 δ^2 + θ) +
                 m (1 + 2 δ - 4 δ^2 + θ)) / (2 - 4 δ^2 + θ + 2 δ θ - θ^2)))))]
```

$$-\frac{(k+m-\alpha)^2(-1+\theta)\left(8\delta^3+(1+\theta)^2-4\delta^2(2+\theta)\right)}{(1+4\delta+\theta)\left(2-4\delta^2+\theta+2\delta\theta-\theta^2\right)^2}$$

```
FullSimplify[Solve[(α (-1 + 2 δ - θ) (-1 + θ) + k (1 + 2 δ - 4 δ^2 + θ) + m (1 + 2 δ - 4 δ^2 + θ)) /
     (2 - 4 δ^2 + θ + 2 δ θ - θ^2) ==
    (α (-1 + θ) (-12 δ^2 + 2 δ (2 + θ) + (1 + θ) (2 + θ)) + k (24 δ^3 - (1 + θ) (2 + θ) - 2 δ (4 + 5 θ)) +
       m (8 δ^3 (1 + 2 λ) - (1 + θ) (θ + 2 λ) + 4 δ^2 (-1 + θ + λ - θ λ) - 2 δ (1 + 2 θ + 3 (1 + θ) λ))) /
     (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ)), λ]]
```

$$\left\{\left\{\lambda\to\frac{m(2+2\delta-\theta)\left(-1-2\delta+4\delta^2-\theta\right)-2(k-\alpha)\delta(-1+\theta)^2}{m(1+2\delta)\left(4\delta^2-2\delta\theta+(-2+\theta)(1+\theta)\right)}\right\}\right\}$$

```
FullSimplify[Reduce[(α (-1 + 2 δ - θ) (-1 + θ) + k (1 + 2 δ - 4 δ^2 + θ) + m (1 + 2 δ - 4 δ^2 + θ)) /
      (2 - 4 δ^2 + θ + 2 δ θ - θ^2) <
     (α (-1 + θ) (-12 δ^2 + 2 δ (2 + θ) + (1 + θ) (2 + θ)) + k (24 δ^3 - (1 + θ) (2 + θ) - 2 δ (4 + 5 θ)) +
        m (8 δ^3 (1 + 2 λ) - (1 + θ) (θ + 2 λ) + 4 δ^2 (-1 + θ + λ - θ λ) - 2 δ (1 + 2 θ + 3 (1 + θ) λ))) /
      (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ)) &&
   m > 0 && k > 0 && 0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < (k + m) < α, λ]]
```

$$\delta>0\;\&\&\;\delta<\theta<1\;\&\&\;m>0\;\&\&\;k>0\;\&\&\;k+m<\alpha\;\&\&$$
$$\frac{-m(2+2\delta-\theta)\left(-1-2\delta+4\delta^2-\theta\right)+2(k-\alpha)\delta(-1+\theta)^2}{m(1+2\delta)\left(4\delta^2-2\delta\theta+(-2+\theta)(1+\theta)\right)}+\lambda>0$$

```
FullSimplify[Solve[(α (-1 + 2 δ - θ) (-1 + θ) + k (1 + 2 δ - 4 δ^2 + θ) + m (1 + 2 δ - 4 δ^2 + θ)) /
     (2 - 4 δ^2 + θ + 2 δ θ - θ^2) ==
    (-α (-1 + 2 δ - θ) (-1 + θ) (2 + 6 δ + θ) + k (1 + 2 δ) (6 δ (-1 + 2 δ) - (1 + θ) (2 + θ)) +
       m (1 + 2 δ) (-2 δ (2 + λ) + 4 δ^2 (2 + λ) - (1 + θ) (2 + θ λ))) /
     (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ)), λ]]
```

$$\left\{\left\{\lambda\to\frac{m(2+2\delta-\theta)\left(-1-2\delta+4\delta^2-\theta\right)-2(k-\alpha)\delta(-1+\theta)^2}{m(1+2\delta)\left(4\delta^2-2\delta\theta+(-2+\theta)(1+\theta)\right)}\right\}\right\}$$

```
FullSimplify[D[(m (2 + 2 δ - θ) (-1 - 2 δ + 4 δ^2 - θ) - 2 (k - α) δ (-1 + θ)^2)/(m (1 + 2 δ) (4 δ^2 - 2 δ θ + (-2 + θ) (1 + θ))), δ]]

(2 (k + m - α) (-1 + θ)^2 (2 + 16 δ^3 - 4 δ^2 (-1 + θ) + θ - θ^2))/(m (1 + 2 δ)^2 (2 - 4 δ^2 + θ + 2 δ θ - θ^2)^2)

Reduce[(2 (k + m - α) (-1 + θ)^2 (2 + 16 δ^3 - 4 δ^2 (-1 + θ) + θ - θ^2))/(m (1 + 2 δ)^2 (2 - 4 δ^2 + θ + 2 δ θ - θ^2)^2) < 0 &&
  m > 0 && k > 0 && 0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < (k + m) < α]

0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < m < α && 0 < k < -m + α

FullSimplify[D[(m (2 + 2 δ - θ) (-1 - 2 δ + 4 δ^2 - θ) - 2 (k - α) δ (-1 + θ)^2)/(m (1 + 2 δ) (4 δ^2 - 2 δ θ + (-2 + θ) (1 + θ))), θ]]

-(2 (k + m - α) δ (-1 + θ) (-5 + 8 δ^2 + θ - 2 δ (1 + θ)))/((m + 2 m δ) (2 - 4 δ^2 + θ + 2 δ θ - θ^2)^2)

Reduce[-(2 (k + m - α) δ (-1 + θ) (-5 + 8 δ^2 + θ - 2 δ (1 + θ)))/((m + 2 m δ) (2 - 4 δ^2 + θ + 2 δ θ - θ^2)^2) > 0 &&
  m > 0 && k > 0 && 0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < (k + m) < α]

0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < m < α && 0 < k < -m + α

FullSimplify[D[(m (2 + 2 δ - θ) (-1 - 2 δ + 4 δ^2 - θ) - 2 (k - α) δ (-1 + θ)^2)/(m (1 + 2 δ) (4 δ^2 - 2 δ θ + (-2 + θ) (1 + θ))), α]]

(2 δ (-1 + θ)^2)/(m (1 + 2 δ) (4 δ^2 - 2 δ θ + (-2 + θ) (1 + θ)))

Reduce[(2 δ (-1 + θ)^2)/(m (1 + 2 δ) (4 δ^2 - 2 δ θ + (-2 + θ) (1 + θ))) < 0 &&
  m > 0 && k > 0 && 0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < (k + m) < α]

0 < θ < 1 && 0 < δ < θ && α > 0 && 0 < m < α && 0 < k < -m + α

FullSimplify[D[(m (2 + 2 δ - θ) (-1 - 2 δ + 4 δ^2 - θ) - 2 (k - α) δ (-1 + θ)^2)/(m (1 + 2 δ) (4 δ^2 - 2 δ θ + (-2 + θ) (1 + θ))), k]]

-(2 δ (-1 + θ)^2)/(m (1 + 2 δ) (4 δ^2 - 2 δ θ + (-2 + θ) (1 + θ)))

Reduce[-(2 δ (-1 + θ)^2)/(m (1 + 2 δ) (4 δ^2 - 2 δ θ + (-2 + θ) (1 + θ))) > 0 &&
  m > 0 && k > 0 && 0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < (k + m) < α]

0 < θ < 1 && 0 < δ < θ && α > 0 && 0 < m < α && 0 < k < -m + α
```

```
FullSimplify[D[(m (2 + 2 δ - θ) (-1 - 2 δ + 4 δ^2 - θ) - 2 (k - α) δ (-1 + θ)^2)/(m (1 + 2 δ) (4 δ^2 - 2 δ θ + (-2 + θ) (1 + θ))), m]]
```

$$\frac{2\ (k-\alpha)\ \delta\ (-1+\theta)^2}{m^2\ (1+2\,\delta)\ \left(4\,\delta^2-2\,\delta\,\theta+(-2+\theta)\ (1+\theta)\right)}$$

```
Reduce[(2 (k - α) δ (-1 + θ)^2)/(m^2 (1 + 2 δ) (4 δ^2 - 2 δ θ + (-2 + θ) (1 + θ))) > 0 &&
  m > 0 && k > 0 && 0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < (k + m) < α]
```

$$0<\theta<1\ \&\&\ 0<\delta<\theta\ \&\&\ \alpha>0\ \&\&\ 0<m<\alpha\ \&\&\ 0<k<-m+\alpha$$

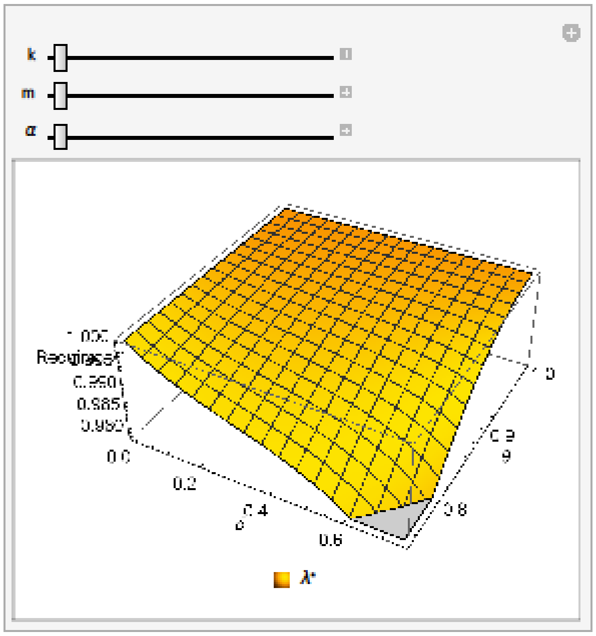
```
Reduce[1 > (m (2 + 2 δ - θ) (-1 - 2 δ + 4 δ^2 - θ) - 2 (k - α) δ (-1 + θ)^2)/(m (1 + 2 δ) (4 δ^2 - 2 δ θ + (-2 + θ) (1 + θ))) > (m - k)/(2 * m) &&
  m > 0 && k > 0 && 0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < α, Reals]
```

$$0<\theta<1\ \&\&\ 0<\delta<\theta\ \&\&\ \alpha>0\ \&\&\ \Bigg(\bigg(0<m\le\frac{-4\,\alpha\,\delta+8\,\alpha\,\delta\,\theta-4\,\alpha\,\delta\,\theta^2}{-2-8\,\delta+4\,\delta^2+8\,\delta^3-\theta+4\,\delta\,\theta-4\,\delta^2\,\theta+\theta^2-2\,\delta\,\theta^2}\ \&\&$$
$$\left(2\,m+8\,m\,\delta-4\,\alpha\,\delta-4\,m\,\delta^2-8\,m\,\delta^3+m\,\theta-4\,m\,\delta\,\theta+8\,\alpha\,\delta\,\theta+4\,m\,\delta^2\,\theta-m\,\theta^2+2\,m\,\delta\,\theta^2-4\,\alpha\,\delta\,\theta^2\right)\Big/$$
$$\left(-2-8\,\delta+4\,\delta^2+8\,\delta^3-\theta+4\,\delta\,\theta-4\,\delta^2\,\theta+\theta^2-2\,\delta\,\theta^2\right)<k<-m+\alpha\bigg)\ ||$$
$$\left(\frac{-4\,\alpha\,\delta+8\,\alpha\,\delta\,\theta-4\,\alpha\,\delta\,\theta^2}{-2-8\,\delta+4\,\delta^2+8\,\delta^3-\theta+4\,\delta\,\theta-4\,\delta^2\,\theta+\theta^2-2\,\delta\,\theta^2}<m<\alpha\ \&\&\ 0<k<-m+\alpha\right)\Bigg)$$

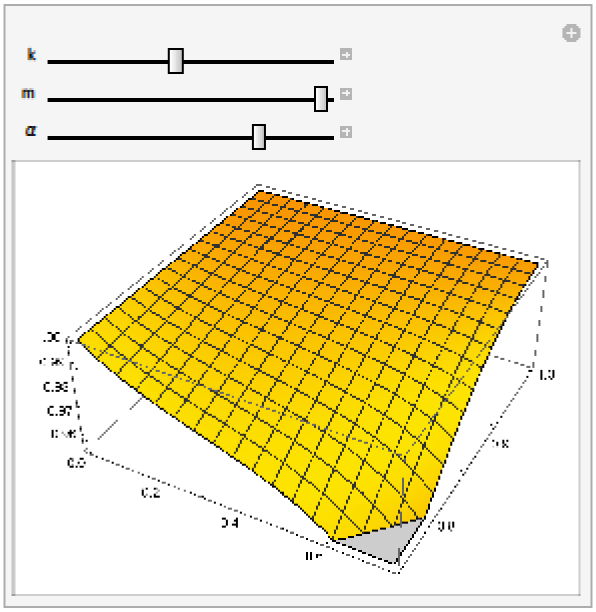
```
Manipulate[
 Plot3D[{(-(((c - α) (1 + 2 δ + θ) (4 δ^2 + (-2 + θ) (1 + θ) + δ (-6 + 4 θ))) / (2 (1 + 4 δ + θ)
          (-2 + 4 δ^2 + 8 δ^3 - 3 θ + θ^3 + 2 δ (-3 + θ) (1 + θ))))) -
     (-(((c - α) (1 + 2 δ) (12 δ^2 - 2 δ (2 + θ) - (1 + θ) (2 + θ))) / ((1 + 4 δ + θ)
          (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))))), 0},
  {δ, 0, .75}, {θ, .75, 1}, PlotLabel → "", AxesLabel → {"δ", "θ", "Rel. Qty."},
  AxesStyle → Larger,
  LabelStyle → Bold,
  PlotLegends →
   Placed[{"Difference: Q1(Case IV)-Q1(Case II)", "Plane at 0"}, Below]],
 {c, 1, 1}, {α, 1.5, 1.5}, {s, 1, 1}]

(*Graphing lambda star*)
```

```
Manipulate[
 Plot3D[{(m (2 + 2 δ - θ) (-1 - 2 δ + 4 δ² - θ) - 2 (k - α) δ (-1 + θ)²) / (m (1 + 2 δ) (4 δ² - 2 δ θ + (-2 + θ) (1 + θ)))}, {δ, 0, .75}, {θ, .75, 1},
  PlotLabel → "", AxesLabel → {"δ", "θ", "Required λ*"}, AxesStyle → Medium,
  PlotLegends → Placed[{"λ*"}, Below]], {k, 1, 1}, {m, 1, 1}, {α, 3, 3}]
```



$$\text{Manipulate}\left[\text{Plot3D}\left[\left\{\frac{m\,(2+2\,\delta-\theta)\,\left(-1-2\,\delta+4\,\delta^2-\theta\right)-2\,(k-\alpha)\,\delta\,(-1+\theta)^2}{m\,(1+2\,\delta)\,\left(4\,\delta^2-2\,\delta\,\theta+(-2+\theta)\,(1+\theta)\right)}\right\},\ \{\delta,\ 0,\ .75\},\ \{\theta,\ .75,\ 1\}\right],\ \{k,\ 0,\ 1\},\ \{m,\ 0,\ 1\},\ \{\alpha,\ 2,\ 4\},\ \text{Axes}\to\text{True},\ \text{AxesOrigin}\to\{0,\ 0\}\right]$$



(*Scope*)

```
((k + m) * (q1) + (k + m) * (q2) + (k + m) * (q3) - (k + λ * m) * (q1 + q2 + q3)) /
  ((k + λ * m) * (q1 + q2 + q3))
```

$$\frac{(k+m)\,q_1+(k+m)\,q_2+(k+m)\,q_3-(k+m\,\lambda)\,(q_1+q_2+q_3)}{(k+m\,\lambda)\,(q_1+q_2+q_3)}$$

$$\text{FullSimplify}\left[D\left[\frac{(k+m)\,q_1+(k+m)\,q_2+(k+m)\,q_3-(k+m\,\lambda)\,(q_1+q_2+q_3)}{(k+m\,\lambda)\,(q_1+q_2+q_3)},\ \lambda\right]\right]$$

$$-\frac{m\,(k+m)}{(k+m\,\lambda)^2}$$

$$\text{Solve}\left[\frac{(k+m)\,q_1+(k+m)\,q_2+(k+m)\,q_3-(k+m\,\lambda)\,(q_1+q_2+q_3)}{(k+m\,\lambda)\,(q_1+q_2+q_3)}==1,\ \lambda\right]$$

$$\left\{\left\{\lambda\to\frac{-k+m}{2\,m}\right\}\right\}$$

$$\text{Solve}\left[\frac{(k+m)\,q_1+(k+m)\,q_2+(k+m)\,q_3-(k+m\,\lambda)\,(q_1+q_2+q_3)}{(k+m\,\lambda)\,(q_1+q_2+q_3)}==0,\ \lambda\right]$$

$$\{\{\lambda\to 1\}\}$$

```
Reduce[0 < ((k + m) q1 + (k + m) q2 + (k + m) q3 - (k + m λ) (q1 + q2 + q3)) / ((k + m λ) (q1 + q2 + q3)) < 1 &&
  m > 0 && k > 0 && 0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < (k + m) < α, λ]
```

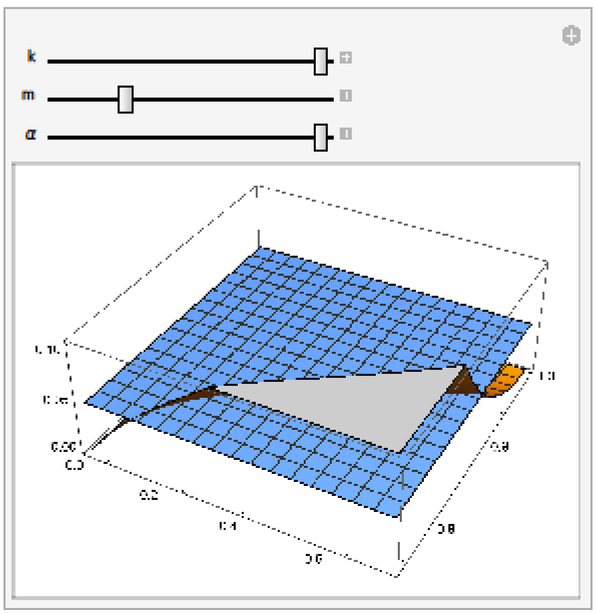
$(q_3 \mid q_2 \mid q_1) \in$ Reals && $0 < \theta < 1$ && $0 < \delta < \theta$ && $m > 0$ && $k > 0$ && $\alpha > k + m$ && $\frac{-k+m}{2m} < \lambda < 1$

Sub in 1 star

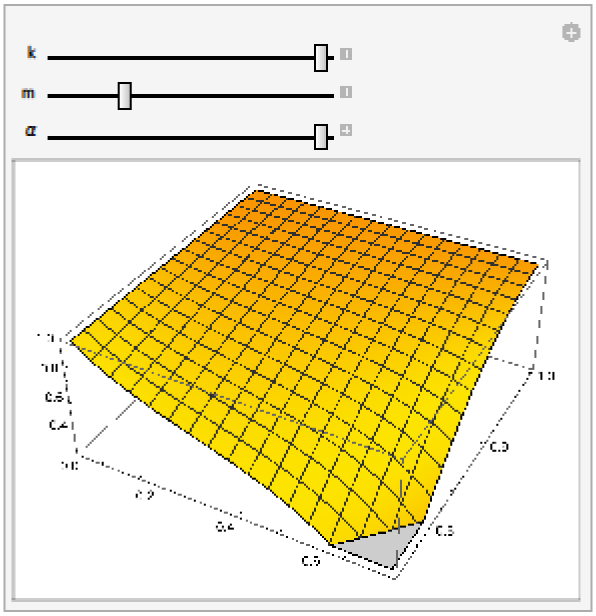
```
FullSimplify[((k + m) q1 + (k + m) q2 + (k + m) q3 -
    (k + m ((m (2 + 2 δ - θ) (-1 - 2 δ + 4 δ^2 - θ) - 2 (k - α) δ (-1 + θ)^2) /
          (m (1 + 2 δ) (4 δ^2 - 2 δ θ + (-2 + θ) (1 + θ))))) (q1 + q2 + q3)) /
  ((k + m ((m (2 + 2 δ - θ) (-1 - 2 δ + 4 δ^2 - θ) - 2 (k - α) δ (-1 + θ)^2) /
        (m (1 + 2 δ) (4 δ^2 - 2 δ θ + (-2 + θ) (1 + θ))))) (q1 + q2 + q3))]
```

$(2\ (k+m-\alpha)\ \delta\ (-1+\theta)^2) \big/ (-2\ (k+m) + 2\ (-3\ (k+m)+\alpha)\ \delta + 4\ (k+m)\ \delta^2 + 8\ (k+m)\ \delta^3 - (k+m+4\,\alpha\,\delta+4\ (k+m)\ \delta^2)\ \theta + (k+m+2\,\alpha\,\delta)\ \theta^2)$

```
Manipulate[Plot3D[
  {(2 (k + m - α) δ (-1 + θ)^2) / (-2 (k + m) + 2 (-3 (k + m) + α) δ + 4 (k + m) δ^2 + 8 (k + m) δ^3 -
      (k + m + 4 α δ + 4 (k + m) δ^2) θ + (k + m + 2 α δ) θ^2), .05}, {δ, 0, .75}, {θ, .75, 1}],
 {k, 0, 1}, {m, 0, 1}, {α, 2, 12.5}, Axes → True, AxesOrigin → {0, 0}]
```



```
Manipulate[Plot3D[{(m (2 + 2 δ - θ) (-1 - 2 δ + 4 δ^2 - θ) - 2 (k - α) δ (-1 + θ)^2) / (m (1 + 2 δ) (4 δ^2 - 2 δ θ + (-2 + θ) (1 + θ)))}, {δ, 0, .75},
  {θ, .75, 1}], {k, 0, 1}, {m, 0, 1}, {α, 2, 10}, Axes → True, AxesOrigin → {0, 0}]
```



```
(*Price Comparison Tables*)

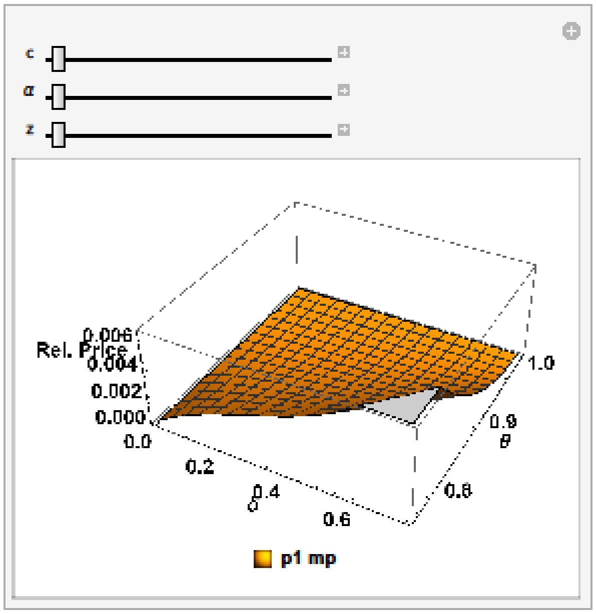
(*PRICES*)

(*Compstat: multiproduct135 p1 vs. p1 premerger &&0<δ<1&&δ<θ<1&&α>0&&0<c<α*)

Reduce[
 ((α (-1 + θ) (-12 δ^2 + 2 δ (2 + θ) + (1 + θ) (2 + θ)) + c (24 δ^3 - (1 + θ) (2 + θ) - 2 δ (4 + 5 θ))) /
     (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))) >
   ((α (-1 + 2 δ - θ) (-1 + θ) + c (1 + 2 δ - 4 δ^2 + θ)) / (2 - 4 δ^2 + θ + 2 δ θ - θ^2)) && 0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < c < α]

0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < c < α
```

```
Manipulate[Plot3D[
  {((α (-1 + θ) (-12 δ^2 + 2 δ (2 + θ) + (1 + θ) (2 + θ)) + c (24 δ^3 - (1 + θ) (2 + θ) - 2 δ (4 + 5 θ))) /
      (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))) -
    ((α (-1 + 2 δ - θ) (-1 + θ) + c (1 + 2 δ - 4 δ^2 + θ)) / (2 - 4 δ^2 + θ + 2 δ θ - θ^2))}, {δ, 0, .75}, {θ, .75, 1},
  PlotLabel → "", AxesLabel → {"δ", "θ", "Rel. Price"}, AxesStyle → Larger,
  LabelStyle → Bold, PlotLegends → Placed[{"p1 mp", "p1 pre"}, Below]],
 {c, 1, 1}, {α, 1.5, 1.5}, {z, 1, 1}]
```




```
(*Compstat: Conglom p1 vs. Premerger p1*)
Reduce[((c + α + 2 c δ - α θ) / (2 + 2 δ - θ)) > ((α (-1 + 2 δ - θ) (-1 + θ) + c (1 + 2 δ - 4 δ^2 + θ)) / (2 - 4 δ^2 + θ + 2 δ θ - θ^2)) &&
  0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < c < α]
```

0 < θ < 1 && 0 < δ < θ && α > 0 && 0 < c < α

