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Sublimation Rate of Vertically Oriented Naphthalene Cylinders in a Natural Convection Environment

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SUBLIMATION RATE OF VERTICALLY ORIENTED NAPHTHALENE CYLINDERS IN A NATURAL CONVECTION ENVIRONMENT

by

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Abstract

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The naphthalene sublimation technique was used to determine the rate of mass transfer from three solid naphthalene cylinders in a natural convection environment. The cylinders' diameters measured 1, 1.5, and 2 inches nominally, and sublimation rates were found to be 1.42×10^{-8}, 1.89×10^{-8}, and 2.28×10^{-8} kg/s, respectively. The mass transfer coefficients were determined to be 1.64×10^{-3}, 1.45×10^{-3}, and 1.31×10^{-3} respectively. Correlations were developed for the Sherwood vs. Rayleigh numbers, Sherwood vs. Grashof numbers, and mass transfer coefficient vs. diameter.

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Chapter 1. Introduction

1.1 Objective

This study experimentally determined the mass transfer coefficient for a naphthalene cylinder vertically oriented in a natural convection environment. Three cylinders with different diameters were used to develop a correlation between the cylinder diameter and the mass transfer coefficient. Correlations between the Rayleigh and Sherwood and also Grashof and Sherwood numbers were determined.

1.2 Literature Review

Determination of the heat transfer coefficient for a specific geometry is necessary for the analysis of many heat transfer systems. Unfortunately, finding this coefficient by means of a heat transfer experiment may require complex, expensive instrumentation and complicated measurements. Instead, it is possible to conduct a mass transfer experiment and convert the results into heat transfer results by means of an analogy. Mass transfer experiments conducted using the sublimation technique are simple, accurate, and inexpensive. They also eliminate the problem of heat leakage by radiation and conduction, and avoid uncertainties associated with possible preheating of fluid along the lateral edges of the specimen [5,6].

Mass transfer studies using the naphthalene sublimation technique may be conducted in forced or natural convection environments. The scope of this experiment is limited to mass transfer that is driven by buoyancy forces in a natural convection environment. These buoyancy forces are created as a boundary layer comprised of vapor is formed on the surface of the specimen [5]. Since the vapor density is greater than that of air, the vapor flows downward from the specimen.

Goldstein conducted several studies using the naphthalene sublimation technique. Goldstein and Lau [6] investigated the influence of extensions at the plate's edges on the rate of mass transfer. Six different flat plate geometries were investigated in a natural convection environment. The rate of mass loss was determined. Numerical solutions for the problem were also determined using finite differences. It was determined that extensions added to the plates horizontally had no affect on the mass loss, however the extensions added vertically lowered the mass transfer rate significantly.

Goldstein, Sparrow, and Jones [5] conducted a mass transfer study on three solid naphthalene plates in a natural convection environment. The plates were circular, square, and rectangular. The mass transfer coefficients were determined, as well as the Sherwood number, which was related to the Rayleigh number. A correlation for the characteristic length of the plates was developed.

Anderson [1] determined the sublimation rate for three sizes of paradichlorobenzene spheres. Solid paradichlorobenzene spheres were cast using rubber tennis ball molds, and hung underneath a scale. The spheres were allowed to sublimate in a natural convection environment until distorted. The rate of mass lost over time was determined, and an equation was produced that related the resulting Sherwood and Rayleigh numbers.

Bautista [2] studied two different size paradichlorobenzene cylinders hung horizontally underneath a balance in a natural convection environment. The sublimation rate was determined for each followed by the Grashof number. The mass flux and heat flux were determined for the cylinders.

Snapp [12] hung paradichlorobenzene cylinders of three different diameters vertically in a natural convection environment. The sublimation rate for the cylinders was determined and equations relating the Sherwood and Rayleigh numbers were developed. The same procedure from the Snapp experiment was used for this work in order to investigate possible sources of error. In addition, future work may be done to compare the results of this work to Snapp's results in a dimensionless parameter study.

Chapter 2. Theory

2.1 Equations

The objective of a mass transfer experiment is to determine the dimensionless Schmidt and Sherwood numbers for a specific geometry. To begin the calculations for this study, the rate of sublimation for each cylinder was calculated by taking the change in mass (kg) lost from the cylinder over the time (seconds) of the experiment

$$\dot{m} = \frac{\Delta m}{\Delta t} \tag{3}$$

Where $\Delta m/\Delta t$ was measured for three different cylinders. Next, the density of the naphthalene at the surface of the cylinder was calculated from the ideal gas law. The vapor pressure over the ambient temperature and the gas constant for naphthalene yield the density at the surface.

