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Analysis of multi-attribute multi-unit procurement auctions and capacity-constrained sequential auctions

Zhuoxiu Zhang

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ANALYSIS OF MULTI-ATTRIBUTE MULTI-UNIT PROCUREMENT AUCTIONS

AND CAPACITY-CONSTRAINED SEQUENTIAL AUCTIONS

By

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ANALYSIS OF MULTI-ATTRIBUTE MULTI-UNIT PROCUREMENT AUCTIONS

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This dissertation examines an iterative multi-attribute auction for multi-unit procurement in the first part. A multi-unit allocation problem that allows order split among suppliers is formulated to improve the market efficiency. Suppliers are allowed to provide discriminative prices over units based on their marginal costs.

A mechanism called Iterative Multiple-attribute Multiple-unit Reverse Auction (IMMRA) is proposed based on the assumption of the modified myopic best-response strategies. Numerical experiment results show that the IMMRA achieves market efficiency in most instances. The inefficiency occurs occasionally on the special cases when cost structures are significantly different among suppliers. Numerical results also show that the IMMRA results in lower buyer payments than the Vickrey-Clarke-Grove (VCG) payments in most cases.

In the second part, two sequential auctions with the Vickrey-Clarke-Grove (VCG) mechanism are proposed for two buyers to purchase multiple units of an identical item.

The invited suppliers are assumed to have capacity constraints of providing the required demands. Three research problems are raised for the analysis of the sequential auctions: the suppliers' expected payoff functions, the suppliers' bidding strategies in the first auction, and the buyers' procurement costs. Because of the intrinsic complexity of the problems, we limit our study to a duopoly market environment with two suppliers. Both suppliers' dominant bidding strategies are theoretically derived. With numerical experiments, suppliers' expected profits and buyers' expected procurement costs are empirically analyzed.

DEDICATION

I would like to dedicate this research to my parents, and my family.

ACKNOWLEDGEMENTS

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CHAPTER I

INTRODUCTION

This chapter consists of four sections. Section1.1 addresses the motivation to

investigate the online business-to-business reverse auctions. Section 1.2 introduces the

multi-attribute multi-unit reverse auctions that will be investigated in chapters 2 through

4 in this dissertation. Section 1.3 introduces another type of reverse auctions, sequential

auctions with capacity constraints, which will be discussed in chapters 5 through 7.

Section 1.4 presents the organization of this dissertation.

1.1 Motivation

Online business-to-business (B2B) auctions have rapidly emerged as an effective

methodology to make sales or procurements through an electronic marketplace. The

convenient and fast communication provided by the Internet helps to significantly reduce

the transaction time, cost and effort compared with traditional auctions. The rapid

development of the eCommerce prompts researchers' interest on the mechanism design

and analysis for online auctions. One special type of online auction is the reverse auction,

which has become one of the major competitive purchasing tools available for companies

to reduce procurement costs. In a reverse auction, a buyer wants to auction off his

procurement contracts to a series of suppliers. One or multiple winners will be awarded

the contracts based on the auctions' mechanisms. The reverse auctions have a large

proportion in the online B2B auctions because the number of suppliers is much larger than the number of buyers in the industrial market. FreeMarkets.com reports a total of 125 buyers compared with 21,000 suppliers involved in the business transactions in the first quarter of year 2002. Hoffman estimates that 43 percent of all the Fortune 2000 companies plan to conduct the reverse auctions for their procurement transactions [1]. Until today, many major corporations, like General Electric (GE), General Motor (GM), Motorola, Boeing, and etc., have begun to use online reverse auctions on a regular basis. The study in this dissertation will focus on two types of the reverse auctions, the multi-attribute multi-unit auction and the sequential auctions.

1.2   Multi-attribute Multi-unit Reverse Auctions

Multi-attribute reverse auctions are proposed as "electronic request for quotation" (eRFQ) buying processes in the auction literature [2]. In an RFQ process, a corporate buyer announces a set of negotiable attributes, such as quality, lead-time, and technical specifications, for the bidding product. Each attribute has several possible levels. Potential suppliers are invited to submit multi-attribute bids on one or several attribute-level bundles. An attribute-level bundle has one and only one level for each product attribute. The traditional RFQ process terminates with an outcome of one winner and a single selected attribute-level bundle. There are two computational challenges in traditional RFQ buying processes: the evaluation of bids and the winner determination.

With the rapid growth of Internet technologies, automatic eRFQ has become common in online Business-to-Business (B2B) auctions. Online auction mechanisms in the literature fall into three categories: multi-item auctions, multi-unit auctions, and

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multi-attribute auctions. The first two auction forms focus on price-only negotiations. The optimal solution finds the supplier(s) with the lowest cost by assuming that the value of the goods or services for the buyer is pre-determined. In reality, in addition to price, other attributes such as quality and specifications may influence the buyer's utility and, thus, her preferences. In such situations, the multi-attribute reverse auctions closely represent the eRFQ buying process with a series of negotiable attributes besides price.

Most research papers on multi-attribute reverse auctions assume that the suppliers' marginal costs of each attribute-level bundle are fixed and independent from the quantity. Under this assumption, a multi-attribute reverse auction with multiple units is typically reduced to the single-item procurement, and a single supplier wins the entire sourcing contract. However, the marginal costs are usually variable over units because of setup costs, variable costs, and capacity constraints in reality [3]. Under variable marginal costs, allowing the split of the sourcing contract among multiple suppliers may achieve more cost-efficient outcomes for both the buyer and the overall system.

In this dissertation, an iterative multi-unit multi-attribute reverse auction mechanism is proposed for multi-attribute procurements with variable marginal costs. The mechanism allows the contract to be split among winning suppliers. However, the mechanism requires all winners to provide the materials or services on the same attribute-level bundle. Most corporate buyers have this homogeneous requirement to simplify management, reduce future maintenance cost, and preserve the same add-on values for their customers. Parkes and Kalagnanam design an iterative Additive & Discrete (AD) Auction for a special case of the multi-attribute allocation problem with the assumption of additive structure on the buyer's valuation and suppliers' costs [4]. The

AD Auction quotes prices on each attribute. We generalize it into the auction with bidding on each attribute-level bundle by relaxing the assumptions of additive structure and preferential independence [5]. Introducing the concept of attribute-level bundle increases the computational complexity for suppliers to evaluate their bids but extends the auction to a more general setting. In this dissertation, we formulate a mathematical programming model to optimize the efficient market allocation. The results of computer-based numerical experiments are used to compare the performance of the multi-attribute reverse auction with the reverse Vickrey-Clarke-Grove (VCG) auctions.

1.3   Capacitated Sequential Reverse Auctions

The multi-attribute multi-unit auctions studied in the first part of our study are one-stage auctions. The suppliers are assumed to participate in only one auction. However, in reality, it is possible that multiple auctions will be conducted in a sequence for the same set of suppliers to compete for the procurement contracts. For example, two automotive manufacturers want to purchase large amounts of tires through the electronic market. The same set of auto part suppliers will be invited to participate in the two auctions. This market environment including the buyers, a set of suppliers and a third-party auctioneer is defined as sequential auctions. In our study, we focus on the sequential auctions with two suppliers only, which form a duopoly market environment. There are numerous duopoly examples, including Pepsi and Coca-Cola in the soft drink market, Airbus and Boeing in the commercial jet aircraft market, Intel and AMD in the microprocessor market, and etc.

From the viewpoint of the suppliers, the two auctions are not independent to each other and the suppliers have to consider both together to determine their proper bidding strategies. The sequential auction studied in this dissertation considers multiple units of a homogeneous item to purchase in the two auctions. When the suppliers have limited capacities, the suppliers have to consider the loss of opportunity in the later auction because of the capacity occupation in the early auction. Jofre-Bonet and Pesendorfer study the auctions of highway paving contracts run by the California Department of Transportation between 1994 and 2002 [6]. It is found that the losing companies in the earlier auction are more aggressive in the subsequent auctions than those winners. In this dissertation, we propose to use both theoretical analysis and numerical experiments to analyze the suppliers' bidding strategies, payoff functions, and the buyers' procurement costs. Another type of sequential auctions is for the procurements of heterogeneous components. Besides the suppliers' capacity restrictions, the two auctions may also be relevant in the suppliers' production costs. Because of economies and/or diseconomies of scale, winning the earlier auction may decrease or increase a supplier's production cost for the later auctioned components. However, variable production costs will not be studied in this research.

This dissertation particularly focuses on a sequence of two Vickrey-Clarke-Grove (VCG) procurement auctions of a homogenous item. We investigate the impact of suppliers' capacity restrictions on the bidders' behaviors and profits of the sequential auctions. In the auction literature, no study has considered order splitting in the sequential auctions for the procurements of multiple units. Our auction mechanism allows multiple winners in each auction to achieve more efficient allocation of the procurement contracts.

1.4 Dissertation Organization

Chapter 2 reviews the literature related to multi-attribute reverse auctions and multi-unit auctions. The mathematical programming models that solve the winner allocation problem in the proposed multi-attribute multi-unit reverse auction are presented in Chapter 3. Chapter 4 proposes the Iterative Multi-attribute Multi-unit Reserve Auction (IMMRA) mechanism design and experimental conclusions. Chapter 5 summarizes the literature review for the sequential auctions. Chapter 6 documents the model of the two-stage capacitated sequential reverse auctions. Chapter 7 proposes both the theoretical and empirical analysis of sequential reverse auctions and summarizes conclusions and future extensions. Reference list and appendices are attached at the end of the document.

CHAPTER II

LITERATURE REVIEW OF MULTI-ATTRIBUTE AND MULTI-UNIT AUCTIONS

2.1   Multi-Attribute Auctions

Che first presents two-dimensional reverse auctions in which a group of suppliers bid on both price and quality [7]. The bids are evaluated by an ex ante scoring rule announced by the buyer. By defining each supplier's cost structure as an increasing function in quality with an unknown parameter, Che develops three sealed-bid auction mechanisms to maximize the expected buyer profits. With her strong commitment power, the buyer can implement the optimal scoring rule. Branco relaxes Che's assumption of independent supplier cost functions and studies the impact of cost correlation on the multi-attribute auctions [8]. Considering three product attributes: price, quality, and lead time, Chen et al. compare the multi-attribute auctions with the price-only auctions [9]. If the quality and lead time utility functions are known to the auctioneer, the multi-attribute auctions outperform the price-only auctions always on the buyer's profit and occasionally on sellers' profits in the standard English auctions. Beil and Wein extend Che's auction to a more general iterative mechanism [10]. In their paper, a supplier's cost function of each attribute is assumed to have P parameters. It is also assumed that the structure of suppliers' cost functions is exposed to the auctioneer while the P parameters are private information held by the suppliers. An iterative auction mechanism with $P+1$

rounds is designed for the auctioneer to derive the P parameters in suppliers' cost

functions of attributes. With all revealed information, the buyer determines the optimal

scoring functions in the $(P+1)^{st}$ round to maximize her expected profit.

Based on the second-score auctions [7], Parkes and Kalagnanam develop an

iterative price-based reverse auction that provides an equilibrium outcome of the

modified Vickrey-Clarke-Groves (VCG) auctions [4]. Instead of focusing on the buyer's

profits, they consider an efficient design for the market that includes the buyer and all the

suppliers. Under the assumptions of additive cost components [11] and preferential

independence [5], all suppliers submit bids in the forms of additive price parts for each

attribute level after evaluating the ask price from the buyer and their own cost structures.

By assuming fixed marginal costs, it is proposed that a single-item multi-attribute auction

can be easily extended to homogeneous multi-unit procurement [4]. For heterogeneous

items, the combinatorial allocation problem (CAP) is studied as multi-item auctions in

literature [12, 13, 14, 15].

Bichler and Kalagnanam study multi-attribute reverse auctions in the case of

multiple sourcing rather than one single supplier [11]. They also extend the

multi-attribute reverse auctions to the concept of configurable offers. The authors develop

the mathematical models for the winner determination problems under different situations

and analyze the computational complexity. However, there is no discussion about the

implemental mechanism design for multi-attribute reverse auctions in their work. Mishra

and Veeramani propose a descending-price multi-attribute reverse auction mechanism for

single outsourcing and study strategic behaviors of both the buyer and suppliers [16]. The

mechanism achieves nearly efficient allocation and nearly competitive final prices. To the

best of our knowledge, no previous literature has proposed a mechanism for multi-attribute reverse auctions with order splitting among multiple suppliers.

2.2   Multi-unit Auctions

Some researchers study several special assumptions under which the traditional single-unit auction can be extended into multi-unit schemes [17, 18]. Teich et al. present a traditional multi-unit auction with one seller and multiple buyers [19]. They design an algorithm to reduce the price discrimination. Wolfram focuses on the bidding strategies in multi-unit auctions [20]. Tenorio first considers a multi-unit reverse auction with non-linear cost structures of suppliers [21]. Jin et al. generalize the cost functions in Tenorio's work to a U-shaped curve to capture the economies and diseconomies of scale [3]. By investigating the properties of such multi-unit reverse auctions and bidders' behaviors, Jin et al. develop an iterative auction mechanism with three tie-breaking rules for multiple optimal solution cases.

CHAPTER III

MULTI-ATTRIBUTE MULTI-UNIT ALLOCATION PROBLEM (MMAP)

This dissertation studies the multi-attribute multi-unit allocation problem (MMAP) and aims to design a reverse auction mechanism that efficiently allocates multiple units of a certain product with multiple attributes among suppliers with different cost structures. The proposed model relaxes the common assumption that the suppliers' marginal costs for products are constant or monotonic in units. In our study, the suppliers may incur variable marginal costs for their products. We first present a mathematical programming model formulation for the MMAP that optimizes product allocation based on market efficiency. The optimal allocation from the perspective of the integrated market is an important criterion for the performance of auctions in commercial procurement. In an industrial outsourcing scheme, repeated buying is common. A sustainable procurement practice requires long-time partnerships between buyers and suppliers. If the auctioneer focuses only on maximizing the buyer's profits, the suppliers may be discouraged in the long run. Therefore, the market efficiency is considered a primary target of the mechanism design in this paper. In general, maximizing the integrated market net profits provides an important benchmark.

Next we provide the formulation of the winner determination problem in iterative reverse auctions. This formulation represents the buyer's decision model that maximizes

her net profits by optimally determining the winning supplier(s) of each round based on the current submitted bids.

3.1   Market Efficiency Model

In the MMAP, a buyer requests D units of a product that has m attributes. The number of possible levels of attribute $j \in \{1,...,m\}$ is defined as L_j. An attribute-level bundle b is mathematically defined as the set of attribute levels $b=\{l_j | j=1,...,m\}$, where $l_j \in \{1,...,L_j\}$. Therefore, the total number of possible *bundles* is calculated as $B = \prod_{j=1}^{m} L_j$.

For example, suppose a buyer who seeks to procure desktop computers considers the computers' processing speeds and memory as the two main attributes. If there are two levels of processing speed (e.g., 1.66GHz, 3.0GHz) and three levels of memory (e.g., 128MB, 256MB, and 512MB), then there are $2 \times 3 = 6$ possible attribute bundles. It is assumed that n suppliers participate in the auction to compete for the contract. Each supplier can bid for any one of the bundles. Supplier $i \in \{1,...,n\}$ incurs the marginal cost c_{ibk} to produce the $k^{th} \in \{1,...,D\}$ unit of the product with attribute-level bundle $b \in \{1,...,B\}$. The marginal cost structures are private information held only by suppliers. The buyer's unit valuation of attribute-level bundle b is v_b, which is also private information of the buyer. At each iteration, the buyer selects a unique attribute-level bundle for all the products to be purchased. As such, even though the buyer can distribute the winning bids among multiple suppliers, all the winning bids must be from the same attribute bundle. We define a binary variable x_b, which equals one if attribute-level bundle b is selected and zero otherwise. Another binary variable y_{ik} is one if k units are allocated to supplier i

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and zero otherwise. Let F_i denote the total production cost for supplier i. The resulting formulation is an integer linear programming (ILP) model that maximizes the market efficiency (ME) through optimal attribute selection and demand allocation among suppliers:

Max $\qquad D \cdot \sum_{b=1}^{B} v_b \cdot x_b - \sum_{i=1}^{n} F_i$ $\qquad\qquad\qquad$ [MMAP-ME]

s.t. $\quad \displaystyle\sum_{i=1}^{n} \sum_{k=1}^{D} k \cdot y_{ik} = D$ $\qquad\qquad\qquad\qquad\qquad$ (1)

$\displaystyle\sum_{b=1}^{B} x_b = 1$ $\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ (2)

$\displaystyle\sum_{k=1}^{D} y_{ik} \leq 1$ $\qquad\qquad\qquad\qquad\quad \forall i$ $\qquad\quad$ (3)

$F_i \geq \displaystyle\sum_{k=1}^{D} y_{ik} \cdot \left(\sum_{r=1}^{k} c_{ibr} \right) + M_{ib} \cdot (x_b - 1)$ $\qquad \forall i,b$ $\qquad\quad$ (4)

$x_b, y_{ik} \in \{0,1\}; F_i \geq 0$

where $\quad M_{ib} = \displaystyle\sum_{k=1}^{D} c_{ibk}$ $\qquad\qquad\qquad\qquad\qquad$ (5)

The model maximizes the total net profits of the market by determining the *efficient* demand allocations and the best attribute-level bundle. Constraint set (1) ensures that the required demand from the buyer is met. Constraint set (2) ensures that exactly one attribute-level bundle is selected because the buyer wants to procure homogeneous items. Constraint set (3) ensures that a unique amount is allocated to each supplier. For example, consider the case where a supplier submits three bids for one particular

attribute-level bundle: ($2, 2 units), ($3, 3 units), ($4, 5 units). Though the combination

of the first two bids is better than the third one for the buyer, it is infeasible because this

supplier accepts only $4 each for 5 units. Constraint set (4) determines F_i by imposing a

positive right-hand side only if attribute-level bundle b is selected. M_{ib} serves as a big

number in constraints (4), and its value can be determined by equation (5).

3.2   Winner Determination Problem

The mathematical programming model that we use to solve the winner

determination problem in an iterative auction is a variation of the MMAP-ME model.

