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Learning from errors in the adaptive mathematics tutoring system

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LEARNING FROM ERRORS IN THE ADAPTIVE MATHEMATICS TUTORING SYSTEM

by

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Abstract

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Errors are considered to play a crucial role in facilitating self-reflection and knowledge acquisition. However, whether help is superior to practice for learning from errors is still open to debate. The goal of this dissertation is to systematically explore how students use help and practice to learn from errors in ALEKS (i.e. Assessment and LEarning in Knowledge Spaces), an adaptive math learning system.

The main results suggested that students tended to utilize help and practice together after making errors. Students were inclined to use the strategy of requesting a worked example and then solving a problem after an error in the low-skill phase, the strategy of solving a problem and then requesting a worked example after an error in the medium-skill phase, and practice in the high-skill phase. For students with the low prior knowledge, the strategies tended to transition to practice after the next error whereas students with the high prior knowledge were apt to transition to the strategy of requesting a worked example and then solving a problem.

College students' delayed performance benefited from help, the strategy of requesting a worked example and then solving a problem after an error, the strategy of giving two wrong answers after an error, and the strategy of giving a wrong answer and then a correct answer after an error. However, delayed performance was hindered by the strategy of giving a wrong answer and then requesting a worked example after an error, the strategy of giving a correct answer and then a wrong answer, and the strategy of giving two correct answers after an error. In the low-skill phase, the shifts to practice strategies helped college students' delayed performance. In the medium-skill and the high-skill phases, delayed performance was boosted by the shifts to strategies involving requesting worked examples.

The study helped us to discover students' strategies to learn from errors in the adaptive learning system and to build a foundation for a finer investigation on students' strategies to learn from errors. The findings expected to fuel insights to understand students' learning strategies and improve the effectiveness of intelligent tutoring systems.

Keywords: learning from errors, worked example, practice, learning phase, prior knowledge, error types, topic difficulty, learning outcomes, math, adaptive learning system,

TABLE OF CONTENTS

LISTS OF TABLES

Tables Page

LISTS OF ABBREVATIONS

E	Requesting worked examples
W	Giving a wrong answer to problems
C	Giving a correct answer to problems
S	Mastering a topic: Three C's in a row within a single topic
F	Failing a topic: Five W's in a row within a single topic
EE/ Help strategy	Requesting worked examples in the next two steps after an error
Example-practice	Requesting a worked example then solving a problem in the next two steps after an error
Practice-example	Solving a problem then requesting a worked example in the next two steps after an error
Practice strategies	Solving problems in the next two steps after an error
EW/ Example-wrong	Requesting a worked example then giving a wrong answer in the next two steps after an error
EC/ Example-correct	Requesting a worked example then giving a correct answer in the next two steps after an error
WE/ Wrong-example	Giving a wrong answer then requesting a worked example in the next two steps after an error
CE/ Correct-example	Giving a correct answer then requesting a worked example in the next two steps after an error
WW/ Wrong-wrong	Giving wrong answers in the next two steps after an error
WC/ Wrong-correct	Giving a wrong answer then a correct answer in the next two steps after an error
CW/ Correct-wrong	Giving a correct answer then a wrong answer in the next two steps after an error
CC/ Correct-correct	Giving correct answers in the next two steps after an error

Chapter 1: Introduction

Educators and learning scientists have a longstanding interest in the impact of errors that occur during the learning. Their research on errors has covered broad branches of learning, such as reading (Tsai, Ouyang, & Chang, 2016), mathematics (Loibl & Rummel, 2014), physics (Horiguchi, Imai, Toumoto, & Hirashima, 2014), and language acquisition (James, 1998). Two divergent interpretations of the effects of errors on learning have emerged from a sizable literature of past studies. On one hand, it is thought that errors lead to anxiety (Heissen, Glass, & Knight, 1987), and undermine learners' self-efficacy (Compeau & Higgins, 1995). However, in the recent decades, a more positive view of errors during learning has developed. For example, researchers found that errors can boost long-term retention and transfer (Lee, 2012), and should be treated as an essential component of effective learning (Bjork, Dunlosky, & Kornell, 2013). Therefore, overall learners could learn from errors even though errors may cause temporary negative impacts on learning.

Endeavoring to understand the effect of errors on learning is not the only interest of educators and learning scientists. They are eager to know why errors can promote learning as well. Abundant existing studies on errors agree that self-explanation on errors is essential for fostering learning from errors. Effective self-explanation has been observed to enable learners to be aware of their knowledge gaps and assist in regulating their subsequent learning (Atkinson, Renkl, & Merrill, 2003; Chi et al., 1989). Furthermore, the worked examples approach can effectively prompt learners to generate self-explanations (Große & Renkl, 2007). However, self-explanations are difficult to observe in a natural learning environment, because they are internally generated by the student. Therefore, in lieu of observing self-explanation, this study investigated learning from errors from the perspective of overt learning behaviors, which involved help-seeking and practice.

According to the existing literature, not many studies have examined learning from errors from the standpoint of external learning behaviors within e-learning settings (Shut, 2008). In addition, worked-example is considered as effective instructions in math (Große & Renkl, 2007). As a result, the current study sought to take the first step by systematically investigating how students learned from errors by using help and practice in a tutoring system that has worked examples. By taking this behavioral approach to understand learning from errors, this study should provide guidance for better tracking student's learning process in e-learning systems. Furthermore, this study should also better inform the interventions used by e-learning systems, ultimately resulting in increased learning gains.

Chapter 2: Literature review

Taxonomy of errors

This study explored learning errors within the mathematics domain. This section first illustrates how the taxonomies of errors evolved from a general setting to a specific e-learning setting. As well, the classifications of math errors are described in this section.

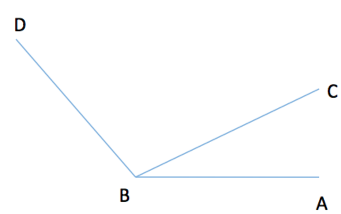
Generic taxonomy of errors

Almost everyone experiences errors on a daily basis. The definition and impact of these errors shift based on their domains. In the Oxford dictionary, an error is defined as, "the state or condition of being wrong in conduct or judgment." This definition clearly illustrates that behavioral errors should be distinguished from cognitive errors. In a similar fashion, Norman (1981) proposed a definition of "error" that influenced the development of a taxonomy for errors. He stated an error caused by inappropriate intention as a mistake. For example, when solving "x+2=5", a student needs to subtract 2 on both sides of the equation to find the correct answer. If he or she meant to add 2 on both sides of the equation, then the wrong answer resulting from this intention should be considered a mistake. Another type of error, a "slip", is an error resulting from actions that deviate from the original intention. For example, if a student wrote 5 instead of 6, or wrote "+" instead of "-" unintentionally, then the error would be considered a slip. Thus, Norman divided errors into two general categories: mistakes and slips.

Reason (1990) refined Norman's (1981) taxonomy of errors in terms of the skill, rules, and knowledge-based (SRK) approach. In Reason's definition, errors are classified into three types: skill-based level errors, rule-based level errors, and knowledge-based level errors (Figure 1). Skill-based level errors occur unconsciously in high skilled or well-prepared activities. A skill-based level error is considered as a slip due to a misapplied skill. For example, in the problem of “Solumedrol 1.5 mg/kg is ordered for a child weighing 73.6 lb. Solumedrol is available as 125 mg / 2mL. How many mL must the nurse administer?”, if a student who knew how to calculate dose did not notice weight being measured in lb instead of kg, the student is very likely to make a slip by multiplying 73.6 to 1.5 directly.

Reason separated mistakes into rule-based level and knowledge-based level errors. Rule-based level errors are caused by an incorrect diagnostic rule. For example, in the problem of:

> If the measurement of Angle ABC is 35°, can you estimate the measurement of CBD? If yes, write down your answer. If no, write “no”.



A student provided a wrong answer by stating that angle CBD was 145° (i.e. 180°-35°) because the total measurement of the two angles was unknown in the problem. Therefore, the student made a rule-based level error due to misuse of geometry rules. Knowledge-based level mistakes were induced by a lack of expertise. Individuals tend to make knowledge-based level mistakes when in high stress or unfamiliar situations. Consider the word problem of:

> Jane bought fresh cookies in Whole foods. The price per cookie was \$0.75. Also, Jane bought a box that was \$0.3 to contain cookies. In total, she paid \$3.54 to the cashier. How many cookies did Jane buy?

A student who does not solve this algebra word problem may fail because the student does not know how to convert this word problem into an equation. Currently, Reason’s taxonomy of error has been expanded and is widely used in the aviation field.

Taxonomy of errors in e-learning settings

The definitions of errors are continually refined within elearning settings, particularly due to the thriving field of computer-based and assisted learning. Priem (2010) proposed a taxonomy of errors in e-learning on the basis of Norman’s taxonomy of errors (1981). Priem categorized errors in elearning settings to learning mistakes and support mistakes (Figure 2). Within this taxonomy, learning mistakes refer to conscious errors that directly link to the learning goals. These errors are caused by the failure of domain expertise or a lack of knowledge in the learner. Learning mistakes refer to the mistakes described in Norman’s taxonomy (1981) or the rule-based and knowledge-based level errors in Reason’s taxonomy of errors (1990).

Support mistakes are subcategorized into two types: general support mistakes and technology support mistakes. General support mistakes are conscious errors that are irrelevant to domain knowledge and occur in the tasks of supporting learning. A substantial amount of general support mistakes link to metacognition and executive functions such as time-management and maintaining attention. For example, a student did not submit homework on time because he or she spent much time on surfing the internet. Priem categorized slip as general support mistakes because he thought that slips are preceded by a high-level decision. For instance, consider a scenario where a student tended to ignore small print during reading because she or he thought this strategy helped gather important information quickly. But when this student solved problems, this “ignoring small text” strategy resulted in ignoring the important instructions. Thus, in this case, the student would make mistakes on problems due to a misapplication of consciously high-level heuristics. Another reason to treat slips as conscious mistakes is that slips can be reduced by explicit training. For example, a student could reduce his or her common misspellings by proofreading his or her work. The proofreading process would bring to the student's attention words he or she struggles to spell correctly.

Within the Priem’s taxonomy, technology support mistakes are defined as errors that occur during tasks which are supported by e-learning systems. An example would include a situation where a student uploaded a wrong format of an assignment to the learning system. Technology support mistakes can be decreased by providing effective instructions.

Taxonomy of errors in mathematics learning

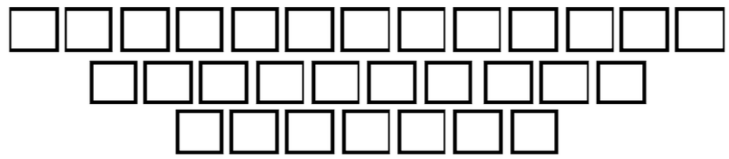
In mathematics, researchers are interested in students’ specific misconceptions behind the errors. Therefore, the errors defined in mathematics vary according to different topics, grades, and problem forms. The following section will elaborate on some mathematics errors that are the focus of past and more recent studies.

The first mathematics error to be discussed is the reversal error in the “student-and-professor” type of problem. A typical “student-and-professor” problem is to write an equation to represent the statement, e.g., “There are six times as many students as professors at this university. Use S for the number of students and P for the number of professors.” (Clement, 1982, p.17). In his analysis, Clement found that 68% of the errors were the reversal errors which students used 6S= P instead of S= 6P. In Clement’s study (1982), this type of error is tremendously stable, in that it occurs among students with different levels of performance, and in problems that involve words, pictures or tables. This reversal error persisted even when students were reminded that some students could put the number (i.e. 6) on the wrong side of the equation. In another study (Rosnick & Clement, 1980), students who majored in math received 15- to 30-minute training, which included worked examples and practice related to the type of “student- and -professor” problems still exhibited the reversal error. Clement thought these errors were not just errors simply caused by carelessness. He found that there were two main reasons leading to the reversal error based on the interview about students' problem-solving process (Clement, 1982). One is that some students map the order of the number and symbols (e.g. S or P) in the equation according to their order in the problem statement. The other reason is that students treat the symbols as the representation of the group (e.g. students or professors) in lieu of the number of the group. So students put the number (e.g. 6) on the side of the larger group (e.g. students) which means that the student group contains more people than the professor group.

Secondly, similar to the reversal error, another type of error also occurs when students have trouble correctly translating word problems into math expressions. In these types of word problems, simply adding or subtracting two numbers can lead to one or more incorrect answers. For example, "In September 1995 the city's youth orchestra had its first concert. What year will the orchestra have its fifth concert if it holds one concert every year?" (Verschaffel, Corte, & Vierstraete, 1999, p. 268). Verschaffel, Corte, and Vierstraete (1999) assigned fifth and sixth graders to two types of the nonstandard additive word problems. One type of problems was backward or forward count starting from a small number or large number (see the previous example). The other type of problems was the differences between two numbers. For example, "Last October (1995) I participated for the first time in the great city running race that is held every year. This race was held for the first time in October 1991. How many times had the race already been held when I participated for the first time?" (Verschaffel, Corte, & Vierstraete, 1999, p. 268). The results indicated that fifth and sixth graders had great difficulty in solving those nonstandard additive word problem (the average percentage of correct answers was 23.5%). Another interesting finding from their study was that students who found solutions to the nonstandard additive word problems were inclined to overgeneralize the solutions to the problems in which straightforward adding or subtracting two numbers yields a correct answer (e.g. In January 1995 a youth orchestra was set up in our city. In what year will the orchestra have its fifth anniversary?).

Overgeneralization is also a common error committed in proportional reasoning problems. An example of a typical problem that yields proportional reasoning errors is the theater seat problem (see Figure 1). Students could double the seat number of the 10th row to find the seat number of the 20th row, which is an incorrect answer. Lannin, Baker, and Townsend (2007) classified the fifth graders' tendency to commit proportional reasoning errors into four levels: "not an error" which means that students do not realize their errors; instance-level errors which means that students only view the errors occurring in particular or limited number of instances (e.g. the student thinks proportional reasoning as an error when doubling the seat number of 10th row to find the seat number of 20th row, but does not think it as an error for other rows); problem-level errors which means that students view the error occurring only in a specific problem context (e.g. the student considers proportion reasoning as an error only applicable to the theater seat problem); cross-problem level errors which means that students view the error occurring in a particular group of problems (e.g. the student regards proportional reasoning as an error applicable to the problem type of the theater seat problem). Lannin et al. (2007) found that students' recognition on the general error of proportional reasoning evolved gradually from the level of "not an error" to instance-level, problem-level, and cross-problem level errors.

In a theater there are 7 seats in the first row. The increase in the number of seats is the same from row to row. Below is a diagram of the first three rows in the theater.



How many seats are there in the 4th row of the theater? In the 5th row? In the 10th row? In the 23rd row? In the 38th row? In the 138th row of the theater? Write a rule that would allow you to calculate the number of seats in any row.

Figure 1. The theater seats problem (Lannin, Baker, & Townsend, 2007)

Although the mathematics errors come in various forms, a few generic types of math errors have been proposed. For example, Borasi (1987) put forward five generic math errors based on the degree of wrongness of the answers. The five types of errors: include incorrect definition, wrong result, unsatisfactory models, right results reached by an unsatisfactory procedure, and approximate results. Borasi hoped that by labeling all math errors into these five generic categories it would help teachers raise questions, and to better understand their students' errors and effectively correct them.

Summary

The review of taxonomies of errors demonstrates that researchers have defined the errors from the generic perspective to specific domains. Although each taxonomy has its own characteristics, they all have latent relationships and are overlapping. The general taxonomies of errors provide a theoretical foundation for the subdivision of errors in relation to specific settings or domains. For example, Reason's (1990) and Priem's (2010) taxonomies are the extensions of Norman's (1981). Both Reason's and Priem's taxonomies confirm that errors include mistakes and slips. Although math errors have various representations due to varying topics, the main types of errors can be summarized to errors caused by an over-generalization of concepts, which occurs when knowledge is used beyond its productive range of application (Smith, diSessa, & Roschelle, 1993), and procedural errors (or "bugs") which are systematic arithmetic errors (e.g. the reversal error).

Of the aforementioned error taxonomies, this study will emphasize Norman's taxonomy. That is, this study is going to focus on the effect of mistakes and slips during learning. The choice to focus on Norman's taxonomy is threefold. First, this taxonomy provides a foundation for the majority of taxonomies developed after it. The definitions for each type of error in Norman's taxonomy are both clear and wieldy. Secondly, a statistical model to detect slips has been created and found to be reliable (Baker, Corbett, & Aleven, 2008a; Baker, Corbett, & Aleven, 2008b). The statistical model can estimate the probability that an error is a slip, which provides a helpful tool for investigating errors during learning. This model is will be applied in the current research to detect the probability of slips. Third, the data collected for this research does not include the specific problems solved by students or the students' solutions to the problems. Therefore, the data limits the capability to investigate more refined types of errors such as knowledge-based level errors or rules-based errors. In the future studies, more in-depth and refined investigations can be conducted on errors should more detailed data become available.

The effects of errors on learning

The previous section introduced what errors are and various taxonomies that help to categorize them. Researchers have already defined errors in terms of the reasons causing errors. The current most valuable question for educational scholars addresses now, how do errors influence learning? That is, do errors undermine learning or benefit learning? How should researchers view errors during learning? This section reviewed the theories and studies addressing the effect that errors have on learning.

The negative effect of errors on learning

In a variety of educational theories, errors are considered as inadequacies of the learner. Inadequacies that learners attempt to avoid are due to fear, and teachers aim to eradicate inadequacies during the learning process. Similarly, behaviorism viewed errors as incorrect behaviors that should be replaced by correct behaviors. Therefore, according to the behaviorist perspective, teachers should provide positive reinforcement for correct answers while performing negative reinforcement or holding back positive reinforcement for incorrect answers during the learning process (Miller, 1983). In this perspective, in addition to reinforcement, teachers should design various means to control errors, such as clearly defining concepts or terms, repeating correct procedures of problem-solving (i.e. worked examples), and breaking down procedures to smallest parts for students (Lannin, Barker, & Townsend, 2007). Hence, the behaviorist perspective emphasizes how to minimize errors through a practical teaching process instead of why errors represent learners' inadequacies.

Information processing theory investigates these reasons from the perspective of cognitive processes. Information processing theory posits that unfamiliar situations and limitations of working memory lead learners to make mistakes (Kintsch & Greeno, 1985; VanLehn, 1983). Ayres and Sweller (1990), Ayres (1993) reported that students performed well on a specific geometry theorem if the theorem occurred in familiar problems. However, students made errors on the same theorem if the theorem was presented in less familiar problems. VanLehn (1983) viewed those errors as students' attempts to cope with unfamiliar situations. When facing unfamiliar or new situations, students would adopt the strategies or methods that are considered appropriate according to their prior knowledge. Although the methods established in their prior knowledge may not solve the problems correctly, students would still attempt to transfer these methods to an unfamiliar or a new situation. For example, when calculating “2/3+1/4”, the student may employ the addition rules of whole numbers to “repair” the missing procedures to reach the correct answers. That is, the student may simply add the two numerators (2+1) as the new numerator and the two denominators (3+4) as the new denominators. The view that errors occur when students unsuccessfully attempt to apply prior knowledge in unfamiliar settings is referred to as repair theory (Van Lehn, 1983, 1988; Woodward & Howard, 1994). Here, it is thought that students attempt to "repair" their knowledge gaps of unfamiliar problems by transferring prior knowledge to the new situation.

According to the information processing theory, errors also occur because of working memory limitations. Information process theory argues that working memory load leads to the decay of information storage (Ayres, 2001). The decay in the storage of problem information is thought to be linked to calculation errors, especially mental arithmetic errors (Adams & Hitch, 1997; Hitch, 1978; Logie, Gilhooly, & Wynn, 1994). For example, when a student calculates 376+198 internally, he or she needs to maintain the numbers as well as process information in working memory. After finishing the calculation, the student also needs to retrieve the numbers that are maintained in working memory as the final result. Therefore, the ability to maintain and process information is directly related to accuracy and calculation speed.

In order to investigate the impact of working memory on calculation errors, Ayres (2001) required students to solve the algebra problems involving two brackets, negative numbers and multiplication (e.g. -3(-4-5x)-2(-3x-4)). If working memory load indeed limits arithmetic calculation, then more errors should occur in the second operation (i.e. -5x) than the first operation (i.e. -4). Because the negative symbol plays dual roles as the symbol for the negative number (-5) and linking 5x with -4, the combination of the two functions should generate larger working memory load. It follows from this argument that the second bracket may exert a heavier load on working memory than the first bracket. So more errors may occur in the second bracket than the first bracket. In the similar fashion, in the second bracket, the fourth operation (i.e. -4) leads to more errors than the third operation (i.e. -3x). The findings of Ayres (2001) supported the above assumptions, that students made more errors on the second operation, and in the second bracket especially on the fourth operation. In the other math areas, researchers found the impact of working memory on math such as arithmetic word problems (Fayol, Abdi, & Gombert, 1987) and geometry (Ayres, 1993; Ayres & Sweller,1990).

The above-stated theories have claimed that errors are negative experiences that, ideally, should be eliminated entirely from learning. Meanwhile, a large number of empirical studies have investigated the negative effect of errors on other individual factors besides learning outcome. The negative effect of errors on individuals stressed most by researchers is that errors may lead to math anxiety. Math anxiety, which is defined as the tension and fear occurring during math-related activities (Richardson & Suinn, 1972), may reduce working memory span (Ashcraft & Kirk, 2001), impede math performance (Mattarella-Micke, Mateo, Kozak, Foster, & Beilock, 2011), and even repel students to avoid math-related career (Ashcraft & Krause, 2007). Math-related negative experiences, especially making errors, is considered the main culprit for producing math anxiety (Ashcraft, Krause, & Hopko, 2007; Meece, Wigfield, & Eccles, 1990; Wang et., 2014). Recently, studies in neuroscience have found that anxiety relates to an increased amplitude of the error-related negativity (ERN) of the human event-related brain potential (ERP) (Moser, Moran, Schroder, Donnellan, & Yeung, 2013; Olvet & Hajcak, 2008).

That is, anxiety is associated with an enhanced error monitoring function, which involves the detection of errors to regulate behaviors across tasks and situations. This indicates that when students detect more errors, they are more likely to feel strong anxiety in learning.

The positive effect of errors on learning

However, the constructivist view of errors differs from the behaviorist view by arguing that errors promote student self-reflection, and facilitates knowledge acquisition. According to the constructivist learning theory, teachers should take advantage of students' errors to pose relevant questions, and guide the students through interpreting their mistakes (DeVries, Zan, Hildebrandt, Edmiaston, & Sales, 2002). Borasi (1987) provides a good example of mistakes being used by teachers to elicit deeper understanding from their students. Borasi thought that errors can unveil the weakness of the strategies chosen to solve problems, as well as the strengths and drawbacks of available strategies. Errors enable students to rethink their strategies and refine them to reach the predetermined goal. Therefore, in addition to the diagnosis and remediation of learning difficulties, teachers could use errors to help students raise questions from perspectives. Thus, students can benefit from the interpretations of errors, which ultimately motivates them to explore mathematics. For example, for the addition problem a/b + c/d, students usually use the wrong addition rule resulting in (a+c)/(b+d). A teacher could facilitate a deeper understanding of the concept by asking a question such as, "Can the wrong addition rule be correct in some situations?"