$$\rho_s = \frac{p_v}{RT} \tag{4}$$

Where p_v is the vapor pressure of naphthalene, R is the gas constant of naphthalene vapor, and T is the temperature in absolute units. The ratio of momentum and mass diffusivities, or Schmidt number, was calculated by dividing the kinematic viscosity of naphthalene by the mass diffusivity, or diffusion coefficient.

$$Sc = \frac{v_n}{\mathcal{D}_n} \tag{5}$$

Where v_n is the kinematic viscosity of naphthalene and $\mathcal{D}_n$ is the mass diffusivity. The Grashof number, was calculated from the definition:

$$Gr = \frac{g\,(\rho_s - \rho_\infty) D^3 \rho_{air}}{\mu_{air}^2} \tag{6}$$

Where g is gravity, D is the diameter of the cylinder, ρ_s is the surface density of naphthalene, ρ_{air} is the density of air, and μ_{air} is the dynamic viscosity of air. The

Grashof number was calculated for each cylinder. The Rayleigh number was also calculated for each cylinder. The Rayleigh number is the product of the Grashof number and the Schmidt number.

$$Ra = Gr * Sc \tag{7}$$

The mass transfer coefficient for each cylinder was calculated next using the equation:

$$h_m = \frac{\dot{m}}{A_s(\rho_s - \rho_\infty)} \tag{8}$$

Where A_s is the surface area. The Sherwood number was also calculated for each cylinder.

$$Sh = \frac{h_m D}{\mathcal{D}_n} \tag{9}$$

The constants used for these calculations are shown in Table 1.

Table 1. Constants used in the calculations.

Vapor Pressure of Naphthalene [7]	p_v	8.206 Pa
Ambient Temperature	T	72 (295 K)
Density of Air [9]	ρ_{air}	$1.265 \frac{kg}{m^3}$
Dynamic Viscosity of air [9]	μ_{air}	$1.79 \text{x} 10^{-5}$ Ns/m^2
Kinematic Viscosity of Naphthalene [9]	νn	$1.41 x 10^{-5}\ m^2/s$
Mass Diffusivity [11]	$\mathcal{D}_n$	$0.059\ cm^2/s$
Gas Constant of Naphthalene Vapor [7]	R	64.89 J/mol*K
Gravity	g	9.81 m/s^2

2.2 Assumptions

Several assumptions were made in this experiment. First, it was assumed that the physical properties of naphthalene vapor behave as a perfect gas. In addition, it was assumed that the density of naphthalene is zero at a significant distance from the cylinder. This assumption was permissible because the facility in which that experiment took place was significantly large. The surface of the cylinder was assumed to be isothermal, and was assumed to be the same temperature as the surrounding air. It was also assumed that there was no heat transfer occurring simultaneously during the mass transfer, and no chemical reactions were taking place. Finally, mass diffusion due to gradients of substances other than naphthalene was assumed to be negligible. Sparrow [15] proved that the concentration of water in the air (relative humidity) does not affect the mass transfer rate of naphthalene. In the same study, Sparrow also showed that the transverse velocity of the fluid flow at the surface of the naphthalene is negligible.

Chapter 3. Procedure

3.1 Molding

The experimental procedure began with the assembly of the aluminum molds. Each mold consisted of two halves each with six bolts, a wax plug with a hole to hold a wooden skewer, and an aluminum end cap, see Appendix C. Before assembling the molds, the inside of each mold half was cleaned with rubbing alcohol and then sprayed with Pam cooking spray. The Pam left a slick coating on the mold surface so that the cylinder could be removed from the mold without sticking and breaking. Next the wax plugs and aluminum end caps were put in place, and the wooden skewers were inserted into the holes in the wax plugs. The end caps and plugs were put in place before bolting the molds together to ensure proper alignment of the halves of the molds as well as to secure a snug fit for the wax plugs. Finally, the bolts were inserted and tightened with a hex wrench.

Next the naphthalene was melted and the molds were filled. The naphthalene was melted in a standard aluminum quart size pot with a glass lid using a propane camping stove. The mold halves were also warmed on the stove to prevent flash cooling of the naphthalene. To fill, the assembled molds were positioned vertically and the aluminum end caps were removed. A plastic funnel was used to fill the mold with molten naphthalene from the pot on the stove. Extra naphthalene was added to the top of the mold after the naphthalene cooled for a minute or two to ensure that the mold was completely full. Finally, the aluminum cap was inserted, and the wooden skewer was placed through the hole in the cap for proper vertical alignment of the skewer in the mold. The mold was allowed to sit for 4 to 5 hours to allow the specimen to sufficiently solidify and cool to the temperature of the surrounding air.