The basic difference is that the suppliers' private marginal costs are replaced with their

bidding prices. The bids submitted by supplier i are represented as s_{ibk} which denotes the

unit bidding price to provide k units of the products with attribute bundle b. Suppliers are

allowed to ask for discriminative bidding prices for different bidding units. That is, their

unit bidding prices depend on the bidding amount. Supplier i choose to bid on one or

more attribute-level bundles with equivalent maximal utilities by evaluating her marginal

costs and the buyer's current ask prices. The bidders' behaviors will be discussed in

detail in later sections. Based on the model MMAP-ME, the winner determination

problem maximizing the buyer's profit is formulated as follows:

$$\text{Max} \quad D \cdot \sum_{b=1}^{B} v_b \cdot x_b - \sum_{i=1}^{n} \sum_{b=1}^{B} F_i \qquad \qquad \text{[MMAP-BP]}$$

$$\text{s.t.} \quad \sum_{i=1}^{n} \sum_{k=1}^{D} k \cdot y_{ik} = D \qquad \qquad (6)$$

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$$\sum_{b=1}^{B} x_b = 1 \tag{7}$$

$$\sum_{k=1}^{D} y_{ik} \leq 1 \qquad \forall i \tag{8}$$

$$F_i \geq \sum_{k=1}^{D} k \cdot s_{ibk} \cdot y_{ik} + M_{ib} \cdot (x_b - 1) \qquad \forall i, b \tag{9}$$

$$x_b, y_{ik} \in \{0,1\}; F_i \geq 0$$

$$where \quad M_{ib} = \max_{r \in \{1...D\}} \sum_{k=1}^{r} k \cdot s_{ibk} \tag{10}$$

Here, F_i represents total payment to supplier i. The model MMAP-BP maximizes the buyer's profit by selecting the lowest-cost suppliers and the best attribute-level bundle. Equations (6) – (10) indicate the same underlying constraints as those in the model MMAP-ME.

CHAPTER IV

IMMRA MECHANISM DESIGN AND ANALYSIS

This chapter presents an Iterative Multi-attribute Multi-unit Reverse Auction
(IMMRA) mechanism for the MMAP defined in Chapter 3. At the beginning of the
auction, the auctioneer collects unit valuation v_b for each bundle from the buyer. At each
round t, the auctioneer sets a flat unit ask price p_b^t for each attribute-level bundle
regardless of volume. This flat unit ask price is due to the fact that each product with a
certain attribute-level bundle has a single unit value for the buyer independent from the
volume of the purchase. Maintaining a flat unit ask price simplifies the winner
determination problem for the buyer. It also simplifies the suppliers' bid evaluation
process. The price p_b^1 at the first round is initialized to be greater than v_b. This
encourages higher supplier participation as the suppliers' costs are typically lower than
the buyer's valuations. At each round t, supplier i submits one or more bids s_{ibk}^t to
indicate the acceptable unit price of being allocated k units with attribute-level bundle b.
At any round, suppliers can change their bidding and may bid for a completely different
attribute-level bundle and quantity from the previous round. However, we assume the
no-regret rule that the suppliers are not allowed to increase their bids (i.e., ask higher
prices) for a specific quantity and attribute-level bundle. This is ensured
by defining $\bar{s}_{ibk}^t = \min_{r=1..t}\{s_{ibk}^r\}$. The auctioneer solves the winner determination problem

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MMAP-BP at round t with parameter $s_{ibk} = \bar{s}^t_{ibk}$. At the next round, ask prices are updated

as $p_b^{t+1} = \min_{i=1..n, k=1..D, r=1..t} \{s^r_{ibk}\} \leq p_b^t$. The winning bids of the previous round are retained

automatically even if the bidding prices are lower than the current ask prices. For the

losing bids or new quantity and bundle pairs, the prices have to be the same or lower than

the current ask price. However, each supplier is offered a last opportunity to place her

final bids with prices higher than the current ask prices on any preferred bundles if all

prices are higher than her expectation and yield negative profits. Once their *final* bids are

submitted, suppliers cannot change their bids further. The auction terminates if no

supplier changes her bids. At the end of the auction the final winners receive their

payments for winning bids based on prices that they have offered at the previous rounds.

4.1 Bidding Strategies

The suppliers determine their bids based on their cost structures and the current

ask prices without knowing cost information of other competing suppliers. However,

based on the rule of $p_b^{t+1} = \min_{i=1,...,n, k=1,...,D, r=1,...,t} \{s^r_{ibk}\} \leq p_b^t$, the auctioneer decreases an

attribute-level bundle's ask price at round $t+1$ if any supplier submits a bidding price

lower than the attribute-level bundle's ask price at round t. Thus, the ask prices reveal

partial bidding information of the competition, which subsequently intensifies the

competition among suppliers. To analyze suppliers' bidding strategies in iterative

auctions, recent papers on auction mechanisms such as [22, 23, 10] employ the *myopic*

best response (MBR) concept. The *myopic best response* (MBR) is defined as a supplier's

best bids at round t that maximize her profits assuming all other suppliers' bids at round

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t-1 stay unchanged [22]. Under this assumption, in single-unit auctions, the suppliers are assumed to act as if round *t* is the *last round* before the auction terminates as no other suppliers are expected to change their bids. This assumption is reasonable for single-unit auctions. However, multi-unit auctions may have multiple provisional winners at round *t-1*. As such, a supplier may need to change her bids more than once to achieve her highest utility, even if other suppliers keep their bids unchanged. Therefore, assuming that round *t* will be the last round before termination is unreasonable in multi-unit auctions. To account for this problem, we define a *modified myopic best response* (MMBR).

Definition 1. *Modified myopic best response (MMBR) is the supplier's best bidding strategy for round t and the following rounds to maximize her final profits, assuming that other suppliers do not change their bids beyond round t-1.*

The solution of the winner determination problem MMAP-BP at round *t* is defined by $\{ x^t_{b^t*} = 1, y^t_{ik^t_i*} = 1 \}$, which indicates that attribute-level bundle b^t* is selected and supplier *i* is allocated k^t_i* units. We define a minimal bid decrement ε, which is a positive pre-determined parameter used for updating ask prices in the auction. At round *t*, the positive largest utility of supplier *i* at ε below the ask price is defined as

$$U_{\max i}{}^t = \max_{\forall b, \forall k}\{(p^t_b - \varepsilon) \cdot k - \sum_{r=1}^{k} c_{ibr}\}.$$ A non-negative value of $U_{\max i}{}^t$ indicates that supplier *i* is currently an active bidder in the auction. If supplier *i* has a negative $U_{\max i}{}^t$, she becomes an inactive supplier and has no option but to submit her *final* bids. Consequently, assuming MMBR we can make the following conclusion:

Theorem 1 *When minimal bid decrement* ε *approaches zero, active supplier i's*

modified myopic best response (MMBR) in the AuctionMM at round t+1 is :

$$\begin{cases} s_{ibk}^{t+1} = p_b^{t+1} - \varepsilon & \text{if } ((p_b^{t+1} - \varepsilon) \cdot k - \sum_{r=1}^{k} c_{ibr}) = U_{\max i}^{t+1} > F_{i^{t*}} - \sum_{r=1}^{k_i^{t*}} c_{ib^{t*}r}; \\ \text{no bids} & \text{other } (b,k) \text{ pairs.} \end{cases} \quad (11)$$

The detailed proof to Theorem 1 is provided in the Appendix. If supplier i is not a

winner at round t, then $F_{i^{t*}} - \sum_{r=1}^{k_i^{t*}} c_{ib^{t*}r} = 0$ since no unit is allocated to her (i.e. $k_i^t{}^* = 0$,

$F_i^t{}^* = 0$). The first condition of Theorem 1 is satisfied if the losing supplier has a positive

$U_{\max i}^{t+1}$. She will place a bid on all pairs of *(b,k)* that incur the largest positive utility at

the prices ε below the ask prices of round *t+1*. If supplier i is a winner at round t (i.e.

$k_i^t{}^* > 0$, $F_i^t{}^* > 0$), $F_{i^{t*}} - \sum_{r=1}^{k_i^{t*}} c_{ib^{t*}r} \geq U_{\max i}^t$ must be true so that the bids at round t can be

justified. Since $p_b^{t+1} \leq p_b^t$ is always true, the first condition of Theorem 1 is violated for

this winning supplier. As such, the MMBR bidding strategy for a winner is to retain the

bids from the previous round automatically and submit no new bids.

Basically, at the first round, supplier i has no incentive to improve the initial ask

prices at this point and determines her initial bid, s_{ibk}^1, based on the MMBR assumption as

follows:

$$\begin{cases} s_{ibk}^1 = p_b^1 & \text{if } (k \cdot p_b^1 - \sum_{r=1}^{k} c_{ibr}) = \max_{\forall b', \forall k'} \{k' p_{b'}^1 - \sum_{r=1}^{k'} c_{ib'r}\} > 0; \\ \text{no bids} & \text{other } (b,k) \text{ pairs.} \end{cases} \quad (12)$$

Supplier i places the bids at the initial ask prices on the pairs *(b, k)* that maximize her

profit under the initial ask prices. She does not place bids on other *(b, k)* pairs that have

positive utilities at the initial ask prices to avoid unnecessary competition against herself. Based on the MMBR assumption, she does not lose any opportunity to update her bids at later rounds. However, this assumption may lead to a special scenario where there is no feasible solution to satisfy the homogeneous constraint. If this happens, the auctioneer updates the ask prices without allocating any units. Therefore, all suppliers are losers at this round and continue to bid following equation (11) in the next round.

For an inactive supplier i at round t (i.e., $U_{\max i}{}^t = \max\limits_{\forall b, \forall k}\{(p_b^t - \varepsilon)\cdot k - \sum\limits_{r=1}^{k} c_{ibr}\} \leq 0$), if she is a winner, the bids submitted by her are retained automatically. However, when the inactive supplier is a loser, she is no more competitive compared to her opponents. In such a case, she releases her *final* bids with bidding prices above the ask prices. The auctioneer recognizes the bids higher than the current ask prices as *final* bids. Once a supplier submits *final* bids, she is not allowed to change her bids any more. Therefore, for *final* bids, the supplier is assumed to bid on every pair *(b, k)* with minimum integer *q>0*

that satisfies $(p_b^t + q \cdot \varepsilon)\cdot k - \sum\limits_{r=1}^{k} c_{ibr} \geq 0$ to increase her winning possibility. Based on

the *no-regret rule*, once a feasible allocation incurs at one round, it is retained if there is no better one. As long as no feasible allocation arises, all suppliers act as losers to continue decreasing the ask prices until *final* bids are submitted. Based on the assumption of *final* bids, it is guaranteed that the auction will terminate with a feasible allocation if one exists. We note that the *final* bids may reveal partial information regarding the bidding supplier's marginal costs. In practice, these *final* bids are recorded by the auctioneer, who has the responsibility to protect the privacy of all players.

4.2 An Illustrative Example

Consider a buyer who is to purchase three desktop personal computers. The buyer is specifically interested in two attributes of the computers. One attribute is the processor, which has two levels as 2.66GHz, 3.0GHz. The other attribute is the memory, with two levels as 256MB, 512MB. There are four possible attribute-level bundles that can be provided by three suppliers as listed in Table 1. Both the buyer's unit valuation for each bundle and the suppliers' marginal costs to produce each unit of every attribute-level bundle are given in Table 1.

Table 1   An Instance with Two Attributes and Three Units

		Units	Bundle 1	Bundle 2	Bundle 3	Bundle 4
			2.66GHz 256MB	3.0GHz 256MB	2.66GHz 512MB	3.0GHz 512MB
	Value/unit	1	$14	$46	$28	$60
Supplier 1	**Marginal**	1	$8	$10	$17	$19
		2	$6	$10	$13	$17
	cost/unit	3	$6	$8	$15	$17
Supplier 2	**Marginal**	1	$6	$9	$13	$16
		2	$9	$12	$18	$21
	cost/unit	3	$10	$12	$21	$23
Supplier 3	**Marginal**	1	$7	$11	$13	$17
		2	$8	$13	$13	$18
	cost/unit	3	$10	$14	$19	$23

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Based on the buyer valuation and the supplier cost information, we can employ the model MMAP-ME to solve the winner allocation problem using any optimization solver (such as ILOG CPLEX). The optimal solution selects the bundle of (3.0GHz, 512MB) with the allocation of (0, 1, 2), indicating that supplier 1 wins nothing, supplier 2 wins 1 unit, and supplier 3 wins 2 units, respectively. The maximum market gain from this trade is:

$$(3)(\$60) - (\$16) - (\$17 + \$18) = \$129.$$

The IMMRA for this example is simulated in Table 2. The minimum decrement ε is set to $6. In reality, this value could be as small as $1. Since most online auctions are implemented by computer agents automatically(e.g. Strecker et al., 2004), the number of rounds is not a considerable computational issue and, as such, the decrements can be kept small. The initial ask prices are set to $p_b^1 = v_b + \varepsilon$. All the suppliers follow the bidding strategies as described in subsection 4.1. At each round, the auctioneer solves the MMAP-BP model to determine the optimal attribute-level bundle and allocation. In case of multiple optimal solutions, the winners of the last round are favored. In the following IMMRA example, bids are expressed as *s[k]* under preferred attribute-level bundles from suppliers, where *s* indicates the bid price and *k* is the desired unit under the price. There might be multiple desired units that yield the same potential profits to the supplier, which is expressed as *s[k₁,k₂,…]*.

Table 2　An IMMRA Example. ("*" indicates the provisional allocation.)

		B1			B2			B3			B4		
	value	14			46			28			60		
R		S1	S2	S3	S1	S2	S3	S1	S2	S3	S1	S2	S3
1	price		20			52			34			66	
	bid										66*[3]	66[3]	66[3]
2	price		20			52			34			66	
	bid										66[3]	60*[3]	60[3]
3	price		20			52			34			60	
	bid				46[3]							60[3]	54*[3]
4	price		20			46			34			54	
	bid				40[3]	40[3]							54*[3]
5	price		20			40			34			54	
	bid										48*[3]	48[3]	54[3]
6	price		20			40			34			48	
	bid					34[3]					48[3]		42*[3]
7	price		20			34			34			42	
	bid				28[3]	28[3]							42*[3]
8	price		20			28			34			42	
	bid										36*[3]	36[3]	42[3]
9	price		20			28			34			36	
	bid					22[3]				28[3]	36*[3]		
10	price		20			22			28			36	
	bid										36[3]	30[3]	30*[3]
11	price		20			22			28			30	
	bid	14[3]	14[3]										30*[3]
12	price		14			22			28			30	
	bid					16[3]		22[3]					30*[3]
13	price		14			16			22			30	
	bid										24*[3]	24[3]	30[3]
14	price		14			16			22			24	
	bid								16[1]	16[2]	24*[3]		
15	price		14			16			16			24	
	bid		8[1]	8[1,2]							24[3]	18*[1]	18[1,2*]
16	price		8			16			16			18	
	bid		8[1]	8[1,2]	10[3]							18*[1]	18[1,2*]
17	price		8			10			16			18	
	bid	8[1,2,3]	8[1]	8[1,2]	10[1,2,3]			**22[1]** 16[2,3]			**24[1]** 18[2,3]	18*[1]	18[1,2*]
18	price		8			10			16			18	
	bid	8[1,2,3]	8[1]	8[1,2]	10[1,2,3]			**22[1]** 16[2,3]			**24[1]** 18[2,3]	18*[1]	18[1,2*]

Round 1. Initial prices are set to be $\varepsilon=\$6$ greater than the values of the buyer. Initial

bidding prices from the suppliers are determined based on condition (12). All suppliers

bid at price \$66 on attribute-level bundle 4 of 3 units (i.e., the pair of (4, 3)) since this pair provides the maximal utility to every supplier. Supplier 1 is arbitrarily selected to be the winner.

Round 2. Winning supplier 1 keeps the same bids. Losing suppliers 2 and 3 update their bids following Theorem 1. If they decrease the ask prices by □□\$6 for all pairs, their maximal utilities still occur on the pair of (4, 3). Therefore, both suppliers 2 and 3 bid at \$60 on pair (4, 3). Solving the winner determination problem MMAP-BP, supplier 2 is selected to be the winner.

Round 3 - 15. Ask prices are updated to match the lowest bidding prices at the last round. Following Theorem 1, losing suppliers update their bids, while winners keep their bids unchanged. A provisional allocation of (0, 1, 2) is the result of round 15 with attribute-level bundle 4 selected.

Round 16-17. If the current ask prices are reduced by \$6 more, the maximum utility is negative for every supplier. Suppliers 2 and 3, who are winners, stay on their current bids. However, the loser, supplier 1, who is inactive, submits her *final* bids. The bids marked in bold that are higher than the ask prices are recognized as *final* bids by the auctioneer. Therefore, supplier 1 is forbidden to update her bids anymore. Consequently, the auction terminates with bundle 4, and the payments of (0, \$18, \$36) to the three suppliers are made respectively.

To evaluate the outcome payments further from the simulated IMMRA, we use the traditional Vickrey-Clarke-Groves (VCG) payment as a benchmark. Let Z^* denote the optimal value of the model MMAP-ME. Z_{-i}^* is the alternative optimal value without supplier i. The marginal value of supplier i is defined as $Z^*-Z_{-i}^*$, which is her contribution

to the market. The VCG mechanism terminates with a payment of the winner's marginal

value plus her cost. Thus, a winner's profit is computed as her marginal value. In this

example, marginal value of supplier 2 is calculated as $Z^* - Z_{-2}^* = \$129 - \$127 = \$2$,

while that of supplier 1 is calculated as $Z^* - Z_{-1}^* = \$129 - \$128 = \$1$. Therefore, the

VCG payments to suppliers 2 and 3 are $16(cost)+$2(margin)=$18 and

$35(cost)+$1(margin)=$36, respectively. In this example, the IMMRA has the same

payments as the VCG auction. In the next section, more numerical experiments are

conducted to compare the performance of the IMMRA to the VCG auction. The analysis

shows that the IMMRA terminates with lower payments than the VCG auction does in

most cases.

4.3   Experimental Results and Discussions

Numerical experiments are conducted on a problem with 4 suppliers, 4 attributes,

and 4 levels per attribute. It is assumed that the suppliers follow the strategies described

in subsection 4.1 throughout the auction. In the experiments, data are generated randomly

with two positive parameters α^{Sell} and α^{Buy}. A weight $w_b \sim U(0,1)$ is randomly selected

for each attribute–level bundle b. All weights are normalized to satisfy $\sum_{b=1}^{B} w_b = 1$. For

the buyer's valuation, we randomly generate B values from $U(0, \alpha^{Buy} \cdot B)$ and sort these

values. Sorted values are multiplied by the weight w_b to generate the buyer's valuation

v_b for attribute-level bundle b. For suppliers' marginal costs, we first create the mean

value $\bar{c}_{ib}$ over all units similarly as the buyer's valuation. For supplier i, B random

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values are generated from $U(0, \alpha^{Sell} \cdot B)$. These values are sorted and multiplied by the

normalized weight w_b to generate $\bar{c}_{ib}$. For supplier i, D coefficients are randomly

generated from $u_k \sim U(-0.5, 0.5)$ for each bundle to compute the marginal cost as

$c_{ibk} = \bar{c}_{ib} + \bar{c}_{ib} \cdot u_k$. The two constants are chosen to satisfy $\alpha^{Buy} \geq \alpha^{Sell}$ to guarantee that

the buyer's valuation for a bundle is greater than the marginal cost of an average supplier.

