8-12-2016

Optimization Models and Algorithms for Vulnerability Analysis and Mitigation Planning of Pyro-Terrorism

Eghbal Rashidi

Optimization models and algorithms for vulnerability analysis and

mitigation planning of pyro-terrorism

By

Eghbal Rashidi

A Dissertation
Submitted to the Faculty of
Mississippi State University
in Partial Fulfillment of the Requirements
for the Degree of Doctor of Philosophy
in Industrial and Systems Engineering
in the Department of Industrial and Systems Engineering

Mississippi State, Mississippi

August 2016

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By

Eghbal Rashidi

Approved:

____

Hugh R. Medal
(Major Professor)

____

Sandra D. Eksioglu
(Committee Member)

____

Xiaopeng Li
(Committee Member)

____

Mohammad Marufuzzaman
(Committee Member)

____

Stanley F. Bullington
(Graduate Coordinator)

____

Jason M. Keith
Dean
Bagley College of Engineering

Name: Eghbal Rashidi

Date of Degree: August 12, 2016

Institution: Mississippi State University

Major Field: Industrial and Systems Engineering

Major Professor: Dr. Hugh R. Medal

Title of Study: Optimization models and algorithms for vulnerability analysis and mitigation planning of pyro-terrorism

Pages in Study 116

Candidate for Degree of Doctor of Philosophy

In this dissertation, an important homeland security problem is studied. With the focus on wildfire and pyro-terrorism management. We begin the dissertation by studying the vulnerability of landscapes to pyro-terrorism. We develop a maximal covering based optimization model to investigate the impact of a pyro-terror attack on landscapes based on the ignition locations of fires. We use three test case landscapes for experimentation. We compare the impact of a pyro-terror wildfire with the impacts of naturally-caused wildfires with randomly located ignition points. Our results indicate that a pyro-terror attack, on average, has more than twice the impact on landscapes than wildfires with randomly located ignition points.

In the next chapter, we develop a Stackelberg game model, a min-max network interdiction framework that identifies a fuel management schedule that, with limited budget, maximally mitigates the impact of a pyro-terror attack. We develop a decomposition algorithm called MinMaxDA to solve the model for three test case landscapes, located in Western U.S. Our results indicate that fuel management, even when conducted on a small scale (when 2% of a landscape is treated), can mitigate a pyro-terror attack by 14%, on average, comparing to doing nothing. For a fuel management plan with 5%, and 10% budget, it can reduce the damage by 27% and 43% on average.

Finally, we extend our study to the problem of suppression response after a pyro-terror attack. We develop a max-min model to identify the vulnerability of initial attack resources when used to fight a pyro-terror attack. We use a test case landscape for experimentation and develop a decomposition algorithm called Bounded Decomposition Algorithm (BDA) to solve the problem since the model has bilevel max-min structure with binary variables in the lower level and therefore not solvable by conventional methods. Our results indicate that although pyro-terror attacks with one ignition point can be controlled with an initial attack, pyro-terror attacks with two and more ignition points may not be controlled by initial attack. Also, a faster response is more promising in controlling pyro-terror fires.

Key words: Homeland security, Stackelberg game, decomposition algorithms, bilevel programming

DEDICATION

To my beloved family.

ACKNOWLEDGEMENTS

I would like to thank Dr. Hugh R. Medal for being such a great adviser, supporting my research, guiding me through this challenging journey, and always being there whenever I needed him. He not only has been an invested adviser, but a supportive and caring friend. I would also like to thank my committee members Dr. Sandra Eksioglu, Dr. Xiaopeng Li, and Dr. Mohammad Marufuzamman who have guided me with my research and dissertation.

I have been very fortunate to have great friends and coworkers. Special thanks to Dr. Satish Vadlamani, Apurba Nandi, Yang Chen, Adindu Emelogu, Mohsen Hosseini, Dr. Md. Roni, Dr. Mohammad Marufuzaman, Dr. Gokce Palak, and Dr. Abdullah Al Khaled. I am graceful for having a group of very supportive friends during my study at MSU. Special thanks to Dr. Amin Kargarian and Dr. Mohsen Eshraghi. I am thankful of every advice they have provided me.

My heartfelt thanks to my family, my mother and brothers. I will always remember all the sacrifices they had to go through to get me where I am today. I would like to thank my sister in law, Maryam, and my lovely nephews, Shayan and Mahan for being in my life so I could work as hard.

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CHAPTER I

INTRODUCTION

Over the last ten years, there has been an average of 75,000 wildfires per year in the United States and an average of 7.2 million acres has burned each year [1]. Billions of dollars are spent annually by the U.S. Forest Service for wildfire suppression [2]. The cost of wildfires is not restricted to monetary cost, but environmental and socioeconomic costs as well. In addition, the loss of human lives is a tremendous societal cost over the course of a wildfire. Destructive wildfires become a primary concern in places where major cities are located close to highly flammable vegetation areas as in Western and Southern U.S., as well as Australia and Mediterranean Europe [3]. About 32 percent of housing units including homes, apartments and buildings in the U.S. and 10 percent of all lands with houses are situated in the wildland-urban interface (WUI), which is the zone of transition between natural land and human development [4], and WUI is expected to continue to grow [5]. With human populations reaching further into wildlands, wildfire risk has further increased.

Along with increasing wildfire risk, the costs associated with wildfire management have also increased. The United States Department of Agriculture (USDA) reported that more than $1.6 billion is spent annually by state forestry agencies on wildfire protection, prevention, and suppression and this cost is increasing [2]. Despite increased investments in wildfire prevention and suppression, wildfire related destruction is a problem that appears to be worsening [6]. Increased wildfire activities have been observed in the U.S. [7], Canada [8], Mediterranean Europe [9] and Australia [10].

Because of the increase in fire activities, their significant short and long term threats to forest ecosystems, and the danger they pose to public safety and property, wildfires have received increased public attention. There are some concerns that the destructive power of wildfires may attract terrorist organizations to use them as a weapon of mass destruction [11–13]. Indeed, pyro-terrorism events have been documented in France, Spain, and Greece [11,12,14]. Pyro-terrorism is the use of large-scale arson attacks by non-state organizations to terrorize, intimidate or coerce a government, the civilian population, or any segment in order to advance, political or social objectives [13]. According to Baird [12], pyro-terrorism possesses the four generally accepted elements of terrorism: targeting of noncombatants, political motivation, violence with psychological impact and organized perpetrators. As a result, both the Department of Homeland Security (DHS) and the Federation Bureau of Investigation (FBI) are concerned about this novel threat [15,16]. It is important for decision makers in these departments to anticipate potential threats, and implement countermeasures to avoid a potential devastating domestic attack. However, no study has been done to investigate the impact of this threat and the vulnerability of landscapes and our suppression resources in confronting such threat; nor there has been any study investigating ways to mitigate against it. Considering the destructive power of natural wildfires, what is the potential impact of a human-made wildfire, i.e., a pyro-terrorism attack?

Wildfire incidences require the co-occurrence of three factors: fire-conductive-weather, source of ignition and fuels, (i.e. flammable vegetation) [17]. In pyro-terrorism, the adversaries can cause the joint occurrence of these three factors by providing the sources of ignitions and choosing where, when and how many fires to start. This makes pyro-terrorism a more destructive threat than natural wildfires. Knowing this, is a way to mitigate pyro-terrorism? If so, how? Are existing resources available to fight natural wildfires sufficient to fight pyro-terrorism? This dissertation explores the use of Operations Research (OR) methods in analyzing these three questions. Specifically, this dissertation has the following main goals: analyze the pyro-terrorism threat and investigate the vulnerability of landscapes to such threats, examine the capability of our resources in suppressing those fires, and investigate a way to mitigate such a hostile activity.

Risk assessment has increasingly become a key input to the wildfire prevention and mitigation decision making processes [18–21]. Determining the vulnerability of a system is an important component of risk assessment, which is employed to help develop risk mitigation strategies to counter risks [22]. Vulnerability assessment studies identify weak points in the system, and focus on defined threats that could compromise the system's ability to meet its intended function. To our knowledge, no risk assessment study has considered the worst-case scenario wildfires based on fire ignition locations and there has not been any pilot risk assessment for a potential pyro-terror attack that utilizes coordinated multiple ignition points. The results of such study can be used in strategic planning efforts for risk mitigation against a threat, especially when available resources and funds are limited. This study is demonstrated in Chapter 2.

Fuels, weather condition and topography of a landscape are the three factors that influence fire behavior, and fuel is the only factor that can be managed in the short run

[10]. To reduce the flammability of landscape and decrease the risk of (natural) wildfires, fire managers use fuels management programs. Fuels management is the process of altering the amount and structure of fuels through the construction of fuel breaks or applications of fuel treatments such as prescribed burning, commercial harvesting and mechanical thinning, to reduce the spread and intensity of wildfires before they occur [23,24]. Modeling methods have been used to design efficient fuels management programs over a landscape. Researchers have used heuristic methods [25–28] and optimization models such as mixed integer programming [17,29–31] and stochastic dynamic programming [32,33] for the spatial allocation of fuels management over a landscape. All these fuels management models, however, have been developed for reducing the impact of naturally-caused wildfires, not arson-induced fires neither pyro-terrorism. To our knowledge, there is no study investigating the effectiveness of fuels management in mitigating pyro-terrorism. Chapter 3 studies the use of fuels management in mitigating pyro-terrorism.

Forest fire management agencies are responsible for dealing with wildland fires and their impacts on people and forest ecosystems [34]. There are certain measures that these management agencies take to deal with fires. The term initial attack (IA) is used by forest fire managers to refer to the first suppression action taken on a wildfire [34]. Initial attack is the primary attempt in suppressing a wildfire within the first several hours of fire discovery [35] to contain the fire before it grows large and becomes difficult to control. Although the majority of wildfire incidents between 1970 and 2002 have been reported to be contained by initial attack, the small percentage of escaped wildfires reportedly have caused more than 97% of the total area burned [36]. Therefore developing more efficient suppression strategies including initial attack is very important for wildland management agencies [37,38]. A number of researchers have developed two stage stochastic programming models for addressing initial attack decision making procedure. In these models, the acquisition and deployment decisions take place in the first stage of the model, and to support a robust decision in the first stage, the dispatching of the resources are decided in the second stage of the model [35,39–44]. With the increasing rate of wildfire incidences and their severity, it is important to assess the capacity of IA in responding to severe wildfires, specifically worst-case scenario wildfires. On the other hand, given the existence of the threat of pyro-terrorism, it is important to evaluate the capability of our IA against such a threat. However, to the best of our knowledge, there have not been any studies addressing this capability. We present a vulnerability analysis of IA capability against pyro-terrorism in Chapter 4.

Thus, the proposed contributions of this dissertation are as follows:

1. In this research the first vulnerability assessment of landscapes to pyro-terrorism is studied. The purpose of the vulnerability assessment study is to help wildfire managers identify critical locations whose protection yields a fire management system robust against possible worst-case scenarios, or potential pyro-terrorism. This study can be used in identifying these highly vulnerable areas for wildfire risk mitigation planning such as fuels treatment scheduling and fire suppression preparedness planning to reduce potential worst-case scenario wildfires. To our knowledge, no risk assessment study has considered worst-case scenario wildfires, and there has not been any pilot risk assessment for a potential arson attack that utilizes coordinated multiple ignition points.

2. After identifying the most vulnerable areas in a landscape, and evaluating the impact of a pyro-terror attack, a model is developed for planning a fuels management layout that can be used for mitigating pyro-terrorism.

3. In addition, a vulnerability analysis of initial attack suppression resources is developed for worst-case scenario wildfires and pyro-terrorism. We examine the initial attack (IA) capacity in responding to the worst-case scenario wildfires and pyro-terrorism. The managerial insights extracted from this research can raise awareness and help decision makers improve fire suppression programs.

The reset of this dissertation is organized as follows: In CHAPTER II the vulnerability of landscapes to pyro-terrorism is studied. A mathematical programming model is developed to assess the maximum damage that a fire can cause on a landscape by optimally locating the ignition points. The model is used to examine the impact of wildfire on a landscape when fire can start from multiple locations. Three case studies are used to investigate the wildfire impacts using this model. A manuscript based on the contents of this chapter [45] was submitted to the European Journal of Operational Research in October 2015.

After assessing the vulnerability of landscapes to pyro-terrorism, in CHAPTER III a mitigation strategy using fuels management is proposed to reduce the impact of a pyro-terror attack on a landscape. This problem is modeled as a Stackelberg game problem in which a fire manager, acting first, finds optimal locations for fuels treatments, and the adversaries, acting second, locate ignition points to maximize the damage. Experiments are conducted on three landscape case studies. A manuscript based on the contents of this chapter [46] was submitted to the IIE Transactions in February 2016.

In 3.6.2, a vulnerability analysis for assessing the capacity of the current initial attack suppression resources in the face of a pyro-terror attack is presented. This study will go a step further than the first vulnerability assessment study presented in chapter 2 and will incorporate the capacity of suppression resources in response to a pyro-terror attack. In case of a pyro-terror attack, the adversaries are aware of the resources deployed to fire stations for an initial attack. Therefore, they can plan accordingly to maximize the impact of their attack such that the initial attack would not be able to control the wildfire and reduce the damage. A manuscript based on this research is under preparation for submission to European Journal of Operational Research.

CHAPTER II

A MAXIMAL COVERING LOCATION-BASED MODEL FOR ANALYZING THE VULNERABILITY OF LANDSCAPES TO PYRO-TERRORISM

2.1 Introduction

Although natural fires are part of many terrestrial ecosystems [47], uncontrolled wildfires can be destructive and can cause loss of human life and property [3]. Destructive wildfires are a primary concern in places where major cities are located close to highly flammable vegetation areas, such as the Western and Southern U.S., Australia and Mediterranean Europe [3]. There has been a sharp increase in fire events across the globe [10], and the destruction caused by wildfires appears to be worsening [6]. From 2002 through 2011, wildfires in the U.S. accounted for $13.7 billion in total economic losses, a $6.9 billion increase from the previous decade [48]. The deaths of 19 firefighters in 2013 the largest such loss since 1933, were part of a general trend of rising threats to lives as well as properties [48].

Wildfire risk has increased with human populations reaching further into wildlands. About 32 percent of housing units including homes, apartments and buildings in the U.S. and 10 percent of all lands with houses are situated in the wildland-urban interface (WUI; the zone of transition between natural land and human development) [4], and WUI is expected to continue to grow[5]. Homes located in the WUI have a high probability of exposure to wildfire, regardless of vegetation type or potential fire size [2].

Along with increasing wildfire risk, the costs associated with wildfire management are increasing. The United States Department of Agriculture (USDA) reported that more than $ 1.6 billion is spent annually by state forestry agencies on wildfire protection, prevention, and suppression [2]. To reduce the consequences of catastrophic wildfires, planning an effective mitigation programs is essential.

Risk assessment has increasingly become a key input to wildfire prevention and mitigation decision making processes [18–20,49]. Determining the vulnerability of a system is an important component of risk assessment, which is employed to help develop risk mitigation strategies to counter risks [22]. Vulnerability assessment studies identify weak points in the system, and focus on defined threats that could compromise the system's ability to meet its intended function. To our knowledge, no risk assessment study has considered the worst-case scenario wildfires, and there has not been any pilot risk assessment for a potential arson attack that utilizes coordinated multiple ignition points. The results of such study can be used in strategic planning efforts for risk mitigation against a threat, especially when available resources and funds are limited. This paper aims to fill this gap by proposing a mathematical programming model to study the vulnerability of landscapes to wildfires in the worst-case scenario.

Operations Research (OR) specialists have worked with fire managers to develop decision support systems that can help improve fire management; however, there remain substantial gaps between wildfire managers' needs and the decision support systems used [34]. Linear programming and mixed integer programming (MIP) have been frequently used in wildfire management (e.g., [50–52]). Other approaches such as heuristics [25–27,53], goal programming [54], stochastic programming [40], stochastic dynamic programming [32,33], and robust optimization [39,55] also have been used in wildfire management. In this research, we develop a mathematical programming model to evaluate the maximum impact of a wildfire on a landscape. We use the model to analyze the vulnerability of landscapes to wildfires based on the impact of the worst-case scenario ignition locations.

Although wildfires can start from anywhere on a landscape, the location and number of ignition points can be an important factor that impact the resulting wildfire spread. Using our developed model, we investigate the effect of ignition locations on wildfires and identify the potential ignition locations which result in a wildfire with the maximum impact on a landscape. To model wildfires' behavior on a landscape, we use FlamMap [56], a fire behavior mapping and analysis program. We consider wildfires that contain a single and multiple ignition points, such as the case in lightning-caused wildfires [57]. The proposed model is then used to evaluate the impact of wildfire on three landscape cases from three national forests in the Western U.S.

We believe this to be the first study that analyzes the vulnerability of landscapes to worst-case wildfires with regard to the location of ignition sites. Our ultimate goal in this research is to help wildfire managers identify critical locations whose protection yields a fire management system robust against possible worst-case scenarios, or potential arson attacks. This study can be used in identifying these highly vulnerable areas for wildfire risk mitigation planning such as fuels treatment scheduling and fire suppression preparedness planning to reduce potential worst-case scenario wildfires.

2.2 Problem description and model formulation

Our objective is to identify ignition locations of a wildfire that pose the maximum damage to the landscape. Damage or impact (used interchangeably through this paper) can be evaluated as the percentage of the landscape burned, or the value lost to fire. For the latter, the value of vegetation type, e.g. commercial timber, and the value of wildland-urban interface (WUI), if any, is used. We consider a landscape divided into a number of raster cells, and use FlamMap to model fire spread characteristics in each cell. If X is the set of vector x indicating the cell(s) that a fire originates from, and $f(x)$ is a function representing the corresponding impact of the fire on the landscape, then the research problem can be defined as identifying the ignition points, represented by vector x, of a fire that has the largest impact on the landscape, or equivalently to find x for which $f(x)$ is the maximum. We formulate the problem as a network optimization problem and later in section 3 test it on three landscape cases.

2.2.1 Modeling the spread of wildfire

To model the spread of wildfire as a network optimization problem, we represent a landscape with a raster map divided into grid cells. If we represent the center of each cell as a node, and connect neighboring cells with directed arcs, then the landscape can be represented with a directed network (see Figure 2.1).

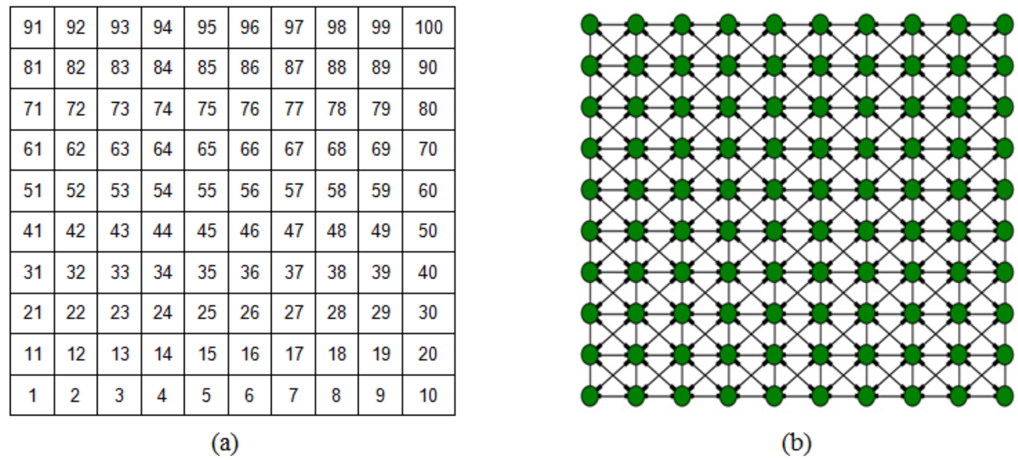


Figure 2.1 Rasterized landscape

(a) A landscape modeled as a 10 by 10 raster cells,
(b) The network representation of the landscape

To model the spread of fire in the landscape, we use the minimum travel time algorithm (MTT) [58]. We study a case that multiple wildfires start at the same time across a landscape. Therefore, to apply the MTT algorithm, we need to calculate the minimum travel time from any cell in the network (potential ignition points) to any other cell in the network. This requires calculating minimum travel time for a network problem with multiple sources and multiple sinks (a source is the starting point of a travel path, and a sink is the ending point; see [59]). In order to facilitate the construction of our model, we convert the problem to a single-source shortest path problem by adding a dummy super source to the network. The dummy super source represents the primary ignition source of fire. We then use the shortest path formulation to compute the minimum travel time from the super source to any cell in the network. The super source, cell 0 , is connected to every cell in the network with 0 travel time. Since we hypothetically assume wildfires start at the dummy super source, the 0 travel time assumption is legitimate. We assume that for any cell i (an ignition point), and any cell j

that fire can reach from , a single fire flow is sent from the super source to cell i . Then the model identifies the shortest path for sending a fire flow unit from cell i to cell j (in the shortest path formulation, it is assumed that a flow unit is sent from the source to the sink; in our formulation, hypothetically, we assume fire flow units are sent from an ignition point to any point in a landscape). An example of this process is shown in Figure 2.2.

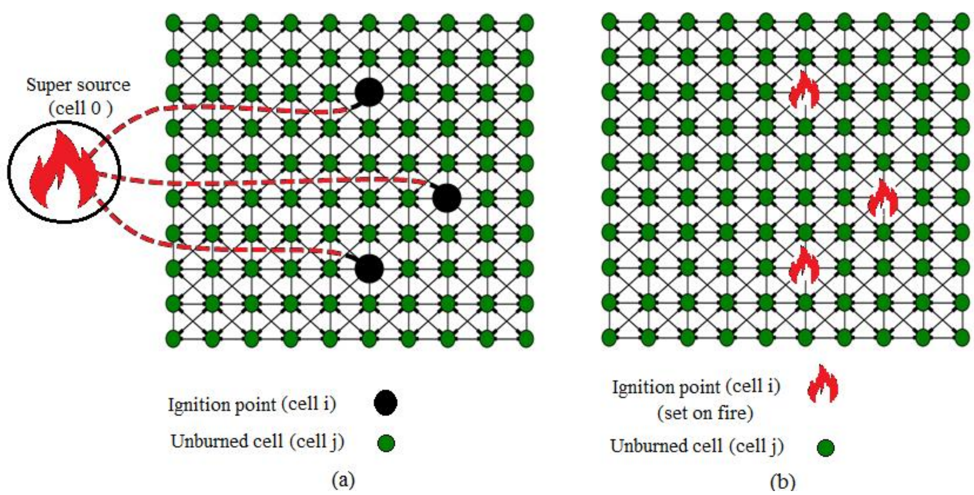


Figure 2.2 Modeling fire spread

The simulated wildfire (a) starts at the super source (cell 0) and arrives at ignition points at time 0, (b) sets fire on the ignition points and spreads through the landscape

In this example, wildfire starts at the super source, travels to the three sample ignition points, arrives at the ignition points at time 0 (simulation time) and from there spreads through the landscape. The travel time from the ignition points to other cells are computed based on the length of the respective shortest path. Using this structure, we can simulate the spread of wildfire in a landscape.

2.2.2 Mathematical formulation

The primary assumptions for the research problem are as follows:

1. The ignition points of wildfires are randomly distributed across the landscape;
2. Multiple fires can start at any location in the landscape; however, for simplicity, we assume that the interaction of fires is negligible;
3. The areas outside the boundaries are unburnable;
4. When wildfire reaches the center of a cell, that cell is assumed burned;
5. Fire spreads in an elliptical shape within each cell.