The cylinders were removed from the molds by removing the aluminum end caps, unbolting the mold halves, and laying the molds on their side. The molds were gently tapped until the cylinders slid out. The wax plugs remained attached to the cylinder, and had to be removed with caution as to not rotate or loosen the wooden skewer. A paper towel was used to wipe the surface of the cylinders to remove any Pam residue.

3.2 Testing

The cylinder was hung from the balance by attaching a clamp to the skewer, and then attaching a steel wire from the clamp to the undercarriage of the scale. Care was taken to make sure that the cylinder hung from the scale with its axis completely vertical. Nylon string was used as a precaution to ensure that the cylinder did not change position in the air or fall during the experiment.

The scale was zeroed and connected to a computer that recorded a weight measurement every minute, and LabX Software automatically input the data to a spread sheet. Since the initial weight measurement was taken at the start of the experiment. Errors that may have resulted due to transient mass loss between the initial measurement and beginning of the experiment were thus eliminated. The cylinder was left attached to the scale in a room held at constant temperature, and free of drafts. Calipers were used to periodically measure the cylinders' diameter at three different locations in horizontal plane (x) and three locations in the vertical plane (y) (see Figure 1). The experiment was completed after a cross sectional measurement showed 0.1 inch loss, selected arbitrarily, from the initial diameter measurement. The cylinder was allowed to deform by no more than 0.1 inch so that the flow pattern was not modified as a result of a change in shape of the cylinder [14].

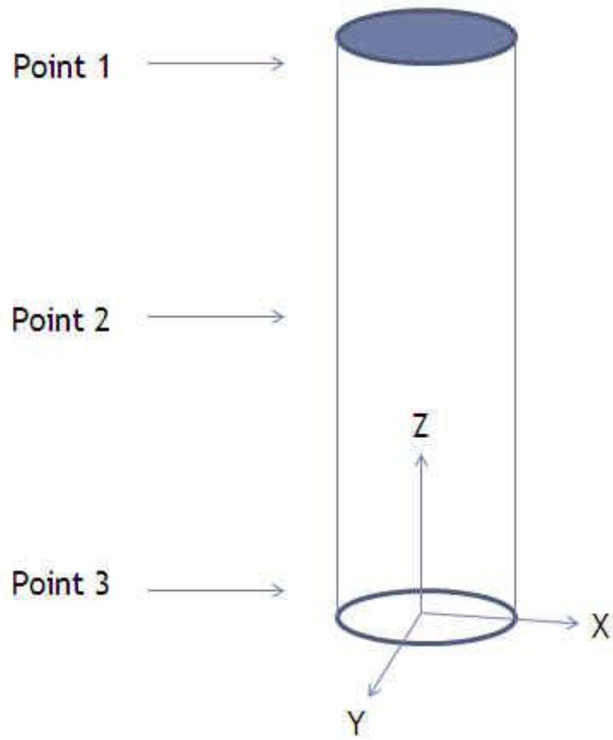

Figure 1. Illustration of cylinder diameter measurements. The measurements were taken in the X and Y planes at 3 points along z axis periodically during experiment. A ruler was used to measure from the top edge of the cylinder to the location for each point to ensure consistency. Points 1, 2, and 3 were taken at 1, 5, and 9 inches respectively.

Chapter 4. Results and Discussion

4.1 Experimental Results

The initial diameter of each cylinder was measured with calipers, and the surface area was calculated, see Table 2. After completion of this procedure, data collected were used to perform the required calculations to find the mass transfer coefficient (h_m) and other required dimensionless numbers (Sh, Ra, Gr). The calculated data for each cylinder is provided in Table 3.

Table 2. Measured and calculated dimensions for each cylinder.

	Nominal Cylinder Diameter (in)		
	1.0	1.5	2.0
Diameter (m)	0.025	0.0373	0.0499
Surface Area(m^2)	0.0204	0.0329	0.0426

Table 3. Calculated values for each cylinder after completion of experimental procedure.

	Nominal Cylinder Diameter(in)		
	1.0	1.5	2.0
$\dot{m}$ (kg/s)	$1.42x10^{-8}$	$1.89x10^{-8}$	$2.28x10^{-8}$
Gr	259	861	2060
Ra	620	2060	4930
$h_m\ (m/s)$	$1.63x10^{-3}$	$1.34x10^{-3}$	$1.25x10^{-3}$
Sh	6.89	8.48	10.6

A correlation was determined for the cylinder diameter and sublimation rate by plotting the corresponding values and finding a trend line (see Figure 1). In a similar manner, correlations were developed for the Sherwood and Rayleigh numbers as well as the Sherwood and Grashof numbers, see Figures 2 and 3 respectively. For all three correlations, the coefficient of determination (r^2) was greater than or equal to 0.95, an acceptable range of variation verifying that the plotted values fit a linear relationship. A sample calculation is provided in Appendix B.