```
(*Compstat:submarket Monopoly p1 vs. premerger p1*)
```

```
Reduce[(1/2 (c + α + (4 (c - α) (-1 + δ) δ (1 + 4 δ + θ))/(-2 + 4 δ^2 + 8 δ^3 - 3 θ + θ^3 + 2 δ (-3 + θ) (1 + θ)))) >
   ((α (-1 + 2 δ - θ) (-1 + θ) + c (1 + 2 δ - 4 δ^2 + θ))/(2 - 4 δ^2 + θ + 2 δ θ - θ^2)) && 0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < c < α]
```

```
0 < θ < 1 && 0 < δ < θ && α > 0 && 0 < c < α
```

```
(*compstat: submarket Monop p6 vs. Premerger p6*)
```

```
Reduce[((-α (-1 + 2 δ - θ) (-1 + θ) (1 + 3 δ + θ) + c (-1 + δ) (1 + 2 δ + θ) (1 + 4 δ + θ)) /
     (-2 + 4 δ^2 + 8 δ^3 - 3 θ + θ^3 + 2 δ (-3 + θ) (1 + θ))) >
   ((α (-1 + 2 δ - θ) (-1 + θ) + c (1 + 2 δ - 4 δ^2 + θ))/(2 - 4 δ^2 + θ + 2 δ θ - θ^2)) && 0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < c < α]
```

```
0 < θ < 1 && 0 < δ < θ && α > 0 && 0 < c < α
```

```
(*compstat: p1 protein monop vs p1 premerger*)
```

```
Reduce[((c + α)/2) > ((α (-1 + 2 δ - θ) (-1 + θ) + c (1 + 2 δ - 4 δ^2 + θ))/(2 - 4 δ^2 + θ + 2 δ θ - θ^2)) &&
  0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < c < α]
```

```
0 < θ < 1 && 0 < δ < θ && α > 0 && 0 < c < α
```

```
(*compstat: conglom p1 vs  Multiproduct p1*)
```

```
Reduce[((c + α + 2 c δ - α θ)/(2 + 2 δ - θ)) >
   ((α (-1 + θ) (-12 δ^2 + 2 δ (2 + θ) + (1 + θ) (2 + θ)) + c (24 δ^3 - (1 + θ) (2 + θ) - 2 δ (4 + 5 θ))) /
     (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))) &&
  0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < c < α]
```

```
0 < θ < 1 && 0 < δ < θ && α > 0 && 0 < c < α
```

```
(*compstat: conglom p2 vs. multiproduct p2*)
```

```
Reduce[((c + α + 2 c δ - α θ)/(2 + 2 δ - θ)) >
   ((-α (-1 + 2 δ - θ) (-1 + θ) (2 + 6 δ + θ) + c (1 + 2 δ) (6 δ (-1 + 2 δ) - (1 + θ) (2 + θ))) /
     (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))) &&
  0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < c < α]
```

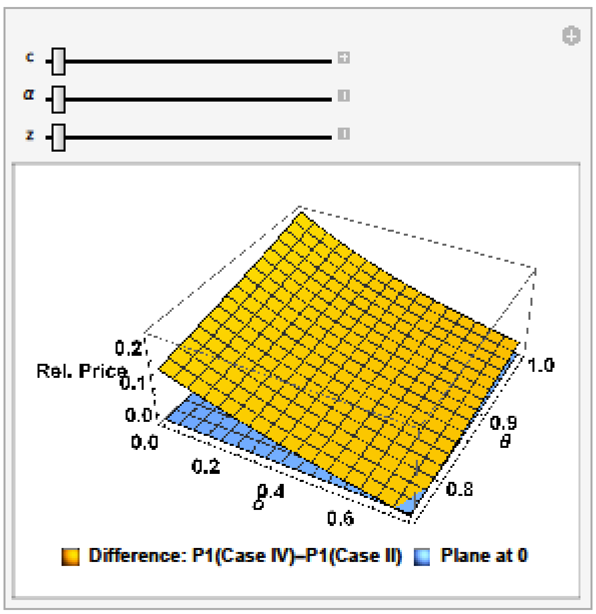
```
0 < θ < 1 && 0 < δ < θ && α > 0 && 0 < c < α
```

```
(*compstat: submarket monop p1 vs  Multiproduct p1*)
```

```
Reduce[(1/2 (c + α + (4 (c - α) (-1 + δ) δ (1 + 4 δ + θ))/(-2 + 4 δ^2 + 8 δ^3 - 3 θ + θ^3 + 2 δ (-3 + θ) (1 + θ)))) >
    ((α (-1 + θ) (-12 δ^2 + 2 δ (2 + θ) + (1 + θ) (2 + θ)) + c (24 δ^3 - (1 + θ) (2 + θ) - 2 δ (4 + 5 θ))) /
     (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))) &&
  0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < c < α]

0 < θ < 1 &&
 0 < δ < Root[-4 θ - 12 θ^2 - 11 θ^3 - θ^4 + 3 θ^5 + θ^6 + (8 - 16 θ - 54 θ^2 - 24 θ^3 + 10 θ^4 + 4 θ^5) #1 +
      (48 + 20 θ - 8 θ^2 + 16 θ^3 - 4 θ^4) #1^2 + (24 + 144 θ + 112 θ^2 - 16 θ^3) #1^3 +
      (-208 + 112 θ - 48 θ^2) #1^4 + (-96 - 192 θ) #1^5 + 192 #1^6 &, 2] && α > 0 && 0 < c < α
```

```
Manipulate[
 Plot3D[{((1/2 (c + α + (4 (c - α) (-1 + δ) δ (1 + 4 δ + θ))/(-2 + 4 δ^2 + 8 δ^3 - 3 θ + θ^3 + 2 δ (-3 + θ) (1 + θ))))) - ((α (-1 + θ)
        (-12 δ^2 + 2 δ (2 + θ) + (1 + θ) (2 + θ)) + c (24 δ^3 - (1 + θ) (2 + θ) - 2 δ (4 + 5 θ))) /
      (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))), 0},
  {δ, 0, .75}, {θ, .75, 1}, PlotLabel → "", AxesLabel → {"δ", "θ", "Rel. Price"},
  AxesStyle → Larger, LabelStyle → Bold, PlotLegends →
   Placed[{"Difference: P1(Case IV)-P1(Case II)", "Plane at 0"}, Below]],
 {c, 1, 1}, {α, 1.5, 1.5}, {z, 1, 1}]
```



(*compstat: submarket monop p2 vs. multiproduct p2*)

$$\text{Reduce}\left[\left(\frac{1}{2}\left(c+\alpha+\frac{4\,(c-\alpha)\,(-1+\delta)\,\delta\,(1+4\,\delta+\theta)}{-2+4\,\delta^2+8\,\delta^3-3\,\theta+\theta^3+2\,\delta\,(-3+\theta)\,(1+\theta)}\right)\right) > \left(\left(-\alpha\,(-1+2\,\delta-\theta)\,(-1+\theta)\,(2+6\,\delta+\theta)+c\,(1+2\,\delta)\,(6\,\delta\,(-1+2\,\delta)-(1+\theta)\,(2+\theta))\right)\Big/\left(24\,\delta^3-12\,\delta^2\,(-1+\theta)+(-2+\theta)\,(1+\theta)\,(2+\theta)+2\,\delta\,(-6+(-4+\theta)\,\theta)\right)\right)\,\&\&\; 0<\delta<1\,\&\&\,\delta<\theta<1\,\&\&\,\alpha>0\,\&\&\,0<c<\alpha\right]$$

$$\left(0<\theta\le\frac{2}{19}\left(3+2\sqrt{7}\right)\,\&\&\,0<\delta<\theta\,\&\&\,\alpha>0\,\&\&\,0<c<\alpha\right)\;||$$
$$\left(\frac{2}{19}\left(3+2\sqrt{7}\right)<\theta<1\,\&\&\,0<\delta<\text{Root}\left[4\,\theta+8\,\theta^2+3\,\theta^3-2\,\theta^4-\theta^5+\left(20\,\theta+34\,\theta^2+4\,\theta^3-10\,\theta^4\right)\#1+\left(-24+12\,\theta+48\,\theta^2-24\,\theta^3\right)\#1^2+\left(-88-32\,\theta^2\right)\#1^3-48\,\theta\,\#1^4+96\,\#1^5\,\&,\,4\right]\,\&\&\,\alpha>0\,\&\&\,0<c<\alpha\right)$$

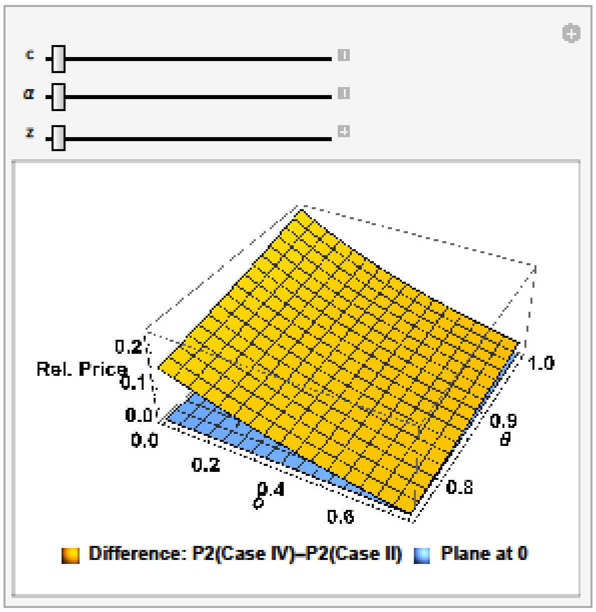
(*compstat: submarket monop p2 vs. multiproduct p2*)

$$\text{Reduce}\left[\left(\frac{1}{2}\left(c+\alpha+\frac{4\,(c-\alpha)\,(-1+\delta)\,\delta\,(1+4\,\delta+\theta)}{-2+4\,\delta^2+8\,\delta^3-3\,\theta+\theta^3+2\,\delta\,(-3+\theta)\,(1+\theta)}\right)\right) > \left(\left(-\alpha\,(-1+2\,\delta-\theta)\,(-1+\theta)\,(2+6\,\delta+\theta)+c\,(1+2\,\delta)\,(6\,\delta\,(-1+2\,\delta)-(1+\theta)\,(2+\theta))\right)\Big/\left(24\,\delta^3-12\,\delta^2\,(-1+\theta)+(-2+\theta)\,(1+\theta)\,(2+\theta)+2\,\delta\,(-6+(-4+\theta)\,\theta)\right)\right)\,\&\&\; 0<\delta<1\,\&\&\,\delta<.75<\theta<1\,\&\&\,\alpha>0\,\&\&\,0<c<\alpha\right]$$

Reduce::ratnz : Reduce was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result. >>

$$0<\delta<0.75\,\&\&\,0.75<\theta<1.\,\&\&\,\alpha>0\,\&\&\,0<c<\alpha$$

```
Manipulate[Plot3D[{(1/2 (c + α + (4 (c - α) (-1 + δ) δ (1 + 4 δ + θ))/(-2 + 4 δ^2 + 8 δ^3 - 3 θ + θ^3 + 2 δ (-3 + θ) (1 + θ)))) -
    ((-α (-1 + 2 δ - θ) (-1 + θ) (2 + 6 δ + θ) + c (1 + 2 δ) (6 δ (-1 + 2 δ) - (1 + θ) (2 + θ))) /
      (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))), 0},
  {δ, 0, .75}, {θ, .75, 1}, PlotLabel → "", AxesLabel → {"δ", "θ", "Rel. Price"},
  AxesStyle → Larger, LabelStyle → Bold,
  PlotLegends → Placed[{"Difference: P2(Case IV)-P2(Case II)", "Plane at 0"},
    Below]], {c, 1, 1}, {α, 1.5, 1.9
 },
 {z,
  1,
  1}]
```

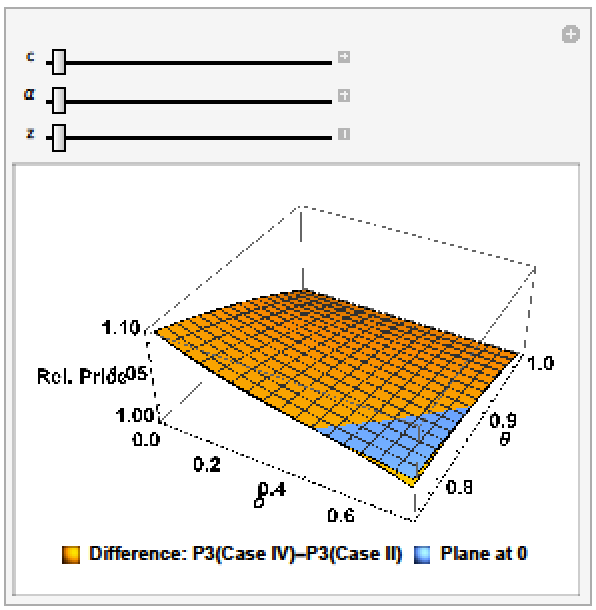



```
(*submarket submarket monop fringe p3 vs. multiproduct p3*)
```

```
Reduce[((-α (-1 + 2 δ - θ) (-1 + θ) (1 + 3 δ + θ) + c (-1 + δ) (1 + 2 δ + θ) (1 + 4 δ + θ)) /
     (-2 + 4 δ^2 + 8 δ^3 - 3 θ + θ^3 + 2 δ (-3 + θ) (1 + θ))) >
   ((α (-1 + θ) (-12 δ^2 + 2 δ (2 + θ) + (1 + θ) (2 + θ)) + c (24 δ^3 - (1 + θ) (2 + θ) - 2 δ (4 + 5 θ))) /
     (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))) &&
  0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < c < α]
```

```
-3 + √13 < θ < 1 &&
 0 < δ < Root[-4 - 2 θ + 9 θ^2 + 8 θ^3 + θ^4 + (-16 + 4 θ + 28 θ^2 + 8 θ^3) #1 + (-4 - 16 θ - 16 θ^2) #1^2 +
     (32 - 80 θ) #1^3 + 48 #1^4 &, 3] && α > 0 && 0 < c < α
```

```
Manipulate[
 Plot3D[{((-α (-1 + 2 δ - θ) (-1 + θ) (1 + 3 δ + θ) + c (-1 + δ) (1 + 2 δ + θ) (1 + 4 δ + θ)) /
     (-2 + 4 δ^2 + 8 δ^3 - 3 θ + θ^3 + 2 δ (-3 + θ) (1 + θ))),
   ((α (-1 + θ) (-12 δ^2 + 2 δ (2 + θ) + (1 + θ) (2 + θ)) + c (24 δ^3 - (1 + θ) (2 + θ) - 2 δ (4 + 5 θ))) /
     (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ)))},
  {δ, 0, .75}, {θ, .75, 1}, PlotLabel → "", AxesLabel → {"δ", "θ", "Rel. Price"},
  AxesStyle → Larger, LabelStyle → Bold, PlotLegends →
   Placed[{"Difference: P3(Case IV)-P3(Case II)", "Plane at 0"}, Below]],
 {c, 1, 1}, {α, 1.5, 1.5}, {z, 1, 1}]
```




```
(*meat market p1 vs. multiproduct p1*)
```

```
Reduce[((c + α)/2) >
    ((α (-1 + θ) (-12 δ^2 + 2 δ (2 + θ) + (1 + θ) (2 + θ)) + c (24 δ^3 - (1 + θ) (2 + θ) - 2 δ (4 + 5 θ))) /
      (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))) &&
   0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < c < α]
```

```
0 < θ < 1 && 0 < δ < θ && α > 0 && 0 < c < α
```

```
(*meat market p2 vs. multiproduct p2*)
```

```
Reduce[((c + α)/2) >
    ((-α (-1 + 2 δ - θ) (-1 + θ) (2 + 6 δ + θ) + c (1 + 2 δ) (6 δ (-1 + 2 δ) - (1 + θ) (2 + θ))) /
      (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))) &&
   0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < c < α]
```

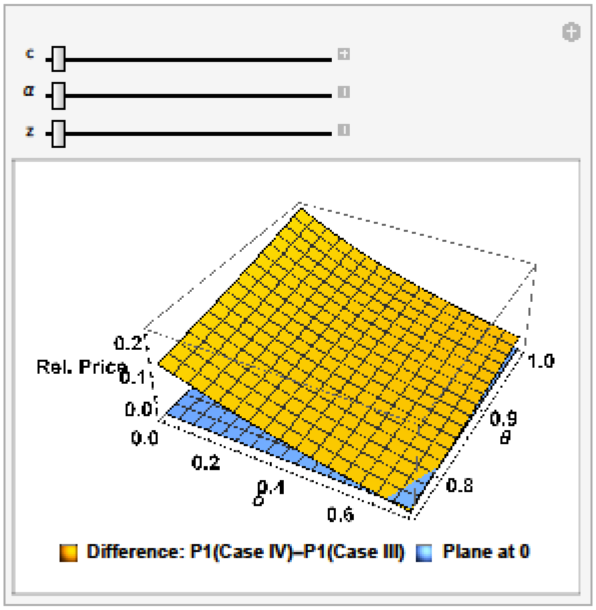
```
0 < θ < 1 && 0 < δ < θ && α > 0 && 0 < c < α
```

```
(*submarket monop p1 vs. conglom p1*)
```

```
Reduce[((1/2) (c + α + (4 (c - α) (-1 + δ) δ (1 + 4 δ + θ))/(-2 + 4 δ^2 + 8 δ^3 - 3 θ + θ^3 + 2 δ (-3 + θ) (1 + θ)))) > ((c + α + 2 c δ - α θ)/(2 + 2 δ - θ)) &&
   0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < c < α]
```

```
0 < θ < 1 && 0 < δ <
   Root[2 θ + 3 θ^2 - θ^4 + (-4 + 8 θ + 8 θ^2 - 4 θ^3) #1 + (-20 + 16 θ - 8 θ^2) #1^2 - 16 θ #1^3 + 16 #1^4 &,
    3] && α > 0 && 0 < c < α
```

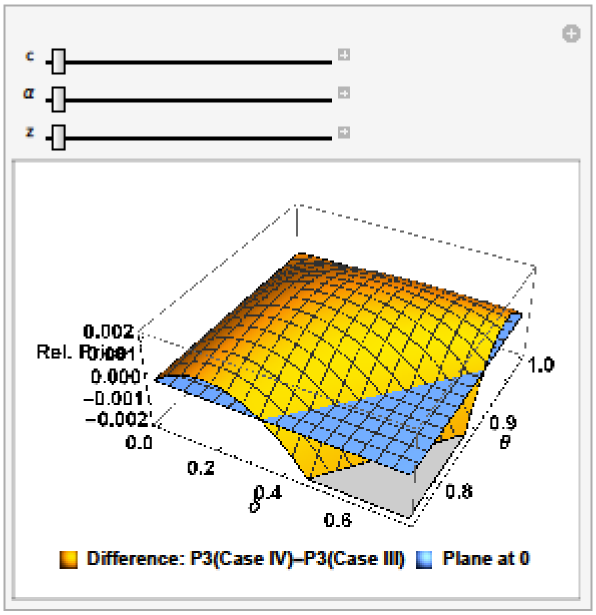
```
Manipulate[
 Plot3D[{(1/2 (c + α + (4 (c - α) (-1 + δ) δ (1 + 4 δ + θ))/(-2 + 4 δ^2 + 8 δ^3 - 3 θ + θ^3 + 2 δ (-3 + θ) (1 + θ)))) - ((c + α + 2 c δ - α θ)/(2 + 2 δ - θ)), 0},
  {δ, 0, .75}, {θ, .75, 1}, PlotLabel → "", AxesLabel → {"δ", "θ", "Rel. Price"},
  AxesStyle → Larger, LabelStyle → Bold, PlotLegends →
   Placed[{"Difference: P1(Case IV)-P1(Case III)", "Plane at 0"}, Below]],
 {c, 1, 1}, {α, 1.5, 1.5}, {z, 1, 1}]
```




```
(*submarket monop p3 vs. conglom p3*)
Reduce[((-α (-1 + 2 δ - θ) (-1 + θ) (1 + 3 δ + θ) + c (-1 + δ) (1 + 2 δ + θ) (1 + 4 δ + θ)) /
     (-2 + 4 δ^2 + 8 δ^3 - 3 θ + θ^3 + 2 δ (-3 + θ) (1 + θ))) >
   ((c + α + 2 c δ - α θ)/(2 + 2 δ - θ)) && 0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < c < α]
```

$\frac{2}{3} < \theta < 1 \ \&\&\ 0 < \delta < \frac{1}{4}(-3 + 4\theta) + \frac{1}{4}\sqrt{1 - 20\theta + 28\theta^2} \ \&\&\ \alpha > 0 \ \&\&\ 0 < c < \alpha$

```
Manipulate[
 Plot3D[{((-α (-1 + 2 δ - θ) (-1 + θ) (1 + 3 δ + θ) + c (-1 + δ) (1 + 2 δ + θ) (1 + 4 δ + θ)) /
      (-2 + 4 δ² + 8 δ³ - 3 θ + θ³ + 2 δ (-3 + θ) (1 + θ))) - ((c + α + 2 c δ - α θ)/(2 + 2 δ - θ)), 0},
  {δ, 0, .75}, {θ, .75, 1}, PlotLabel → "", AxesLabel → {"δ", "θ", "Rel. Price"},
  AxesStyle → Larger, LabelStyle → Bold, PlotLegends →
   Placed[{"Difference: P3(Case IV)-P3(Case III)", "Plane at 0"}, Below]],
 {c, 1, 1}, {α, 1.5, 1.5}, {z, 1, 1}]
```




```
(*meat market p1 vs. conglom p1*)

Reduce[((c + α)/2) > ((c + α + 2 c δ - α θ)/(2 + 2 δ - θ)) && 0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < c < α]

0 < θ < 1 && 0 < δ < θ && α > 0 && 0 < c < α

(*meat market p1 vs submarket monop p1*)

Reduce[((c + α)/2) > (1/2 (c + α + (4 (c - α) (-1 + δ) δ (1 + 4 δ + θ))/(-2 + 4 δ² + 8 δ³ - 3 θ + θ³ + 2 δ (-3 + θ) (1 + θ)))) &&
  0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < c < α]

0 < θ < 1 && 0 < δ < θ && α > 0 && 0 < c < α

(*meat market p3 vs. submarket monop p3*)
```

```
Reduce[
 ((c + α)/2) > ((-α (-1 + 2 δ - θ) (-1 + θ) (1 + 3 δ + θ) + c (-1 + δ) (1 + 2 δ + θ) (1 + 4 δ + θ)) / (-2 +
       4 δ^2 + 8 δ^3 - 3 θ + θ^3 + 2 δ (-3 + θ) (1 + θ))) && 0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < c < α]
```

```
0 < θ < 1 && 0 < δ < θ && α > 0 && 0 < c < α
```

```
(*multiproduct 2,4,6  vs base*)
```

```
Reduce[((-α (-1 + 2 δ - θ) (-1 + θ) (2 + 6 δ + θ) + c (1 + 2 δ) (6 δ (-1 + 2 δ) - (1 + θ) (2 + θ))) /
     (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))) >
   ((α (-1 + 2 δ - θ) (-1 + θ) + c (1 + 2 δ - 4 δ^2 + θ)) / (2 - 4 δ^2 + θ + 2 δ θ - θ^2)) && 0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < c < α]
```

```
0 < θ < 1 && 0 < δ < θ && α > 0 && 0 < c < α
```

```
(*multiproduct 135 p1 vs multiproduct p2*)
```

```
Reduce[
 ((α (-1 + θ) (-12 δ^2 + 2 δ (2 + θ) + (1 + θ) (2 + θ)) + c (24 δ^3 - (1 + θ) (2 + θ) - 2 δ (4 + 5 θ))) /
     (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))) >
   ((-α (-1 + 2 δ - θ) (-1 + θ) (2 + 6 δ + θ) + c (1 + 2 δ) (6 δ (-1 + 2 δ) - (1 + θ) (2 + θ))) /
     (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))) &&
  0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < c < α]
```

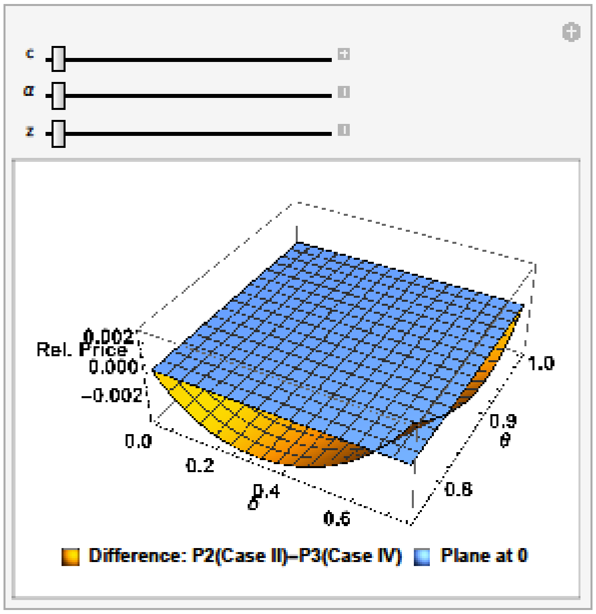
```
0 < θ < 1 && 0 < δ < θ && α > 0 && 0 < c < α
```