We use FlamMap to calculate the Rate of Spread (ROS) along with the major fire spread direction in each cell. The major fire spread direction in each cell represents the direction in that cell for which fires spread with the fastest speed. Fires would also spread along other directions, but at slower speed [31]. We use formulas (2.1) and (2.2) to calculate ROS along other directions:

$$ROS = \frac{b^2 - c^2}{b - c \times COS(\theta)} \quad \text{for } 0 \leq \theta < \frac{\pi}{2} \tag{2.1}$$

$$ROS = \frac{b^2 - c^2}{b + c \times COS(\pi - \theta)} \quad \text{for } \frac{\pi}{2} \leq \theta < \pi \tag{2.2}$$

θ is the angle between the major fire spread direction in each cell computed by FlamMap and the fire spread direction from this cell to the center of adjacent cells. In this formula b and c are outputs of FlamMap and are standard parameters used to describe the ellipse of fire spread. For more information we refer the reader to [60]. Two mixed-integer programming formulations are developed for this problem and are presented in this section. The models use the following notations:

Table 2.1 Notations

Sets and indices	
d	is the expected fire duration
C	is the set of raster cells in a landscape indexed with r, i and j
N_i	is the set of raster cells adjacent to cell i
Parameters	
$F_{i,j}$	distance (meters) from the center of cell i to the center of cell j
$R_{i,j}$	rate of fire spread (meters per minute) from cell i to cell j
$t_{i,j}$	the fire spread time (minutes) from cell i to cell j, $t_{i,j} = \frac{F_{i,j}}{R_{i,j}}$
B	is the number of ignition points
V_r	is the value of cell r lost to the fire
Variables	
$x_{r,i,j}$	1 if the shortest path from an ignition point to cell r passes through link (i,j); 0 otherwise
z_j	1 if a fire starts at cell j; 0 otherwise
y_r	1 if cell r is reached by a fire within duration d; 0 otherwise

The objective of this model is to locate wildfire ignition points with the largest impact on the landscape. In wildfires, it is not only how much of the landscape that is burned and damaged that matters, but also wildfire losses. Therefore, the objective function of the model should also compute the total damage including a monetary value lost to fire. The model identifies the optimal locations of ignition points such that the resulting wildfire has the maximum impact on the landscape based on the value lost. The MIP model is as follows:

$$SPWVA\text{: } \max f = \sum_{r\in C} V_r y_r \tag{2.3}$$

$$\sum_{j\in N_i} x_{r,i,j} - \sum_{j\in N_i} x_{r,j,i} = \begin{cases} 1 & i = 0 \\ 0 & i \neq 0, r \\ -1 & i = r \end{cases} \quad \forall i, r \in C \tag{2.4}$$

$$x_{r,0,j} \leq z_j \quad \forall j, r \in C \tag{2.5}$$

$$y_r \leq \frac{d}{\sum_{i\in C, j\in N_i} x_{r,i,j} t_{i,j}} \quad \forall\ r \in C \tag{2.6}$$

$$\sum_{j\in C} z_j \leq B \tag{2.7}$$

$$x_{r,i,j} \in \{0,1\} \quad \forall\ r,j \in C, i \in C \cup \{0\} \tag{2.8}$$

$$y_r \in \{0,1\} \quad \forall\ r \in C \tag{2.9}$$

$$z_j \in \{0,1\} \quad \forall\, j \in C \tag{2.10}$$

We term the model "shortest path-based wildfire vulnerability assessment" or SPWVA. The objective function (2.3) maximizes the total loss due to wildfire within duration d. Constraints (2.4) ensure that one unit fire flow is sent from the super source to every cell. These constraints are called the flow conservative constraints (see [59]). Constraints (2.5) ensure that the fire spreads to cell j from the super source only when cell j is selected as an ignition point. Constraints (2.6) are the burn constraints, and set the values of the binary variables y_r . These variables are used to track whether cell r is reached by wildfire and, therefore, burned within duration d. If the minimum travel time from a fire to cell r is less than or equal to the duration d, which is $\sum_{i\in C, j\in N_i} x_{r,i,j} t_{i,j} \leq d$, then $\frac{d}{\sum_{i\in C, j\in N_i} x_{r,i,j} t_{i,j}} \geq 1$ and therefore y_r will be equal to 1 (the objective is maximizing on y_r). Otherwise if $\sum_{i\in C, j\in N_i} x_{r,i,j} t_{i,j} > d$, then $\frac{d}{\sum_{i\in C, j\in N_i} x_{r,i,j} t_{i,j}} < 1$, and therefore y_r has to be 0 as it is a binary variable. It is noteworthy that the model maximizes on y_r, and, therefore based on constraint (2.6) minimizes $\sum_{i\in C, j\in N_i} x_{r,i,j} t_{i,j}$, which is the fire travel time to cell r. This along with constraints (2.4) and (2.5) form a travel time minimization problem, or the shortest path problem. Constraint (2.7) controls the number of ignition points.

To linearize constraints (2.6) we introduce another binary variable $\mathrm{w_{r,i,j}} = \mathrm{y_r} \times \mathrm{x_{r,i,j}}$ and add the following constraints to the model:

$$\sum_{i\in C, j\in N_i} w_{r,i,j} t_{i,j} \leq d \quad \forall\, r \in C \tag{2.11}$$

$$w_{r,i,j} \leq y_r \quad \forall\, r, i, j \in C \tag{2.12}$$

$$w_{r,i,j} \leq x_{r,i,j} \quad \forall\, r, i, j \in C \tag{2.13}$$

$$w_{r,i,j} \geq x_{r,i,j} + y_r - 1 \quad \forall\, r, i, j \in C \tag{2.14}$$

$$x_{r,i,j} \in \{0,1\} \quad \forall\, r, i, j \in C \tag{2.15}$$

Doing so would increase the size of the model, however, and would make it more difficult to solve. An alternative way to formulate constraints (2.6) is as follows:

$$y_r \leq \frac{d - \sum_{i\in C, j\in N_i} x_{r,i,j} t_{i,j}}{M} + 1 \quad \forall\, r \in C \tag{2.16}$$

M is the length of the travel time path $\left(M = \max\{\sum_{i\in C, j\in N_i} x_{r,i,j} t_{i,j}, \forall\, r \in C\}\right)$.

Constraints (2.16) perform the same function as constraints (2.6) while they are linear, and, therefore, unlike constraints (2.6), they do not require adding extra constraints and variables for linearization. To illustrate how constraints (2.16) work, assume that the length of the shortest path to r is less than or equal to duration d , which means cell r is considered burned, then $d - \sum_{(i,j)} x_{r,i,j} t_{i,j}$ is non-negative, say $\frac{d - \sum_{i\in C, j\in N_i} x_{r,i,j} t_{i,j}}{M} = \varepsilon > 0$, thus . $y_r \leq 1 + \varepsilon$, and y_r can be 1, otherwise $y_r \leq 1 - \varepsilon$, and y_r must be 0.

The model selects the optimum potential cells for starting a fire that can reach and burn the maximum number of cells in the landscape. In this model, the ignition points are selected, and then, the shortest paths between the ignition points and every cell in the network are calculated and, accordingly, the number of cells reachable by the fire within duration d is computed. Our preliminary tests with hypothetical landscapes, similar to those used in [31], reveal a drawback of SPWVA model. The model is difficult to solve for sample instances larger than 100 cells. To overcome this problem, and solve large landscape cases more efficiently, we develop an additional model.

2.2.3 A maximal covering location-based formulation

To overcome the difficulty of solving the shortest path problems as part of the original problem, we develop another model based on the idea of the maximal covering location problem [61]. This model represents the cover of wildfire in a landscape in a given time when fire uses shortest path to spread. In this model, the shortest paths are calculated prior to solving the model and entered into the model as input parameters. This way we no longer require shortest path problems as part of the original problem. To present the model, we define a new parameter, $H_{r,j}$, which is 1 if the length of the shortest path from cell j to cell r is less than d , and 0 otherwise. For any cell r in the landscape, $H_{r,j}$ implies whether cell r is reached within duration d by a wildfire that starts at cell. The model is as follows:

$$\text{MCWVA: } \max f = \textstyle\sum_{r \in C} V_r \tag{2.17}$$

$$y_r \leq \textstyle\sum_{j \in C} H_{r,j} \times z_j \quad \forall\ r \in C \tag{2.18}$$

$$\textstyle\sum_{j \in C} z_j \leq B \tag{2.19}$$

$$y_r \in \{0,1\} \quad \forall\ r \in C \tag{2.20}$$

$$z_j \in \{0,1\} \quad \forall\ j \in C \tag{2.21}$$

We term the model “maximal covering location-based wildfire vulnerability assessment” or MCWVA. MCWVA uses the same variable y_r as was used in SPWVA. Since the shortest paths are already given, MCWVA has fewer variables and constraints than SPWVA. In fact, SPWVA has $n^3 + n^2 + 2n$ variables and $2n^2 + 2n + 1$ constraints (n is the number of cells in the network), while MCWVA has only $2n$ variables and $n + 1$ constraints. This is without considering variable type constraints (2.8)-(2.10) and (2.20)-(2.21). Therefore, we expect MCWVA to be solved faster.

The objective function (2.17) maximizes the total loss of the landscape due to wildfires. Constraints (2.18) are the burn constraints, and set the values of the binary variables y_r. Constraint (2.19) controls the number of ignition points. Constraints (2.20) - (2.21) restrict the variables to binary values. The model can consider unburnable cells or treated cells (e.g. cells with fuel breaks) if such data are available. These considerations need to be made for fire behavior in each cell. For example, if cell i is a treated cell then this affects the fire spread time $t_{i,j}$ from cell i to any adjacent cell j. We can increase $t_{i,j}$ by a constant greater than d so that it lengthens the paths that go through cell i, and, therefore, prohibits wildfires from spreading through cell i. One can also define the ignition probability for each cell in the landscape such that for unburnable cells or treated cells, the corresponding ignition probability is zero. There might be parts of the landscape that have more fire incidences, so those cells should have higher ignition probabilities. For this reason, historical wildfire records can be used to estimate average annual wildfire occurrence rates in each cell [28]. In the next section, we use MCWVA to investigate the impact of wildfires with optimally located ignition points. We also compute the average impact of wildfires over all possible ignition location scenarios. The current model can be extended to compute the expected loss due to wildfires across a possible fire duration distribution [62], instead of a fixed fire duration. Given the probability for each fire duration, it can be added to the objective function.

2.3 Model demonstration

In this section, we use MCWVA model to assess the impacts of the worst-case scenario wildfires. A preliminary experiment using hypothetical landscapes, of sizes 7×7, 8×8 and 10×10, indicates that MCWVA can be solved efficiently. To use more realistic-sized networks, we use three case studies located in the western USA, where large wildfires are common. For these landscapes, we compare two scenarios: worst-case wildfires with optimally located ignition points and wildfires with randomly located ignition points. For the former, we use our MCWVA model to compute the maximum impact of wildfires, based on their ignition locations, and for the latter we compute the average impact of wildfires with ignition points randomly located across the landscape. For this reason, we conduct a series of experiments to consider the impact of wildfires on different landscapes, with different fire duration, and different wind speed scenarios. We also run a series of experiments to compute the impact of wildfires in presence of WUI in a landscape. These experiments are discussed in details in the following sections. Since we use case study landscapes for experimentation (which are much larger than the hypothetical landscapes used in the preliminary tests), we do not report the results of the preliminary tests on the hypothetical landscapes.

We used the LANDFIRE database to obtain landscape files (LCP) for the landscapes under study. LANDFIRE data are commonly used in wildland fire simulation modeling, as they are standardized, and updated regularly to adjust to disturbances such as wildfires, fuels treatment and urban development [20]. Landscape files (LCP) contain spatial data themes such as fuel models, elevation, slope, aspect, and canopy characteristics. We use these data as inputs of FlamMap to model fire behavior and spread in each cell of the landscapes. FlamMap inputs these data, along with wind speed, wind direction, and fuel moisture conditions to compute rate of spread and the major fire spread direction in each cell. We use the outputs of FlamMap (the rate of spread, and the major fire spread direction in each cell) to model fire spread in the landscapes using minimum travel time algorithm. The details of the landscape cases are discussed in the following section.

2.3.1 Case studies

The first case is the 6307 km^2 Santa Fe National Forest in northern New Mexico. A prevailing west to east wind is assumed for this case with 300 Azimuth at 12 miles per hour (19.31 km per hour). The second case is the 3979 km^2 Umpqua National Forest at the western slopes of Cascade Mountains in Oregon. The same wind condition is assumed. The third case is the 3334 km^2 San Bernardino National Forest located in the San Bernardino Mountains in southern California. For this case a prevailing east to west wind with 270 Azimuth at 12 miles per hour is assumed. However, we also study this case under slower and faster wind speed conditions. Figure 2.3 Figure 2.2 shows the approximate locations of these case study landscapes.

Figure 2.3 Test case locations

The approximate locations of the case study landscapes in the US (retrieved from [63])

Although modeling these cases into rasterized networks with high number of cells makes the model more accurate, as the size of the networks increases, the model becomes more difficult to solve. According to Minas et al [17], landscapes divided into several hundred to a thousand management units are of practical interest for fuels management purposes. We clip an area of 3 km×3 km from the first and second landscapes. To test the capability of the model for a larger landscape, we clip an area of 4.2 km×4.2 km from the third landscape and rasterize them into networks with 25×25 (625) square cells, each 120 m × 120 m wide, for the first two landscapes, and 35×35 (1225) square cells, each 120 m × 120 m wide, for the third landscape. To quantify fire behavior on these landscapes we use FlamMap 5.0 to calculate the rate of spread and fire spread directions.

Table 2.2 Initial fuel moisture conditions used in FlamMap

1 hour initial moisture	6
10 hour initial moisture	7
100 hours initial moisture	8
Herbaceius fuel moisture	60
Live woody fuel moisture	90

We use the same initial fuel moisture scenarios for all three cases in our study (Table 2.2). FlamMap uses GIS data, landscape characteristics, fuel moisture, and wind conditions and outputs fire behavior for each cell. In this section, we run a set of experiments to find the effect of the locations of ignition points on the damage that wildfires can cause. Therefore, we compare two scenarios: (1) wildfires with random ignition points ("random wildfires"), and (2) wildfires with optimally located ignition points ("worst-case wildfires"). In the worst-case wildfires, the ignition locations are selected optimally through solving MCWVA model. Figure 2.4 shows the fire foot print after 24 hours for a sample random wildfire and the worst-case wildfire with one ignition point for the Santa Fe landscape. The worst-case wildfire with an optimally located ignition point has much larger impact on the landscape than the sample random wildfire (see Figure 2.4).

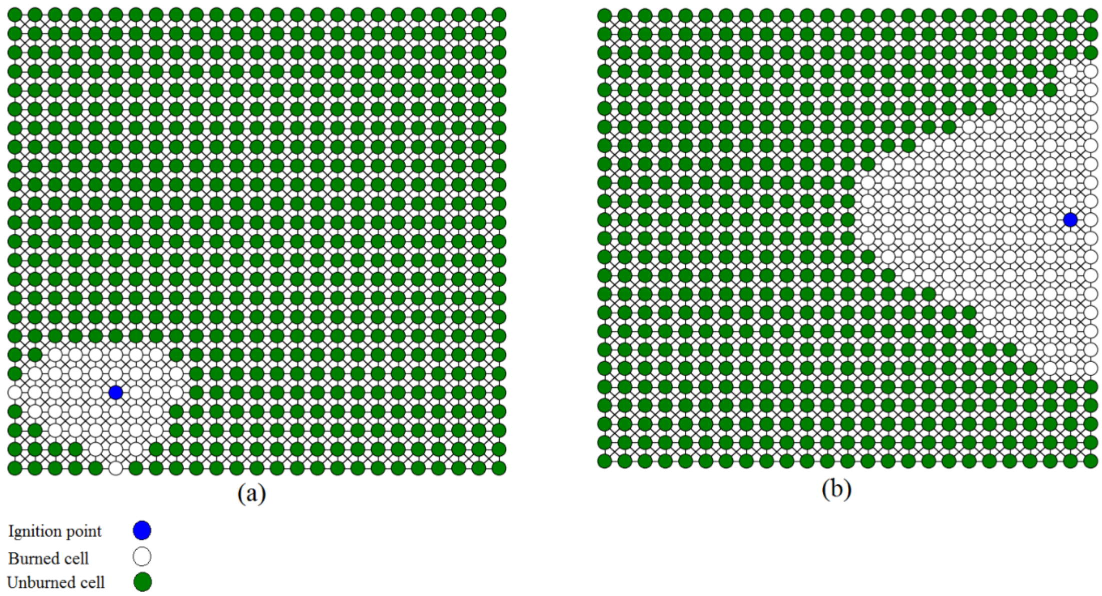


Figure 2.4 Fire footprint example

Fire footprint after 24 hours for the Santa Fe National Forest landscape for
(a) a sample random wildfire with single ignition point,
(b) the worst-case wildfire with single ignition point

To compare these wildfires, we conduct a series of experiments by which we also test the effect of other parameters. In the first set of experiments we assume cells have the same value across all the landscapes. We compute the impact of wildfires as percentages of landscapes burned. Through these experiments, we can see the impact of wildfires on different landscapes as well. In the second set of experiments, we only focus on the largest landscape and test the effect of wind speed on wildfires impact. In the last set of experiments, we assume that part of the landscape is occupied by WUI, and, therefore, not all cells have equal value. In this experiment, we test the impact of worst-case scenario wildfires in presence of WUI.

To calculate the impact of wildfires with optimally located ignition points, we solve the MCWVA model for the three landscape cases. We implement the model formulation using Python 2.7 and solve it with Gurobi 6.0 [64]. All tests are performed on a computer with Intel Core i5 2520M processor at 2.5 GHz and 8 GB RAM. By solving the model to optimality, it gives us the optimal location(s) of ignition point(s) for a wildfire with the maximum damage it can cause.

In all of the following experiments, we compare the two wildfire cases (random wildfires and worst-case wildfires) for different number of ignition point scenarios, by systematically increasing the number of ignition points from one to five. To calculate the impact of random wildfires, in which the ignition points are randomly located, we compute the average impact of wildfires, for all scenarios of ignition locations, for one and two ignition points. However, for three and more ignition points, computing the average impact of wildfires requires tremendous computational effort. For example, for a three ignition point scenario, we would need to compute the average impact of wildfires for C_3^{625} scenarios (number of 3-combination from a set with 625 elements), which entails more than 40 million scenarios for the first two landscapes, and more than 300 million scenarios for the third landscape (C_3^{1225}). Therefore, we use Monte Carlo simulation for 3, 4, and 5 ignition point scenarios. We take a random sample of 5,000 possible ignition location scenarios, and after finding the average and standard deviation of the impact of wildfires for each case, we build 95% confidence intervals for comparison. The experiments are discussed in the following sections.

2.3.2 The impact of wildfires on different landscapes

In this section, we run a set of experiments on the three landscape case studies to investigate the impact of two cases of wildfires, random wildfires, and worst-case wildfires. We compute the impacts of these wildfires under three fire duration scenarios, 12, 18 and 24 hours. For random wildfires, we compute the average impact, and the 95% confidence intervals for 5,000 randomly selected Monte Carlo samples. We assume that all cells are homogeneous and have equal values (V_r=1.. $\forall\, r \in C$). Thus, the impacts of wildfires can be presented as the percentages of the landscape burned. Table 2.3 shows the percentages of each landscape burned by worst-case wildfires with X number of ignition points (represented by WCWF(X)), and the average percentages of landscapes burned by random wildfires with X number of ignition points (represented by RWF(X)). The 95% confidence intervals for random wildfires are presented in Table 2.4

Table 2.3 The percentages of landscape burned

Fire duration	Landscape name	WCWF (1)	RWF (1)	WCWF (2)	RWF (2)	WCWF (3)	RWF (3)	WCWF (4)	RWF (4)	WCWF (5)	RWF (5)
12 hours	Santa Fe	7.84	2.72	14.72	5.28	20.96	8.00	25.92	10.56	30.56	12.96
	Umpqua	8.96	2.40	16.64	4.80	22.24	7.20	27.20	9.44	32.16	11.68
	San Bernardino	11.84	4.73	20.24	9.31	28.24	13.63	36.00	17.63	42.69	21.47
18 hours	Santa Fe	13.92	5.12	23.84	11.36	33.28	16.48	42.40	21.28	49.44	25.76
	Umpqua	17.60	5.28	28.16	10.24	36.48	15.04	44.32	19.36	52.00	23.52
	San Bernardino	20.73	10.20	36.33	19.27	49.71	27.27	61.63	34.45	72.90	40.82
24 hours	Santa Fe	21.12	9.92	35.84	18.88	48.32	26.88	59.20	33.92	68.80	40.00
	Umpqua	25.60	9.28	38.56	17.44	50.08	24.80	60.64	31.36	69.76	36.80
	San Bernardino	31.84	17.06	55.43	30.94	76.49	87.02	42.12	51.51	93.71	59.02
Average		17.72	17.72	7.41	29.97	14.20	40.64	20.16	49.37	25.50	56.89

The percentages of study landscapes burned with the worst-case wildfires with X number of ignition points (represented by WCWF(X)), and the average percentages of landscapes burned by random wildfires with X number of ignition points (represented by RWF(X)) for different numbers of ignition points and under different fire duration scenarios.

Table 2.4 The 95% confidence interval

Duration	Landscape name	RWF(1)		RWF(2)		RWF(3)		RWF(4)		RWF(5)	
		LB	UB	LB	UB	LB	UB	LB	UB	LB	UB
12 hours	Santa Fe	2.61	2.83	5.23	5.33	7.94	8.06	10.49	10.63	12.88	13.04
	Umpqua	2.26	2.54	4.73	4.87	7.12	7.28	9.35	9.63	11.58	11.87
	San Bernardino	4.42	4.66	9.23	9.38	13.54	13.37	17.53	17.74	21.35	21.59
18 hours	Santa Fe	4.90	5.34	11.26	11.46	16.36	16.60	21.15	21.41	25.63	25.89
	Umpqua	5.00	5.56	10.11	10.37	14.90	15.18	19.21	19.51	23.36	23.68
	San Bernardino	10.02	10.39	19.15	19.38	27.09	27.44	34.26	34.64	40.62	41.02
24 hours	Santa Fe	9.57	10.27	18.72	19.04	26.71	27.05	33.74	34.10	39.81	40.19
	Umpqua	8.84	9.72	17.25	17.63	24.60	25.00	31.15	31.57	36.60	37.00
	San Bernardino	16.81	17.31	30.79	31.09	41.87	42.38	51.24	51.78	58.75	59.29
Average		7.16	7.63	14.05	14.28	20.01	20.30	25.35	25.66	30.06	30.39

The 95% confidence interval for percentages of landscapes burned by RWF wildfires for different number of ignition points, and fire duration scenarios.

Table 2.5 The ratios of percentages of landscapes burned

Duration	Landscape name	$\frac{WCWF(1)}{RWF(1)}$	$\frac{WCWF(2)}{RWF(2)}$	$\frac{WCWF(3)}{RWF(3)}$	$\frac{WCWF(4)}{RWF(4)}$	$\frac{WCWF(5)}{RWF(5)}$	Average
12 hours	Santa Fe	2.88	2.79	2.62	2.45	2.36	2.62
	Umpqua	3.73	3.47	3.09	2.88	2.75	3.18
	San Bernardino	2.50	2.17	2.07	2.04	1.99	2.16
18 hours	Santa Fe	2.72	2.10	2.02	1.99	1.92	2.15
	Umpqua	3.33	2.75	2.43	2.29	2.21	2.60
	San Bernardino	2.03	1.89	1.82	1.79	1.79	1.86
24 hours	Santa Fe	2.13	1.90	1.80	1.75	1.72	1.86
	Umpqua	2.76	2.21	2.02	1.93	1.90	2.16
	San Bernardino	1.87	1.79	1.82	1.69	1.59	1.75
Average		2.66	2.34	2.19	2.09	2.02	2.26

The ratios of percentages of landscapes burned with the worst-case wildfires with X number of ignition points (represented by WCWF(X)), and the average percentages of landscapes burned by random wildfires with X number of ignition points (represented by RWF(X)) for different numbers of ignition points and under different fire duration scenarios.