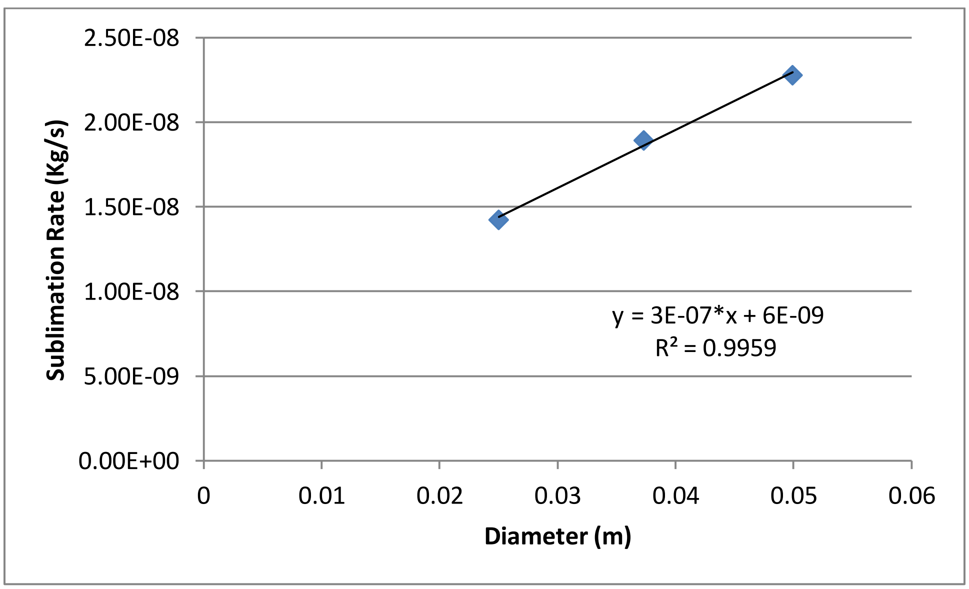

Figure 2. Sublimation rate correlated to the cylinder diameter. In the equation, y corresponds to the sublimation rate ($\dot{m}$) and the x to the diameter (D).

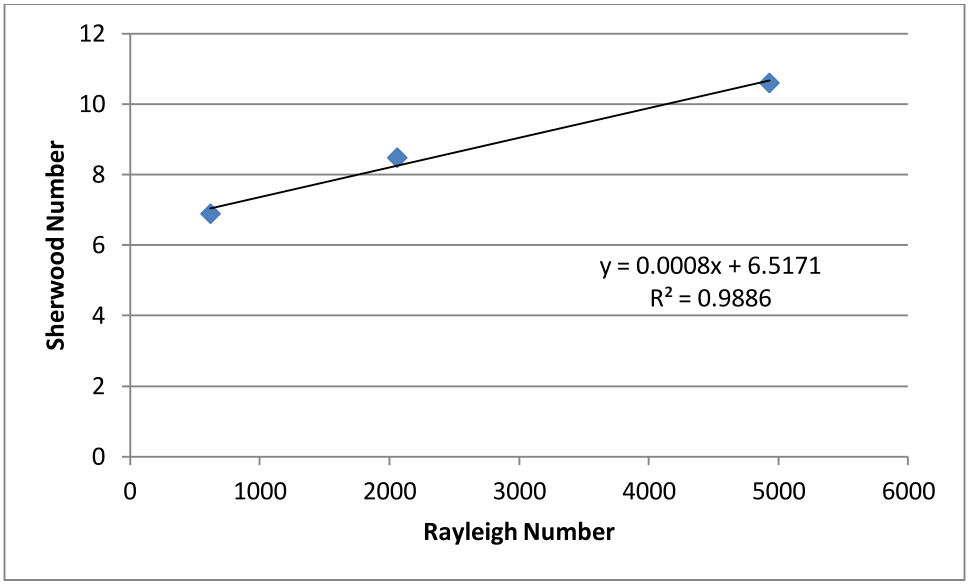

Figure 3. Sherwood number as a function of the Rayleigh number. In the equation, the y corresponds to the Sherwood Number (Sh) and the x to the Rayleigh number (Ra).