Many other researchers support the view that errors can help students gain a deeper understanding of their learning materials. Ohlsson (1996) pointed out that students can detect and correct their performance errors during skill practice because many errors are assumed to be caused by knowledge overgeneralization. While practicing, students can detect the conflicts between correct problem solutions and their inappropriate knowledge structures that are thought to be true by students. These conflicts enable students to refine their knowledge structures and learn to employ existing structures in appropriate situations. Mott, Callaway, Zettlemoyer, Lee, and Lester (1999) proposed narrative-centered learning environments to encourage students to self-monitor and interpret their errors. They stated that students could benefit from trial-and-error while exploring strategies to solve problems. Furthermore, after reviewing the past literature pertinent to errors, Bjork, Dunlosky, and Kornell (2013) summarized that errors caused by desirable difficulty are essential for promoting learning. Likewise, avoiding errors may eliminate opportunities to deepen understanding. A learning task with desirable difficulty requires students to take significant but the desirable amount of effort, and can improve long-term performance (Bjork, 1994).

While the studies of Bjork (1994) and Ohlsson (1996) demonstrated that errors are beneficial to deeper learning at a broad level, Roll, Baker, Aleven, and Koedinger (2014) examined errors and their effect on learning at a finer-grained level. Their findings indicated that repeated errors on low-skill steps raised the probability of success on the subsequent steps. The low-skill steps were defined as the steps when the probability of knowing the knowledge is lower than .4 -- indicating areas where students need the most assistance. Roll et al. (2014) interpreted these results by explaining that errors help students self-explain their own answers and make sense of instructional explanations (Kapur, 2008; Roll, Aleven, McLaren, & Koedinger, 2011; Schwartz & Martin, 2004; Westermann & Rummel, 2012). It should be noted that previous research demonstrated that instructional feedback can undermine students' interpretations of their own errors (Mathan & Koedinger, 2005; Schworm & Renkl, 2006). Therefore, errors on low-skills steps, which are treated as students' struggling in the early phase of learning, were beneficial to the subsequent steps, whereas the instructional feedback was ineffective at grounding domain knowledge. The results of Roll, Baker, Aleven, and Koedinger (2014) confirms the phenomenon of "time for telling" (Schwartz & Bransford, 1998) to some degree, which means that the failed attempts before learning materials can help students differentiate their prior knowledge structures and promote students learning.

The previous studies also demonstrated that errors caused by desirable difficulty could improve students' performance on knowledge retention, although errors slow down the learning process. In the study of Taylor and Rohrer (2010), students were required to calculate the numbers of faces, edges, corners, and angles of a prism based on the number of base sides. Students were asked to calculate the four different numbers in either an interleaved order or a blocked order. In the practice phase, the students in the interleaved condition made more errors than those in the blocked condition. However, in the test given one day later, students in the interleaved condition outperformed those in the blocked condition. This finding indicates that errors caused by desirable difficulty may slow learning, but promote longer-term performance. That is, students in the interleaved condition frequently switched between different types of problems (i.e. calculate face, edge, corner, and angle). Their errors required them to pay more attention to how to pair every problem with the correct procedures. However, in the blocked condition, it was not necessary for students to discriminate the problems due to continuous occurrence of the same type of problem instances. Similarly, Simon and Bjork (2001) found that in a task that required students to remember particular keystroke sequences, students made more errors in the "random condition". In this condition, different types of sequences were displayed randomly, compared to those in the blocked condition where the same type of sequences occurred continuously. In the test given one day after the acquisition phase, students in the random condition exceeded those in the blocked condition.

In reverse, manipulations to rule out errors impair students to skill acquisition, retention, and transfer. The results of some experiments found that when learning a new task, learners who were allowed to make mistakes without any penalties made better gains in the task than those who were told to avoid errors (Chillarege et al., 2003; Keith & Frese, 2005).

Other manipulations to avoid errors, such as easy retrieval of to-be-learned information or intense support, are found to impede learning in some empirical studies. In the experiment on testing the effect of rereading on text understanding, Rawson and Kintsch (2005) found that students in the massed condition (i.e. read texts twice in immediate succession) performed better on both recall and comprehension tasks in an immediate test than counterparts in the distributed condition (i.e. the second reading trial delayed 1 week after the first reading trial). However, in the delayed test given two days after the second reading trial, students in distributed condition outperformed those in the massed condition on both tasks. The reason for the students' discrepant performance between the two tests may be that rereading in a massed fashion focused on memorization of the text information rather than providing enough time integrating and organizing the text information. When compared to the massed condition, students in distributed condition had more time to process the text deeply and form an organized representation of the text. The closely accurate representation is more beneficial on recall after a delay than shallow memorization of text information, although shallow memorization enables students to recall text information easily in an immediate test. A study that compared DragonBox (a commercialized education game) with Lynnette (a web-based linear Equation Tutor), Long and Aleven (2014) found that students made fewer errors in DragonBox and finished solving all equation problems faster than Lynnette when taking the tests provided by each system. Nevertheless, students using DragonBox performed worse than those using Lynnette in the transfer posttest. Long and Aleven (2014) attributed these findings as to DragonBox offering immediate feedback after each step and using concrete context to represent equations. These features provide much support in learning and hide the connection to standard algebraic notation with transformation rules. The lack of a deep understanding of abstract equation hinders students' ability to transfer knowledge out of DragonBox.

Summary

The reviewed literature revealed two seemingly opposing standpoints of the effects of errors on learning. How errors impact learning depends on how students or teachers help them to cope with errors. The ways to optimize the positive effect of errors on learning include focusing on desirable difficulty and self-explanation. Manipulating the difficulty of learning materials enables students to master knowledge after making an effort, which could also prevent students from acquiring a potential domain anxiety. Instructions should be designed to prompt students to interpret their own answers instead of simply identifying an error. Through those two ways, errors can promote a deeper-level understanding of the content and learn from errors.

This study tends to adopt the view that errors can foster learning. The reasons involve two aspects. First, ALEKS (Assessment and LEarning in Knowledge Spaces), the learning environment used in this study, is designed by the principle of desirable difficulty. The core mechanism of ALEKS is knowledge spaces theory (KST), which imitates expert teachers to assess students' prior knowledge. In KST, student's prior knowledge is referred to as the "inner fringe" (Falmagne, & Doignon, 2010). KST also defines students' outer fringes, which are the topics most ready to learn compared to inner fringe which students have already mastered

(Falmagne, & Doignon, 2010). While ALEKS updates its student model with changes in prior knowledge, ALEKS also updates students' outer fringe to assure that the topic difficulty dynamically matches students' prior knowledge (i.e. inner fringes). Thereby, ALEKS is designed in a way of desirable difficulty. The second reason this research adopting the view that errors can foster learning is also grounded in the ALEKS learning system. ALEKS provides worked examples to facilitate students to activate prior knowledge and interpret their own errors. According to the design of ALEKS, it was hypothesized that errors should have a more positive impact on learning than a negative one. That is, students were more likely to learn from errors in ALEKS.

The strategies to learn from errors

The aforementioned research on learning from errors provided significant support for the idea that errors bolster learning, but how students learn from their errors is equally important. Given the research on the positive effect of errors on learning, many educational scientists agree that it is the students' explanations on their answers that enable them to deepen their understanding (Aleven & Koedinger, 2002; Hiebert et al., 1997; Lannin, Barker, & Townsend, 2007; Siegler, 2003). However, in the traditional classroom setting, self-explanation on errors are difficult to observe naturally. The specific cognitive mechanism driving the positive effects of self-explanation remains unclear (Heemsoth & Heinze, 2016). Educators or practitioners may infer students' self-explanation by observing learning behaviors or think aloud activities, and also by applying instructions to help facilitate their reflections on errors. Hence, the following paragraphs illustrate students' strategies to learn from errors and the external aspects relevant to self-explanation.

Self- explanation

The existing studies of errors during the learning process specify that self-explanation on self-made errors or errors in worked examples fosters effective learning (Ramdass & Zimmerman, 2008). Self-explanation on errors requires students to reconstruct their knowledge (Heemsoth & Heinze, 2016) and tune their strategies to solve problems (Borasi, Fonzi, Smith, & Rose, 1999). For example, in a field experiment, Heemsoth and Heinze (2016) required students to reflect on their fraction errors in two ways. The first reflection method consisted of an error-centered manner, in which students described their answers and errors, recalled and explained their thought process, modified their errors, and generated a problem which a similar error may occur and solved it correctly. The second reflection method consisted of a solution-centered manner, in which students described their answers, explained why the steps in worked examples pertained to the problem were correct, and then revised their answers. Students in the error-centered condition outperformed those in the solution-centered condition in the fraction posttest and delayed test. Aleven and Koedinger (2002) also found that students who were required to explain the reasoning behind their problem- solving steps outperformed those who were only required to solve the problems in the posttest.

Although researchers treat self-explanation on errors as an effective learning strategy, so far most of the findings on this view have been borrowed from studies on individual reflection on incorrect worked examples. Incorrect worked examples unveil a student's whole problem-solving process, including incorrect steps occurring during learning. Students are usually not aware of their existing incorrect prior knowledge that lead to wrong answers (van Loon, Bruin, van Gog, & van Merriënboer, 2013), so their incorrect prior knowledge is difficult to replace (Große & Renkl, 2007). Explicitly addressing errors and misconceptions brings the inaccurate prior knowledge to the student's attention, which results in improving learning outcomes. (van Dooren, de Bock, Hessels, & Verschaffel, 2004). According to social psychology theories, students tend to attribute poor grades to external factors, such as difficult test or bad luck, instead of a reflection on their misconceptions on the problems (Forsyth, 2007). Therefore, presenting errors in worked examples may be a way to facilitate students to face incorrect prior knowledge and revise their insufficient knowledge structure. Siegler (2002) discovered that reflection on both incorrect and correct worked examples fosters better learning outcomes in children compared to only reflecting on correct worked examples. Furthermore, Curry (2004) found that self-explanation on incorrect worked examples helped students learn more than simply reflecting on correct worked examples. Likewise, Große & Renkl (2007) found similar results, but their results indicated that positive effect on learning from reflecting on incorrect worked examples favored high prior knowledge students.

A central finding from the above-mentioned research is that self-explanation promotes knowledge acquisition. Nevertheless, self-explanation is difficult to observe in a natural learning process (e.g., classroom setting). Researchers often employ a verbal protocol to observe self-explanation (e.g. think aloud) in experimental or quasi-experimental settings (Chi, Bassok, Lewis, Reimann, & Glaser, 1989; Magliano, Trabasso, & Graesser, 1999). This method allows researchers to know a student's thought process behind their self-explanation but does not unveil the cognitive process of the self-explanation. Currently, few empirical studies provide insight as to why self-explanations on errors facilitate knowledge acquisition (Heemsoth & Heinze, 2016). Based on the limitations of investigations on self-explanation, this study sought to investigate learning from errors from an external behavioral aspect in place of observing self-explanation on errors. This research examined how students utilize worked example-based explanations to learn from their errors.

Practice and help-seeking

From a behavioral standpoint, the debate on how to learn from errors may concentrate on which practice strategies and help-seeking strategies foster learning. Constructivists think that students should explore learning by themselves with minimal instructions, which is learning by doing (Schwartz & Bransford, 1998). The practical implementations of constructivism include discovery learning, problem-based learning, inquiry learning, experiential learning, and constructivist learning. This theory assumes that students constructing solutions by themselves is the most effective learning experience, and this experience is the best way to acquire knowledge in the disciplines that emphasize procedures, such as math and medicine (Kirschner, Sweller, & Clark, 2006). Supporters of constructivist argue that instructions provided during learning may interfere students' natural learning process that is presented in various forms due to students' own prior knowledge and learning styles (Bernstein, Penner, Clarke-Stewart, Roy, & Wickens, 2003). Alternatively, behaviorism and cognitive theory suggest it is best to generate information-rich instructions for eliminating errors and recognize that practice can minimize errors (Woodward & Howard, 1994). The direct instructions proposed by behaviorism and cognitive theory involves interpreting concepts and procedures thoroughly related to learning and providing guidance on learning strategies such as worked examples, immediate or delayed feedback, discrimination practice, and clear presentation of concepts.

The constructivist approach, which emphasizes the positive effects of practice on learning, have been supported by empirical studies. The existing study suggest that the number of consecutive tests on recall are negatively related to the amount of forgetting reading materials

(Wheeler & Roediger,1992). Similarly, Roediger and Karpicke (2006) found that consecutive repetition of recall tests on reading materials without feedback, increase students' performance on a delayed recall test more so than restudying the materials repeatedly. The findings relevant to reading may indicate that practice facilitates the memorization of facts in learning. In the math domain, Christianson, Mestre, and Luke (2012) discovered that the amount of practice was a significant positive predictor to the accuracy of solving "student- and -professor" algebra word problems. This type of algebra problems requires students to use algebraic equations to present the ratio relationship between two variables. For example, the problem used by Christanson et al. (2012) is:

> The window display at Jack's boot shop showcases both black and tan leather boots. There are three more black boots on display than there are tan boots. Using B to represent the number of black boots on display and T to represent the number of tan boots, write an equation that describes the situation in the window display at Jack's (p.822).

The behaviorism approach, which thinks that appropriate instructions foster students' learning, is supported by existing studies as well (Aleven, McLaren, Roll, & Koedinger, 2006; Renkl, 2002; VanLehn, 2006; Van Gog, Kester, & Paas, 2011; Wood & Wood, 1999). In reality, students have been observed to misuse guidance in the learning system. Help-seeking error, including help avoidance and help overuse, made up 73% of students' help-seeking strategies in Cognitive tutor (Aleven, McLaren, Roll, and Koedinger, 2006). It may imply that students often lack the capability to self-correct during learning. These suboptimal strategies could undermine students' performance. Therefore, Aleven and his colleagues thought that it was necessary to design adaptive instructions to guide students on help-seeking strategies. They developed a model of help-seeking for the Cognitive tutor (Figure 2). In this model, appropriate help-seeking behaviors should largely depend on how well the students' skill related to the problems are mastered. That is, if student feels familiar with the problem and knows somewhat how to solve the problem, he or she will attempt to solve the problem otherwise request help. Roll, Aleven, McLaren, and Koedinger (2011) integrated immediate metacognitive feedback on help-seeking misuse in Geometry Cognitive Tutor. The results showed that students made fewer errors in the ITS (intelligent tutoring system) that integrated feedback on help-seeking strategies. This ITS outperformed other ITSs that did not provide feedback on help-seeking strategies in the delayed transfer test.

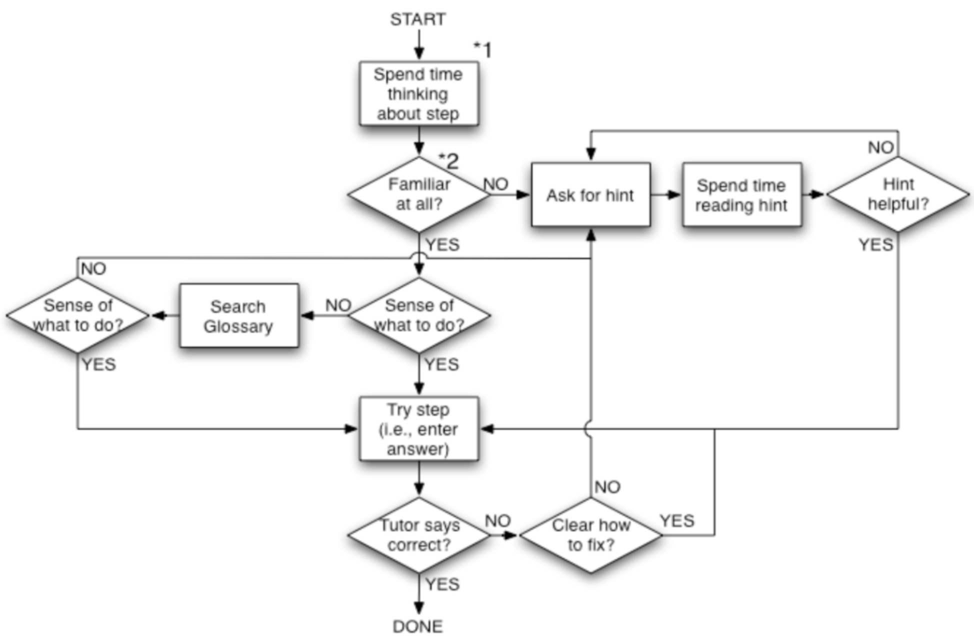


Figure. 2 The model of ideal help-seeking behaviors (Aleven, McLaren, Roll, & Koedinger, 2006)

Worked examples have been found to be another form of effective instructions during learning, which displays an expert's problem solution step by step, so learners can study and emulate the expert's problem-solving model (Atkinson, Derry, Renkl, & Wortham, 2000). According to cognitive load theory, long-term memory plays a role as internal instructions for cognitive activities, and worked examples provide detailed problem solutions as external instructions in learning (Kalyuga, 2007). Through worked examples, students are able to activate relevant cognitive schemas in long-term memory more easily, so their intrinsic loads decrease in working memory. This allows students to have more available cognitive resources to process information, compared to solving problems without worked examples. As a consequence of activating prior knowledge and decreasing working memory, students can more easily refine their problematic solutions. Therefore, it follows the logic of cognitive load theory that worked example should be more effective than only solving problems in learning. Indeed, Renkl & Atkinson (2003) found that worked examples foster better performance than only problem solving when students lack domain knowledge. In a similar fashion, Van Gog, Kester, and Paas (2011) found that novices who either only read worked examples or read worked examples interleaved with practice, outperformed those students who did neither on transfer tests on electric circuit troubleshooting. In addition, among the interleaving conditions, reading worked examples before problem-solving fostered better learning than reading worked examples after problem-solving. Nevertheless, in their following experiments, Van Gog and Kester (2012) found that reading worked examples before problem-solving boosted students' performance in the immediate retention test but pure reading worked examples promoted students' performance in the delayed near transfer test.

According to past studies on practice and help-seeking, both of those two strategies are suggested to be effective in learning, although some inconsistent findings exist. For that reason, currently, researchers are more concerned about when and how to provide students help to optimize learning. Based on a meta-analysis on a synthesis set of 228 meta-analyses on learning strategies, Hattie and Donoghue (2016) emphasized that "...the optimal strategies depend on where in the learning cycle the student is located." (p.9). That is, strategies should be embedded into learning cycles in lieu of separate learning sessions targeting specific skills or knowledge. They also mentioned differences of strategies in different learning phases, which is very similar to the learning phase theory proposed by VanLehn (1996). VanLehn defined three phases during skill acquisition: early, intermediate, and late phases. During the early phase, students attempt to ask for more help and gain a basic understanding of the domain knowledge, without needing to apply knowledge in practice. During intermediate phase, students pay more attention in learning how to solve the problems as the domain knowledge grows after accumulation in the early phase. They try to correct their errors or misunderstandings existing in the prior knowledge structures. Then students enter the late phase and focus on applying knowledge to solve problems to increase the speed and accuracy of their problem-solving. The application of learning phase theory results in a fading design of worked examples, which more and more steps of the worked example are successively removed as the learning continues until students are required to solve the problem by their own. Faded worked examples are widely employed in teaching and suggested to promote near transfer, and in combination with self-explanation, also facilitates far transfer (Atkinson, Renkl, & Merrill, 2003; Koedinger, & Aleven, 2007; Nievelstein, Van Gog, Van Dijck, & Boshuizen, 2013; Renkl, Atkinson, & Große, 2004; Salden, Aleven, Schwonke, & Renkl, 2010). This study adopted the learning phase theory as the framework for investigating practice and help-seeking during different learning phases.

Temporal patterns of strategies to learn from errors

Per the view of Hattie and Donoghue (2016) on learning strategies, strategies can change as a function of time. Many existing intelligent tutoring systems allow students to regulate their learning themselves, which is referred to as self-regulated learning. In self-regulated learning,

learning strategies usually evolve with time. The changes in strategies over time, such as randomness and transition patterns of strategies, become important features to describe learning strategies during students' interactions with tutoring systems.

Shannon entropy is applied to measure disorder or uncertainty in many different fields such as language (Shannon, 1951), consumer's choices (Fasolo, Hertwig, Huber, & Ludwig, 2009), and educational data mining (San Pedro, Snow, Baker, McNamara, & Heffernan, 2015). High entropy values indicate an uncertainty for predicting the occurrence of targeted objects. That is, the occurring order of the objects is more disordered. More disordered learning behaviors are linked to lower performance in learning (Snow, Jackson, & McNamara, 2014; Zimmerman & Martinez-Ponz, 1986). For instance, Snow, Allen, Jacovina, and McNamara (2015) applied Shannon entropy to observe student-controlled patterns of choice on game-based features in an ITS of comprehension training. When students presented a more disordered pattern of choices, their quality of self-explanation texts was lower. This work led to the hypothesis that more disordered pattern of choices on strategies to learn from errors is correlated with lower performance. Additionally, students' choices on learning behaviors may evolve over time as they gain more experience (Bandura, 1991), so it is assumed the pattern of choices on strategies to learn from errors become more ordered as students gain more knowledge.

Summary

Given the above discussion on the various strategies for learning from errors, it is clear that self-explanation is an effective way to prompt students to reconstruct their knowledge structures after making mistakes. Self-explanation in combination with worked examples and practice can boost the positive effects of those strategies on learning (e.g. Aleven & Koedinger, 2002; Hilbert, Renkl, Kessler, & Reiss, 2008). However, self-explanation is a metacognitive activity which is hard to naturally observed in traditional learning settings. The difficulty of observing self-explanations is why currently there are few empirical studies that interpret the cognitive process of self-explanation. Most existing studies simply indicate its benefit on learning. Hence, this study attempts to examine students' learning from error strategies by observing errors from an explicitly behavioral aspect. That is, how students utilize practice and help to learn from errors.

The previous research implied that both practice and help-seeking favors knowledge acquisition, such as testing effect (Roediger & Karpicke, 2006) and worked example (Van Gog, Kester, & Paas, 2011). However, those strategies have their shortcomings. For example, practice without guidance may cause frustration and more errors, which may slow down learning process (Koedinger & Aleven, 2007). Overuse of worked examples may lead to redundant information, which may interfere with learning especially for high prior knowledge students (Kalyuga, Chandler, Tuovinen, & Sweller, 2001). Therefore, based on cognitive load theory, VanLehn (1996) proposed the learning phase theory, which in application results in the transition from help to practice that occurs gradually during the learning process. ALEKS matches topic difficulty with students' prior knowledge so most of the students should start in the intermediate phase in ALEKS. Also according to VanLehn (1996), during the intermediate phase, students prefer to study worked examples rather than other help-instructions during problem-solving. So it leads us to assume that ALEKS is constructed in a way that best allows for students to adopt a "mixed" strategy.