In the experiments, we set $\alpha^{Buy} = 30$ and $\alpha^{Sell} = 40$ by default. Table 3 shows the

performance comparison of the VCG auction and the IMMRA for 20 instances with the

minimal bid decrement of $1.

Table 3   Comparison of IMMRA and the VCG Auction

	The VCG Auction			IMMRA		
Instance	System utility	Total payment	Buyer utility	System utility	Total payment	Buyer utility
1	354	56	298	*347*	47	300
2	371	44	327	371	41	330
3	332	42	290	332	37	295
4	153	13	140	153	3	150
5	468	78	390	468	73	395
6	256	77	179	256	71	185
7	323	73	250	323	68	255
8	437	93	344	437	92	345
9	332	24	308	332	*27*	*305*
10	329	28	301	329	*29*	*300*
11	342	23	319	342	22	320
12	278	15	263	278	13	265
13	390	14	376	390	10	380
14	168	35	133	168	33	135
15	430	59	371	430	55	375
16	640	77	563	640	*80*	*560*
17	782	120	662	782	*122*	*660*
18	820	108	712	*818*	38	780
19	776	99	677	776	96	680
20	997	383	614	997	382	615

In the results listed in Table 3, 2 out of 20 instances yield inefficient system utility marked in bold and italics. To further verify the efficiency of IMMRA, we run the experiments for 100 instances and get 27 cases with inefficient solutions. In other words, 73 percent of the instances result in efficient allocations and the best bundle selections for the market.

We study the following simple case that does not result in the first best solution under the proposed IMMRA procedure to investigate the underlying reasons of the inefficiency. Suppose the buyer wants to procure three units of a product with two attribute-level bundles. Three suppliers are involved in the auction to compete for the order. The buyer's valuations and suppliers' marginal costs are provided in Table 4. The simulation of IMMRA is conducted round by round in Table 5 with the minimal decrement of $10. Solving the optimization model MMAP-ME, we show that the efficient solution is to allocate 3 units to supplier 1 with bundle 1. However, the outcome of the auction selects bundle 2 with the same allocation. In this case, the system utility is degraded from the optimal value of $114 to $92.

Table 4   An Inefficient Instance

		units	*Bundle 1*	*Bundle 2*
	Value/unit	1	$70	$80
supplier 1	marginal	1	$34	$58
		2	$34	$54
	cost/unit	3	$28	$36
supplier 2	Marginal	1	$38	$62
		2	$44	$60
	cost/unit	3	$60	$56

Table 5   An Inefficient IMMRA Example. ("*" indicates the provisional allocation.)

		B1		B2	
	value	70		80	
R		S1	S2	S1	S2
1	price	80		90	
	bid	80*[3]	80[3]		
2	price	80		90	
	bid	80[3]	70*[3]		
3	price	70		90	
	bid		70*[3]	80[3]	
4	price	70		80	
	bid	60*[3]	70[3]		
5	price	60		80	
	bid	60*[3]			70[3]
6	price	60		70	
	bid	60*[3]	50[2]		
7	price	50		70	
	bid	60[3]	40[1]		60*[3]
8	price	40		60	
	bid		40[1]	50*[3]	60[3]
9	price	40		50	
	bid		40[1] 50[2,3]	50*[3]	70[1,2] 60[3]
10	Price	40		50	
	Bid		40[1] 50[2,3]	50*[3]	70[1,2] 60[3]

At rounds 6 and 7, supplier 2 reduces her bidding prices of 2 units or 1 unit for bundle 1. She is not selected to be a winner because no feasible solution including her bids exists. However, the buyer's ask prices are triggered to decrease because of the lowest bidding prices from supplier 2. Supplier 1 automatically retains her bids as a provisional winner. Therefore, at round 8, bundle 1 with an ask price at $40 is no longer the best choice for supplier 1 to improve her bids based on Theorem 1. After supplier 1 reduces her bidding price for bundle 2 to $50, supplier 2 is no longer competitive and has to submit her *final* bids. The auction terminates with the outcome of allocating 3 units to supplier 1 and selecting bundle 2. This situation occurs when the ask prices for the efficient bundle decrease too quickly because a losing supplier has an especially low production cost for an amount that is less than the demand and her bids for that bundle do not lead to a feasible allocation for the buyer. Nevertheless, the ask price is still decreased due to diminishing bids. Meanwhile, there exists another competitive bundle with a better utility for the previous winner for which the best feasible allocation is achieved. Consequently, the ask prices that are decreased inefficiently lead to less-than-optimal results.

The primary purpose of the buying corporations to participate in online auctions is to save their costs by reducing their payments to suppliers. The numerical analysis reveals that, under the IMMRA mechanism, auctions terminate with lower payments to the winners compared to the VCG auctions in most instances. In the experimental results, we observe that the buyer saves money in procurement costs by participating in IMMRA in 80 percent instances. Recall that the VCG payment to supplier i is defined as the

margin value $Z^*-Z_{-i}^*$. Here, Z_{-i}^* indicates the optimal market utility based on the optimal

allocation in the absence of supplier i. This allocation is the second best solution in a

single item multi-attribute auction. However, since we allow the order split in the

IMMRA, which considers multiple units, the second best solution may still include

supplier i as a winner with different winning allocation. Therefore, the IMMRA may

result in a payment less than the margin value $Z^*-Z_{-i}^*$ because of a better second-best

solution that includes supplier i. If the second-best solution for the IMMRA does not

include any winning supplier from the best allocation, the payment to the winners should

be at least as good as the VCG payment when the minimal decrement ε approaches

zero. The four payments marked in bold and italics in Table 3 are greater than the VCG

payments because the minimal decrement ε is not small enough.

4.4   Conclusions

The iterative multi-attribute multi-unit reverse auction (IMMRA) is proposed in

the previous chapters for procurement auctions involving multiple units of products or

services with a series of negotiable attributes besides price. The goal of the proposed

mechanism is to incorporate the suppliers' variable marginal costs efficiently in the

multi-attribute reverse auctions, where suppliers prefer to bid with discriminative prices

over units and buyers can split their purchases among bidding suppliers. Under this

mechanism, splitting the procurement contract to more than one supplier often leads to

higher buyer profits. To our knowledge, our work is the first to investigate the mechanism

design of a multi-attribute auction that allows for multiple unit procurement with order

splits.

We first proposed an integer programming model for the IMMRA to solve the winner determination problem. To study the bidders' behaviors, we tailor the myopic best response (MBR) approach for the multi-attribute case and develop the *modified myopic best response* (MMBR). The latter approach ensures a proper bidding strategy for the order-splitting situation in the proposed IMMRA mechanism. The suppliers' MMBR bidding strategies for the IMMRA are discussed in detail and illustrated in Section 4. The experimental results show that the market efficiency is achieved in most instances. The inefficiency occurs occasionally for special cases where bidding suppliers have significantly different cost structures. Numerical results show that, in general, the buyer pays less in the IMMRA mechanism compared to the VCG payments.

In the future, it will be interesting to study the information revelation and computational complexity issues of the IMMRA discussed in this paper. Fixing the inefficiencies that were observed in the experimental results is another interesting future topic. One idea is to allow the auctioneer to maintain discriminative ask prices over units, meaning that we may avoid the excess price reductions on the efficient allocation but increase the computational complexity. Another interesting future direction is to relax the assumption for placing the *final* bids and find out a more robust bidding strategy without compensating of efficiency.

CHAPTER V

LITERATURE REVIEW OF SEQUENTIAL AUCTIONS

In the following chapters 6 and 7, we will investigate another type of online B2B

auctions, sequential reverse auctions, which is roughly introduced in section 1.3. Chapter

5 reviews different aspects of literature in this research field. Section 5.1 summarizes

existing study about morning and afternoon effects of prices in sequential auctions. Both

theoretical analysis and empirical observations of the price trend have been studied in

literature. Section 5.2 provides a review of the information structure of sequential

auctions presented in articles. Section 5.3 discusses two types of bidders' valuation

structures, complementarity and substitutes, and reviews the literature relative to these

two configurations. Section 5.4 reviews articles in which the sequential auctions with

capacity or budget constraints are investigated.

5.1   Morning and Afternoon Effects in Sequential Auctions

Milgrom and Weber studied sequential auctions for selling identical objects to a

set of bidders who each has unit demand [18]. They compared simultaneous auctions and

sequential auctions regarding the differences in equilibrium characterizations,

bidding strategies, expected price series, and expected revenues. The authors theoretically

presented that the equilibrium expected prices in sequential auctions would follow an

upward-drifting trend, called "morning effect", by assuming symmetric risk neutral

bidders with independent private valuations. However, empirical evidence shows that actual prices may either decline or increase through sequential auctions. In the famous empirical phenomenon called "afternoon effect", an obvious decline in prices has been observed over sequential auctions for various items. Ashenfelter observed the "afternoon effect" in sequential wine auctions [24]. Later, Ashenfelter and Genesove reported the similar "price-decline-anomaly" phenomenon in auctions of apartments [25]. McAfee and Vincent investigated the Chicago wine auctions to develop theoretical models [26]. They obtained the "afternoon effect" in their model with the assumption of non-decreasing risk-averse bidders. On the contrary, Gandal observed increasing prices in the cable TV license auctions in Israel [27]. In sequential wool auctions, the prices were noticed to either increase or decrease [28].

In the theoretical literature of sequential auctions, these anomalous price trends are explained in different ways. Picci and Scorcu used dynamic panel data econometric techniques to analyze the price trend in sequential auctions of heterogeneous art objects [29]. They took the auctioneer behaviors into consideration to achieve the efficient allocation and maximized revenue. Under this environment, no "afternoon effect" was observed but a slight price increase seemed to be present. They summarized the literature presenting the anomaly "afternoon effect" and the possible underlying assumptions leading to decline prices. Von der Fehr took the participation cost into consideration to explain "afternoon effect" [30]. Black and de Meza presented the "right to choose" option to allow the first-auction winner to select his/her preferred object [31]. The other object left was to be auctioned in the second auction by remaining bidders. This option could also lead to the "afternoon effect" in sequential auctions. Gale and Hausch compared

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price tendencies between sealed-bid sequential auctions and "right to choose" sequential auctions [32]. With two unit-demand bidders and two identical objects for sale, they presented the conjecture that declining prices would be possibly caused by bidders' conservative strategies. Under the assumption of the complete information of the future object's valuation, sealed-bid sequential auctions may yield a non-efficient allocation while "right to choose" sequential auctions can achieve the efficiency. When the information of reservation prices and other bidders' behaviors is available to all bidders, a model in which two heterogeneous objects are auctioned off sequentially presents the "afternoon effect" and further demonstrates that it is optimal for the auctioneer to order the objects by declining estimated values [32]. The impact of the information structure on equilibrium bidding strategies and the allocation is another hot topic in the sequential auction research.

5.2   Information Structure in Sequential Auctions

The assumptions on information structure play an important role in the theoretical analysis of sequential auctions. Most researches in the sequential auction literature assume identical objects. Bidders are typically assumed to have the complete information of future goods. In other words, they know the quantity of identical objects that will be auctioned in later auctions. Besides the research by Milgrom and Weber [18] and Black and de Meza [31] mentioned in subsection 5.1, Katzman examined a second-price sequential auction for multi-unit identical objects with the assumption of diminishing marginal valuations of bidders [33]. Both situations of complete and incomplete information about other bidders' behaviors were investigated. The equilibrium allocation

under the complete information situation may be inefficient and lead to declining prices (afternoon effect). Under the incomplete information, however, symmetric equilibrium accomplishes the efficient outcome and increasing prices (morning effect). In practice, many sequential auctions have heterogeneous objects to sell. Benoit and Krishna proposed common value sequential auctions for two heterogeneous objects and three budget-constrained bidders [12]. Under a complete information setting, it is demonstrated that selling the more valuable object first improves the expected revenue. This result can be extended to the cases with two objects and n budget-constrained bidders. In sequential English auctions for common value objects in an incomplete information environment, bidders' budget constraints affect the optimal bidding behaviors and the revenue [34].

In addition to common value objects, sequential auctions of heterogeneous private value objects also got interest from researchers. Pitchik and Schotter studied the budget-constrained bidders who were willing to purchase multiple units of both goods [35]. With the perfect information about future goods, the authors investigated the effect of budget constraints on bidders' behaviors. Elmaghraby complemented the studies of sequential auctions for heterogeneous private value objects under incomplete information about future goods and other bidders' behavior [36]. By using the second-price sealed-bid auction mechanism, the author analyzed how the ordering of objects in sequential auctions would affect the bidding behaviors and the expected revenue. Fatima et al. presented their study of sequential auctions in uncertain information settings [37]. Both computational and economic properties of the equilibrium solution were analyzed based on three sources of uncertainty: valuation of the objectives for sale, the number of objects for sale, and the number of participating bidders.

5.3 Complements vs. Substitutes

Winning an object in the first auction may affect a bidder's valuations in the second auction. This influence commonly happens in sequential auctions of heterogeneous objects in practice. If winning previous auction decreases the winner's values of objects in the later auction, these objects being sold are classified as substitutes. On the contrary, complementarity means that winning previous auction increases the winner's values of objects in the later auction. Many empirical evidences show that complements and substitutes both exist in sequential auctions. Complementarity usually exists in sequential procurement auctions of related construction contracts. Winning multiple contracts can reduce the total cost because of synergistic tasks. Cramton studied the Federal Communications Commission (FCC) spectrum auctions and showed that synergies among heterogeneous objects would affect both bidders' complementarities between goods and the efficiency of the goods allocation [38]. The synergies could directly stimulate winning bidders to bid more aggressively in sequential first-price auctions [39]. Wolfram investigated sequential electricity auctions with bids including the fixed setup cost and idle cost and reported that complementarities are observed in the electricity generation for consecutive time periods [20]. Anton and Yao documented that there were complementarities in sequential auctions for defense contracts because accumulating higher experience in the earlier project would help to reduce training and learning costs in the future contract [40]. Gandal observed complementarities in cable TV license auctions [27]. Pesendorfer proposed the incumbency advantage in winning adjacent school milk contracts because of cost savings in tank investments and the daily

delivery [41]. Substitution usually happens in industrial procurement auctions where the bidders (suppliers) have limited capacities. Therefore, winning additional objects (project contracts) may require extension of capacities, which may yield extra fixed cost. Zulehner observed the negative correlation between bids in the first auction and the subsequent auctions from the same bidder in sequential cattle auctions [42]. Jofre-Bonet and Pesendorfer theoretically showed that the winning bidder would increase the bid mark-up of substitutes [6]. The effect of this substitution was supported by the empirical evidence described in the paper. List et al. investigated sequential timber auctions and found the similar characterization of substitutes [43]. Jofre-Bonet and Pesendorfer studied substitutes in sequential highway-paving procurement auctions and summarized characterizations of complementarities and substitutes to provide theoretical analysis of two-period reverse sequential auctions with a single contract offered in each period [6]. They concluded that the buyer would prefer sequential first-price auctions for substitutes while standard English auctions, equivalent to sealed-bid second-price auctions, are preferred by the buyer for complementarities. Since the preference of the buyer minimizes procurement costs and therefore hurts the profits of bidders, bidders prefer the contrary to maximize their revenue. In the next subsection 5.4, we will review papers studying budget-constrained sequential auctions as a special case of substitutes.

5.4   Auctions with Budget or Capacity Constraints

Gallien and Wein proposed a multi-item procurement auction based on optimal winner determination models [23]. In their model, the buyer is willing to buy a set of objects from suppliers who have capacity constraints. To efficiently allocate the buyer's

demands among suppliers, order splitting is allowed to purchase one object from multiple

suppliers. The authors analyzed the bidding behaviors and the incentive compatibility for

the revelation of suppliers' capacity and cost information. Capacity constraints are

necessary to be considered in the order splitting environment to guarantee the feasibility

of the allocation. In the literature of sequential auctions for identical objects, budget

constraints are typically simplified to the assumption that each bidder wants only one

object. For heterogeneous objects, Pitchik studied the budget-constrained sequential

sealed-bid auctions based on private valuation models [44]. The author investigated the

impact of bidders' income budgets on their bidding behaviors and the expected revenues.

Since two auctions in the sequence were assumed to be offered by the same selling agent,

the adjustable sequence of objects would also affect the expected revenue and prices.

In this dissertation, we investigate suppliers' bidding behaviors under the

two-stage sequential reverse auctions. We extend Gallien and Wein's work of

procurement auctions with capacity constraints for one single project to multi-unit items

procurement auctions. In our market environment defined in the next chapter, it is

assumed that suppliers have complete information about other suppliers' costs and

capacities, demand for the first auction, and demand distribution for the second auction.

Under this information structure, suppliers' payoff functions and buyers' procurement

costs are empirically studied in chapter 7. To the best of our knowledge, the reverse

procurement sequential auctions in which the suppliers have limited manufacturing

capacities have not been studied in the literature.

CHAPTER VI

FORMULATIONS OF CAPACITATED REVERSE SEQUENTIAL AUCTIONS

6.1 Market Environment and Problem Description

In our market environment, we assume two corporations play as buyers in two sequential auctions respectively. Buyers A and B want to purchase D^A and D^B units of one homogenous industrial component. Each auction offers one buyer's procurement contracts to suppliers by using the Vickrey-Clarke-Groves (VCG) mechanism. This study considers a duopoly supply market in which two monopolistic suppliers compete for the two buyers' procurement opportunities. The two suppliers dominate the market with cost competitiveness. Supplier 1 and supplier 2 are characterized by their respective unit costs, c_1 and c_2, and their fixed total supply capacities, O_1 and O_2, respectively. Without the loss of generality, supplier 1 is assumed to be more price-competitive with smaller unit cost, (i.e., $c_1 \leq c_2$). It is assumed that the capacities of the two suppliers can be used and only be used for the two opportunities and any unused capacity is assumed to have zero salvage value. In other words, the supply capacity is time-sensitive and cannot be carried over periods. It is also assumed that suppliers cannot expand their capacity during and after auctions, so overstating his capacity cannot increase a supplier's profit under any circumstances and may cause heavy penalty because of violating procurement contract and losing long-term goodness from buyers.

In the first auction, buyer A announces her demand, D^A, collects prices and available quantities from both suppliers, and then determines the optimal assignments of her contract to minimize the procurement costs. Order splitting is assumed acceptable for the buyers to obtain low costs. When the total submitted amount from the two suppliers is not enough for the demand, the buyer can fulfill the unmet demand from a spot market at the price of c_m, which is higher than both c_1 and c_2. In the second auction, buyer B and the two suppliers follow the same procedures. We assume that when submitting prices and available amounts in the first auction, the two suppliers do not exactly know buyer B's demands. However, it is also assumed that the future demand in the second auction can be forecasted based on historical data. A common cumulative distribution function $F(d)$ for the demand in the second auction is public to all suppliers before the first auction. Therefore, when suppliers make decisions about their quantities and prices to be submitted in the first procurement auction, the uncertainty of the future demand from buyer B is necessary to be considered. We assume that in a duopoly market environment, the two suppliers monopolize most market share and have enough knowledge of each other's capacity and cost information based on historical experience. Knowing complete information except the actual future demand of buyer B, the suppliers need to sequentially decide quantities and prices in both auctions to maximize their total profits. Under the risk neutral assumption, the suppliers are assumed to solely focus on the expected total profits.