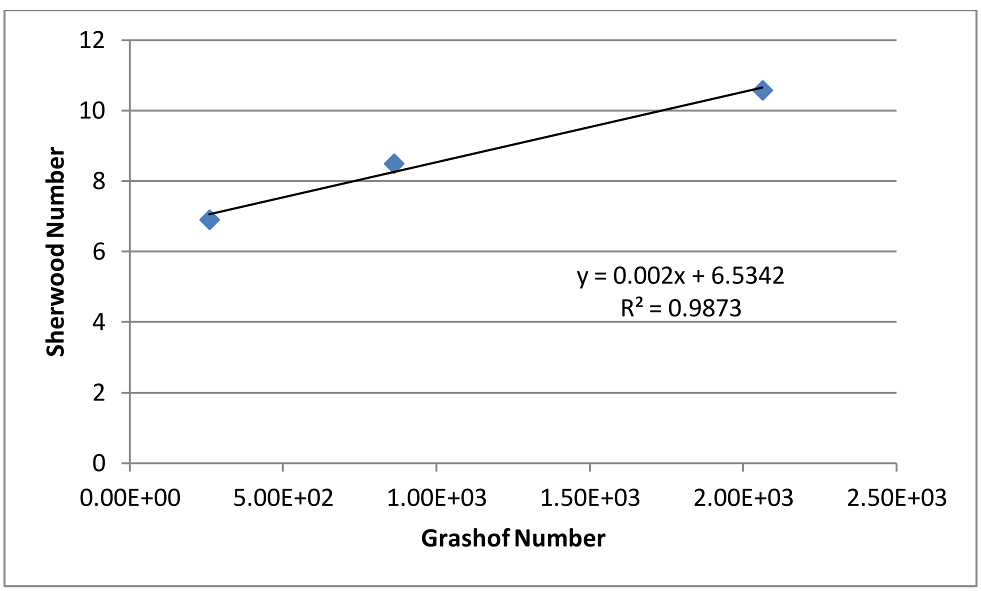

Figure 4. Sherwood number as a function of the Grashof number. In the equation, the y corresponds to the Sherwood number (Sh) and the x to the Grashof number (Gr).

An uncertainty analysis was performed in a method described by Moffat [10]. The calculated values are shown in Table 4.

Table 4. Calculated uncertainties (+/-) for all of the measured and calculated values.

	Nominal Cylinder Diameter (in)					
	1.0		1.5		2.0	
	-	+	-	+	-	+
$D\ (m)$	1.27E-05	1.27E-05	1.27E-05	1.27E-05	1.27E-05	1.27E-05
$A_s\ (m^2)$	5.11E-4	5.11E-4	2.24E-3	1.22E-3	3.63E-4	6.61E-4
$\dot{m}$ (kg/s)	3.20E-14	2.28E-14	4.93E-10	4.93E-10	4.28E-11	4.28E-11
Gr	.395	.395	.879	.879	1.57	1.58
Ra	.900	.989	1.96	2.25	3.41	4.11
$h_m(m/s)$	4.31E-5	3.84E-5	6.2E-5	1.63E-5	9.40E-6	2.00E-5
Sh	.179	.104	.386	.106	.076	.170

4.2 Discussion

Several potential sources of error exist surrounding this study. Natural convection environments are very difficult to keep perfectly constant and fluctuations in temperature or air flow can affect the results. For example, a 1° temperature change can result in a 10% change in the vapor pressure of the naphthalene at the surface of the cylinder [14]. Also, the effects of changes in atmospheric pressure were not evaluated in this study. It is possible that slight atmospheric pressure fluctuations occurred as each cylinder took over a week to reach the desired level of deformation. In addition, slight unseen deformations in the surface of the naphthalene could have occurred during the molding and manufacture of the cylinder specimen. Error could have occurred in the suspension of the cylinder. If the cylinder was not perfectly vertical it may have slightly changed the flow field. While care was taken to remove the Pam residue with a paper towel, minute amounts remaining may have affected the sublimation process.

It was observed that the surface deformations occurred most quickly at the top of the cylinder (position 1, see Figure 1). This was likely due to the accumulation of naphthalene vapor near the bottom of the cylinder as it fell from the top of the cylinder to the floor. This accumulation resulted in a lesser mass fraction gradient at the bottom of the cylinder than the top.

4.3 Recommendations for future work

In future work, the data collected in this study may be converted to heat transfer data using one of the heat and mass transfer analogy equations. Also, this experiment may be conducted using other various geometries and orientations. In addition, the results from this experiment could be compared to the Snapp [12] experiment in a dimensionless parameter study.