```
(*multiproduct p2 vs. submarket fringe p3*)
```

```
Reduce[((-α (-1 + 2 δ - θ) (-1 + θ) (2 + 6 δ + θ) + c (1 + 2 δ) (6 δ (-1 + 2 δ) - (1 + θ) (2 + θ))) /
     (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))) >
   ((-α (-1 + 2 δ - θ) (-1 + θ) (1 + 3 δ + θ) + c (-1 + δ) (1 + 2 δ + θ) (1 + 4 δ + θ)) /
     (-2 + 4 δ^2 + 8 δ^3 - 3 θ + θ^3 + 2 δ (-3 + θ) (1 + θ))) &&
  0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < c < α]
```

```
(0 < θ ≤ Root[-1 + 26 #1 - 94 #1^2 + 180 #1^3 + 51 #1^4 &, 2] &&
   Root[-3 θ^2 - 3 θ^3 + (4 - 4 θ - 18 θ^2) #1 + (20 - 20 θ) #1^2 + 24 #1^3 &, 3] < δ < θ &&
   α > 0 && 0 < c < α) || (Root[-1 + 26 #1 - 94 #1^2 + 180 #1^3 + 51 #1^4 &, 2] < θ < 1 &&
   Root[-3 θ^2 - 3 θ^3 + (4 - 4 θ - 18 θ^2) #1 + (20 - 20 θ) #1^2 + 24 #1^3 &, 1] < δ < θ &&
   α > 0 && 0 < c < α)
```

```
Manipulate[Plot3D[
  {(((-α (-1 + 2 δ - θ) (-1 + θ) (2 + 6 δ + θ) + c (1 + 2 δ) (6 δ (-1 + 2 δ) - (1 + θ) (2 + θ))) /
        (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))) -
      ((-α (-1 + 2 δ - θ) (-1 + θ) (1 + 3 δ + θ) + c (-1 + δ) (1 + 2 δ + θ) (1 + 4 δ + θ)) /
        (-2 + 4 δ^2 + 8 δ^3 - 3 θ + θ^3 + 2 δ (-3 + θ) (1 + θ)))), 0}, {δ, 0, .75},
  {θ, .75, 1}, PlotLabel → "", AxesLabel → {"δ", "θ", "Rel. Price"},
  AxesStyle → Larger,
  LabelStyle → Bold, PlotLegends →
   Placed[{"Difference: P2(Case II)-P3(Case IV)", "Plane at 0"}, Below]],
 {c, 1, 1}, {α, 1.5, 1.5}, {z, 1, 1}]
```




```
(*submarket fringe p3 vs submarket p1*)

Reduce[((-α (-1 + 2 δ - θ) (-1 + θ) (1 + 3 δ + θ) + c (-1 + δ) (1 + 2 δ + θ) (1 + 4 δ + θ)) /
     (-2 + 4 δ^2 + 8 δ^3 - 3 θ + θ^3 + 2 δ (-3 + θ) (1 + θ))) >
   (1/2 (c + α + (4 (c - α) (-1 + δ) δ (1 + 4 δ + θ)) / (-2 + 4 δ^2 + 8 δ^3 - 3 θ + θ^3 + 2 δ (-3 + θ) (1 + θ)))) &&
  0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < c < α]

False

(*QUANTITY COMPARISON TABLE*)

(*Quantities*)
```

```
(*Compstat: multiproduct135 q1 vs. q1 premerger*) (*&&0<δ<1&&δ<θ<1&&α>0&&0<c<α*)
Reduce[(-(((c-α) (1+2δ) (12δ^2-2δ (2+θ)-(1+θ) (2+θ)))/((1+4δ+θ)
        (24δ^3-12δ^2 (-1+θ)+(-2+θ) (1+θ) (2+θ)+2δ (-6+(-4+θ) θ))))) >
   (-((c-α) (-1-2δ+4δ^2-θ))/((1+4δ+θ) (4δ^2-2δθ+(-2+θ) (1+θ)))) &&0 < δ < 1 &&δ < θ <
   1 &&α > 0 &&0 < c < α]
False
```

```
(*Compstat: Conglom q1 vs. Premerger q1*)
Reduce[(-((c-α) (1+2δ))/((2+2δ-θ) (1+4δ+θ))) > (-((c-α) (-1-2δ+4δ^2-θ))/((1+4δ+θ) (4δ^2-2δθ+(-2+θ) (1+θ)))) &&
  0 < δ < 1 &&δ < θ < 1 &&α > 0 &&0 < c < α]
False
```

```
(*Compstat:Monopoly q1 vs. premerger q1*)
Reduce[(-(((c-α) (1+2δ+θ) (4δ^2+(-2+θ) (1+θ)+δ (-6+4θ)))/
       (2 (1+4δ+θ) (-2+4δ^2+8δ^3-3θ+θ^3+2δ (-3+θ) (1+θ))))) >
   (-((c-α) (-1-2δ+4δ^2-θ))/((1+4δ+θ) (4δ^2-2δθ+(-2+θ) (1+θ)))) &&0 < δ < 1 &&δ < θ < 1 &&α > 0 &&0 < c < α]
False
```

```
(*compstat: Monop q6 vs. Premerger q6*)
Reduce[(-(((c-α) (-1-2δ+4δ^2-θ) (1+3δ+θ))/
       ((1+4δ+θ) (-2+4δ^2+8δ^3-3θ+θ^3+2δ (-3+θ) (1+θ))))) >
   (-((c-α) (-1-2δ+4δ^2-θ))/((1+4δ+θ) (4δ^2-2δθ+(-2+θ) (1+θ)))) &&0 < δ < 1 &&δ < θ < 1 &&α > 0 &&0 < c < α]
0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < c < α
```

```
(*compstat: q1 protein monop vs q1 premerger*)
Reduce[((-c+α)/(2 (1+4δ+θ))) > (-((c-α) (-1-2δ+4δ^2-θ))/((1+4δ+θ) (4δ^2-2δθ+(-2+θ) (1+θ)))) &&
  0 < δ < 1 &&δ < θ < 1 &&α > 0 &&0 < c < α]
False
```

```
(*compstat: conglom q1 vs  Multiproduct q1*)
```

```
Reduce[(-((c - α) (1 + 2 δ))/((2 + 2 δ - θ) (1 + 4 δ + θ))) >
   (-(((c - α) (1 + 2 δ) (12 δ^2 - 2 δ (2 + θ) - (1 + θ) (2 + θ))) / ((1 + 4 δ + θ)
        (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))))) &&
  0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < c < α]

0 < θ < 1 && 0 < δ < θ && α > 0 && 0 < c < α

(*compstat: conglom q2 vs. multiproduct q2*)

Reduce[
 (-((c - α) (1 + 2 δ))/((2 + 2 δ - θ) (1 + 4 δ + θ))) > (-(((c - α) (-1 - 2 δ + 4 δ^2 - θ) (2 + 6 δ + θ)) / ((1 + 4 δ + θ)
        (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))))) &&
  0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < c < α]

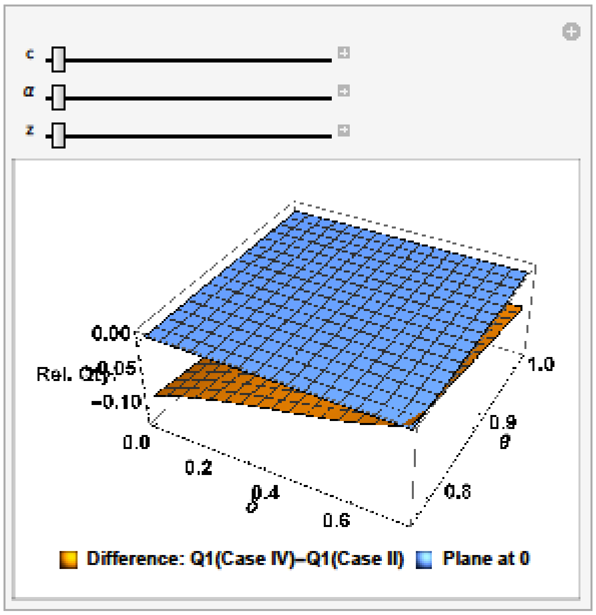
False

(*compstat: submarket monop q1 vs  Multiproduct q1*)

Reduce[(-(((c - α) (1 + 2 δ + θ) (4 δ^2 + (-2 + θ) (1 + θ) + δ (-6 + 4 θ))) /
        (2 (1 + 4 δ + θ) (-2 + 4 δ^2 + 8 δ^3 - 3 θ + θ^3 + 2 δ (-3 + θ) (1 + θ))))) >
   (-(((c - α) (1 + 2 δ) (12 δ^2 - 2 δ (2 + θ) - (1 + θ) (2 + θ))) / ((1 + 4 δ + θ)
        (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))))) &&
  0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < c < α]

(0 < θ ≤ Root[-323 584 + 1 961 728 #1 + 28 813 824 #1^2 + 94 403 584 #1^3 +
       263 944 704 #1^4 + 683 816 944 #1^5 + 534 514 964 #1^6 - 913 874 480 #1^7 -
       1 387 986 349 #1^8 + 356 268 511 #1^9 + 1 013 865 120 #1^10 - 2 821 312 #1^11 -
       282 280 888 #1^12 - 25 337 648 #1^13 + 24 635 440 #1^14 + 4 256 272 #1^15 &, 5] &&
   Root[4 θ + 12 θ^2 + 11 θ^3 + θ^4 - 3 θ^5 - θ^6 + (-8 + 24 θ + 82 θ^2 + 48 θ^3 - 14 θ^4 - 12 θ^5) #1 +
       (-64 + 28 θ + 152 θ^2 + 32 θ^3 - 28 θ^4) #1^2 + (-136 - 128 θ + 48 θ^3) #1^3 +
       (48 - 432 θ + 48 θ^2) #1^4 + (352 - 256 θ) #1^5 + 192 #1^6 &, 5] < δ < θ && α > 0 && 0 < c < α) ||
 (Root[-323 584 + 1 961 728 #1 + 28 813 824 #1^2 + 94 403 584 #1^3 + 263 944 704 #1^4 +
       683 816 944 #1^5 + 534 514 964 #1^6 - 913 874 480 #1^7 - 1 387 986 349 #1^8 +
       356 268 511 #1^9 + 1 013 865 120 #1^10 - 2 821 312 #1^11 - 282 280 888 #1^12 -
       25 337 648 #1^13 + 24 635 440 #1^14 + 4 256 272 #1^15 &, 5] < θ < 1 &&
   Root[4 θ + 12 θ^2 + 11 θ^3 + θ^4 - 3 θ^5 - θ^6 + (-8 + 24 θ + 82 θ^2 + 48 θ^3 - 14 θ^4 - 12 θ^5) #1 +
       (-64 + 28 θ + 152 θ^2 + 32 θ^3 - 28 θ^4) #1^2 + (-136 - 128 θ + 48 θ^3) #1^3 +
       (48 - 432 θ + 48 θ^2) #1^4 + (352 - 256 θ) #1^5 + 192 #1^6 &, 3] < δ < θ && α > 0 && 0 < c < α)
```

```
Manipulate[
 Plot3D[{(-(((c - α) (1 + 2 δ + θ) (4 δ² + (-2 + θ) (1 + θ) + δ (-6 + 4 θ))) / (2 (1 + 4 δ + θ)
          (-2 + 4 δ² + 8 δ³ - 3 θ + θ³ + 2 δ (-3 + θ) (1 + θ))))) -
    (-(((c - α) (1 + 2 δ) (12 δ² - 2 δ (2 + θ) - (1 + θ) (2 + θ))) / ((1 + 4 δ + θ)
          (24 δ³ - 12 δ² (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))))), 0},
  {δ, 0, .75}, {θ, .75, 1}, PlotLabel → "", AxesLabel → {"δ", "θ", "Rel. Qty."},
  AxesStyle → Larger,
  LabelStyle → Bold,
  PlotLegends →
   Placed[{"Difference: Q1(Case IV)-Q1(Case II)", "Plane at 0"}, Below]],
 {c, 1, 1}, {α, 1.5, 1.5}, {z, 1, 1}]
```




```
(*compstat: submarket monop q2 vs. multiproduct q2*)
Reduce[(-(((c - α) (1 + 2 δ + θ) (4 δ² + (-2 + θ) (1 + θ) + δ (-6 + 4 θ))) /
        (2 (1 + 4 δ + θ) (-2 + 4 δ² + 8 δ³ - 3 θ + θ³ + 2 δ (-3 + θ) (1 + θ))))) >
   (-(((c - α) (-1 - 2 δ + 4 δ² - θ) (2 + 6 δ + θ)) / ((1 + 4 δ + θ)
          (24 δ³ - 12 δ² (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))))) &&
  0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < c < α]
False
```

```
(*submarket submarket monop fringe q3 vs. multiproduct q3*)
Reduce[(-(((c-α) (-1-2δ+4δ^2-θ) (1+3δ+θ))/
        ((1+4δ+θ) (-2+4δ^2+8δ^3-3θ+θ^3+2δ (-3+θ) (1+θ))))) >
   (-(((c-α) (1+2δ) (12δ^2-2δ (2+θ) - (1+θ) (2+θ)))/((1+4δ+θ)
          (24δ^3-12δ^2 (-1+θ) + (-2+θ) (1+θ) (2+θ) + 2δ (-6+(-4+θ) θ))))) &&
  0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < c < α]

0 < θ < 1 && 0 < δ < θ && α > 0 && 0 < c < α

(*meat market q1 vs. multiproduct q1*)
Reduce[
 ((-c+α)/(2 (1+4δ+θ))) > (-(((c-α) (1+2δ) (12δ^2-2δ (2+θ) - (1+θ) (2+θ)))/((1+4δ+θ)
          (24δ^3-12δ^2 (-1+θ) + (-2+θ) (1+θ) (2+θ) + 2δ (-6+(-4+θ) θ))))) &&
  0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < c < α]

False

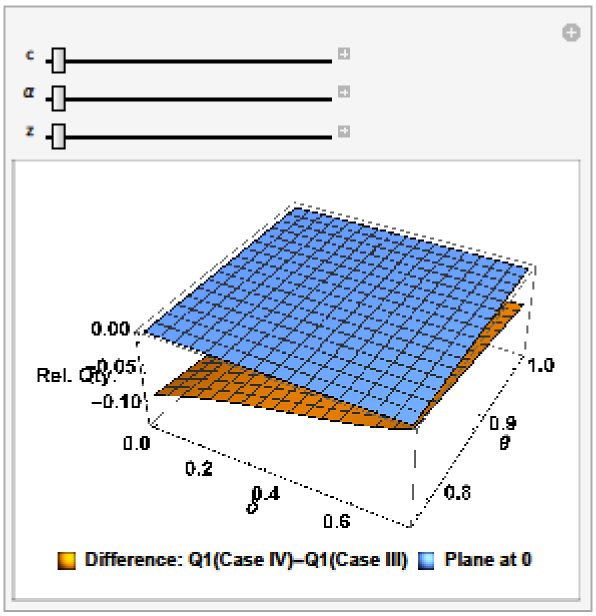
(*meat market q2 vs. multiproduct q2*)
Reduce[((-c+α)/(2 (1+4δ+θ))) > (-(((c-α) (-1-2δ+4δ^2-θ) (2+6δ+θ))/((1+4δ+θ)
          (24δ^3-12δ^2 (-1+θ) + (-2+θ) (1+θ) (2+θ) + 2δ (-6+(-4+θ) θ))))) &&
  0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < c < α]

False

(*submarket monop q1 vs. conglom q1*)
Reduce[(-(((c-α) (1+2δ+θ) (4δ^2+(-2+θ) (1+θ) + δ (-6+4θ)))/
        (2 (1+4δ+θ) (-2+4δ^2+8δ^3-3θ+θ^3+2δ (-3+θ) (1+θ))))) >
   (-((c-α) (1+2δ))/((2+2δ-θ) (1+4δ+θ))) && 0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < c < α]

0 < θ < 1/34 (3+√145) && Root[
   -2θ-3θ^2+θ^4 + (4-16θ-12θ^2+8θ^3) #1 + (20-40θ+8θ^2) #1^2 + (32-16θ) #1^3 + 16 #1^4 &,
   2] < δ < θ && α > 0 && 0 < c < α
```

```
Manipulate[
 Plot3D[{(-(((c - α) (1 + 2 δ + θ) (4 δ^2 + (-2 + θ) (1 + θ) + δ (-6 + 4 θ))) / (2 (1 + 4 δ + θ)
        (-2 + 4 δ^2 + 8 δ^3 - 3 θ + θ^3 + 2 δ (-3 + θ) (1 + θ))))) - (-((c - α) (1 + 2 δ))/((2 + 2 δ - θ) (1 + 4 δ + θ))),
   0}, {δ, 0, .75}, {θ, .75, 1}, PlotLabel → "", AxesLabel → {"δ", "θ", "Rel. Qty."},
  AxesStyle → Larger, LabelStyle → Bold, PlotLegends →
   Placed[{"Difference: Q1(Case IV)-Q1(Case III)", "Plane at 0"}, Below]],
 {c, 1, 1}, {α, 1.5, 1.5}, {z, 1, 1}]
```




```
(*submarket monop q3 vs. conglom q3*)

Reduce[(-(((c - α) (-1 - 2 δ + 4 δ^2 - θ) (1 + 3 δ + θ)) /
      ((1 + 4 δ + θ) (-2 + 4 δ^2 + 8 δ^3 - 3 θ + θ^3 + 2 δ (-3 + θ) (1 + θ))))) >
   (-((c - α) (1 + 2 δ))/((2 + 2 δ - θ) (1 + 4 δ + θ))) && 0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < c < α]

0 < θ < 1 && 0 < δ < θ && α > 0 && 0 < c < α

(*meat market q1 vs. conglom q1*)
```

```
Reduce[
 ((-c + α)/(2 (1 + 4 δ + θ))) > (-((c - α) (1 + 2 δ))/((2 + 2 δ - θ) (1 + 4 δ + θ))) && 0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < c < α]
```

```
False
```

```
(*meat market q1 vs submarket monop q1*)
```

```
Reduce[((-c + α)/(2 (1 + 4 δ + θ))) > (-(((c - α) (1 + 2 δ + θ) (4 δ^2 + (-2 + θ) (1 + θ) + δ (-6 + 4 θ)))/
      (2 (1 + 4 δ + θ) (-2 + 4 δ^2 + 8 δ^3 - 3 θ + θ^3 + 2 δ (-3 + θ) (1 + θ))))) &&
  0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < c < α]
```

```
False
```

```
(*meat market q3 vs. submarket monop q3*)
```

```
Reduce[((-c + α)/(2 (1 + 4 δ + θ))) > (-(((c - α) (-1 - 2 δ + 4 δ^2 - θ) (1 + 3 δ + θ))/
      ((1 + 4 δ + θ) (-2 + 4 δ^2 + 8 δ^3 - 3 θ + θ^3 + 2 δ (-3 + θ) (1 + θ))))) &&
  0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < c < α]
```

```
False
```

```
(*multiproduct 2,4,6 q vs base*)
```

```
Reduce[(-(((c - α) (-1 - 2 δ + 4 δ^2 - θ) (2 + 6 δ + θ))/((1 + 4 δ + θ)
        (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))))) >
   (-((c - α) (-1 - 2 δ + 4 δ^2 - θ))/((1 + 4 δ + θ) (4 δ^2 - 2 δ θ + (-2 + θ) (1 + θ)))) && 0 < δ < 1 && δ < θ <
   1 && α > 0 && 0 < c < α]
```

```
0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < c < α
```

```
(*multiproduct 135 q1 vs multiproduct q2*)
```

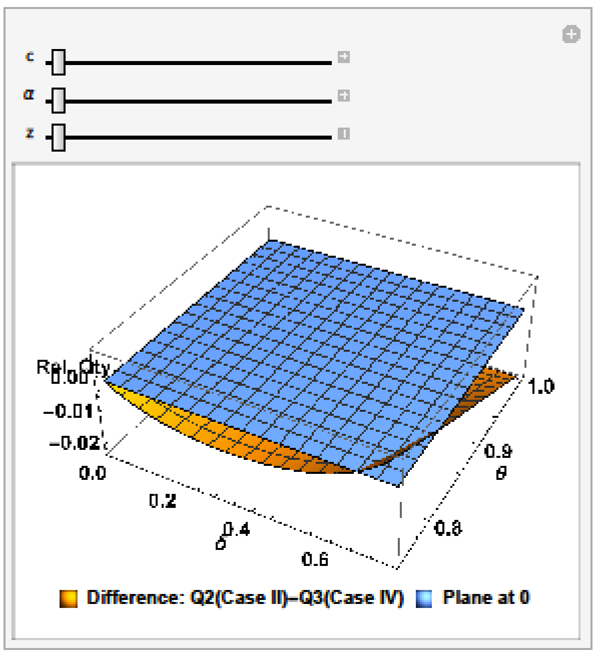
```
Reduce[(-(((c - α) (1 + 2 δ) (12 δ^2 - 2 δ (2 + θ) - (1 + θ) (2 + θ)))/((1 + 4 δ + θ)
        (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))))) >
   (-(((c - α) (-1 - 2 δ + 4 δ^2 - θ) (2 + 6 δ + θ))/((1 + 4 δ + θ)
        (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))))) &&
  0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < c < α]
```

```
False
```

```
(*multiproduct q2 vs. submarket fringe q3*)
```

```
Reduce[(-(((c - α) (-1 - 2 δ + 4 δ^2 - θ) (2 + 6 δ + θ)) / ((1 + 4 δ + θ)
        (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))))) >
   (-(((c - α) (-1 - 2 δ + 4 δ^2 - θ) (1 + 3 δ + θ)) / ((1 + 4 δ + θ) (-2 + 4 δ^2 + 8 δ^3 - 3 θ +
          θ^3 + 2 δ (-3 + θ) (1 + θ))))) && 0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < c < α]
0 < δ < 1 && δ < θ < Root[-4 δ - 20 δ^2 - 24 δ^3 + (4 δ + 20 δ^2) #1 + (3 + 18 δ) #1^2 + 3 #1^3 &, 3] &&
 α > 0 && 0 < c < α

Manipulate[
 Plot3D[{((-(((c - α) (-1 - 2 δ + 4 δ^2 - θ) (2 + 6 δ + θ)) / ((1 + 4 δ + θ) (24 δ^3 - 12 δ^2 (-1 + θ) +
            (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))))) -
     (-(((c - α) (-1 - 2 δ + 4 δ^2 - θ) (1 + 3 δ + θ)) / ((1 + 4 δ + θ)
            (-2 + 4 δ^2 + 8 δ^3 - 3 θ + θ^3 + 2 δ (-3 + θ) (1 + θ)))))), 0},
  {δ, 0, .75}, {θ, .75, 1}, PlotLabel → "", AxesLabel → {"δ", "θ", "Rel. Qty."},
  AxesStyle → Larger,
  LabelStyle → Bold,
  PlotLegends →
   Placed[{"Difference: Q2(Case II)-Q3(Case IV)", "Plane at 0"}, Below]],
 {c, 1, 1}, {α, 1.5, 1.5},
 {z, 1, 1}]
```




```
(*submarket fringe q3 vs submarket q1*)
```

```
Reduce[(-(((c - α) (-1 - 2 δ + 4 δ^2 - θ) (1 + 3 δ + θ)) /
        ((1 + 4 δ + θ) (-2 + 4 δ^2 + 8 δ^3 - 3 θ + θ^3 + 2 δ (-3 + θ) (1 + θ))))) >
   (-(((c - α) (1 + 2 δ + θ) (4 δ^2 + (-2 + θ) (1 + θ) + δ (-6 + 4 θ))) /
        (2 (1 + 4 δ + θ) (-2 + 4 δ^2 + 8 δ^3 - 3 θ + θ^3 + 2 δ (-3 + θ) (1 + θ))))) &&
  0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < c < α]

0 < θ < 1 && 0 < δ < θ && α > 0 && 0 < c < α

(*PROFIT COMPARISON TABLE*)

(*Profit*)

(*Compstat: multiproduct135 profit vs. profit 1 premerger*)
(*&&0<δ<1&&δ<θ<1&&α>0&&0<c<α*)

Reduce[(-((3 (c - α)^2 (1 + 2 δ) (-1 + θ) (-12 δ^2 + 2 δ (2 + θ) + (1 + θ) (2 + θ))^2) / ((1 + 4 δ + θ)
        (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))^2))) >
   (-((c - α)^2 (-1 + 2 δ - θ) (-1 - 2 δ + 4 δ^2 - θ) (-1 + θ)) / ((1 + 4 δ + θ) (2 - 4 δ^2 + θ + 2 δ θ - θ^2)^2)) && 0 < δ < 1 && δ <
   θ < 1 && α > 0 && 0 < c < α]

0 < θ < 1 && 0 < δ < θ && α > 0 && 0 < c < α

(*Compstat: multiproduct2,4,or6 profit vs. profit 1 premerger*)