6.2   Models

In both auctions, the Vickrey-Clarke-Groves (VCG) mechanism is assumed to be used to determine the payment to winning suppliers. In the first procurement auction for buyer A, suppliers 1 and 2 set their quantities and prices at (Q_1^A, P_1^A) and (Q_2^A, P_1^A) respectively. Based on the suppliers' offers, the optimal assignment to minimize the total cost to fulfill the demand for buyer A is defined by the following optimization model SAP-1:

$$\min \quad Z^A = \sum_i P_i^A x_i^A + c_m y_m \qquad \text{[SAP-1]}$$

$$\text{s.t.} \quad x_i^A + y_m = D^A \qquad i =1 \text{ or } 2; \qquad (13)$$

$$x_i^A \le O_i \qquad i =1 \text{ or } 2; \qquad (14)$$

$$x_i^A \ge 0 \qquad i =1 \text{ or } 2. \qquad (15)$$

The solution to problem SAP-1 is well defined by

$$x_1^A = \begin{cases} \min\{Q_1^A, D^A\} & P_1^A \le P_2^A \\ \min\{Q_1^A, D^A - x_2^A\} & P_1^A > P_2^A \end{cases}, \text{ and} \qquad (16)$$

$$x_2^A = \begin{cases} \min\{Q_2^A, D^A\} & P_1^A > P_2^A \\ \min\{Q_2^A, D^A - x_1^A\} & P_1^A \le P_2^A \end{cases}. \qquad (17)$$

Here, x_1^A, x_2^A, and y_m are the quantities of buyer A's demands allocated to supplier 1, supplier 2, and spot market respectively. Let Z_{-i}^A as the objective function value of the optimal solution to model SAP-1 after setting $Q_i^A = 0$. Based on the generalized VCG auction mechanism, VCG payments to supplier i would be formulated as $Pay_1^A = P_1^A x_1^A + Z_{-i}^A - Z^A$. The same VCG mechanism deciding allocation and payment

is also applied for the procurement auction of buyer B. Shown by Vickrey (1961), the dominating bidding strategy for a determinant VCG auction is to bid a supplier's true cost. In this sequential auction analysis, suppliers are assumed to be well educated about the VCG mechanism and have incentive to set prices as their true costs (i.e., $P_i^A = P_i^B = c_i$). Since suppliers are willing to submit their true cost information as prices, allocations of buyer A's demand between the two suppliers are simplified to:

$$x_1^A = Q_1^A, \text{ and}$$

$$x_2^A = \min\{Q_2^A, D^A - Q_1^A\}, \tag{18}$$

$$\text{where } Q_1^A \le O_1; Q_2^A \le O_2.$$

After the first auction, suppliers' capacities have been occupied by the assigned units x_i^A. The available capacity of supplier i for the second auction are reduced to $O_i - x_i^A$. A supplier prefers to submit his true remaining capacity in the second auction because he wants to utilize his capacity as much as possible due to the assumption of zero salvage values for unused capacity. With submitted quantities of $O_1 - Q_1^A$ and $O_2 - x_2^A$ from the two suppliers, allocations of buyer B's demand in the second auction are:

$$x_1^B = \min\{O_1 - Q_1^A, \tilde{D}_B\} \text{ and}$$

$$x_2^B = \min\{[Q_2^A - D_A + Q_1^A]^+, [\tilde{D}_B - O_1 + Q_1^A]^+\}, \tag{19}$$

where random variable $\tilde{D}^B$ indicates buyer B's demand, which is not deterministic during the first auction when suppliers need to decide bidding quantities in buyer A's procurement auction. The uncertainty of future demand in the second auction brings a challenge to suppliers: whether to submit all available capacities in auction 1 or save some capacities for auction 2 in which higher payments may be accomplished. The

obvious risk for saving capacities is to encounter some profit loss if demand in the second auction is too low to fulfill the remaining capacities. The suppliers' overall objective is to maximize their total profits, denoted by π_i, from the two buyers' procurement contracts by participating in both auctions. Supplier i's optimal strategy to determine the bidding quantity Q_i^A in the first auction is defined by the following optimization model [SAP-2]:

$$\max \quad \pi_i = Pay_i^A - c_i x_i^A + Pay_i^B - c_i x_i^B \qquad \text{[SAP-2]}$$

$$\text{s.t.} \quad Q_i^A \leq O_i; \qquad\qquad\qquad . \qquad\qquad (20)$$

$$Q_i^A \geq 0.$$

The two payment terms are defined by the objective function value Z_i^A from model [SAP-1], which makes solving the problem [SAP-2] nontrivial. Further more, random variable $\widetilde{D}^B$ is involved in the model, which brings additional challenges to solve Q_i^A directly. In the next Chapter, we investigate suppliers' bidding strategies with theoretical analysis.

CHAPTER VII

ANALYSIS OF SEQUENTIAL AUCTIONS

7.1 Suppliers' Bidding Strategies

As discussed in Chapter 6, both suppliers have incentive to reveal their true costs

in the two sequential VCG auctions. Buyers would prefer to allocate the demands to

supplier 1 until supplier 1's submitted quantities are fulfilled. Then, buyers assign

remaining units to supplier 2 with VCG payments $Pay_2^A = c_m x_2^A$ and $Pay_2^B = c_m x_2^B$ in

the two auctions respectively.

Theorem 2. Supplier 2 has dominant strategies in both auctions as follows:

$$Q_2^A = \min\{O_2, D^A\} \text{ and}$$
$$Q_2^B = O_2 - x_2^A \tag{21}$$

to maximize his total assigned allocations x_2^A and x_2^B by submitting his true available

capacities in both auctions.

With the knowledge of VCG payments and supplier 2's dominant strategies,

supplier 1's profit in the first auction is formulated as functions in Q_1^A as follows:

$$\pi_1^A(Q_1^A) = Pay_i^A - c_i x_i^A$$

$$= \begin{cases} D^A(c_2 - c_1) & Q_1^A = D^A \le O_2 \\ O_2 c_2 + (D^A - O_2)c_m - D^A c_1 & Q_1^A = D^A > O_2 \\ Q_1^A(c_2 - c_1) & Q_1^A < D^A, D^A - O_2 \le 0 \\ (O_2 - D^A + Q_1^A)(c_2 - c_1) + (D^A - O_2)(c_m - c_1) & D^A > Q_1^A \ge D^A - O_2 > 0 \\ Q_1^A(c_m - c_1) & 0 < Q_1^A < D^A - O_2. \end{cases} \quad (22)$$

In the first two scenarios of (22), supplier 1 chooses to provide as many as buyer A's demand to win the whole procurement contract and leaves nothing to supplier 2 in buyer A's procurement auction. In the third and forth scenarios, supplier 1's submitted quantity is not enough to cover buyer A's procurement demand and supplier 2 has enough capacity to meet the remaining demand. In the third scenario, supplier 2 can meet buyer A's demand even without supplier 1's submitted capacity, which results in a unit payment $c_2 - c_1$ to supplier 1. However, in the fourth scenario, price for part of the allocation to supplier 1 is determined by the spot market price because of the VCG mechanism. In the last scenario, even the sum of the two suppliers submitted capacities is not enough to meet buyer A's demand so that part of the demand has to be fulfilled by the spot market with the price of c_m.

Supplier 1's profit in the second auction can be formulated in Q_1^A as well. First of all, we define three scenarios based on the relationships among capacities, demands, and submitted demands Q_1^A.

Scenario (a): $Q_1^A \ge D^A - O_2$ and $Q_1^A \le O_1 - \tilde{D}^B$;

Scenario (b): $Q_1^A \ge D^A - O_2$ and $Q_1^A > O_1 - \tilde{D}^B$; and

Scenario (c): $Q_1^A < D^A - O_2$.

The close forms for supplier 1's profit $\pi_1^B(Q_1^A) = Pay_i^B - c_i x_i^B$ in buyer B's procurement auction are derived for the three above situations respectively.

Under scenario (a), the two conditions guarantee that demand from buyer A can be covered by the sum of two suppliers' submitted quantities $Q_1^A + O_2$. Meanwhile, the supplier 1's remaining capacity $O_1 - Q_1^A$ is greater than buyer B's demand $\widetilde{D}^B$. Then the allocation to supplier 1 in the second auction is $x_1^B = \widetilde{D}^B$. Therefore, the profit of supplier 1 is:

$$\pi_1^B(Q_1^A) = \begin{cases} \widetilde{D}^B c_2 - \widetilde{D}^B c_1 & O_2 - D^A + Q_1^A \geq \widetilde{D}^B \\ (O_2 + Q_1^A - D^A)c_2 + (\widetilde{D}^B + D^A - O_2 - Q_1^A)c_m - \widetilde{D}^B c_1 & O_2 - D^A + Q_1^A < \widetilde{D}^B. \end{cases} \quad (23)$$

(23) shows the profit of supplier 1 in buyer B's procurement depends on whether supplier 2 has enough capacity to satisfy buyer B's demand alone without the participation of supplier 1.

Under scenario (b), the two suppliers submit enough capacities to meet buyer A's demand in the first auction. However, buyer B's demand $\widetilde{D}^B$ cannot be completely covered by supplier 1's remaining capacity and therefore be partially allocated to supplier 2 and, if necessary, the spot market. The allocation to supplier 1 in the second auction is $x_1^B = O_1 - Q_1^A$ in this scenario. The profit of supplier 1 in buyer B's procurement is determined by (24).

$$\pi_1^B(Q_1^A)$$
$$= \begin{cases} (O_1 - Q_1^A)c_2 - (O_1 - Q_1^A)c_1 & O_2 - D^A + Q_1^A \geq \widetilde{D}^B \\ (O_1 + O_2 - D^A - \widetilde{D}^B)c_2 + (D^A + \widetilde{D}^B - O_2 - Q_1^A)c_m - (O_1 - Q_1^A)c_1 & O_2 - D^A + Q_1^A < \widetilde{D}^B, O_1 + O_2 \geq D^A + \widetilde{D}^B \\ (O_1 - Q_1^A)c_m - (O_1 - Q_1^A)c_1 & O_1 + O_2 < D^A + \widetilde{D}^B. \end{cases} \quad (24)$$

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In (24), when supplier 2 can completely meet the demand of buyer B alone without the contribution from supplier 1, the profit of supplier 1 in buyer B's procurement is decided by the cost difference between the two suppliers because the spot market is not involved even in SAP-1 to calculate Z_{-1}^{B}. When the total capacity of the two suppliers is greater than the total demands in the two procurements and supplier 2's remaining capacity is not enough to completely cover the demand of buyer B without the contribution from supplier 1, the spot market is involved in SAP-1 to calculate Z_{-1}^{B} but not involved in SAP-1 to calculate Z^{B}. When the total capacity of the two suppliers is less than the total demands in the two procurements, the spot market is involved in the actual allocation of buyer B's demand so that the profit of supplier 1 in buyer B's procurement is decided by the cost difference between supplier 1 and the spot market.

Under scenario (c), the two suppliers' total submitted capacities are not sufficient for buyer A's demand. Therefore, supplier 2 utilizes all his capacity in buyer A's procurement and will not participate in buyer B's procurement. The profit for supplier 1 in buyer B's procurement is decided by (25).

$$\pi_1^{B}(Q_1^{A}) = \begin{cases} \widetilde{D}^{B}c_m - \widetilde{D}^{B}c_1 & Q_1^{A} \leq O_1 - \widetilde{D}^{B} \\ (O_1 - Q_1^{A})c_m - (O_1 - Q_1^{A})c_1 & Q_1^{A} > O_1 - \widetilde{D}^{B}. \end{cases} \tag{25}$$

The allocation to supplier 1 in buyer B's procurement is $x_1^{B} = \min\{\widetilde{D}^{B}, O_1 - Q_1^{A}\}$ and the profit rate is decided by the cost difference between supplier 1 and the spot market.

Since demand from buyer B is uncertain during the procurement auction for buyer A, demand $\widetilde{D}^{B}$ is defined as a random variable following a distribution with a density function $f(x)$. By summarizing the profits from the two auctions, the expected total profit

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$E[\pi_1(Q_1^A)]$ for supplier 1 when he decides the value of Q_1^A is calculated for the following three intervals.

(1) When $Q_1^A > \dfrac{1}{2}(O_1 + D^A - O_2)$,

$$E[\pi_1(Q_1^A)] = \pi_1^A(Q_1^A) + \int_0^{O_1-Q_1^A}(c_2 - c_1)x dF(x)$$

$$+ \int_{O_1-Q_1^A}^{Q_1^A+O_2-D^A}(c_2 - c_1)(O_1 - Q_1^A)dF(x)$$

$$+ \int_{Q_1^A+O_2-D^A}^{O_1+O_2-D^A}[(O_1 + O_2 - D^A - x)c_2 + (D^A + x - O_2 - Q_1^A)c_m - (O_1 - Q_1^A)c_1]dF(x)$$

$$+ \int_{O_1+O_2-D^A}^{\infty}(c_m - c_1)(O_1 - Q_1^A)dF(x)$$

$$= \pi_1^A(Q_1^A) - \int_0^{O_1-Q_1^A}(c_2 - c_1)F(x)dx - \int_{Q_1^A+O_2-D^A}^{O_1+O_2-D^A}(c_m - c_2)F(x)dx + (c_m - c_1)(O_1 - Q_1^A);$$

(26)

(2) When $D^A - O_2 \le Q_1^A \le \dfrac{1}{2}(O_1 + D^A - O_2)$

$$E[\pi_1(Q_1^A)] = \pi_1^A(Q_1^A) + \int_0^{Q_1^A+O_2-D^A}(c_2 - c_1)x dF(x)$$

$$+ \int_{Q_1^A+O_2-D^A}^{O_1-Q_1^A}[(O_2 - D^A + Q_1^A)c_2 + (D^A + x - O_2 - Q_1^A)c_m - xc_1]dF(x)$$

$$+ \int_{O_1-Q_1^A}^{O_1+O_2-D^A}[(O_1 + O_2 - D^A - x)c_2 + (D^A + x - O_2 - Q_1^A)c_m - (O_1 - Q_1^A)c_1]dF(x)$$

$$+ \int_{O_1+O_2-D^A}^{\infty}(c_m - c_1)(O_1 - Q_1^A)dF(x)$$

$$= \pi_1^A(Q_1^A) - \int_0^{Q_1^A+O_2-D^A}(c_2 - c_1)F(x)dx$$

$$- \int_{Q_1^A+O_2-D^A}^{O_1-Q_1^A}(c_m - c_1)F(x)dx - \int_{O_1-Q_1^A}^{O_1+O_2-D^A}(c_m - c_2)F(x)dx$$

$$+ (c_m - c_1)(O_1 - Q_1^A); \text{ and}$$

(27)

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(3) When $Q_1^A < D^A - O_2$

$$E[\pi_1(Q_1^A)] = (c_m - c_1)[Q_1^A + \int_0^{O_1 - Q_1^A} x dF(x) + (O_1 - Q_1^A)(1 - F(O_1 - Q_1^A))]$$

$$= (c_m - c_1)(O_1 - \int_0^{O_1 - Q_1^A} F(x) dx).$$

(28)

Please note that $E[\pi_1(Q_1^A)]$ is continuous at any $Q_1^A \in [0, \min(O_1, D^A)]$. It is

differentiable except at the point of $D^A - O_2$ if $D^A \geq O_2$ and $D^A - O_2 \geq O_1$. Therefore,

the optimal bidding quantity for supplier 1 in buyer A's procurement, Q_1^{*A}, will either

satisfy the first-order condition $\dfrac{dE[\pi_1(Q_1^A)]}{dQ_1^A} = 0$ or incurs at the boundary points

$0, \min(O_1, D^A)$, or $D^A - O_2$ (if $D^A \geq O_2$). The derivative of the expected profit function is

calculated as follows if $D^A \geq O_2$:

$$\frac{dE[\pi_1(Q_1^A)]}{dQ_1^A} = \begin{cases} (c_2 - c_1)F(O_1 - Q_1^A) - (c_m - c_2)(1 - F(Q_1^A + O_2 - D^A)) & Q_1^A > D^A - O_2 \quad (29\text{-}1) \\ (c_m - c_1)F(O_1 - Q_1^A) & Q_1^A < D^A - O_2. \quad (29\text{-}2) \end{cases}$$

Theorem 3. Supplier 1's dominant strategy can be determined by

$$Q_1^{*A} = \begin{cases} O_1 & D^A \geq O_1 + O_2 \\ \underset{Q_1^A = \min(O_1, D^A), \hat{Q}_1^A, \max(D^A - O_2, 0)}{\arg\max} E[\pi_1(Q_1^A)] & D^A \leq O_1 + O_2, \end{cases}$$

(30)

where $\hat{Q}_1^A$, if exists, is any $Q_1^A \in [\max(0, D^A - O_2), \min(O_1, D^A)]$ that can make (29-1) be

zero. There may be multiple or no $\hat{Q}_1^A$ existing under certain distributions and cost

structures.

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When $O_1 + O_2 \leq D^A$, we have $Q_1^A \leq D^A - O_2$ for any $Q_1^A \in [0, O_2]$. Based on

(29-2), the first-order condition $\dfrac{dE[\pi_1(Q_1^A)]}{dQ_1^A} = (c_m - c_1)F(O_1 - Q_1^A) \geq 0$ for any

$Q_1^A \in [0, O_2]$. Therefore, the optimal bid for supplier 1 in buyer A's auction is

$Q_1^{*A} = O_1$ when $O_1 + O_2 \leq D^A$. This result is straightforward because the condition of

$O_1 + O_2 \leq D^A$ indicates that the two suppliers do not have any remaining capacity after

satisfying buyer A's demand so that the unit profit of supplier 1 in buyer A's

procurement is decided by the cost difference between supplier 1 and the spot market (i.e.,

$c_m - c_1$), which is already the best marginal profit $(c_m\text{-}c_1)$. Because of the uncertainty of

the demand from buyer B, the optimal strategy for supplier 1 is to bid $Q_1^{*A} = O_1$ in the

first auction to fully utilize his capacity.

When $O_1 + O_2 > D^A$, a feasible bidding from supplier 1 in buyer A's auction

Q_1^A could be greater than $D^A - O_2$. Therefore, we need to consider both intervals in (29).