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Appendix

Appendix A. Nomenclature

Appendix B. Sample Calculations

Appendix C. Illustrations, Photographs, and Additional Graphs

Appendix A: Nomenclature

A_s	Surface area of specimen (m^2)
D	Diameter of specimen (m)
$\mathcal{D}$	Mass diffusivity (m^2/s)
$\mathcal{D}_n$	Mass diffusivity of naphthalene (m^2/s)
g	Gravity (m/s^2)
Gr	Grashof number
h_m	Coefficient of mass transfer (m/s)
$\dot{m}$	Rate of mass loss or sublimation rate (kg/s)
Δm	Change in mass (kg)
μ_{air}	Dynamic viscosity of air (Ns/m^s)
p_v	Vapor pressure of naphthalene (Pa)
ρ	Density (Pa)
ρ_s	Surface density of naphthalene (kg/m^3)
ρ_∞	Density of naphthalene at a significant distance from the cylinder
ρ_{air}	Density of air (kg/m^3)
R	Gas constant for naphthalene vapor ($J/Kmol$)
Ra	Rayleigh number
Sc	Schmidt number
Sh	Sherwood number
T	Temperature (K)
Δt	Change in time (s)
ν_n	Kinematic viscosity of naphthalene (m^2/s)
ω	Mass fraction

Appendix B: Sample Calculations

Sample calculations for the 1 inch cylinder

$$\dot{m} = \frac{\Delta m}{\Delta t} = \frac{(157.7 - 144.37)g}{15603\ min} * \frac{(1\ kg)(1\ \text{min}\quad)}{(1000g)(60s)} = 1.424x10^{-8}\ kg/s$$

$$Sc = \frac{\nu_n}{\mathcal{D}_n} = \frac{1.41x10^{-5}\ m^2/s}{.059\ cm^2/s} * (\frac{100\ cm}{1\ m})^2 = 2.39$$

$$Gr = \frac{g\ (\rho_s - \rho_\infty)D^3\rho_{air}}{\mu_{air}^2}$$

$$= \frac{\left(9.81\frac{m}{s^2}\right)(4.28\ x\ 10^{-4}\ kg/m^3 - 0\ )(.0254\ m)^3(1.265\frac{kg}{m^3})}{(1.79x10^{-5}\frac{Ns}{m^2})^2} = 259$$

$$Ra = Gr * Sc = (2.39) * (\ 259) = 618$$

$$h_m = \frac{\dot{m}}{A_s(\rho_s - \rho_\infty)} = \frac{1.424x10^{-8}\ kg/s}{(.02027\ m^2)(4.28\ x\ 10^{-4}\ kg/m^3 - 0\ )} = 0.00163\ m/s$$

$$Sh = \frac{h_m D}{\mathcal{D}_n} = \frac{(.00163\ m/s) * (.0254\ m)}{.059\ cm^2/s} * (\frac{100\ cm}{1\ m})^2 = 6.89$$

Sample Calculation for Uncertainty Analysis of Grashof Number

$$\delta_{caliper} = .5 * smallest\ increment = 1.27\ x\ 10^{-5}\ \text{m}$$

$$Gr_{upper} = \frac{g\ (\rho_s - \rho_\infty)(D + \delta_{caliper)}{}^3 \rho_{air}}{\mu_{air}^2}$$

$$= \frac{\left(9.81\frac{m}{s^2}\right)(4.28\ x\ 10^{-4}\ kg/m^3 - 0\ )(.0254\ m + 1.27\ x10^{-5}m)^3(1.265\frac{kg}{m^3})}{(1.79x10^{-5}\frac{Ns}{m^2})^2} = 260$$

$$Gr_{lower} = \frac{g\ (\rho_s - \rho_\infty)(D - \delta_{caliper)}{}^3 \rho_{air}}{\mu_{air}^2}$$

$$= \frac{\left(9.81\frac{m}{s^2}\right)(4.28\ x\ 10^{-4}\ kg/m^3 - 0\ )(.0254\ m - 1.27\ x10^{-5}m)^3(1.265\frac{kg}{m^3})}{(1.79x10^{-5}\frac{Ns}{m^2})^2} = 259$$

Appendix C: Illustrations, Photographs, and Additional Graphs

Figure 5. Material used in experiment. 99.95% pure naphthalene in the form of moth balls. Available at hardware stores.