Reduce[(-(((c - α)^2 (-1 + 2 δ - θ) (-1 - 2 δ + 4 δ^2 - θ) (-1 + θ) (2 + 6 δ + θ)^2) / ((1 + 4 δ + θ)
        (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))^2))) >
   (-((c - α)^2 (-1 + 2 δ - θ) (-1 - 2 δ + 4 δ^2 - θ) (-1 + θ)) / ((1 + 4 δ + θ) (2 - 4 δ^2 + θ + 2 δ θ - θ^2)^2)) && 0 < δ < 1 && δ <
   θ < 1 && α > 0 && 0 < c < α]

0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < c < α

(*Compstat: Conglom profit135 vs. Premerger profit 1*)

Reduce[
 (-(3 (c - α)^2 (1 + 2 δ) (-1 + θ)) / ((-2 - 2 δ + θ)^2 (1 + 4 δ + θ))) > (-((c - α)^2 (-1 + 2 δ - θ) (-1 - 2 δ + 4 δ^2 - θ) (-1 + θ)) / ((1 + 4 δ + θ) (2 - 4 δ^2 + θ + 2 δ θ - θ^2)^2)) &&
  0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < c < α]

0 < θ < 1 && 0 < δ < θ && α > 0 && 0 < c < α

(*Compstat:submarket monopoly profit12 vs. premerger profit 1*)
```

```
Reduce[(-(((c - α)^2 (-1 + 2 δ - θ) (1 + 2 δ + θ) (4 δ^2 + (-2 + θ) (1 + θ) + δ (-6 + 4 θ))^2) /
      (2 (1 + 4 δ + θ) (-2 + 4 δ^2 + 8 δ^3 - 3 θ + θ^3 + 2 δ (-3 + θ) (1 + θ))^2))) >
  (-((c - α)^2 (-1 + 2 δ - θ) (-1 - 2 δ + 4 δ^2 - θ) (-1 + θ)) / ((1 + 4 δ + θ) (2 - 4 δ^2 + θ + 2 δ θ - θ^2)^2)) &&
  0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < c < α]

0 < θ < 1 && 0 < δ < θ && α > 0 && 0 < c < α
```

```
(*compstat: submarket monopoly fringe2,4,5,or 6 profit vs. Premerger profit 1*)
```

```
Reduce[(-(((c - α)^2 (-1 + 2 δ - θ) (-1 - 2 δ + 4 δ^2 - θ) (-1 + θ) (1 + 3 δ + θ)^2) /
      ((1 + 4 δ + θ) (-2 + 4 δ^2 + 8 δ^3 - 3 θ + θ^3 + 2 δ (-3 + θ) (1 + θ))^2))) >
  (-((c - α)^2 (-1 + 2 δ - θ) (-1 - 2 δ + 4 δ^2 - θ) (-1 + θ)) / ((1 + 4 δ + θ) (2 - 4 δ^2 + θ + 2 δ θ - θ^2)^2)) &&
  0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < c < α]

0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < c < α
```

```
(*compstat: meat monopolist profit vs profit 1 premerger*)
```

```
Reduce[(3 (c - α)^2 / (2 (1 + 4 δ + θ))) > (-((c - α)^2 (-1 + 2 δ - θ) (-1 - 2 δ + 4 δ^2 - θ) (-1 + θ)) / ((1 + 4 δ + θ) (2 - 4 δ^2 + θ + 2 δ θ - θ^2)^2)) &&
  0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < c < α]

0 < θ < 1 && 0 < δ < θ && α > 0 && 0 < c < α
```

```
(*compstat:Multiproduct profit 135 vs multiproduct profit2,4,or6*)
```

```
Reduce[(-((3 (c - α)^2 (1 + 2 δ) (-1 + θ) (-12 δ^2 + 2 δ (2 + θ) + (1 + θ) (2 + θ))^2) / ((1 + 4 δ + θ)
      (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))^2))) >
  (-(((c - α)^2 (-1 + 2 δ - θ) (-1 - 2 δ + 4 δ^2 - θ) (-1 + θ) (2 + 6 δ + θ)^2) / ((1 + 4 δ + θ)
      (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))^2))) &&
  0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < c < α]

0 < θ < 1 && 0 < δ < θ && α > 0 && 0 < c < α
```

```
(*compstat: Multiproduct profit 135 vs. conglomerate135 profit*)
```

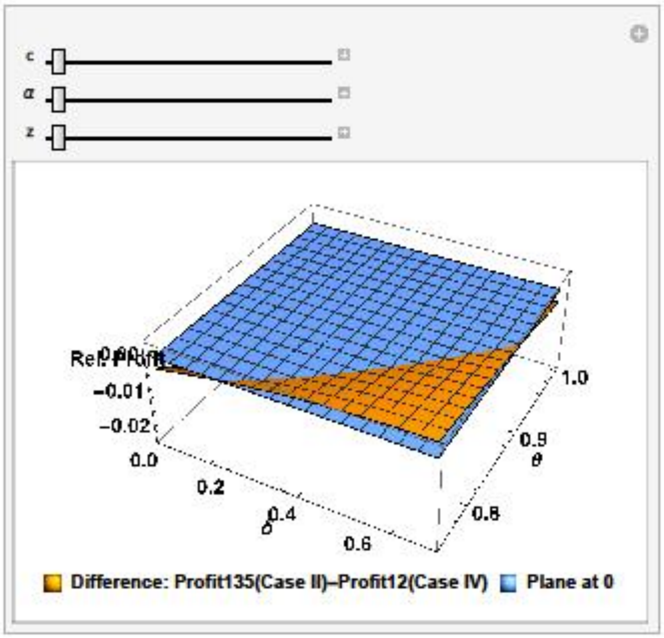
```
Reduce[(-((3 (c - α)^2 (1 + 2 δ) (-1 + θ) (-12 δ^2 + 2 δ (2 + θ) + (1 + θ) (2 + θ))^2) / ((1 + 4 δ + θ)
      (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))^2))) >
  (-(3 (c - α)^2 (1 + 2 δ) (-1 + θ)) / ((-2 - 2 δ + θ)^2 (1 + 4 δ + θ))) && 0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < c < α]

False
```

```
(*compstat: Multiproduct profit 135 vs. Submarket monopolist12 profit*)
```

```
Reduce[(-((3 (c - α)^2 (1 + 2 δ) (-1 + θ) (-12 δ^2 + 2 δ (2 + θ) + (1 + θ) (2 + θ))^2) / ((1 + 4 δ + θ)
        (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))^2))) >
  (-(((c - α)^2 (-1 + 2 δ - θ) (1 + 2 δ + θ) (4 δ^2 + (-2 + θ) (1 + θ) + δ (-6 + 4 θ))^2) /
      (2 (1 + 4 δ + θ) (-2 + 4 δ^2 + 8 δ^3 - 3 θ + θ^3 + 2 δ (-3 + θ) (1 + θ))^2))) &&
   0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < c < α]
(0 < θ ≤ -1 + √3 && 0 < δ < θ && α > 0 && 0 < c < α) ||
 (-1 + √3 < θ < 1 && Root[32 + 160 θ + 256 θ^2 - 16 θ^3 - 534 θ^4 - 630 θ^5 - 183 θ^6 + 180 θ^7 + 159 θ^8 +
        26 θ^9 - 17 θ^10 - 8 θ^11 - θ^12 + (384 + 1632 θ + 1968 θ^2 - 1176 θ^3 - 4980 θ^4 - 4224 θ^5 -
           372 θ^6 + 1488 θ^7 + 804 θ^8 - 108 θ^10 - 24 θ^11) #1 + (1376 + 4800 θ + 4168 θ^2 -
           4632 θ^3 - 11 708 θ^4 - 7816 θ^5 + 384 θ^6 + 2984 θ^7 + 1012 θ^8 - 232 θ^9 - 128 θ^10) #1^2 +
        (-192 - 1600 θ + 1136 θ^2 + 10 256 θ^3 + 9184 θ^4 - 2384 θ^5 - 4880 θ^6 - 592 θ^7 + 512 θ^8 + 80 θ^9)
         #1^3 + (-9728 - 28 160 θ - 80 θ^2 + 64 256 θ^3 + 56 224 θ^4 -
           3424 θ^5 - 17 568 θ^6 - 2432 θ^7 + 1392 θ^8) #1^4 +
        (-9216 - 29 376 θ - 13 248 θ^2 + 35 200 θ^3 + 36 288 θ^4 + 1600 θ^5 - 6912 θ^6 - 512 θ^7) #1^5 +
        (25 408 + 41 216 θ - 102 016 θ^2 - 149 248 θ^3 + 11 904 θ^4 + 36 864 θ^5 - 4096 θ^6) #1^6 +
        (31 488 + 103 936 θ - 105 984 θ^2 - 133 632 θ^3 + 14 336 θ^4 + 13 824 θ^5) #1^7 +
        (-28 416 + 44 544 θ + 222 976 θ^2 - 60 416 θ^3 - 5888 θ^4) #1^8 +
        (-46 080 - 79 872 θ + 273 408 θ^2 - 55 296 θ^3) #1^9 + (-9216 - 135 168 θ + 24 576 θ^2) #1^10 +
        (36 864 - 73 728 θ) #1^11 + 36 864 #1^12 &, 6] < δ < θ && α > 0 && 0 < c < α)
```

```
Manipulate[Plot3D[
  {(((- ((3 (c - α)^2 (1 + 2 δ) (-1 + θ) (-12 δ^2 + 2 δ (2 + θ) + (1 + θ) (2 + θ))^2) / ((1 + 4 δ + θ)
          (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))^2))) -
     (- (((c - α)^2 (-1 + 2 δ - θ) (1 + 2 δ + θ) (4 δ^2 + (-2 + θ) (1 + θ) + δ (-6 + 4 θ))^2) /
         (2 (1 + 4 δ + θ) (-2 + 4 δ^2 + 8 δ^3 - 3 θ + θ^3 + 2 δ (-3 + θ) (1 + θ))^2)))), 0},
  {δ, 0, .75}, {θ, .75, 1}, PlotLabel → "", AxesLabel → {"δ", "θ", "Rel. Profit"},
  AxesStyle → Larger,
  LabelStyle → Bold,
  PlotLegends →
   Placed[{"Difference: Profit135(Case II)-Profit12(Case IV)", "Plane at 0"},
    Below]], {c, 1, 1}, {α, 1.5, 1.5}, {z, 1, 1}]
```




```
(*compstat: Multiproduct profit 135 vs  submarket monopoly fringe2,
4,5,or 6 profit*)
```

```
Reduce[(-((3 (c - α)^2 (1 + 2 δ) (-1 + θ) (-12 δ^2 + 2 δ (2 + θ) + (1 + θ) (2 + θ))^2) / ((1 + 4 δ + θ)
      (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))^2))) >
   (-(((c - α)^2 (-1 + 2 δ - θ) (-1 - 2 δ + 4 δ^2 - θ) (-1 + θ) (1 + 3 δ + θ)^2) /
      ((1 + 4 δ + θ) (-2 + 4 δ^2 + 8 δ^3 - 3 θ + θ^3 + 2 δ (-3 + θ) (1 + θ))^2))) &&
  0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < c < α]
```

```
0 < θ < 1 && 0 < δ < θ && α > 0 && 0 < c < α
```

```
(*compstat: Multiproduct profit 135 vs. meatmonopolist profit*)
```

```
Reduce[(-((3 (c - α)^2 (1 + 2 δ) (-1 + θ) (-12 δ^2 + 2 δ (2 + θ) + (1 + θ) (2 + θ))^2) / ((1 + 4 δ + θ)
      (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))^2))) >
   (3 (c - α)^2 / (2 (1 + 4 δ + θ))) && 0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < c < α]
```

```
False
```

```
(* multiproduct profit2,4,or6 vs. conglomerate135 profit*)
```

```
Reduce[(-(((c - α)^2 (-1 + 2 δ - θ) (-1 - 2 δ + 4 δ^2 - θ) (-1 + θ) (2 + 6 δ + θ)^2) / ((1 + 4 δ + θ)
      (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))^2))) >
   (-(3 (c - α)^2 (1 + 2 δ) (-1 + θ)) / ((-2 - 2 δ + θ)^2 (1 + 4 δ + θ))) && 0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < c < α]
```

```
False
```

```
(*multiproduct profit2,4,or6 vs. Submarket monopolist12 profit*)
```

```
Reduce[(-(((c - α)^2 (-1 + 2 δ - θ) (-1 - 2 δ + 4 δ^2 - θ) (-1 + θ) (2 + 6 δ + θ)^2) / ((1 + 4 δ + θ)
      (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))^2))) >
   (-(((c - α)^2 (-1 + 2 δ - θ) (1 + 2 δ + θ) (4 δ^2 + (-2 + θ) (1 + θ) + δ (-6 + 4 θ))^2) /
      (2 (1 + 4 δ + θ) (-2 + 4 δ^2 + 8 δ^3 - 3 θ + θ^3 + 2 δ (-3 + θ) (1 + θ))^2))) &&
  0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < c < α]
```

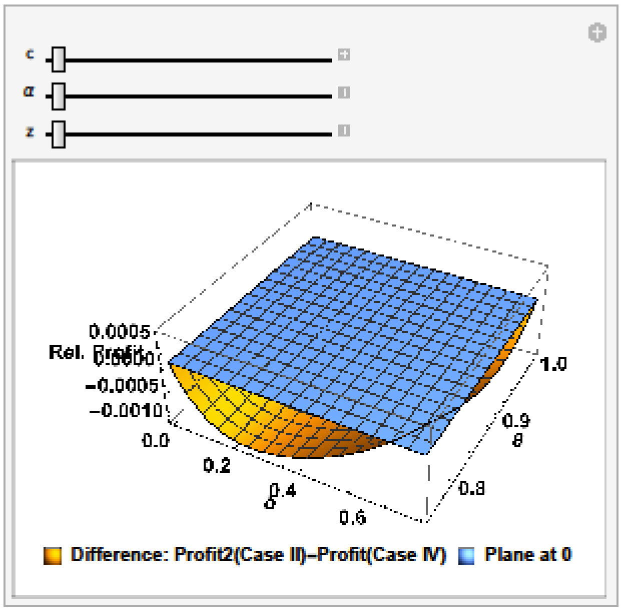
```
False
```

```
(*multiproduct profit2,4,or6 vs. submarket monopoly fringe2,4,5,or 6 profit*)
```

```
Reduce[(-(((c - α)^2 (-1 + 2 δ - θ) (-1 - 2 δ + 4 δ^2 - θ) (-1 + θ) (2 + 6 δ + θ)^2) / ((1 + 4 δ + θ)
      (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))^2))) >
   (-(((c - α)^2 (-1 + 2 δ - θ) (-1 - 2 δ + 4 δ^2 - θ) (-1 + θ) (1 + 3 δ + θ)^2) /
      ((1 + 4 δ + θ) (-2 + 4 δ^2 + 8 δ^3 - 3 θ + θ^3 + 2 δ (-3 + θ) (1 + θ))^2))) &&
  0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < c < α]
```

```
0 < δ < 1 && δ < θ < Root[-4 δ - 20 δ^2 - 24 δ^3 + (4 δ + 20 δ^2) #1 + (3 + 18 δ) #1^2 + 3 #1^3 &, 3] &&
 α > 0 && 0 < c < α
```

```
Manipulate[
 Plot3D[{((-(((c - α)² (-1 + 2 δ - θ) (-1 - 2 δ + 4 δ² - θ) (-1 + θ) (2 + 6 δ + θ)²) / ((1 + 4 δ + θ)
          (24 δ³ - 12 δ² (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))²))) -
     (-(((c - α)² (-1 + 2 δ - θ) (-1 - 2 δ + 4 δ² - θ) (-1 + θ) (1 + 3 δ + θ)²) /
         ((1 + 4 δ + θ) (-2 + 4 δ² + 8 δ³ - 3 θ + θ³ + 2 δ (-3 + θ) (1 + θ))²)))), 0},
  {δ, 0, .75}, {θ, .75, 1}, PlotLabel → "", AxesLabel → {"δ", "θ", "Rel. Profit"},
  AxesStyle → Larger,
  LabelStyle → Bold,
  PlotLegends →
   Placed[{"Difference: Profit2(Case II)-Profit(Case IV)", "Plane at 0"}, Below]],
 {c, 1, 1}, {α, 1.5, 1.5}, {z, 1, 1}]
```



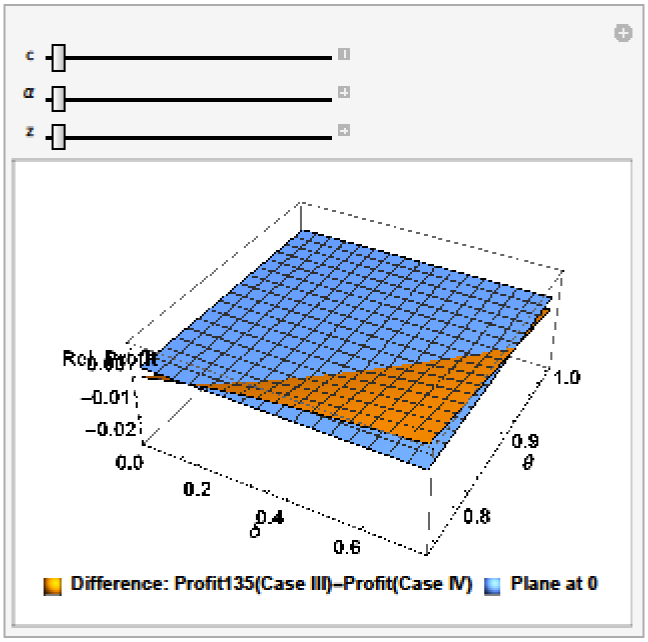

```
(*multiproduct profit2,4,or6 1 vs. meatmonopolist profit*)
Reduce[(-(((c - α)² (-1 + 2 δ - θ) (-1 - 2 δ + 4 δ² - θ) (-1 + θ) (2 + 6 δ + θ)²) / ((1 + 4 δ + θ)
       (24 δ³ - 12 δ² (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))²))) >
   (3 (c - α)² / (2 (1 + 4 δ + θ))) && 0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < c < α]
False

(*conglomerate135 profit vs. Submarket monopolist12 profit*)
```

$$\text{Reduce}\Big[\left(-\frac{3\,(c-\alpha)^2\,(1+2\,\delta)\,(-1+\theta)}{(-2-2\,\delta+\theta)^2\,(1+4\,\delta+\theta)}\right) >$$
$$\Big(-\Big(\Big((c-\alpha)^2\,(-1+2\,\delta-\theta)\,(1+2\,\delta+\theta)\,\big(4\,\delta^2+(-2+\theta)\,(1+\theta)+\delta\,(-6+4\,\theta)\big)^2\Big)\Big/$$
$$\Big(2\,(1+4\,\delta+\theta)\,\big(-2+4\,\delta^2+8\,\delta^3-3\,\theta+\theta^3+2\,\delta\,(-3+\theta)\,(1+\theta)\big)^2\Big)\Big)\Big)\;\&\&$$
$$0<\delta<1\;\&\&\;\delta<\theta<1\;\&\&\;\alpha>0\;\&\&\;0<c<\alpha\Big]$$

$$\left(0<\theta\leq -1+\sqrt{3}\;\&\&\;0<\delta<\theta\;\&\&\;\alpha>0\;\&\&\;0<c<\alpha\right)\;||$$
$$\Big(-1+\sqrt{3}<\theta<1\;\&\&\;\text{Root}\big[8+16\,\theta-10\,\theta^2-38\,\theta^3-13\,\theta^4+16\,\theta^5+8\,\theta^6-$$
$$2\,\theta^7-\theta^8+\left(64+104\,\theta-108\,\theta^2-244\,\theta^3-20\,\theta^4+108\,\theta^5+16\,\theta^6-16\,\theta^7\right)\#1+$$
$$\left(184+200\,\theta-360\,\theta^2-384\,\theta^3+152\,\theta^4+120\,\theta^5-40\,\theta^6\right)\#1^2+$$
$$\left(208-240\,\theta+96\,\theta^2+352\,\theta^3-192\,\theta^4\right)\#1^3+\left(-48-1248\,\theta+2400\,\theta^2-384\,\theta^3-192\,\theta^4\right)\#1^4+$$
$$\left(-256-1024\,\theta+1728\,\theta^2-576\,\theta^3\right)\#1^5+\left(128-640\,\theta-128\,\theta^2\right)\#1^6+$$
$$(512-512\,\theta)\,\#1^7+256\,\#1^8\,\&,\,4\big]<\delta<\theta\;\&\&\;\alpha>0\;\&\&\;0<c<\alpha\Big)$$

```
Manipulate[Plot3D[{((-(3 (c - α)^2 (1 + 2 δ) (-1 + θ))/((-2 - 2 δ + θ)^2 (1 + 4 δ + θ))) -
    (-(((c - α)^2 (-1 + 2 δ - θ) (1 + 2 δ + θ) (4 δ^2 + (-2 + θ) (1 + θ) + δ (-6 + 4 θ))^2)/
      (2 (1 + 4 δ + θ) (-2 + 4 δ^2 + 8 δ^3 - 3 θ + θ^3 + 2 δ (-3 + θ) (1 + θ))^2)))), 0},
  {δ, 0, .75}, {θ, .75, 1}, PlotLabel → "", AxesLabel → {"δ", "θ", "Rel. Profit"},
  AxesStyle → Larger,
  LabelStyle → Bold, PlotLegends →
   Placed[{"Difference: Profit135(Case III)-Profit(Case IV)", "Plane at 0"},
    Below]], {c, 1, 1}, {α, 1.5, 1.5}, {s, 1, 1}]
```




```
(*conglomerate135 profit  vs. submarket monopoly fringe2,4,5,or 6 profit*)
Reduce[(-(3 (c - α)^2 (1 + 2 δ) (-1 + θ))/((-2 - 2 δ + θ)^2 (1 + 4 δ + θ))) >
   (-(((c - α)^2 (-1 + 2 δ - θ) (-1 - 2 δ + 4 δ^2 - θ) (-1 + θ) (1 + 3 δ + θ)^2)/
      ((1 + 4 δ + θ) (-2 + 4 δ^2 + 8 δ^3 - 3 θ + θ^3 + 2 δ (-3 + θ) (1 + θ))^2))) &&
  0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < c < α]
0 < θ < 1 && 0 < δ < θ && α > 0 && 0 < c < α
```

```
(*conglomerate135 profit vs meatmonopolist profit*)
```

$$\text{Reduce}\left[\left(-\frac{3(c-\alpha)^2(1+2\delta)(-1+\theta)}{(-2-2\delta+\theta)^2(1+4\delta+\theta)}\right) > \left(\frac{3(c-\alpha)^2}{2(1+4\delta+\theta)}\right) \,\&\&\, 0<\delta<1 \,\&\&\, \delta<\theta<1 \,\&\&\, \alpha>0 \,\&\&\, 0<c<\alpha\right]$$

False

```
(*Submarket monopolist12 profit vs. submarket monopoly fringe2,4,5,or 6 profit*)
```

$$\text{Reduce}\Big[\Big(-\Big(\big((c-\alpha)^2(-1+2\delta-\theta)(1+2\delta+\theta)(4\delta^2+(-2+\theta)(1+\theta)+\delta(-6+4\theta))^2\big)\Big/ \big(2(1+4\delta+\theta)(-2+4\delta^2+8\delta^3-3\theta+\theta^3+2\delta(-3+\theta)(1+\theta))^2\big)\Big)\Big) > \Big(-\Big(\big((c-\alpha)^2(-1+2\delta-\theta)(-1-2\delta+4\delta^2-\theta)(-1+\theta)(1+3\delta+\theta)^2\big)\Big/ \big((1+4\delta+\theta)(-2+4\delta^2+8\delta^3-3\theta+\theta^3+2\delta(-3+\theta)(1+\theta))^2\big)\Big)\Big) \,\&\&\, 0<\delta<1 \,\&\&\, \delta<\theta<1 \,\&\&\, \alpha>0 \,\&\&\, 0<c<\alpha\Big]$$

$0<\theta<1 \,\&\&\, 0<\delta<\theta \,\&\&\, \alpha>0 \,\&\&\, 0<c<\alpha$

```
(*Submarket monopolist12 profit vs meatmonopolist profit*)
```

$$\text{Reduce}\Big[\Big(-\Big(\big((c-\alpha)^2(-1+2\delta-\theta)(1+2\delta+\theta)(4\delta^2+(-2+\theta)(1+\theta)+\delta(-6+4\theta))^2\big)\Big/ \big(2(1+4\delta+\theta)(-2+4\delta^2+8\delta^3-3\theta+\theta^3+2\delta(-3+\theta)(1+\theta))^2\big)\Big)\Big) > \left(\frac{3(c-\alpha)^2}{2(1+4\delta+\theta)}\right) \,\&\&\, 0<\delta<1 \,\&\&\, \delta<\theta<1 \,\&\&\, \alpha>0 \,\&\&\, 0<c<\alpha\Big]$$

False

```
(*submarket monopoly fringe2,4,5,or 6 profit vs meatmonopolist profit*)
```

$$\text{Reduce}\Big[\Big(-\Big(\big((c-\alpha)^2(-1+2\delta-\theta)(-1-2\delta+4\delta^2-\theta)(-1+\theta)(1+3\delta+\theta)^2\big)\Big/ \big((1+4\delta+\theta)(-2+4\delta^2+8\delta^3-3\theta+\theta^3+2\delta(-3+\theta)(1+\theta))^2\big)\Big)\Big) > \left(\frac{3(c-\alpha)^2}{2(1+4\delta+\theta)}\right) \,\&\&\, 0<\delta<1 \,\&\&\, \delta<\theta<1 \,\&\&\, \alpha>0 \,\&\&\, 0<c<\alpha\Big]$$

False