The first-order necessary condition

$(c_2 - c_1)F(O_1 - Q_1^A) - (c_m - c_2)(1 - F(Q_1^A + O_2 - D^A)) = 0$ yields the local minimum or

maximum solutions $\hat{Q}_1^A$, which is feasible only if it belongs to the interval of

$[\max(0, D^A - O_2), \min(O_1, D^A)]$. Therefore, under the condition of $O_2 \leq D^A \leq O_1 + O_2$,

supplier 1's global optimal bidding quantity Q_1^{*A} in the first auction will happen at

either feasible $\hat{Q}_1^A$ or one of the boundary points $\min(O_1, D^A)$ and $D^A - O_2$. If

$D^A \leq O_2$, interval for (29-2) disappears. Boundary point $Q_1^A = 0$ replaces the point of

$D^A - O_2$ as a possible candidate for the optimal bidding quantity. Based on all the

analysis, supplier 1's dominant strategy of bidding quantity Q_1^A can be well defined.

7.2 Numerical Illustrations with Uniform Demand

In this section, we numerically analyze the sequential auctions by considering the

market environment under different scenarios. The parameters, which are public

information in numerical examples, include suppliers' costs, capacities, and buyer A's

demand. It is assumed that buyer B's demand follows a uniform distribution

$U(\underline{d}, \overline{d})$, which is also assumed to be public information for players. In subsection 7.2.1,

we provide examples to show how supplier 1's dominant bidding strategy changes along

with the different settings of parameters. In subsection 7.2.2, we conduct sensitivity

analysis to demonstrate the impacts of market configurations on suppliers' optimal

expected profits with applying dominant strategies.

7.2.1 Supplier 1's Bidding Strategy

It is shown in section 7.1 that supplier 2's dominant strategy is to bid his true

available capacities in both auctions. In the following analysis, it is assumed that supplier

2 always follows his dominant strategy. Both suppliers have incentive to reveal their true

costs because VCG payments are applied in both auctions. Based on the analysis in

section 7.1, supplier 1's expected profit can be formulated as a function in his bidding

quantity Q_1^A. We provide numerical examples for supplier 1's expected profit function

$E[\pi_1(Q_1^A)]$ under the five scenarios listed in Table 6.

Table 6   Market Configurations of Scenarios for Expected Profit Function Simulation

Scenarios	c_1	c_2	c_m	O_1	O_2	D^A	$\underline{d}$	$\overline{d}$
1	8	10	15	70	50	80	30	80
2	8	10	15	70	50	80	0	20
3	8	19	20	70	50	80	30	80
4	8	10	15	70	50	40	30	80
5	8	10	15	70	50	40	20	30
6	8	10	15	70	50	40	20	40
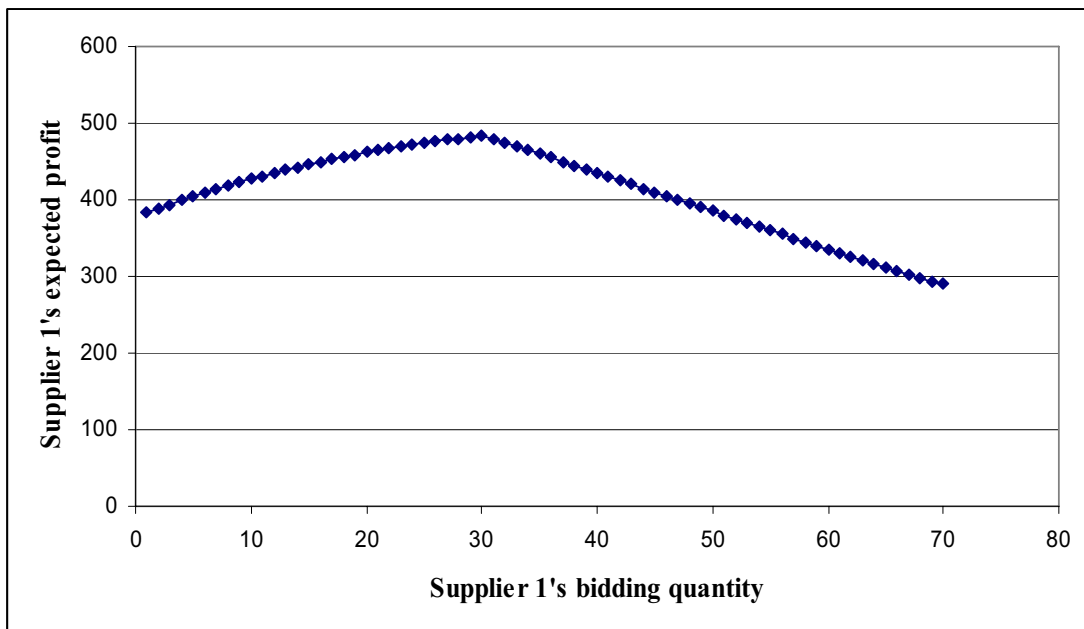
Figure 1   Supplier 1's Expected Profit Function (Scenario 1)

For scenario 1, feasible bidding quantity of supplier 1 falls in the region of [0, 70] because it can not exceed the available capacity O_1. Figure 1 shows that the maximal expected profit happens at $Q_1^A* = D^A - O_2 = 30$. This bidding quantity leaves exactly O_2 units of buyer A's demand in the first auction to supplier 2 and forces supplier 2 to utilize all his capacity in buyer A's auction. By doing it, supplier 1 receives the best unit

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payment c_m in both auctions that maximizes his expected profit. However, this bidding

quantity is no longer the best choice when buyer B's demand is too low to compensate

supplier 1's loss of opportunity in the first auction, or when the price c_m from spot market

is not much higher than supplier 2's cost c_2. Scenarios 2 and 3 exemplify these two
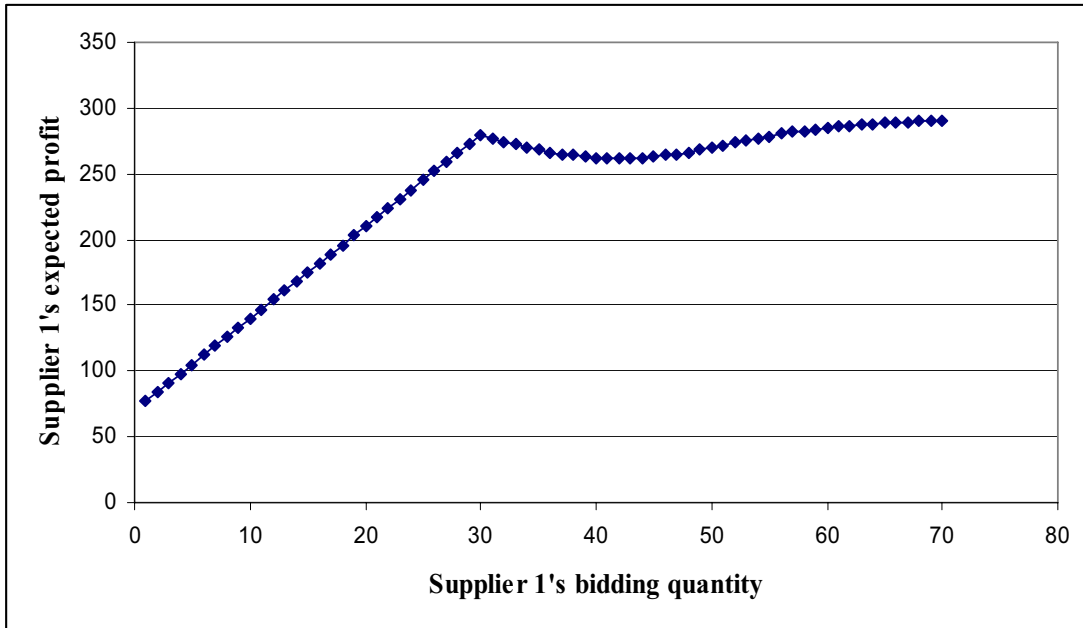
possibilities respectively.

Figure 2   Supplier 1's Expected Profit Function (Scenario 2)

In Figure 2, the optimal expected profit of supplier 1 happens at the point

$Q_1^A* = O_1 = 70$ because of buyer B's low demand represented by the distribution U(0,

20). In Figure 3, $Q_1^A* = 35$ gives supplier 1 the best expected profit as \$831.25. This

optimal bidding quantity is one of the solutions to the first-order necessary condition

$\nabla E[\pi_1(Q_1^A)] = 0$ in the interval of $[D^A - O_2, O_1]$ (i.e., [30, 70]). In scenarios 4, 5, and 6,

buyer A's demand is lower than supplier 2's capcity (i.e., $D^A - O_2 < 0$). Figure 4 shows

that the optimal bidding strategy for supplier 1 in this scenario is not to participate in the first auction and save all his capacity for the second auction to receive higher payment as long as buyer B's demand is high enough. If buyer B's demand decreases as in scenarios 5 and 6, supplier 1's expected profit has a U-shape function in his bidding quantity and the optimum occurs at $Q_1^A = O_1$ or $Q_1^A = 0$ as shown in Figure 5 and 6 respectively.
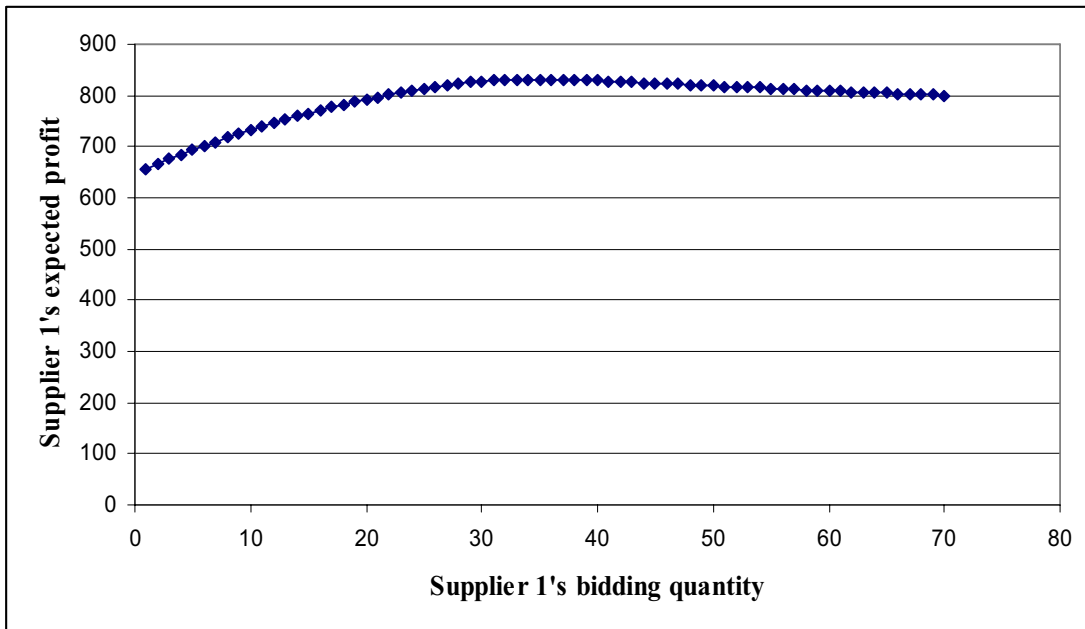

Figure 3   Supplier 1's Expected Profit Function (Scenario 3)
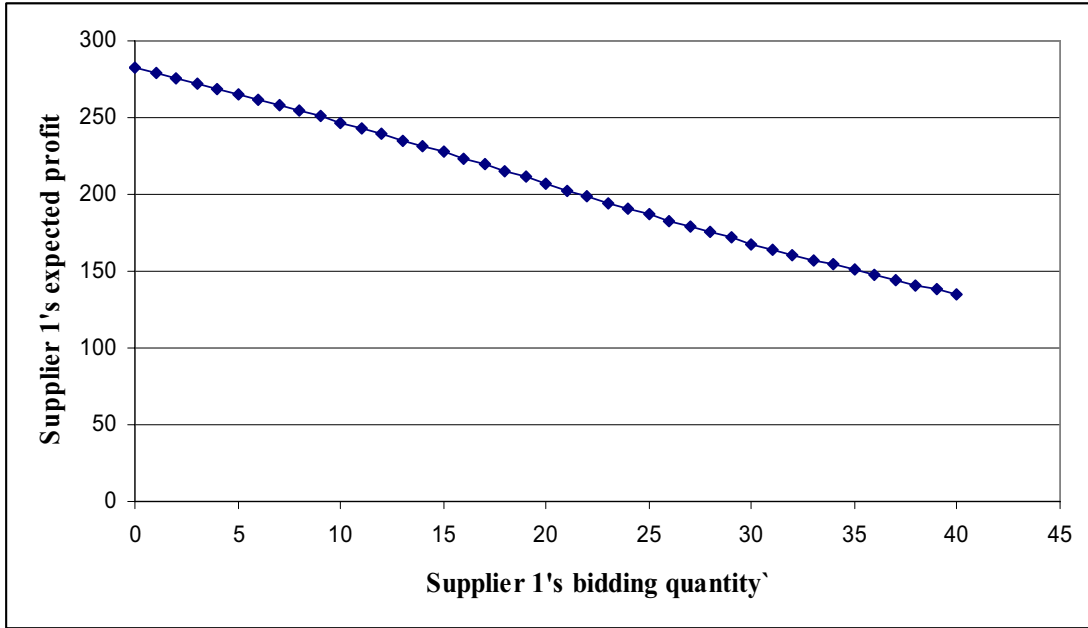
Figure 4   Supplier 1's Expected Profit Function (Scenario 4)

Figure 5   Supplier 1's Expected Profit Function (Scenario 5)
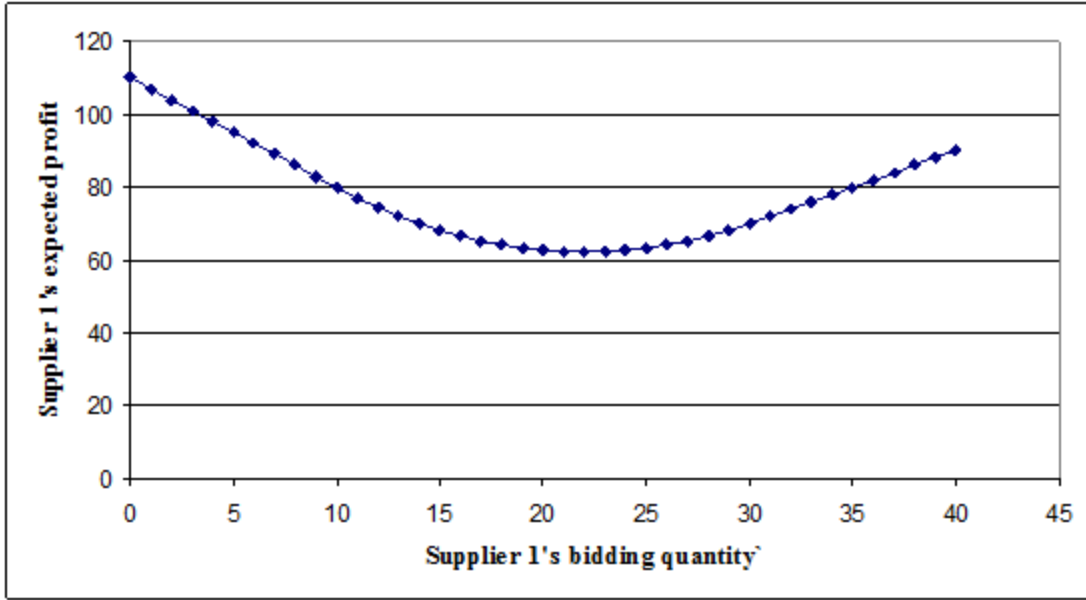
Figure 6   Supplier 1's Expected Profit Function (Scenario 6)

The six examples numerically demonstrate the Theorem 3 in section 7.1. It shows that unique dominant strategy does exist for supplier 1. However, it varies for different market configurations. Comparisons of supplier 1's expected profits among boundary values of Q_1^A at 0, $\min\{D^A, O_1\}$, $D^A - O_2$ and feasible solutions to the first-order necessary condition $\nabla E[\pi_1(Q_1^A)] = 0$ are necessary to determine his dominant strategy.

7.2.2   Sensitivity Analysis

In this section, we vary suppliers' costs, capacities, and buyers' demands to illustrate how these given parameters affect supplier 1's optimal expected profit when both suppliers apply their dominant strategies. Table 7 shows the parameters we use to conduct the sensitivity analysis.