Throughout the analysis of this research, learning was divided into three phases: low-skill phase, medium-skill phase, and high-skill phase corresponding to the learning phase theory (VanLehn, 1996). Low and medium skill phases corresponded to the intermediate phase. The high-skill phase corresponded to the late learning phase. Strategies used to learn from errors were categorized into the following groups: practice, help strategy, and mixed strategy which combined help strategy and practice.

The factors impacting learning from errors

The strategies used to learn from errors and their effect on learning are affected by individual factors and learning materials. The following section covered some potentially influential factors of strategies to learn from errors, including prior knowledge, errors types, and topic difficulty.

Prior knowledge

From the cognitive perspective, the learning process can be considered as a process of modifying an existing knowledge structure. That is, learning requires activation of prior knowledge to detecting missing or inaccurate knowledge in long-term memory so that the existing knowledge structure can be refined by including this missing or correct knowledge. Students with the high prior knowledge tend to solve problems by themselves and seek help when they are in the impasse (Wood & Wood, 1999). Fyfe, Rittle- Johnson, and DeCaro (2012) conducted an experiment to examine the moderating role of prior knowledge in learning. They assigned students to three conditions: no feedback, outcome feedback (e.g. “Great job! You got the correct answer”), and strategy feedback (e.g. “Great job! That is one correct way to solve the problems”). Students received a procedural posttest measured by the number of problems being solved successfully, and conceptual posttest measured by a correct explanation on equal sign and structure of equation. The findings indicated that students with the low prior knowledge benefited more from outcome feedback and strategy feedback than no feedback in both the procedural and conceptual posttest. Also, they did not have significant differences in performance on those two feedback types. On the contrary, students whose prior knowledge is above average benefited more from no feedback than outcome feedback and strategy feedback in both the procedural and a conceptual posttest. Additionally, strategy feedback hindered their learning on conceptual knowledge. The results suggested that prior knowledge may influence the effect of help on learning.

Prior knowledge has also been found to influence the effect of worked examples on learning. Worked examples are especially beneficial for novices or low prior knowledge students (Kalyuga, Chandler, Tuovinen, & Sweller, 2001; Pass & Van Gog, 2006; Renkl, 1997). Nevertheless, for students with the more prior knowledge, worked examples hinder their acquisition of skills or knowledge (Kalyuga, 2007; Renkl & Atkinson, 2003). This phenomenon is named the expertise reversal effect. In order to examine this effect, Kalyuga, Chandler, Tuovinen, and Sweller (2001) assigned students three sequential training sessions to learn how to write basic programmable logic controller programs for relay circuits. Students received two different sets of instructions: one is included worked examples as well as a general introduction to learning content; the other focused on problem-solving in addition to the same introductions on learning content. After each session, students were provided test and self-rating scales on topic difficulty. Instruction efficacy was measured by using students' performance and ratings on the material difficulty in the two instructions conditions. The findings indicated that as students went through the training sessions, the instructions efficacy increased in the problem-solving condition, whereas the instruction efficacy decreased in worked-example condition. Finally, it was observed that as domain knowledge increased, student performance in the problem-solving condition exceeded performance in the worked-example condition. Many researchers interpret the expertise reversal effect as a result of the cognitive load theory (Pass, Renkl, & Sweller, 2003;

Renkl & Atkinson, 2003). That is, information provided by worked examples may provide redundant information for students with higher prior knowledge. When these students devote working memory to process the redundant information increases the "cognitive load", hindering their acquisition of knowledge and skills.

The influence of prior knowledge on learning is also reflected in the degree of students' ability to control their learning behaviors. Snow, Jackson, and McNamara (2014) found that low reading ability students exhibited a more disordered choice pattern of learning behaviors than high ability students. However, this differences only took place when they began to interact with new features of the intelligent tutoring system. As their interaction with new features continued, the differences vanished. This suggests that prior reading ability affects students' control on learning behaviors during the learning processes. This influence may disappear as students gain more knowledge in learning the domain materials.

Error types

According to the learning phase theory (VanLehn, 1996), different strategies are employed to learn from different types of errors. That is, when students make mistakes due to impasses, they seek help to learn from errors. Conversely, students are more likely to choose to solve problems after making careless errors. Furthermore, in the initial interaction with new topics, students' learning behaviors are more random due to a lack of experience with the topic (Snow, Jackson, & McNamara, 2014). Extending this logic, it follows that during this initial phase, the strategies students use to learn from errors may be more varied. As domain knowledge grows, students utilize strategies in more ordered patterns to learn from errors that tend to be careless errors.

Topic difficulty

Within the learning sciences domain, learning material difficulty is an inevitable factor that affects learning outcomes. Materials that are too easy lead to a ceiling effect, whereas learning materials that are too difficult result in a floor effect. Difficult learning materials require more cognitive skills for students to understand the knowledge illustrated by the materials. Similarly, more cognitive skills are required to implement the knowledge in similar situations or transfer to a complex or new situation. Although ALEKS makes an effort to balance the impact of math topics' difficulty on learning (i.e., matches students' prior knowledge with topics' difficulty), some topics are more difficult in coding during information processes and strategies of problem-solving (e.g., an increase on working memory demands; Kalyuga, Chandler, Tuovinen, & Sweller, 2001). Those topics, naturally, may take students' more time or effort to master them.

Worked examples have been considered as an effective way to decrease working memory demands (Kalyuga, 2007; Renkl & Atkinson, 2003; Van Gog & Kester, 2012). Compared to mean-ends strategy, students may ask for worked examples to help them find the means to solve problems with difficult topics. Also, students may lack experience on those problems because the existing knowledge in long-term memory is more difficult to directly implement problems of the difficult topics. Inexperience on topics may cause students to exert more random learning behaviors, especially given the findings of Snow, Jackson, and McNamara (2014).

Summary

Given the above discussion on the influence of individual factors on learning from errors and the influence of the learning materials on learning from errors, it may be discovered that the reasons for these effects are due to students' cognitive load in learning. For instance, students with the high prior knowledge may avoid worked examples when solving problems because the information provided in worked examples are likely to be considered redundant. Conversely, more difficult topics demand more cognitive resources to process information, so students may tend to ask for more worked examples to decrease working memory demands. As for careless errors, students are more likely to practice on similar problems because they do not need worked examples to decrease the cognitive load to process knowledge.

Chapter 3: Research questions

Previous research on errors have already defined taxonomies of errors and determined that self-explanation is a key factor to improve learning from errors. However, due to the limitations on natural observations of self-explanation, this study investigated students' strategies for learning from errors from external learning behaviors.

Requesting worked examples and practice are two main types of students' external learning behaviors. Therefore, the first big question of this study was how students used help and practice as strategies to learn from errors. This question aimed to reveal the patterns of using help and practice to learn from errors. Learning strategies can change as a function of time (Hattie & Donoghue, 2016), so the first question attempted to answer how students' strategies changed overtime as well. Therefore, the first big question can be decomposed into three subquestions in order to understand students' strategies to learn from errors.

1a. What strategies did students utilize to learn from errors?

1b. How did students use strategies to learn from errors? That is, what patterns of strategies are used?

1c. How did those strategies vary overtime?

According to existing literature (Kalyuga, Chandler, Tuovinen, & Sweller, 2001; Wood & Wood, 1999; VanLehn, 1996), students' learning strategies may be impacted by characteristics of students and learning materials. Therefore, the second question of this study targeted to answer what factors influenced students' strategies of learning from errors. Prior knowledge, error types, and topic difficulty were the three important factors to impact students' learning strategies based on the previous literature. Therefore, the second question was composed of the following three subquestions.

2a. How did prior knowledge influence strategies being utilized to learn from errors?

2b. How did error types influence strategies being utilized to learn from errors?

2c. How did topic difficulty influence strategies being utilized to learn from errors?

After understanding how students learn from errors, an inevitable question would be how the learning strategies related to students' learning performance. Furthermore, the impacts of help and practice on learning are still open to debate (Aleven, McLaren, Roll, and Koedinger, 2006; Christianson, Mestre, & Luke, 2012; Kirschner, Sweller, & Clark, 2006; van Gog, Kester, & Paas, 2011). Thus the third major question this study was how strategies for learning from errors relate to learning outcomes. In addition, the relationships of strategies on learning performance might be influenced by prior knowledge and topic difficulty. Therefore, the third question could be broken down into three subquestions.

3a. How did strategies to learn from errors relate to learning outcomes (i.e. immediate and delayed learning outcomes)?

3b. How did prior knowledge influence the relationship between strategies and learning outcomes?

3c. How did topic difficulty influence the relationship between strategies and learning outcomes?

Chapter 4: Methods

Participants

The participants were sampled from two groups of students. One group included 204 sixth graders which were recruited from five intermediate schools in West Tennessee from 2012 August to 2014 April. Specifically, there were 9 white males, 64 white females, 15 African American males, and 77 African American females. The students volunteered to participate in an after-school math program aimed to improve math. The program provided various incentives to increase retention in the program and to engage students in math learning such as pizza parties, games, and drawings for gifts. Students attended the program for one academic year.

The other group involved 179 college students sampled from 11 college classes that used ALEKS for developmental mathematics in Fall 2016. The demographic information was not available for this group.

Data sample

ALEKS is a self-paced intelligent tutoring system. Students are able to choose the topics in the knowledge pie to learn. The knowledge pie represents the topics that students are most ready to learn, as well as the learning progress (see Figure 3). When learning a selected topic, students can choose to read explanations or solve problems. In ALEKS, each topic has unlimited problem instances. Thus, students can have enough practice on a specific topic without seeing a duplicate problem. Worked examples are applied as help to problem instances. The worked example of a specific instance can only be read once.

The existing literature supported the conclusion that ALEKS is an effective intelligent tutoring system. For example, it can compete with expert teachers (Craig et. al., 2013; Hu et. al., 2012;) and other math intelligent tutoring systems (Sabo, Atkinson, Barrus, Joseph, & Perez, 2013). Additionally, ALEKS was found to shrink the disparities in math achievement that exists between African American and white students (Hu, Xu, Hall, Walker, & Okwumabua, 2013; Huang, Craig, Xie, Graesser, & Hu, 2016). The interface that students interacted with ALEKS is displayed in Figure 4.

This study collected student's log data in ALEKS, pretest and posttest. The log data included the following information:

(1) Students' identification

(2) Topic name that the attempted problem instance belongs to

(3) Students' activities on a specific problem instance, which included "correct" (C), "wrong" (W), mastering a topic (S: three C's in a row within a single topic), failing a topic (F: five W's in a row within a single topic) and explanations (E: requesting an worked example)

(4) Timestamp for each activity

For 6th graders, the 5th-grade math score of Tennessee Comprehensive Assessment Program (TCAP) was used as the pretest. The 6th-grade math score of TCAP was applied as 6th graders' posttest. Tennessee Comprehensive Assessment Program involves standardized tests for math and reading. A student's score in math signifies his or her percentile compared to the counterparts in the state. For college students, their first assessment score in ALEKS was used as the pretest, and the last assessment score in ALEKS was utilized as the posttest.

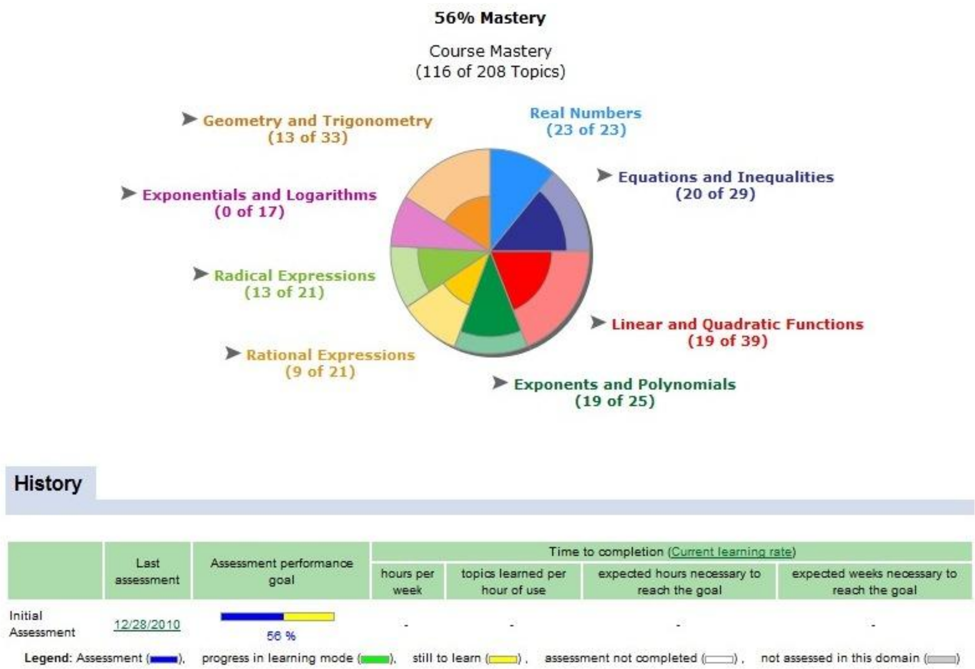


Figure 3. The knowledge pie of ALEKS

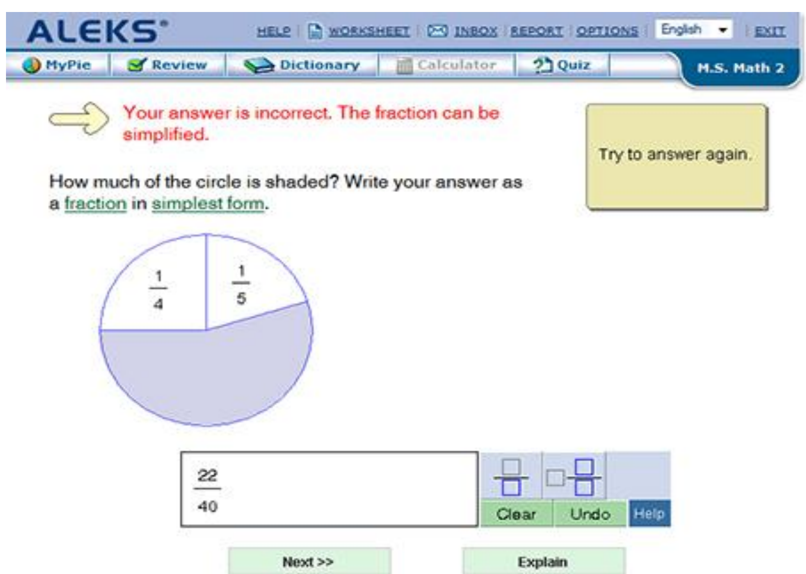


Figure 4. The interface of ALEKS

Materials

Measurements

In order to understand the measurements in this section, two definitions needed to be clarified in advance. The first definition was a "step" in ALEKS, which meant solving a problem or requesting a worked example. An error was defined as a step failing to solve a problem.

Strategies to learn from errors

The strategies of learning from errors were classified into three categories: help (student only requested worked examples in the next two steps after an error); practice (student only solved problems in the next two steps after an error); and mixed (student requested a worked example and solved a problem in the next two steps after an error). Furthermore, the mixed strategies were divided into two subtypes: a) student requested a worked example firstly and then solved a problem (example-practice), and b) student solved a problem and then requested a worked example (practice-example).

In order to clearly observe students' strategies after an error, those four strategies were further divided into subcategories based on the results of practice. Thus, example-practice included example-wrong (EW) and example-correct (EC). Practice-example involved wrong-example (WE) and correct-example (CE). Practice strategies comprised wrong-wrong (WW), wrong-correct (WC), correct-wrong (CW), correct-correct (CC). Help strategy was shortened as EE in the rest of the study.

Learning phases

In order to investigate the changes of strategies, the learning process was separated into three phases: low-skill phase, medium-skill phase, and high-skill phase. The probability of knowing the concept in the Bayesian Knowledge Tracing model ($P(L_n)$) was applied to distinguish the learning phases (Roll, Baker, Aleven, & Koedinger, 2014). Based on the method utilized by Roll, Baker, Aleven, and Koedinger (2014), the three phases of the learning process were: (1) low-skill phase ($0<P(L_n)<0.4$); (2) medium-skill phase ($0.4<P(L_n)<0.6$); (3) high-skill phase ($P(L_n)>0.6$).

Bayesian knowledge tracing model is used to model the change of each student's knowledge during learning within the intelligent tutoring system. It assumes that if students give correct answers, it signifies that they know the knowledge, otherwise they do not know the knowledge. Sometimes students may guess to obtain the correct answers or give wrong answers due to carelessness. Bayesian knowledge tracing model considers the probabilities of guess (P(G)) and slips (P(S)) as fixed variables in learning. It generates the probability of knowing the knowledge ($P(L_n)$) based of the performance on the previous action. Formula 1 is to produce $P(L_n)$.

$$P(L_{n-1}|Correct_n) = \frac{P(L_{n-1}) * (1 - P(S))}{P(L_{n-1}) * \left(1 - P(S)\right) + \ (1 - P(L_{n-1})) * (P(G))}$$

$$P(L_{n-1}|Incorrect_n) = \frac{P(L_{n-1}) * P(S)}{P(L_{n-1}) * P(S) + \ (1 - P(L_{n-1})) * (1 - P(G))}$$

$$P(L_n|Action_n) = \ P(L_{n-1}|Action_n) + \left((1 - P(L_{n-1}|Action_n)\right) * P(T))$$

Bayesian Knowledge Tracing model (1)

Probabilities of strategies occurring after errors

Probabilities of strategies occurring after errors were measured by the likelihood metric of D'Mello and Graesser (2012) (see Formula 2). This method balances the influence of an event's basic probability of its related event. The formula was listed below. Here, C represents an error, and X represents a strategy to learn from errors. When the value generated by the formula is larger than zero, it indicates that the probability that the strategy being used after an error is above and beyond the base rate of the strategy. When the value equals to zero, the probability of strategy is by chance. When the value is smaller than zero, the likelihood of strategy being used after an error is much lower than the base rate of the strategy.

$$L[C \rightarrow X] = \frac{\frac{\Pr[X \cap C]}{\Pr[C]} - \Pr[X]}{1 - \Pr[X]}$$

The likelihood metric formula (2)

Strategy shifts

This study examined the specific transition patterns between strategies in the temporal sequence. The likelihood metric of D'Mello and Graesser (2012) were also used to measure those strategy shifts. In the process of calculating strategy shifts, C represented the strategy occurring after the current error, and X represents the strategy occurring after the next error. If the value obtained by the formula was higher than zero, then the strategy shift took place in the higher probability than the base rate of the strategy occurring after the current error. If the value equaled to zero, then the strategies shift occurred by chance. If the value was lower than zero, then the strategy shift occurred in lower probability than the base rate of the strategy occurring after the current error.

Randomness

Randomness was used to measure whether students apply strategies to learn from errors in an ordered manner. It was measured by Shannon entropy (1951). In the formula of entropy (see Formula 3), x_i represents a strategy being used after errors. When the formula produces a high value of entropy, this indicates that students tend to use strategies more randomly. When the formula produces low value, this indicates that students employ strategies in a more ordered fashion.

$$H(X) = -\sum_{i=1}^{n} p(x_i) \log p(x_i)$$

The formula of Shannon entropy (3)

Prior knowledge

Prior knowledge was measured by students' pretest. For 6th graders, 5th- grade math score of TCAP was considered as prior knowledge. For college students, the first assessment score in ALEKS was treated as prior knowledge.

Slip probability

Slip probability is applied to measure whether errors are caused by carelessness. This study introduced the contextual slip model (Baker, Corbett, & Aleven, 2008a; Baker, Corbett, & Aleven, 2008b) to estimate slip probability for each error. The contextual slip model evolves from Bayesian knowledge tracing model that assumes slip probability for all errors is the same. The contextual slip model employs the correctness of the two actions afterward (*n+1, n+2*) to estimate the slip probability of the current error. For example, if the student gives correct answers in the next two steps, then it is very likely that the current error is caused by carelessness. Formula 4 is to generate slip probability.

$$P(L_n|\, A_{+1+2}) = \frac{P(A_{+1+2}|\, L_n) * P(L_n)}{P(A_{+1+2})}$$

The formula of slip probability (4)

In the formula, $P(L_n)$ is the probability of knowing the knowledge obtained from Bayesian knowledge tracing model. $P(A_{+1+2})$ is the probability of actions at step A+1 and A+2 (see Formula 5). $P(A_{+1+2}|L_n)$ is the probability of actions at step A+1 and A+2 when a student knows the knowledge at current error step. $P(A_{+1+2}|{\sim}L_n)$ is the probability of actions at step A+1 and A+2 when a student does not know the knowledge at current error step.

$$P(A_{+1+2}) = P(L_n) * P(A_{+1+2}|\, L_n) + (1 - P(L_n)) * P(A_{+1+2}|\, {\sim}L_n)$$

$$P(A_{+1+2} = C, C|\, L_n) = P({\sim}S)^2 \qquad P(A_{+1+2} = C, {\sim}C|\, L_n) = P(G)P({\sim}S)$$

$$P(A_{+1+2} = {\sim}C, C|\, L_n) = P(G)P({\sim}S) \qquad P(A_{+1+2} = {\sim}C, {\sim}C|\, L_n) = P(G)^2$$

$$P(A_{+1+2} = C, C|\, L_n) = P({\sim}S)^2 \qquad P(A_{+1+2} = C, {\sim}C|\, L_n) = P(G)P({\sim}S)$$

$$P(A_{+1+2} = {\sim}C, C|\, L_n) = P(G)P({\sim}S) \qquad P(A_{+1+2} = {\sim}C, {\sim}C|\, L_n) = P(G)^2$$

$$P(A_{+1+2} = C, C|\, {\sim}L_n) = P(T)P({\sim}S)^2 + P({\sim}T)P(T)P(G)P({\sim}S) + P({\sim}T)^2P(G)^2$$

$$P(A_{+1+2} = C, {\sim}C|\, {\sim}L_n) = P(T)P({\sim}S)P(S) + P({\sim}T)P(T)P(G)\big(P(S)\big) + P({\sim}T)^2P(G)P({\sim}G)$$

$$P(A_{+1+2} = {\sim}C, C|\, {\sim}L_n) = P(T)P(S)P({\sim}S) + P({\sim}T)P(T)P({\sim}G)P({\sim}S) + P({\sim}T)^2P({\sim}G)P(G))$$

$$P(A_{+1+2} = {\sim}C, {\sim}C|\, {\sim}L_n) = P(T)P(S)^2 + P({\sim}T)P(T)P({\sim}G)P(S) + P({\sim}T)^2P({\sim}G)^2$$

$$P(A_{+1+2} = C, C|\, {\sim}L_n) = P(T)P({\sim}S)^2 + P({\sim}T)P(T)P(G)P({\sim}S) + P({\sim}T)^2P(G)^2$$

$$P(A_{+1+2} = C, {\sim}C|\, {\sim}L_n) = P(T)P({\sim}S)P(S) + P({\sim}T)P(T)P(G)\big(P(S)\big) + P({\sim}T)^2P(G)P({\sim}G)$$

$$P(A_{+1+2} = {\sim}C, C|\, {\sim}L_n) = P(T)P(S)P({\sim}S) + P({\sim}T)P(T)P({\sim}G)P({\sim}S) + P({\sim}T)^2P({\sim}G)P(G))$$

$$P(A_{+1+2} = {\sim}C, {\sim}C|\, {\sim}L_n) = P(T)P(S)^2 + P({\sim}T)P(T)P({\sim}G)P(S) + P({\sim}T)^2P({\sim}G)^2$$

Formulae of the probability of actions at step A+1 and A+2 (5)

However, the estimated slip probability by using the next two steps is hard to apply to the model in real time. Additionally, in some situations, the Bayesian knowledge tracing model may produce an over-estimated slip probability. Therefore, the extreme high slip probabilities need to be removed in the calculations. Then the rest of the estimated slip probabilities are applied as training labels of careless errors to produce a less noisy model. That is, those filtered slip probabilities are applied to train the linear regression model built on the past learning behaviors. The final slip probability is the prediction values produced by the linear regression model.