Using the confidence intervals, we can see whether there is a significant difference between the impact of random wildfires and the worst-case wildfires on each landscape. For the three landscape cases, the differences between the average impacts of random wildfires (based on the number of ignition points) are statistically significant at 95% significance level (none of the computed confidence intervals overlap, see Table 2.4). For wildfires with the same number of ignition points and under the same fire duration scenario, the differences between the impacts of the worst-case wildfires and the average impacts of random wildfires on each landscape case are statistically significant at 95% significance level. For wildfires with the same number of ignition points, the worst-case wildfires cause more than twice the damage than random wildfires (Figure 2.5). This difference is marked for wildfires with only one ignition point; the WCWF(1) causes approximately three times more damage to the landscapes than RWF(1), when wildfire last for 12 hours. When the number of ignition points increases, the difference between the two wildfire cases decreases slightly (Figure 2.5). The worst-case wildfires over random wildfires ratio goes from 2.66 for wildfires with one ignition point to 2.02 for wildfires with five ignition points.

The worst-case wildfires have higher impacts on the landscapes than random wildfires (Figure 2.5, Figure 2.7 and Figure 2.8). Wildfires have different impacts on different landscapes. The worst-case wildfires and random wildfires both have higher impact on the San Bernardino case landscape than the other two landscape cases (Figure 2.5, Figure 2.7 and Figure 2.8). Also, the difference between the impact of the worst-case wildfires and the average impact of random wildfires is greater for the Umpqua landscape case than the San Bernardino landscape case (0). These differences are likely due to landscape characteristics which impact the rate of spread and major fire spread direction.

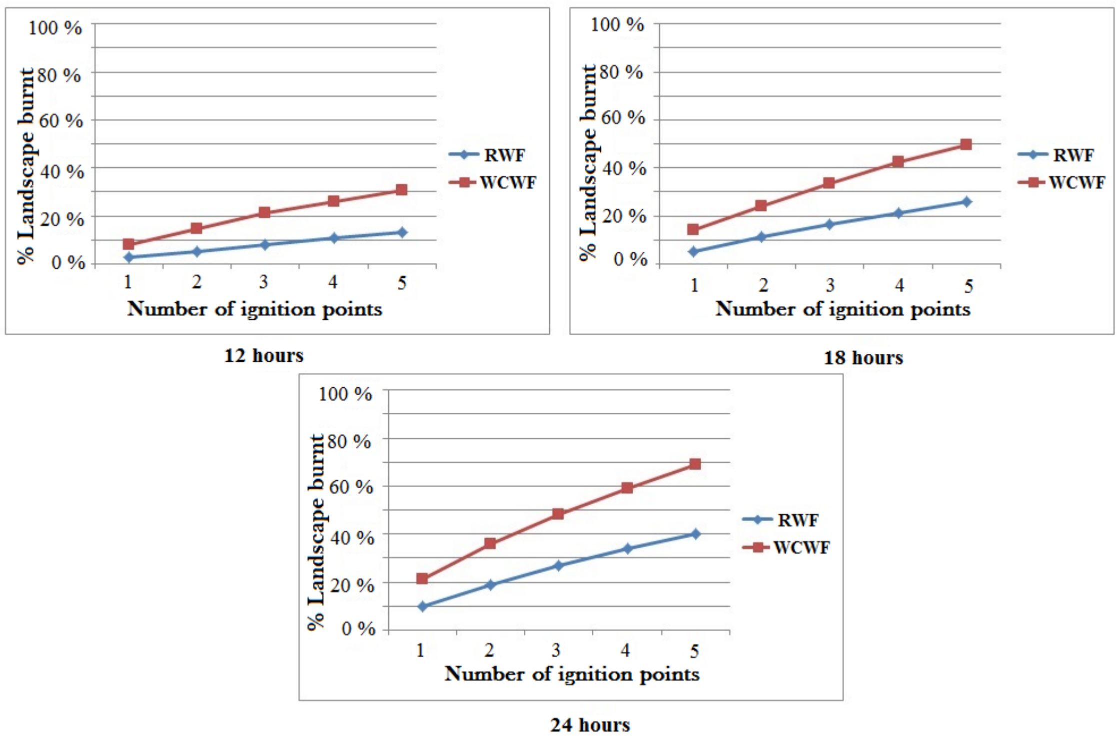


Figure 2.5	The percentage of Santa Fe landscape burned

The percentage of Santa Fe National Forest landscape case burned with random wildfires (represented by RWF) and the worst-case wildfires (represented by WCWF) under different number of ignition points; in which fire lasts: (a) 12 hours, (b) 18 hours, and (c) 24 hours

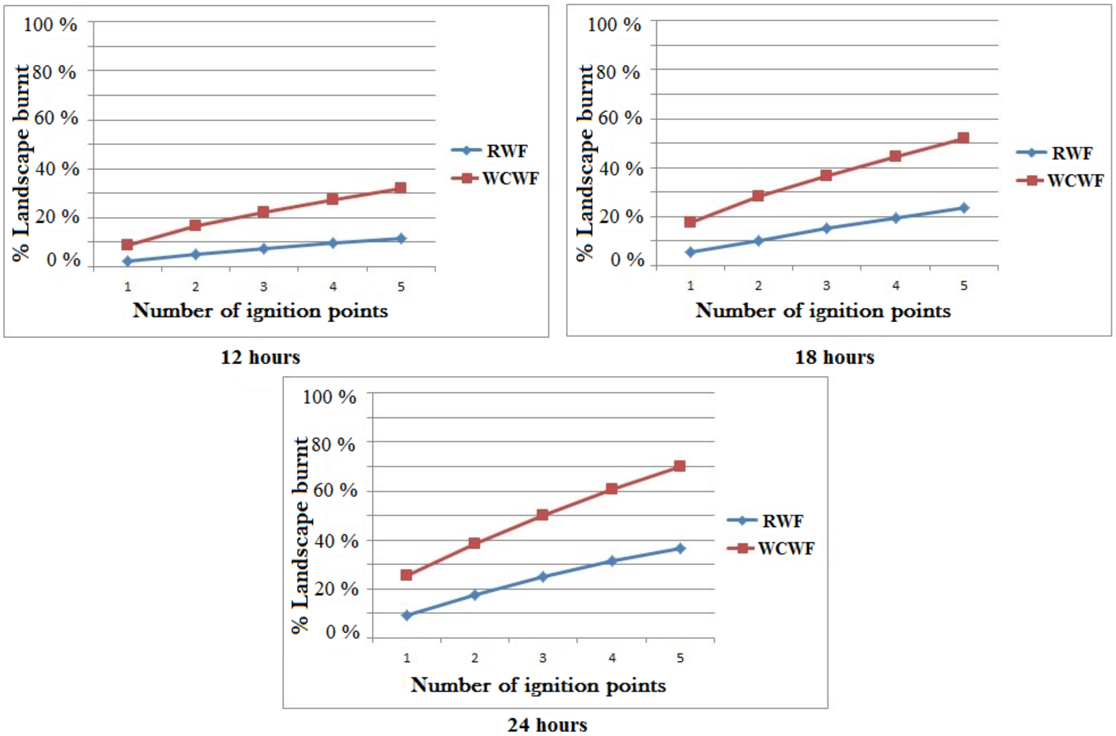


Figure 2.6 The percentage of Umpqua landscape burned

The percentage of Umpqua National Forest landscape case burned with random wildfires (represented by RWF) and the worst-case wildfires (represented by WCWF) under different number of ignition points; in which fire lasts: (a) 12 hours, (b) 18 hours, and (c) 24 hours

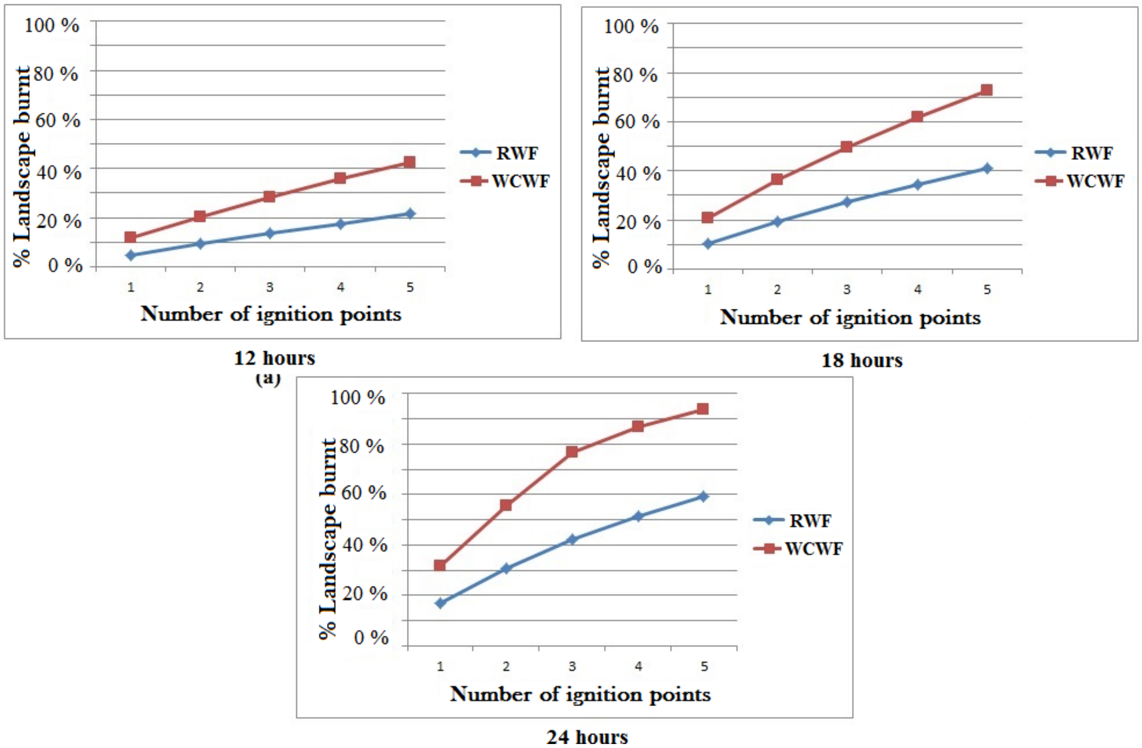


Figure 2.7 The percentage of San Bernardino landscape burned

The percentage of San Bernardino National Forest landscape case burned with random wildfires (represented by RWF) and the worst-case wildfires (represented by WCWF) under different number of ignition points; in which fire lasts: (a) 12 hours, (b) 18 hours, and (c) 24 hours

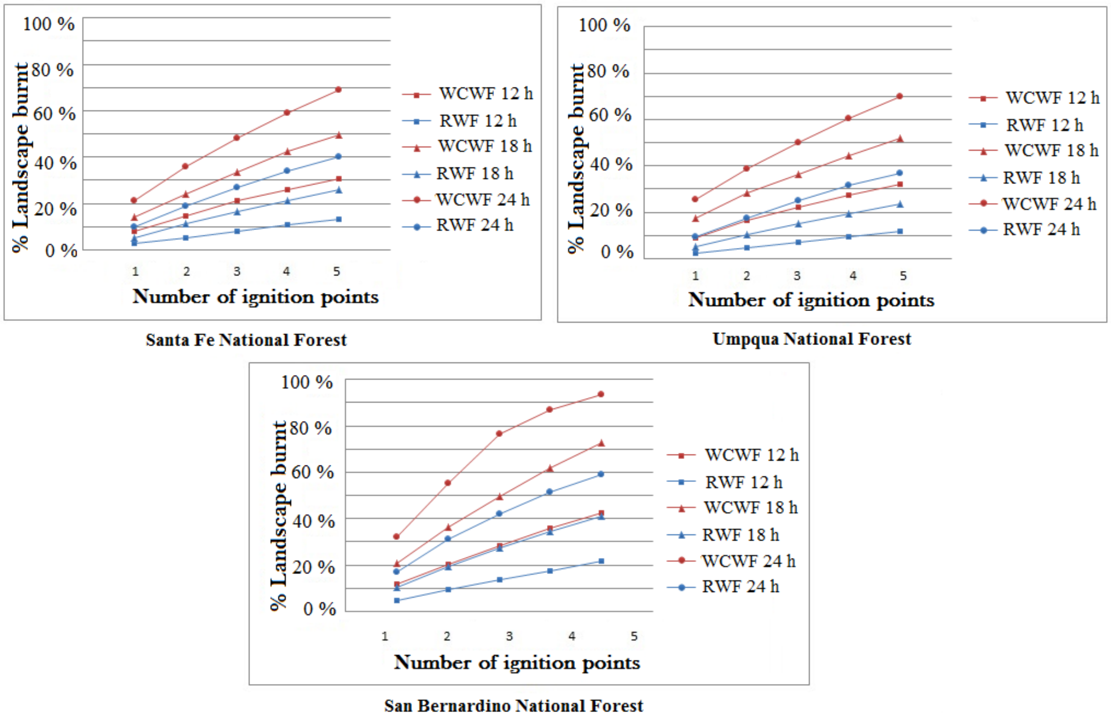


Figure 2.8 The percentage of three landscape burned

The percentage of the three landscape cases burned with random wildfires (represented by RWF) and the worst-case wildfires (represented by WCWF) under different number of ignition points, and different fire duration scenario

The worst-case wildfires and random wildfires both cause more damage on landscapes when fires last longer; however, the worst-case wildfires on average spread faster and cause more damage over shorter times than random wildfires cause over longer times (Figure 2.8). For example, the impact of the worst-case wildfires over 12 hours and 18 hours are respectively more than the impact of random wildfires over 18 hours and 24 hours.

2.3.3 The impact of wildfires under different wind speed scenarios

In addition to landscape characteristics, wind speed also has a major impact on fire behavior [65]. In the previous set of experiments, we assumed the same wind speed conditions for all three landscape cases. In this section, we test the impact of wildfires under three different wind speed scenarios. By doing so, we can obtain a more robust conclusion about the effect of ignition locations on the impact of wildfires on landscapes. For this reason, we run a set of experiments on the San Bernardino landscape case (the largest landscape with 35 × 35 cells) to investigate the impact of wildfires under three different wind speed scenarios: 8 , 12 and 16 *mph* (12.87 , 19.31 , 25.75 *kph* respectively). As we discussed before, of the three cases, the San Bernardino case has the least difference between worst-case wildfires and random wildfires (we pick the weakest case for this experiment). The results show that for higher speed winds, wildfires cause more damage; the higher the wind speed, the more damage the wildfires cause (Table 2.6 and Figure 2.9). In this experiment, under different wind speed scenarios, the worst-case wildfires still have a greater impact on the landscape than random wildfires (Table 2.6 and Table 2.8; for 95% confidence intervals for random wildfires see Table 2.7).

Table 2.6 The impact of wind speed, the percentages burned

Fire duration	Wind speed	WCWF (1)	RWF (1)	WCWF (2)	RWF (2)	WCWF (3)	RWF (3)	WCWF (4)	RWF (4)	WCWF (5)	RWF (5)
12 hours	8	11.67	3.92	20.08	7.67	26.29	11.18	32.00	14.61	37.47	17.88
	12	11.84	4.73	20.24	9.31	28.24	13.63	36.00	17.63	42.69	21.47
	16	13.06	5.88	23.67	11.51	33.71	16.65	43.18	21.47	50.53	26.04
18 hours	8	20.24	8.33	32.73	15.84	43.27	27.61	53.71	28.73	62.53	34.29
	12	20.73	10.20	36.33	19.27	49.71	27.27	61.63	34.45	72.90	40.82
	16	21.63	12.57	42.29	23.43	60.82	32.73	74.04	40.90	84.49	47.92
24 hours	8	31.18	13.96	48.24	25.63	62.37	35.43	72.65	43.59	83.67	50.69
	12	31.84	17.06	55.43	30.94	76.49	87.02	42.12	51.51	93.71	59.02
	16	35.10	20.82	62.61	36.90	84.33	49.39	94.12	59.27	96.65	66.86
Average		21.92	10.83	37.96	20.06	51.69	33.43	56.61	34.68	69.40	40.55

The percentages of the San Bernardino landscape burned with the worst-case wildfires with X number of ignition points (represented by WCWF(X)), and the average percentages of landscapes burned by random wildfires with X number of ignition points (represented by RWF(X)) for different numbers of ignition points and under different fire duration and wind speed scenarios.

Table 2.7 The impact of wind speed, the 95% confidence interval

Fire duration	Landscape name	RWF(1)		RWF(2)		RWF(3)		RWF(4)		RWF(5)	
		LB	UB	LB	UB	LB	UB	LB	UB	LB	UB
12 hours	Santa Fe	2.61	2.83	5.23	5.33	7.94	8.06	10.49	10.63	12.88	13.04
	Umpqua	2.26	2.54	4.73	4.87	7.12	7.28	9.35	9.63	11.58	11.87
	San Bernardino	4.42	4.66	9.23	9.38	13.54	13.37	17.53	17.74	21.35	21.59
18 hours	Santa Fe	4.90	5.34	11.26	11.46	16.36	16.60	21.15	21.41	25.63	25.89
	Umpqua	5.00	5.56	10.11	10.37	14.90	15.18	19.21	19.51	23.36	23.68
	San Bernardino	10.02	10.39	19.15	19.38	27.09	27.44	34.26	34.64	40.62	41.02
24 hours	Santa Fe	9.57	10.27	18.72	19.04	26.71	27.05	33.74	34.10	39.81	40.19
	Umpqua	8.84	9.72	17.25	17.63	24.60	25.00	31.15	31.57	36.60	37.00
	San Bernardino	16.81	17.31	30.79	31.09	41.87	42.38	51.24	51.78	58.75	59.29
Average		7.16	7.63	14.05	14.28	20.01	20.30	25.35	25.66	30.06	30.39

The 95% confidence interval for percentages of San Bernardino landscape burned by RWF wildfires for different number of ignition points, and under different fire duration and wind speed scenarios.

Table 2.8 The impact of wind speed, ratios

Duration	Wind speed	$\frac{WCWF(1)}{RWF(1)}$	$\frac{WCWF(2)}{RWF(2)}$	$\frac{WCWF(3)}{RWF(3)}$	$\frac{WCWF(4)}{RWF(4)}$	$\frac{WCWF(5)}{RWF(5)}$	Average
12 hours	8	2.98	2.62	2.35	2.19	2.10	2.45
	12	2.50	2.17	2.07	2.04	1.99	2.16
	16	2.22	2.06	2.02	2.01	1.94	2.05
18 hours	8	2.43	2.07	1.91	1.87	1.82	2.02
	12	2.03	1.89	1.82	1.79	1.79	1.86
	16	1.72	1.80	1.86	1.81	1.76	1.79
24 hours	8	2.23	1.88	1.76	1.67	1.65	1.84
	12	1.87	1.79	1.82	1.69	1.59	1.75
	16	1.69	1.70	1.71	1.59	1.45	1.62
Average		2.19	2.00	1.93	1.85	1.79	1.95

The ratios of percentages of the San Bernardino landscape burned with the worst-case wildfires with X number of ignition points (represented by WCWF(X)), and the average percentages of landscapes burned by random wildfires with X number of ignition points (represented by RWF(X)) for different numbers of ignition points and under different fire duration and wind speed scenarios.

For wildfires with the same number of ignition points, and for the same fire duration scenario, the worst-case wildfires under low wind speed condition have higher impact on the landscape than random wildfires under higher wind speed condition (Figure 2.9). For example, the worst-case wildfires with the 8 *mph* wind condition have higher impact on the landscape than random wildfires with the 16 *mph* wind condition.

For wildfires with one and two ignition points, the impact of worst-case wildfires is on average twice the impact of random wildfires (Table 2.8). This difference decreases as the number of ignition points and the fire duration increases.

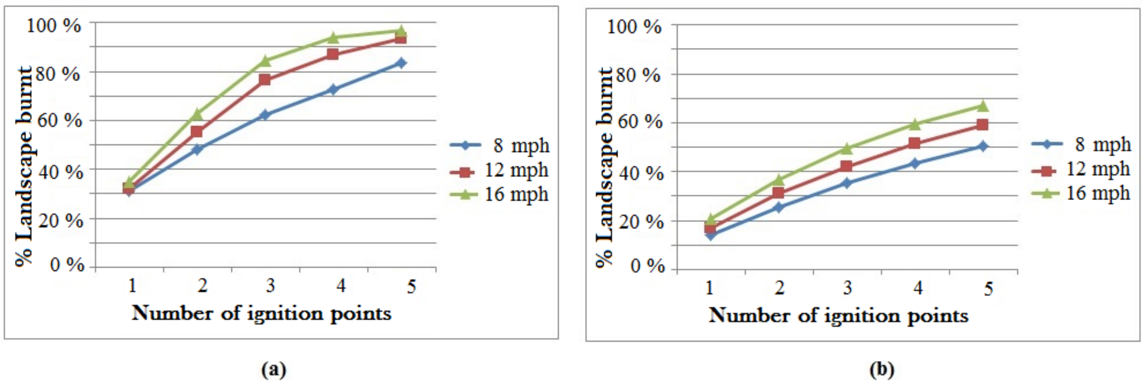
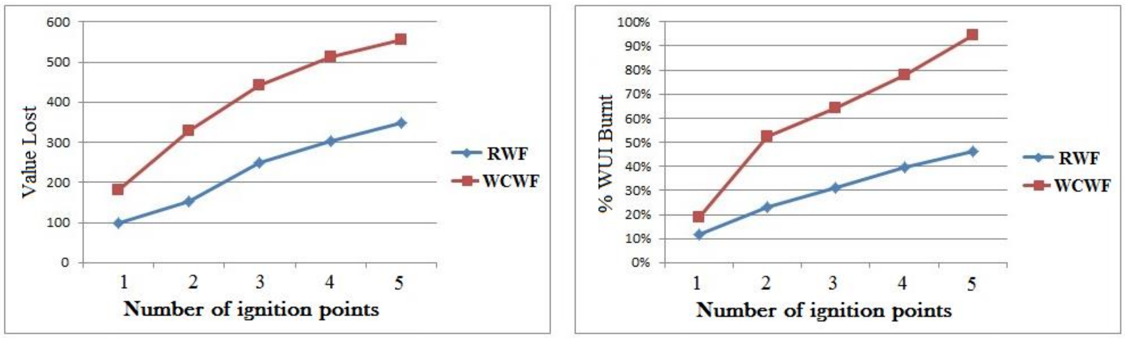


Figure 2.9 The impact of wind speed, San Bernardino case

The percentages of the San Bernardino landscape case burned with: (a) the worst-case wildfires, and (b) random wildfires; for different number of ignition points when wildfires last for 24 hours.

2.3.4 The impact of wildfires in presence of wildland-urban interface

To investigate the impact of wildfires on landscapes in the presence of WUI, we run another set of experiments on San Bernardino landscape (the largest landscape with 35 by 35 cells). In this set of experiments, we assume that about ten percent of the landscape contains intermix WUI. In intermix WUI, as opposed to interface WUI, houses mingle with wildland fuels [2], allowing the cells containing WUI to be ignitable points. To address WUI losses due to wildfires, we include the value of each cell in the model. By doing so, we can also address cases where cells have different values depending on the vegetation type. In this experiment, WUI locations are distributed arbitrarily through the landscape. To set a value for each cell in the corresponding network, we assume a non-WUI cell has a value of 0.4, the same value that Wei [31] uses for non-commercial timber forest. As it is difficult to estimate the damage to a WUI cell, including damage to properties and human life, we follow Wei [31] and use a value of 1.4 for cells containing WUI (and non-commercial timber). These values are unit-less. However, the RAVAR [66] resource evaluation method along with the real locations of WUI and vegetation types can be used to assign a value to each cell. We assume that all wildfires burn for 24 hours. The objective of the mathematical optimization model is to locate the ignition points of a wildfire that causes the maximum damage. Therefore, we expect the model to locate the ignition points adjacent to cells with higher values (WUI cells), and thus the resulting worst-case wildfire causes more damage to WUI cells than random wildfires causes. Figure 2.10 (a) shows the value lost due to wildfires that last for 24 hours considering different numbers of ignition points, and Figure 2.10 (b) shows the percentage of WUI cells that are burned by the two types of wildfires, the worst-case wildfires and random wildfires. As expected, the worst-case wildfires still have higher impact on the landscape and pose more risk (more than two times on average) to WUI than random wildfires (Figure 2.10 (b)).