```
(*multiproduct 2,4,6 profit vs premerger profit 1*)
```

$$\text{Reduce}\Big[\Big(-\Big(\big((c-\alpha)^2(-1+2\delta-\theta)(-1-2\delta+4\delta^2-\theta)(-1+\theta)(2+6\delta+\theta)^2\big)\Big/ \big((1+4\delta+\theta)(24\delta^3-12\delta^2(-1+\theta)+(-2+\theta)(1+\theta)(2+\theta)+2\delta(-6+(-4+\theta)\theta))^2\big)\Big)\Big) > \left(-\frac{(c-\alpha)^2(-1+2\delta-\theta)(-1-2\delta+4\delta^2-\theta)(-1+\theta)}{(1+4\delta+\theta)(2-4\delta^2+\theta+2\delta\theta-\theta^2)^2}\right) \,\&\&\, 0<\delta<1 \,\&\&\, \delta<\theta<1 \,\&\&\, \alpha>0 \,\&\&\, 0<c<\alpha\Big]$$

$0<\delta<1 \,\&\&\, \delta<\theta<1 \,\&\&\, \alpha>0 \,\&\&\, 0<c<\alpha$

```
(*multiproduct 135 profit vs multiproduct profit24 or6*)
```

```
Reduce[(-((3 (c - α)^2 (1 + 2 δ) (-1 + θ) (-12 δ^2 + 2 δ (2 + θ) + (1 + θ) (2 + θ))^2) / ((1 + 4 δ + θ)
      (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))^2))) >
  (-(((c - α)^2 (-1 + 2 δ - θ) (-1 - 2 δ + 4 δ^2 - θ) (-1 + θ) (2 + 6 δ + θ)^2) / ((1 + 4 δ + θ)
      (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))^2))) &&
  0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < c < α]
```

```
0 < θ < 1 && 0 < δ < θ && α > 0 && 0 < c < α
```

```
(*multiproduct profit 2,4,6 vs. submarket fringe profit 3*)
```

```
Reduce[(-(((c - α)^2 (-1 + 2 δ - θ) (-1 - 2 δ + 4 δ^2 - θ) (-1 + θ) (2 + 6 δ + θ)^2) / ((1 + 4 δ + θ)
      (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))^2))) >
  (-(((c - α)^2 (-1 + 2 δ - θ) (-1 - 2 δ + 4 δ^2 - θ) (-1 + θ) (1 + 3 δ + θ)^2) /
     ((1 + 4 δ + θ) (-2 + 4 δ^2 + 8 δ^3 - 3 θ + θ^3 + 2 δ (-3 + θ) (1 + θ))^2))) &&
  0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < c < α]
```

```
0 < δ < 1 && δ < θ < Root[-4 δ - 20 δ^2 - 24 δ^3 + (4 δ + 20 δ^2) #1 + (3 + 18 δ) #1^2 + 3 #1^3 &, 3] &&
 α > 0 && 0 < c < α
```

```
(*submarket fringe profit3 vs submarket profit 12*)
```

```
Reduce[(-(((c - α)^2 (-1 + 2 δ - θ) (-1 - 2 δ + 4 δ^2 - θ) (-1 + θ) (1 + 3 δ + θ)^2) /
     ((1 + 4 δ + θ) (-2 + 4 δ^2 + 8 δ^3 - 3 θ + θ^3 + 2 δ (-3 + θ) (1 + θ))^2))) >
  (-(((c - α)^2 (-1 + 2 δ - θ) (1 + 2 δ + θ) (4 δ^2 + (-2 + θ) (1 + θ) + δ (-6 + 4 θ))^2) /
     (2 (1 + 4 δ + θ) (-2 + 4 δ^2 + 8 δ^3 - 3 θ + θ^3 + 2 δ (-3 + θ) (1 + θ))^2))) &&
  0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < c < α]
```

```
False
```

```
(*PRODUCER SURPLUS COMPARISON TABLE*)
```

```
(*Comparison tables: Producer Surplus  (*&&0<δ<1&&δ<θ<1&&α>0&&0<c<α*) *)
```

```
(*Case 2 vs case 1*)
```

```
Reduce[((3 (c - α)^2 (-1 + θ) (-(-1 + 2 δ - θ) (-1 - 2 δ + 4 δ^2 - θ) (2 + 6 δ + θ)^2 -
      (1 + 2 δ) (-12 δ^2 + 2 δ (2 + θ) + (1 + θ) (2 + θ))^2)) /
    ((1 + 4 δ + θ) (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))^2)) >
  (-(6 (c - α)^2 (-1 + 2 δ - θ) (-1 - 2 δ + 4 δ^2 - θ) (-1 + θ)) /
     ((1 + 4 δ + θ) (2 - 4 δ^2 + θ + 2 δ θ - θ^2)^2)) &&
  0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < c < α]
```

```
0 < θ < 1 && 0 < δ < θ && α > 0 && 0 < c < α
```

```
(*Case 3 vs case 1*)
```

```
Reduce[
 (-(6 (c - α)^2 (1 + 2 δ) (-1 + θ))/((-2 - 2 δ + θ)^2 (1 + 4 δ + θ))) > (-(6 (c - α)^2 (-1 + 2 δ - θ) (-1 - 2 δ + 4 δ^2 - θ) (-1 + θ))/((1 + 4 δ + θ) (2 - 4 δ^2 + θ + 2 δ θ - θ^2)^2)) &&
  0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < c < α]
0 < θ < 1 && 0 < δ < θ && α > 0 && 0 < c < α

(*Case 4 vs. case 1*)

Reduce[
 (-(((c - α)^2 (-1 + 2 δ - θ) (32 δ^5 + 368 δ^4 (-1 + θ) + (1 + θ)^3 (12 + (-12 + θ) θ) + 2 δ (1 + θ)^2
          (48 + 5 (-10 + θ) θ) + 8 δ^3 (-7 + 34 (-1 + θ) θ) +
         4 δ^2 (1 + θ) (51 + 2 θ (-35 + 9 θ))))/
      (2 (1 + 4 δ + θ) (-2 + 4 δ^2 + 8 δ^3 - 3 θ + θ^3 + 2 δ (-3 + θ) (1 + θ))^2))) >
   (-(6 (c - α)^2 (-1 + 2 δ - θ) (-1 - 2 δ + 4 δ^2 - θ) (-1 + θ))/((1 + 4 δ + θ) (2 - 4 δ^2 + θ + 2 δ θ - θ^2)^2)) && 0 <
   δ <
   1 && δ <
   θ <
   1 && α >
   0 && 0 < c < α]
0 < θ < 1 && 0 < δ < θ && α > 0 && 0 < c < α

(*Case 5 vs case 1*)

Reduce[((3 (c - α)^2)/(2 (1 + 4 δ + θ))) > (-(6 (c - α)^2 (-1 + 2 δ - θ) (-1 - 2 δ + 4 δ^2 - θ) (-1 + θ))/((1 + 4 δ + θ) (2 - 4 δ^2 + θ + 2 δ θ - θ^2)^2)) &&
  0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < c < α]
0 < θ < 1 && 0 < δ < θ && α > 0 && 0 < c < α

(*case 3 vs case 2*)

Reduce[
 (-(6 (c - α)^2 (1 + 2 δ) (-1 + θ))/((-2 - 2 δ + θ)^2 (1 + 4 δ + θ))) > ((3 (c - α)^2 (-1 + θ) (-(-1 + 2 δ - θ) (-1 - 2 δ + 4 δ^2 - θ)
          (2 + 6 δ + θ)^2 - (1 + 2 δ) (-12 δ^2 + 2 δ (2 + θ) + (1 + θ) (2 + θ))^2))/
     ((1 + 4 δ + θ) (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))^2)) &&
  0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < c < α]
0 < θ < 1 && 0 < δ < θ && α > 0 && 0 < c < α

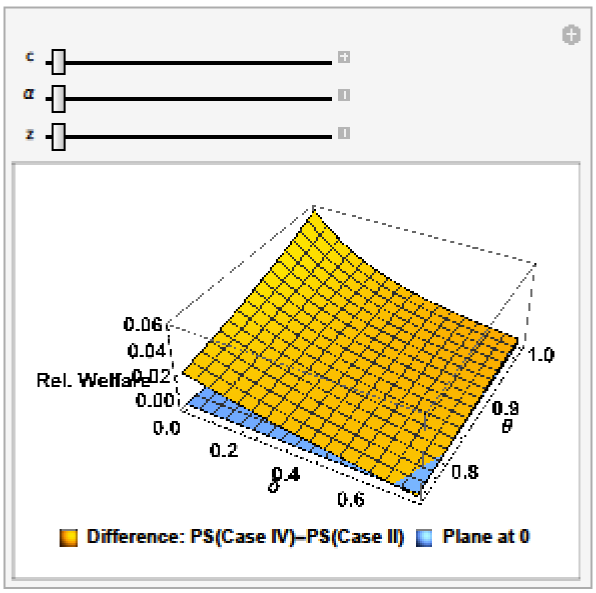
(*case 4 vs case 2*)
```

```
Reduce[
 (-(((c - α)^2 (-1 + 2 δ - θ) (32 δ^5 + 368 δ^4 (-1 + θ) + (1 + θ)^3 (12 + (-12 + θ) θ) + 2 δ (1 + θ)^2
            (48 + 5 (-10 + θ) θ) + 8 δ^3 (-7 + 34 (-1 + θ) θ) +
           4 δ^2 (1 + θ) (51 + 2 θ (-35 + 9 θ))))/
       (2 (1 + 4 δ + θ) (-2 + 4 δ^2 + 8 δ^3 - 3 θ + θ^3 + 2 δ (-3 + θ) (1 + θ))^2))) >
   ((3 (c - α)^2 (-1 + θ) (-(-1 + 2 δ - θ) (-1 - 2 δ + 4 δ^2 - θ) (2 + 6 δ + θ)^2 -
         (1 + 2 δ) (-12 δ^2 + 2 δ (2 + θ) + (1 + θ) (2 + θ))^2))/
     ((1 + 4 δ + θ) (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))^2)) &&
  0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < c <
   α]
(0 < θ ≤
   Root[320 052 180 054 162 401 512 493 285 376 - 712 821 260 621 503 764 172 114 624 512 #1 -
       28 429 041 543 775 694 149 357 026 607 104 #1^2 -
       104 103 491 269 631 754 015 335 408 762 880 #1^3 +
       49 889 437 406 185 377 687 083 205 083 136 #1^4 +
       42 496 825 581 139 607 463 086 370 938 880 #1^5 -
       1 220 765 402 136 313 069 390 732 642 430 976 #1^6 +
       624 477 468 482 469 616 215 504 370 706 432 #1^7 +
       7 469 393 620 370 352 161 626 228 777 318 912 #1^8 +
       1 020 405 213 507 296 675 016 424 931 909 632 #1^9 -
       19 796 336 624 432 721 587 649 894 639 227 904 #1^10 -
       10 800 382 577 723 856 452 316 723 042 187 264 #1^11 +
       27 464 578 531 330 666 433 316 301 614 447 424 #1^12 +
       24 903 815 307 410 074 380 336 384 973 894 592 #1^13 -
       19 364 036 579 520 858 755 314 574 984 531 616 #1^14 -
       28 307 880 052 647 168 559 392 580 501 839 040 #1^15 +
       3 631 156 007 122 584 323 238 794 206 413 248 #1^16 +
       16 815 187 397 328 442 270 543 926 519 426 464 #1^17 +
       3 116 228 825 743 911 480 500 634 971 372 016 #1^18 -
       4 813 565 582 930 730 488 870 944 926 207 040 #1^19 -
       1 017 260 353 738 069 318 024 946 908 418 240 #1^20 +
       1 417 262 994 701 649 216 903 975 350 483 824 #1^21 -
       160 283 342 791 213 171 849 833 240 495 456 #1^22 -
       1 347 147 694 741 121 079 833 265 272 848 096 #1^23 -
       819 287 022 552 404 090 469 918 344 172 284 #1^24 +
       169 183 427 586 432 643 741 721 225 348 232 #1^25 +
       613 831 206 767 747 724 964 430 318 569 424 #1^26 +
       469 893 239 984 595 320 347 622 375 921 656 #1^27 +
       104 368 400 754 351 711 776 165 935 572 840 #1^28 -
       175 614 427 346 509 243 532 385 070 284 532 #1^29 -
       178 955 035 783 124 157 737 663 559 100 196 #1^30 -
```

```
24 044 680 547 044 393 358 342 832 540 416 #1^31 +
31 622 376 932 256 484 488 123 186 833 860 #1^32 -
6 360 870 011 513 052 836 434 919 184 336 #1^33 -
17 812 261 188 818 885 541 378 406 336 657 #1^34 -
3 032 042 253 074 924 175 132 757 844 768 #1^35 +
2 606 832 898 812 389 220 906 949 486 738 #1^36 + 1 055 382 848 073 542 323 948 250 906 356
  #1^37 + 102 657 824 398 868 456 876 060 339 491 #1^38 &, 5] &&
0 < δ < Root[-16 θ^2 - 96 θ^3 - 232 θ^4 - 272 θ^5 - 121 θ^6 + 58 θ^7 + 89 θ^8 + 28 θ^9 - 7 θ^10 -
    6 θ^11 - θ^12 + (-32 θ - 432 θ^2 - 1656 θ^3 - 2756 θ^4 - 1824 θ^5 + 444 θ^6 + 1328 θ^7 + 564 θ^8 -
        96 θ^9 - 124 θ^10 - 24 θ^11) #1 + (288 + 320 θ - 3208 θ^2 - 9592 θ^3 - 8820 θ^4 + 1256 θ^5 +
        7256 θ^6 + 3816 θ^7 - 276 θ^8 - 696 θ^9 - 136 θ^10) #1^2 + (3392 + 4800 θ - 9328 θ^2 -
        17 136 θ^3 + 4048 θ^4 + 19 456 θ^5 + 8624 θ^6 - 1520 θ^7 - 976 θ^8 + 160 θ^9) #1^3 +
    (11 872 + 16 032 θ + 3760 θ^2 + 21 120 θ^3 + 27 648 θ^4 - 7392 θ^5 - 16 000 θ^6 + 480 θ^7 + 2960 θ^8)
     #1^4 + (-2688 + 18 624 θ + 77 888 θ^2 + 37 376 θ^3 - 65 920 θ^4 - 52 864 θ^5 - 2368 θ^6 + 3776 θ^7)
     #1^5 + (-87 488 + 11 648 θ + 72 704 θ^2 - 99 712 θ^3 - 40 704 θ^4 + 16 128 θ^5 - 12 544 θ^6) #1^6 +
    (-90 624 - 53 504 θ - 106 752 θ^2 + 77 568 θ^3 + 101 888 θ^4 - 4608 θ^5) #1^7 +
    (192 768 - 256 512 θ + 112 384 θ^2 + 68 608 θ^3 + 55 552 θ^4) #1^8 +
    (261 120 - 98 304 θ + 15 360 θ^2 - 86 016 θ^3) #1^9 + (-156 672 + 270 336 θ - 233 472 θ^2) #1^10 +
    (-184 320 + 147 456 θ) #1^11 + 36 864 #1^12 &, 6] && α > 0 && 0 < c < α) ||
(Root[320 052 180 054 162 401 512 493 285 376 - 712 821 260 621 503 764 172 114 624 512 #1 -
    28 429 041 543 775 694 149 357 026 607 104 #1^2 -
    104 103 491 269 631 754 015 335 408 762 880 #1^3 +
    49 889 437 406 185 377 687 083 205 083 136 #1^4 +
    42 496 825 581 139 607 463 086 370 938 880 #1^5 -
    1 220 765 402 136 313 069 390 732 642 430 976 #1^6 +
    624 477 468 482 469 616 215 504 370 706 432 #1^7 +
    7 469 393 620 370 352 161 626 228 777 318 912 #1^8 +
    1 020 405 213 507 296 675 016 424 931 909 632 #1^9 -
    19 796 336 624 432 721 587 649 894 639 227 904 #1^10 -
    10 800 382 577 723 856 452 316 723 042 187 264 #1^11 +
    27 464 578 531 330 666 433 316 301 614 447 424 #1^12 +
    24 903 815 307 410 074 380 336 384 973 894 592 #1^13 -
    19 364 036 579 520 858 755 314 574 984 531 616 #1^14 -
    28 307 880 052 647 168 559 392 580 501 839 040 #1^15 +
    3 631 156 007 122 584 323 238 794 206 413 248 #1^16 +
    16 815 187 397 328 442 270 543 926 519 426 464 #1^17 +
    3 116 228 825 743 911 480 500 634 971 372 016 #1^18 -
    4 813 565 582 930 730 488 870 944 926 207 040 #1^19 -
    1 017 260 353 738 069 318 024 946 908 418 240 #1^20 +
    1 417 262 994 701 649 216 903 975 350 483 824 #1^21 -
    160 283 342 791 213 171 849 833 240 495 456 #1^22 -
    1 347 147 694 741 121 079 833 265 272 848 096 #1^23 -
    819 287 022 552 404 090 469 918 344 172 284 #1^24 +
```

```
169 183 427 586 432 643 741 721 225 348 232 #1^25 +
613 831 206 767 747 724 964 430 318 569 424 #1^26 +
469 893 239 984 595 320 347 622 375 921 656 #1^27 +
104 368 400 754 351 711 776 165 935 572 840 #1^28 -
175 614 427 346 509 243 532 385 070 284 532 #1^29 -
178 955 035 783 124 157 737 663 559 100 196 #1^30 -
24 044 680 547 044 393 358 342 832 540 416 #1^31 +
31 622 376 932 256 484 488 123 186 833 860 #1^32 -
6 360 870 011 513 052 836 434 919 184 336 #1^33 -
17 812 261 188 818 885 541 378 406 336 657 #1^34 -
3 032 042 253 074 924 175 132 757 844 768 #1^35 +
2 606 832 898 812 389 220 906 949 486 738 #1^36 + 1 055 382 848 073 542 323 948 250 906 356
 #1^37 + 102 657 824 398 868 456 876 060 339 491 #1^38 &, 5] < θ < 1 &&
0 < δ < Root[-16 θ^2 - 96 θ^3 - 232 θ^4 - 272 θ^5 - 121 θ^6 + 58 θ^7 + 89 θ^8 + 28 θ^9 - 7 θ^10 -
  6 θ^11 - θ^12 + (-32 θ - 432 θ^2 - 1656 θ^3 - 2756 θ^4 - 1824 θ^5 + 444 θ^6 + 1328 θ^7 + 564 θ^8 -
    96 θ^9 - 124 θ^10 - 24 θ^11) #1 + (288 + 320 θ - 3208 θ^2 - 9592 θ^3 - 8820 θ^4 + 1256 θ^5 +
    7256 θ^6 + 3816 θ^7 - 276 θ^8 - 696 θ^9 - 136 θ^10) #1^2 + (3392 + 4800 θ - 9328 θ^2 -
    17 136 θ^3 + 4048 θ^4 + 19 456 θ^5 + 8624 θ^6 - 1520 θ^7 - 976 θ^8 + 160 θ^9) #1^3 +
  (11 872 + 16 032 θ + 3760 θ^2 + 21 120 θ^3 + 27 648 θ^4 - 7392 θ^5 - 16 000 θ^6 + 480 θ^7 + 2960 θ^8)
   #1^4 + (-2688 + 18 624 θ + 77 888 θ^2 + 37 376 θ^3 - 65 920 θ^4 - 52 864 θ^5 - 2368 θ^6 + 3776 θ^7)
   #1^5 + (-87 488 + 11 648 θ + 72 704 θ^2 - 99 712 θ^3 - 40 704 θ^4 + 16 128 θ^5 - 12 544 θ^6) #1^6 +
  (-90 624 - 53 504 θ - 106 752 θ^2 + 77 568 θ^3 + 101 888 θ^4 - 4608 θ^5) #1^7 +
  (192 768 - 256 512 θ + 112 384 θ^2 + 68 608 θ^3 + 55 552 θ^4) #1^8 +
  (261 120 - 98 304 θ + 15 360 θ^2 - 86 016 θ^3) #1^9 + (-156 672 + 270 336 θ - 233 472 θ^2) #1^10 +
  (-184 320 + 147 456 θ) #1^11 + 36 864 #1^12 &, 4] && α > 0 && 0 < c < α)
```

```
Manipulate[
 Plot3D[{((((-(((c - α)² (-1 + 2 δ - θ) (32 δ⁵ + 368 δ⁴ (-1 + θ) + (1 + θ)³ (12 + (-12 + θ) θ) +
             2 δ (1 + θ)² (48 + 5 (-10 + θ) θ) + 8 δ³ (-7 + 34 (-1 + θ) θ) +
             4 δ² (1 + θ) (51 + 2 θ (-35 + 9 θ))))) /
          (2 (1 + 4 δ + θ) (-2 + 4 δ² + 8 δ³ - 3 θ + θ³ + 2 δ (-3 + θ) (1 + θ))²)))) -
      (((3 (c - α)² (-1 + θ) (-(-1 + 2 δ - θ) (-1 - 2 δ + 4 δ² - θ) (2 + 6 δ + θ)² -
            (1 + 2 δ) (-12 δ² + 2 δ (2 + θ) + (1 + θ) (2 + θ))²)) / ((1 + 4 δ + θ)
          (24 δ³ - 12 δ² (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))²)))),
   0}, {δ, 0, .75}, {θ, .75, 1}, PlotLabel → "", AxesLabel →
   {"δ",
    "θ",
    "Rel. Welfare"},
  AxesStyle → Larger, LabelStyle →
   Bold,
  PlotLegends →
   Placed[
    {"Difference: PS(Case IV)-PS(Case II)", "Plane at 0"}, Below]],
 {c, 1, 1}, {α, 1.5, 1.5}, {z, 1,
  1}]
```




```
(*case 5 vs case 2*)
```

Reduce[

$$\left(\frac{3\,(c-\alpha)^2}{2\,(1+4\,\delta+\theta)}\right) > \left(\left(3\,(c-\alpha)^2\,(-1+\theta)\,\left(-(-1+2\,\delta-\theta)\,(-1-2\,\delta+4\,\delta^2-\theta)\,(2+6\,\delta+\theta)^2 - (1+2\,\delta)\,\left(-12\,\delta^2+2\,\delta\,(2+\theta)+(1+\theta)\,(2+\theta)\right)^2\right)\right) \Big/ \left((1+4\,\delta+\theta)\,\left(24\,\delta^3-12\,\delta^2\,(-1+\theta)+(-2+\theta)\,(1+\theta)\,(2+\theta)+2\,\delta\,(-6+(-4+\theta)\,\theta)\right)^2\right)\ \&\&$$

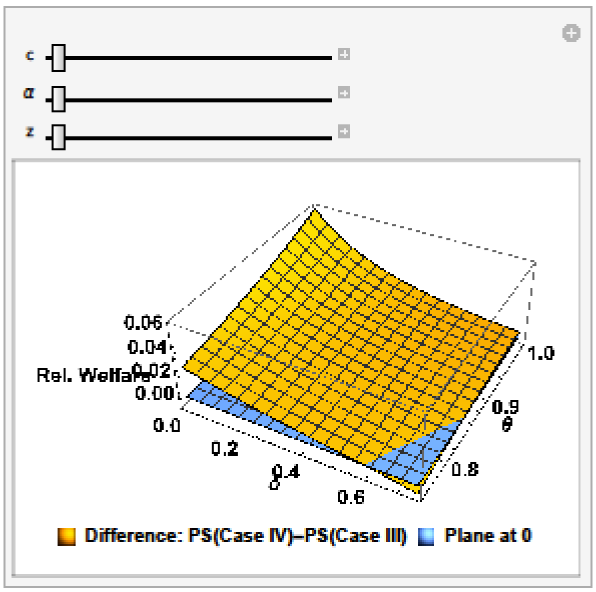
$0 < \delta < 1\ \&\&\ \delta < \theta < 1\ \&\&\ \alpha > 0\ \&\&\ 0 < c < \alpha$]

$0 < \theta < 1\ \&\&\ 0 < \delta < \theta\ \&\&\ \alpha > 0\ \&\&\ 0 < c < \alpha$

(*case 4 vs case 3*)