Topic difficulty

In ALEKS not all the topics are equally attempted by every student because students have the freedom to choose or quit the topics in the knowledge pies. This study firstly chose the topics whose attempt frequencies were higher than the average value. This study assumed that higher mastery on the topics signified that the topics were easier. Thus the chosen topics were sorted by mastery percentage (i.e. the number of students mastering the topic/the number of students attempting the topic) in descending order. The topics with mastery percentage not less than the average mastery were defined as difficult topics, whereas the other topics were defined as easy topics.

Learning outcomes

Learning outcomes included students' immediate performance and delayed performance. The immediate performance was measured by the probability of correctness in the next step after a strategy. That is, for each student, immediate performance= (the number of correct responses on the instances after a specific strategy being used/ the number of instances in which this strategy was used). For example, if a student utilized practice strategy after errors for 20 times, and in 16 times the student obtained correct answers in the next problem following practice strategy, then the probability of correctness in the next attempt was computed as 16/20=0.80. This study treated reading an explanation in the next step as failure to solve a problem on that step.

The delayed performance was measured by students' posttest. For 6th graders, 6th grade TCAP math score was used as delayed performance. The last assessment score in ALEKS was applied to college students' delayed performance.

Analysis methods

This study used one-way ANOVA to compare differences between means. This analysis method was applied to compare (1) the probabilities of the strategies and strategy shift in the whole learning process as well as in different learning phases; (2) the likelihood of strategies, randomness, and strategy shift in different levels of topic difficulty; (3) immediate performance after different strategies as well as immediate performance after different strategies in each level of prior knowledge; (4) immediate performance after different strategies in each level of topics' difficulty.

Pearson's correlations were applied to test the relationships between two independent variables. It was utilized to estimate the relationship of the likelihood of strategies, randomness, and strategy shift with prior knowledge as well as the relationship of the likelihood of strategies, randomness, and strategy shift with slip probability.

Multiple linear regressions were conducted to examine the relationships of factors with learning outcomes. The regressions were used to examine: (1) the relationships of the likelihood of strategies, and randomness with delayed performance; (2) the relationships of strategy shifts with delayed performance in different learning phases; (3) the relationships likelihood of strategies and strategy shifts with delayed performance in each level of prior knowledge.

The individual student's data in this study was nested in different classes but the mixed effects model was not applied in this study. The reason was that the number of students was less than 10 in many classes so that the matrix containing students' strategies and classes was sparse. Therefore, there were not enough observations to conduct the mixed effects model.

Expected Results

Question 1: How do students learn from errors?

Q 1a: What strategies do students utilize to learn from errors?

As the discussion in the literature, students were assumed to start from immediate phase because of the adaptive design of students' prior knowledge in ALEKS. Therefore, mixed strategies (i.e. a combination of practice and requesting worked examples) would be the most frequent strategies used by students (**Hypothesis 1**).

Q 1b: How do students use strategies to learn from errors?

Based on the model of help-seeking behavior (Aleven, McLaren, Roll, & Koedigner, 2006), students request help when feeling unfamiliar with the problems and attempted to solve problems after having a sense of how to solve the problem. Furthermore, the learning phase theory (VanLehn, 1996) claimed that student allocated attentions to practice after gaining a basic understanding of domain knowledge through teachers' or the learning system's help. But at this moment practice was not the primary behavior because student focused more on accumulating domain knowledge instead of applying knowledge. Therefore, it followed this argument that ideally after using help strategy, the student should attempt to request a worked example and then solved a problem (example-practice) when making the next error (**Hypothesis 2**).

When students used mixed strategies after the current error, he or she may stay in intermediate phase according to the learning phase theory. The student may continue to stay in intermediate phase or prepared to move forward to the late phase. Therefore, after using mixed strategy on the current error, the student should be likely to use mixed or practice strategies when making the next error (**Hypothesis 3**).

In a similar fashion, when the student attempted two sequential problems after the current error (practice strategy), he or she was assumed in the late phase of learning. Thus, after using practice strategy on the current error, the student may focus on problem-solving (practice strategies) to help troubleshoot the solution of the problem when making the next error (**Hypothesis 4**).

Q 1c: How do those strategies evolve overtime?

According to learning phase theory (VanLehn, 1996), students' learning behaviors would gradually transit from seeking help to practice as their domain knowledge was accumulated from the early learning phase to the late learning phase. Because ALEKS is designed to adapt students' domain knowledge on time, hence this study assumed the low-skill phase as the starting of the intermediate phase. In the low-skill phase, student gained some domain knowledge but still needed help to accumulate related knowledge to solve problems successfully. Therefore, in low-skill phase students may tend to use more strategies for example-practice (**Hypothesis 5)**. In the medium-skill phase which student was preparing to enter late phase, students gradually took practice as the main goal. Therefore, students may use more strategies of practice-example (**Hypothesis 6**). In the high-skill phase which student focused on practice, therefore, students may tend to use practice strategies after errors (**Hypothesis 7**).

Similarly, in the low-skill phase, students may transit from the strategy used after the current error to example-practice after the next errors (**Hypothesis 8**). In the medium-skill phase, students may transit from the strategy used after the current error to practice- example after the next errors because they paid more attention to practice (**Hypothesis 9**). In the high-skill phase students focus on practice, therefore, they may transit from the strategy used after the current error to practice strategies after the next errors (**Hypothesis 10**).

Question 2: What factors influence the strategies of learning from errors?

Q 2a: How does prior knowledge influence strategies being utilized to learn from errors?

Based on expertise reversal effect (Renkl & Atkinson, 2003; Pass, Renkl, & Sweller, 2003), prior knowledge was assumed to positively relate to the probability of practice strategy (**Hypothesis 11**). Ordered pattern of strategies (low entropy value of randomness) was assumed to positively associate with prior knowledge according to the finding of the existing study (Snow, Jackson, & McNamara, 2014) (**Hypothesis 12**).

According to the model of help-seeking (Aleven, McLaren, Roll, & Koedigner, 2006) and relationship between worked example and prior knowledge (Kalyuga, Chandler, Tuovinen, & Sweller, 2001; Pass & Van Gog, 2006), students with the low prior knowledge may more rely on help to have a sense of how to solve the problems before solving problems. Therefore, students with the low prior knowledge were assumed to be more likely to transit from the strategy used after the current error to example-practice (i.e. request a worked example before solving a problem) after the next error (**Hypothesis 13**).

Q2b: How does error type influence strategies being utilized to learn from errors?

According to discussions in the literature about the relationship between error types and learning from errors, slip probability was assumed to positively relate to the probability of practice strategies (**Hypothesis 14**) and ordered pattern of strategies (low entropy value of randomness) (**Hypothesis 15**).

Q2c: How does topic difficulty influence strategies being utilized to learn from errors?

Based on the impact of worked examples on learning (Renkl & Atkinson, 2003; Van Gog, Kester, & Paas, 2011), difficult topics were assumed to present with a higher probability of example-practice (**Hypothesis 16**). Similarly, difficult topics were hypothesized to present the transition from the current strategies to example-practice (i.e. request a worked example after solving a problem) (**Hypothesis 17**).

Additionally, a disordered pattern of strategies (high entropy value of randomness) was assumed to positively related to difficult topics (Snow, Jackson, & McNamara, 2014) (**Hypothesis 18**).

Question 3: How do strategies of learning from errors relate to learning outcomes?

Q 3a: How do strategies to learn from errors relate to learning outcomes (i.e. immediate and delayed learning outcomes)?

According to learning phase theory, ideally, practice was assumed to be used in the late learning phase to learn from errors to increase the accuracy and speed of problem-solving. Therefore, it was hypothesized that the likelihood of correctness after practice strategies was highest among four strategies (**Hypothesis 19**).

Based on the findings of Van Gog, Kester, and Paas (2011) that pure reading worked examples and worked examples interleaved with practice fostered better learning, hence it was assumed that help strategy and mixed strategies positively linked to delayed performance (**Hypothesis 20**).

Ordered pattern of strategies (low entropy value of randomness) was assumed to positively associate to delayed performance based on the finding of Snow, Jackson, and McNamara (2014) (**Hypothesis 21**).

The ideal transition occurring in each learning phase should benefit learning according to the learning phase theory (VanLehn, 1996). Specifically, the transitions from the current strategies to example-practice strategies (request a worked example before solving a problem) in the low-skill phase was assumed to benefit delayed performance (**Hypothesis 22**).

In medium-skill phase, the transitions from the current strategies to practice-example strategies (i.e. request a worked example after solving a problem) was assumed to boost delayed performance (**Hypothesis 23**).

In high-skill phase, the transitions from the current strategies to practice strategies (i.e. solve two problems) was hypothesized to favor delayed performance (**Hypothesis 24**).

Q 3b: How does prior knowledge influence the relationship between strategies and learning outcomes?

Students with the low prior knowledge need more help (Fyfe, Rittle- Johnson, & DeCaro, 2012) and benefit more from worked examples than high prior knowledge (Pass & Van Gog, 2006). But overuse of worked examples occupy limited working memory to process redundant information (Renkl & Atkinson, 2003). Therefore, example-practice (i.e. request a worked example before solving a problem) was assumed to boost the likelihood of correctness in the next step for students with the low prior knowledge (**Hypothesis 25**). Similarly, example-practice was also supposed to increase the delayed performance of students with the low prior knowledge (**Hypothesis 26**).

In similar fashion, the transitions from the current strategies to example-practice strategies (i.e. request a worked example before solving a problem) was hypothesized to benefit delayed performance of low prior knowledge students (**Hypothesis 27**).

Students with the high prior knowledge tend to learn by themselves (Fyfe, Rittle- Johnson, & DeCaro, 2012) and are impeded by too much help (Pass, Renkl, & Sweller, 2003). Therefore, the correctness after practice strategies (i.e. solve two sequential problems) was assumed to be higher than the correctness after other strategies for the high prior knowledge level (**Hypothesis 28**).

In like manner, practice strategies were supposed to positively link to posttest of students with the high prior knowledge (**Hypothesis 29**).

It followed the above argument that the transitions from the current strategies to practice strategies (i.e. solve two sequential problems) was assumed to favor posttest of high prior knowledge students (**Hypothesis 30**).

Q 3c: How does topic difficulty influence the relationship between strategies and learning outcomes?

According to the relationship between topic difficulty and worked example in the existing literature (Kalyuga, 2007; Pass & Van Gog, 2006; Renkl & Atkinson, 2003; Van Gog & Kester, 2012), the correctness after example-practice was hypothesized to be higher than other strategies on difficult topics (**Hypothesis 31**). However, the correctness after practice strategies was hypothesized to be highest among the strategies for easy topics (**Hypothesis 32**).

Chapter 5: Results

The systematic investigations on strategies after making errors yielded abundant results. Therefore, in order to clearly illustrate the findings, two tables were used to separately display the hypothesis supported by results and the hypothesis that were not supported by the results (See Table 1 and Table 2).

Table 1.

The hypothesis that were supported by the results.

Hypothesis	Theoretical basis	Percentage of results to support hypothesis
Mixed strategies occurred most frequently after errors after adjusting base rate.	Learning phase theory (VanLehn, 1996)	100%
Example-practice strategies were used more frequently than other strategies in the low-skill phase.	Learning phase theory (VanLehn, 1996)	50%
Practice-example strategies were more likely to be used in the medium-skill phase.	Learning phase theory (VanLehn, 1996)	28.6%
Practice strategies occurred more frequently than others in the high-skill phase.	Learning phase theory (VanLehn, 1996)	40%
Mixed strategies tended to transition to mixed strategies or practice strategies.	Learning phase theory (VanLehn, 1996)	58.3%
Practice strategies were apt to transition to practice strategies.	Learning phase theory (VanLehn, 1996)	70%
Strategies were prone to transition to example-practice in the low-skill phase.	Learning phase theory (VanLehn, 1996)	21%

Table 1 (Continued)

Strategies tended to transition to practice-example in the medium-skill phase.	Learning phase theory (VanLehn, 1996)	20%
Strategies were more likely to transition to practice strategies in the high-skill phase.	Learning phase theory (VanLehn, 1996)	38%
Students tended to use example-practice after errors on difficult topics than easy topics.	Worked examples foster better performance than problem solving when students lack knowledge (Renkl & Atkinson, 2003)	30%
Help strategy and mixed strategies were beneficial to posttest.	Learning phase theory (VanLehn, 1996)	60%
In the low-skill phase the transitions from the current strategies to example-practice strategies benefited students' posttest.	Learning phase theory (VanLehn, 1996)	15.4%
The transitions between practice-example benefited students' posttest in the medium-skill phase.	Learning phase theory (VanLehn, 1996)	18.2%
The transitions from practice to practice were boost students' posttest in the high-skill phase.	Learning phase theory (VanLehn, 1996)	8.3%
The correctness after practice strategies was higher than the correctness after other strategies in high prior knowledge level.	Learning phase theory (VanLehn, 1996)	100%
Example-practice strategies benefited students in the low prior knowledge.	Learning phase theory (VanLehn, 1996)	33.3%
Practice strategies favored posttest scores of students in the high prior knowledge.	Learning phase theory (VanLehn, 1996)	40%

Table 1 (Continued)

The transitions from the current strategies to example-practice strategies benefited posttest of students with low prior knowledge.	Learning phase theory (VanLehn, 1996)	6.25%
Transitions from the current strategies to practice strategies were beneficial to students with high prior knowledge.	Learning phase theory (VanLehn, 1996)	55.6%
The correctness after practice strategies was highest on easy topics.	Learning phase theory (VanLehn, 1996)	100%

Table 2.

The hypothesis that were not supported by the results

Hypothesis	Theoretical basis
Help strategy tended to transition to example-practice after the next error.	Learning phase theory (VanLehn, 1996)
Mixed strategies tended to transition to mixed strategies or practice strategies.	Learning phase theory (VanLehn, 1996)
Practice strategies were apt to transition to practice strategies.	Learning phase theory (VanLehn, 1996)
Students with higher prior knowledge were more likely to utilize practice strategies after the error.	Expertise reversal effect (Renkl & Atkinson, 2003)
Students with high prior knowledge exerted more ordered pattern when using strategies after errors.	Students in the high prior knowledge exerted an ordered pattern when using strategies (Snow, Jackson, & McNamara, 2014)
Students with lower prior knowledge tended to transition from the current strategy to example-practice.	The model of help-seeking (Aleven, McLaren, Roll, & Koedigner, 2006)

Table 2 (Continued)

Students were more likely to use practice strategies after making careless errors.	Learning phase theory (VanLehn, 1996)
Students used strategies after errors in an ordered pattern when making careless errors.	Students in the high prior knowledge exerted an ordered pattern when using strategies (Snow, Jackson, & McNamara, 2014)
Strategies occurred in a more disordered pattern on difficult topics (high entropy value) than easy topics.	Students in the high prior knowledge exerted an ordered pattern when using strategies (Snow, Jackson, & McNamara, 2014)
Strategies were more likely to transition to example-practice strategies on difficult topics than easy topics.	Worked examples foster better performance than problem solving when students lack knowledge (Renkl & Atkinson, 2003)
The randomness of strategies used after errors was negatively associated with posttest.	An ordered pattern of strategies was positively associated to learning performance (Snow, Jackson, and McNamara, 2014)
The correctness after example-practice was higher than the correctness after other strategies in low prior knowledge level.	Worked examples facilitated learning performance of students in the low prior knowledge (Pass & Van Gog, 2006); expertise reversal effects (Renkl & Atkinson, 2003)
The correctness after example-practice was highest among the strategies on difficult topics	Worked examples foster better performance than problem solving when students lack knowledge (Renkl & Atkinson, 2003; Pass & Van Gog, 2006)

The patterns of strategies utilized to learn from errors

The overall likelihood of strategies occurring after errors in learning

The results of a one-way ANOVA implied that for 6th graders, there were significant differences among the strategies' likelihood of occurrence after an error, $F(1,1834) = 276.2$, $p < .000$, $\eta^2 = 0.13$. Table 3 shows the likelihood of occurrence after an error in the 6th graders' group as well as each strategy's base rate and conditional percentage after an error.

Multiple comparisons of the nine substrategies suggested that likelihood of example-wrong (EW, $M = 0.08$) occurring after an error was significantly higher than other strategies. The likelihood of example-correct (EC, $M = 0.05$) was significantly lower than example-wrong but significantly higher than the rest of strategies. That is, the likelihood of example-practice occurred more frequently than help strategy, practice-example, and practice strategy.

There were no significant differences between the likelihood of help strategy (EE, $M = 0.004$), correct-example (CE, $M = -0.002$), and wrong-wrong (WW, $M = 0.01$). Their likelihood was significantly lower than the likelihoods of EW and EC, but higher than the likelihoods of the remaining strategies. The likelihoods of wrong-example (WE, $M = -0.02$), wrong-correct (WC, $M = -0.03$), correct-wrong (CW, $M = -0.01$), and correct-correct (CC, $M = -0.10$) were significantly lower than the other strategies, but there were no significant differences between them.

Table 3

The strategy's likelihood of occurrence after an error, base rate, and conditional percentage after an error (6th graders)

	Help	Example-practice		Practice-example		Practice			
	EE	EW	EC	WE	CE	WW	WC	CW	CC

Table 3 (Continued)

Base rate	0.06	0.10	0.07	0.16	0.01	0.19	0.10	0.10	0.22
Conditional percentage after an error	0.06	0.18	0.12	0.14	0.01	0.20	0.08	0.09	0.14
Likelihood of occurrence after an error (compared to base rate)	0.004	0.08***	0.05***	-0.02***	-0.002***	0.01***	-0.03***	-0.01***	-0.10***

Note: A one-sample t test ($\mu = 0$) was used to examine whether the likelihood of occurrence after an error was different from the base rate. If the likelihood was significantly below the base rate, then the value would be negative and marked with *. If the likelihood was significantly beyond the base rate, then the value would be positive and marked with *. If the likelihood was not significantly different than the base rate, then the value would NOT be marked with *. *, $p < 0.006$, **, $p < 0.001$, ***, $p < 0.0001$

The results also showed that for college students, the likelihoods of strategies were significantly different, $F(1,1609) = 674.9$, $p < .000$, $\eta^2 = 0.29$. Table 4 illustrates the likelihoods of strategies occurring after an error in the college students' group as well as each strategy's base rate and conditional percentage after an error.

In a similar fashion, the results of multiple comparisons indicated that likelihood of wrong-example (WE) ($M = 0.17$) occurring after an error was significantly higher than other strategies including correct-example (CE) ($M = -0.01$). Therefore, WE dominated practice-example and was the college students' most frequently used strategy. Additionally, the likelihood of example-correct (EC) ($M = 0.10$) was significantly lower than WE but was higher than the likelihoods of the rest of the strategies. Based on the above results, practice-example (dominated by WE) was the most frequently used strategy and example-practice was the second most frequently used strategy for college students.

By combining the results from 6th graders and college students, it was found that mixed strategies were the most frequently used strategies after an error. The findings confirmed **Hypothesis 1** which predicted that mixed strategies were the most frequently used strategies by students after making an error.

Table 4

The strategy's likelihood of occurrence after an error, base rate, and conditional probability after an error (college students)

	Help	Example-practice		Practice-example		Practice			
	EE	EW	EC	WE	CE	WW	WC	CW	CC
Base rate	0.05	0.08	0.11	0.15	0.02	0.08	0.07	0.09	0.34
Conditional percentage after an error	0.06	0.12	0.19	0.30	0.01	0.01	0.01	0.08	0.22
Likelihood of occurrence after an error (compared to base rate)	0.01 ***	0.05 ***	0.09 ***	0.17 ***	-0.01 ***	-0.07 ***	-0.07 ***	-0.01 ***	-0.20 ***

Note: A one-sample t test was used to examine whether the likelihood of occurrence after an error was different from the base rate. If the likelihood was significantly below the base rate, then the value would be negative and marked with *. If the likelihood was significantly beyond the base rate, then the value would be positive and marked with *. If the likelihood was not significantly different than the base rate, then the value would NOT be marked with *. *, $p <$ 0.006, **, $p < 0.001$, ***, $p < 0.0001$

The likelihood of strategies occurring after errors in different learning phases

The likelihood of strategies in the low-skill phase (PL<0.4)

The results of comparisons among the likelihood of strategies occurring after errors for 6th graders in the low-skill phase showed significant differences, $F(1,1834) = 198.8$, $p < 0.000$, $\eta^2 = 0.10$. The multiple comparisons indicated that EW ($M = 0.05$) and CC ($M = 0.03$) were the most frequent strategies, and WE ($M = -0.09$) and WC ($M = -0.05$) occurred less than other strategies in the low-skill phase. Table 1 in the Appendix A lists the descriptions of the likelihood of strategies for 6th graders in the low-skill phase.

For college students, there were no significant differences were found between the likelihoods of strategies occurring after errors in the low-skill phase, $F(1,1609) = 1.21$, $p = 0.27$, $\eta^2 = 0.001$. Table 2 in the Appendix A lists the descriptions of the likelihood of strategies for college students in the low-skill phase.

After combining the results of example-practice strategies for 6th graders and college students, EW was found to be used most frequently only by 6th graders. Therefore, **Hypothesis 5**, which predicted that students would use more example-practice strategies than other strategies in the low-skill phase, gained narrow support from the results.

The likelihood of strategies in the medium-skill phase (0.4<PL<0.6)

For 6th graders, the results showed significant differences among the likelihood of strategies occurring after errors in the medium-skill phase, $F(1,1630) = 900$, $p < 0.000$, $\eta^2 = 0.36$. The multiple comparisons indicated that EE ($M = 0.05$), EW ($M = 0.06$), EC ($M = 0.07$), WE ($M = 0.06$), and WW ($M = 0.07$) occurred more frequently than other strategies, but CC ($M = -0.28$) occurred less frequently relative to other strategies. Table 3 in the Appendix A lists the descriptions of the likelihood of strategies for 6th graders in the medium-skill phase.