Table 7   Parameters Settings for Sensitivity Analysis

c_1	c_2	c_m	O_1	O_2	D^A	$\underline{d}$	$\bar{d}$
4	[5, 14]	15	70	50	80	30	80
8	10	[11, 22]	70	50	80	30	80
8	10	15	[10, 120]	50	80	30	80
8	10	15	70	[10, 150]	80	30	80
8	10	15	70	50	[10, 120]	30	80
8	10	15	70	50	80	[10, 65]	80
8	10	15	70	50	80	30	[35, 90]

In Figure 7, supplier 1's optimal expected profit does not change when supplier 2's unit production cost changes from \$5 to \$13. After we investigate supplier 1's profit function in Q_1^A, supplier 1's dominant strategy under these cost values happens at $Q_1^A = D^A - O_2$. When $Q_1^A = D^A - O_2$ is the dominant strategy, we showed that supplier 1's unit payment is determined by the spot market price c_m and not related to supplier 2's unit production cost. Therefore, supplier 1's optimal expected profit keeps the same. The sudden increment from $c_2 = \$13$ to $c_2 = \$14$ happens because $Q_1^A = D^A - O_2$ is no longer the optimal bidding quantity when $c_2 = \$14$, which is similar to scenario 3 in section 8.1. When the difference between c_2 and c_m is small enough, feasible solution Q_1^A to the first-order condition $\nabla E[\pi_1(Q_1^A)] = 0$ may yield a better expected profit for supplier 1.
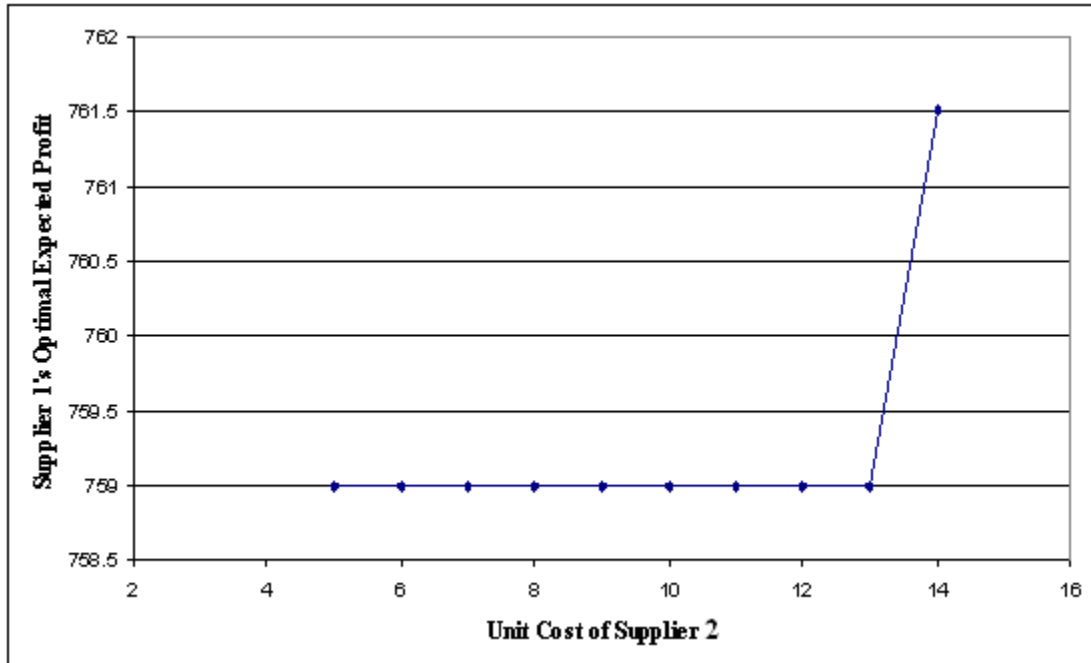
Figure 7   Supplier 1's Optimal Expected Profit changes in c_2

As shown in Figure 8, the impact of spot market unit price on the supplier 1's expected profit follows an obvious increasing linear function based on the expected profit function presented in section 7.1.
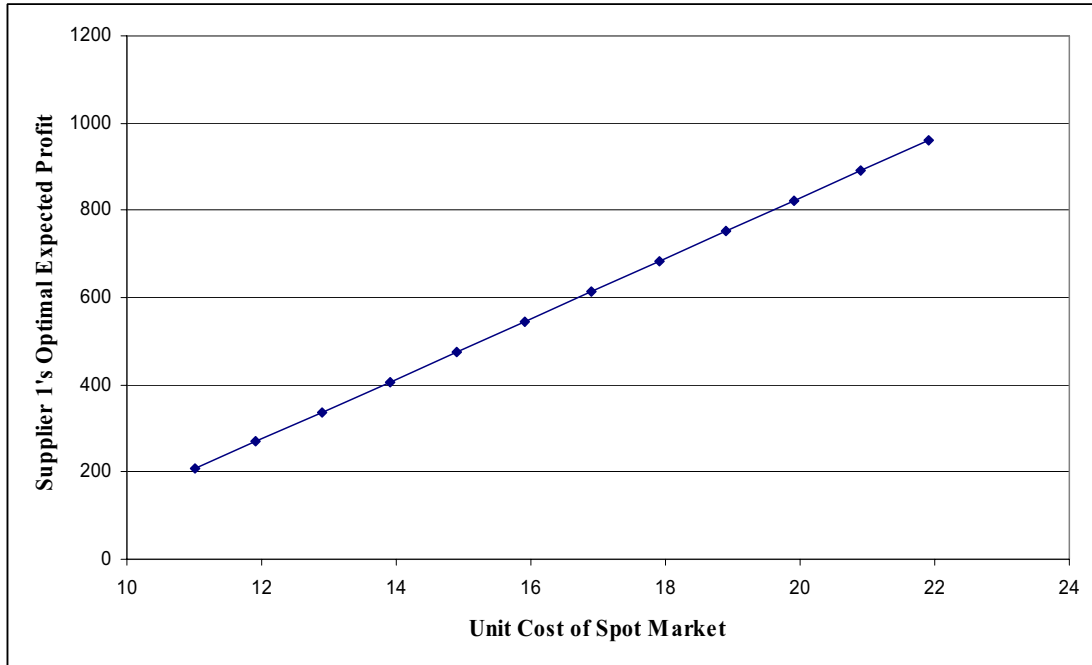
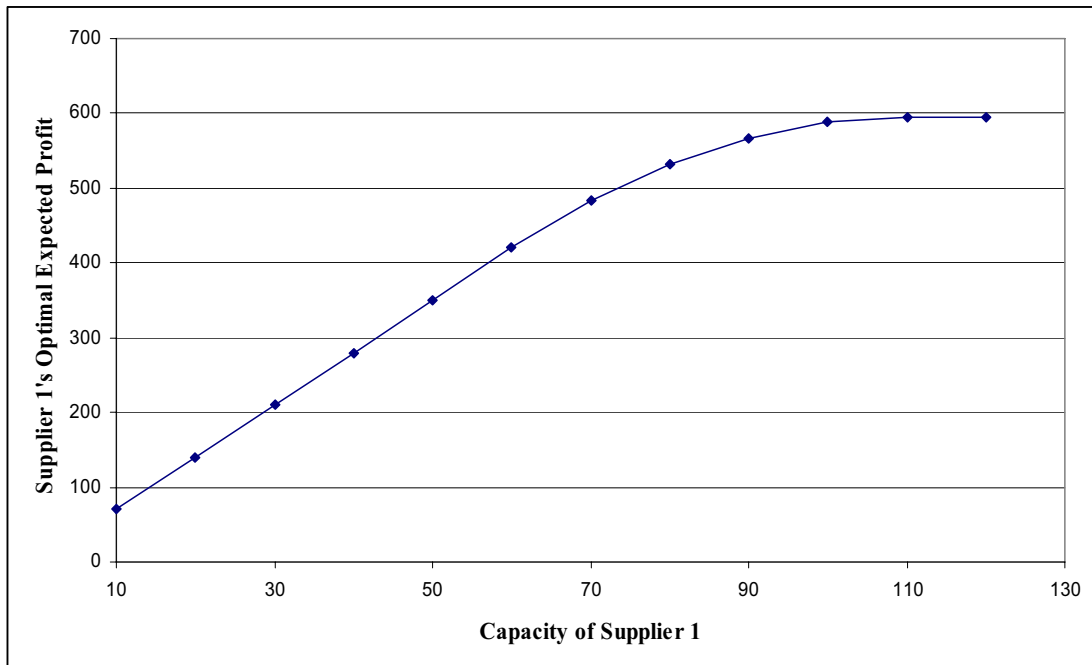Figure 8   Supplier 1's Optimal Expected Profit changes in c_m

Figure 9   Supplier 1's Optimal Expected Profit changes in O_1

Figure 9 and 10 show us the impact of suppliers' capacities on supplier 1's expected profit. In Figure 9, supplier 1's optimal expected profit increases when his capacity increases. But the increment rate decreases as capacity becomes higher because more units allocated to supplier 1 have a unit payment of c_2. Finally, O_1 will reach a high enough value after which supplier 1's optimal expected profit is not affected by further increasing O_1. The impact of changes in O_1 always follows this shape though the decrement in slope varies based on other parameters. If there is any opportunity for supplier 1 to extend his capacity, this sensitivity analysis provides enough information to help supplier 1 to determine whether it benefits the expected profit or not.
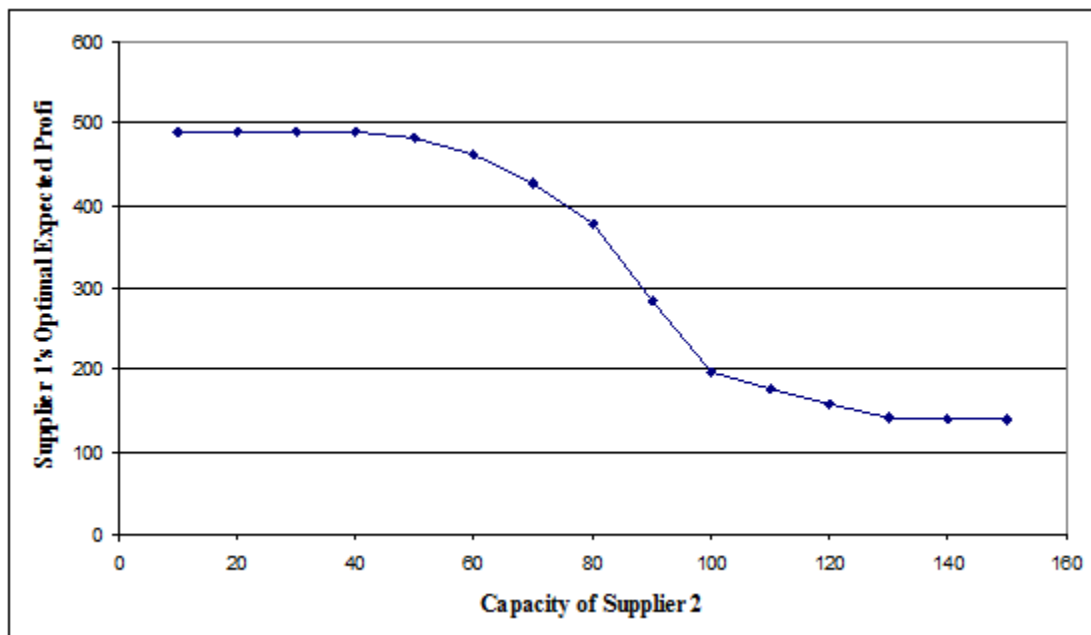

Figure 10   Supplier 1's Optimal Expected Profit changes in O_2

In Figure 10, supplier 1's optimal expected profit decreases when supplier 2's capacity increases. The more supplier 2's capacity is, there is less opportunity for supplier 1 to receive a payment determined by spot market price. When the capacity from supplier 2 is small enough to make supplier 1 always receive unit payment c_m, the changes in O_2 has no impact on the optimal expected profit of supplier 1 as shown in Figure 10. Similarly, when supplier 2's capacity is large enough to make supplier 1's unit payment always be c_2, supplier 1's optimal expected profit is not affected by further increment of O_2.

The impact of buyer A's demand on supplier 1's optimal expected profit is shown in Figure 11, which has a similar shape as Figure 9. Increment in buyer A's demand definitely brings more profit to supplier 1 as long as he has enough capacity. However, even if D^A is greater than O_1, the optimal expected profit keeps increasing with smaller slope because the demand will utilize more supplier 2's capacity, which may increase supplier 1's VCG payments.
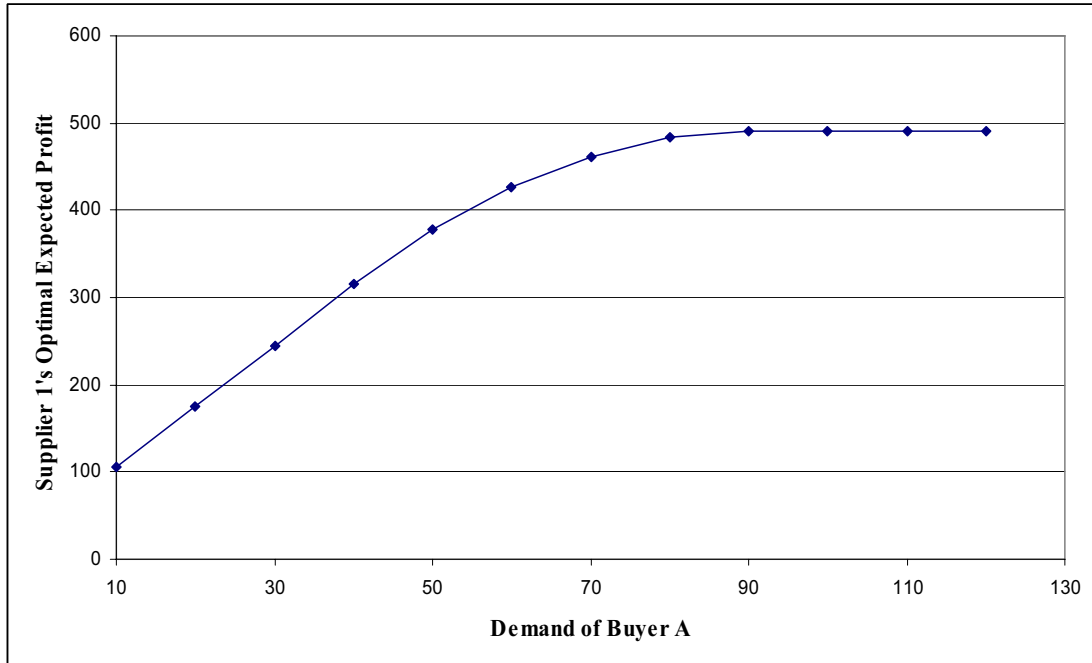
Figure 11   Supplier 1's Optimal Expected Profit changes in D^A

Since we assume the demand from buyer B follows a uniform distribution

$U(\underline{d}, \overline{d})$, we want to investigate how the two bounds of the distribution will affect

supplier 1's optimal expected profit. Figure 12 shows that it increases slightly as $\underline{d}$

increases while Figure 13 shows that the upper bound $\overline{d}$ has more significant impact.

However, the increment rate is also related to all other parameters, especially depends on

suppliers' capacities based on the expected profit functions derived for uniform
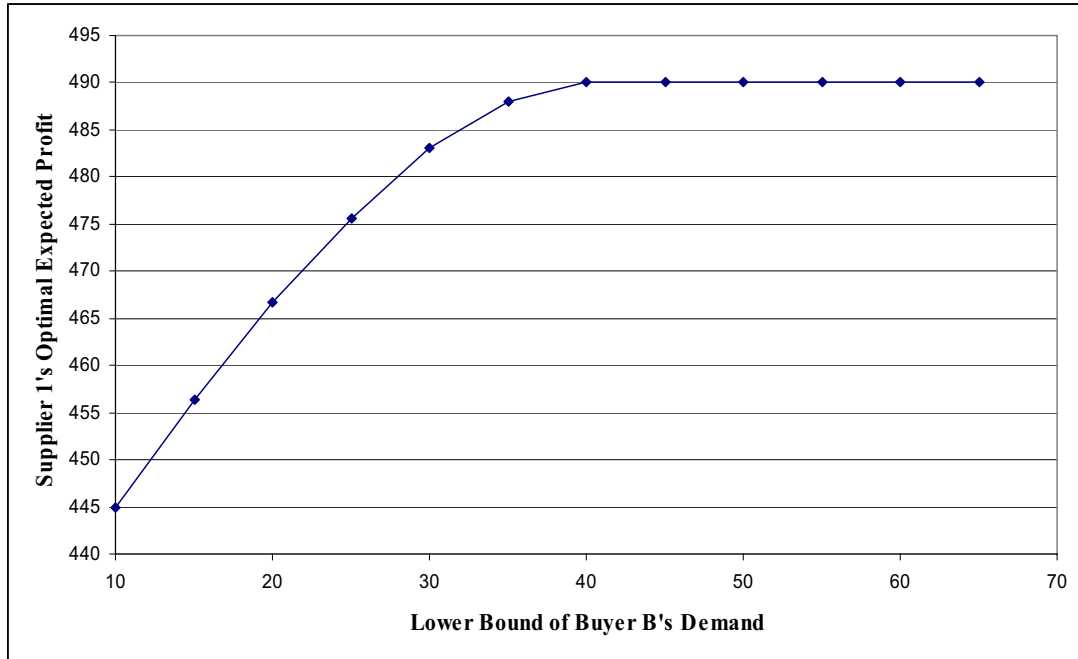
distribution demand (Appendix B).

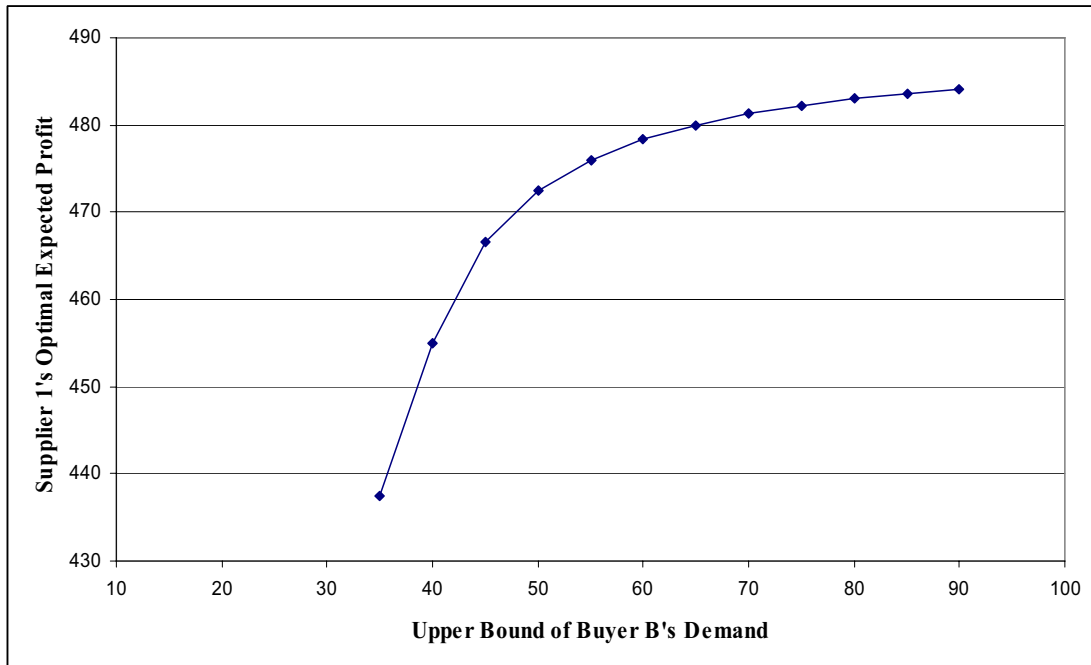Figure 12   Supplier 1's Optimal Expected Profit changes in $\underline{d}$

Figure 13   Supplier 1's Optimal Expected Profit changes in $\overline{d}$

7.2.3 Buyers' Procurement Costs

In this section, we investigate the sequential auctions from the buyers' viewpoint. When both suppliers implement their dominant strategies, we numerically simulate the difference between two buyers' procurement costs under different market configurations to make recommendations on the auction sequence if the two buyers' demands are symmetric. For the first two scenarios listed in Table 8, upper bound and lower bound of buyer B's demand distribution are varied to study the impact of this variation on buyers' procurement.

Table 8   Market Configurations of Scenarios for Buyers' Procurement Cost Simulation

Scenarios	c_1	c_2	c_m	O_1	O_2	D^A	$\underline{d}$	$\overline{d}$
1	8	10	15	70	50	80	0	[2,46]
2	8	10	15	70	50	80	[0, 20]	20
3	8	10	15	70	50	[65,120]	0	20
4	8	10	15	[10,120]	50	80	0	20
5	8	10	15	70	[10,75]	80	0	20

As shown in Figure 14, buyer A's unit procurement cost is lower than buyer B's when the upper bound $\overline{d}$ is smaller than 25 units. We observe that the optimal bidding quantity of supplier 1 happens at $Q_1^A* = \min\{O_1, D^A\} = 70$, which yields this morning effect in procurement price because buyer A has more opportunity to allocate his demand to supplier 1 with lower cost. However, when upper bound $\overline{d}$ is greater than 25 units, supplier 1's dominant strategy changes to $Q_1^A* = D^A - O_2$ which makes the payment to supplier 1 from buyer A increases to the spot market price $c_m = \$15$. So there is no

morning or afternoon effect in price when the supplier 1's optimal bidding quantity is

$Q_1^{A}* = D^{A} - O_2$. Figure 15 illustrates the similar impact of $\underline{d}$ as shown in Figure 14.