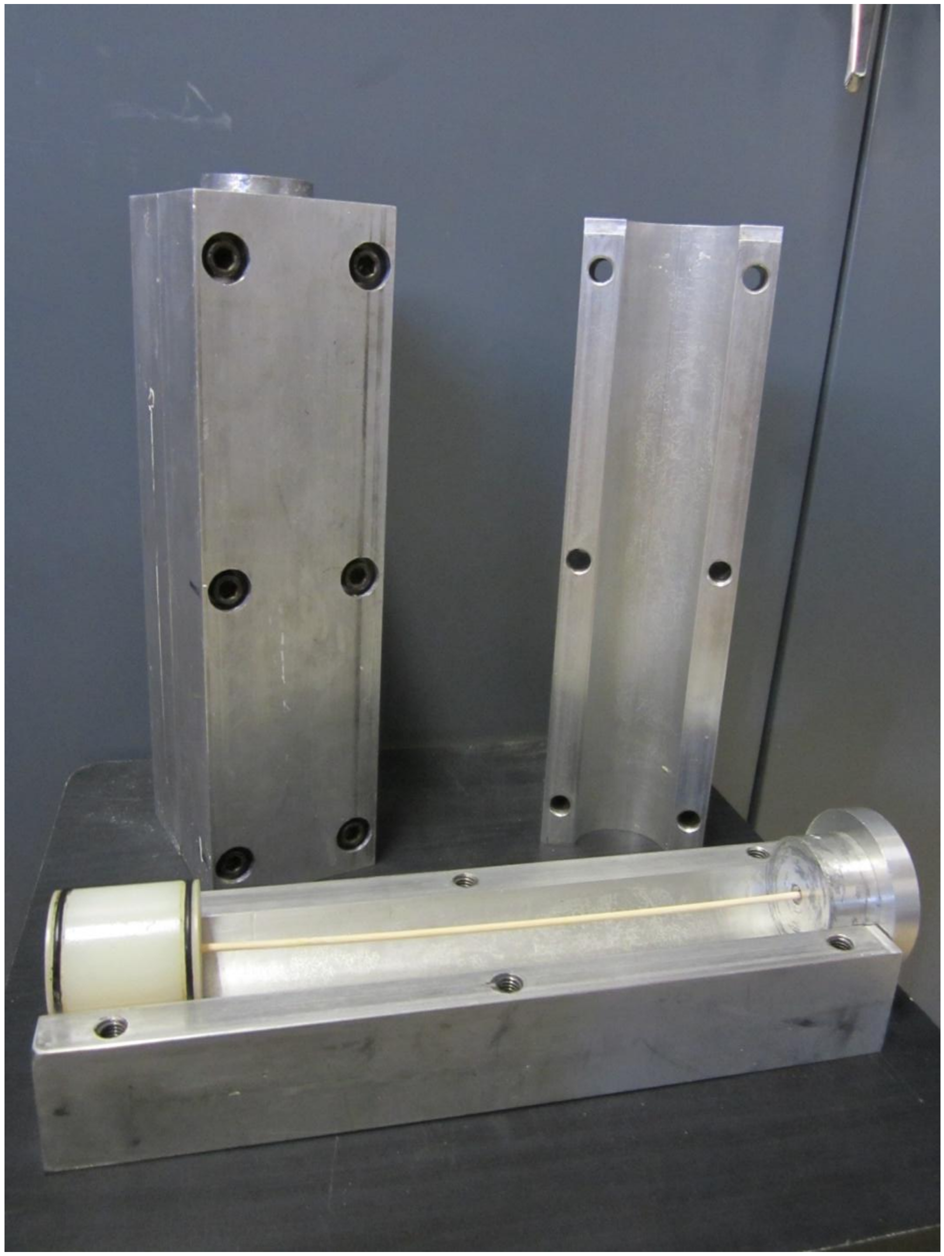

Figure 6. Aluminum molds used in experiment. Pictured on the back left is the mold fully assembled.

Figure 7. Experimental set up in the lab.

Figure 8. Cylinder hanging vertically underneath scale during procedure.

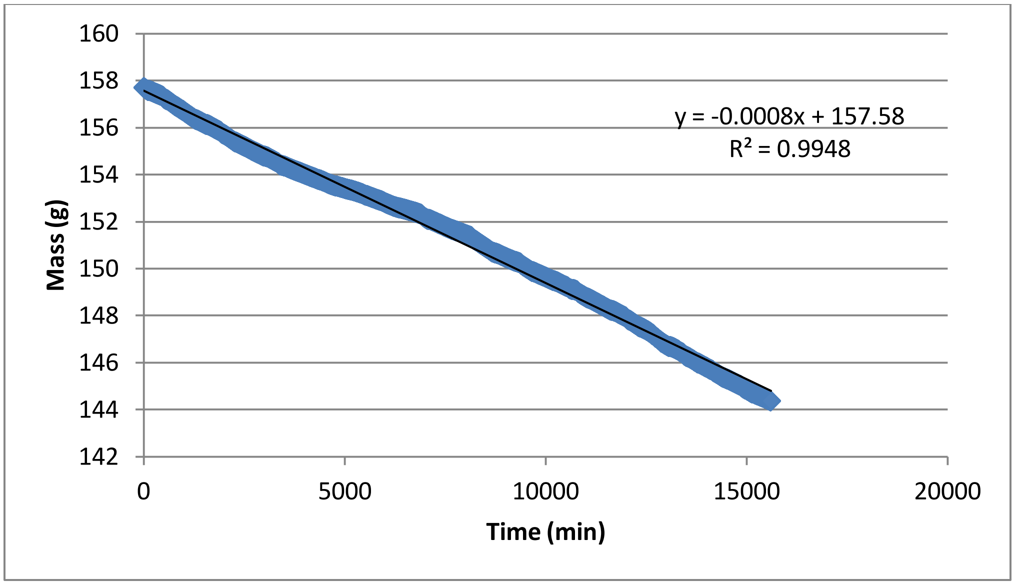

Figure 9. Sublimation rate for the 1 inch nominal diameter cylinder. A scatter plot of experimental data with trend line.