```
Reduce[
 (-(((c - α)^2 (-1 + 2 δ - θ) (32 δ^5 + 368 δ^4 (-1 + θ) + (1 + θ)^3 (12 + (-12 + θ) θ) + 2 δ (1 + θ)^2
          (48 + 5 (-10 + θ) θ) + 8 δ^3 (-7 + 34 (-1 + θ) θ) +
         4 δ^2 (1 + θ) (51 + 2 θ (-35 + 9 θ)))) /
      (2 (1 + 4 δ + θ) (-2 + 4 δ^2 + 8 δ^3 - 3 θ + θ^3 + 2 δ (-3 + θ) (1 + θ))^2))) >
  (-(6 (c - α)^2 (1 + 2 δ) (-1 + θ)) / ((-2 - 2 δ + θ)^2 (1 + 4 δ + θ))) && 0 < δ < 1 && δ <
  θ <
  1 && α >
  0 && 0 <
  c <
  α]
(0 < θ ≤ Root[
     3 863 771 921 520 - 60 321 588 016 256 #1 + 415 543 526 192 984 #1^2 - 1 703 486 268 398 248
        #1^3 + 4 674 979 693 212 380 #1^4 - 9 134 497 053 688 400 #1^5 + 13 236 884 740 895 890 #1^6 -
       14 778 406 076 272 382 #1^7 + 13 318 063 934 633 713 #1^8 - 10 160 531 190 817 622 #1^9 +
       6 576 956 136 799 639 #1^10 - 3 225 739 409 339 818 #1^11 + 768 203 455 982 297 #1^12 +
       332 883 217 177 886 #1^13 - 402 228 075 865 892 #1^14 + 167 784 757 113 928 #1^15 -
       30 920 704 212 812 #1^16 + 540 713 271 424 #1^17 + 426 022 473 481 #1^18 &, 3] &&
   0 < δ < Root[-4 θ^2 - 12 θ^3 - 9 θ^4 + 4 θ^5 + 6 θ^6 - θ^8 +
       (16 θ - 56 θ^2 - 160 θ^3 - 12 θ^4 + 124 θ^5 + 20 θ^6 - 28 θ^7) #1 +
       (144 + 16 θ - 608 θ^2 - 264 θ^3 + 520 θ^4 + 184 θ^5 - 120 θ^6) #1^2 +
       (992 - 1120 θ - 960 θ^2 + 1280 θ^3 + 80 θ^4 - 48 θ^5) #1^3 +
       (1808 - 3232 θ + 2944 θ^2 - 1088 θ^3 + 96 θ^4) #1^4 +
       (-320 + 832 θ + 640 θ^2 - 1280 θ^3) #1^5 + (-2560 + 3968 θ - 2048 θ^2) #1^6 +
       (-1024 + 1024 θ) #1^7 + 256 #1^8 &, 6] && α > 0 && 0 < c < α) ||
 (Root[3 863 771 921 520 - 60 321 588 016 256 #1 + 415 543 526 192 984 #1^2 -
       1 703 486 268 398 248 #1^3 + 4 674 979 693 212 380 #1^4 - 9 134 497 053 688 400 #1^5 +
       13 236 884 740 895 890 #1^6 - 14 778 406 076 272 382 #1^7 +
       13 318 063 934 633 713 #1^8 - 10 160 531 190 817 622 #1^9 + 6 576 956 136 799 639 #1^10 -
       3 225 739 409 339 818 #1^11 + 768 203 455 982 297 #1^12 + 332 883 217 177 886 #1^13 -
       402 228 075 865 892 #1^14 + 167 784 757 113 928 #1^15 - 30 920 704 212 812 #1^16 +
       540 713 271 424 #1^17 + 426 022 473 481 #1^18 &, 3] < θ < 1 &&
   0 < δ < Root[-4 θ^2 - 12 θ^3 - 9 θ^4 + 4 θ^5 + 6 θ^6 - θ^8 +
       (16 θ - 56 θ^2 - 160 θ^3 - 12 θ^4 + 124 θ^5 + 20 θ^6 - 28 θ^7) #1 +
       (144 + 16 θ - 608 θ^2 - 264 θ^3 + 520 θ^4 + 184 θ^5 - 120 θ^6) #1^2 +
       (992 - 1120 θ - 960 θ^2 + 1280 θ^3 + 80 θ^4 - 48 θ^5) #1^3 +
       (1808 - 3232 θ + 2944 θ^2 - 1088 θ^3 + 96 θ^4) #1^4 +
       (-320 + 832 θ + 640 θ^2 - 1280 θ^3) #1^5 + (-2560 + 3968 θ - 2048 θ^2) #1^6 +
       (-1024 + 1024 θ) #1^7 + 256 #1^8 &, 4] && α > 0 && 0 < c < α)
```

```
Manipulate[
 Plot3D[{(((-(((c - α)^2 (-1 + 2 δ - θ) (32 δ^5 + 368 δ^4 (-1 + θ) + (1 + θ)^3 (12 + (-12 + θ) θ) +
           2 δ (1 + θ)^2 (48 + 5 (-10 + θ) θ) + 8 δ^3 (-7 + 34 (-1 + θ) θ) +
           4 δ^2 (1 + θ) (51 + 2 θ (-35 + 9 θ))))/
       (2 (1 + 4 δ + θ) (-2 + 4 δ^2 + 8 δ^3 - 3 θ + θ^3 + 2 δ (-3 + θ) (1 + θ))^2)))) -
    (-(6 (c - α)^2 (1 + 2 δ) (-1 + θ))/((-2 - 2 δ + θ)^2 (1 + 4 δ + θ)))), 0}, {δ, 0, .75}, {θ, .75, 1},
  PlotLabel → "", AxesLabel → {"δ", "θ", "Rel. Welfare"},
  AxesStyle →
   Larger, LabelStyle →
   Bold, PlotLegends →
   Placed[{"Difference: PS(Case IV)-PS(Case III)", "Plane at 0"}, Below]],
 {c, 1, 1}, {α, 1.5, 1.5},
 {z,
  1,
  1}]
```




```
(*case 5 vs case 3*)
```

```
Reduce[
 (3 (c - α)^2/(2 (1 + 4 δ + θ))) > (-(6 (c - α)^2 (1 + 2 δ) (-1 + θ))/((-2 - 2 δ + θ)^2 (1 + 4 δ + θ))) && 0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < c < α]
```

```
0 < θ < 1 && 0 < δ < θ && α > 0 && 0 < c < α
```

```
(*case 5 vs case 4*)
```

```
Reduce[(3 (c - α)^2/(2 (1 + 4 δ + θ))) >
   (-(((c - α)^2 (-1 + 2 δ - θ) (32 δ^5 + 368 δ^4 (-1 + θ) + (1 + θ)^3 (12 + (-12 + θ) θ) +
            2 δ (1 + θ)^2 (48 + 5 (-10 + θ) θ) + 8 δ^3 (-7 + 34 (-1 + θ) θ) +
            4 δ^2 (1 + θ) (51 + 2 θ (-35 + 9 θ))))/
        (2 (1 + 4 δ + θ) (-2 + 4 δ^2 + 8 δ^3 - 3 θ + θ^3 + 2 δ (-3 + θ) (1 + θ))^2))) &&
  0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < c <
   α]
```

```
0 < θ < 1 && 0 < δ < θ && α > 0 && 0 < c < α
```

```
(*CONSUMER SURPLUS COMPARISON TABLE*)

(*Comparison tables: Consumer Surplus  (*&&0<δ<1&&δ<θ<1&&α>0&&0<c<α*) *)

(*Case 2 vs case 1*)
```

```
Reduce[
 ((s (1 + 4 δ + θ) (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))^2 + 3
         c^2 (1 + 2 δ) (-144 δ^4 + 288 δ^5 + 2 δ (1 + θ) (2 + θ) (7 + 5 θ) +
           (2 + 3 θ + θ^2)^2 - 24 δ^3 (6 + θ (8 + θ)) - 2 δ^2 (-13 + θ (-9 + θ (3 + θ)))) -
        6 c α (1 + 2 δ) (-144 δ^4 + 288 δ^5 + 2 δ (1 + θ) (2 + θ) (7 + 5 θ) + (2 + 3 θ + θ^2)^2 -
           24 δ^3 (6 + θ (8 + θ)) - 2 δ^2 (-13 + θ (-9 + θ (3 + θ)))) +
        3 α^2 (1 + 2 δ) (-144 δ^4 + 288 δ^5 + 2 δ (1 + θ) (2 + θ) (7 + 5 θ) + (2 + 3 θ + θ^2)^2 -
           24 δ^3 (6 + θ (8 + θ)) - 2 δ^2 (-13 + θ (-9 + θ (3 + θ)))))/
      ((1 + 4 δ + θ) (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))^2)) >
   ((3 c^2 (1 + 2 δ - 4 δ^2 + θ)^2 - 6 c α (1 + 2 δ - 4 δ^2 + θ)^2 + 3 α^2 (1 + 2 δ - 4 δ^2 + θ)^2 +
        s (1 + 4 δ + θ) (2 - 4 δ^2 + θ + 2 δ θ - θ^2)^2)/
      ((1 + 4 δ + θ) (2 - 4 δ^2 + θ + 2 δ θ - θ^2)^2)) && 0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < c < α]
```

```
False
```

```
(*Case 3 vs case 1*)
```

```
Reduce[(s + (-6 c α (1 + 2 δ)^2 + 3 (c + 2 c δ)^2 + 3 (α + 2 α δ)^2)/((-2 - 2 δ + θ)^2 (1 + 4 δ + θ))) >
   ((3 c^2 (1 + 2 δ - 4 δ^2 + θ)^2 - 6 c α (1 + 2 δ - 4 δ^2 + θ)^2 +
       3 α^2 (1 + 2 δ - 4 δ^2 + θ)^2 + s (1 + 4 δ + θ) (2 - 4 δ^2 + θ + 2 δ θ - θ^2)^2) /
     ((1 + 4 δ + θ) (2 - 4 δ^2 + θ + 2 δ θ - θ^2)^2)) && 0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < c < α]

False

(*Case 4 vs. case 1*)

Reduce[
  (s + ((c - α)^2 (1 + 2 δ + θ) (480 δ^5 + 48 δ^4 (-11 + 7 θ) + 4 δ^2 (1 + θ) (43 + 4 (-11 + θ) θ) + 6 δ
          (1 + θ)^2 (16 + (-6 + θ) θ) + 8 δ^3 (-31 + 10 (-6 + θ) θ) + (1 + θ)^3 (12 + (-4 + θ) θ))) /
      (4 (1 + 4 δ + θ) (-2 + 4 δ^2 + 8 δ^3 - 3 θ + θ^3 + 2 δ (-3 + θ) (1 + θ))^2)) >
   ((3 c^2 (1 + 2 δ - 4 δ^2 + θ)^2 - 6 c α (1 + 2 δ - 4 δ^2 + θ)^2 + 3 α^2 (1 + 2 δ - 4 δ^2 + θ)^2 +
       s (1 + 4 δ + θ) (2 - 4 δ^2 + θ + 2 δ θ - θ^2)^2) /
     ((1 + 4 δ + θ) (2 - 4 δ^2 + θ + 2 δ θ - θ^2)^2)) && 0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < c < α]

False

(*Case 5 vs case 1*)

Reduce[(s + (3 (c - α)^2)/(4 (1 + 4 δ + θ))) > ((3 c^2 (1 + 2 δ - 4 δ^2 + θ)^2 - 6 c α (1 + 2 δ - 4 δ^2 + θ)^2 +
       3 α^2 (1 + 2 δ - 4 δ^2 + θ)^2 + s (1 + 4 δ + θ) (2 - 4 δ^2 + θ + 2 δ θ - θ^2)^2) /
     ((1 + 4 δ + θ) (2 - 4 δ^2 + θ + 2 δ θ - θ^2)^2)) && 0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < c < α]

False

(*case 3 vs case 2*)

Reduce[(s + (-6 c α (1 + 2 δ)^2 + 3 (c + 2 c δ)^2 + 3 (α + 2 α δ)^2)/((-2 - 2 δ + θ)^2 (1 + 4 δ + θ))) >
   (s + (3 (c - α)^2 (1 + 2 δ) (-144 δ^4 + 288 δ^5 + 2 δ (1 + θ) (2 + θ) (7 + 5 θ) +
          (2 + 3 θ + θ^2)^2 - 24 δ^3 (6 + θ (8 + θ)) - 2 δ^2 (-13 + θ (-9 + θ (3 + θ))))) /
      ((1 + 4 δ + θ) (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))^2)) &&
  0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < c < α]

False

(*case 4 vs case 2*)
```

```
Reduce[
 (z + ((c - α)^2 (1 + 2 δ + θ) (480 δ^5 + 48 δ^4 (-11 + 7 θ) + 4 δ^2 (1 + θ) (43 + 4 (-11 + θ) θ) + 6 δ
          (1 + θ)^2 (16 + (-6 + θ) θ) + 8 δ^3 (-31 + 10 (-6 + θ) θ) + (1 + θ)^3 (12 + (-4 + θ) θ))) /
     (4 (1 + 4 δ + θ) (-2 + 4 δ^2 + 8 δ^3 - 3 θ + θ^3 + 2 δ (-3 + θ) (1 + θ))^2)) >
   (z + (3 (c - α)^2 (1 + 2 δ) (-144 δ^4 + 288 δ^5 + 2 δ (1 + θ) (2 + θ) (7 + 5 θ) +
          (2 + 3 θ + θ^2)^2 - 24 δ^3 (6 + θ (8 + θ)) - 2 δ^2 (-13 + θ (-9 + θ (3 + θ))))) /
     ((1 + 4 δ + θ) (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))^2)) &&
  0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < c < α]

z ∈ Reals &&
 ((0 < θ ≤ Root[-44 550 383 342 148 065 872 134 144 + 99 861 193 070 176 314 056 504 459 264 #1 +
        2 441 105 753 043 633 233 045 862 436 864 #1^2 +
        16 822 226 614 865 141 168 377 416 187 904 #1^3 +
        17 142 310 939 900 416 641 191 387 378 688 #1^4 -
        68 464 786 712 700 144 817 478 413 362 176 #1^5 +
        18 027 574 928 300 203 836 030 227 263 232 #1^6 +
        234 486 779 497 514 750 937 498 427 136 512 #1^7 -
        966 709 801 318 351 910 270 449 144 515 328 #1^8 -
        1 668 102 593 629 555 080 440 820 647 181 440 #1^9 +
        3 960 895 160 375 427 190 696 723 894 748 736 #1^10 +
        7 361 906 563 724 338 604 765 505 142 711 872 #1^11 -
        6 569 267 532 994 431 399 261 453 971 636 976 #1^12 -
        17 610 822 021 606 736 331 652 778 219 374 560 #1^13 +
        2 768 505 958 508 040 220 197 724 808 032 088 #1^14 +
        24 445 234 410 644 993 212 274 628 064 886 104 #1^15 +
        7 336 913 135 337 441 892 626 052 420 547 816 #1^16 -
        18 630 113 820 050 620 900 402 046 180 801 960 #1^17 -
        12 529 327 396 410 341 157 742 958 476 240 852 #1^18 +
        4 432 240 564 820 955 461 083 727 955 891 364 #1^19 +
        4 191 173 092 037 803 094 622 819 213 282 200 #1^20 +
        1 158 695 670 579 049 131 778 170 062 884 636 #1^21 +
        5 709 904 465 179 775 374 205 620 867 746 708 #1^22 +
        3 861 811 480 500 267 233 659 007 097 857 080 #1^23 -
        4 300 971 493 463 319 889 762 119 779 976 349 #1^24 -
        4 909 677 416 785 197 839 961 007 623 008 619 #1^25 +
        554 422 082 521 391 597 654 035 769 371 875 #1^26 +
        2 387 983 223 043 185 207 420 370 216 634 581 #1^27 +
        573 585 034 928 307 823 554 183 948 924 099 #1^28 -
        499 864 512 844 852 997 410 663 242 877 611 #1^29 -
        274 512 863 082 962 161 263 807 458 000 877 #1^30 +
        13 759 377 836 343 339 912 267 762 574 633 #1^31 +
        46 938 532 534 250 480 985 929 158 136 489 #1^32 +
```

$13\,850\,468\,909\,871\,843\,298\,820\,523\,620\,959\,\#1^{33} -$
$415\,800\,007\,175\,085\,992\,189\,875\,332\,459\,\#1^{34} -$
$1\,537\,548\,046\,804\,567\,624\,286\,399\,761\,053\,\#1^{35} -$
$497\,076\,991\,932\,676\,524\,798\,249\,557\,631\,\#1^{36} -$
$58\,919\,759\,871\,482\,511\,967\,156\,485\,381\,\#1^{37} + 4\,217\,266\,806\,953\,326\,153\,856\,460\,873$
$\#1^{38} + 1\,653\,582\,912\,561\,189\,586\,077\,107\,679\,\#1^{39}\ \&,\ 7\Big]\ \&\&$
$\text{Root}\Big[-64\,\theta - 368\,\theta^2 - 832\,\theta^3 - 856\,\theta^4 - 212\,\theta^5 + 353\,\theta^6 + 298\,\theta^7 + 23\,\theta^8 - 56\,\theta^9 - 17\,\theta^{10} +$
$2\,\theta^{11} + \theta^{12} + \big(192 - 224\,\theta - 3984\,\theta^2 - 9144\,\theta^3 - 7304\,\theta^4 + 972\,\theta^5 + 4728\,\theta^6 + 1916\,\theta^7 -$
$504\,\theta^8 - 444\,\theta^9 - 40\,\theta^{10} + 12\,\theta^{11}\big)\,\#1 + \big(2656 + 3392\,\theta - 14\,648\,\theta^2 - 34\,600\,\theta^3 -$
$16\,660\,\theta^4 + 15\,600\,\theta^5 + 16\,568\,\theta^6 + 1576\,\theta^7 - 2644\,\theta^8 - 656\,\theta^9 + 40\,\theta^{10}\big)\,\#1^2 +$
$\big(12\,480 + 24\,576\,\theta - 7280\,\theta^2 - 34\,576\,\theta^3 + 1344\,\theta^4 + 30\,096\,\theta^5 + 12\,432\,\theta^6 -$
$2928\,\theta^7 - 1696\,\theta^8 + 112\,\theta^9\big)\,\#1^3 + \big(13\,472 + 53\,152\,\theta + 94\,544\,\theta^2 +$
$67\,072\,\theta^3 - 14\,272\,\theta^4 - 34\,400\,\theta^5 - 4416\,\theta^6 + 4896\,\theta^7 + 1392\,\theta^8\big)\,\#1^4 +$
$\big(-65\,856 + 4736\,\theta + 228\,672\,\theta^2 + 84\,224\,\theta^3 - 153\,472\,\theta^4 - 72\,960\,\theta^5 + 11\,392\,\theta^6 + 4736\,\theta^7\big)$
$\#1^5 + \big(-180\,672 - 152\,832\,\theta + 26\,240\,\theta^2 - 49\,024\,\theta^3 - 49\,152\,\theta^4 - 8960\,\theta^5 - 5504\,\theta^6\big)$
$\#1^6 + \big(33\,792 - 286\,464\,\theta - 306\,432\,\theta^2 + 231\,168\,\theta^3 + 120\,576\,\theta^4 - 20\,736\,\theta^5\big)\,\#1^7 +$
$\big(484\,608 - 185\,856\,\theta - 37\,632\,\theta^2 + 215\,040\,\theta^3 + 42\,240\,\theta^4\big)\,\#1^8 +$
$\big(208\,896 + 267\,264\,\theta - 147\,456\,\theta^2 - 52\,224\,\theta^3\big)\,\#1^9 +$
$\big(-451\,584 + 423\,936\,\theta - 331\,776\,\theta^2\big)\,\#1^{10} + (-221\,184 + 110\,592\,\theta)\,\#1^{11} +$
$110\,592\,\#1^{12}\ \&,\ 6\Big] < \delta < \theta\ \&\&\ \alpha > 0\ \&\&\ 0 < c < \alpha\big)\ ||$
$\big(\text{Root}\Big[-44\,550\,383\,342\,148\,065\,872\,134\,144 + 99\,861\,193\,070\,176\,314\,056\,504\,459\,264\,\#1 +$
$2\,441\,105\,753\,043\,633\,233\,045\,862\,436\,864\,\#1^2 +$
$16\,822\,226\,614\,865\,141\,168\,377\,416\,187\,904\,\#1^3 +$
$17\,142\,310\,939\,900\,416\,641\,191\,387\,378\,688\,\#1^4 -$
$68\,464\,786\,712\,700\,144\,817\,478\,413\,362\,176\,\#1^5 +$
$18\,027\,574\,928\,300\,203\,836\,030\,227\,263\,232\,\#1^6 +$
$234\,486\,779\,497\,514\,750\,937\,498\,427\,136\,512\,\#1^7 -$
$966\,709\,801\,318\,351\,910\,270\,449\,144\,515\,328\,\#1^8 -$
$1\,668\,102\,593\,629\,555\,080\,440\,820\,647\,181\,440\,\#1^9 +$
$3\,960\,895\,160\,375\,427\,190\,696\,723\,894\,748\,736\,\#1^{10} +$
$7\,361\,906\,563\,724\,338\,604\,765\,505\,142\,711\,872\,\#1^{11} -$
$6\,569\,267\,532\,994\,431\,399\,261\,453\,971\,636\,976\,\#1^{12} -$
$17\,610\,822\,021\,606\,736\,331\,652\,778\,219\,374\,560\,\#1^{13} +$
$2\,768\,505\,958\,508\,040\,220\,197\,724\,808\,032\,088\,\#1^{14} +$
$24\,445\,234\,410\,644\,993\,212\,274\,628\,064\,886\,104\,\#1^{15} +$
$7\,336\,913\,135\,337\,441\,892\,626\,052\,420\,547\,816\,\#1^{16} -$
$18\,630\,113\,820\,050\,620\,900\,402\,046\,180\,801\,960\,\#1^{17} -$
$12\,529\,327\,396\,410\,341\,157\,742\,958\,476\,240\,852\,\#1^{18} +$
$4\,432\,240\,564\,820\,955\,461\,083\,727\,955\,891\,364\,\#1^{19} +$
$4\,191\,173\,092\,037\,803\,094\,622\,819\,213\,282\,200\,\#1^{20} +$
$1\,158\,695\,670\,579\,049\,131\,778\,170\,062\,884\,636\,\#1^{21} +$
$5\,709\,904\,465\,179\,775\,374\,205\,620\,867\,746\,708\,\#1^{22} +$
$3\,861\,811\,480\,500\,267\,233\,659\,007\,097\,857\,080\,\#1^{23} -$

4 300 971 493 463 319 889 762 119 779 976 349 $\#1^{24}$ -
4 909 677 416 785 197 839 961 007 623 008 619 $\#1^{25}$ +
554 422 082 521 391 597 654 035 769 371 875 $\#1^{26}$ +
2 387 983 223 043 185 207 420 370 216 634 581 $\#1^{27}$ +
573 585 034 928 307 823 554 183 948 924 099 $\#1^{28}$ -
499 864 512 844 852 997 410 663 242 877 611 $\#1^{29}$ -
274 512 863 082 962 161 263 807 458 000 877 $\#1^{30}$ +
13 759 377 836 343 339 912 267 762 574 633 $\#1^{31}$ +
46 938 532 534 250 480 985 929 158 136 489 $\#1^{32}$ +
13 850 468 909 871 843 298 820 523 620 959 $\#1^{33}$ -
415 800 007 175 085 992 189 875 332 459 $\#1^{34}$ -
1 537 548 046 804 567 624 286 399 761 053 $\#1^{35}$ -
497 076 991 932 676 524 798 249 557 631 $\#1^{36}$ -
58 919 759 871 482 511 967 156 485 381 $\#1^{37}$ + 4 217 266 806 953 326 153 856 460 873
$\#1^{38}$ + 1 653 582 912 561 189 586 077 107 679 $\#1^{39}$ &, 7] < θ <
Root[- 44 550 383 342 148 065 872 134 144 + 99 861 193 070 176 314 056 504 459 264 $\#1$ +
2 441 105 753 043 633 233 045 862 436 864 $\#1^{2}$ +
16 822 226 614 865 141 168 377 416 187 904 $\#1^{3}$ +
17 142 310 939 900 416 641 191 387 378 688 $\#1^{4}$ -
68 464 786 712 700 144 817 478 413 362 176 $\#1^{5}$ +
18 027 574 928 300 203 836 030 227 263 232 $\#1^{6}$ +
234 486 779 497 514 750 937 498 427 136 512 $\#1^{7}$ -
966 709 801 318 351 910 270 449 144 515 328 $\#1^{8}$ -
1 668 102 593 629 555 080 440 820 647 181 440 $\#1^{9}$ +
3 960 895 160 375 427 190 696 723 894 748 736 $\#1^{10}$ +
7 361 906 563 724 338 604 765 505 142 711 872 $\#1^{11}$ -
6 569 267 532 994 431 399 261 453 971 636 976 $\#1^{12}$ -
17 610 822 021 606 736 331 652 778 219 374 560 $\#1^{13}$ +
2 768 505 958 508 040 220 197 724 808 032 088 $\#1^{14}$ +
24 445 234 410 644 993 212 274 628 064 886 104 $\#1^{15}$ +
7 336 913 135 337 441 892 626 052 420 547 816 $\#1^{16}$ -
18 630 113 820 050 620 900 402 046 180 801 960 $\#1^{17}$ -
12 529 327 396 410 341 157 742 958 476 240 852 $\#1^{18}$ +
4 432 240 564 820 955 461 083 727 955 891 364 $\#1^{19}$ +
4 191 173 092 037 803 094 622 819 213 282 200 $\#1^{20}$ +
1 158 695 670 579 049 131 778 170 062 884 636 $\#1^{21}$ +
5 709 904 465 179 775 374 205 620 867 746 708 $\#1^{22}$ +
3 861 811 480 500 267 233 659 007 097 857 080 $\#1^{23}$ -
4 300 971 493 463 319 889 762 119 779 976 349 $\#1^{24}$ -
4 909 677 416 785 197 839 961 007 623 008 619 $\#1^{25}$ +
554 422 082 521 391 597 654 035 769 371 875 $\#1^{26}$ +
2 387 983 223 043 185 207 420 370 216 634 581 $\#1^{27}$ +
573 585 034 928 307 823 554 183 948 924 099 $\#1^{28}$ -