(a) (b)

Figure 2.10 The impact of wildfires on WUI

(a) Value lost (unit-less) for the San Bernardino case with random wildfires (represented by RWF) and the worst-case wildfires (represented by WCWF) for different ignition point scenarios, (b) The percentage of WUI burnt in the San Bernardino case with random wildfires and the worst-case wildfires for different number of ignition point scenarios when fire last for 24 hours.

2.4 Discussion and conclusions

Wildfires can have serious and long-lasting impacts on ecological, social and economic systems [49]. It is necessary to identify and understand these impacts, and to develop cost effective mitigation strategies accordingly. In this paper, we studied the vulnerability of landscapes to wildfire threats considering the impact of fire ignition locations – the worst case scenario. We compared the impacts of wildfires with optimally located ignition points, the worst-case wildfires, with the impacts of wildfires with randomly located ignition points, random wildfires. We used FlamMap to model fire behavior using landscape data, wind condition and fuel moisture data, and developed a mixed integer programming model to find the maximum impact of wildfires and their optimal ignition locations. Three landscape cases were used for experimentation and the impacts of various factors such as the number and location of ignition points, fire durations, and wind speeds were investigated. The proposed model is compact, and yet it can incorporate a variety of features such as the presence of fuel breaks and unburnable cells, and fire duration distribution.

The major contribution of this work is the development of a compact model for assessing the vulnerability of landscapes to wildfires regarding the location and number of ignition points – the worst case scenario. The model can be used for assessing the vulnerability of landscapes to arson-induced wildfires, for identifying high vulnerability areas in a landscape. This is especially important for wildfire management and mitigation planning. Thus far, no other research has attempted to provide such assessment. Our results show that the worst-case wildfires cause more damage (more than two times on average) to the landscapes than random wildfires. This is also true when WUI exists in the landscape. The worst-case wildfires cause more than two times, on average, damage to WUI lands than random wildfires. Although higher wind speed can exacerbate the impact of wildfires [67], our study shows that even under low wind speed condition, the worst-case wildfires have higher impact on landscapes than random wildfires would have under high wind speed condition. The worst-case wildfires spread faster and cause more damage in shorter period of time than random wildfires can cause in longer period of time. Within 12 hours, a worst-case wildfire with one ignition point can cause three times, on average, more damage to a landscape than a random wildfire with one ignition point. This indicates the need for a faster response to the worst-case wildfires than random wildfires would require. Thus, controlling the worst-case wildfires would require a faster and larger initial attack than random wildfires would need.

For arson-induced wildfire cases, it is not only the location of ignition points that can be determined, but the number of ignition points can also be determined. Therefore, arson-induced wildfires can have more ignition points (multiple fires) than natural wildfires; which makes arson-induced wildfires more catastrophic and difficult to suppress. Our results indicate that the worst-case wildfires with five ignition points respectively cause 7 and 4 times more damage to a landscape than random wildfires with one and two ignition points (Table 2.3). This difference can grow even larger if more ignition points are chosen in an arson-induced wildfire, which makes arson-induced wildfires even more catastrophic. Thus, the resources currently used for mitigating and suppressing natural wildfires are probably insufficient for controlling a potential arson-induced wildfire.

As illustrated in this research, the worst-case wildfires have different impacts on different landscapes. This is likely due to differences in landscapes and vegetation characteristics that influence rate of spread, and major fire spread direction; which makes a landscape more vulnerable to arson-induced wildfires. Therefore, our model can be used to assess the vulnerability of a particular landscape to these wildfires. The model can identify high priority areas for wildfire risk mitigation planning such as fuels treatment scheduling and fire suppression preparedness planning to reduce the spread and intensity of the potential worst-case wildfires. Kim and Bettinger [68] illustrated that a fuels management program across a broad landscape may have limited impact on human-caused wildfires. We suspect this is also true for arson-induced wildfires. In a fuels management program planned for mitigating arson-induced wildfires, the high priority areas should be prioritized for fuels treatments. This can reduce the vulnerability of landscapes, and mitigate the impact of these wildfires. The same prioritization is also suggested for suppression preparedness planning. However, a more extensive analysis may be required for investigating the merits of these plans.

There can be extensions to this research for future studies that are not addressed in this paper. We investigated the impact of wildfires based on how long they last before suppressed; assuming that the suppression efforts can successfully control wildfires. One might investigate the impact of wildfires while also taking fire response into account, knowing how many resources and fire-response crews are available at various points in the landscape. This can be especially helpful in assessing the risk of arson attacks in which adversaries are aware of fire response resources and their locations, so that they can plan accordingly. This research can be further extended to study the mitigation of potential arson attacks with fuels management. Although prioritizing high vulnerable areas of a landscape for fuels management is suggested, it is not the optimal approach. Since the fuels management program is visible to arsonists, they can act accordingly by attacking other vulnerable areas. In that case, a network interdiction approach [69] might be more effective. Another extension to this work is to consider the interaction effects of multiple fires, which have been assumed negligible in this research. Fire behavior and characteristics can dramatically change in the presence of another fire [70], and, therefore, they can cause more damage than it is shown in this research. Therefore, one can also take the fire interaction effects into account. We also did not include spot fires in this study; they can increase wildfires risks by helping them spread faster [71]. For more accurate assessment, a study can include spot fires into account as well.

In this research we have developed a mathematical programming model to the combinatorially complex problem of landscape vulnerability assessments to arson-induced wildfires (worst-case wildfires). Our hope is that this study can fill the gap in the literature, and assist landscape and wildfire managers in developing a fire management system resilient to potential arson-induced wildfire threats.

CHAPTER III

MITIGATING A PYRO-TERROR ATTACK USING FUEL MANAGEMENT

3.1 Introduction

In this paper, we study the mitigation of a potential pyro-terror attack using fuel management. Fuel management is used to reduce the flammability of a landscape and decrease the risk of wildfires. Wildfire managers use fuel management to reduce the spread rate and intensity of wildfires and therefore mitigate their impacts. Our goal is to plan a fuel management on a landscape that minimizes the impact of a possible pyro-terror attack.

To reduce the flammability of a landscape and decrease the risk of wildfires, fire managers use fuel management programs. Fuel management is the process of altering the amount and structure of fuel through the construction of fuel breaks or applications of fuel treatments such as prescribed burning, commercial harvesting and mechanical thinning, to reduce the spread and intensity of wildfires before they occur.

Over the last ten years, there has been an average of 75,000 wildfires per year and an average of 7.2 million acres have burned in the U.S. [1]. The U.S. Forest Service spends billions of dollars annually for wildfire suppression [2]. Moreover, wildfires also incur tremendous environmental and socioeconomic costs as well as the loss of human life. In particular, destructive wildfires become a primary concern in places where major cities are located close to highly flammable vegetation areas such as in the western and southern U.S. along with Australia and Mediterranean Europe [3]. Due to the significant short- and long-term threats of wildfires to forest ecosystems, and due to public safety and property concerns, wildfires have been receiving increased public attention [72].

Wildfires can be categorized into two general categories: natural wildfires, and arson-induced wildfires. Although arson-induced wildfires (which are mostly unintentional) occur more often than natural wildfires, natural wildfires are more likely to escape containment and become severe [34]. The destructive power of wildfires makes them a viable option for adversaries as in pyro-terrorism. Pyro-terrorism is the use of large-scale arson attacks by non-state organizations to terrorize, intimidate or coerce a government or the civilian population in order to advance political or social objectives [13]. According to [12], pyro-terrorism possesses the four generally accepted elements of terrorism: targeting of noncombatants, political motivation, violence with psychological impact, and organized perpetrators. Previous studies of pyro-terrorism have demonstrated that it is a realistic threat [11,12,14]. Pyro-terrorism events have been documented in France, Spain, and Greece [11,12]. As a result, both the Department of Homeland Security (DHS) and the Federal Bureau of Investigations (FBI) are concerned about this novel threat [15,16]. It is important for decision makers in these government agencies to anticipate potential threats and implement countermeasures to avoid a potentially devastating domestic attack. However, no previous study has investigated how to mitigate the threat of pyro-terrorism. In this study we investigate how to mitigate pyro-terrorism using fuel treatment, a popular approach for mitigating natural wildfires.

Wildfire incidences require the co-occurrence of three factors: fire-conducive-weather, a source of ignition, and fuel (i.e. flammable vegetation) [17]. In pyro-terrorism, the arsonist(s) can facilitate this process by providing a source of ignition; the location, time, and quantity of fires are decisions for the arsonist(s) to make. Because of this ability to optimally choose wildfire conditions, pyro-terrorism can be a more destructive threat than natural wildfires. Rashidi et al. [45] conducted a vulnerability assessment of landscapes to the worst-case wildfires, finding that a pyro-terror attack with a single fire could be twice as destructive as natural wildfire.

Fuel, weather conditions and topography of a landscape are the three factors that influence fire behavior, and fuel is the only factor that can be managed in the short run [10]. To reduce the flammability of a landscape and decrease the risk of wildfires, fire managers use fuel management programs. Fuel management is the process of altering the amount and structure of fuel through the construction of fuel breaks or applications of fuel treatments such as prescribed burning, commercial harvesting and mechanical thinning, to reduce the spread and intensity of wildfires before they occur [23,24]. Modeling methods have been used to design efficient fuel management programs over a landscape. Researchers have used heuristic methods [25–28,53] and optimization models such as mixed integer programming [3,29–31], and stochastic dynamic programming [32,33] for spatial allocation of fuel management over a landscape. However, all of these fuel management models have been developed for reducing the spread and resulting impact of natural wildfires. Thus, more understanding is needed of how effective fuel management is at mitigating pyro-terror attacks and worst-case wildfires.

Although there is a rich literature on using fuel management programs to mitigate natural wildfires, no previous study has investigated mitigation of the worst-case wildfires. Human-caused wildfires account for a large majority of all wildfire incidences.

In the Mediterranean region and in Southern California, human-caused wildfires account for more than 95% of all fires [73–75]. A study in Spain found that more than 71% of all wildfires are caused by people [76]. Of those human-caused wildfires, only 22.5% (16% of all wildfires) were due to negligence while 77.5% (55% of all wildfires) were intentional [76]. Pyro-terrorism can be considered a worst-case arson-induced wildfire. Some studies have shown that arson-induced wildfires cannot be mitigated effectively with fuel management programs designed for mitigating natural wildfires [27]. In this paper, we investigate the mitigation of pyro-terrorism using a constrained fuel management program.

In this paper, we use a network interdiction model for planning a fuel management program that mitigates the impact of a single-ignition-point pyro-terror attack. However, the results of this study can also be used for worst-case wildfires regardless of the cause of the wildfire. In the worst-case wildfire the ignition points are placed at the worst possible locations in a landscape such that it results in a wildfire that causes the maximum damage to the landscape. We model a natural landscape as a grid network and model the spread of fire in the landscape as a network optimization problem. We assume that fire uses paths with the minimum travel time (i.e. shortest path) to spread through the network. For this reason, we use the minimum travel time algorithm (MTT) to model fire growth in the landscape [77]. MTT has also been used in wildfire simulation models such as FlamMap [56], FsPro [78], and FSim [79]. When a wildfire starts in a cell in the network, it uses its adjacent cell to spread through the network and reach other cells. Therefore, the spread of wildfire in the network within a given time limit (for example d hours) can be modeled as a one-to-all shortest path problem. For any given cell in the network, if the length of the shortest path from the fire ignition point to that cell is less than *d*, then we assume that the cell can be reached and burned by the fire. This process can equivalently be viewed as computing the "cover" of a wildfire given that fire uses the shortest paths to spread. This methodology gives a basis for interdicting the spread of fire in the network through interdicting the wildfire spread paths (i.e. shortest paths) using fuel management. Given that we know the location of fire ignition and that we are constrained on the number of interdictions (*b* is the maximum number of cells that we can interdict), the problem becomes identifying *b* cells to interdict in order to minimize the number of cells that will be reached (burned) by the fire.

Network interdiction models are network-based bilevel optimization programs in which the objective of the upper level model is to impair the objective of the lower level model. Wollmer (1964), McMasters and Mustin (1970), and Ghareh et al. (1971) were the earliest to study network interdiction. Network interdiction has received extensive attention in literature because of its utility in modeling practical applications in homeland security problems such as delaying an adversary's development of a first nuclear weapon [82], securing a border against smuggling nuclear material [83–85], drug enforcement optimization [86]; and other applications such as electrical grid analysis [87–90], preventing hospital infections [91], conflict resolution [92], multicommodity flow networks [93,94], and optimizing the placement of stationary monitors [95].

One of the classic examples of network interdiction is interdicting the shortest path between a source node and a sink node [96,97]. A discrete version of this problem in which the interdicted arcs are removed from the network is called the *k-most-vital-arcs* problem [98–100]. In the *k-most-vital-arcs* problem, the objective is to identify a fixed number of arcs that, if removed, would cause the largest increase in the length of the shortest path between two specified nodes. Israeli and Wood [101] considered a generalization of the *k-most-vital-arcs* problem. Maximizing the shortest path through interdicting nodes also has been studied [99,102]. In all of these cases, the objective is to maximize the length of the shortest path between two given nodes. In our case, however, the objective is to identify a fixed number of nodes (considering a limited budget for interdiction) that, if interdicted, would delay the spread of fire and minimize the number of nodes that can be reached by fire within a given time. This problem is similar to the r-interdiction covering problem which was studied by Church et al. [103]. However, in their problem they only considered interdiction of source nodes (i.e. facilities), not the intermediate nodes that build paths through which demands are met. In this research, however, we also consider the interdiction of intermediate nodes.

The pyro-terrorism mitigation problem can be interpreted as identifying the *b-most-vital-nodes* (*b* is the budget for interdiction) in a one-to-all shortest path problem whose interdiction would minimize the number of nodes reachable from the source node within a critical time limit (i.e. nodes whose shortest path's length from the source node is within a critical value - suppression time *d*). Based on this idea, we develop a network interdiction model for mitigating a pyro-terror attack using fuel management. The model is considered as a Stackelberg leader-follower game [104] in which fire managers, acting first, identify optimal locations for fuel management (with the limited budget *b*), and terrorists, acting second with complete information of the fuel management locations, identify the optimal ignition point for a pyro-terror attack to inflict the maximum damage. Assuming that in *d* hours the adversary-ignited fire can be suppressed, the goal of the fire managers is to delay the spread of fire by spatially allocating fuel management, with limited budget b (number of treated cells), across the landscape to delay the spread of the fire and minimize damage caused by the fire (the number of cells burned in d hours). This work is the first attempt to develop a model for mitigating pyro-terrorism using fuel management. The major contribution of this work is developing a computational method for optimizing the spatial allocation of fuel management for mitigating a pyro-terror attack. Since the model is a min-max model in which the inner level is a mixed-integer programming model, the model cannot be directly solved using any commercial solver; therefore, we utilize a decomposition algorithm to solve the model. The proposed decomposition algorithm alternates between a master problem for fuel managers and a sub problem for pyro-terrorists. The master problem identifies an optimal interdiction strategy for a fixed pyro-terror attack (a known attack). The sub problem chooses an optimal pyro-terror attack that identifies the ignition point of an attack given a fuel management program. The algorithm iteratively solves the two problems generating lower bounds and upper bounds for the problem, until the two bounds converge.

3.2 Pyro-terrorism mitigation problem (PTMP)

The pyro-terrorism mitigation problem is as follows. Wildfire managers choose a fuel treatment plan $\boldsymbol{H}$ (a spatial allocation of fuel treatment) that will mitigate the impact of a potential pyro-terror attack on the landscape. Specifically, if a landscape area is treated, fire cannot spread through that area. Next, the adversary, having seen the fuel treatment applied by the wildfire managers, chooses a pyro-terror attack $\boldsymbol{F}$ by selecting the most vulnerable area in the landscape to start a fire at a single ignition point in order to maximize the total value of acreage burned by the fire.

In this section we develop a bi-level integer program for PTMP. In our model both the wildfire managers' problem and the adversary's problem problems are modeled as network optimization problems. We also model fire behavior in a landscape as a shortest path problem through a composite network formed by the landscape characteristics.

3.2.1 Modeling a landscape as a raster cell

We consider a landscape divided into a number of raster cells representing potential fire ignition locations and candidate locations for fuel management (Figure 3.1), and we use FlamMap [56], a fire behavior mapping and analysis program, to model the rate of fire spread in each cell. FlamMap uses Geographic Information Systems (GIS) data, landscape characteristics, fuel moisture, and wind conditions and computes the rate of spread (ROS) and the major fire spread direction for each cell. These data are then used to model fire spread behavior in each cell. The major fire spread direction in each cell is the direction of the fastest fire spread for that cell. In reality, fire would also spread along other directions but at a slower speed [31]. We use equations (3.1) and (3.2) to calculate ROS along the other directions.

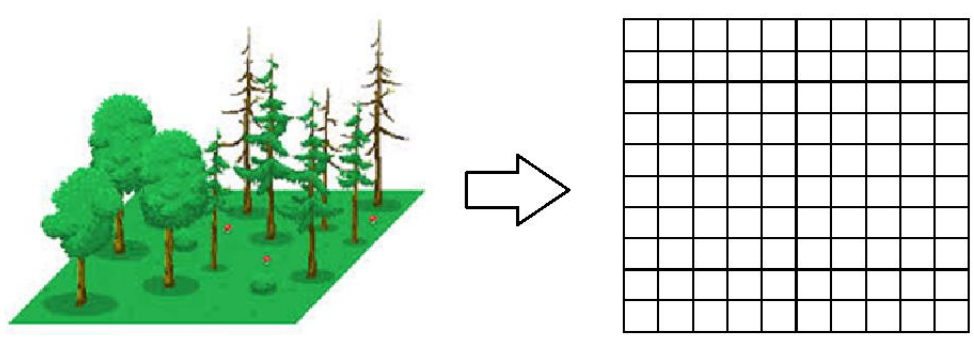


Figure 3.1 A landscape modeled as a 10 by 10 raster cell.

$$R_{q,r} = \frac{b^2 - c^2}{b - c \times cos\theta}, \quad 0 \le \theta < \frac{\pi}{2}, q \in C, r \in N_q \tag{3.1}$$

$$R_{q,r} = \frac{b^2 - c^2}{b + c \times cos(\pi - \theta)}, \quad \frac{\pi}{2} \le \theta < \pi, q \in C, r \in N_q \tag{3.2}$$

θ is the angle between the major fire spread direction in cell q, computed by FlamMap, and the fire spread direction from cell q to the center of the adjacent cell r. The values of b and c are outputs of FlamMap and are standard parameters used to describe the ellipse of fire spread. For a more detailed description, we refer the reader to Green et al. [60].

3.2.2 Problem description and model formulation

In our model the adversary has complete knowledge about weather and the topography of the landscape. They also are aware of the location of all treated cells. Having this complete knowledge, they identify an optimal ignition point in the landscape to ignite a fire with maximum total damage in terms of the value of landscape burned. The wildfire managers act before the adversary and take a proactive approach, identifying the optimal locations for fuel treatment. The wildfire manager's objective is to minimize the total damage of an attack in terms of the value of landscape burned; thus, the two-player game is symmetric. In our model the wildfire manager seeks to mitigate against the attack with the worst possible damage. Thus, the optimal objective value returned by our model is a lower bound on the case in which the adversary does not have complete knowledge.

The damage of a pyro-terror attack can be evaluated as a percent of the landscape burned or the value lost to fire (such as value of vegetation depending on the type, e.g. commercial timber) and the value of wild-land urban interface (WUI) lost. In this research, we measure the damage as the percent of the landscape burned. However, our model can also be directly used for cases in which landscape areas have different values.

The primary assumptions of this research are the following: (1) Fire travels using paths with the minimum travel time to spread through the landscape. (2) When a fire reaches the center of a cell, that cell is completely burned. (3) If fuel treatment is conducted in a cell, it delays the fire from spreading to the adjacent cells; however, it will not prevent this cell from burning. (4) We only focus on the landscape, and ignore the effect of fire on the areas outside the boundary. (5) The fire managers are able to suppress the fire in 24 hours. (6) The pyro-terrorist only starts a wildfire in one cell.

Although a multiple-ignition-point pyro-terror attack is more destructive than a single-ignition-point pyro-terrorism event [45], in this paper we assume the adversary conducts single-ignition-point attack. The single-ignition-point pyro-terror attack is a reasonable assumption due to the fact that although the adversary wants to maximize the damage of his attack, he or she also may wish to avoid detection. Starting multiple man-made fires (i.e. using a multiple-ignition-point pyro-terror attack) increases the likelihood of being seen by authorities or civilians.

The notations used in the model are as follows:

Table 3.1 Notations

Sets and indices	
C	is the set of raster cells in a landscape indexed with r, i and j
N_q	is the set of raster cells adjacent to cell q
Parameters	
$\tau_{i,j}$	the fire spread time from cell i to cell j
v_r	value of cell r
M	a big number which is bigger than the largest shortest travel time path from any point to any other point in the landscape
$\Delta_{q,r}$	the distance between cells r and q
$R_{q,r}$	the rate of spread from q to r
Γ	the delay in fire spread time in a cell caused by treating the cell
b	the fuel management budget
d	the duration of pyro-terrorism wildfire
Variables	
$X_{r,i,j}$	1 if the shortest path for fire passes from cell i to cell j to reach cell r, 0 otherwise (vector $\boldsymbol{X}$)
F_j	1 if the adversary ignites a fire at cell j, otherwise 0 (vector $\boldsymbol{F}$ is the pyro-terror attack)
H_j	1 if cell j is treated, otherwise 0 (vector $\boldsymbol{H}$ is the fuel management program used as an mitigation plan)
$T_{s,r}$	the fire arrival time for cell r when fire has started from cell s
Y_r	1 if fire reaches cell r, otherwise 0 (vector $\boldsymbol{Y}$)
$Y_{s,r}$	1 if fire that is ignited at cell s reaches cell r, otherwise 0 (vector $\boldsymbol{Y}$)

The mathematical formulation for the pyro-terrorism mitigation problem (PTMP) is as follows:

$$PTMP: Z^* = \min_{\boldsymbol{H}\in\Omega}\left(Max_{(\boldsymbol{X},\boldsymbol{F})\in\Psi(\boldsymbol{H})} \textstyle\sum_r v_r Y_r\right) \tag{3.3}$$

Where the set Ω is defined as the set of all $\boldsymbol{H}$ such that

$$\textstyle\sum_i H_i \leq b \tag{3.4}$$

$$H \in \{0,1\}, \quad \forall\ i \in C \tag{3.5}$$

and the set $\Psi(\boldsymbol{H})$ is defined by

$$\sum_{j \in N_i} X_{r,i,j} - \sum_{j \in N_i} X_{r,j,i} = \begin{cases} 1 & i = 0 \\ 0 & i \neq 0, r \\ -1 & i = r \end{cases} \quad \forall\, r \in C, i \in C \cup \{0\} \tag{3.6}$$

$$X_{r,0,j} \leq F_j \quad \forall\, r, j \in C \tag{3.7}$$

$$\sum_{j \in C} F_j \leq 1 \tag{3.8}$$

$$Y_r \leq \frac{d - \sum_{(i,j)} (\tau_{i,j} + H_i \Gamma) X_{r,i,j}}{M} + 1 \quad \forall\, r \in C \tag{3.9}$$

$$Y_i \in \{0, 1\}, \quad \forall\; i \in C \tag{3.10}$$

$$X_{r,j,i} \in [0, 1], \quad \forall\; r \in C, j, i \in C \cup \{0\} \tag{3.11}$$

$$F_j \in \{0, 1\}, \quad \forall\; j \in C \tag{3.12}$$

This is a bilevel optimization model with fire managers' problem in the upper level, and the adversary's problem in the lower level. The objective function (3.3) represents the fire managers' desire to choose a fuel management program $\boldsymbol{H}$ (i.e., a mitigation plan) which limits the fire spread and thereby minimizes the damage caused by the pyro-terror attack; simultaneously, it also represents the adversary's desire to maximize the damage of a pyro-terror attack by choosing the optimal ignition point of fire in the landscape. The fire managers' mitigation plan is restricted by constraints (3.4) and (3.5). Constraint (3.4) is the restriction on the budget for fuel management; it is the number of cells to which fuel management can be applied.