For college students, the results revealed significant differences among the likelihood of strategies occurring after errors in the medium-skill phase, $F(1,1251) = 725.9$, $p < 0.000$, $\eta^2 = 0.37$. The multiple comparisons of college students' strategies in the medium-skill phase showed that WE ($M = 0.11$) and EC ($M = 0.10$) tended to be used after errors. However, CC ($M = -0.22$) was less likely to occur after errors relative to other strategies. Table 4 in the Appendix A lists the descriptions of college students' likelihood of strategies in the medium-skill phase.

Based on the results of both 6th graders and college students across the practice-example strategies, only WE was found to be frequently used by students. Therefore, the results provided limited support for **Hypothesis 6** that practice-example strategies were more likely to be used in the medium-skill phase.

The likelihood of strategies in the high-skill phase (P > 0.6)

In the high-skill phase, the results of comparisons among 6th graders' likelihood of strategies revealed significant differences, $F(1.1630) = 690.4$, $p < 0.000$, $\eta^2 = 0.30$. The multiple comparisons indicated that EE (M=0.07), EW ($M = 0.07$), EC ($M = 0.10$), WE ($M = 0.09$), WW ($M = 0.10$), and WC ($M = 0.07$) occurred more frequently than other strategies, and CC (M = -1.16) was less likely to occur. Table 5 in the Appendix A lists the descriptions of the likelihood of strategies for 6th graders in the high-skill phase.

For college students, the results of comparisons among the likelihood of strategies in the high-skill phase showed significant differences, $F(1,1251) = 762.8$, $p < 0.000$, $\eta^2 = 0.38$. Further comparisons among the strategies indicated that EE ($M = 0.06$), EW ($M = 0.07$), EC ($M = 0.14$), and WE ($M = 0.13$) were more likely to occur, and CC ($M = -1.22$) occurred the least. Table 6 in the Appendix A lists the descriptions of the likelihood of strategies for college students in the high-skill phase.

According to the results of 6th graders and college students, WW, WC, CW, EE and some of example-practice strategies (e.g. EW, EC, and WE) tended to occur more frequently than the rest of strategies. Therefore, the results partially supported **Hypothesis 7** that practice strategies occurred more frequently than other strategies in the high-skill phase.

The patterns of strategy shifts occurring after errors

The overall patterns of strategy shifts occurring after errors

Strategy shifts from help strategy

The one-way ANOVA results of transitional likelihood between strategies used after two sequential errors by 6th graders showed that there were significant differences between the transitions from the help strategy (EE) used after the current errors to the strategies used after the next errors (EE→ next strategy), $F(8,1827) = 30.73$, $p < 0.000$, $\eta^2 = 0.12$. Results of further comparisons indicated that when using EE after the current error, students were more likely to continually adopt help strategies after the next error (EE→ EE, $M = 0.04$) relative to other strategies. The help strategy was least likely to change to wrong-wrong (EE→ WW, $M = -0.03$) or correct-correct (EE→ CC, M=-0.03) after the next error occurred. Table 7 in the Appendix A displays 6th graders' base rate and conditional probability for all strategy shifts. Table 8 in the Appendix A shows 6th graders' likelihood of strategy shifts from all strategies.

Similar to 6th graders, significant differences were found for college students' transitional likelihood from EE to the next strategy (EE→ next strategy), $F(8,1602) = 49.49$, $p < 0.000$, $\eta^2 = 0.20$. The multiple comparisons indicated that EE→EE ($M = 0.02$) occurred more often than other transitions. EE→WE ($M = -0.02$), EE→CC ($M = -0.04$), EE→EC ($M = -0.04$), and EE→EW ($M = -0.03$) were less likely to occur relative to other transitions from EE. Table 9 in the Appendix A shows college students' base rate and conditional probability for all strategy shifts. Table 10 in the Appendix A illustrates college students' likelihood of strategy shifts from all strategies.

Based on the results from both 6th graders and college students, **Hypothesis 2**, which predicted that help strategy tended to transition to example-practice after the next error, was not supported.

Strategy shifts from mixed strategies

As for 6th graders' example-practice strategies (i.e. example-wrong/EW and example-correct/EC) used after the current errors, there was significant differences between the transitional likelihood from EW/EC to the strategies used after the next errors, $F(8,1827) = 28,22$, $p < 0.000$, $\eta^2 = 0.11$ (EW→ next strategy), $F(8,1827) = 30.03$, $p < 0.000$, $\eta^2 = 0.12$ (EC→ next strategy). The results of multiple comparisons indicated that among the transitions from EW to another strategy (EW→ next strategy), EW→WW ($M = 0.04$) occurred most while EW→CE ($M = -0.02$) and EW→CC ($M = -0.02$) were observed less frequently than other transitions. Among the transitions from EC to the next strategy (EC→ next strategy), EC→CC ($M = 0.04$) tended to occur more than other transitions while EC→ EW ($M = -0.04$) was least likely to occur. According to the results for 6th graders, example-practice strategies were more likely to transition to some specific practice strategies (EW→WW, EC→CC).

For 6th graders, significant differences were also found among the transitional likelihood from practice-example strategies (i.e. wrong-example/WE and correct-example /CE) to the strategies used after the next errors, $F(8,1827) =14.14$, $p < 0.000$, $\eta^2 = 0.06$ (WE→ next strategy), $F(8,1827) =154.4$, $p < 0.000$, $\eta^2 = 0.40$ (CE→ next strategy). The results of multiple comparisons showed that for transitions from WE to the next strategy (WE→ next strategy), WE→ WE ($M = 0.02$) and WE→ WW ($M = 0.03$) occurred most. Among the transitions from CE to the next strategy (CE→ next strategy), CE→CE (*M* = 0.06) tended to occur more frequently whereas CE→ EW (*M* = -0.05) and CE→ WW (*M* = -0.06) were less likely to occur. Therefore, the results indicated that 6th graders may tend to repeat the same practice-example strategies used after the current error on the next error (WE→ WE, CE→CE).

The comparisons of college students' transitional likelihood from example-practice (i.e. example-wrong/EW and example-correct/EC) to the strategies used after the next errors showed significant differences, $F(8,1602) = 28.22$, $p < 0.000$, $\eta^2 = 0.08$ (EW→ next strategy), $F(8,1602) = 10.49$, $p < 0.000$, $\eta^2 = 0.05$ (EC→ next strategy). Specifically, among the transitions from EW to the next strategy (EW→ next strategy), EW→WE (*M* = 0.04) was most likely to occur. Multiple comparisons of the transitions from EC to the next strategy (EC→ next strategy) revealed that EC→EC (*M* = 0.01), and EC→CE (*M* = 0.01) occurred more often. However, EC→EW (*M* = -0.04) was less likely to occur compared to other transitions. Therefore, the results may imply that example-practice tended to transition to some specific mixed strategies (EW→WE, EC→EC, EC→CE) when students made prior errors.

The comparisons of college students' transitional likelihood from practice-example (i.e. wrong-example/WE and correct-example /CE) to the strategies used after the next errors showed significant differences, $F(8,1602) = 211.7$, $p < 0.000$, $\eta^2 = 0.51$(WE→ next strategy), $F(8,1602) = 237.1$, $p < 0.000$, $\eta^2 = 0.54$ (CE→ next strategy). Multiple comparisons of the transitions from WE to the next strategy (WE→ next strategy) showed that WE→WE (*M* = 0.13) was more likely to occur relative to other transitions, but WE→EC (*M* = -0.10) occurred less than other transitions. Among the transitions from CE to the next strategy (CE→ next strategy), CE→ CE (*M* = 0.05), CE→WW (*M* = 0.05), and CE→WC (*M* = 0.05) were more likely to occur compared to other transitions and the transition of CE→EC (*M* = -0.10) occurred the least. Therefore, practice-example may tend to transition to specific mixed or practice strategies (WE→WE, CE→ CE, CE→WW, CE→WC).

Based on the results for 6th graders and college students, the mixed strategies may be more likely to transition to mixed or practice strategies. Therefore, the results confirmed **Hypothesis 3**, which predicted that mixed strategies tended to transition to mixed strategies or practice strategies.

Strategy shifts from practice strategies

The one-way ANOVA results revealed significant differences among 6th graders' transitional likelihood from practice strategies (i.e. wrong-wrong/ WW, wrong-correct/ WC, correct-wrong/ CW, correct-correct/ CC) to the strategies used on the next error, $F(8,1827) = 461.1$, $p < 0.000$, $\eta^2 = 0.67$ (WW→ next strategy), $F(8,1827) = 134.2$, $p < 0.000$, $\eta^2 = 0.37$ (WC→ next strategy), $F(8,1827) = 169.8$, $p < 0.000$, $\eta^2 = 0.42$ (CW→ next strategy), $F(8,1827) = 108.7$, $p < 0.000$, $\eta^2 = 0.32$ (CC→ next strategy). Multiple comparisons of transitions from WW to the next strategy (WW→ next strategy) indicated that WW→EW ($M = 0.18$) occurred most often and WW→CC ($M = -0.08$) was observed least often. Among the transitions from WC to the next strategy (WC→ next strategy), WC→WW ($M = 0.04$), WC→WC ($M = 0.05$), WC→CW ($M = 0.04$), and WC→CC ($M = 0.04$) were more likely to occur. Compared to the transitions from CW to other strategies used after the next strategy (CW→ next strategy), CW→WW ($M = 0.07$), CW→WC ($M = 0.05$), and CW→CC ($M = 0.08$) were more likely to be observed. In the transitions from CC to the next strategy (CC→ next strategy), CC→CC ($M = 0.07$) occurred most often. Based on the results, 6th graders continuously tended to adopt practice strategies after the next errors.

The results of comparisons between college students' transitions from practice strategies (i.e. wrong-wrong/ WW, wrong-correct/ WC, correct-wrong/ CW, correct-correct/ CC) revealed significant differences, $F(8,1602) = 111.9$, $p < 0.000$, $\eta^2 = 0.15$ (WW→ next strategy), $F(8,1602) = 289$, $p < 0.000$, $\eta^2 = 0.59$ (WC→ next strategy), $F(8,1602) = 169.8$, $p < .000$, $\eta^2 = 0.52$ (CW→ next strategy), $F(8,1602) = 141$, $p < 0.000$, $\eta^2 = 0.41$ (CC→ next strategy). Among the transitions from WW to the next strategy (WW→ next strategy), WW→ EE ($M = 0.02$), WW→ EW ($M = 0.03$), WW→ CE ($M = 0.04$), WW→ WW ($M = 0.04$), and WW→ WC ($M = 0.04$) were more likely to occur relative to other transitions. However, WW→ CC ($M = -0.05$), WW→ EC ($M = -0.08$), and WW→ WE ($M = -0.08$) occurred less than other transitions.

Multiple comparisons of the transitions from wrong-correct to the next strategy (WC→ next strategy) for college students showed that WC→ WC ($M = 0.05$), WC→ CW ($M = 0.05$), and WC→ CE ($M = 0.05$) tended to be used more often than other strategies. WC→ EC ($M = -0.11$) was least likely to occur.

The results of multiple comparisons among the transitions from correct-wrong to the next strategy (CW→ next strategy) indicated that CW→ WE (M=0.11), and CW→ CC (M=0.11) were more likely to occur, but CW→ EC (M=-0.10) was less likely to occur in college students' learning processes.

Among the transitions from correct-correct to the next strategy (CC→ next strategy), CC→ CC (M=0.06) was more likely to occur while CC→EC (M=-0.11) was least likely to occur. Based on the results above of college students' practice strategy, only some the practice strategies may transition to practice-example (WW→ CE, WC→ CE, and CW→ WE).

Based on the results of 6th graders and college students, 6th graders were only inclined to transit between practice strategies. College students, on the other hand, not only tended to transitioned between practice strategies but were also prone to transition from practice strategies to mixed strategies. Therefore, **Hypothesis 4**, which predicted that practice strategies were apt to transition to practice strategies, was partially supported. Table 5 summarizes the strategy shifts for both 6^{th} graders and college students.

Table 5

Summary of 6th graders and college students' strategy shifts

Hypothesis	6th graders Results	Supported or not?	College students Results	Supported or not?
Hypothesis 2: Help strategy was more likely to transit to example-practice.	EE→EE	No	EE→EE	No
Hypothesis 3: Mixed strategies were prone to transit to mixed strategies or practice strategies.	EW→WW, EC→CC, WE→WE, WE→WW, CE→ CE	Yes	EW→WE, EC→EC, EC→CE, WE→WE, CE→ CE, CE→WW, CE→WC	Yes
Hypothesis 4: Practice strategies tended to transit to practice strategies.	WW→EW, WC→WW, WC→WC, WC→CW, WC→CC, CW→WW, CW→WC, CW→CC, CC→CC	Partially	WW→ EE, WW→EW, WW→ CE, WW→ WW, WW→ WC, WC→ WC, WC→ CW, WC→ CE, CW→ WE, CW→ CC, CC→ CC	Partially

The patterns of strategies occurring after errors in different learning phases

Strategy shifts in the low-skill phase (PL<0.4)

In the low-skill phase, the results of comparing 6th graders' transitions from help strategies to the strategies occurring after the next error (EE→ next strategy) showed significant differences, $F(8,1827) = 26.03$, $p < 0.000$, $\eta^2 = 0.10$. Specifically, among EE→ next strategy, EE→ EE ($M = 0.04$) and EE→ CE ($M = 0.02$) were more likely to occur than other transitions, but EE→WW ($M = -0.04$) occurred least frequently.

For the transitions from the 6th graders' example-practice (i.e. example-wrong/ EW and example-correct/EC) to the next strategies in the low-skill phase, the comparison results revealed significant differences, $F(8,1827) = 24.84$, $p < 0.000$, $\eta^2 = 0.07$ (EW→ next strategy), $F(8,1827) = 40.32$, $p < 0.000$, $\eta^2 = 0.43$ (EC→ next strategy). Among EW→ next strategy, EW→WW ($M = 0.04$) was found to occur more than other transitions. EC→CC ($M = 0.05$) was more likely to occur than other EC→ next strategy.

The comparisons results of 6th graders' transitions from practice-example (i.e. wrong-example/WE and correct-example/CE) to the next strategies in the low-skill phase showed significant differences, $F(8,1827) = 16.56$, $p < 0.000$, $\eta^2 = 0.10$ (WE→ next strategy), $F(8,1827) = 173.6$, $p < 0.000$, $\eta^2 = 0.43$, (CE→ next strategy). The further multiple comparisons results indicated that WE→ WE ($M = 0.02$) and WE→WW ($M = 0.03$) tended to occur more frequently than other WE→ next strategy. CE→ CE ($M = 0.06$) was more likely to occur than other CE→ next strategy.

Significant differences were also found among the 6th graders' transitions from practice (i.e. wrong-wrong/WW, wrong-correct/WC, correct-wrong/CW, correct-correct/CC) to the next strategies in the low-skill phase, $F(8,1827) = 413.5$, $p < 0.000$, $\eta^2 = 0.64$ (WW→ next strategy) $F(8,1827) = 146.7$, $p < 0.000$, $\eta^2 = 0.37$ (WC→ next strategy), $F(8,1827) = 172.9$, $p < 0.000$, $\eta^2 = 0.43$ (CW→ next strategy), $F(6,1421) = 118.3$, $p < 0.000$, $\eta^2 = 0.34$ (CC→ next strategy). Multiple comparisons indicated that WW→ EW ($M = 0.17$) occurred more frequently than WW→ next strategy. Among WC→ next strategy, WC→ WW ($M = 0.04$), WC→ WC ($M = 0.04$), WC→ CW ($M = 0.04$), and WC→ CC ($M = 0.04$) were found to be more likely to occur relative to other strategies. CW→ WW ($M = 0.07$), CW→ WC ($M = 0.05$), and CW→ CC ($M = 0.07$) tended to be observed more frequently than other CW→ next strategy. CC→ CC ($M = 0.07$) was more likely to occur compared to other CC→ next strategy. Table 11 in the Appendix A shows 6th graders' base rate and the conditional probability for all strategy shifts in the low-skill phase. Table 12 in the Appendix A illustrates 6th graders' likelihood of strategy shifts from all strategies in the low-skill phase.

For college students, the results of comparing the transitions from help strategy to the strategies occurring after the next errors in the low-skill phase (EE→ next strategy) showed significant differences, $F(8,1602) = 60.88$, $p < 0.000$, $\eta^2 = 0.23$. Multiple comparisons revealed that EE→EE ($M = 0.03$), EE→EW ($M = 0.01$), EE→EC ($M = 0.01$), EE→CE ($M = 0.02$), EE→WW ($M = 0.02$), and EE→WC ($M = 0.02$) occurred more frequently than other transitions.

The results of comparing college students' transitions from example-practice (i.e. example-wrong/EW and example-correct/EC) to the next strategies in the low-skill phase revealed significant differences, $F(7,1424) = 38.32$, $p < 0.000$, $\eta^2 = 0.16$ (EW→ next strategy), $F(7,1424) = 48.69$, $p < 0.000$, $\eta^2 = 0.19$ (EC→ next strategy). The results of multiple comparisons revealed that among EW→ next strategy, EW→EW ($M = 0.04$), and EW→EC (M

= 0.06) were more likely to occur than others. EC→EC ($M = 0.08$) was found to occur more frequently than other EC→ next strategy.

The results also showed significant differences among the college students' transitions from practice-example (i.e. wrong-example/WE and correct-example/CE) to the next strategies in the low-skill phase, $F(8,1602) = 93.1$, $p < 0.000$, $\eta^2 = 0.32$ (WE→ next strategy), $F(6,1246) = 95.08$, $p < 0.000$, $\eta^2 = 0.31$ (CE→ next strategy). The results of multiple comparison showed that WE→WE ($M = 0.08$) was more likely to occur than other WE→ next strategy. CE→CE ($M = 0.08$) tended to occur more frequently than other CE→ next strategy.

The results of comparing college students' transitions from practice strategies (i.e. wrong-wrong/WW, wrong-correct/WC, correct-wrong/CW, and correct-correct/CC) to the next strategies in the low-skill phase showed significant differences, $F(7,1424) = 100.6$, $p < 0.000$, $\eta^2 = 0.33$ (WW→ next strategy), $F(8,1602) = 152.6$, $p < 0.000$, $\eta^2 = 0.46$ (WC→ next strategy), $F(8,1602) = 100.8$, $p < 0.000$, $\eta^2 = 0.33$ (CW→ next strategy), $F(5,1068) = 47.66$, $p < 0.000$, $\eta^2 = 0.18$ (CC→ next strategy). The results of multiple comparisons revealed that WW→EW ($M = 0.08$) was more likely to occur than other WW→ next strategy. WC→ CE ($M = 0.05$), WC→ WW ($M = 0.05$), and WC→ WC ($M = 0.05$) were observed more frequently than other WC→ next strategy. CW→WE ($M = 0.07$) and CW→CC ($M = 0.08$) occurred more frequently than other CW→ next strategy. CC→ CC ($M = 0.13$) was more likely to occur than other CC→ next strategy. Table 13 in the Appendix A shows the base rate and the conditional probability for all strategy shifts for college students in the low-skill phase. Table 14 in the Appendix A illustrates college students' likelihood of strategy shifts from all strategies in the low-skill phase.

After combining the above results from 6th graders and college students, only some specific types of strategy shifts would transition from the current strategies to example-practice.

For example, WW→ EW was found to occur more frequently than other WW→ next strategy for 6th graders. For college students, EE→EC was one of the most frequent transitions from the help strategy to the next strategy; EW→EW and EW→EC were more likely to occur than other EW→ next strategy. Therefore, the results partially supported **Hypothesis 8** that the current strategies were prone to transition to example-practice in the low-skill phase.

Strategy shifts in the medium-skill phase ($0.4<PL<0.6$)

In the medium-skill phase, the results of comparing 6th graders' transitions from help strategy to the strategies after the next error indicated significant differences, $F(7,1608) = 187.1$, $p < 0.000$, $\eta^2 = 0.45$ (EE→ next strategy). The results of multiple comparisons showed that EE→WC ($M = 0.02$) occurred more frequently than other transitions from the help strategy.

The results of comparing 6th graders' transitions from example-practice (i.e. example-wrong/EW and example-correct/EC) to the next strategies in the medium-skill phase also revealed significant differences, $F(7,1608) = 53.61$, $p < 0.000$, $\eta^2 = 0.16$ (EW→ next strategy), $F(7,1608) = 85.58$, $p < 0.000$, $\eta^2 = 0.27$ (EC→ next strategy). Further multiple comparisons revealed that EW→ WC ($M = 0.02$) was more likely to occur than other EW→ next strategy. EC→WC ($M = 0.02$) was found to occur more frequently than other EC→ next strategy.

The comparison results of 6th graders' transitions from wrong-example to the next strategies (WE→ next strategy) in the medium-skill phase revealed significant differences, $F(7,1608) = 53.61$, $p < 0.000$, $\eta^2 = 0.19$. Multiple comparison results indicated that WE→WC ($M = 0.02$) tended to occur more frequently than other transitions from wrong-example.

Significant differences were found among 6th graders' transitions from practice strategies to the next strategies (i.e. wrong-wrong/WW, wrong-correct/WC, correct-wrong/CW, and correct-correct/CC) in the medium-skill phase, $F(7,1608) = 75.74$, $p < 0.000$, $\eta^2 = 0.25$ (WW→

next strategy), $F(5,1206) = 39.58$, $p < 0.000$, $\eta^2 = 0.14$ (WC→ next strategy), $F(7,1608) = 29.55$, $p < 0.000$, $\eta^2 = 0.11$ (CW→ next strategy), $F(7,1608) = 70.18$, $p < 0.000$, $\eta^2 = 0.23$ (CC→ next strategy). The results of multiple comparison showed that WW→EW ($M = 0.04$), and WW→ EC ($M = 0.03$) were more likely to occur than other WW→ next strategy. WC→ WC ($M = 0.07$) occurred more frequently than other WC→ next strategy. Among CW→ next strategy, CW→ WC ($M = 0.03$) was found to occur more frequently than others. CC→WC ($M = 0.02$) tended to occur more frequently than other CC→ next strategy. Table 15 in the Appendix A shows 6th graders' base rate and the conditional probability for all strategy shifts in the medium-skill phase. Table 16 in the Appendix A illustrates 6th graders' likelihood of strategy shifts from all strategies in the medium-skill phase.

For college students, the results of comparing the transitions from help strategies to the strategies occurring after the next errors in the medium-skill phase showed significant differences, $F(5,1056) = 111.3$, $p < 0.000$, $\eta^2 = 0.35$. Multiple comparisons results indicated that EE→EE ($M = 0.05$), and EE→EW ($M = 0.04$) were more likely to occur than other EE→ next strategy.