```
      499 864 512 844 852 997 410 663 242 877 611 #1^29 -
      274 512 863 082 962 161 263 807 458 000 877 #1^30 +
      13 759 377 836 343 339 912 267 762 574 633 #1^31 +
      46 938 532 534 250 480 985 929 158 136 489 #1^32 +
      13 850 468 909 871 843 298 820 523 620 959 #1^33 -
      415 800 007 175 085 992 189 875 332 459 #1^34 -
      1 537 548 046 804 567 624 286 399 761 053 #1^35 -
      497 076 991 932 676 524 798 249 557 631 #1^36 -
      58 919 759 871 482 511 967 156 485 381 #1^37 + 4 217 266 806 953 326 153 856 460 873
       #1^38 + 1 653 582 912 561 189 586 077 107 679 #1^39 &, 8] &&
  Root[-64 θ - 368 θ^2 - 832 θ^3 - 856 θ^4 - 212 θ^5 + 353 θ^6 + 298 θ^7 + 23 θ^8 - 56 θ^9 - 17 θ^10 +
      2 θ^11 + θ^12 + (192 - 224 θ - 3984 θ^2 - 9144 θ^3 - 7304 θ^4 + 972 θ^5 + 4728 θ^6 + 1916 θ^7 -
          504 θ^8 - 444 θ^9 - 40 θ^10 + 12 θ^11) #1 + (2656 + 3392 θ - 14 648 θ^2 - 34 600 θ^3 -
          16 660 θ^4 + 15 600 θ^5 + 16 568 θ^6 + 1576 θ^7 - 2644 θ^8 - 656 θ^9 + 40 θ^10) #1^2 +
      (12 480 + 24 576 θ - 7280 θ^2 - 34 576 θ^3 + 1344 θ^4 + 30 096 θ^5 + 12 432 θ^6 -
          2928 θ^7 - 1696 θ^8 + 112 θ^9) #1^3 + (13 472 + 53 152 θ + 94 544 θ^2 +
          67 072 θ^3 - 14 272 θ^4 - 34 400 θ^5 - 4416 θ^6 + 4896 θ^7 + 1392 θ^8) #1^4 +
      (-65 856 + 4736 θ + 228 672 θ^2 + 84 224 θ^3 - 153 472 θ^4 - 72 960 θ^5 + 11 392 θ^6 + 4736 θ^7)
       #1^5 + (-180 672 - 152 832 θ + 26 240 θ^2 - 49 024 θ^3 - 49 152 θ^4 - 8960 θ^5 - 5504 θ^6)
       #1^6 + (33 792 - 286 464 θ - 306 432 θ^2 + 231 168 θ^3 + 120 576 θ^4 - 20 736 θ^5) #1^7 +
      (484 608 - 185 856 θ - 37 632 θ^2 + 215 040 θ^3 + 42 240 θ^4) #1^8 +
      (208 896 + 267 264 θ - 147 456 θ^2 - 52 224 θ^3) #1^9 +
      (-451 584 + 423 936 θ - 331 776 θ^2) #1^10 + (-221 184 + 110 592 θ) #1^11 +
      110 592 #1^12 &, 4] < δ < θ && α > 0 && 0 < c < α) ||
(Root[-44 550 383 342 148 065 872 134 144 + 99 861 193 070 176 314 056 504 459 264 #1 +
      2 441 105 753 043 633 233 045 862 436 864 #1^2 +
      16 822 226 614 865 141 168 377 416 187 904 #1^3 +
      17 142 310 939 900 416 641 191 387 378 688 #1^4 -
      68 464 786 712 700 144 817 478 413 362 176 #1^5 +
      18 027 574 928 300 203 836 030 227 263 232 #1^6 +
      234 486 779 497 514 750 937 498 427 136 512 #1^7 -
      966 709 801 318 351 910 270 449 144 515 328 #1^8 -
      1 668 102 593 629 555 080 440 820 647 181 440 #1^9 +
      3 960 895 160 375 427 190 696 723 894 748 736 #1^10 +
      7 361 906 563 724 338 604 765 505 142 711 872 #1^11 -
      6 569 267 532 994 431 399 261 453 971 636 976 #1^12 -
      17 610 822 021 606 736 331 652 778 219 374 560 #1^13 +
      2 768 505 958 508 040 220 197 724 808 032 088 #1^14 +
      24 445 234 410 644 993 212 274 628 064 886 104 #1^15 +
      7 336 913 135 337 441 892 626 052 420 547 816 #1^16 -
      18 630 113 820 050 620 900 402 046 180 801 960 #1^17 -
      12 529 327 396 410 341 157 742 958 476 240 852 #1^18 +
      4 432 240 564 820 955 461 083 727 955 891 364 #1^19 +
```

4 191 173 092 037 803 094 622 819 213 282 200 $\#1^{20}$ +
1 158 695 670 579 049 131 778 170 062 884 636 $\#1^{21}$ +
5 709 904 465 179 775 374 205 620 867 746 708 $\#1^{22}$ +
3 861 811 480 500 267 233 659 007 097 857 080 $\#1^{23}$ −
4 300 971 493 463 319 889 762 119 779 976 349 $\#1^{24}$ −
4 909 677 416 785 197 839 961 007 623 008 619 $\#1^{25}$ +
554 422 082 521 391 597 654 035 769 371 875 $\#1^{26}$ +
2 387 983 223 043 185 207 420 370 216 634 581 $\#1^{27}$ +
573 585 034 928 307 823 554 183 948 924 099 $\#1^{28}$ −
499 864 512 844 852 997 410 663 242 877 611 $\#1^{29}$ −
274 512 863 082 962 161 263 807 458 000 877 $\#1^{30}$ +
13 759 377 836 343 339 912 267 762 574 633 $\#1^{31}$ +
46 938 532 534 250 480 985 929 158 136 489 $\#1^{32}$ +
13 850 468 909 871 843 298 820 523 620 959 $\#1^{33}$ −
415 800 007 175 085 992 189 875 332 459 $\#1^{34}$ −
1 537 548 046 804 567 624 286 399 761 053 $\#1^{35}$ −
497 076 991 932 676 524 798 249 557 631 $\#1^{36}$ −
58 919 759 871 482 511 967 156 485 381 $\#1^{37}$ + 4 217 266 806 953 326 153 856 460 873
$\#1^{38}$ + 1 653 582 912 561 189 586 077 107 679 $\#1^{39}$ &, 8] $\leq \theta \leq$
Root[− 44 550 383 342 148 065 872 134 144 + 99 861 193 070 176 314 056 504 459 264 #1 +
2 441 105 753 043 633 233 045 862 436 864 $\#1^{2}$ +
16 822 226 614 865 141 168 377 416 187 904 $\#1^{3}$ +
17 142 310 939 900 416 641 191 387 378 688 $\#1^{4}$ −
68 464 786 712 700 144 817 478 413 362 176 $\#1^{5}$ +
18 027 574 928 300 203 836 030 227 263 232 $\#1^{6}$ +
234 486 779 497 514 750 937 498 427 136 512 $\#1^{7}$ −
966 709 801 318 351 910 270 449 144 515 328 $\#1^{8}$ −
1 668 102 593 629 555 080 440 820 647 181 440 $\#1^{9}$ +
3 960 895 160 375 427 190 696 723 894 748 736 $\#1^{10}$ +
7 361 906 563 724 338 604 765 505 142 711 872 $\#1^{11}$ −
6 569 267 532 994 431 399 261 453 971 636 976 $\#1^{12}$ −
17 610 822 021 606 736 331 652 778 219 374 560 $\#1^{13}$ +
2 768 505 958 508 040 220 197 724 808 032 088 $\#1^{14}$ +
24 445 234 410 644 993 212 274 628 064 886 104 $\#1^{15}$ +
7 336 913 135 337 441 892 626 052 420 547 816 $\#1^{16}$ −
18 630 113 820 050 620 900 402 046 180 801 960 $\#1^{17}$ −
12 529 327 396 410 341 157 742 958 476 240 852 $\#1^{18}$ +
4 432 240 564 820 955 461 083 727 955 891 364 $\#1^{19}$ +
4 191 173 092 037 803 094 622 819 213 282 200 $\#1^{20}$ +
1 158 695 670 579 049 131 778 170 062 884 636 $\#1^{21}$ +
5 709 904 465 179 775 374 205 620 867 746 708 $\#1^{22}$ +
3 861 811 480 500 267 233 659 007 097 857 080 $\#1^{23}$ −
4 300 971 493 463 319 889 762 119 779 976 349 $\#1^{24}$ −

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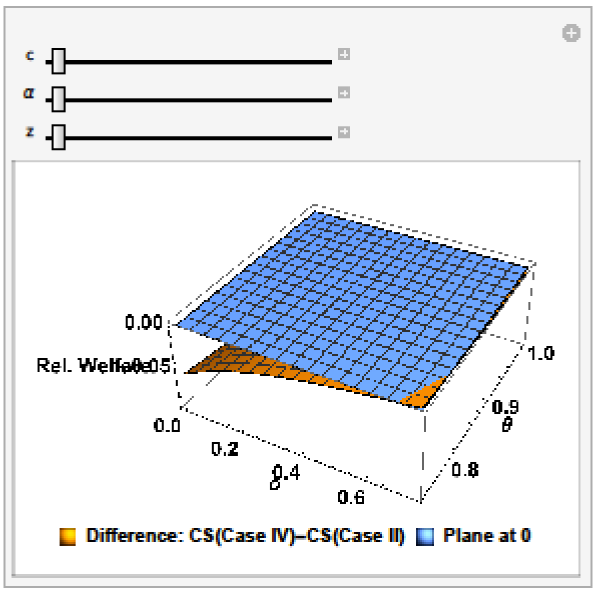
      4 909 677 416 785 197 839 961 007 623 008 619 #1^25 +
      554 422 082 521 391 597 654 035 769 371 875 #1^26 +
      2 387 983 223 043 185 207 420 370 216 634 581 #1^27 +
      573 585 034 928 307 823 554 183 948 924 099 #1^28 -
      499 864 512 844 852 997 410 663 242 877 611 #1^29 -
      274 512 863 082 962 161 263 807 458 000 877 #1^30 +
      13 759 377 836 343 339 912 267 762 574 633 #1^31 +
      46 938 532 534 250 480 985 929 158 136 489 #1^32 +
      13 850 468 909 871 843 298 820 523 620 959 #1^33 -
      415 800 007 175 085 992 189 875 332 459 #1^34 -
      1 537 548 046 804 567 624 286 399 761 053 #1^35 -
      497 076 991 932 676 524 798 249 557 631 #1^36 -
      58 919 759 871 482 511 967 156 485 381 #1^37 + 4 217 266 806 953 326 153 856 460 873
       #1^38 + 1 653 582 912 561 189 586 077 107 679 #1^39 &, 9] &&
  Root[-64 θ - 368 θ^2 - 832 θ^3 - 856 θ^4 - 212 θ^5 + 353 θ^6 + 298 θ^7 + 23 θ^8 - 56 θ^9 - 17 θ^10 +
      2 θ^11 + θ^12 + (192 - 224 θ - 3984 θ^2 - 9144 θ^3 - 7304 θ^4 + 972 θ^5 + 4728 θ^6 + 1916 θ^7 -
         504 θ^8 - 444 θ^9 - 40 θ^10 + 12 θ^11) #1 + (2656 + 3392 θ - 14 648 θ^2 - 34 600 θ^3 -
         16 660 θ^4 + 15 600 θ^5 + 16 568 θ^6 + 1576 θ^7 - 2644 θ^8 - 656 θ^9 + 40 θ^10) #1^2 +
      (12 480 + 24 576 θ - 7280 θ^2 - 34 576 θ^3 + 1344 θ^4 + 30 096 θ^5 + 12 432 θ^6 -
         2928 θ^7 - 1696 θ^8 + 112 θ^9) #1^3 + (13 472 + 53 152 θ + 94 544 θ^2 +
         67 072 θ^3 - 14 272 θ^4 - 34 400 θ^5 - 4416 θ^6 + 4896 θ^7 + 1392 θ^8) #1^4 +
      (-65 856 + 4736 θ + 228 672 θ^2 + 84 224 θ^3 - 153 472 θ^4 - 72 960 θ^5 + 11 392 θ^6 + 4736 θ^7)
       #1^5 + (-180 672 - 152 832 θ + 26 240 θ^2 - 49 024 θ^3 - 49 152 θ^4 - 8960 θ^5 - 5504 θ^6)
       #1^6 + (33 792 - 286 464 θ - 306 432 θ^2 + 231 168 θ^3 + 120 576 θ^4 - 20 736 θ^5) #1^7 +
      (484 608 - 185 856 θ - 37 632 θ^2 + 215 040 θ^3 + 42 240 θ^4) #1^8 +
      (208 896 + 267 264 θ - 147 456 θ^2 - 52 224 θ^3) #1^9 +
      (-451 584 + 423 936 θ - 331 776 θ^2) #1^10 + (-221 184 + 110 592 θ) #1^11 +
      110 592 #1^12 &, 6] < δ < θ && α > 0 && 0 < c < α) ||
(Root[-44 550 383 342 148 065 872 134 144 + 99 861 193 070 176 314 056 504 459 264 #1 +
      2 441 105 753 043 633 233 045 862 436 864 #1^2 +
      16 822 226 614 865 141 168 377 416 187 904 #1^3 +
      17 142 310 939 900 416 641 191 387 378 688 #1^4 -
      68 464 786 712 700 144 817 478 413 362 176 #1^5 +
      18 027 574 928 300 203 836 030 227 263 232 #1^6 +
      234 486 779 497 514 750 937 498 427 136 512 #1^7 -
      966 709 801 318 351 910 270 449 144 515 328 #1^8 -
      1 668 102 593 629 555 080 440 820 647 181 440 #1^9 +
      3 960 895 160 375 427 190 696 723 894 748 736 #1^10 +
      7 361 906 563 724 338 604 765 505 142 711 872 #1^11 -
      6 569 267 532 994 431 399 261 453 971 636 976 #1^12 -
      17 610 822 021 606 736 331 652 778 219 374 560 #1^13 +
      2 768 505 958 508 040 220 197 724 808 032 088 #1^14 +
      24 445 234 410 644 993 212 274 628 064 886 104 #1^15 +
```

7 336 913 135 337 441 892 626 052 420 547 816 $\#1^{16}$ −
18 630 113 820 050 620 900 402 046 180 801 960 $\#1^{17}$ −
12 529 327 396 410 341 157 742 958 476 240 852 $\#1^{18}$ +
4 432 240 564 820 955 461 083 727 955 891 364 $\#1^{19}$ +
4 191 173 092 037 803 094 622 819 213 282 200 $\#1^{20}$ +
1 158 695 670 579 049 131 778 170 062 884 636 $\#1^{21}$ +
5 709 904 465 179 775 374 205 620 867 746 708 $\#1^{22}$ +
3 861 811 480 500 267 233 659 007 097 857 080 $\#1^{23}$ −
4 300 971 493 463 319 889 762 119 779 976 349 $\#1^{24}$ −
4 909 677 416 785 197 839 961 007 623 008 619 $\#1^{25}$ +
554 422 082 521 391 597 654 035 769 371 875 $\#1^{26}$ +
2 387 983 223 043 185 207 420 370 216 634 581 $\#1^{27}$ +
573 585 034 928 307 823 554 183 948 924 099 $\#1^{28}$ −
499 864 512 844 852 997 410 663 242 877 611 $\#1^{29}$ −
274 512 863 082 962 161 263 807 458 000 877 $\#1^{30}$ +
13 759 377 836 343 339 912 267 762 574 633 $\#1^{31}$ +
46 938 532 534 250 480 985 929 158 136 489 $\#1^{32}$ +
13 850 468 909 871 843 298 820 523 620 959 $\#1^{33}$ −
415 800 007 175 085 992 189 875 332 459 $\#1^{34}$ −
1 537 548 046 804 567 624 286 399 761 053 $\#1^{35}$ −
497 076 991 932 676 524 798 249 557 631 $\#1^{36}$ −
58 919 759 871 482 511 967 156 485 381 $\#1^{37}$ + 4 217 266 806 953 326 153 856 460 873
$\#1^{38}$ + 1 653 582 912 561 189 586 077 107 679 $\#1^{39}$ &, 9] $< \theta < 1$ &&
Root[$-64\,\theta - 368\,\theta^2 - 832\,\theta^3 - 856\,\theta^4 - 212\,\theta^5 + 353\,\theta^6 + 298\,\theta^7 + 23\,\theta^8 - 56\,\theta^9 - 17\,\theta^{10} +$
$2\,\theta^{11} + \theta^{12} + (192 - 224\,\theta - 3984\,\theta^2 - 9144\,\theta^3 - 7304\,\theta^4 + 972\,\theta^5 + 4728\,\theta^6 + 1916\,\theta^7 -$
$504\,\theta^8 - 444\,\theta^9 - 40\,\theta^{10} + 12\,\theta^{11})\ \#1 + (2656 + 3392\,\theta - 14\,648\,\theta^2 - 34\,600\,\theta^3 -$
$16\,660\,\theta^4 + 15\,600\,\theta^5 + 16\,568\,\theta^6 + 1576\,\theta^7 - 2644\,\theta^8 - 656\,\theta^9 + 40\,\theta^{10})\ \#1^2 +$
$(12\,480 + 24\,576\,\theta - 7280\,\theta^2 - 34\,576\,\theta^3 + 1344\,\theta^4 + 30\,096\,\theta^5 + 12\,432\,\theta^6 -$
$2928\,\theta^7 - 1696\,\theta^8 + 112\,\theta^9)\ \#1^3 + (13\,472 + 53\,152\,\theta + 94\,544\,\theta^2 +$
$67\,072\,\theta^3 - 14\,272\,\theta^4 - 34\,400\,\theta^5 - 4416\,\theta^6 + 4896\,\theta^7 + 1392\,\theta^8)\ \#1^4 +$
$(-65\,856 + 4736\,\theta + 228\,672\,\theta^2 + 84\,224\,\theta^3 - 153\,472\,\theta^4 - 72\,960\,\theta^5 + 11\,392\,\theta^6 + 4736\,\theta^7)$
$\#1^5 + (-180\,672 - 152\,832\,\theta + 26\,240\,\theta^2 - 49\,024\,\theta^3 - 49\,152\,\theta^4 - 8960\,\theta^5 - 5504\,\theta^6)$
$\#1^6 + (33\,792 - 286\,464\,\theta - 306\,432\,\theta^2 + 231\,168\,\theta^3 + 120\,576\,\theta^4 - 20\,736\,\theta^5)\ \#1^7 +$
$(484\,608 - 185\,856\,\theta - 37\,632\,\theta^2 + 215\,040\,\theta^3 + 42\,240\,\theta^4)\ \#1^8 +$
$(208\,896 + 267\,264\,\theta - 147\,456\,\theta^2 - 52\,224\,\theta^3)\ \#1^9 +$
$(-451\,584 + 423\,936\,\theta - 331\,776\,\theta^2)\ \#1^{10} + (-221\,184 + 110\,592\,\theta)\ \#1^{11} +$
110 592 $\#1^{12}$ &, 4] $< \delta < \theta$ && $\alpha > 0$ && $0 < c < \alpha$))

```
Manipulate[Plot3D[
  {((((z + ((c - α)^2 (1 + 2 δ + θ) (480 δ^5 + 48 δ^4 (-11 + 7 θ) + 4 δ^2 (1 + θ) (43 + 4 (-11 + θ) θ) +
            6 δ (1 + θ)^2 (16 + (-6 + θ) θ) + 8 δ^3 (-31 + 10 (-6 + θ) θ) + (1 + θ)^3 (12 + (-4 + θ)
               θ))) / (4 (1 + 4 δ + θ) (-2 + 4 δ^2 + 8 δ^3 - 3 θ + θ^3 + 2 δ (-3 + θ) (1 + θ))^2))) -
     ((z + (3 (c - α)^2 (1 + 2 δ) (-144 δ^4 + 288 δ^5 + 2 δ (1 + θ) (2 + θ) (7 + 5 θ) + (2 + 3 θ + θ^2)^2 -
            24 δ^3 (6 + θ (8 + θ)) - 2 δ^2 (-13 + θ (-9 + θ (3 + θ))))) / ((1 + 4 δ + θ)
          (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))^2)))),
   0}, {δ, 0, .75}, {θ, .75, 1}, PlotLabel → "", AxesLabel →
   {"δ",
    "θ",
    "Rel. Welfare"},
  AxesStyle → Larger, LabelStyle →
   Bold,
  PlotLegends →
   Placed[
    {"Difference: CS(Case IV)-CS(Case II)", "Plane at 0"}, Below]],
 {c, 1, 1}, {α, 1.5, 1.5}, {z, 1,
  1}]
```




```
(*case 5 vs case 2*)
```

```
Reduce[
 (z + (3 (c - α)^2)/(4 (1 + 4 δ + θ))) > (z + (3 (c - α)^2 (1 + 2 δ) (-144 δ^4 + 288 δ^5 + 2 δ (1 + θ) (2 + θ) (7 + 5 θ) +
          (2 + 3 θ + θ^2)^2 - 24 δ^3 (6 + θ (8 + θ)) - 2 δ^2 (-13 + θ (-9 + θ (3 + θ)))))/
      ((1 + 4 δ + θ) (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))^2)) &&
  0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < c < α]

False

(*case 4 vs case 3*)

Reduce[
 (z + ((c - α)^2 (1 + 2 δ + θ) (480 δ^5 + 48 δ^4 (-11 + 7 θ) + 4 δ^2 (1 + θ) (43 + 4 (-11 + θ) θ) + 6 δ
          (1 + θ)^2 (16 + (-6 + θ) θ) + 8 δ^3 (-31 + 10 (-6 + θ) θ) + (1 + θ)^3 (12 + (-4 + θ) θ)))/
      (4 (1 + 4 δ + θ) (-2 + 4 δ^2 + 8 δ^3 - 3 θ + θ^3 + 2 δ (-3 + θ) (1 + θ))^2)) >
   (z + (-6 c α (1 + 2 δ)^2 + 3 (c + 2 c δ)^2 + 3 (α + 2 α δ)^2)/((-2 - 2 δ + θ)^2 (1 + 4 δ + θ))) &&
  0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < c < α]

z ∈ Reals &&
 ((0 < θ ≤ Root[-48 726 217 260 000 + 804 129 728 780 000 #1 - 5 439 395 755 622 400 #1^2 +
          21 097 684 572 520 320 #1^3 - 53 421 733 819 197 024 #1^4 + 94 343 663 707 788 448 #1^5 -
          121 154 479 259 181 780 #1^6 + 115 629 091 150 331 200 #1^7 - 76 764 366 788 216 454 #1^8 +
          13 772 459 757 183 462 #1^9 + 53 952 538 263 369 107 #1^10 - 94 638 586 568 671 222 #1^11 +
          90 046 349 473 590 135 #1^12 - 57 089 193 605 172 108 #1^13 + 25 543 797 726 685 606 #1^14 -
          8 328 429 882 977 772 #1^15 + 1 993 867 713 297 772 #1^16 - 331 014 665 995 242 #1^17 +
          34 355 119 946 283 #1^18 - 3 502 063 304 910 #1^19 + 561 632 676 179 #1^20 &, 2] &&
      Root[-16 θ - 44 θ^2 - 24 θ^3 + 25 θ^4 + 20 θ^5 - 6 θ^6 - 4 θ^7 + θ^8 +
          (96 - 112 θ - 544 θ^2 - 104 θ^3 + 396 θ^4 + 68 θ^5 - 92 θ^6 + 4 θ^7) #1 +
          (880 - 672 θ - 2224 θ^2 + 832 θ^3 + 1200 θ^4 - 352 θ^5 - 48 θ^6) #1^2 +
          (2624 - 2368 θ - 2224 θ^2 + 2864 θ^3 - 64 θ^4 - 160 θ^5) #1^3 +
          (1968 - 1408 θ + 1568 θ^2 - 512 θ^3 - 32 θ^4) #1^4 +
          (-3200 + 5760 θ - 1792 θ^2 - 1152 θ^3) #1^5 + (-4864 + 6144 θ - 3200 θ^2) #1^6 +
          (-768 + 768 θ) #1^7 + 768 #1^8 &, 4] < δ < θ && α > 0 && 0 < c < α) ||
    (Root[-48 726 217 260 000 + 804 129 728 780 000 #1 - 5 439 395 755 622 400 #1^2 +
          21 097 684 572 520 320 #1^3 - 53 421 733 819 197 024 #1^4 + 94 343 663 707 788 448 #1^5 -
          121 154 479 259 181 780 #1^6 + 115 629 091 150 331 200 #1^7 - 76 764 366 788 216 454 #1^8 +
          13 772 459 757 183 462 #1^9 + 53 952 538 263 369 107 #1^10 - 94 638 586 568 671 222 #1^11 +
          90 046 349 473 590 135 #1^12 - 57 089 193 605 172 108 #1^13 + 25 543 797 726 685 606 #1^14 -
          8 328 429 882 977 772 #1^15 + 1 993 867 713 297 772 #1^16 - 331 014 665 995 242 #1^17 +
          34 355 119 946 283 #1^18 - 3 502 063 304 910 #1^19 + 561 632 676 179 #1^20 &, 2] < θ <
      Root[-48 726 217 260 000 + 804 129 728 780 000 #1 - 5 439 395 755 622 400 #1^2 +
          21 097 684 572 520 320 #1^3 - 53 421 733 819 197 024 #1^4 + 94 343 663 707 788 448 #1^5 -
```