For any specific fuel management program $\boldsymbol{H}$ chosen by the wildfire managers ($\boldsymbol{H}$ should be viewed as data when viewing the adversary's problem) the adversary's pyro-terror plan is restricted to set $\Psi(\boldsymbol{H})$, as defined by constraints (3.6) through (3.12). In addition to choosing the optimal ignition location, the lower level problem identifies paths with the minimum travel time for modeling fire spread in the landscape. The model contains the one-to-all shortest path formulation. Since the starting point of fire is unknown, we use a dummy super source (cell 0) to represent the starting point of fire. The super source is a hypothetical cell and is connected to every cell in the landscape, with zero travel time. For any target cell (a potential cell for the wildfire to reach and burn) in the landscape, one unit fire flow is sent from the super source through the ignition point (cell i) and from there to the target cell, using the shortest path (Figure 3.2)

The set of flow conservative constraints in (3.6) requires that one unit of fire flow is sent from the super source to any cell r. The set of constraints (3.7) ensures that the fire spreads to cell j from the super source only when cell j is selected as the ignition point by the adversary. Constraint (3.8) enforces the assumption that the adversary only starts a wildfire in one cell. The constraints presented in the equation (3.9) are the burn constraints and set the values of the binary variables Y_r. These variables are used to track whether the fire reaches cell r and therefore burned within duration d. Also, the mitigation impact of fuel management is implemented in constraints (3.9). For example, if fuel management is conducted in cell i ($H_i = 1$), through these constraints the fire travel time from cell i to each adjacent cell j increases by Γ. Constraints (3.9) resembles the node-interdiction version of the shortest path network interdiction problem [101].

Fire managers aim to interdict fire growth by lengthening fire travel time, assuming that fire travels along paths with the minimum travel time (i.e., the shortest path) [58]. Constraints (3.6) through (3.9) form a one-to-all shortest path problem and are used to identify paths with the minimum travel time.

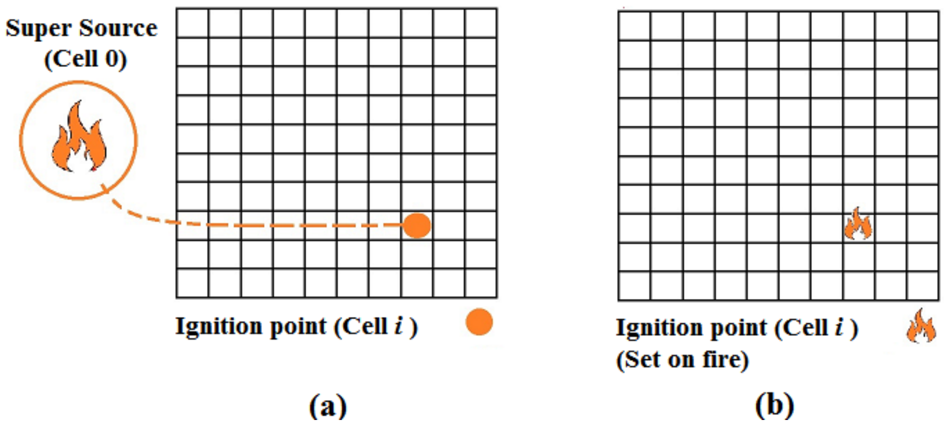


Figure 3.2 Modeling fire spread using shortest paths

A dummy super source (cell 0) is connected to each cell with 0 travel time: (a) A wildfire would start at cell 0; (b) would set fire on the ignition point at time 0 and from there would spread through the landscape using shortest paths.

3.3 A solution approach

The pyro-terrorism mitigation problem (PTMP) has a bilevel "min-max" structure with a mixed integer programming (MIP) problem as the lower level; thus, the lower level problem is not guaranteed a nonzero duality gap. Therefore, the problem does not readily lend itself to the common approach of taking the dual of the inner problem, resulting in a single-level minimization problem. In this section, we present a decomposition algorithm, called Min-Max Decomposition Algorithm ($MinMaxDA$), to solve $PTMP$. $MinMaxDA$, like the decomposition algorithm proposed by Brown et al. [82] alternates between (i) a master problem, where the wildfire managers identify an optimal mitigation strategy through a fuel management program for a fixed pyro-terror attack with the starting point of the fire being known, and (ii) a sub problem, where the pyro-terrorist (adversary) identifies an optimal pyro-terror attack to start the fire for a fixed fuel management program. In $MinMaxDA$, unlike the standard Benders decomposition algorithm, the sub problem is a MIP, not a linear program, and it is not totally unimodular to be relaxed.

The sub problem and master problem of our decomposition algorithm are discussed in the following sections. To present the algorithm we first need to introduce new notations:

Table 3.2 Additional notations

Sets and indices	
ϕ	the set of pyro-terror attack plans (cells considered to start a pyro-terror wildfire) used in the decomposition algorithm
Ω	the set of feasible solutions for the upper level problem (the fire managers' problem)
$\Psi(.)$	the set of feasible solutions for the lower level problem (the pyro-terrorist's problem)

3.3.1 The pyro-terrorists' problem (PTP) for a known fuel management program

For a fixed fuel management program $\widehat{\boldsymbol{H}} \in \Omega$, we denote the resulting pyro-terrorism model for the adversary as $PTP(\widehat{\boldsymbol{H}})$:

$$\text{PTP}(\widehat{\boldsymbol{H}}): Z_{max}(\widehat{\boldsymbol{H}}) = \max \textstyle\sum_r v_r Y_r \qquad (3.13)$$

With constraints (3.6) through (3.12), in which the variable H_j is replaced with $\widehat{H}_j \in \widehat{\boldsymbol{H}}$ for all $j \in C$ (i.e. the locations of the cells that underwent fuel management is known to the adversary)

The PTP model presented above selects the optimum potential cell to start a fire that can cause the maximum damage to the landscape given an observed fuel management program $\widehat{\boldsymbol{H}}$. For any potential ignition point, the model computes the number of cells that the fire is able to reach (using the paths with minimum travel time) within duration d and selects the ignition point that, if fire starts from that cell, causes the maximum damage to the landscape. A major computation burden of this problem is calculating the minimum travel time paths (shortest paths) for any potential ignition point. For a given fuel management program $\widehat{\boldsymbol{H}}$, if the shortest paths were known, the problem would be equivalent to a maximal covering location problem [61]. For example, say $L_{j,r}(\widehat{\boldsymbol{H}})$ is 1 if the length of the shortest path from cell *j* to cell *r* is less than or equal to *d*, and 0 otherwise. This implies whether the fire ignited at cell *j* can reach (cover) cell *r* within duration *d*. The equivalent maximal covering-based pyro-terrorism model (MCPTP) is as follows:

$$\mathrm{MCPTP}\left(\widehat{\boldsymbol{H}}\right): Z_{max}\left(\widehat{\boldsymbol{H}}\right) = \max \sum_r v_r Y_r \tag{3.14}$$

$$Y_r \leq \sum_{j \in C} L_{j,r}(\widehat{\boldsymbol{H}}) F_j \quad \forall\, r \in C \tag{3.15}$$

$$\sum_{j \in C} F_j \leq 1 \tag{3.16}$$

$$Y_r \in \{0, 1\}, \quad \forall\ r \in C \tag{3.17}$$

$$F_j \in \{0, 1\}, \quad \forall\ j \in C \tag{3.18}$$

$\mathrm{MCPTP}\left(\widehat{\mathbf{H}}\right)$ is a smaller model than $\mathrm{PTP}\left(\widehat{\mathbf{H}}\right)$ in that it has fewer variables and constraints. In fact, $\mathrm{PTP}\left(\widehat{\mathbf{H}}\right)$ has $n^3 + n^2 + 2n$ variables and $2n^2 + 2n + 1$ constraints (n is the number of cells in the network), without considering sign restriction constraints (3.10) through (3.12). $\mathrm{MCPTP}\left(\widehat{\mathbf{H}}\right)$ on other hand has only $2n$ variables and $n + 1$ constraints without sign restriction constraints (3.17) through (3.18). Our preliminary experiments verify that $\mathrm{MCPTP}\left(\widehat{\mathbf{H}}\right)$ can be solved faster than $\mathrm{PTP}\left(\widehat{\mathbf{H}}\right)$.

To solve MCPTP$(\widehat{\mathbf{H}})$, we first compute $L_{j,r}(\widehat{\boldsymbol{H}})$ for $\forall\ j,r \in C$. The mitigation effect of a fuel management program $\widehat{\boldsymbol{H}}$ is imposed in calculating $L_{j,r}(\widehat{\boldsymbol{H}})$. For this reason, we assume that treated cells can block the spread of fire (treated cells are impassable); therefore the fire cannot use any treated cell to spread through the landscape. To compute $P_{j,r(\widehat{\boldsymbol{H}})}$, we independently solve the all-to-all shortest paths problem, and compute $T_{j,r}$ (the fire arrival time to cell r when fire has started from cell j) for all cells in the network. If a cell is treated, the time required for the fire to spread through this cell will be given a value larger than d; therefore, the fire cannot spread out of this cell to any adjacent cell within duration d. Solving MCPTP$(\widehat{\mathbf{H}})$ renders the ignition point of the optimal pyro-terror attack for the given fuel management program $\widehat{\boldsymbol{H}}$**.**

Knowing the adversary's problem for a specific fuel management program, as shown above, the pyro-terrorism mitigation problem $PTMP$ is equivalent to:

$$Z^* = \min Z_{max}(\boldsymbol{H}) \qquad \boldsymbol{H} \in \Omega \tag{3.19}$$

Theoretically, we could solve (3.19) by enumerating the finite set of fuel management mitigation plans $\widehat{\boldsymbol{H}} \in \Omega$, solving MCPTP$(\widehat{\mathbf{H}})$ for each plan, and choosing the plan that results in the least value of $Z_{max}(\widehat{\boldsymbol{H}})$. However, in reality, Ω is too large to enumerate. For a landscape rasterized into a 100 cells network and for a 10% fuel management budget, there are $C_{10}^{100} \cong 1.73 \times 10^{13}$ fuel management programs. Therefore, we solve (3.19) with the decomposition algorithm described below.

3.3.2 Optimal mitigation of a known pyro-terror attack using fuel management program

To solve PTMP we use a procedure that computes a lower bound on the objective value (the effect of an optimal fuel management on controlling fire growth). This lower bound is an optimistic value of how much the wildfire managers can reduce the damage caused by fire by interdicting the fire growth with a fuel management program. For this reason, we formulate an optimization model denoted by IPMin $(\hat{\mathbf{F}})$ that can determine an optimal mitigation of any fixed pyro-terror attack plan $(\hat{\mathbf{F}})$ using an optimal fuel management program. A solution to this model is a lower bound to $PTMP$ because the adversary's plan is restricted to $(\hat{\mathbf{F}})$. This model is adopted from [31].

$$\text{IPMin: } Z_{\min}(\widehat{\boldsymbol{F}}) \equiv \min_{\boldsymbol{H}\in\Omega} Z \tag{3.20}$$

$$s.t.\;\; Z \geq \textstyle\sum_r v_r Y_{s,r} \qquad \forall\; s \in \phi \tag{3.21}$$

$$T_{s,s} = 0 \qquad \forall\; s \in \phi \tag{3.22}$$

$$T_{s,r} \leq T_{s,q} + \frac{\Delta_{q,r}}{R_{q,r}} + \Gamma H_q \quad \forall\; r, q \in C,\; s \in \phi \tag{3.23}$$

$$Y_{s,r} \geq \frac{d - T_{s,r}}{d} \qquad \forall s \in \phi, r \in C \tag{3.24}$$

$$\textstyle\sum_r H_r \leq b \tag{3.25}$$

$T_{s,r}$ = the fire arrival time from the ignition point s to cell r

In this model ϕ is the set of ignition points of the optimal pyro-terror attacks found up to the current point in the algorithm's progression. For the given pyro-terror attack $\hat{F}$, if $\hat{s}$ is the index of the ignition point in $\hat{F}$ ($\hat{s} = \{s \in C \,|\widehat{F_s} = 1\}$), we add $\hat{s}$ to ϕ. The objective function (3.20) represents the loss caused by fire and is denoted by a new variable Z and is bounded in constraint (3.21). Constraints (3.22) set the fire arrival time of cell s to zero when cell s has been chosen by the adversary (pyro-terrorist) as the ignition point. Constraints (3.23) apply the minimum travel time algorithm to track the earliest time that the fire can reach cell r from any of its adjacent cells when the fire starts from cell s; this is where the fuel management based interdiction strategy can be imposed to mitigate the damage of fire by delaying the fire's growth. This formulation also resembles a node-based shortest path network interdiction problem; it is used to delay fire growth by means of the treated cells. The amount of time delay is defined by a parameter Γ. If Γ is set to a value larger than the maximum fire duration, then constraint (3.23) implies that fuel management can block the fire from spreading out of the treated cells.

Constraints (3.24) track whether fire has reached cell r within duration d. If the fire can reach the center of a cell within duration d ($T_{s,r} \leq d$) then that cell is considered burned or lost and the binary variable $Y_{s,r}$ is set to 1. The model is minimizing the amount of loss due to the fire represented by $Y_{s,r}$ variables, and, therefore, through constraints (3.24), it maximizes the fire arrival time (calculated with the MTT algorithm) by allocating fuel management resources in the landscape (through H_q in constraints (3.23)). Constraint (3.25) is the budget constraint restricting the number of fuel management cells due to limited resources.

3.3.3 Decomposition algorithm: MinMaxDA

We define a set of pyro-terror attacks ϕ. At iteration k of $MinMaxDA$, a pyro-terror attack $\widehat{F^k}(k = 1, \dots, K)$, will have been generated from the sub problem which identifies an optimal ignition point. The set of pyro-terror attacks ϕ^k is updated by adding the index of the newly found attack $\widehat{F^k}$ ($\widehat{s^k} = \{s \in C \mid \widehat{F_s^k} = 1\}$). Then the master problem *IPMin* is updated to include new constraints and variables associated with the newly found ignition plan $\widehat{F^k}$. In particular, constraint (3.21) is replaced with constraints (3.26) to impose a lower bound on the fire loss due to the fire that is generated by all pyro-terror attacks found so far, including $\widehat{F^k}$. Also, constraints $(3.22) - (3.24)$ are updated to include the newly found ignition point ($\widehat{s^k} = \{s \in C \mid \widehat{F_s^k} = 1\}$).

$$Z \geq \sum_r v_r Y_{s,r} \qquad \forall\ s \in \phi^k \tag{3.26}$$

Constraints (3.26) are analogous to Benders cuts. Let's call the master problem at iteration k that contains these cuts $IPMin^k$.

3.3.3.1 Theorem 1

The optimal objective value for $IPMin^k$ provides a valid lower bound for Z^*.

This is true because $IPMin^k$ determines an optimal mitigation plan for any fixed pyro-terror attack limited to ϕ^k while Z^* in PTMP (equation (3.3)) considers all possible pyro-terror attacks. This is similar to solving Z^* for $\boldsymbol{H} \in \Omega$ when $\Psi(\boldsymbol{H})$ (the lower level problem) is restricted on the set of potential ignition points. ■

3.3.3.2 Theorem 2: ${Z^{k+1}}_{min} \geq {Z^k}_{min}$.

Since $\phi^k \subseteq \phi^{k+1}$, therefore, $IPMin^{k+1}$ is more restricted than $IPMin^k$. ■

The lower bound from $IPMin^k$ converges to Z*. Since the lower bound is non-decreasing (theorem 2), if the solution of the master problem does not repeat, $IPMin^k$ converges to Z*. We use the solution elimination constraints (3.27) [82] to prohibit any mitigation plan from being repeated and ensure the convergence of the algorithm.

$$\sum_{q \mid \widehat{H_q^k = 0}} H_q + \sum_{q \mid \widehat{H_q^k = 1}} (1 - H_q) \geq 1 \qquad k = 1, \dots, K \tag{3.27}$$

3.3.3.3 Algorithm $MinMaxDA$:

At iteration k of this algorithm, we refer to the optimal objective value of the master problem $IPMin^k$ as $Z^k{}_{min}$, and the optimal objective value of the sub problem as $Z_{max}(\widehat{\boldsymbol{H}^k})$;

Step (1) Initialize upper bound and lower bound: $\bar{Z}_{UB} = \infty$, $\underline{Z}_{LB} = 0$, and set the iteration counter $k = 1$. Set the mitigation plan and the pyro-terror attack plan to null: $\widehat{\boldsymbol{H}^k} = \{\}$, $\phi^k = \{\}$. Set the current mitigation plan as the best found so far: $\boldsymbol{H}^* = \widehat{\boldsymbol{H}^k}$;

Step (2) Given the mitigation fuel management program $\widehat{\boldsymbol{H}^k}$, compute the values of $L_{r,j}(\widehat{\boldsymbol{H}})$ and then solve the sub-problem $MCPTP\left(\widehat{\boldsymbol{H}^k}\right)$ to find the optimal pyro-terror attack represented by a fire ignition point $\widehat{F^k}$. The bound on the damage caused by the corresponding pyro-terror attack is $Z^k{}_{max}$;

Step (3) If $Z^k{}_{max} < \bar{Z}_{UB}$, then set $\bar{Z}_{UB} = Z^k{}_{max}$ and update the best fuel management program $\boldsymbol{H}^* = \widehat{\boldsymbol{H}^k}$;

Step (4) If $\bar{Z}_{UB} - \underline{Z}_{LB} \leq \varepsilon$; stop, the algorithm has converged to an $\varepsilon - optimal$ mitigation plan;

Step (5) Update the set of pyro-terror attack ϕ^k with the newly found $\widehat{F^k}$, and then update the master problem $IPMin^k$: all the constraints (3.21) through (3.24) need to be updated to include the new attack $\widehat{F^k}$, which is added as a new ignition point to the model, and, accordingly, a new set of constraints (3.21) through (3.24) will be generated. Next, solve the updated master problem, $IPMin^k$, for a new mitigation plan and find

$Z^k{}_{min}$ and a new mitigation plan $\widehat{\boldsymbol{H}^k}$;

Step (6) If $Z^k{}_{min} > \underline{Z}_{LB}$, then set $\underline{Z}_{LB} = Z^k{}_{min}$;

Step (7) If $\bar{Z}_{UB} - \underline{Z}_{LB} \leq \varepsilon$, stop, the algorithm has converged to an $\varepsilon - optimal$ mitigation plan;

Step (8) set k = k + 1 and go to step (2);

Note: $MCPTP(\widehat{\boldsymbol{H}^k})$ is always feasible. Even when every cell in the landscape has been treated (interdicted); starting a fire at any cell will burn that cell, and $Z_{max}(\widehat{\boldsymbol{H}^k}) = 1$.

Note: $IPMin^k$ is always feasible. By setting $H_r = 0\ \forall r \in C$, and $Y_{s,r} = 1\ \forall\ r \in C, \forall\ s \in \phi$, the model is linear; for the remaining variables, $T_{s,r}\ \forall\ r \in C, \forall\ s \in \phi$, a feasible solution can be found by solving the shortest path problem for each origin $\forall\ s \in \phi$ and destination $\forall\ r \in C$.

Whenever $\underline{Z}_{LB}$ or $\bar{Z}_{UB}$ is updated, we need to check whether the $\varepsilon - optimal$ mitigation plan has been achieved; that is why the stopping criteria is checked both in step (4) and in step (7).

Note: The master problem grows in size as more cuts are added and more constraints and variables are added to the model. At step (5), however, when adding constraints (3.23) and (3.24) for each new ignition point, instead of considering all target points $\forall\ r \in C$, we can only consider the target points $r \in C$ that are reachable by the fire. The one-to-all shortest path problem can be solved for this purpose. Only those target cells whose shortest path's length is less than or equal to duration d (cells reachable by fire within duration d) need to be considered. This realization eliminates some unnecessary variables and constraints and speeds up the algorithm.

3.4 Experimentation

Fuel management programs can influence fire behavior differently based on their spatial layouts [31]. Identifying the best fuel treatment layout to interdict pyro-terrorism and mitigate its consequences is a challenging problem. We have arbitrarily picked three small landscapes from the U.S. national forests for experimentation. Here, we implement our pyro-terrorism mitigation model on these test landscapes to identify the optimal fuel management plan that can optimally mitigate a pyro-terror attack.

3.4.1 Test landscapes

The first test case is taken from the landscape of Santa Fe National Forest in northern New Mexico. A prevailing west to east wind is assumed for this case at 12 miles per hour (19.31 *km* per hour). The second test case is taken from the landscape of Umpqua National Forest located at the western slopes of the Cascade Mountains in Oregon. The same wind condition is assumed. The third test case is taken from the landscape of San Bernardino National Forest located in the San Bernardino Mountains of California. For this test case, a prevailing west to east wind at 12 miles per hour is assumed. Figure 3.3 shows the approximate locations of these landscapes.

Figure 3.3 The approximate locations of the test case landscapes

Similar to a study by [31], we clip an area of 1.8 *km* by 1.8 *km* (the area is arbitrarily chosen) from these landscapes and rasterize them into 10×10 square cells, each 180 *m* by 180 *m*. To calculate the rate of spread and major fire spread directions under these conditions for these three landscapes, we use FlamMap 5.0 with the LANDFIRE database [105] providing the landscape files (LCP) for these test cases. LANDFIRE data are commonly used in wildland fire simulation modeling, as they are standardized and updated regularly to adjust to disturbances such as wildfires, fuel treatment, and urban development [20]. Landscape files (LCP) contain spatial data themes such as fuel models, elevation, slope, aspect and canopy characteristics. FlamMap inputs these data along with wind speed, wind direction, and fuel moisture conditions to compute rate of spread (ROS) and the major direction of fire spread in each cell.