The results of comparing college students' transitions from example-practice (i.e. example-wrong/EW and example-correct/EC) to the next strategies in the medium-skill phase also indicated significant differences, $F(5,1056) = 29.85$, $p < 0.000$, $\eta^2 = 0.12$ (EW→ next strategy), $F(6,1232) = 110.7$, $p < 0.000$, $\eta^2 = 0.35$ (EC→ next strategy). The results of multiple comparison showed that EW→EE ($M = 0.04$), EW→EW ($M = 0.03$), and EW→EC ($M = 0.02$) were more likely to occur than other EW→ next strategy. EC→CE ($M = 0.05$) occurred more frequently than other EC→ next strategy.

The comparison results also indicated significant differences among college students' transitions from wrong-example to the next strategies in the medium-skill phase, $F(4,880) = 12.82$, $p < 0.000$, $\eta^2 = 0.06$ (WE→ next strategy). The results of multiple comparison showed that WE→WE ($M = 0.08$), WE→EW ($M = 0.05$), and WE→CC ($M = 0.05$) were more likely to occur than other WE→ next strategy.

For college students, WW→ next strategy and WC→ next strategy were not observed in the medium-skill phase. The results of comparing college students' transitions from practice strategies (i.e. correct-wrong/CW, and correct-correct/CC) to the next strategies in the medium-skill phase indicated significant differences, $F(5,1056) = 28.76$, $p < 0.000$, $\eta^2 = 0.12$ (CW→ next strategy), $F(4,880) = 181$, $p < 0.000$, $\eta^2 = 0.45$ (CC→ next strategy) (Note that WW and WC were not be observed in the medium-skill phase for college students).Multiple comparisons revealed that CW→ CE ($M = 0.06$) was more likely to occur than other CW→ next strategy. Among CC→ next strategy, CC→ CE ($M = 0.06$) occurred more frequently than the other strategies. Table 17 in the Appendix A shows college students' base rate and the conditional probability of all strategy shifts in the medium-skill phase. Table 18 in the Appendix A illustrates college students' likelihood of strategy shifts from all strategies in the medium-skill phase.

Based on the results of 6^{th} graders' and college students' strategy shifts in the medium-skill phase, only college students' data showed that some strategies were apt to transition to practice-example (e.g. EC→CE, WE→WE, CW→ CE, CC→ CE). Therefore, the results provided limited support for **Hypothesis 9** that the strategies would transition to practice-example after the next error in the medium-skill phase.

Strategy shifts in the high-skill phase (PL>0.6)

For 6th graders, the results of comparing the transitions from the help strategy to the strategies after the next error in the high-skill phase indicated significant differences, $F(4,985) = 528.8$, $p < 0.000$, $\eta^2 = 0.68$ (EE→ next strategy). Multiple comparison results showed that EE→ EE ($M = 0.13$) occurred more frequently than other EE→ next strategy.

The results of comparisons of 6th graders' transitions from example-practice (i.e. example-wrong/EW and example-correct/EC) to the next strategies in the high-skill phase showed significant differences, $F(7,1567) = 464.4$, $p < 0.000$, $\eta^2 = 0.67$ (EW→ next strategy), $F(3,788) = 172.4$, $p < 0.000$, $\eta^2 = 0.40$ (EC→ next strategy). The results of multiple comparison revealed that EW→ WC ($M = 0.08$) occurred more frequently than other EW→ next strategy. EC→ EC ($M = 0.17$) was more likely to occur than other EC→ next strategy.

The results of comparing 6th graders' transitions from wrong-example to the next strategies in the high-skill phase showed significant differences, $F(7,1576) = 507.4$, $p < 0.000$, $\eta^2 = 0.69$ (WE→ next strategy). Further multiple comparison results revealed that WE→WC ($M = 0.08$) was more likely to occur than other WE→ next strategy.

The results of comparing 6th graders' transition from practice strategies (i.e. wrong-wrong/WW, wrong-correct/WC, correct-wrong/CW, and correct-correct/CC) to the next strategies in the high-skill phase indicated significant differences, $F(7,1576) = 242.1$, $p < 0.000$, $\eta^2 = 0.52$ (WW→ next strategy), $F(5,1182) = 413.9$, $p < 0.000$, $\eta^2 = 0.64$ (WC→ next strategy), $F(6,1379) = 283$, $p < 0.000$, $\eta^2 = 0.55$ (CW→ next strategy), $F(5,1182) = 808.3$, $p < 0.000$, $\eta^2 = 0.77$ (CC→ next strategy). The results of multiple comparison revealed that WW→ EC ($M = 0.06$) and WW→ WC ($M = 0.07$) were more likely to occur than other WW→ next strategy. WC→ WC ($M = 0.12$) occurred more frequently than other WC→ next strategy. Among CW→

next strategy, CW→ WC (M = 0.10) was found to occur more frequently than other transitions. CC→ WC (M = 0.12) was more likely to occur than other CC→ next strategy. Table 19 in the Appendix A shows 6th graders' base rate and the conditional probability of all strategy shifts in the high-skill phase. Table 20 in the Appendix A illustrates 6th graders' likelihood of strategy shifts from all strategies in the high-skill phase.

For college students, the results of comparing the transitions from the help strategy to the strategies occurring after the next error in the high-skill phase were significantly different, $F(5,1026) = 547.4$, $p < 0.000$, $\eta^2 = 0.73$ (EE→ next strategy). The results of multiple comparisons indicated that EE→CE (M = 0.05) was more likely to occur than other transitions from the help strategy.

The results of comparing college students' transitions from example-practice (i.e. example-wrong/EW and example-correct/EC) to the next strategies in the high-skill phase showed significant differences, $F(6,1197) = 281.3$, $p < 0.000$, $\eta^2 = 0.59$ (EW→ next strategy), $F(5,1026) = 139$, $p < 0.000$, $\eta^2 = 0.43$ (EC→ next strategy). The results of multiple comparisons indicated that EW→ CE (M = 0.08) occurred more frequently than other EW→ next strategy. EC→ EE (M = 0.04) and EC→ CW (M = 0.04) were more likely than other EC→ next strategy.

The comparison results of college students' transitions from practice-example (i.e. wrong-example/WE and correct-example/CE) to the next strategies in the high-skill phase showed significant differences, $F(6,1197) = 114.9$, $p < 0.000$, $\eta^2 = 0.36$ (WE→ next strategy), $F(1,342) = 52.45$, $p < 0.000$, $\eta^2 = 0.13$ (CE→ next strategy). Multiple comparison results indicated that WE→ CE (M = 0.07) occurred more frequently than other WE→ next strategy. CE→ EC (M = 0.40) was more likely to occur than other CE→ next strategy.

For college students, WW→ next strategy and WC→ next strategy were not observed in the high-skill phase. Therefore, college students' transitions from practice strategies to the next strategies in the high skill phase only involved CW→ next strategy and CC→ next strategy. The results of comparing those transitions showed significant differences, $F(4,855) = 14.24$, $p < 0.000$, $\eta^2 = 0.06$ (CW→ next strategy), $F(3,684) = 371.2$, $p < 0.000$, $\eta^2 = 0.77$ (CC→ next strategy). Multiple comparison results indicated that CW→ EE ($M = 0.06$) and CW→ CC ($M = 0.07$) were more likely to occur than other CW→ next strategy. CC→ CE ($M = 0.19$) tended to occur more frequently than other CC→ next strategy. Table 21 in the Appendix A shows college students' base rate and the conditional probability of all strategy shifts in the high-skill phase. Table 22 in the Appendix A illustrates college students' likelihood of strategy shifts from all strategies in the high-skill phase.

After examining the results from both 6th graders and college students in the high-skill phase, 6th graders tended to transition from the current strategies to WC, while college students seldom transitioned to practice strategies (e.g. EC→CW was more likely to occur than other EC→ next strategy). Therefore, the results partially support **Hypothesis 10**, which predicted that the strategies were more likely to transition to practice strategies in the high-skill phase. Table 6 summarizes the strategy shifts in different learning phases for both 6th graders and college students.

Table 6

Summary of strategy shifts in different learning phases

Hypothesis	6th graders Results	Support or not?	College students Results	Support or not?
Hypothesis 8: In the low-skill phase, the current strategy was more likely to transition to example-practice strategies.	EE→ EE, EE→ CE, EW→WW, EC→CC, WE→ WE, WE→WW, CE→ CE, WW→ EW, WC→ WW, WC→ WC, WC→ CW, WC→ CC, CW→ WW, CW→ WC, CW→ CC, CC→ CC	Very limitedly	EE→EE, EE→EW, EE→EC, EE→CE, EE→WW, EE→WC, EW→EW, EW→EC, EC→EC, WE→WE, CE→CE, WW→EW, WC→ CE, WC→ WW, WC→ WC, CW→WE, CW→CC, CC→ CC	Partially
Hypothesis 9: In the medium-skill phase, the current strategy was apt to transition to practice-example strategies.	EE→WC, EW→ WC, EC→WC, WE→WC, WW→EW, WW→ EC, WC→ WC, CW→ WC, CC→WC	No	EE→EE, EE→EW, EW→EE, EW→EW, EW→EC, EC→CE, WE→WE, WE→EW, WE→CC, CW→ CE, CC→ CE,	Very limitedly
Hypothesis 10: In the high-skill phase, the current strategy tended to transition to practice strategies.	EE→ EE, EW→ WC, EC→ EC, WE→WC, WW→ EC, WW→ WC, WC→ WC, CW→ WC, CC→ WC,	Partially	EE→CE, EW→ CE, EC→ EE, EC→ CW, WE→ CE, CE→ EC, CW→ EE, CW→ CC, CC→ CE,	Very limitedly

The factors impacting the strategies

Prior knowledge

Prior knowledge and the likelihood of strategies

The results of Pearson correlations showed significant and negative correlations between 6th graders' prior knowledge and EW, $r = -0.25$, $n = 204$, $p < 0.000$. 6th graders with lower prior knowledge were more likely to exert the strategy of EW after an error. In addition, there was a significant and negative correlation between 6th graders' prior knowledge and CC, $r = -0.23$, $n = 204$, $p = 0.001$. This might indicate that 6th graders with lower prior knowledge tended to solve two problems successfully in a row after the error. Conversely, there was significant and positive correlation between 6th graders' prior knowledge and WE, $r = 0.26$, $n = 204$, $p < 0.000$. 6th graders with higher prior knowledge were more likely to use the strategy of WE after errors.

For college students, the results of Pearson correlations showed that there was a significant and negative correlation between prior knowledge and WC, $r = -0.22$, $n = 179$, $p = 0.003$. College students with lower prior knowledge may have tended to exert the strategy of WC after an error.

Based on the above results, the use of practice strategies may negatively correlate with students' prior knowledge. That is, students with lower prior knowledge tended to use practice strategies after an error. Therefore, the results did not support **Hypothesis 11**, which predicted that students with higher prior knowledge were more likely to utilize practice strategies after an error.

Prior knowledge and randomness of strategies

The results of Pearson correlations revealed that there was a significant and positive correlation between prior knowledge and the entropy value of strategies, $r = 0.36$, $n = 204$, $p <$

0.000 (6th graders), $r = 0.38$, $n = 179$, $p < 0.000$ (college students). This meant that both 6th graders and college students with lower prior knowledge exerted more ordered patterns when using strategies after errors. Therefore, the results did not support **Hypothesis 12**, which stated that students with the high prior knowledge exerted more ordered patterns when using strategies after errors.

Prior knowledge and strategy shifts

The results of Pearson correlation showed that there was a significant and positive correlation between 6th graders' prior knowledge and CE→EW, $r = 0.63$, $n = 204$, $p < 0.000$. That is, 6th graders with higher prior knowledge were more likely to exert CE→EW. Additionally, there was a significant and positive correlation between 6th graders' prior knowledge and WW→EC, $r = 0.26$, $n = 204$, $p < 0.000$. It meant that 6th graders with higher prior knowledge were more likely to exert WW→EC.

For college students, the results of Pearson correlations indicated that there was a significant and positive correlation between prior knowledge and CC→EW, $r = 0.25$, $n = 179$, $p < 0.000$. That is, college students with higher prior knowledge tended to exert CC→EW. However, there was a significant and negative correlation between college students' prior knowledge and CC→CC, $r = -0.26$, $n = 179$, $p < 0.000$. This meant that college students with lower prior knowledge were more inclined to exert CC→CC.

According to the results from 6th graders and college students, it was found that students with higher prior knowledge were more likely to exert the transition from the current strategy to example-practice after the next error. Therefore, the results did not support **Hypothesis 13**, which predicted that students with lower prior knowledge would transition from the current strategy to example-practice after the next error.

Error types

Error types and strategies

The results of Pearson correlation showed that there was a significant and positive correlation between 6th graders' EE and slip probability, $r = 0.25$, $n = 204$, $p < 0.000$. That is, when 6th graders made careless errors, they tended to use EE after those errors. However, for 6th graders there was a significant and negative correlation between slip probability and CC, $r = -0.4$, $n = 204$, $p < 0.000$. This may indicate that 6th graders were more likely to use CC after making errors due to lack of knowledge.

For college students, the results of Pearson correlations indicated that there was a significant and positive correlation between EE and slip probability, $r = 0.25$, $n = 179$, $p < 0.006$. Similarly, the results showed that there was significant and positive correlation between college students' slip probability and EC, $r = 0.36$, $n = 179$, $p < 0.000$. Additionally, the results revealed significant and positive correlations between college students' slip probability and WW, $r = 0.36$, $n = 179$, $p < 0.000$. That is, when students made careless errors, they were inclined to use EE, EC, or WW strategies after those errors. However, the results showed significant and negative correlations between college students' slip probability and WE ($r = -0.26$, $n = 179$, $p < 0.006$), slip probability and WC ($r = -0.37$, $n = 179$, $p < 0.000$), and slip probability and CC ($r = -0.32$, $n = 179$, $p < 0.000$). That is, when students made errors due to lack of knowledge, they tended to exert WE, WC, or CC.

Based on the above results of 6th graders and college students, students may tend to use practice strategies when making errors due to lack of knowledge. Therefore, the results did not support **Hypothesis 14**, which stated that students would be more likely to use practice strategies after making careless errors.

Error types and randomness of strategies

For 6th graders, the results of Pearson correlations showed that there was a significant and positive correlation between the entropy value of strategies and slip probability, $r = 0.65$, $n = 204$, $p < 0.000$. That is, when 6th graders made more careless errors, they were inclined to use strategies after making errors in a more varied manner after making errors. Similarly, the results of Pearson correlations indicated a significant positive correlation between college students' entropy value of strategies and slip probability, $r = 0.67$, $n = 179$, $p < 0.000$. When college students made more careless errors, they would also use varied strategies more after errors as well. Therefore, the results did not support **Hypothesis 15**, which stated that students would use strategies after errors in an ordered pattern when making careless errors.

Topic difficulty

For 6th graders, the average number of students attempting one topic was 57.4. Topics that were attempted by more than 57 students ($N = 156$) were chosen to calculated topic difficulty. Topics that student mastery was greater than the mean level of mastery ($M = 0.74$) were labeled as easy topics ($N = 86$, mean of mastery = 0.86, S.D. of mastery = 0.06), and the rest of topics were treated as difficult topics ($N = 69$, mean of mastery = 0.59, S.D. of mastery = 0.12).

For college students, the average number of students attempting one topic was 38.5. The topics that were attempted by more than 38 students ($N = 322$) were chosen to calculated topic difficulty. Topics where student mastery was greater than the mean level of mastery ($M = 0.97$) were labeled as easy topics (N =198, mean of mastery = 0.99, S.D. of mastery = 0.01), and the rest of topics were treated as difficult topics ($N = 124$, mean of mastery = 0.94, S.D. of mastery = 0.04).

Topic difficulty and likelihood of strategies

The results of comparing 6th graders' likelihood of strategies occurring after the errors between easy topics and difficult topics showed that WE was more likely to occur for easy topics (M = -0.005) than difficult topics (M = -0.04), $t(133.53) = 5.00$, $p < 0.000$, $d = 0.79$. However, EW occurred more frequently for difficult topics (M = 0.10) than easy topics (M = 0.08), $t(143.51) = -3.55$, $p < 0.000$, $d = 0.48$. WC, one of the practice strategies, also tended to occur more often for difficult topics (M = -0.02) than easy topics (M = -0.03), $t(151.78) = -3.55$, $p < 0.000$, $d = 0.34$. In addition, CC was occurred significantly more often for difficult topics (M = -0.09) than easy topics (M = -0.16), $t(123.07) = -4.10$, $p < 0.000$, $d = 0.63$. Table 1 in the Appendix B lists the descriptions of 6th graders' likelihood of strategies for easy topics. Table 2 in the Appendix B illustrates the descriptions of 6th graders' likelihood of strategies for difficult topics.

For college students, the results of comparing strategies occurring after errors between easy topics and difficult topics indicated that WW occurred more frequently for easy topics (M = -0.06) than difficult topics (M = -0.07), $t(240.57) = 3.77$, $p = 0.0006$, $d = 0.31$. Table 3 in the Appendix B lists the descriptions of college students' likelihood of strategies for easy topics. Table 4 in the Appendix B illustrates the descriptions of college students' likelihood of strategies for difficult topics.

Based on the above results of 6th graders and college students, EW was found to occur more often for difficult topics than easy topics only for 6th graders. Therefore, the results provided very limited support for **Hypothesis 16**, which stated that students would use example-practice more often after errors for difficult topics than easy topics.

Topic difficulty and randomness of strategies

The t-tests results of comparing 6th graders' randomness of strategies (entropy value) between easy topics and difficult topics showed that there was no significant differences on entropy value of strategies between easy topics (M = 1.76) and difficult topics (M = 1.76), $t(152.54) = -1.45$, $p = 0.15$, $d = 0.24$. But the results for college students showed that the entropy value of strategies being used for easy topics (M = 1.81) was significantly higher than difficult topics (M = 1.79), $t(181.33) = 6.84$, $p < 0.000$, $d = 1.16$. Therefore, based on the results of 6th graders and college students, the results did not support **Hypothesis 17**, which predicted that strategies would occur after errors in a more varied pattern for difficult topics (high entropy value) relative to easy topics.

Topic difficulty and strategy shifts

For 6th graders, the t-tests comparing the transitions to the strategies occurring after the next errors showed that CE→EW occurred more often for easy topics (M = -0.03) than difficult topics (M = -0.08), $t(104.72) = 5.35$, $p < 0.000$, $d = 0.89$. WC→EE tended to occur more often for easy topics (M = -0.01) than difficult topics (M = -0.03), $t(134.85) = 4.04$, $p < 0.000$, $d = 0.66$. However, CE→CC was more likely to occur for difficult topics (M = -0.01) than easy topics (M = -0.04), $t(152.98) = -6.9$, $p < 0.000$, $d = 1.10$. In addition, WW→CC occurred more often for difficult topics (M = -0.06) than easy topics (M = -0.10), $t(152.67) = -6$, $p < 0.000$, $d = 0.89$. Table 5 and Table 7 in the Appendix B display 6th graders' base rate and the conditional probability of all strategy shifts for easy topics and difficult topics, respectively. Table 6 and Table 8 in the Appendix B show 6th graders' likelihood of strategy shifts for easy topics and difficult topics, respectively.

For college students, results of the t-test comparing the transitions from help strategy to the strategies occurring after the next error (EE→ next strategy) between easy topics and difficult topics showed that EE→CE occurred more frequently for easy topics ($M = 0.02$) than difficult topics ($M = 0.002$), $t(231.8) = 4.69$, $p < 0.000$, $d = 0.62$. EE→WW occurred significantly more often for easy topics ($M = 0.02$) than difficult topics ($M = 0.002$), $t(224.18) = 4.72$, $p < 0.000$, $d = 0.63$. Moreover, EE→ WC occurred more often for easy topics ($M = 0.02$) than difficult topics ($M = 0.004$), $t(236.91) = 4.63$, $p < 0.000$, $d = 0.56$. Table 9 and Table 11 in the Appendix B display college students' base rate and the conditional probability of all strategy shifts for easy topics and difficult topics, respectively. Table 10 and Table 12 in the Appendix B show college students' likelihood of strategy shifts for easy topics and difficult topics, respectively.

The t-test comparing college students' transitions from example-wrong to the next strategy (EW→ next strategy) between easy topics and difficult topics showed that EW→ CE occurred more frequently for easy topics ($M = -0.006$) than difficult topics ($M = -0.02$), $t(231) = 3.69$, $p = 0.0002$, $d = 0.40$. EW→WW occurred significantly more often for easy topics ($M = -0.004$) than difficult topics ($M = -0.02$), $t(255.02) = 4.49$, $p < 0.000$, $d = 0.49$. Additionally, EW→WC occurred more often for easy topics (M=-0.003) than difficult topics ($M = -0.02$), $t(264.3) = 4.63$, $p < 0.000$, $d = 0.53$. However, EW→WE occurred more often for difficult topics ($M = 0.03$) than easy topics ($M = -0.02$), $t(276.01) = -3.90$, $p = 0.0001$, $d = 0.44$.

The t-test comparing college students' transitions from example-correct to the next strategies (EC→ next strategy) between easy topics and difficult topics revealed that EC→WW occurred more often for easy topics ($M = 0.002$) than difficult topics ($M = -0.01$), $t(245.87) = 3.77$, $p = 0.0002$, $d = 0.44$. In addition, the results also showed that EC→WC was more likely to occur for easy topics (*M* = 0.005) than difficult topics (*M* = -0.008), *t*(253.66) = 3.89, *p* = 0.0001, *d* = 0.45.

The results of t-tests comparing college students' transitions from wrong-example to the next strategy (WE→ next strategy) showed that WE→CC occurred more often for easy topics (*M* = 0.05) than difficult topics (*M* = -0.002), *t*(303.88) = 5.07, $p < 0.000$, *d* = 0.57. Conversely, WE→WE occurred more often for difficult topics (*M* = 0.12) than easy topics (*M* = 0.06), *t*(253.74) = -3.96, $p < 0.000$, *d* = 0.46.

The t-test comparing college students' transitions from practice to the next strategy indicated that WW→WC occurred significantly more often for easy topics (*M* = 0.03) than difficult topics (*M* = 0.02), *t*(243.41) = 4.32, $p < 0.000$, *d* = 0.50.

Based on the above results for 6th graders and college students, the transition from the current strategy to the example-practice strategy was not found to occur more often for difficult topics than easy topics. Therefore, the results did not support **Hypothesis 18** that the transitions from the current strategies to example-practice strategies were supposed to occur more often for difficult topics.

Learning performance and strategies occurring after errors

Immediate performance after strategies

The results of comparisons of 6th graders' correctness after strategies indicated that there were significant differences on the correctness among strategies, *F*(8,1819) = 343.1, $p < 0.000$, $\eta^2 = 0.60$. Multiple comparison results showed that the correctness after CC (*M* = 0.77) was higher than the correctness after other strategies. The correctness after WW (*M* = 0.07) was the lowest relative to the correctness after other strategies. Table 7 displays the correctness on the next attempt after using strategies for 6th graders.

In a similar fashion, the results of comparisons of college students' correctness after strategies showed significant differences, $F(8,1600) = 254.2$, $p < 0.000$, $\eta^2 = 0.56$. Multiple comparison results revealed that the correctness after CC ($M = 0.87$) was highest and the correctness after WW ($M = 0.06$) was still lowest. Table 8 displays the correctness on the next attempt after using strategies for college students.