$121\,154\,479\,259\,181\,780\,\#1^{6} + 115\,629\,091\,150\,331\,200\,\#1^{7} - 76\,764\,366\,788\,216\,454\,\#1^{8} +$

$13\,772\,459\,757\,183\,462\,\#1^{9} + 53\,952\,538\,263\,369\,107\,\#1^{10} - 94\,638\,586\,568\,671\,222\,\#1^{11} +$

$90\,046\,349\,473\,590\,135\,\#1^{12} - 57\,089\,193\,605\,172\,108\,\#1^{13} + 25\,543\,797\,726\,685\,606\,\#1^{14} -$

$8\,328\,429\,882\,977\,772\,\#1^{15} + 1\,993\,867\,713\,297\,772\,\#1^{16} - 331\,014\,665\,995\,242\,\#1^{17} +$

$34\,355\,119\,946\,283\,\#1^{18} - 3\,502\,063\,304\,910\,\#1^{19} + 561\,632\,676\,179\,\#1^{20}\ \&,\ 3\big]\ \&\&$

$\text{Root}\big[-16\,\theta - 44\,\theta^{2} - 24\,\theta^{3} + 25\,\theta^{4} + 20\,\theta^{5} - 6\,\theta^{6} - 4\,\theta^{7} + \theta^{8} +$

$\left(96 - 112\,\theta - 544\,\theta^{2} - 104\,\theta^{3} + 396\,\theta^{4} + 68\,\theta^{5} - 92\,\theta^{6} + 4\,\theta^{7}\right)\#1 +$

$\left(880 - 672\,\theta - 2224\,\theta^{2} + 832\,\theta^{3} + 1200\,\theta^{4} - 352\,\theta^{5} - 48\,\theta^{6}\right)\#1^{2} +$

$\left(2624 - 2368\,\theta - 2224\,\theta^{2} + 2864\,\theta^{3} - 64\,\theta^{4} - 160\,\theta^{5}\right)\#1^{3} +$

$\left(1968 - 1408\,\theta + 1568\,\theta^{2} - 512\,\theta^{3} - 32\,\theta^{4}\right)\#1^{4} +$

$\left(-3200 + 5760\,\theta - 1792\,\theta^{2} - 1152\,\theta^{3}\right)\#1^{5} + \left(-4864 + 6144\,\theta - 3200\,\theta^{2}\right)\#1^{6} +$

$(-768 + 768\,\theta)\,\#1^{7} + 768\,\#1^{8}\ \&,\ 2\big] < \delta < \theta\ \&\&\ \alpha > 0\ \&\&\ 0 < c < \alpha)\ ||$

$(\text{Root}\big[-48\,726\,217\,260\,000 + 804\,129\,728\,780\,000\,\#1 - 5\,439\,395\,755\,622\,400\,\#1^{2} +$

$21\,097\,684\,572\,520\,320\,\#1^{3} - 53\,421\,733\,819\,197\,024\,\#1^{4} + 94\,343\,663\,707\,788\,448\,\#1^{5} -$

$121\,154\,479\,259\,181\,780\,\#1^{6} + 115\,629\,091\,150\,331\,200\,\#1^{7} - 76\,764\,366\,788\,216\,454\,\#1^{8} +$

$13\,772\,459\,757\,183\,462\,\#1^{9} + 53\,952\,538\,263\,369\,107\,\#1^{10} - 94\,638\,586\,568\,671\,222\,\#1^{11} +$

$90\,046\,349\,473\,590\,135\,\#1^{12} - 57\,089\,193\,605\,172\,108\,\#1^{13} + 25\,543\,797\,726\,685\,606\,\#1^{14} -$

$8\,328\,429\,882\,977\,772\,\#1^{15} + 1\,993\,867\,713\,297\,772\,\#1^{16} - 331\,014\,665\,995\,242\,\#1^{17} +$

$34\,355\,119\,946\,283\,\#1^{18} - 3\,502\,063\,304\,910\,\#1^{19} + 561\,632\,676\,179\,\#1^{20}\ \&,\ 3\big] \le \theta < 1\ \&\&$

$\text{Root}\big[-16\,\theta - 44\,\theta^{2} - 24\,\theta^{3} + 25\,\theta^{4} + 20\,\theta^{5} - 6\,\theta^{6} - 4\,\theta^{7} + \theta^{8} +$

$\left(96 - 112\,\theta - 544\,\theta^{2} - 104\,\theta^{3} + 396\,\theta^{4} + 68\,\theta^{5} - 92\,\theta^{6} + 4\,\theta^{7}\right)\#1 +$

$\left(880 - 672\,\theta - 2224\,\theta^{2} + 832\,\theta^{3} + 1200\,\theta^{4} - 352\,\theta^{5} - 48\,\theta^{6}\right)\#1^{2} +$

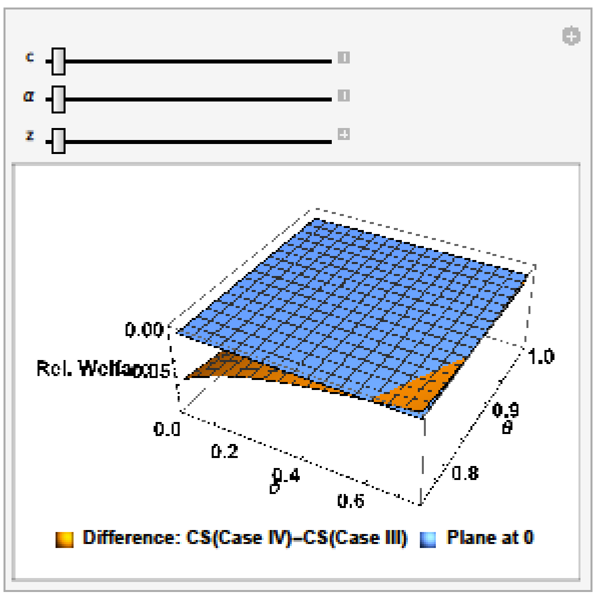
$\left(2624 - 2368\,\theta - 2224\,\theta^{2} + 2864\,\theta^{3} - 64\,\theta^{4} - 160\,\theta^{5}\right)\#1^{3} +$

$\left(1968 - 1408\,\theta + 1568\,\theta^{2} - 512\,\theta^{3} - 32\,\theta^{4}\right)\#1^{4} +$

$\left(-3200 + 5760\,\theta - 1792\,\theta^{2} - 1152\,\theta^{3}\right)\#1^{5} + \left(-4864 + 6144\,\theta - 3200\,\theta^{2}\right)\#1^{6} +$

$(-768 + 768\,\theta)\,\#1^{7} + 768\,\#1^{8}\ \&,\ 4\big] < \delta < \theta\ \&\&\ \alpha > 0\ \&\&\ 0 < c < \alpha))$

```
Manipulate[Plot3D[
  {[(((z + ((c - α)² (1 + 2 δ + θ) (480 δ⁵ + 48 δ⁴ (-11 + 7 θ) + 4 δ² (1 + θ) (43 + 4 (-11 + θ) θ) +
            6 δ (1 + θ)² (16 + (-6 + θ) θ) + 8 δ³ (-31 + 10 (-6 + θ) θ) + (1 + θ)³ (12 + (-4 + θ)
            θ))) / (4 (1 + 4 δ + θ) (-2 + 4 δ² + 8 δ³ - 3 θ + θ³ + 2 δ (-3 + θ) (1 + θ))²))) -
     (z + (-6 c α (1 + 2 δ)² + 3 (c + 2 c δ)² + 3 (α + 2 α δ)²) / ((-2 - 2 δ + θ)² (1 + 4 δ + θ)))], 0}, {δ, 0, .75},
  {θ, .75, 1}, PlotLabel → "", AxesLabel →
   {"δ", "θ", "Rel. Welfare"},
  AxesStyle → Larger, LabelStyle → Bold,
  PlotLegends →
   Placed[{"Difference: CS(Case IV)-CS(Case III)", "Plane at 0"}, Below]],
 {c, 1, 1}, {α, 1.5, 1.5},
 {z,
  1,
  1}]
```




```
(*case 5 vs case 3*)
```

$$\text{Reduce}\left[\left(z+\frac{3\,(c-\alpha)^2}{4\,(1+4\,\delta+\theta)}\right)>\left(z+\frac{-6\,c\,\alpha\,(1+2\,\delta)^2+3\,(c+2\,c\,\delta)^2+3\,(\alpha+2\,\alpha\,\delta)^2}{(-2-2\,\delta+\theta)^2\,(1+4\,\delta+\theta)}\right)\;\&\&\;0<\delta<1\;\&\&\;\delta<\theta<1\;\&\&\;\alpha>0\;\&\&\;0<c<\alpha\right]$$

False

(*case 5 vs case 4*)

$$\text{Reduce}\left[\left(z+\frac{3\,(c-\alpha)^2}{4\,(1+4\,\delta+\theta)}\right)>\left(z+\left((c-\alpha)^2\,(1+2\,\delta+\theta)\,\left(480\,\delta^5+48\,\delta^4\,(-11+7\,\theta)+4\,\delta^2\,(1+\theta)\,(43+4\,(-11+\theta)\,\theta)+6\,\delta\,(1+\theta)^2\,(16+(-6+\theta)\,\theta)+8\,\delta^3\,(-31+10\,(-6+\theta)\,\theta)+(1+\theta)^3\,(12+(-4+\theta)\,\theta)\right)\right)\Big/\left(4\,(1+4\,\delta+\theta)\,\left(-2+4\,\delta^2+8\,\delta^3-3\,\theta+\theta^3+2\,\delta\,(-3+\theta)\,(1+\theta)\right)^2\right)\right)\;\&\&\;0<\delta<1\;\&\&\;\delta<\theta<1\;\&\&\;\alpha>0\;\&\&\;0<c<\alpha\right]$$

False

(*TOTAL WELFARE COMPARISON TABLE*)

(*Comparison tables: Total welfare (*&&0<δ<1&&δ<θ<1&&α>0&&0<c<α*) *)

(*Case 2 vs case 1*)

$$\text{Reduce}\Big[\left(z+\left(3\,(c-\alpha)^2\,\left(576\,\delta^6-576\,\delta^5\,(-1+\theta)-(1+\theta)^2\,(2+\theta)^2\,(-3+2\,\theta)+96\,\delta^4\,(-6+(-4+\theta)\,\theta)-2\,\delta\,(1+\theta)\,(2+\theta)\,(-21+\theta\,(-7+\theta\,(9+\theta)))+4\,\delta^3\,(-103+\theta\,(-39+\theta\,(51+19\,\theta)))+2\,\delta^2\,(43+\theta\,(85+\theta\,(41-\theta\,(5+2\,\theta))))\right)\right)\Big/\left((1+4\,\delta+\theta)\,\left(24\,\delta^3-12\,\delta^2\,(-1+\theta)+(-2+\theta)\,(1+\theta)\,(2+\theta)+2\,\delta\,(-6+(-4+\theta)\,\theta)\right)^2\right)\right)>\left(\frac{1}{(1+4\,\delta+\theta)\,\left(2-4\,\delta^2+\theta+2\,\delta\,\theta-\theta^2\right)^2}\left(-6\,(c-\alpha)^2\,(-1+2\,\delta-\theta)\,\left(-1-2\,\delta+4\,\delta^2-\theta\right)\,(-1+\theta)+3\,c^2\,\left(1+2\,\delta-4\,\delta^2+\theta\right)^2-6\,c\,\alpha\,\left(1+2\,\delta-4\,\delta^2+\theta\right)^2+3\,\alpha^2\,\left(1+2\,\delta-4\,\delta^2+\theta\right)^2+z\,(1+4\,\delta+\theta)\,\left(2-4\,\delta^2+\theta+2\,\delta\,\theta-\theta^2\right)^2\right)\right)\;\&\&\;0<\delta<1\;\&\&\;\delta<\theta<1\;\&\&\;\alpha>0\;\&\&\;0<c<\alpha\Big]$$

False

(*Case 3 vs case 1*)

$$\text{Reduce}\Big[\Big(z+\frac{3\,(c-\alpha)^2\,(1+2\,\delta)\,(3+2\,\delta-2\,\theta)}{(-2-2\,\delta+\theta)^2\,(1+4\,\delta+\theta)}\Big)>\Big(\frac{1}{(1+4\,\delta+\theta)\,(2-4\,\delta^2+\theta+2\,\delta\,\theta-\theta^2)^2}\Big(-6\,(c-\alpha)^2\,(-1+2\,\delta-\theta)\,(-1-2\,\delta+4\,\delta^2-\theta)\,(-1+\theta)+3\,c^2\,(1+2\,\delta-4\,\delta^2+\theta)^2-6\,c\,\alpha\,(1+2\,\delta-4\,\delta^2+\theta)^2+3\,\alpha^2\,(1+2\,\delta-4\,\delta^2+\theta)^2+z\,(1+4\,\delta+\theta)\,(2-4\,\delta^2+\theta+2\,\delta\,\theta-\theta^2)^2\Big)\Big)\ \&\&\ 0<\delta<1\ \&\&\ \delta<\theta<1\ \&\&\ \alpha>0\ \&\&\ 0<c<\alpha\Big]$$

False

(*Case 4 vs. case 1*)

$$\text{Reduce}\Big[\Big(z+\Big((c-\alpha)^2\,\big(832\,\delta^6+64\,\delta^5\,(15-4\,\theta)+(1+\theta)^4\,(36+\theta\,(-28+3\,\theta))+4\,\delta\,(1+\theta)^3\,(66+\theta\,(-49+6\,\theta))+16\,\delta^4\,(-96+\theta\,(-4+9\,\theta))+4\,\delta^2\,(1+\theta)^2\,(97+3\,\theta\,(-34+11\,\theta))+16\,\delta^3\,(1+\theta)\,(-52+\theta\,(-16+23\,\theta))\big)\Big)\Big/\Big(4\,(1+4\,\delta+\theta)\,\big(-2+4\,\delta^2+8\,\delta^3-3\,\theta+\theta^3+2\,\delta\,(-3+\theta)\,(1+\theta)\big)^2\Big)\Big)>\Big(\frac{1}{(1+4\,\delta+\theta)\,(2-4\,\delta^2+\theta+2\,\delta\,\theta-\theta^2)^2}\Big(-6\,(c-\alpha)^2\,(-1+2\,\delta-\theta)\,(-1-2\,\delta+4\,\delta^2-\theta)\,(-1+\theta)+3\,c^2\,(1+2\,\delta-4\,\delta^2+\theta)^2-6\,c\,\alpha\,(1+2\,\delta-4\,\delta^2+\theta)^2+3\,\alpha^2\,(1+2\,\delta-4\,\delta^2+\theta)^2+z\,(1+4\,\delta+\theta)\,(2-4\,\delta^2+\theta+2\,\delta\,\theta-\theta^2)^2\Big)\Big)\ \&\&\ 0<\delta<1\ \&\&\ \delta<\theta<1\ \&\&\ \alpha>0\ \&\&\ 0<c<\alpha\Big]$$

False

(*Case 5 vs case 1*)

$$\text{Reduce}\Big[\Big(z+\frac{9\,(c-\alpha)^2}{4\,(1+4\,\delta+\theta)}\Big)>\Big(\frac{1}{(1+4\,\delta+\theta)\,(2-4\,\delta^2+\theta+2\,\delta\,\theta-\theta^2)^2}\Big(-6\,(c-\alpha)^2\,(-1+2\,\delta-\theta)\,(-1-2\,\delta+4\,\delta^2-\theta)\,(-1+\theta)+3\,c^2\,(1+2\,\delta-4\,\delta^2+\theta)^2-6\,c\,\alpha\,(1+2\,\delta-4\,\delta^2+\theta)^2+3\,\alpha^2\,(1+2\,\delta-4\,\delta^2+\theta)^2+z\,(1+4\,\delta+\theta)\,(2-4\,\delta^2+\theta+2\,\delta\,\theta-\theta^2)^2\Big)\Big)\ \&\&\ 0<\delta<1\ \&\&\ \delta<\theta<1\ \&\&\ \alpha>0\ \&\&\ 0<c<\alpha\Big]$$

False

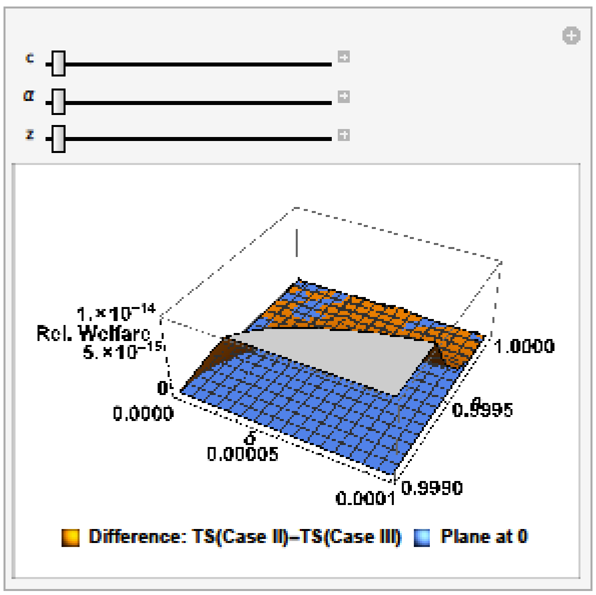
(*case 3 vs case 2*)

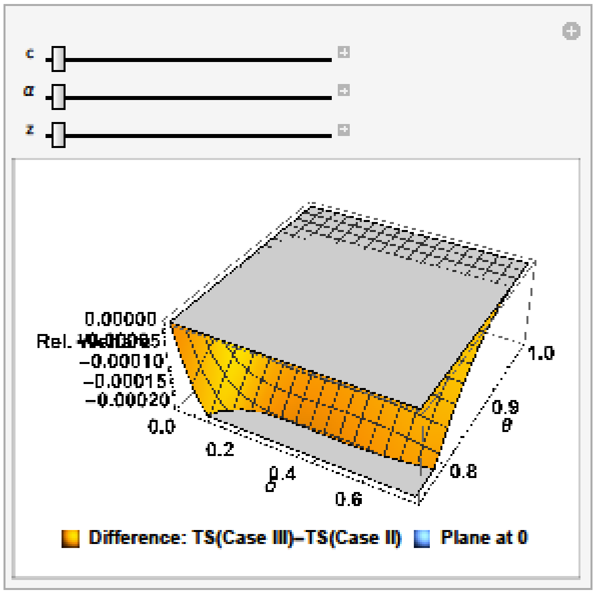
```
Reduce[(z + (3 (c - α)^2 (1 + 2 δ) (3 + 2 δ - 2 θ))/((-2 - 2 δ + θ)^2 (1 + 4 δ + θ))) >
    (z + (3 (c - α)^2 (576 δ^6 - 576 δ^5 (-1 + θ) - (1 + θ)^2 (2 + θ)^2 (-3 + 2 θ) +
            96 δ^4 (-6 + (-4 + θ) θ) - 2 δ (1 + θ) (2 + θ) (-21 + θ (-7 + θ (9 + θ))) +
            4 δ^3 (-103 + θ (-39 + θ (51 + 19 θ))) + 2 δ^2 (43 + θ (85 + θ (41 - θ (5 + 2 θ)))))) /
        ((1 + 4 δ + θ) (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))^2)) &&
  0 < δ < 1 && δ < θ < 1 && α > 0 && 0 < c < α]
```

```
z ∈ Reals && Root[-8 - 56 #1 - 78 #1^2 + 86 #1^3 + 83 #1^4 &, 4] < θ < 1 &&
 Root[-8 - 12 θ - 2 θ^2 + 3 θ^3 + θ^4 + (-44 - 52 θ - θ^2 + 8 θ^3) #1 +
      (-24 - 72 θ - 6 θ^2) #1^2 + (156 - 72 θ) #1^3 + 152 #1^4 &, 4] < δ < θ && α > 0 && 0 < c < α
```

Case 2 - case 3

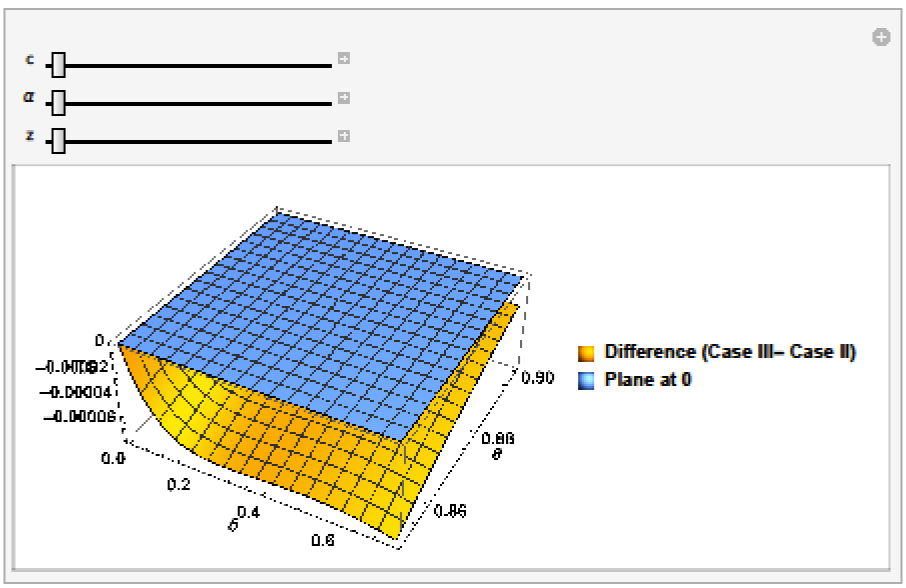
```
Manipulate[
 Plot3D[{(((z + (3 (c - α)^2 (576 δ^6 - 576 δ^5 (-1 + θ) - (1 + θ)^2 (2 + θ)^2 (-3 + 2 θ) + 96 δ^4
              (-6 + (-4 + θ) θ) - 2 δ (1 + θ) (2 + θ) (-21 + θ (-7 + θ (9 + θ))) + 4 δ^3 (-103 + θ
                 (-39 + θ (51 + 19 θ))) + 2 δ^2 (43 + θ (85 + θ (41 - θ (5 + 2 θ))))))/((1 + 4 δ +
            θ) (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))^2))) -
     (z + (3 (c - α)^2 (1 + 2 δ) (3 + 2 δ - 2 θ))/((-2 - 2 δ + θ)^2 (1 + 4 δ + θ))), 0}, {δ, 0, .0001}, {θ,
   .999, 1}, PlotLabel →
   "",
  AxesLabel → {"δ", "θ", "Rel. Welfare"},
  AxesStyle →
   Larger, LabelStyle →
   Bold, PlotLegends →
   Placed[{"Difference: TS(Case II)-TS(Case III)", "Plane at 0"}, Below]],
 {c, 1, 1}, {α, 1.5, 1}, {z,
  1,
  1}]
```




c
α
z
0.00000
Rel. Welfare
-0.00005
-0.00010
-0.00015
-0.00020
0.0
0.2
0.4
α
0.6
0.8
0.9
θ
1.0
Difference: TS(Case III)–TS(Case II)
Plane at 0

3, 2

```
Manipulate[Plot3D[{(z + (3 (c - α)^2 (1 + 2 δ) (3 + 2 δ - 2 θ))/((-2 - 2 δ + θ)^2 (1 + 4 δ + θ))) -
    (z + (3 (c - α)^2 (576 δ^6 - 576 δ^5 (-1 + θ) - (1 + θ)^2 (2 + θ)^2 (-3 + 2 θ) +
          96 δ^4 (-6 + (-4 + θ) θ) - 2 δ (1 + θ) (2 + θ) (-21 + θ (-7 + θ (9 + θ))) +
          4 δ^3 (-103 + θ (-39 + θ (51 + 19 θ))) + 2 δ^2 (43 + θ (85 + θ (41 - θ (5 + 2 θ))))))/
       ((1 + 4 δ + θ) (24 δ^3 - 12 δ^2 (-1 + θ) + (-2 + θ) (1 + θ) (2 + θ) + 2 δ (-6 + (-4 + θ) θ))^2)),
   0}, {δ, 0, .75}, {θ, .85, .9}, PlotLabel → "", AxesLabel →
   {"δ", "θ", "TS"},
  AxesStyle → Medium, LabelStyle → Bold,
  PlotLegends →
   {"Difference (Case III- Case II)", "Plane at 0"}],
 {c, 1, 1}, {α, 1.5, 1.5},
 {z,
  1,
  1}]
```




```
(*case 4 vs case 2*)
```

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