We use the same initial fuel moisture conditions for the three test cases in our study (Table 3.3). The outputs of FlamMap (the rate of spread and the major direction of fire spread in each cell) are used to model fire spread in the landscapes using a minimum travel time algorithm. Figure 3.4, Figure 3.5 and Figure 3.6 show major fire spread directions (*degree*) and rate of spread (*meter/minute*) along the major fire spread directions for the three test cases. To calculate the impact of wildfires with optimally located ignition points, we implement the model using Python 2.7 and solve it with Gurobi 6.0 [64]. All tests are performed on a computer with an Intel Core i5 2520M processor at 2.5 GHz and 8 GB RAM

Table 3.3 Initial fuel moisture conditions used in FlamMap

1 hour initial moisture	6
10 hour initial moisture	7
100 hours initial moisture	8
Herbaceius fuel moisture	60
Live woody fuel moisture	90

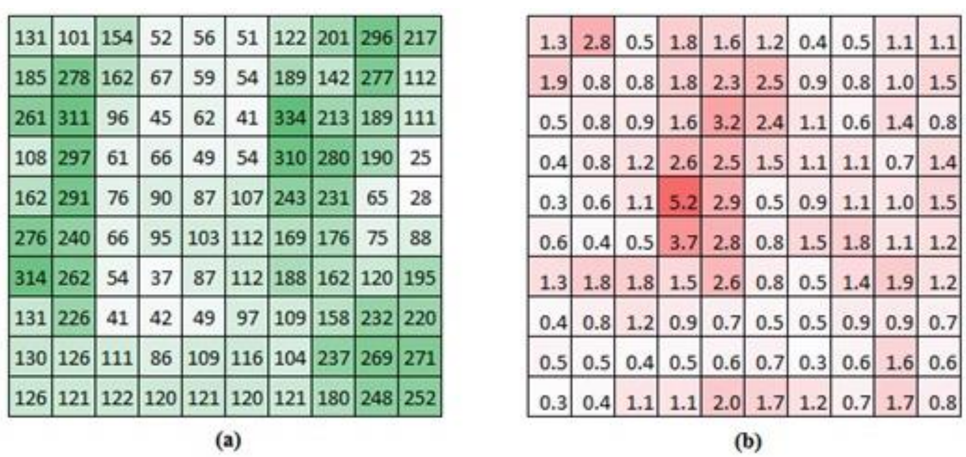


Figure 3.4 Santa Fe data heat map

Santa Fe landscape test case; (a) Major fire spread direction (*degree*),
(b) Rate of spread (*meter/minute*) along the major fire spread directions

131	101	154	52	56	51	122	201	296	217
185	278	162	67	59	54	189	142	277	112
261	311	96	45	62	41	334	213	189	111
108	297	61	66	49	54	310	280	190	25
162	291	76	90	87	107	243	231	65	28
276	240	66	95	103	112	169	176	75	88
314	262	54	37	87	112	188	162	120	195
131	226	41	42	49	97	109	158	232	220
130	126	111	86	109	116	104	237	269	271
126	121	122	120	121	120	121	180	248	252

(a)

1.3	2.8	0.5	1.8	1.6	1.2	0.4	0.5	1.1	1.1
1.9	0.8	0.8	1.8	2.3	2.5	0.9	0.8	1.0	1.5
0.5	0.8	0.9	1.6	3.2	2.4	1.1	0.6	1.4	0.8
0.4	0.8	1.2	2.6	2.5	1.5	1.1	1.1	0.7	1.4
0.3	0.6	1.1	5.2	2.9	0.5	0.9	1.1	1.0	1.5
0.6	0.4	0.5	3.7	2.8	0.8	1.5	1.8	1.1	1.2
1.3	1.8	1.8	1.5	2.6	0.8	0.5	1.4	1.9	1.2
0.4	0.8	1.2	0.9	0.7	0.5	0.5	0.9	0.9	0.7
0.5	0.5	0.4	0.5	0.6	0.7	0.3	0.6	1.6	0.6
0.3	0.4	1.1	1.1	2.0	1.7	1.2	0.7	1.7	0.8

(b)

Figure 3.5 Umpqua data heat map

Umpqua landscape test case; (a) Major fire spread direction (*degree*),
(b) Rate of spread (*meter/minute*) along the major fire spread directions

100	90	86	70	93	92	92	81	66	66
96	101	91	66	96	112	93	98	97	99
83	86	93	91	69	76	106	98	99	96
92	99	100	151	118	115	76	75	76	71
102	96	117	116	104	95	91	97	94	93
96	106	108	109	87	91	98	91	88	86
93	101	110	101	105	114	100	89	95	119
114	112	97	100	103	87	92	98	136	132
99	89	89	90	94	96	97	99	198	150
88	85	78	82	84	76	88	115	131	210

(a)

0.9	2.0	1.3	2.3	2.2	2.1	1.5	1.6	1.2	2.0
1.5	1.7	1.4	1.8	2.1	1.9	1.2	1.3	3.4	1.0
1.6	1.8	1.7	1.2	0.9	1.5	1.8	2.1	1.8	1.9
1.7	1.6	2.0	1.7	1.8	1.2	1.0	1.3	1.1	1.0
1.4	1.7	1.9	1.6	1.2	1.8	2.2	3.3	4.6	1.9
1.2	1.2	1.1	1.0	1.5	1.7	2.5	6.1	5.1	2.3
1.2	1.1	1.3	1.3	1.2	1.3	1.4	1.5	1.8	0.9
1.2	1.3	0.8	1.6	1.2	1.2	2.1	1.7	1.0	1.0
1.2	0.8	1.3	1.5	1.7	1.7	1.5	1.3	0.7	1.9
1.4	2.0	1.7	3.8	2.3	1.7	1.8	1.5	1.0	1.3

(b)

Figure 3.6 San Bernardino data heat map

San Bernardino landscape test case; (a) Major fire spread direction (*degree*),
(b) Rate of spread (*meter/minute*) along the major fire spread directions

The Umpqua test case has an average ROS of 1.2 (meter/minute), which is higher than that of the Santa Fe test case, with an average ROS of 0.4 (meter/minute). The San Bernardino test case has an average ROS of 1.7 (meter/minute), the highest of these test cases. The higher the ROS for a landscape, the faster fire grows and the more damage it causes to the landscape before it is suppressed.

3.4.2 Computational experiments

3.4.2.1 Optimal pyro-terrorism fuel management plan (PFMP)

By solving our pyro-terrorism mitigation model to optimality, we can compute the percentage of the area burned and find the optimal spatial allocation for the fuel management strategy to mitigate a pyro-terror attack. The results indicate that the optimal fuel management plan can mitigate the impact of pyro-terrorism on the three landscape test cases (Table 3.4). As expected, the San Bernardino test case has been affected the most with pyro-terrorism (100 % burned when no fuel management is conducted). This is likely due to the characteristics of this landscape that influence the rate of spread and the major fire spread direction. As mentioned earlier, the San Bernardino test case has the highest ROS.

We term the fuel management plan that resulted from solving our model the Pyro-terrorism Fuel Management Plan (PFMP). Following Minas et al. [10], we consider three scenarios for the fuel treatment budget: 2%, 5% and 10%. The results indicate that the PFMP with a 10% budget can mitigate the impact of a pyro-terror attack by more than 42%, 46% and 41% respectively for the Santa Fe, Umpqua, and San Bernardino test cases (Table 3.4). Even the PFMP with a small budget (2%) can mitigate the impact of pyro-terrorism by 13.9% on average over the three landscape test cases.

Table 3.4 The impact of PFMP

Landscape	% landscape treated		
	2%	5%	10%
Santa Fe	25.00	30.77	42.31
Umpqua	13.70	32.88	46.58
San Bernardino	3.00	18.00	41.00
Average	13.9	27.22	43.30

The percent improvement in area burned when the PFMP is implemented

Table 3.5 Computation time

Landscape	% landscape treated		
	2%	5%	10%
Santa Fe	7	37	2,139
Umpqua	8	171	4,716
San Bernardino	3	570	32,722

Computation times (seconds) for solving the pyro-terrorism mitigation problem for test landscapes with different fuel management budgets

Table 3.5 shows the computation time for solving the pyro-terrorism mitigation problem using the $MinMaxDA$. As the budget for fuel management grows, the computation time for solving the PTMP model increases.

3.4.2.2 A Wildfire Fuel Management Plan (WFMP) v.s. Pyro-terrorism Fuel Management Plan (PFMP)

A fuel management program designed for mitigating a wildfire does not consider the threat of a pyro-terror attack, and, therefore, we hypothesize that it cannot optimally mitigate a pyro-terror attack. To investigate this hypothesis, we conduct an experiment to compare the effectiveness of a fuel management program developed for wildfires (WFMP) with the fuel management program developed for pyro-terrorism (PFMP); we use Wei's model [31] to develop the WFMP. The results indicate that with a low fuel management budget, the difference between the two fuel management programs is small, averaging 2.55% (Table 3.6). However, with a larger budget, the differences are more significant: 7.85% and 10.49%, respectively, for fuel management programs with 5% and 10% budgets.

The PFMP is designed to mitigate pyro-terrorism and, as is shown in Table 3.6, it is more effective in mitigating pyro-terrorism than the WFMP. However, the PFMP may not be as effective in mitigating natural wildfires (we use natural wildfires and wildfires interchangeably). To investigate this hypothesis, we draw a comparison between the effects of the PFMP and those of the WFMP for wildfires. Figure 3.7, Figure 3.8 and Figure 3.9 illustrate the performance of the WFMP and the PFMP for both natural wildfires and pyro-terrorism. These results indicate that the PFMP is also effective in mitigating wildfires but not as effective as the WFMP. As can be seen in Figure 3.9, there is a noticeable difference between the two fuel treatment plans for the San Bernardino landscape test case when a wildfire occurs; however, for the Santa Fe and Umpqua landscape test cases, the differences are small.

Table 3.6 WFMP v.s. PFMP: A comparison

Landscape	Fuel management plan	% landscape treated		
		2%	5%	10%
Santa Fe	WFMP	23.08	23.08	26.92
	PFMP	25.00	30.77	42.31
	Difference	1.92	7.69	15.39
Umpqua	WFMP	10.96	26.03	31.51
	PFMP	13.70	32.88	46.58
	Difference	2.74	6.85	15.07
San Bernardino	WFMP	0.00	9.00	40.00
	PFMP	3.00	18.00	41.00
	Difference	3.00	9.00	1.00
Average difference		2.55	7.85	10.49

A comparison between the WFMP and the PFMP: The percent improvement in landscape burned by a pyro-terror attack when different fuel management programs with different budgets are implemented.

For the San Bernardino test case, not conducting any fuel management leads to a complete burn of the landscape (Figure 3.9), while a fuel management program with 10% budget can mitigate pyro-terrorism by more than 41%, as shown in Table 3.6. Also, for this test case, when the budget is 10%, the difference between the WFMP and the PFMP under pyro-terrorism is small; however, when the budget is 9%, the difference is again noticeable (Figure 3.9).

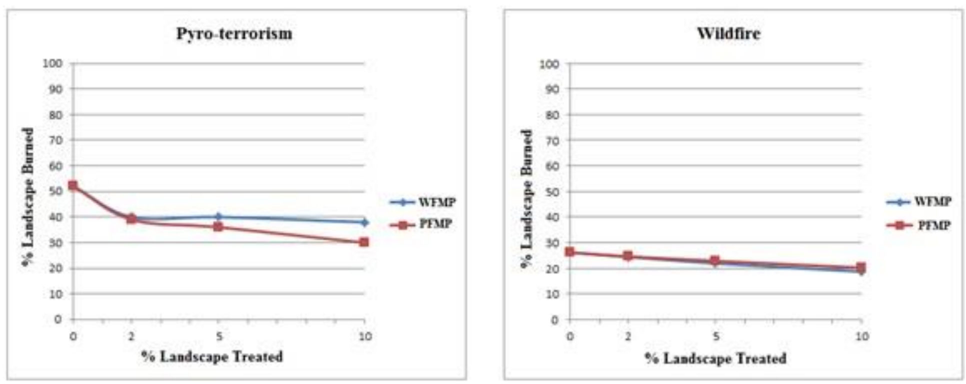


Figure 3.7 Percent area burned Santa Fe

Percent area burned for Santa Fe landscape test case; the WFMP and the PFMP under pyro-terrorism and wildfires

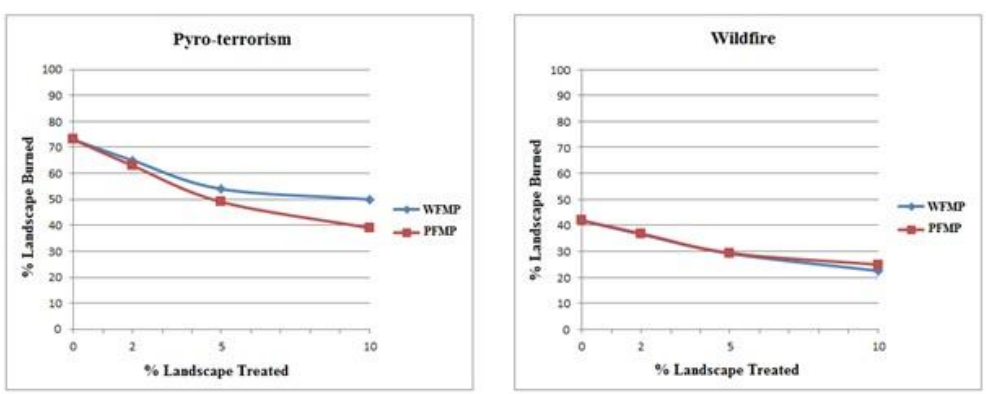


Figure 3.8 Percent area burned Umpqua

Percent area burned for Umpqua landscape test case; the WFMP and the PFMP under pyro-terrorism and wildfires

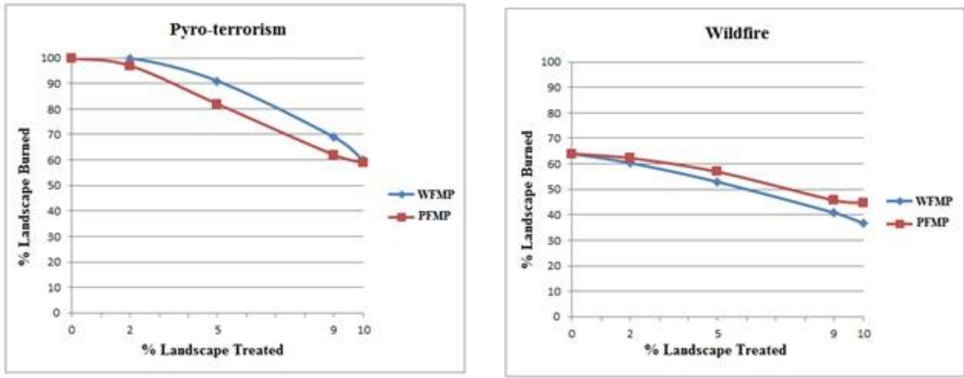


Figure 3.9 Percent area burned San Bernardino

Percent area burned for San Bernardino landscape test case; the WFMP and the PFMP under pyro-terrorism and wildfires

After plotting the spatial distribution of fuel treatment for the WFMP for this case, we realize that, when the budget is 10%, the San Bernardino landscape test case is divided into two halves under this fuel treatment plan (Figure 3.10). Therefore, when the adversary attacks either half, the fire burns that half completely (because of the high ROS in San Bernardino test case); however, the fire cannot reach the other half.

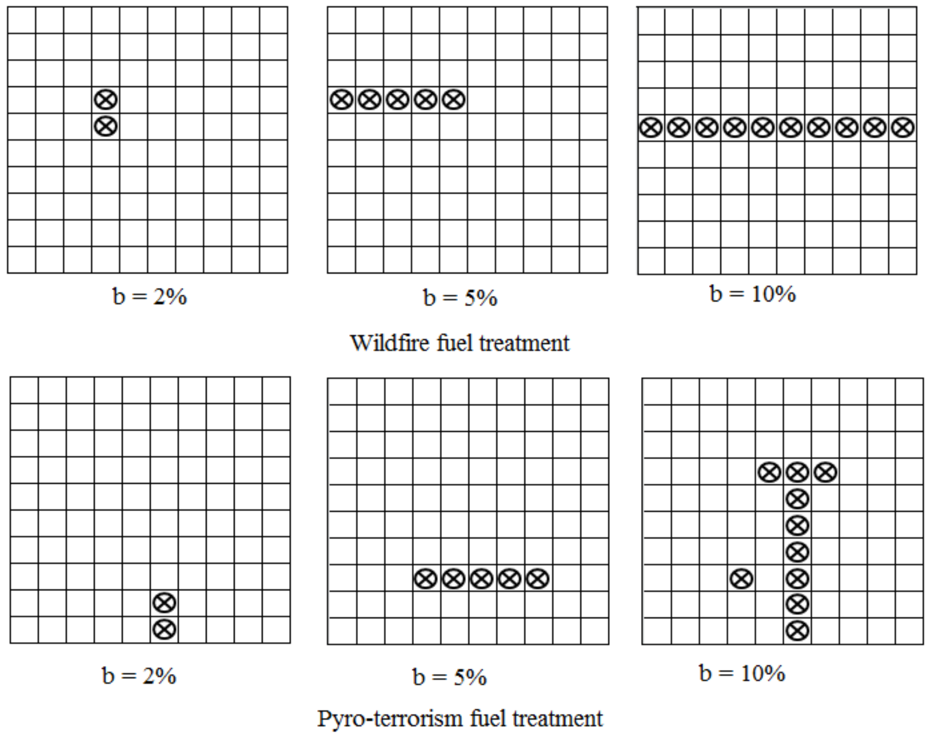


Figure 3.10 San Bernardino fuel management layout

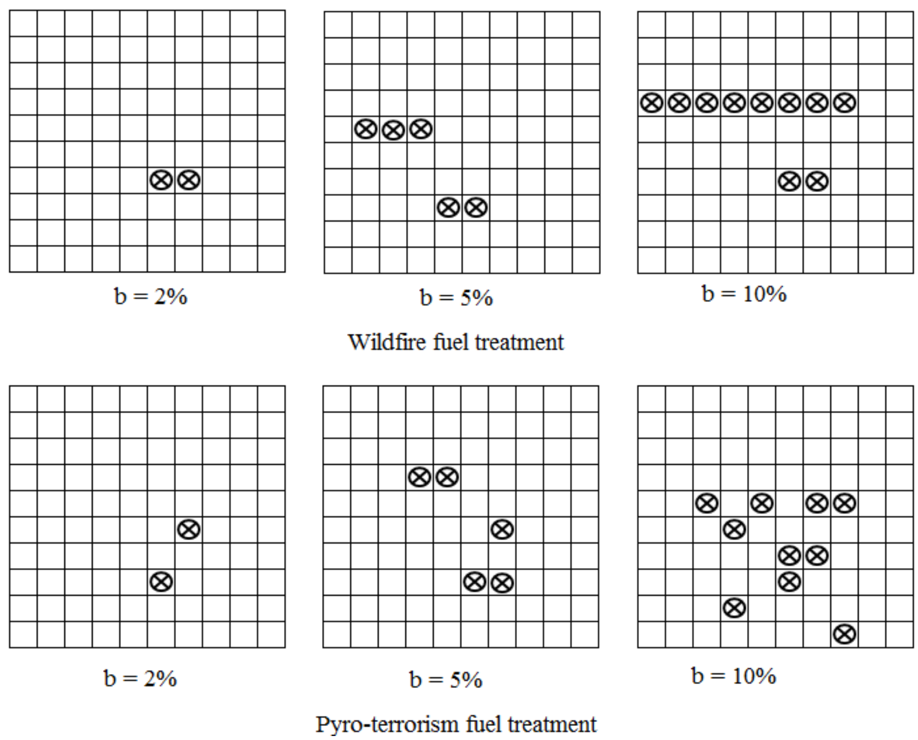


Figure 3.11 Santa Fe fuel management layout

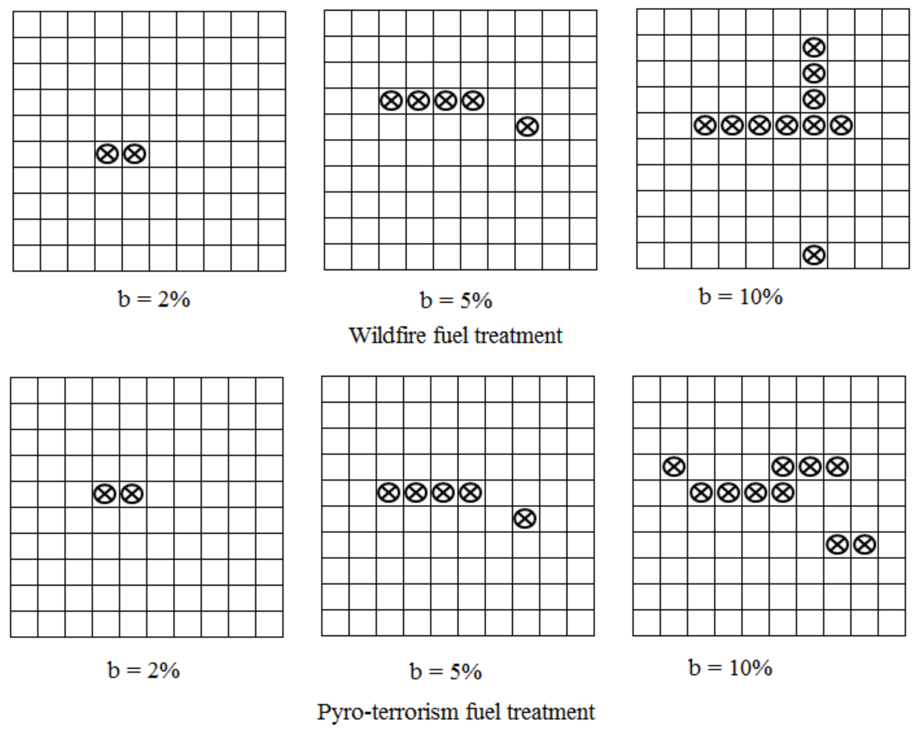


Figure 3.12 Umpqua fuel management layout

As a result, the difference between the PFMP and the WFMP in this case when the budget is 10% is small. However, for smaller budgets, the difference is more significant. Figure 3.11 and Figure 3.12 show the spatial layouts for the WFMP and the PFMP for the two other test cases.

Figure 3.13 shows the fire foot print of a pyro-terror attack on the Santa Fe landscape test case. It shows three scenarios: (1) when no fuel management plan is conducted on the landscape, (2) when the PFMP with 10% budget is conducted on the landscape, and (3) when the WFMP with a 10% budget is conducted on the landscape. As is shown in this figure, the impact of pyro-terrorism is less on this landscape when the

PFMP is conducted, as opposed to when no fuel management plan is conducted at all or when the WFMP is conducted.

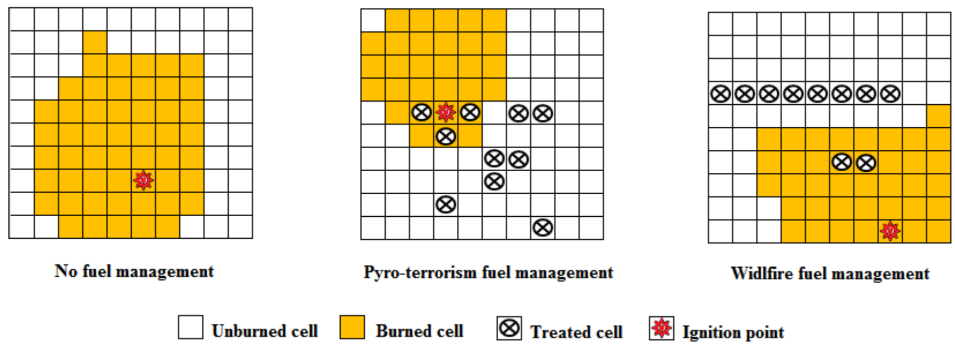


Figure 3.13   Pyro-terrorism fire footprint for Santa Fe test case

Santa Fe landscape test case; fire footprint of pyro-terror attacks with and without fuel management

3.5 Discussions

In this paper, we have presented the first optimization model for mitigating a pyro-terror attack. The model spatially allocates fuel treatment through a landscape such that it mitigates the impact of the resulting wildfire. We have demonstrated the use of this model on three landscape test cases with differing fuel management budgets. Of these three cases, the San Bernardino test case is the most vulnerable to pyro-terrorism. When no fuel treatment is scheduled, a pyro-terror attack will burn 100% of this landscape; proportionally, this massive devastation is two times greater than that of the Santa Fe test case. This significant increase in damage is likely due to a higher rate of fire spread; the higher the rate of spread, the faster the fire can grow, and the more damaging the fire can be to the landscape in a given amount of time. The average rate of spread for the San Bernardino case is 1.7 (*meter/minute*), more than that of Umpqua at 1.2 (*meter/minute*) and Santa Fe at 0.4 (*meter/minute*). However, these landscapes also have different major fire spread directions, and this difference can affect their vulnerability to wildfires as well.