Figure 16 simulate the third scenario in which buyer A's demand is varied. Similarly as in Figure 14, when buyer A's demand is less than 90 units, supplier 1's dominant strategy is $Q_1^{A} *= \min\{O_1, D^{A}\}$ which results in the morning effect shown in Figure 14 and Figure 15. When D^{A} is large enough to make $Q_1^{A}* = D^{A} - O_2$, both buyers have the same unit procurement cost as c_m=\$15.
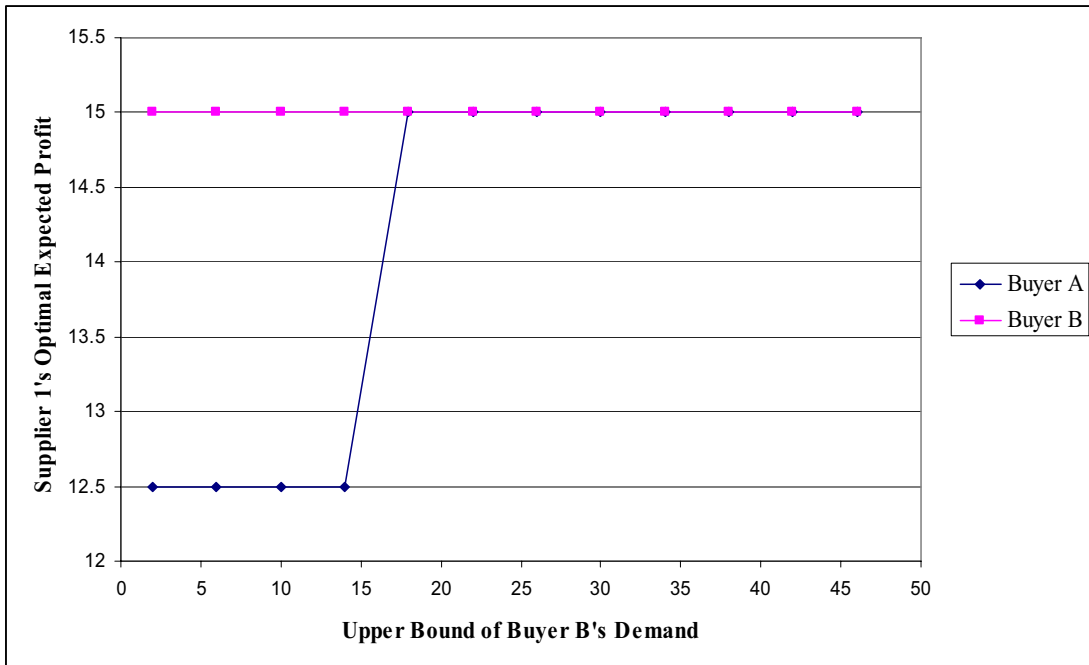

Figure 14   Buyers' Procurement Costs change in $\overline{d}$

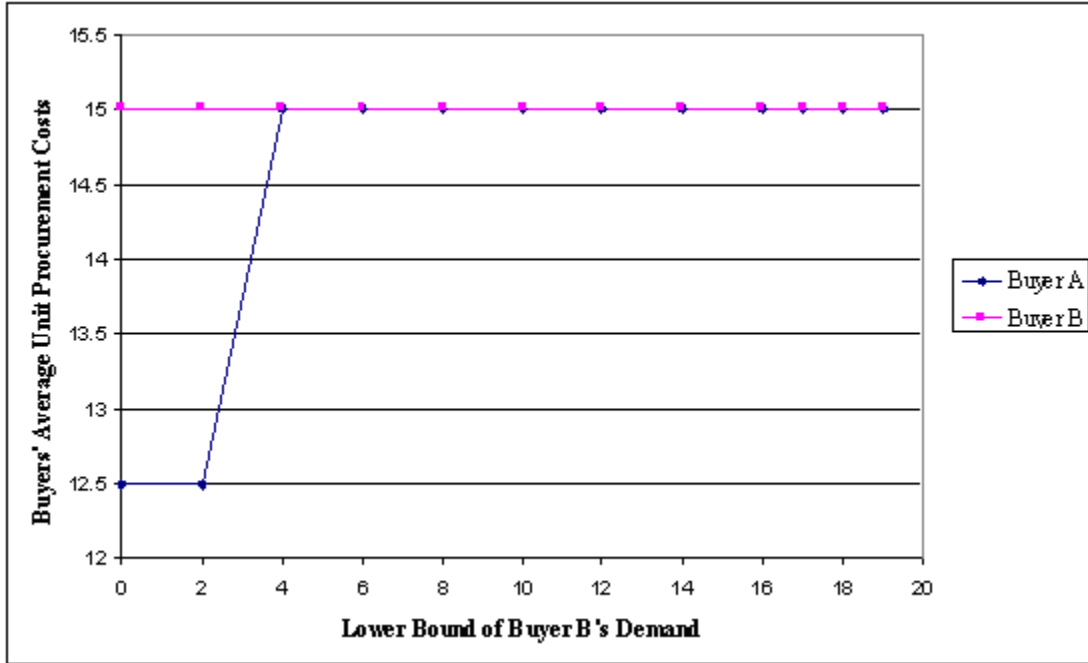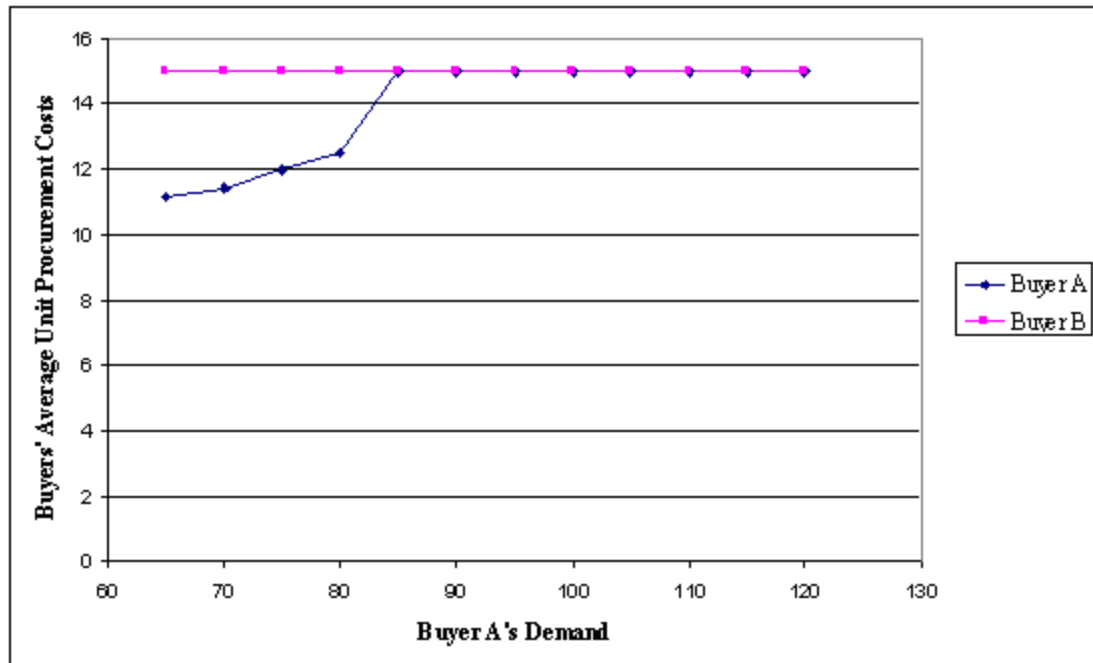Figure 15   Buyers' Procurement Costs change in $\underline{d}$

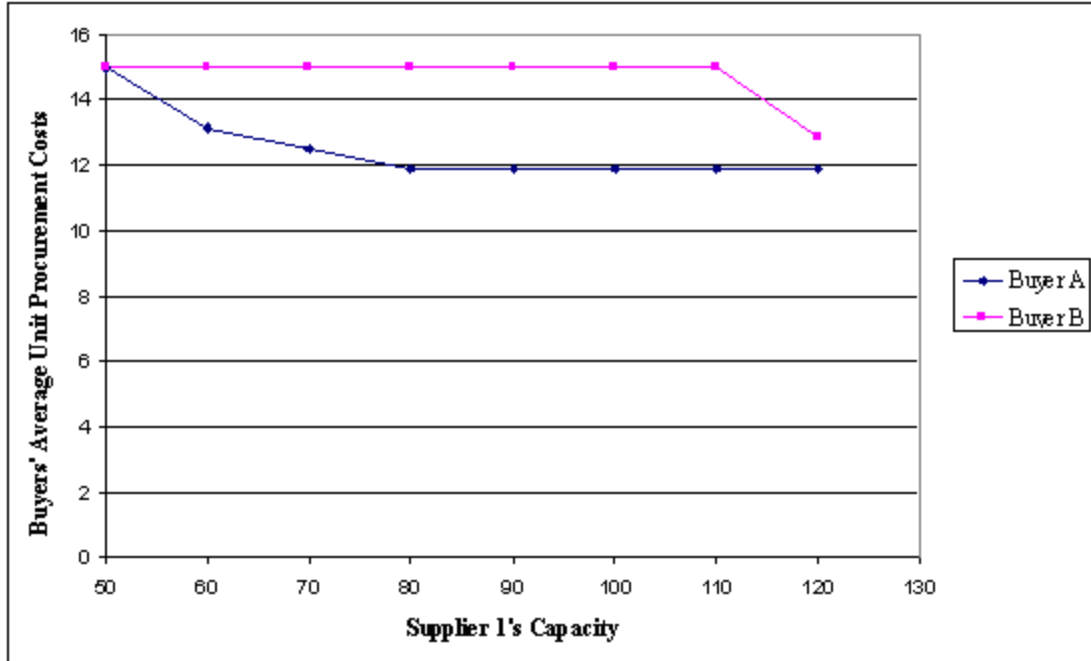Figure 16   Buyers' Procurement Costs change in D^A

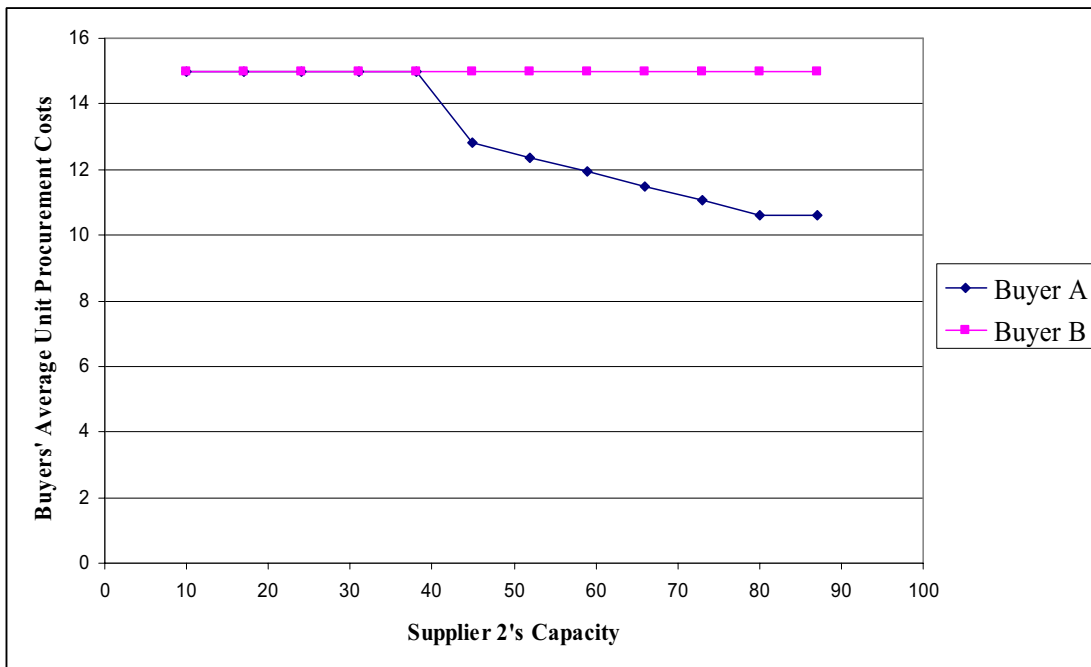Figure 17   Buyers' Procurement Costs change in O_1

Figure 18   Buyers' Procurement Costs change in O_2

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Both Figure 17 and Figure 18 under scenarios 4 and 5 respectively show that morning effect of buyers' procurement costs dose exist under some market configurations. Based on the numerical experiments conducted, we can empirically conclude that, in average, buyer B's unit procurement cost is higher than Buyer A's.

7.3   Conclusions

In the previous several chapters, the capacity constrained reverse sequential auctions are proposed. The procurement auctions involving two buyers and two suppliers are defined in chapter 6. It is assumed that suppliers' costs and capacities are well known by each other under the duopoly market environment. The only uncertainty that affect suppliers' bidding strategy is that demand from buyer B is unknown at the beginning of the first auction. However, it is assumed that both suppliers know buyer B's demand distribution $F(x)$. We investigate the sequential auctions from the view point of suppliers and analyze suppliers' bidding behaviors. Since both sequential auctions are assumed to implement VCG mechanism, it is shown that suppliers have the incentive to reveal their true cost as bidding price. Therefore, our study focuses on the other bidding variable, the available capacities submitted by the suppliers. It is shown in Theorem 2 that supplier 2 with higher production cost has a dominant strategy to bid his true capacities in both auctions.

To analyze supplier 1's dominant strategy which is more sophisticated than supplier 2's dominant strategy, supplier 1's expected profit function is derived in section 7.1. By applying the first-order necessary condition, supplier 1's dominant strategy is proposed in Theorem 3. It is shows that, for some situations, it is better for supplier 1 to

save some capacities in the first auction to obtain more expected profits in the second auction. In section 7.2.1, we numerically illustrate how supplier 1's dominant strategy is affected by the market parameters, like suppliers' costs and capacities, and buyers' demands, when buyer B's demand follows a uniform distribution. Furthermore, we conduct sensitivity analysis to study the impacts of these market parameters on supplier 1's optimal expected profit shown in section 7.2.2. Finally, with numerical experiments, we empirically observe a morning effect in buyers' average unit procurement cost under some market configurations. It shows that buyer A's average unit procurement cost is either lower or equal to buyer B's average unit procurement cost. However, we can not conclude that it is always better for a buyer to conduct the auction first because the two buyers are not symmetric in our problem definition. But it could be an interesting future topic to investigate the impact of the sequence of the two auctions.

Besides the sequence of the two auctions, it will be also interesting to relax the assumption of the duopoly market environment and study more than two suppliers that participate into the auctions. Along with this extension, suppliers may no longer know other suppliers' cost and capacity information which brings more uncertainty into the information structures and makes the analysis much more complicate.

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APPENDIX A

PROOF TO THEOREM 1

Because of the *non-regret* rule, $F_{ib^{t^*}} - \sum_{r=1}^{k_i^{t^*}} c_{ib^{t^*}r} \geq U_{\max i}^t$ if the active supplier i is a winner

at round t. Since the auctioneer always decreases or keeps the same ask prices,

$U_{\max i}^{t+1} \leq U_{\max i}^t$ is always true. Therefore, for a winner of round t,

$F_{ib^{t^*}} - \sum_{r=1}^{k_i^{t^*}} c_{ib^{t^*}r} \geq U_{\max i}^t \geq U_{\max i}^{t+1}$ indicates that the current gained profit is higher than or

equal to the potential maximal utility after decreasing the ask prices of round $t+1$ by $\square$.

The modified MBR bidding strategy for a winner of previous round is to automatically

retain the current winning bids, which is the second scenario of expression (11). The first

scenario studies the condition that the potential gain may be increased by reducing the

buyer's ask prices by $\tilde{\square}$ This scenario happens only when an active supplier is a loser at

the last round t. The modified MBR is to reduce the acceptable unit payments by $\square$ for all

pairs that yield the maximal potential utility $U_{\max i}^{t+1}$. If there is another better bidding

strategy that yields a final gain of $F_{i\hat{b}} - \sum_{r=1}^{\hat{k}_i} c_{i\hat{b}r}$ for the pair $(\hat{b}, \hat{k}_i)$, $F_{i\hat{b}} - \sum_{r=1}^{\hat{k}_i} c_{i\hat{b}r} \leq U_{\max i}^{t+1}$

is true since all other suppliers no longer change their bids. If these two terms are equal,

$(\hat{b}, \hat{k}_i) \in \arg\max_{b=1..B, k=1..D} \{(p_b^{t+1} - \varepsilon) \cdot k - \sum_{r=1}^k c_{ibr}\}$ is satisfied and the bid $s_{ib\hat{k}}^{t+1} = p_{\hat{b}}^{t+1} - \varepsilon$ is submitted

to achieve $F_{i\hat{b}} - \sum_{r=1}^{\hat{k}_i} c_{i\hat{b}r}$. The modified MBR bidding strategy reaches the same gain as the

"better" bidding strategy. For $F_{i\hat{b}} - \sum_{r=1}^{\hat{k}_i} c_{i\hat{b}r} < U_{\max i}^{t+1}$, when $\square$ approaches to zero, the

upper bound $U_{\max i}^{t+1}$ is reduced by a very small number δ_{t+1} from $U_{\max i}^t$. Therefore,

this upper bound reaches $F_{i\hat{b}} - \sum_{r=1}^{\hat{k}_i} c_{i\hat{b}r}$ at a later round $t+\square$, when pair $(\hat{b}, \hat{k}_i)$ satisfies

the equation $(\hat{b}, \hat{k}_i) \in \underset{b=1..B, k=1..D}{\arg\max} \{(p_b^{t+\alpha} - \varepsilon) \cdot k - \sum_{r=1}^{k} c_{ibr}\}$ and yields the gain

of $F_{i\hat{b}} - \sum_{r=1}^{\hat{k}_i} c_{i\hat{b}r}$. Therefore, the modified MBR presented in Theorem 1 can achieve an

outcome at least as good as any possible 'better' solution pair $(\hat{b}, \hat{k}_i)$ for a provisional

loser when $\square$ approaches zero.

APPENDIX B

COMPUTATION PROCEDURES FOR UNIFORM DEMAND IN THE SEQUENTIAL

AUCTIONS

Calculations of Supplier 1's Expected profit

(1) $O_1 + O_2 \leq D^A$

Under this condition (1), sum of two suppliers' capacities can not meet buyer A's demand.

So payments to supplier 1 from the two buyers are the same as spot market price c_m.