According to the results of 6th graders and college students, although the correctness after CC was highest and the correctness after WW was lowest. That is, specific types of practice strategies led to higher correctness than other strategies. Therefore, the results narrowly supported **Hypothesis 19** that the correctness after practice strategies would be higher than other strategies.

Table 7

The correctness percentage on the next attempt after using strategies (6th graders)

Strategies to learn from errors		Mean of correctness percentage	Standard deviation of correctness percentage
Help	EE	0.22	0.15
Example-practice	EW	0.19	0.11
	EC	0.56	0.13
Practice-example	WE	0.33	0.11
	CE	0.35	0.33
Practice	WW	0.07	0.05
	WC	0.47	0.16
	CW	0.38	0.15
	CC	0.77	0.09

Table 8

The correctness percentage on the next attempt after using strategies (college students)

Strategies to learn from errors		Mean of correctness percentage	Standard deviation of correctness percentage
Help	EE	0.41	0.25
Example-practice	EW	0.22	0.18
	EC	0.70	0.15
Practice-example	WE	0.51	0.14
	CE	0.51	0.30
Practice	WW	0.06	0.14
	WC	0.52	0.28
	CW	0.45	0.18
	CC	0.87	0.07

Delayed performance and strategies

Delayed performance and the likelihood of strategies

Multiple linear regressions were applied to estimate the relationships between delayed performance and the likelihood of strategies. Each regression included pretest scores and one strategy as predictors of delayed performance. The results of 6th graders' strategies on posttest scores showed that the coefficients of all strategies were not significant. The results also indicated that there was low collinearity between strategies and pretest scores. Table 1 in the Appendix C illustrates the detailed results of the multiple linear regressions for 6th graders.

In order to interpret the relationship between strategies and posttest scores, simple linear regressions on college students' posttest scores that only included one strategy in each regression were compared to the multiple linear regressions on posttest scores. Compared to the relationship direction in the multiple linear regression, the relationship direction of EE and CW with posttest scores was reversible in the simple linear regressions. But the other strategies did not change the relationship direction with posttest scores in the simple linear regression. Table 2 in the Appendix C shows the detailed results of the simple linear regressions. Based on the above results, it may mean that most of the 6th graders' strategies were not significantly related to posttest.

The multiple linear regressions on posttest using by involving pretest and one strategy in each regression were applied to examine the relationship between both college students' strategies and posttest as well. The results revealed that the coefficients of EE, EW, EC, WW, and WC were positive. However, the coefficients of WE, CW, and CC were negative. The results also showed that the collinearity of strategies with pretest was low. The detailed results of multiple linear regressions are illustrated in Table 3 in the Appendix C.

The comparisons of the results of simple linear regressions with multiple linear regressions did not indicate a change in the relationship directions of strategies with college students' posttest. The results of the simple linear regression are shown in Table 4 in the Appendix C. Therefore, the interpretation of the relationships between strategies and college students' posttest might be that EE, EW, EC, WW, and WC benefited college students' posttest. However, WE, CW, and CC appeared to decrease college students' posttest scores.

Based on the results above, strategies were not found to influence 6th graders' posttest, but EE, EW, and EC positively impacted college students' posttest. Additionally, two types of practice strategies (WW and WC) also promoted college students' posttest. Hence, the results partially supported **Hypothesis 20** that the help strategy and mixed strategies would be beneficial to posttest.

Delayed performance and randomness of strategies

The results of multiple linear regressions on 6th graders' posttest by including pretest and randomness of strategies suggested that the coefficient of entropy value of strategies (i.e. randomness) used after the errors was not significant (see Table 5 in the Appendix C). The collinearity of 6th graders' pretest and randomness was low, *VIF* = 1.15. In addition, by comparing the results of the multiple regression with the results of the simple linear regressions on 6th graders' posttest, the relationships directions of pretest and randomness with 6th graders' posttest scores did not change. Table 6 in the Appendix C displays the results of the simple linear regression. The above results indicated that the randomness of strategies used after the errors did not significantly link to 6th graders' posttest.

In a similar fashion, the results of the multiple linear regression on college students' posttest indicated that the coefficient of entropy value of strategies was also not significant (See Table 7 in Appendix C). The results showed that the two predictors in the regression had low collinearity, *VIF* = 1.17. Furthermore, the comparison between the results of the simple linear regression on college students' posttest and the results of multiple linear regression illustrated that the relationship directions of pretest and the entropy value with college students' posttest did not reverse. Table 8 in the Appendix C shows the results of the simple linear regression. Therefore, the randomness of strategies was not significantly associated with college students' posttest.

The above results for 6th graders and college students indicated that randomness of strategies used after errors did not significantly relate to posttest. Therefore, the results did not support **Hypothesis 21** that the entropy value of strategies used after errors would be negatively associated with posttest.

Delayed performance and strategy shifts

Delayed performance and strategy shifts in the low-skill phase

The multiple linear regressions of students (6th graders and college students) strategy shifts on posttest were applied to examine the relationship of strategy transitions in the low-skill phase with posttest. Each multiple linear regression included the pretest and one strategy shift as predictors. The results of the regression on 6th graders' posttest showed that EE→WE, WW→EC, and CW→WC in the low-skill phase was positively associated with 6th graders' posttest. However, EE→WC, and EC→EC in the low-skill phase negatively related to 6th graders' posttest. The results of VIF also indicated that there was low collinearity between pretest and strategy shifts. The detailed results of multiple linear regressions for 6th graders are displayed in Table 9 in Appendix C. The relationship directions between strategy shifts and posttest in the simple linear regression on 6th graders' posttest did not change when compared to the multiple linear regressions. Table 10 in the Appendix C illustrates the results of the simple linear regressions on 6th graders' posttest. Hence, the results may indicate that EE→WE, WW→EC, and CW→WC in the low-skill phase favored 6th graders' posttest. EE→WC and EC→EC in the low-skill phase hindered 6th graders' posttest.

The results of multiple linear regressions in the low-skill phase on college students' posttest indicated that EE→EE, WW→WW, WW→WC, WC→EE, WC→CE, WC→WW, WC→WC, WC→CW, and CC→CC in the low-skill phase were positively associated to college students' posttest. Nevertheless, EE→CC, EW→CC, EC→EC, WW→EC, WW→CC, WC→EW, WC→EC, CW→EE, CW→EC, CC→EE, and CC→CE in the low-skill phase negatively related to college students' posttest scores. The results of the college students' regression on posttest are listed in Table 11 in the Appendix C.

The collinearity and relationship directions from the multiple regression results were examined. The VIF results indicated low collinearity between pretest and strategy shifts in the multiple regressions (Table 12 in the Appendix C). The relationship directions between strategy shifts and college students' posttest scores did not change in the simple linear regression compared to the results of multiple linear regression on college students' posttest. Therefore, the results suggested that EE→EE, EC→EC, WW→WW, WW→WC, WC→EE, WC→CE, WC→WW, WC→WC, WC→CW, and CC→CC in the low-skill phase boosted college students' posttest. EE→CC, EW→CC, WW→EC, WW→CC, WC→EW, WC→EC, CW→EE, CW→EC, CC→EE, and CC→CE in the low-skill phase hindered college students' posttest.

According to the results of multiple regressions on the posttest, many transitions from the current strategies to example-practice strategies in the low-skill phase significantly and negatively linked to students' posttest except WW→EC and EC→EC. Hence, the results provided extremely narrow support for **Hypothesis 22** that assumed the transitions from the current strategies to example-practice strategies in the low-skill phase would benefit students' posttest.

Delayed performance and strategy shifts in the medium-skill phase

In like manner, the multiple linear regressions in the medium-skill phase on posttest were applied to investigate the relationships between strategy shifts and posttest by involving pretest and one strategy in each regression. The results of the multiple linear regressions on 6th graders' posttest found that EE→EW, WE→WW, and CC→EW in the medium-skill phase were positively related to 6th graders' posttest. However, WW→WW in the medium-skill phase was negatively related to 6th graders' posttest. The results of multiple linear regressions on 6th graders' posttest are illustrated in Table 13 in the Appendix C. The results of VIF indicated low collinearity between pretest and strategy shifts (see Table 13 in the Appendix C). The relationship directions between strategy shifts and 6th graders' posttest did not change in the simple linear regression on 6th graders' posttest compared to the results of multiple linear regressions on 6th graders' posttest. The results of simple linear regressions in the medium-skill phase on 6th graders' posttest are shown in Table 14 in the Appendix C. Consequently, these results may suggest that EE→EW, WE→WW, and CC→EW in the medium-skill phase were beneficial to 6th graders' posttest. WW→WW in the medium-skill may inhibit 6th graders' posttest.

The results of multiple linear regressions on college students' posttest revealed that EE→EW, EW→EW, EW→EC, CE→EW, CE→CE, CC→EE, CC→EW, and CC→CE in the medium-skill phase positively linked to college students' posttest. Nevertheless, EE→WE, EE→CC, EW→CW, EW→CC, EC→CW, EC→CC, WE→EC, CE→EC, CE→WE, CE→CC, CW→EC, and CC→CC in the medium-skill phase negatively related to college students' posttest. Table 15 in the Appendix C lists the results of the multiple linear regressions in the medium-skill phase on college students' posttest.

The results of VIF indicated low collinearity between pretest and strategy shifts in the medium-skill phase (see Table 15 in the Appendix C). Compared to the results of multiple linear regressions on college students' posttest, the relationship directions between strategy shifts in the medium-skill phase and college students' posttest did not change in the simple linear regressions on college students' posttest. The results of simple linear regressions are illustrated in Table 16 in the Appendix C. As a result, EE→EW, EW→EW, EW→EC, CE→EW, CE→CE, CC→EE, CC→EW, and CC→CE in the medium-skill benefited college students' posttest. EE→WE,

EE→CC, EW→CW, EW→CC, EC→CW, EC→CC, WE→EC, CE→EC, CE→WE, CE→CC, CW→EC, and CC→CC in the medium-skill phase reduced college students' posttest scores.

According to the results of 6th graders and college students, only CE→CE and CC→CE positively influenced posttest for college students. Hence, the results narrowly support **Hypothesis 23** that the transitions between practice-example would benefit students' posttest.

Delayed performance and strategy shifts in the high-skill phase

By the same token, the multiple linear regressions for students' strategy shifts on posttest were applied to examine the relationships of strategy shifts in the high-skill phase with posttest. The results of 6th graders' multiple linear regressions showed that CW→WC in the high-skill phase positively related to 6th graders' posttest while WC→EW in the high-skill phase negatively linked to 6th graders' posttest. The detailed results are listed in Table 17 in the Appendix C. The results of VIF indicated low collinearity between pretest and strategy shifts (Table 17 in the Appendix C). Compared to the results of multiple linear regressions in the high-skill phase on 6th graders' posttest, the relationship directions of strategy shifts in the high-skill phase with 6th graders' posttest did not change in the simple linear regressions on 6th graders. The full results of simple linear regressions are listed in Table 18 in the Appendix C. Accordingly, the above results may indicate that CW→WC was helpful to 6th graders' posttest scores. However, WC→EW may impede 6th graders' posttest scores.

The results of the multiple linear regressions of strategy shifts in the high-skill phase on college students' posttest specified that EE→EE, EE→EW, EE→WE, EE→CE, EW→EE, EW→EW, EC→EE, EC→EW, WE→EE, WE→EW, and WE→WE in the high-skill phase were positively related to college students' posttest. EE→EC, EE→CC, EW→CC, EC→CC, WE→EC, WE→CC, CE→EC, CE→CC, and CW→EC in the high-skill phase were negatively related to college students' posttest. The complete results of multiple linear regressions in the high-skill phase on college students' posttest are illustrated in Table 19 in the Appendix C.

The results of VIF indicated low collinearity between pretest and college students' strategy shifts in the high-skill phase (Table 19 in the Appendix C). The relationship directions of strategy shifts in the high-skill phase did not change compared to the results of multiple linear regressions in the high-skill phase on college students' posttest. The results of the simple linear regressions in the high-skill phase on college students' posttest are listed in Table 20 in the Appendix C. Thus, EE→EE, EE→EW, EE→WE, EE→CE, EW→EE, EW→EW, EC→EE, EC→EW, WE→EE, WE→EW, and WE→WE in the high-skill phase improved college students' posttest. But EE→EC, EE→CC, EW→CC, EC→CC, WE→EC, WE→CC, CE→EC, CE→CC, and CW→EC reduced college students' posttest scores.

According to the results of 6th graders and college students in the high-skill phase, only CW→WC was found to be helpful to 6th graders' posttest among the transitions from practice to practice strategy. Consequently, the results narrowly supported **Hypothesis 24** that the transitions from practice to practice would boost students' posttest in the high-skill phase. Table 9 summarizes the results of relationships between strategy shifts and delayed performance for both 6th graders and college students in different learning phases.

Table 9

Summary of relationships between strategy shifts and delayed performance

Hypothesis	6th graders Results	Support or not?	College students Results	Support or not?
Hypothesis 22: In the low-skill phase, the transitions from current strategies to example-practice strategies benefited delayed performance.	EE→WE, WW→EC, CW→WC, EE→WC, EC→EC	Very limited support	EE→EE WW→WW, WW→WC, WC→EE, WC→CE, WC→WW, WC→WC, WC→CW, CC→CC, EE→CC, EW→CC, EC→EC, WW→EC, WW→CC, WC→EW, WC→EC, CW→EE, CW→EC, CC→EE, CC→CE	No
Hypothesis 23: In the medium-skill phase, the transitions from current strategies to practice-example strategies benefited delayed performance.	EE→EW, WE→WW, CC→EW, WW→WW	No	EE→EW, EW→EW, EW→EC, CE→EW, CE→CE, CC→EE, CC→EW, CC→CE, EE→WE, EE→CC, EW→CW, EW→CC, EC→CW, EC→CC, WE→EC, CE→EC CE→WE, CE→CC, CW→EC, CC→CC	Very limited support
Hypothesis 24: In the high-skill phase, the transitions from current strategies to practice strategies benefited delayed performance.	CW→WC, WC→EW	Very limited support	EE→EE, EE→EW, EE→WE, EE→CE, EW→EE, EW→EW, EC→EE, EC→EW, WE→EE, WE→EW, WE→WE, EE→EC, EE→CC, EW→CC, EC→CC, WE→EC, WE→CC, CE→EC, CE→CC, CW→EC	No

Note: The strategy shifts in red indicate the shifts that were negatively related to delayed performance; the shifts in green were positively related to delayed performance.

Prior knowledge, strategies, and learning outcomes

Immediate performance for different levels of prior knowledge

The comparisons of 6th graders' correctness after strategies for different levels of prior knowledge showed that there were significant differences on correctness for both the low and the high prior knowledge, $F(1,961) = 197.5$, $p < 0.000$, $\eta^2 = 0.17$ (low prior knowledge), $F(1,863) = 182.8$, $p < 0.000$, $\eta^2 = 0.17$ (high prior knowledge). The multiple comparison results indicated that for both the low and the high prior knowledge, the correctness after CC ($M = 0.76$, low prior knowledge; $M = 0.77$, high prior knowledge) was higher than the correctness after other strategies. For college students, the results of comparisons of correctness after strategies for different levels of prior knowledge also revealed significant differences in the correctness after strategies for both the low and the high prior knowledge, $F(1,861) = 59.9$, $p < 0.000$, $\eta^2 = 0.07$ (low prior knowledge), $F(1,744) = 26.44$, $p < 0.000$, $\eta^2 = 0.10$ (high prior knowledge). The multiple comparisons of correctness after strategies revealed that the correctness after CC ($M = 0.85$, low prior knowledge; $M = 0.90$, high prior knowledge) was higher than the correctness after other strategies. Table 1 and Table 2 in the Appendix D displayed the descriptions of correctness after strategies for 6th graders and college students, respectively.

Based on the results of 6th graders and college students, the correctness after CC was the highest of all the strategies for both the low and the high prior knowledge. Thus, the results did not support **Hypothesis 25** that the correctness after example-practice would be higher than the correctness after other strategies for the low prior knowledge level. However, the results supported **Hypothesis 27** that the correctness after practice strategies would be higher than the correctness after other strategies for the high prior knowledge level.

Delayed performance and likelihood of strategies for different levels of prior knowledge

Multiple linear regressions of students' strategies on posttest were used to investigate the relationship of the likelihood of strategies with posttest for different levels of prior knowledge. Each regression involved pretest and one strategy as predictors of posttest scores. The regression results of 6^{th} graders' strategies on posttest scores for both the low and the high prior knowledge levels revealed no significant coefficients for every strategy. The complete results of 6^{th} multiple linear regressions are displayed in Table 3 in the Appendix D.

In order to interpret the relationships, found in the multiple linear regressions, collinearity between strategies and pretest, and the relationship directions of strategies and posttest were examined. The VIF results indicated low collinearity between strategies and pretest for both the low and the high prior knowledge levels (Table 3 in the Appendix D). The relationship directions between strategies and posttest for both high and low prior knowledge levels did not change in the simple linear regressions compared to the multiple linear regressions. The results of simple linear regressions for 6^{th} graders with different levels of prior knowledge are listed in Table 4 in the Appendix D. As a result, 6^{th} graders' strategies did not influence posttest for both high and low prior knowledge levels.

In a parallel way, the multiple linear regressions were applied to examine the relationship between college students' strategies and posttest for different levels of prior knowledge. The results of multiple linear regressions for the low prior knowledge level showed that EE, EC, WW, and WC positively related to the posttest of college students with the low prior knowledge. However, CC negatively related to the posttest of college students with the low prior knowledge. The detailed results are illustrated in Table 59 in the Appendix. The results of VIF indicated low collinearity between pretest and strategies for low prior knowledge (Table 5 in the Appendix D).

When compared to the results of multiple linear regressions, the relationship directions between strategies and posttest did not change in the simple linear regressions for low prior knowledge except for the relationship directions between WC and posttest. The coefficients of strategies and pretest in the simple linear regressions are displayed in Table 6 in the Appendix D. Therefore, the results may suggest that EE, EC, and WW benefited the posttest of college students with the low prior knowledge. Nevertheless, CC hindered the posttest of college students with the low prior knowledge.

The results of multiple regressions for the high prior knowledge level for college students suggested that EE, EW, EC, WW, and WC positively related to posttest while CW and CC negatively related to posttest. The full results of multiple regressions for high prior knowledge level are displayed in Table 5 in the Appendix D. The results of VIF indicated low collinearity between strategies and pretest (Table 5 in the Appendix D). Compared with the results of multiple linear regressions, the relationship directions between strategies and posttest in the simple linear regression (Table 6 in the Appendix D) did not change for the high prior knowledge level. Consequently, the results may show that EE, EW, EC, WW, and WC favored the posttest of college students with the high prior knowledge. However, WE, CW and CC were harmful to the posttest of college students with the low prior knowledge.

According to the results of 6th graders and college students, strategies did not impact 6th graders' posttest for any level of prior knowledge. The result that EC may boost the posttest of college students with the low prior knowledge partially supported **Hypothesis 26** that example-practice strategies would benefit students with the low prior knowledge. For college students with the high prior knowledge, some specific practice strategies were found to be helpful to posttest (e.g. WW and WC). Therefore, the results partially support **Hypothesis 28** that practice strategies would benefit students with the high prior knowledge.

Delayed performance and strategy shifts in different levels of prior knowledge

The multiple linear regressions on posttest were applied to estimate the relationship between strategy shifts and posttest in different levels of prior knowledge by involving pretest and one strategy shift as predictors in each linear regression.

Delayed performance and strategy shifts for the low prior knowledge level

The results of the multiple linear regression on 6th graders' posttest found that EW→CE, WE →WE, WW→EC, and CW→WC positively related to the posttest of 6th graders with the low prior knowledge. Nevertheless, EE→WC, EC→ EW, EC→ EC, CW→EC, and WW→CC negatively related to the posttest of 6th graders with the low prior knowledge. The full results of multiple linear regressions on the posttest of 6th graders with the low prior knowledge are listed in Table 7 in the Appendix D. In order to interpret the relationship between strategy shifts and posttest in the multiple linear regressions, collinearity between strategy shifts and pretest, and relationship directions of strategy shifts and posttest were examined.

The result of VIF indicated low collinearity between pretest and strategy shifts (Table 7 in the Appendix D). The relationship directions of strategy shifts and the posttest of 6th graders with the low prior knowledge did not change in the simple linear regressions compared to the results of multiple linear regressions. The coefficients of strategy shifts in the simple linear regressions are displayed in Table 8 in the Appendix D. Thus, the results may suggest that EW→CE, WE →WE, WW→EC, and CW→WC were helpful to the posttest of 6th graders with the low prior knowledge while EE→WC, EC→ EW, EC→ EC, CW→EC, and WW→CC hindered the posttest of 6th graders for the low prior knowledge.

The results of multiple linear regressions found that the transition from practice-example to correct-example (i.e. WE→CE, CE→CE), practice-example to wrong-correct (i.e. WE→WC, CE→WC), CE→WW, CE→CW, WW→ CE, WC→CE, CW→CE, WW→WW, WW→WC, WC→WW, WC→WC, and CC→CC positively related to posttest of college students with the low prior knowledge. However, EE→WW, the transition from practice-example to example-correct (WE→EC, CE→EC), CW→EE, WW→EC, WC→EC, CW→EC, WC→EW, CC→EW, and WW→CC negatively related to posttest of college students with the low prior knowledge. Table 9 in Appendix D displays the detailed results of multiple linear regressions on posttest of college students with the low prior knowledge. The results of VIF indicated low collinearity between strategy shifts and pretest (Table 9 in the Appendix D). The relationship direction between strategy shifts and posttest of college students with the low prior knowledge did not reverse in the simple linear regressions. The coefficients of strategy shifts in the simple linear regressions are listed in Table 10 in the Appendix D.

Hence, the results above may suggest that the transition from practice-example to correct-example (i.e. WE→CE, CE→CE), practice-example to wrong-correct (i.e. WE→WC, CE→WC), CE→WW, CE→CW, WW→ CE, WC→CE, CW→CE, WW→WW, WW→WC, WC→WW, WC→WC, and CC→CC boosted posttest of college students with the low prior knowledge. EE→WW, the transition from practice-example to example-correct (WE→EC, CE→EC), CW→EE, WW→EC, WC→EC, CW→EC, WC→EW, CC→EW, and WW→CC may impede posttest of college students with the low prior knowledge.

Based on the results of 6th graders and college students, only WW→EC was found to link to posttest of students with the low prior knowledge among the transitions between example-practice. Therefore, the results provided very narrow support for **Hypothesis 29** that the transitions from the current strategies to example-practice strategies would benefit posttest of students with the low prior knowledge.