Our results indicate that fuel management, even on a small scale, if appropriately allocated on a landscape, can mitigate the impact of pyro-terrorism. An optimal fuel management plan for pyro-terrorism (PFMP) with a 2% budget can mitigate the impact of a pyro-terror attack by 25%, 13% and 3% for the Santa Fe, Umpqua and San Bernardino test cases, respectively. Our results indicate that the rate of spread is also important in the effectiveness of the fuel treatment plan. The PFMP is less effective on the San Bernardino test case than it is on the Umpqua and Santa Fe test cases. However, as the budget for fuel treatment increases, so does the effectiveness of fuel treatment on mitigating pyro-terrorism. The PFMP with a 10% budget can mitigate the impact of pyro-terrorism by 41% for the San Bernardino test case, a significant improvement. Similarly, the PFMP can mitigate pyro-terrorism by 46.58% and 42.31% for the Santa Fe and Umpqua test cases, respectively.

We have compared the effectiveness of the PFMP versus the WFMP (an optimal fuel management plan designed to mitigate natural wildfires) for mitigating pyro-terrorism. The two fuel management plans have different layouts, and our results show that the spatial layout of a fuel treatment allocation is an important factor in its effectiveness in mitigating a pyro-terror attack. As we have illustrated, a fuel management layout designed to mitigate a natural wildfire (i.e. WFMP) is not as effective for mitigating the impact of pyro-terrorism. For example, for the San Bernardino case, the PFMP with 2% budget can mitigate pyro-terrorism by 3%, the WFMP with 2% budget has no impact at all. Although the differences between the effectiveness of the WFMP and the PFMP are small for all test cases, as the budget for fuel treatment increases, the difference between the two fuels treatment plans increases as well. With a 10% fuel treatment budget, the PFMP, on average, is more than 15% more effective in mitigating pyro-terrorism than the WFMP for the Santa Fe and Umpqua test cases. However, there is an exception for the San Bernardino case; the difference between the PFMP and the WFMP is about 1%. This is because with a 10% budget, the WFMP (almost) equally divides the landscape into two pieces and since it is assumed that the adversary only uses single-point pyro-terror attacks, the WFMP can prevent the fire from reaching the other half of the landscape. This layout makes the WFMP (with a 10% budget) almost as effective as the PFMP (with a 10% budget) for the San Bernardino test case. Overall, the PFMP with 10% budget is, on average, more than 10% more effective in mitigating pyro-terrorism than the WFMP with a 10% budget. This indicates that a fuel treatment plan that has been optimally designed to mitigate wildfires (i.e. WFMP) is not as effective in mitigating pyro-terrorism (i.e. arson-induced wildfire). This result is consistent with what Kim et al. [27] reported for arson-induced wildfires.

Our results indicate that for the Santa Fe and Umpqua test cases, the PFMP is effective under both natural wildfires and pyro-terrorism conditions. Although the WFMP is more effective than the PFMP in mitigating natural wildfires, the difference between these two fuel treatment plans are small for the Santa Fe and Umpqua test cases. Considering the weaker performance of the WFMP against pyro-terrorism, the PFMP can be a more robust fuel treatment plan for these landscape test cases. A common characteristic of the Santa Fe and Umpqua test cases is that, on average, they have smaller rate of spread than the San Bernardino case. Therefore, this result might be also applicable to other landscape test cases with low rates of spread; conducting the PFMP might be a better alternative than the WFMP for those cases. For those landscapes, the PFMP is a more robust fuel management plan and makes the landscapes resilient against worst-case wildfires and pyro-terrorism. However, this requires more investigation that is beyond the scope of this work. The San Bernardino landscape test case is the most vulnerable of all under both natural wildfires and pyro-terrorism conditions. Although conducting fuel treatment in a small scale (e.g. 2%) cannot effectively mitigate wildfires and pyro-terrorism for this case, fuel management in a larger scale (e.g. 10%) is effective; we speculate the high rate of spread is the reason for that. For this landscape, the difference between the WFMP and the PFMP under wildfire conditions when the fuel management budget is 10% is larger than that of the Santa Fe and Umpqua landscape test cases. Therefore, for this case, the PFMP cannot be recommended for mitigating wildfires.

The computation time for $MinMaxDA$ is different for each landscape test case and for different budgets. The difficulty of $MinMaxDA$ lies in solving the master problem that allocates fuel management spatially in a landscape. The percentage of landscape treated (which is dependent on the fuel management budget) is an important factor in computation efforts required to solve the master problem. For larger fuel management budgets, the problem is more difficult to solve.

Also, the larger the size of a landscape, the more difficult the pyro-terrorism mitigation problem is. Solving the mitigation problem for larger landscapes (modeled as large networks) requires a more efficient method to solve the corresponding master problem. We leave this challenge for future studies. However, since the problem is a strategic level decision making problem, the computation time can be tolerated depending on the benefit-cost ratio of the solution.

3.6 Conclusions and future work

3.6.1 Conclusion

In this research, we investigated the possibility of mitigating a pyro-terrorism wildfire using fuel management. We modeled this problem as a bilevel min-max optimization problem and developed a decomposition algorithm to solve it. We restricted our study only to acts of pyro-terrorism with one ignition point. Our results indicate that fuel management can effectively be used to mitigate a potential pyro-terrorism attack (or worst-case wildfires).

3.6.2 Future work

The proposed model has been presented in a simple and general form. However, the model could be readily adapted without significantly changing its structure to include a fire duration distribution. Additionally, the model can be extended to include Wildland Urban Interface (WUI) by adjusting cell values used in the objective function (3).

In addition to the ignition location, the number of ignition points also increases the damage caused by a pyro-terrorism attack [45]. One can study the pyro-terrorism mitigation problem with multiple ignition points (multiple concurrent fires); however, such a problem is more difficult to solve. Increasing the number of ignition points causes the master problem to grow exponentially, and the number of iterations can grow as well.

We have investigated the effect of fuel management in mitigating pyro-terrorism assuming that it can be successfully suppressed within a time *d*. This requires an appropriate response from fire managers that are expected to dispatch fire control resources from bases to which they have already been deployed. However, a pyro-terrorist can observe the locations, types, and amount of these resources and plan an attack accordingly. A network interdiction approach can be used for optimally deploying the resources to bases such that the impact of pyro-terrorism can be mitigated.

CHAPTER IV

AN ATTACKER-DEFENDER MODEL FOR ANALYZING THE VULNERABILITY OF INITIAL ATTACK IN FIGHTING THE WORST CASE WILDFIRES AND PYRO-TERRORISM

4.1 Introduction

In this paper, we study the vulnerability analysis of initial attack (IA) in controlling pyro-terrorism and worst case wildfires. IA is used as the primary suppression response to control a wildfire after its discovery. Wildfire managers use initial attack to contain a wildfire before it grows large and becomes difficult to suppress. IA is usually planned using historical wildfires data such as ignition locations and number of fires. However, unlike natural wildfires, in which the ignition locations are located randomly, pyro-terrorists can use coordinated wildfires [45] which are more difficult to control. Our goal in this paper is to evaluate the vulnerability of IA to these worst case wildfires and get managerial insights.

There are two general types of wildfires: natural wildfires and human-caused wildfires. Human caused wildfires account for a large majority of all wildfire incidences: e.g., more than 95% of wildfires in Mediterranean region and in Southern California are caused by humans [73–75]. A study in Spain found that more than 71% of all wildfires are caused by people [76]. Of those human-caused wildfires, only 22.5% (16% of all wildfires) were due to negligence while 77.5% (55% of all wildfires) were intentional

[76]. Because of the destructive power of wildfires, authorities are concerned about the possibility of using wildfires as a means of terrorism [15,16].

Pyro-terrorism is the use of large-scale arson attacks by non-state organizations to terrorize, intimidate or coerce a government or the civilian population in order to advance political or social objectives [13]. Pyro-terrorism possesses the major elements of terrorism: targeting of noncombatants, political motivation, and organized violence with psychological impacts [11,12,14]. Pyro-terrorism has been documented in France, Spain, and Greece [11,12]. Pyro-terror wildfires are more destructive than natural wildfires [45], as the arsonists can make decisions about the location, time, and quantity of fires. It is therefore important for decision makers to anticipate potential threats and implement countermeasures to avoid a potentially devastating disaster. In this study, we investigate the capability of initial attack resources for responding to pyro-terrorism, or worst-case wildfires.

It has been long understood that a vigorous, rapid IA can contain a fire quickly before it grows large and causes substantial damage [43]. IA is the primary suppression attempt in containing a wildfire within the first several hours of fire discovery. Although the majority of wildfire incidents have been reported to be contained by IA, more than 97% of the total area burned by wildfires have been caused by fires that have escaped IA [36]. Therefore, successfully containing a fire using IA is very important [37,38].

Initial attack planning consists of two types of allocation decisions. First is the deployment decision in which wildfire managers assign suppression resources to fire bases[1] to minimize operating costs, subject to fire bases capacity, while meeting resource

[1] The locations where initial attack resources are located.

requirements for suppressing potential fires in coming days, weeks, or months. Second is the dispatch decision for the time when fires occur. Dispatch decisions include determining the number and type of suppression resources to dispatch to fires which are subject to resource availability, and dispatching costs.

To improve the efficiency of IA, researchers have developed several optimization models with different structures and fire containment rules, e.g. a dynamic programming model for IA dispatching decisions [106], a mixed integer programming (MIP) model for dispatching fire suppression resources to a fire across multiple time steps [107], MIP models for containing fires at multiple locations while they are competing for suppression resources [108,109], and a MIP model to allocate control locations one fire to minimize fire loss [110].

All these models are deterministic. To make the IA decisions, the wildfire managers face substantial uncertainty about the number, location, and intensity of fires as resources are deployed to fire bases before the number, location and intensity of fires are known [35]. To incorporate the uncertainties affiliated with wildfires in an IA decision making, a number of researchers have developed two stage stochastic programming models in which the acquisition and deployment decision of resources takes place in the first stage of the model, and the dispatching of those resources to fire locations takes place in the second stage of the model [35,39–44]. In these models, the deployment and dispatching of resources are planned based on the average impact of fires considering multiple ignition location scenarios using historical data.

However, unlike the natural wildfires that are subject to those uncertainties, in pyro-terror wildfires the adversaries can facilitate the conditions for a more severe wildfire. For example, the adversaries can choose to ignite a fire on a day with suitable wind and low humidity, and coordinate fire ignition locations such that the resulting wildfire poses the maximum damage to the land. Rashidi et al [45] showed that coordinating ignition locations can have a significant impact on the severity of a wildfire. However, as suppressing of wildfires is concerned, the distance of fire locations from fire bases also becomes important. The adversaries can identify the optimal locations for fires after observing fire bases and the available fire suppression resources at those fire bases so that it will be more difficult for wildfire managers to suppress the fires. Therefore, we can expect a pyro-terror wildfire with coordinated ignition points, equivalently a worst-case wildfire, to be more devastating and more difficult to suppress than a typical wildfire with average impact. However, the IA decision making in most of recent literature is planned based on the average scenario and not the worst case scenario. In this research, we study the effectiveness of IA in containing worst case wildfires, and pyro-terrorism.

Vulnerability assessment studies identify weak points in the system, and focus on defined threats that could compromise the system's ability to meet its intended function. Determining the vulnerability of a system is an important component of risk assessment, which is employed to help develop risk mitigation strategies to counter risks [22]. Risk assessment has increasingly become a key input to wildfire management [18–20,49]. To our knowledge, no study has been done on vulnerability assessment of IA when responding to pyro-terrorism or worst case wildfires. This paper aims to fill this gap by proposing a mathematical programming model to identify the vulnerability of initial attack. The resulting managerial insights of this study can help wildfire managers in planning an initial attack strategy that is robust against worst case scenarios.

We model a natural landscape as a grid network and model the spread of fires in the landscape and the initial attack as a network optimization problem. To model wildfire's behavior in a landscape, we use FlamMap [56], a fire behavior mapping and analysis program. We develop a Stackelberg game model for analyzing the vulnerability of initial attack when responding to pyro-terrorism or worst-case wildfires in that landscape. The arsonists, acting as the first player, observe the locations and amount of fire suppression resources, and having perfect knowledge of fire spread and weather condition, they start fires across the landscape so that the number of fires that cannot be contained by IA (escaped fires), and the resulting damage to the landscape are maximized. The wildfire managers, on the other hand, observe the location of fires, and optimally dispatch the available suppression resources to contain the fires. We then use the model to evaluate the vulnerability of IA on a test case problem for a landscape clipped out of Santa Fe National forest, located in Western U.S.

We believe this to be the first study that analyzes the vulnerability of initial attack when responding to the worst case wildfires and pyro-terrorism. Identifying the vulnerability of initial attack can help wildfire managers in developing a more robust suppression strategy that can successfully respond to the worst case scenario wildfires, and pyro-terrorism. The contributions of this paper are as follows: (1) Proposing the first mathematical model for studying the vulnerability of initial attack, and (2) developing a decomposition algorithm to solve the model.

4.2 Vulnerability assessment of initial attack problem (VAIAP)

The problem of vulnerability assessment of initial attack is as follows. Adversaries, having observed the location of bases and the suppression resources, choose a pyro-terror attack plan $\boldsymbol{W}$ (a set of specific fire ignition locations), selecting the most vulnerable area in the landscape to start a wildfire that spreads quickly as is difficult to contain; in other words, it causes the maximum damage. Next, the wildfire managers, detecting the location of fires, attack the fires using an optimal dispatching of resources $\boldsymbol{D}$ to contain the fires and minimize the number of escaped fires and the acreage they could burn. In this section we develop a Stackelberg game model for the VAIAP. It is a bi-level integer programming model with the adversaries' problem represented with the outer level model, and the wildfire managers' problem represented with the inner level model. Both the wildfire managers' problem and the adversaries' problem are modeled as network optimization problems.

4.2.1 Modeling fire behavior in a landscape

To model fire behavior in a landscape, we consider a landscape divided into a number of raster cells representing potential fire ignition points. If we represent the center of each cell as a node, and connect neighboring cells with directed arcs (Figure 4.1) we will have a bidirectional network [45]. This bidirectional network implies that fire can burn up and down slopes and with and into the wind. We use FlamMap [56], a fire behavior mapping and analysis program to compute the rate of fire spread on each of those arcs. FlamMap uses Geographic Information Systems (GIS) data, landscape characteristics, fuel moisture, and wind conditions and computes the major fire spread direction and the rate of spread (ROS) along the major fire spread direction for each cell. We then use the Minimum Travel Time algorithm (MTT) [58] to model the spread of fire through the landscape. MTT has also been used in wildfire simulation programs such as FlamMap [56], FsPro [78], and FSim [79].

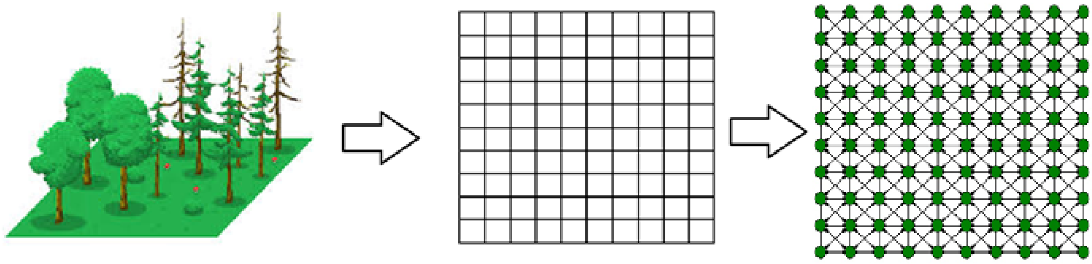


Figure 4.1 A landscape modeled as a bidirectional network

4.2.2 Problem description and model formulation

In our model, the adversaries are assumed to have complete knowledge about weather and the topography of the landscape. They are also aware of the location of fire bases and the available suppression resources at each fire base. Having this complete knowledge, they identify an optimal set of ignition points across the landscape to start fires that are difficult for wildfire managers to contain, using initial attack, and the maximum acreage is burned by the escaped fires[2]. The wildfire managers, after detecting fire locations, initiate an optimal initial attack by optimally dispatching the fire suppression resources from fire bases to fire locations. The wildfire managers' objective is to control the fires and minimize the number of escaped fires and the acreage they could burn; thus, this two-player game is symmetric. In this model, the wildfire managers are assumed to use optimal dispatching of resources which requires wildfire managers to have perfect knowledge about fire suppression requirements and the ability to anticipate the amount of resources needed from each fire base to control the fire. However, in reality, wildfire managers do not have perfect knowledge about weather and the

[2] Escaped fires are those fires that cannot be contained using initial attack.

landscape, and cannot use optimal dispatching of resources. Therefore, the vulnerability assessment gained with our model is a lower bound for the real problem.

The primary assumptions of this research are as follows: (1) The fire area containment rate of the fire suppression resource are known. (2) Wildfire managers have perfect knowledge of the amount of resources required to control each fire. (3) All cells have high fire intensity. (4) There is no interaction between fires.

The notations used in the model are as follows:

Table 4.1 Notations

Sets and indices	
C	is the set of raster cells in a landscape indexed with j
F	is the set of potential fire ignition locations indexed with f
B	is the set of operating bases indexed with b
R	is the set of resources type indexed with r
Parameters	
$\Phi_{b,r,f}$	fire containment area built by T by resource r dispatched from operating base b to fire f
$\pi_{f,j}$	if cell j reached and burned by fire f within fire duration T
φ_j	the area of cell j
Δ_f	total area burned by fire ignited at location f $\left(\sum_{j \in C} \varphi_j \pi_{f,j} w_f\right)$ within fire duration T;
β	the pyro-terrorists' budget (number of fires the terrorists ignite)
$Q_{b,r}$	the number of resources of type r available at operating base b
Variables	
Z_f	1 if the fire ignited at location f is contained with initial attack, 0 otherwise;
W_f	1 if a fire is ignited at location f, 0 otherwise;
$Y_{b,r,f}$	the number of resources of type r dispatched from operating base b to the fire ignited at location f;

The mathematical formulation for the vulnerability assessment of initial attack problem (VAIAP) is as follows:

$$\text{VAIAP: } A^* = \max_{\boldsymbol{W}\in\Xi}\left(min_{\boldsymbol{Y}\in\Psi(\boldsymbol{W})} \sum_{f\in F}\left(1 - Z_f\right)\Delta_f\right) \tag{4.1}$$

Where the set Ξ is defined as the set of all $\boldsymbol{W}$ such that

$$\sum_{f\in F} W_f \leq \beta \tag{4.2}$$

$$W_f \in \{0,1\}, \ \ \forall \ f \in F \tag{4.3}$$

and the set $\Psi(\boldsymbol{W})$ is defined by

$$\Delta_f Z_f W_f \leq \sum_{b\in B}\sum_{r\in R}\Phi_{b,r,f}Y_{b,r,f} \quad \forall \, f \in F \tag{4.4}$$

$$\sum_{f\in F} Y_{b,r,f} \leq Q_{b,r} \quad \forall b \in B, \forall r \in R \tag{4.5}$$

$$Y_{b,r,f} \in \mathbb{Z} \qquad \forall f \in F, \forall b \in B, \forall r \in R \tag{4.6}$$

$$Z_f \in \{0,1\} \qquad\qquad \forall \ f \in F \tag{4.7}$$

This is a bilevel optimization problem with the arsonists' problem in the outer level, and the wildfire managers' problem in the inner level. The objective function (4.1) includes maximizing the area burned (represented by *A*) by the escaped fires caused by a pyro-terror attack in the outer level (the attacker problem) while minimizing that in the inner level (the defender problem). We assume that fire managers can see the terrorists' attack, the number and locations of fires. The arsonists' pyro-terror attack plan is restricted by constraints (4.2) and (4.3). Constraint (4.2) sets the limit for the number of fires the terrorists can start. Constraints (4.3) are variable type constraints.

For any specific pyro-terror attack plan $\boldsymbol{W}$ chosen by the adversaries, the wildfire managers problem is restricted to set $\Psi(\boldsymbol{W})$, as defined by constraints (4.4) through (4.7). Instead of using predefined values for evaluating containment of a fire as in Haight and

Fried, and Lee et al. [39,43], we directly model fire containment condition by comparing the size of fire area at time T and the size of fire containment area built by suppression resources at time T. We use containment area construction instead of line construction as used in [35,42,111]. In our model, computing the area of a fire is simpler than its perimeter, therefore, we do not use perimeter for this reason. The containment condition is enforced using constraints (4). Fire set at location f is considered contained ($Z_f = 1$) if the total containment area built by the suppression resources dispatched to that fire is greater than or equal to the area burnt by that fire by time T. To ensure that no more resources than what is available at each fire base can be dispatched to fires, we enforce constraints (4.5). Constraints (4.6) and (4.7) are variable type constraints.

It should be noted that this model only considers the dispatch decisions and not the resource deployment decisions. In this Stackelberg game model, when attackers plan a pyro-terror wildfire, they can observe the resources that wildfire managers have deployed to each base. When the pyro-terrorists attack a landscape (as the first player in this game), wildfire managers have to dispatch the available resources from fire bases to fires to control them.

4.3 Solution methodology

The vulnerability assessment of initial attack problem (VAIAP) has a bilevel "max-min" structure with an integer programming problem in the lower level; thus, the lower level problem is not guaranteed a nonzero duality gap. Therefore, the problem does not readily lend itself to the common approach of taking the dual of the inner level problem, resulting in a single-level minimization problem. To solve the VAIAP, we develop a decomposition algorithm, called Bounded Decomposition Algorithm (BDA). BDA alternates between (i) a master problem, where the pyro-terrorists identify the ignition locations to start a wildfire, and (ii) a sub problem, where wildfire managers, having observed the location of fires, trigger an IA by optimally dispatching suppression resources to control the fires. The sub-problem and master problem are presented as follows.

4.3.1 The dispatching problem (DP) for a known pyro-terror attack

For a given pyro-terror attack $\widehat{\boldsymbol{W}} \in \Xi$ (the ignition locations are known), we denote the dispatching problem (sub problem) for wildfire managers as DP($\widehat{\boldsymbol{W}}$) which identifies the optimal dispatching of resources for wildfire scenario $\widehat{\boldsymbol{W}}$ and computes the resulting containment area constructed with those resources around the fires:

$$\text{DP}(\widehat{\boldsymbol{W}}):\ A_{min}(\widehat{\boldsymbol{W}}) = \min \textstyle\sum_{f\in F}(1 - Z_f)\,\Delta_f \tag{4.8}$$

$$\Delta_f Z_f \widehat{W}_f \leq \textstyle\sum_{b\in B}\sum_{r\in R}\Phi_{b,r,f}Y_{b,r,f}\ \forall\ f \in F \tag{4.9}$$

$$\textstyle\sum_{f\in F} Y_{b,r,f} \leq Q_{b,r} \qquad \forall b \in B, r \in R \tag{4.10}$$

$$Z_f \in \{0,1\} \qquad \forall\ f \in F \tag{4.11}$$

$$Y_{b,r,f} \in \mathbb{Z} \qquad \forall f \in F, \forall b \in B, \forall r \in R \tag{4.12}$$

$\theta_f = \sum_{b\in B}\sum_{r\in R}\Phi_{b,r,f}Y_{b,r,f}$ is the containment area constructed around fire f by the optimal dispatching of resources in response to fire scenario $\widehat{\boldsymbol{W}}$, and $\Theta = \sum_{\forall f\in F}\theta_f$ is the total containment area constructed by resources. Knowing wildfire managers' optimal dispatching plan for a specific pyro-terror attack, the initial attack vulnerability assessment problem is equivalent to:

$$A^* = \max A_{min}(\boldsymbol{W}) \qquad \boldsymbol{W} \in \Xi \tag{4.13}$$

Theoretically, we could solve (4.13) by enumerating the finite set of pyro-terror attack plans $\widehat{\boldsymbol{W}} \in \Xi$ (representing fire location scenarios), solving $\mathrm{DP}(\widehat{\boldsymbol{W}})$ for each plan, and choosing the pyro-terror plan that results in the maximum acreage burned by the escaped fires $A_{min}(\widehat{\boldsymbol{W}})$. However, in reality, Ξ is too large to enumerate. For example, for a landscape rasterized into a 100,000 cells grid network, and for pyro-terror attacks with 3-ignition points, there are $C_3^{100{,}000} \cong 8.33 \times 10^{22}$ fire ignition scenarios. Therefore, we solve (4.13) with our BDA decomposition algorithm.