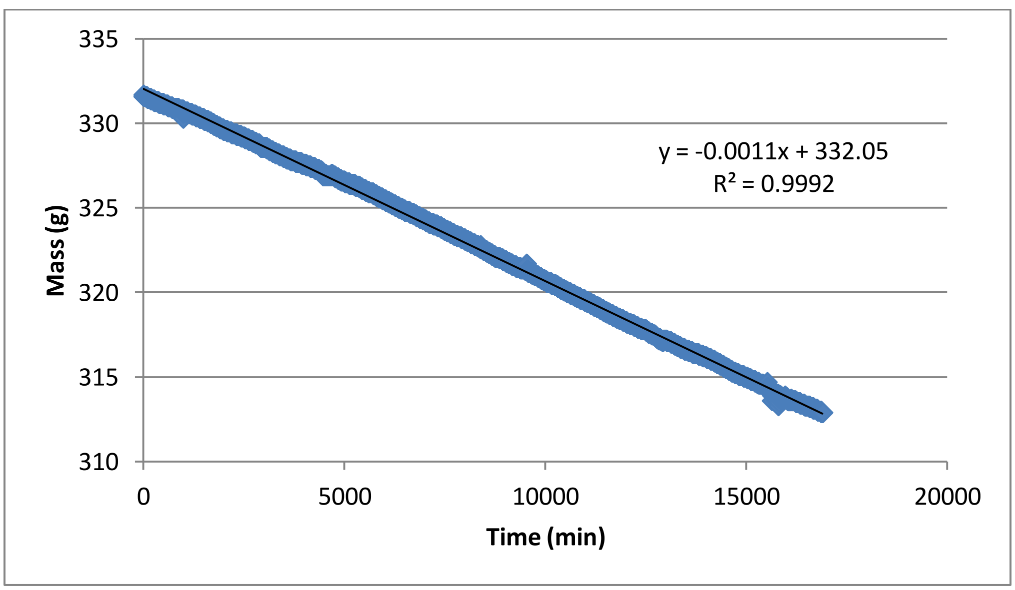

Figure 10. Sublimation rate for the 1.5 inch nominal diameter cylinder. A scatter plot of experimental data with trend line.

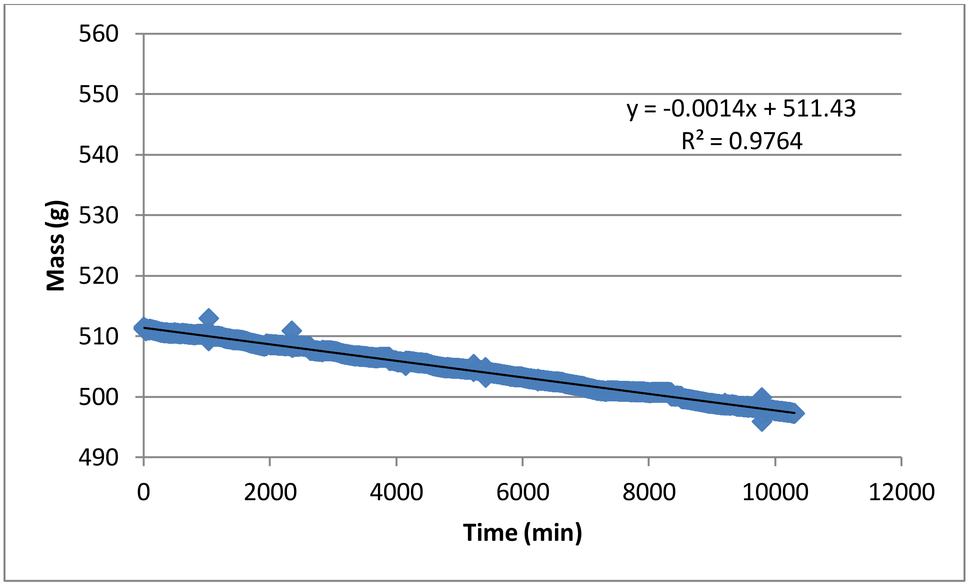

Figure 11. Sublimation rate for the 2 inch nominal diameter cylinder. A scatter plot of experimental data with trend line.