Because of the uncertainty of buyer B's demand, supplier 1's optimal strategy is to bid

full capacity in the first auction to win as many as possible.

$$Q_1^A * = O_1 \qquad Q_2^A * = O_2$$

$$\pi_1 * = O_1(c_m - c_1) \qquad \pi_2 * = O_2(c_m - c_2)$$

(2) $O_2 < D^A < O_1 + O_2$

Under the condition (2), as discussed in section 7.1, supplier 1's expected profit is

derived under three scenarios (2-1), (2-2), (2-3) separately.

(2-1) $Q_1^A \geq \frac{1}{2}(O_1 + D^A - O_2)$

$$\pi_1^A = c_m(O_1 + D^A - O_2 - Q_1^A) + c_2(O_2 - D^A + Q_1^A) - c_1 O_1 ;$$

$$E[\pi_1(Q_1^A)] = \pi_1^A - \int_0^{O_1 - Q_1^A} (c_2 - c_1) F(x) dx - \int_{Q_1^A + O_2 - D^A}^{O_1 + O_2 - D^A} (c_m - c_2) F(x) dx$$

When the uniform distribution for buyer B's demand has different parameters $\underline{d}$ and $\overline{d}$,

calculations of supplier 1's expected profit function are conducted as follows.

(2-1-1) $\overline{d} \leq O_1 - Q_1^A$

$$E[\pi_1(Q_1^A)] = \pi_1^A - \frac{c_2 - c_1}{2(\bar{d} - \underline{d})}(\bar{d} - \underline{d})^2 - (c_2 - c_1)(O_1 - Q_1^A - \bar{d}) - (c_m - c_2)(O_1 - Q_1^A)$$

(2-1-2) $\underline{d} \le O_1 - Q_1^A \le \bar{d} \le Q_1^A + O_2 - D^A$

$$E[\pi_1(Q_1^A)] = \pi_1^A - \frac{c_2 - c_1}{2(\bar{d} - \underline{d})}(O_1 - Q_1^A - \underline{d})^2 - (c_m - c_2)(O_1 - Q_1^A)$$

(2-1-3) $\underline{d} \le O_1 - Q_1^A \le Q_1^A + O_2 - D^A \le \bar{d} \le O_1 + O_2 - D^A$

$$E[\pi_1(Q_1^A)] = \pi_1^A - \frac{c_2 - c_1}{2(\bar{d} - \underline{d})}(O_1 - Q_1^A - \underline{d})^2 - \frac{c_m - c_2}{2(\bar{d} - \underline{d})}[(\bar{d} - \underline{d})^2 - (Q_1^A + O_2 - D^A - \underline{d})^2]$$
$$- (c_m - c_2)(O_1 + O_2 - D^A - \bar{d})$$

(2-1-4) $\underline{d} \le O_1 - Q_1^A \le Q_1^A + O_2 - D^A \le O_1 + O_2 - D^A \le \bar{d}$

$$E[\pi_1(Q_1^A)] = \pi_1^A - \frac{c_2 - c_1}{2(\bar{d} - \underline{d})}(O_1 - Q_1^A - \underline{d})^2 - \frac{c_m - c_2}{2(\bar{d} - \underline{d})}[(O_1 + O_2 - D^A - \underline{d})^2 - (Q_1^A + O_2 - D^A - \underline{d})^2]$$

(2-1-5) $O_1 - Q_1^A \le \underline{d} < \bar{d} \le Q_1^A + O_2 - D^A$

$$E[\pi_1(Q_1^A)] = \pi_1^A - (c_m - c_2)(O_1 - Q_1^A)$$

(2-1-6) $O_1 - Q_1^A \le \underline{d} \le Q_1^A + O_2 - D^A \le \bar{d} \le O_1 + O_2 - D^A$

$$E[\pi_1(Q_1^A)] = \pi_1^A - \frac{c_m - c_2}{2(\bar{d} - \underline{d})}[(\bar{d} - \underline{d})^2 - (Q_1^A + O_2 - D^A - \underline{d})^2] - (c_m - c_2)(O_1 + O_2 - D^A - \bar{d})$$

(2-1-7) $O_1 - Q_1^A \le \underline{d} \le Q_1^A + O_2 - D^A \le O_1 + O_2 - D^A \le \bar{d}$

$$E[\pi_1(Q_1^A)] = \pi_1^A - \frac{c_m - c_2}{2(\bar{d} - \underline{d})}[(O_1 + O_2 - D^A - \underline{d})^2 - (Q_1^A + O_2 - D^A - \underline{d})^2]$$

(2-1-8) $Q_1^A + O_2 - D^A \le \underline{d} \le \bar{d} \le O_1 + O_2 - D^A$

$$E[\pi_1(Q_1^A)] = \pi_1^A - \frac{c_m - c_2}{2(\bar{d} - \underline{d})}(\bar{d} - \underline{d})^2 - (c_m - c_2)(O_1 + O_2 - D^A - \bar{d})$$

(2-1-9) $Q_1^A + O_2 - D^A \le \underline{d} \le O_1 + O_2 - D^A \le \bar{d}$

$$E[\pi_1(Q_1^A)] = \pi_1^A - \frac{c_m - c_2}{2(\bar{d} - \underline{d})}(O_1 + O_2 - D^A - \underline{d})^2$$

(2-1-10) $O_1 + O_2 - D^A \le \underline{d}$

$$E[\pi_1(Q_1^A)] = \pi_1^A$$

(2-2) $D^A - O_2 \le Q_1^A \le \frac{1}{2}(O_1 + D^A - O_2)$

$$\pi_1^A = c_m(O_1 + D^A - O_2 - Q_1^A) + c_2(O_2 - D^A + Q_1^A) - c_1 O_1 ;$$

$$E[\pi_1(Q_1^A)] = \pi_1^A - \int_0^{Q_1^A + O_2 - D^A}(c_2 - c_1)F(x)dx - \int_{Q_1^A + O_2 - D^A}^{O_1 - Q_1^A}(c_m - c_1)F(x)dx - \int_{O_1 - Q_1^A}^{O_1 + O_2 - D^A}(c_m - c_2)F(x)dx$$

Similarly as (2-1), when the uniform distribution for buyer B's demand has different

parameters $\underline{d}$ and $\bar{d}$, calculations of supplier 1's expected profit function are

conducted as follows.

(2-2-1) $\bar{d} \le Q_1^A + O_2 - D^A$

$$E[\pi_1(Q_1^A)] = \pi_1^A - \frac{c_2 - c_1}{2(\bar{d} - \underline{d})}(\bar{d} - \underline{d})^2 - (c_2 - c_1)(Q_1^A + O_2 - D^A - \bar{d}) - (c_m - c_1)(O_1 + D^A - 2Q_1^A - O_2)$$
$$- (c_m - c_2)(Q_1^A + O_2 - D^A)$$

(2-2-2) $\underline{d} \le Q_1^A + O_2 - D^A \le \bar{d} \le O_1 - Q_1^A$

$$E[\pi_1(Q_1^A)] = \pi_1^A - \frac{c_2 - c_1}{2(\bar{d} - \underline{d})}(Q_1^A + O_2 - D^A - \underline{d})^2 - \frac{c_m - c_1}{2(\bar{d} - \underline{d})}[(\bar{d} - \underline{d})^2 - (Q_1^A + O_2 - D^A - \underline{d})^2]$$
$$- (c_m - c_1)(O_1 - Q_1^A - \bar{d}) - (c_m - c_2)(Q_1^A + O_2 - D^A)$$

(2-2-3) $\underline{d} \le Q_1^A + O_2 - D^A \le O_1 - Q_1^A \le \bar{d} \le O_1 + O_2 - D^A$

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$$E[\pi_1(Q_1^A)] = \pi_1^A - \frac{c_2 - c_1}{2(\overline{d} - \underline{d})}(Q_1^A + O_2 - D^A - \underline{d})^2 - \frac{c_m - c_1}{2(\overline{d} - \underline{d})}[(O_1 - Q_1^A - \underline{d})^2 - (Q_1^A + O_2 - D^A - \underline{d})^2]$$

$$- \frac{c_m - c_2}{2(\overline{d} - \underline{d})}[(\overline{d} - \underline{d})^2 - (O_1 - Q_1^A - \underline{d})^2] - (c_m - c_2)(O_1 + O_2 - D^A - \overline{d})$$

(2-2-4) $\underline{d} \leq Q_1^A + O_2 - D^A \leq O_1 - Q_1^A \leq O_1 + O_2 - D^A \leq \overline{d}$

$$E[\pi_1(Q_1^A)] = \pi_1^A - \frac{c_2 - c_1}{2(\overline{d} - \underline{d})}(Q_1^A + O_2 - D^A - \underline{d})^2 - \frac{c_m - c_1}{2(\overline{d} - \underline{d})}[(O_1 - Q_1^A - \underline{d})^2 - (Q_1^A + O_2 - D^A - \underline{d})^2]$$

$$- \frac{c_m - c_2}{2(\overline{d} - \underline{d})}[(O_1 + O_2 - D^A - \underline{d})^2 - (O_1 - Q_1^A - \underline{d})^2]$$

(2-2-5) $Q_1^A + O_2 - D^A \leq \underline{d} < \overline{d} \leq O_1 - Q_1^A$

$$E[\pi_1(Q_1^A)] = \pi_1^A - \frac{c_m - c_1}{2(\overline{d} - \underline{d})}(\overline{d} - \underline{d})^2 - (c_m - c_1)(O_1 - Q_1^A - \overline{d}) - (c_m - c_2)(Q_1^A + O_2 - D^A)$$

(2-2-6) $Q_1^A + O_2 - D^A \leq \underline{d} \leq O_1 - Q_1^A \leq \overline{d} \leq O_1 + O_2 - D^A$

$$E[\pi_1(Q_1^A)] = \pi_1^A - \frac{c_m - c_1}{2(\overline{d} - \underline{d})}(O_1 - Q_1^A - \underline{d})^2 - \frac{c_m - c_2}{2(\overline{d} - \underline{d})}[(\overline{d} - \underline{d})^2 - (O_1 - Q_1^A - \underline{d})^2]$$

$$- (c_m - c_2)(O_1 + O_2 - D^A - \overline{d})$$

(2-2-7) $Q_1^A + O_2 - D^A \leq \underline{d} \leq O_1 - Q_1^A \leq O_1 + O_2 - D^A \leq \overline{d}$

$$E[\pi_1(Q_1^A)] = \pi_1^A - \frac{c_m - c_1}{2(\overline{d} - \underline{d})}(O_1 - Q_1^A - \underline{d})^2 - \frac{c_m - c_2}{2(\overline{d} - \underline{d})}[(O_1 + O_2 - D^A - \underline{d})^2 - (O_1 - Q_1^A - \underline{d})^2]$$

(2-2-8) $O_1 - Q_1^A \leq \underline{d} \leq \overline{d} \leq O_1 + O_2 - D^A$

$$E[\pi_1(Q_1^A)] = \pi_1^A - \frac{c_m - c_2}{2(\overline{d} - \underline{d})}(\overline{d} - \underline{d})^2 - (c_m - c_2)(O_1 + O_2 - D^A - \overline{d})$$

(2-2-9) $O_1 - Q_1^A \leq \underline{d} \leq O_1 + O_2 - D^A \leq \overline{d}$

$$E[\pi_1(Q_1^A)] = \pi_1^A - \frac{c_m - c_2}{2(\overline{d} - \underline{d})}(O_1 + O_2 - D^A - \underline{d})^2$$

(2-2-10) $O_1 + O_2 - D^A \leq \underline{d}$

$$E[\pi_1(Q_1^A)] = \pi_1^A$$

(2-3) $Q_1^A \le D^A - O_2$

$$E[\pi_1(Q_1^A)] = (c_m - c_1)(O_1 - \int_0^{O_1 - Q_1^A} F(x)dx).$$

Under this scenario, supplier 1's expected profit functions are calculated as follows.

(2-3-1) $\overline{d} \le O_1 - Q_1^A$

$$E[\pi_1(Q_1^A)] = (c_m - c_1)O_1 - \frac{c_m - c_1}{2(\overline{d} - \underline{d})}(\overline{d} - \underline{d})^2 - (c_m - c_1)(O_1 - Q_1^A - \overline{d})$$

(2-3-2) $\underline{d} \le O_1 - Q_1^A \le \overline{d}$

$$E[\pi_1(Q_1^A)] = (c_m - c_1)O_1 - \frac{c_m - c_1}{2(\overline{d} - \underline{d})}(O_1 - Q_1^A - \underline{d})^2$$

(2-3-3) $O_1 - Q_1^A \le \underline{d}$

$$E[\pi_1(Q_1^A)] = (c_m - c_1)O_1$$

(3) $O_2 \ge D^A \Leftrightarrow D^A - O_2 \le 0$

Under condition (3), there are two difference scenarios similar as (2-1) and (2-2) while

(2-3) violates the condition $D^A - O_2 \le 0$.

(3-1) $Q_1^A \ge \frac{1}{2}(O_1 + D^A - O_2)$

Supplier 1's profit received from the first auction is changed to:

$$\pi_1^A = (c_2 - c_1)Q_1^A + (c_m - c_1)(O_1 - Q_1^A);$$

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$$E[\pi_1(Q_1^A)] = \pi_1^A - \int_0^{O_1 - Q_1^A}(c_2 - c_1)F(x)dx - \int_{Q_1^A + O_2 - D^A}^{O_1 + O_2 - D^A}(c_m - c_2)F(x)dx$$

Supplier 1's expected profit functions are the same as those derived in (2-1) except the term π_1^A.

(3-2) $\quad D^A - O_2 \leq Q_1^A \leq \dfrac{1}{2}(O_1 + D^A - O_2)$

Supplier 1's profit received from the first auction is changed to:

$$\pi_1^A = (c_2 - c_1)Q_1^A + (c_m - c_1)(O_1 - Q_1^A);$$

$$E[\pi_1(Q_1^A)] = \pi_1^A - \int_0^{Q_1^A + O_2 - D^A}(c_2 - c_1)F(x)dx - \int_{Q_1^A + O_2 - D^A}^{O_1 - Q_1^A}(c_m - c_1)F(x)dx - \int_{O_1 - Q_1^A}^{O_1 + O_2 - D^A}(c_m - c_2)F(x)dx$$

Supplier 1's expected profit functions are the same as those derived in (2-2) except the term π_1^A.

Solutions to the first-order necessary condition $\nabla E[\pi_1(Q_1^A)] = 0$

As discussed in section 7.1, feasible solutions $\hat{Q}_1^A$ to the first-order necessary condition

$(c_2 - c_1)F(O_1 - Q_1^A) - (c_m - c_2)(1 - F(Q_1^A + O_2 - D^A)) = 0$ are candidate for global

optimum. With uniformly distributed buyer B's demand, $\hat{Q}_1^A$ is solved for different

scenarios defined with conditions (2-1), (2-2), (3-1), and (3-2) above.

Under the conditions (2-1) and (3-1) $Q_1^A \geq \dfrac{1}{2}(O_1 + D^A - O_2)$:

(1-a) $\bar{d} \leq O_1 - Q_1^A$

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$\nabla E[\pi_1(Q_1^A)] = 0$ is simplified to $(c_2 - c_1) = 0$ which is infeasible.

(1-b) $\underline{d} \le O_1 - Q_1^A \le \overline{d} \le Q_1^A + O_2 - D^A$

$\nabla E[\pi_1(Q_1^A)] = 0$ is solved to get $\hat{Q}_1^A = O_1 - \underline{d}$. As long as this $\hat{Q}_1^A$ satisfied condition (1-b), the solution is feasible.

(1-c) $\underline{d} \le O_1 - Q_1^A \le Q_1^A + O_2 - D^A \le \overline{d}$

$\nabla E[\pi_1(Q_1^A)] = 0$ is solved to get $\hat{Q}_1^A = \dfrac{(O_1 - \underline{d}) - (c_m - c_2)(\overline{d} - O_2 + D^A)}{2c_2 - c_1 - c_m}$. As long as this

$\hat{Q}_1^A$ satisfied condition (1-c), the solution is feasible.

(1-d) $O_1 - Q_1^A \le \underline{d} < \overline{d} \le Q_1^A + O_2 - D^A$

It is shown that $\nabla E[\pi_1(Q_1^A)]$ is constantly equal to zero under this condition.

(1-e) $O_1 - Q_1^A \le \underline{d} \le Q_1^A + O_2 - D^A \le \overline{d}$

$\nabla E[\pi_1(Q_1^A)] = 0$ is solved to get $\hat{Q}_1^A = \overline{d} - O_2 + D^A$. As long as this $\hat{Q}_1^A$ satisfied

condition (1-e), the solution is feasible.

(1-f) $Q_1^A + O_2 - D^A \le \underline{d}$

$\nabla E[\pi_1(Q_1^A)] = 0$ is simplified to $(c_m - c_2) = 0$ which is infeasible.

Under the conditions (2-2) and (3-2) $D^A - O_2 \le Q_1^A \le \dfrac{1}{2}(O_1 + D^A - O_2)$:

(2-a) $\overline{d} \le Q_1^A + O_2 - D^A$

$\nabla E[\pi_1(Q_1^A)] = 0$ is simplified to $(c_2 - c_1) = 0$ which is infeasible.

(2-b) $\underline{d} \le Q_1^A + O_2 - D^A \le \overline{d} \le O_1 - Q_1^A$

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$\nabla E[\pi_1(Q_1^A)] = 0$ is solved to get $\hat{Q}_1^A = \overline{d} - \dfrac{(c_2 - c_1)(\overline{d} - \underline{d})}{(c_m - c_2)} - O_2 + D^A$. As long as this

$\hat{Q}_1^A$ satisfied condition (2-b), the solution is feasible.

(2-c) $\underline{d} \le Q_1^A + O_2 - D^A \le O_1 - Q_1^A \le \overline{d}$

$\nabla E[\pi_1(Q_1^A)] = 0$ is solved to get $\hat{Q}_1^A = \dfrac{(O_1 - \underline{d}) - (c_m - c_2)(\overline{d} - O_2 + D^A)}{2c_2 - c_1 - c_m}$. As long as this

$\hat{Q}_1^A$ satisfied condition (2-c), the solution is feasible.

(2-d) $Q_1^A + O_2 - D^A \le \underline{d} < \overline{d} \le O_1 - Q_1^A$

It is shown that $\nabla E[\pi_1(Q_1^A)]$ is constantly equal to zero under this condition.

(2-e) $Q_1^A + O_2 - D^A \le \underline{d} \le O_1 - Q_1^A \le \overline{d}$

$\nabla E[\pi_1(Q_1^A)] = 0$ is solved to get $\hat{Q}_1^A = O_1 - \dfrac{(c_m - c_2)(\overline{d} - \underline{d})}{(c_2 - c_1)} - \underline{d}$. As long as this $\hat{Q}_1^A$

satisfied condition (2-e), the solution is feasible.

(2-f) $O_1 - Q_1^A \le \underline{d}$

$\nabla E[\pi_1(Q_1^A)] = 0$ is simplified to $(c_m - c_2) = 0$ which is infeasible.