For solving the VAIAP problem, we need to consider two factors: (i) The ROS value at each cell (fires grow at different rate at different locations in the landscape); (ii) Fire base locations and the amount of suppression resources available at each base (the location of fire bases can impact the initial attack's response time at a given fire location). The distance between a fire base and a fire location, along with the number of resources available at each fire base and the resources' speed to reach a target in the landscape can affect the effectiveness of initial attack at each potential fire location. Indeed if we could ignore the travel time required for a resource to reach a fire location from a fire base, then the location of fire bases would not matter at all. In addition to that, we need to consider that fires would compete for resources which add up to the complexity of problem. We take these factors into consideration in developing our BDA decomposition algorithm.

4.3.2 The Bounded Decomposition Algorithm (BDA)

We decompose the VAIAP problem into a master problem, the pyro-terrorist problem (PTP), and a sub problem, the dispatching problem (DP) shown with (4.8) through (4.12). The PTP problem is as follows:

$$\text{PTP: } \Lambda = max \sum_{f \in C} W_f \Delta_f \tag{4.14}$$

$$\sum_{f \in F} W_f \leq \beta \tag{4.15}$$

$$W_f \in \{0, 1\} \quad \forall \; f \in F \tag{4.16}$$

The BDA uses PTP to identify the fire ignition location scenarios that would result in the maximum acreage burned. Those fires, with their large potential acreage burnt, would require a larger containment area construction to be controlled. This will help us prioritize our search space in finding the optimal fire ignition location scenario (i.e. the pyro-terrorism) that IA is vulnerable to.

For any given fire ignition location scenario $\widehat{\boldsymbol{W}}$ found by solving PTP, the BDA solves DP($\widehat{\boldsymbol{W}}$) to compute the optimum dispatching of resources for fighting that fire, and evaluates whether the fire can be contained by an IA or not. If the optimal dispatching of resources for fire scenario $\widehat{\boldsymbol{W}}$ was able to contain the fire, the algorithm would update PTP with a constraint that would exclude $\widehat{\boldsymbol{W}}$ from the search space, and continue for another iteration. Otherwise, if the optimal dispatching of resources was incapable of containing the fire, then the algorithm would stop and conclude that the IA is vulnerable to the pyro-terror attack scenario $\widehat{\boldsymbol{W}}$.

In a situation where IA is not vulnerable to any wildfire scenarios, the BDA algorithm would continue until it exhausts all the fire ignition location scenarios, which would be computationally expensive as explained before. To avoid an exhaustive enumeration in such a situation, we propose a lower bound (LB) on the capability of IA on containing wildfires to bound the search space and use it as a stopping criterion. The LB is computed using the following model:

$$LB: \Theta_{LB} = min \sum_{b,r,f} \Phi_{b,r,f} Y_{b,r,f} \tag{4.17}$$

$$\sum_{f \in F} W_f \leq \beta \tag{4.18}$$

$$\sum_{b \in B} \sum_{r \in R} Y_{b,r,f} \leq \sum_{b \in B} \sum_{r \in R} Q_{b,r} W_f \qquad \forall\ f \in F \tag{4.19}$$

$$\sum_{f \in F} Y_{b,r,f} \geq Q_{b,r} \qquad \forall b \in B, \forall r \in R \tag{4.20}$$

$$Y_{b,r,f} \in \mathbb{Z} \qquad \forall f \in F, \forall b \in B, \forall r \in R \tag{4.21}$$

$$Z_f \in \{0, 1\} \qquad \forall\ f \in F \tag{4.22}$$

The objective function (4.17) is to identify a fire scenario $\boldsymbol{W}$ for which the containment area constructed by the resources is minimum. Constraint (4.18) limits the number of fires in a fire scenario. Constraints (4.19) ensure that the amount of resources dispatched to a fire is less than or equal to the amount those resources that are available at fire bases. Constraints (4.19) also ensure that resources are only dispatched to the fire locations. Constraints (4.20) ensure that all of the available resources are dispatched to fire locations, otherwise the model would converge to a solution that does not dispatch any resources at all which would result to a dispatching solution with a zero containment area. Constraints (4.21) and (4.22) are variable type constraints.

4.3.2.1 Theorem 1

Θ_{LB} is a lower bound on the capability of IA in containing any fire scenario when all the resources are dispatched.

Any dispatching plan $\widehat{D}$ that uses all the resources to construct containment area around a set of fires is a feasible solution for LB, and, therefore would construct a containment area no less than Θ_{LB}. Lets' assume $\widetilde{D}(\boldsymbol{W})$ is the optimal dispatching plan for fire scenario $\boldsymbol{W}$ that would construct the containment area $\tilde{\Theta}(\boldsymbol{W})$ around an arbitrary fire scenario $\boldsymbol{W}$. Since $\widetilde{D}(\boldsymbol{W})$ is a feasible solution for LB, therefore $\Theta_{LB} \leq \tilde{\Theta}(\boldsymbol{W})$. ■

In developing a lower bound for IA capability, as implied in theorem 1 above, we only focus on fire scenarios that require dispatching of all the resources. The reason for this is if there is a fire scenario that cannot be contained with an IA, it must be one that depletes all the resources and, even when all the resources are dispatched, cannot be contained. If, at any given iteration of BDA, the computed maximum acreage burned (Λ) by the optimal pyro-terror attack found by solving PTP was less than or equal to Θ_{LB}, then the BDA algorithm could stop to avoid an exhaustive enumeration. However, it could be the case that while the total acreage burned by a fire scenario was not greater than the total containment area built by an IA ($\Lambda(\boldsymbol{W}) \leq \Theta(\boldsymbol{W})$), there was still at least one fire for which the containment area built was less than the acreage burned by that fire, and therefore not enough to contain it (for example while 3 + 6 < 2 + 8, but still 3 > 2). To avoid this exception, we need to add an additional condition to the BDA algorithm before we use LB as a stopping criterion. That is, if at any iteration of BDA, the first condition is held ($\Lambda \leq \Theta_{LB}$), then the algorithm would check to see whether the maximum acreage burned by each individual fire is less than or equal to the minimum containment area built by the dispatching plan in Θ_{LB}. If so, then the algorithm would stop and conclude that all wildfire scenarios can be contained by IA (i.e., IA is not vulnerable to any wildfire scenarios or pyro-terrorism).

The steps of the BDA is as follows:

Step (1) Solve the lower bound (LB) problem for dispatching model and identify the corresponding fire scenario for which these bounds hold. We refer to the corresponding fire location scenario for which LB is held as $\boldsymbol{W_{LB}}$.

Step (2) Set the iteration counter $k = 1$;

Step (3) Solve the master problem PTP and compute the area burned $\Lambda(\widehat{\boldsymbol{W}^k})$ for the optimal pyro-terror attack $\widehat{\boldsymbol{W}^k}$;

Step (4) If $\Lambda(\widehat{\boldsymbol{W}^k}) < \Theta_{LB}$, then if $Max\left\{\Delta_f \,|\, f \in \{f \in F|\widehat{\boldsymbol{W_f}^k} = 1\}\right\} <$ $Min\left\{\theta_f \mid f \in \{f \in F|\boldsymbol{W_{LB}} = 1\}\right\}$ stop, there is no pyro-terror attack that cannot be contained;

Step (5) Solve the lower level problem $DP(\widehat{\boldsymbol{W}^k})$ for the optimal pyro-terror attack $\widehat{\boldsymbol{W}^k}$.

Step (6) If $A_{min}(\widehat{\boldsymbol{W}^k}) > 0$, stop; the resulting fire cannot be contained and IA is vulnerable to fir scenario $\widehat{\boldsymbol{W}^k}$.

Step (7) If $A_{min}(\widehat{\boldsymbol{W}^k}) = 0$, then set $k = k + 1$, add constraint (2.23) to the master problem and go to Step (3).

$$\sum_{q|\widehat{W_q^k}=1} W_q < \beta \tag{4.23}$$

Constraint (4.23) excludes the latest optimal pyro-terror attack $\widehat{\boldsymbol{W}^k}$ from the search space when we realize that an optimal dispatching of resources can contain the resulting fires.

4.4 Model demonstration

4.4.1 Test case problem

We test our model on a test case problem that is based on a small piece of landscape extracted from the Santa Fe National Forest which is located in northern New Mexico. The test case landscape is about 111.68 km^2 (11.49 *km* long and 9.72 *km* wide). We rasterize the landscape into a grid network with 124,092 (383 by 324) square cells, each 30 *m* by 30 *m* wide. If we represent the center of each cell as a node, and connect adjacent cells with directed edges, then the landscape can be represented with a directed network. The resulting network has 124,092 nodes and 988,498 edges.

To model the spread of fire in this landscape, we use FlamMap 5.0 to compute the rate of spread from any cell in the landscape to any of its adjacent cells. We then use the minimum travel time algorithm (MTT) [77] to compute the time requires for fire to travel from any point in the landscape to any other point. FlamMap requires some input data such as fuel models, elevation, slope, aspect and canopy in addition to wind speed and direction and fuel moisture. For this test case, we assume a prevailing west to east wind at 12 miles per hour (19.31 *km* per hour), similar to [45]. For our test case problem we acquire the landscape files (LCP), containing fuel models, elevation, slope, aspect and canopy cover, from the LANDFIRE database [105]. Table 1 shows the initial fuel moisture conditions that we use for this problem (similar to [45]).

Table 4.2 Initial fuel moisture conditions used in FlamMap

1 hour fuel moisture	6
10 hour fuel moisture	7
100 hour fuel moisture	8
Herbaceous fuel moisture	60
Live woody fuel moisture	90

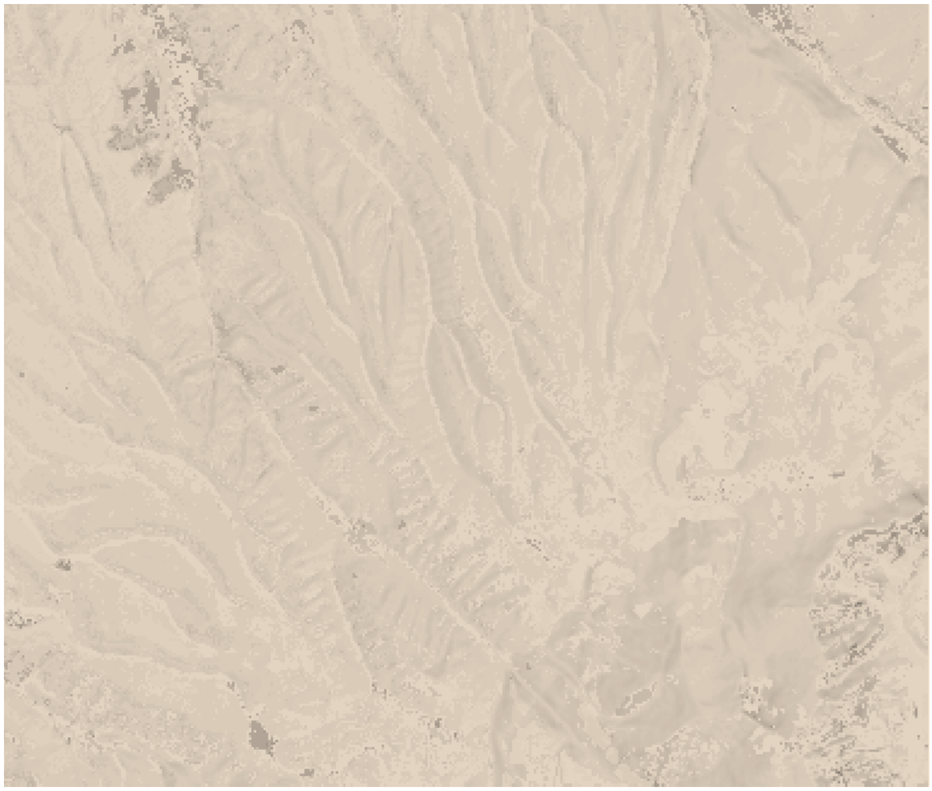


Figure 4.2 The ROS heat map for the test case landscape problem.

The ROS ranges from 0 *meter*/*minute* to 55 *meter*/*minute* (darker area shows higher ROS). Each pixel is 30 *meter* long and 30 *meter* wide.

Initial attack is generally defined as the first 1-8 hours of fire suppression effort, during which the primary objective is to contain all the fires in the shortest possible time [43]. Examples of initial attack resources that are used in suppressing fires are engines, bulldozers, hand crews, and water dropping helicopters. In this test case problem, we use three types of resources in initial attack: hand crew, small engine and large engine. This decision is based on our discussion with Santa Fe National Forest administration. Due to security concern, we do not use real data (such as the number and locations of resources) when modeling the initial attack.

Table 4.3 Initial attack resource characteristics

Resources	Average fire line production rate (*m*/*hour*) [35]
Hand crew	39
Small engine	180
Large engine	317

As mentioned earlier, in this research we use the area of fire containment constructed by resources instead of the perimeter. If the area of fire containment constructed by the dispatched resources was bigger than the area of the fire at the end of initial attack time limit, then the fire is considered contained, otherwise, it is considered escaped. The containment area that a resource can construct depends on its "fire area production rate" and the available time to construct the containment area which is the defined time limit for initial attack (i.e. the response time threshold) deducted by the time that it takes for a resource to travel from a fire base to a fire. Table 4.3 shows the "average fire line production rate" for the resources adapted from [35].

The response times for resources to travel between the fire bases and fire locations are computed based on their approximate Euclidian distance and the speed of the resources, plus a 1-hour delay between fire ignition and dispatch (time required for detecting the fire and initiating a dispatch), similar to We et al [35]. The average speed for these resources is assumed to be 56 *km*/*hour* (933.33 *meter*/*minute*) [35].

To solve the resulting optimization problem, we model the problem with Python 2.7 and use Gurobi Optimizer 6.5 on a desktop computer with 32 GB memory and an Intel (R) Core ™ i7-4770S CPU at 3.1 GH running on a 64-bit Windows 7 Operating System.

We consider three different time limits for initial attack: 4 hours, 6 hours and 8 hours (Table 4.4Table 4.5 and Table 4.6). For all these time limit scenarios, the results show that for pyro-terror attack fires with one ignition point, the IA can successfully contain the fire. However, for pyro-terror attacks with more than one ignition point, IA is incapable of containing the resulting fires.

Table 4.4 Computational results: $T = 4$ hours

Scenario	β	Computation time (sec.)	Θ_{LB}	$\Lambda(\widehat{\boldsymbol{W}})$	$A_{min}(\widehat{\boldsymbol{W}})$	Vulnerable
1	1	2197	972314	611100	996171	No
2	2	2229	985835	1217700	1002817	Yes

Vulnerability analysis of IA when the time limit for IA is 4 hours.

Table 4.5 Computational results: $T = 6$ hours

Scenario	β	Computation time (sec.)	Θ_{LB}	$\Lambda(\widehat{\boldsymbol{W}})$	$A_{min}(\widehat{\boldsymbol{W}})$	Vulnerable
1	1	2243	1491941	933300	1501865	No
2	2	2252	1494237	1855800	1501879	Yes

Vulnerability analysis of IA when the time limit for IA is 6 hours.

Table 4.6 Computational results: $T = 8$ hours

Scenario	β	Computation time (sec.)	Θ_{LB}	$\Lambda(\widehat{\boldsymbol{W}})$	$A_{min}(\widehat{\boldsymbol{W}})$	Vulnerable
1	1	2278	1998344	1309500	2000342	No
2	2	2246	1998354	2614500	2002638	Yes

Vulnerability analysis of IA when the time limit for IA is 8 hours.

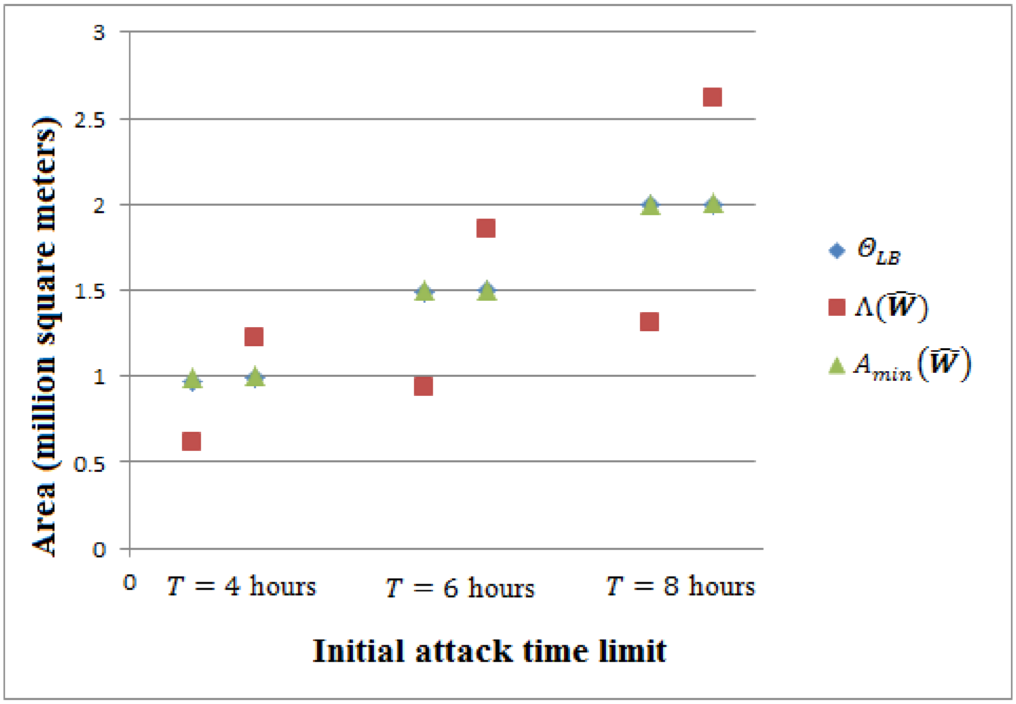


Figure 4.3 Computational results

Computational results for different initial attack time limits

As seen in Figure 4.3, as the allotted time for IA increases, so does the gap between area burned and the containment area built. This is because the rate by which the containment area is built is constant while, the rate by which the fire grows and spreads can be variant, depending especially on ROS. The results suggest a faster IA response is more promising for containing a pyro-terror fire.

For pyro-terror attacks with one ignition point (when $\beta = 1$), the BDA algorithm is terminated by the LB stopping criteria. However, for pyro-terror attacks with more ignition points (when $\beta \geq 2$), since the resulting fire cannot be contained with an IA, the algorithm stops and concludes that the IA is vulnerable to those fires.

4.5 Conclusions and future works

4.5.1 Conclusions

In this paper, we have presented the first optimization model for studying the vulnerability assessment of initial attack to pyro-terror wildfires. The model is a Stackelberg game model with max-min bilevel structure that has integer variables in the lower level. To solve the model, we develop a decomposition algorithm named Bounded Decomposition Algorithm (BDA).

We have demonstrated the use of this model on a test case problem, a landscape that covers a small portion of Santa Fe National Forest, located in New Mexico. For experimentation, we have considered three time limit scenarios for initial attack. The results indicate that although IA can contain a pyro-terror attack with one ignition point, when the number of ignition points increases, initial attack can no longer control the fire. The results also suggest that a quicker response is more effective, as fire grows, it becomes more difficult to control.

4.5.2 Future work

Initial attack includes two decisions: deploying of resources to fire bases, and dispatching of those resources to fires when they occur. In this paper, we study the vulnerability of initial attack when responding to pyro-terrorism based only on the dispatching decision. Researchers have used stochastic programming models, to identify the optimal deployment decision based on various fire location scenarios. However, in pyro-terrorism, the ignition locations are not arbitrarily; rather, they are selected such that the resulting fire can cause the maximum damage, and the likelihood of not being contained is maximized. However, this depends on the resources, types and numbers, that are available at fire stations. One can study an optimal initial attack especially designed for responding to pyro-terrorism by planning an optimal deployment of resources to fire bases such that when dispatch to a pyro-terror fire, can maximally contain it.

CHAPTER V

CONCLUSIONS AND FUTURE WORK

5.1 Conclusion

Wildfires can have serious and long-lasting impacts on ecological, social and economic systems [21]. Because of the increase in fire activities, their significant short and long term threats to forest ecosystems, and the danger they pose to public safety and property, wildfires have received increased public attention. There are some concerns that the destructive power of wildfires may attract terrorist organizations to use them as a weapon of mass destruction [11–13]. Indeed, pyro-terrorism events have been documented in France, Spain, and Greece [11,12,14]. It is, therefore, necessary, to identify and understand the impact of a potential pyro-terror attack on a landscape, and our ability to mitigate and control such a threat. In this dissertation, we study the impact of pyro-terrorism on landscapes, and the effectiveness of initial attack in responding to them. We also study the possibility of mitigating a pyro-terrorism fire using fuel management.

We first study the vulnerability of landscapes to the worst case wildfires (i.e. pyro-terrorism). We develop a maximal covering location-based formulation for the problem. We use FlamMap to model fire behavior, and use the mathematical model to identify the vulnerable areas in a landscape, the potential ignition locations for a pyro-terror attack. We use three test case landscapes for experimentation. Our results indicate that pyro-terrorism wildfires with coordinated ignition points, on average, have more than twice the impact on landscapes than natural wildfires with randomly located ignition points.

Next, we study the problem of mitigating a pyro-terror attack with fuel management. We model the problem as defender attacker Stackelberg game and develop a bilevel min-max model. We develop a decomposition algorithm called *MinMaxDA* to solve the problem, as it is not solvable by conventional methods. Three test case landscapes are used for experimentation. The results indicate that fuel management, even if conducted on small scale, can effectively mitigate the effects of a pyro-terrorism.

Suppressing a pyro-terror fire using initial attack is our focus in the next chapter. We investigate the effectiveness of initial attack in containing a pyro-terror fire by developing an attacker-defender Stackelberg model. The model is a bilevel max-min model with integer variables in the lower level, therefore, we develop a decomposition algorithm called Bounded Decomposition Algorithm (BDA) to solve the problem. We test the model on a test case landscape extracted from the Santa Fe National Forest.

5.2 Future work

For the future research, one can study the pyro-terrorism mitigation problem with multiple ignition points (multiple concurrent fires); however, such a problem is more difficult to solve. Increasing the number of ignition points causes the master problem to grow exponentially, and the number of iterations can grow as well.

When studying multiple ignition points wildfires (concurrent fires), one can take the interaction of fires into consideration, as we neglect the interaction effects. Fire behavior and characteristics can dramatically change in presence of another fire [70], and therefore, they can cause more damage that it was shown in our research, and as a result, they can be more difficult to contain. We also did not include spot fires in this study; they can increase wildfires risks by helping them spread faster [71]. For a more accurate assessment, a study can include spot fires into account as well.

Initial attack includes two decisions: deploying of resources to fire bases, and when fires occur, dispatching of those resources to fires. We studied the vulnerability of initial attack when responding to pyro-terrorism based on dispatching the available resources that have already been deployed to fire bases. Researchers have used stochastic programming models, considering the uncertainty of the ignition locations using various scenarios, to identify the optimal deployment decision. However, in pyro-terrorism, the ignition locations are intelligently selected so that the resulting fire causes the maximum damage, and is more difficult to contain by initial attack. One can study an optimal initial attack especially designed for responding to pyro-terrorism by planning an optimal deployment of resources to fire bases such that when dispatch to a pyro-terror fire, can maximally contain it.

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