Delayed performance and strategy shifts for the high prior knowledge level

The results of the multiple linear regression on posttest of 6^{th} graders for the high prior knowledge level found that EW→ EC and WE→CW positively related to posttest of 6^{th} graders with the high prior knowledge. But EW→WW negatively related to posttest of 6^{th} graders with the high prior knowledge. The detailed results of multiple linear regressions were displayed in Table 11 in the Appendix D. The results of VIF indicated low collinearity between pretest and strategy shifts for the high prior knowledge level (Table 11 in the Appendix D). The relationship directions between strategy shifts and posttest for the high prior knowledge level did not change in the simple linear regressions compared to the results of multiple linear regressions. The coefficients of strategy shifts in the simple linear regressions are shown in Table 12 in the Appendix D. Based on the results, EW→ EC and WE→CW may benefit posttest of 6^{th} graders with the high prior knowledge. EW→WW may hinder posttest of 6^{th} graders with the high prior knowledge.

The results of multiple linear regressions on posttest of college students with the high prior knowledge showed that EE→ EE, EC→ EC, WE→ CE, CE→ CE, WE→ WC, CE→ WW, CE→ WC, CE→ CW, WW→ CE, WC→ CE, WW→ WW, WW→ WC, WC→ WW, WC→ WC, WC→ CW, and CC→ CC positively linked to posttest of college students with the high prior knowledge. However, EC→ EE, EC→ WW, EC→ WC, EC→ CC, WE→ EC, CE→ EC, CC→ EE, WW→ EC, WC→ EC, CW→ EC, CC→ EC, and WW→ CC negatively related to posttest of college students with the high prior knowledge. The full results of multiple linear regressions are listed in Table 13 in the Appendix D. The results of VIF indicated low collinearity between pretest and strategy shifts for the high prior knowledge level (Table 13 in the Appendix D). The relationship directions between strategy shifts and posttest of college students in high knowledge level did not change in the simple linear regressions compared to the results of multiple linear regressions. The coefficients of strategy shifts in the simple linear regressions are illustrated in Table 14 in the Appendix D. According to the results, EE→ EE, EC→ EC, WE→ CE, CE→ CE, WE→ WC, CE→ WW, CE→ WC, CE→ CW, WW→ CE, WC→ CE, WW→ WW, WW→ WC, WC→ WW, WC→ WC, WC→ CW, and CC→ CC were helpful to the posttest of college students with the high prior knowledge. EC→ EE, EC→ WW, EC→ WC, EC→ CC, WE→ EC, CE→ EC, CC→ EE, WW→ EC, WC→ EC, CW→ EC, CC→ EC, and WW→ CC may be harmful to the posttest of college students with the high prior knowledge.

Combining the results of 6th graders and college students with the high prior knowledge, WE→CW impacted the posttest of 6th graders with the high prior knowledge. Some transitions to practice strategies (e.g. WW→ WW, WW→ WC, and WC→ WW) also boosted posttest of college students with the high prior knowledge. Thus, the results partially supported **Hypothesis 30** that transitions from the current strategies to practice strategies would benefit students with the high prior knowledge.

Topic difficulty, strategies, and immediate performance

The results of comparisons of 6th graders' correctness after strategies among easy topics indicated significant differences, $F(1,772) = 117.1$, $p < 0.000$, $\eta^2 = 0.13$. The multiple comparisons found that the correctness after CC ($M = 0.82$) was highest among the correctness among easy topics. The correctness after EC ($M = 0.63$) and WC ($M = 0.60$) was higher than the correctness after the rest of strategies among easy topics. The descriptions of correctness after strategies for easy topics are listed in Table 15 in the Appendix D. In a similar fashion, there were also significant differences among 6th graders' correctness after strategies for difficult topics, $F(1,619) = 124.9$, $p < 0.000$, $\eta^2 = 0.17$. The multiple comparisons showed that the correctness after CC ($M = 0.73$) was higher than correctness after other strategies for difficult topics. The correctness after EC ($M = 0.54$) and WC ($M = 0.44$) was higher than the correctness after the rest of strategies for difficult topics. The descriptions of correctness after strategies for difficult topics are displayed in Table 15 in the Appendix D.

The results of comparisons of college students' correctness after strategies for easy topics revealed significant differences, $F(1,1780) = 54$, $p < 0.000$, $\eta^2 = 0.03$. The multiple comparisons indicated that the correctness after CC ($M = 0.91$) was higher than correctness after other strategies, and the correctness after EC ($M = 0.77$) was higher than the correctness after the rest of strategies for easy topics. The descriptions of college students' correctness after strategies for easy topics are illustrated in Table 16 in the Appendix D. In a like manner, the results of comparisons of correctness after strategies for difficult topics showed significant differences, $F(1,1114) = 56.15$, $p < 0.000$, $\eta^2 = 0.05$. The multiple comparisons revealed that the correctness after CC ($M = 0.71$) was highest among correctness after other strategies, and the correctness after EC ($M = 0.87$) was higher than the correctness after the rest of strategies for difficult topics. The descriptions of college students' correctness after strategies for difficult topics are displayed in Table 16 in the Appendix D.

According to the results of 6th graders and college students, **Hypothesis 31** which predicted that the correctness after example-practice would be highest among the strategies for difficult topics was not supported by the results. However, the results partially supported

Hypothesis 32 which stated that the correctness after practice strategies would be highest for easy topics.

Chapter 6: Discussion

The present study investigated students' strategies occurring after errors and the relationship between strategies and learning outcomes. In addition, this study examined the factors that predicted strategies used to cope with errors and the influence of those factors on the relationship between strategies and learning outcomes.

The likelihood of strategies to occur after errors

The overall likelihood of strategies to occur after errors

The study found that students were apt to use mixed strategies (i.e. EC, EW, WE, and CE) after making errors in the process of learning math. This suggests that the predominant tendency that students in ALEKS did not perform help overuse or help avoidance behaviors after making errors. There are one potential explanation for this phenomenon. ALEKS matches problems' difficulty to students' prior knowledge; hence students were capable of solving the problem. From the viewpoint of the learning phase theory, students actually started learning from the intermediate phase in ALEKS. In the intermediate phase, students have already accumulated knowledge for problem-solving so students not only relied on worked examples but also gradually switched their attention to practice strategies.

The likelihood of strategies occurring after errors in different learning phases

This study unveiled changes in strategy use in different learning phases. Overall the results provided evidence to support students' learning behaviors proposed based on the learning phase theory (VanLehn, 1996). In the low-skill phase, students tended to use the EW strategy after making errors. This may indicate that students were located in the early stage of the intermediate learning phase and needed to accumulate knowledge in order to understand topics. Hence, students had a high probability of failing problems again after requesting an example. In the medium-skill phase, students were inclined to use the WE strategy after making errors. In this phase, students may have been in the intermediate learning phase but had already accumulated some knowledge through the low-skill phase. As a result, students preferred to focus on practice strategies, which led them to attempt problems before requesting worked examples. In the high-skill phase, the WW and WC strategies were observed to occur most frequently. An explanation for this might be that students entered the late learning phase in which they already had enough knowledge to solve problems successfully. Therefore, the students insisted on solving problems to practice the knowledge accumulated in the previous phases.

Although the findings mainly confirmed the learning phase theory, students' strategies after making errors seemed more complicated than described in the theory. One of the interesting points was that EE appeared in the frequent strategies lists of the medium-skill phase and the high-skill phase. According to learning phase theory, students were assumed to rely more on help than practice in the early learning phase. Consequently, it would be understandable that EE was one of the frequent strategies in the low-skill learning phase, which was located in the transitional stage from the early phase to the intermediate phase. However, the present study discovered findings that conflict with the learning phase theory. The contradictory results suggest that students experienced cognitive disequilibrium in situations where they thought they should not make errors. Cognitive disequilibrium often emerged when experience did not match expectation (Craig, Graesser, Sullins, & Gholson, 2004). In order to clarify the causes of errors and rebuild cognitive equilibrium students may have carefully gone through worked examples again. Herein, errors may have led to cognitive disequilibrium that caused students to realize and modify incorrect knowledge on the topics.

Another interesting point was that some practice strategies were also used frequently by students in the low-skill phase and the medium-skill phase. For example, 6th graders tended to exert the WW strategy after making errors in the medium-skill phase. Based on the learning phase theory, students were supposed to frequently use practice strategies in the high-skill phase instead of the phases preceding it. The students who were inclined to use practice strategies after making errors in the first two learning phases may have been overconfident about their ability or exerted the learning style of "time for telling" (Schwartz & Bransford, 1998). Future studies could cluster the students who preferred to use practice strategies after making errors and investigate their characteristics.

The pattern of strategy shifts in learning

The overall pattern of strategy shifts in learning

Besides examining the likelihood of strategies occurring after errors, this study was interested in the temporal pattern of strategies as well. The findings suggested that students were more likely to continuously utilize the help strategy (EE) on the second error after using the EE strategy to cope with a previous error (EE→EE). This finding did not support the hypothesis based on the learning phase theory, which predicted that EE would transition to example-practice strategies. These results may indicate that students behaved conservatively on subsequent errors after requesting worked examples to correct their previous errors. After making the second error, students were apt to completely rely on worked examples again to clarify causes of errors instead of attempting to solve problems under the guidance of worked examples.

In addition, the study found that students were inclined to use mixed strategies or practice strategies on subsequent errors after using mixed strategies to correct the previous errors (Mixed→Mixed/Practice). These results aligned with the hypothesis based on the learning phase theory, which stated that mixed strategies would transition to mixed strategies or practice strategies. When students felt that they had already accumulated enough knowledge on the topics and built-up the confidence to attempt to solve problems, they were more positive that they were able to correct errors under minimal guidance of worked examples. Therefore, students were more likely to choose mixed strategies or practice strategies to correct subsequent errors when they had utilized mixed strategies after the previous error.

This study found that students tended to utilize practice strategies continuously to correct two subsequent errors (Practice→Practice). This finding supported the hypothesis on the basis of the learning phase theory, which predicted that practice strategies would shift to practice strategies. When students attempted practice strategies to correct previous errors, they felt confident enough to figure out the causes of errors by practice and master topics without worked examples. Another possible explanation might be that students constructed solutions by themselves through learning by doing (Kirschner, Sweller, & Clark, 2006). However, some frequent transitions from practice strategies were not to other practice strategies. For example, WW was also apt to shift to EW for 6^{th} graders. For college students, WW was more likely to shift to EE, EW, and CE as well. The reason for transitions from practice to mixed or help strategies might be made clear by looking into changes in strategy shifts during different learning phases, which are discussed in the following section.

The patterns of strategy shifts in different learning phases

In most cases, the transitions from EE did not confirm the hypothesis based on the learning phase theory. In the low-skill phase, after using EE to correct a previous error, students were inclined to choose the EE, CE, WW, or WC strategy. When students entered later learning phases, the transitions from EE to practice disappeared. For example, for 6th graders, the frequent transitions from EE changed from EE→WC (the medium-skill phase) to EE→EE (the high-skill phase). For college students, in the medium-skill phase and the high-skill phase EE did not tend to transition to practice strategies. Table 10 lists the frequent transitions from EE in different learning phases. EE occurring after making errors may indicate cognitive disequilibrium that might promote students to rely on worked examples to puzzle out their confusions. Making errors again after EE might result in increased confusion. Thus, students would be more conservative and might request worked examples again to ensure that they completely understood the knowledge.

For the transitions from mixed strategies or practice strategies, the changes of those transitions in different learning phases overall confirmed the hypothesis based on learning phase theory. That is, mixed or practice strategies were more likely to transition to example-practice in the low-skill phase, practice-example in the medium-skill phase, and practice strategies in the high-skill phase. Table 11 and Table 12 respectively display the transitions from mixed strategies and practice strategies in different learning phases. Like the changes in transitions from EE during the course of learning, transitions from practice strategies exhibited a similar trend. That is, the transitions from practice strategies to mixed strategies or EE occurred frequently in the late learning phases as well, especially for college students. The trend that EE and practice strategies tended to transit to EE or mixed strategies in the late phases might indicate that students experienced confusion when making errors in the medium-skill or high-skill phases. Those errors enabled them to dive into worked examples again to adjust strategies used after errors.

Hence, the results of strategy shifts may imply that students' strategies after making errors mainly followed the viewpoints of learning phase theory, but also became more complicated when errors promoted students to experience confusion. The study investigated students' strategies occurring after errors and the relationship between strategies and learning outcomes. In addition, the study examined the factors impacting strategies to cope with errors and the influence of those factors on the relationship between strategies and learning outcomes.

Table 10

The transitions from help strategy in different learning phases

Hypothesis	6th graders Results	Support or not?	College students Results	Support or not?
Hypothesis 8: In the low-skill phase, the current strategy was more likely to transit to example-practice strategies.	EE→ EE, EE→ CE	No	EE→EE, EE→EW, EE→EC, EE→CE, EE→WW, EE→WC	Partially
Hypothesis 9: In the medium-skill phase, the current strategy was apt to transit to practice-example strategies.	EE→WC	No	EE→EE, EE→EW	No
Hypothesis 10: In the high-skill phase, the current strategy tended to transit to practice strategies.	EE→ EE	No	EE→CE	No

Table 11

The transitions from mixed strategies in different learning phases

Hypothesis	6th graders Results	Support or not?	College students Results	Support or not?
Hypothesis 8: In the low-skill phase, the current strategy was more likely to transit to example-practice strategies.	EW→WW, EC→CC, WE→ WE, WE→WW, CE→ CE	Very limitedly	EW→EW, EW→EC, EC→EC, WE→WE, CE→CE	Partially

Table 11 (Continued)

Hypothesis 9: In the medium-skill phase, the current strategy was apt to transit to practice-example strategies.	EW→ WC, EC→WC, WE→WC	No	EW→EE, EW→EW, EW→EC, EC→CE, WE→WE, WE→EW, WE→CC	Very limitedly
Hypothesis 10: In the high-skill phase, the current strategy tended to transit to practice strategies.	EW→ WC, EC→ EC, WE→WC	Partially	EW→ CE, EC→ EE, EC→ CW, WE→ CE, CE→ EC	No

Table 12

The transitions from practice strategies in different learning phases

Hypothesis	6th graders Results	Support or not?	College students Results	Support or not?
Hypothesis 8: In the low-skill phase, the current strategy was more likely to transit to example-practice strategies.	WW→EW, WC→WW, WC→WC, WC→ CW, WC→ CC, CW→WW, CW→ WC, CW→ CC, CC→ CC	Very limitedly	WW→EW, WC→ CE, WC→ WW, WC→ WC, CW→WE, CW→CC, CC→ CC	Partially
Hypothesis 9: In the medium-skill phase, the current strategy was apt to transit to practice-example strategies.	WW→EW, WW→ EC, WC→ WC, CW→ WC, CC→WC	No	CW→ CE, CC→ CE	Partially
Hypothesis 10: In the high-skill phase, the current strategy tended to transit to practice strategies.	WW→ EC, WW→ WC, WC→ WC, CW→ WC, CC→ WC	Partially	CW→ EE, CW→ CC, CC→ CE	Very limitedly

The factors impacting strategies to occur after errors

Prior knowledge and strategies occurring after making errors

Compared to students with the high prior knowledge, students with the low prior knowledge tended to use practice strategies after making errors, which conflicted with the findings in the existing literature (Fyfe, Rittle- Johnson, & DeCaro, 2012; Pass & Van Gog, 2006). The finding that 6th graders with the high prior knowledge were inclined to use WE might indicate that students preferred to request help after making repeated errors. This finding may be in line with the existing literature that students with the high prior knowledge prefer to request help after making errors (Wood & Wood, 1999).

As for the strategy shifts, students with the low prior knowledge tended to transition to practice strategies, while students with the high prior knowledge were apt to transition to example-practice strategies. This finding was against the hypothesis that students with lower prior knowledge would transition to example-practice after the next error. However, these results were in line with the above-stated findings on strategies being used by students with different levels of prior knowledge.

Based on the results of prior knowledge and strategies, students with the high prior knowledge tended to request worked examples after making errors. However, students with the low prior knowledge insisted on practicing after making errors, which might have been inappropriate according to the model of ideal help-seeking behavior (Aleven, McLaren, Roll, & Koedinger, 2006).

Students with the high prior knowledge were also found to use strategies more variedly after making errors relative to those with the low prior knowledge. This result was contradictory to the findings of Snow, Jackson, and McNamara (2014). A potential explanation for this may be that students with the high prior knowledge were more capable of accurately evaluating their prior relative to students with the low prior knowledge (Tobias, 1994). Hence, after making errors, students with the high prior knowledge flexibly used the best strategies to recover from errors based on an evaluation of their current prior knowledge. However, students with the low prior knowledge might have been too stuck on their previously developed strategies to cope with errors.

Error types and strategies occurring after errors

Students were more likely to adopt EE or example-practice strategies after making careless errors rather than practice strategies. These results went against the hypothesis that students would utilize practice strategies after making careless errors. However, this finding may conform with the finding on strategies used in different learning phases. Careless errors were more likely to occur in the high-skill phase. The finding on strategies used in the high-skill phase indicated that students were inclined to use EE and mixed strategies. Hence, these results may indicate that making careless errors in the high-skill phase causes students to experience cognitive disequilibrium and request worked examples to ensure that they understood the material.

In addition, the finding on error type and strategies unveiled that students behaved in a more varied pattern of strategy use when making careless errors. A potential explanation might be that at times careless errors indicated students' disengagement (San Pedro, Baker, & Rodrigo, 2014). Thus, students may have randomly responded to errors which led to unpredictable learning patterns.

Topic difficulty and strategies occurring after errors

6th graders were inclined to use practice strategies or EW after making errors on difficult topics, whereas they tended to utilize WE after making errors on easy topics. This finding provided limited support for the hypothesis that students would be inclined to adopt example-practice strategies after making errors on difficult topics. An interesting phenomenon was that 6th graders also preferred to use practice strategies to correct errors on difficult topics. A potential explanation is that students may have utilized practice opportunities in order to understand the worked examples when they lacked knowledge on the topic (Schwartz et al., 2007). Furthermore, sometimes instructions may interfere students to self-explain their own answers (Kapur, 2008; Schworm & Renkl, 2006; Shih et al., 2010). Therefore, students avoided requesting worked examples on difficult topics.

The study found that students exerted more varied patterns of strategy use after making errors on easy topics. This finding did not confirm the previous assumption, potentially because easy topics did not enable students to engage in learning and therefore students behaved more randomly to recover from errors.

As for strategy shifts, students tended to transition to the practice-example strategy on difficult topics and preferred to transition to practice strategies on easy topics. Although this finding was not in line with the hypothesis that students were more likely to transition to the example-practice strategy, it was reasonable that students requested worked examples after repeatedly making errors on difficult topics.

The relationship between strategies and learning outcome

The relationship between strategies and immediate learning outcome

The finding that correctness after CC was highest (M>0.71) among all the strategies was stable in students overall, across different levels of prior knowledge, and across different topic difficulty. This meant that students might have the high probability of recovering from errors and mastering topics after solving two problems in a row successfully. In ALEKS, solving three problems in a row successfully was the index for topic mastery. Therefore, CC after making errors might be treated as a predictor of topic mastery.

The relationship between strategies and delayed learning outcomes

The study did not find a significant relationship between strategies and 6^{th} graders' delayed performance. However, the help strategy, example-practice strategies, WW, and WC benefited college students' delayed performance while WE, CW and CC hindered delayed performance. This finding remained stable across different levels of college students' prior knowledge as well. The hypothesis that the help strategy would boost delayed performance was in line with the finding that pure reading worked examples did not facilitate immediate test performance but promoted delayed performance (Van Gog & Kester 2012). The finding that example-practice strategies fostered learning confirmed the existing finding (Van Gog, Kester, & Paas, 2011) that reading worked examples before solving problems fosters better learning relative to solving problems before reading worked examples.

By combining the results for correctness, an interesting finding emerged that CC resulted in high correctness but hindered students' delayed performance. An explanation for this might be that CC occurring after making an error indicates that the problems of the topics were relatively easy to students so they could recover from errors more easily. Based on effects of desirable difficulty (Bjork, Dunlosky, & Kornell, 2013) and errors caused by impasse (VanLehn, Siler, Murray, Yamauchi, & Baggett, 2003) on learning, too easy problems may not provide students with opportunities to deeply understand and transfer knowledge in new situations. Therefore, if students used CC too often after making errors, it might imply that learning materials were easy for them. In this case, the learning system should improve the topic difficulty to help foster students' learning.

The study did not find a significant relationship between randomness of strategies and delayed performance. However, a closer investigation to the relationship between strategy shifts and delayed performance revealed significant relationships, which were different from the expectations based on the learning phase theory. In the low-skill phase, college students benefited from shifts to practice strategies while the shifts to strategies that involved requesting worked examples (i.e. help strategies and mixed strategies) did not facilitate their delayed performance. In the medium-skill phase and the high-skill phase, the shifts to strategies that involved requesting worked examples gradually played positive roles in student learning. The change in the relationship between strategy shifts and delayed performance from the low-skill phase to high-skill phase might be in line with the finding of "time for telling" (Schwartz & Bransford, 1998). That is, the early repeated failures in practice promoted students' performance. A potential explanation for this might be that early repeated failures in practice promoted students more straightforward to activate prior knowledge in mind (Schwartz et al., 2007). Roll, Baker, Aleven, and Koedinger (2014) would also argue that although those seemingly inappropriate practices did not boost students' immediate performance, this practice in the low-skill phase might foster learning for students who were ready to master the knowledge. On the other hand, the early repeated failures provided valuable experience of attempting various solutions to problems. Those early struggle experiences promoted students to differentiate concepts and encouraged them to understand worked examples requested in the following phases. The existing study found that students who experienced multiple failure solutions outperformed those who experienced fewer solutions (Kapur, 2012).

In addition, college students with both low and high prior knowledge benefited from strategy shifts to practice strategies. These results were consistent with learning by doing which exploring math was an effective learning experience for students (Kirschner, Sweller, & Clark, 2006). The potential explanation was that hands-on activities supply students with opportunities to build solutions by themselves (Bernstein, Penner, Clarke-Stewart, Roy, & Wickens, 2003).

Chapter 7: Conclusions and Future Directions

Conclusions

This study sheds light on how students use strategies (i.e. help and practice) to recover from errors in math. The findings revealed that students basically followed learning phases theory to use strategies after making errors. However, a closer examination of the strategies temporal pattern (i.e. strategy shifts) unveiled a more complex view on learning behaviors that students not only used ideal learning strategies based on learning phase theory, but they also generated other learning strategies. These strategies included repeated practice in the low-skill phase and repeated requesting of worked examples in the high-skill phase. Those learning strategies might more delicately reflect students' learning process such as constructing solutions by themselves or relying on worked examples when confronted with cognitive disequilibrium.

Furthermore, some strategies based on learning phase theory fostered learning. Nevertheless, enough repeated practice in the low-skill phase and requests of worked examples benefited students' delayed performance as well. This may indicate that students used the strategies of "Time for telling" or learning by doing in the adaptive learning system in addition to learning through the system's help. These findings suggest that students adapted their learning strategies based on the corresponding learning phases.

This study also examined the factors impacting strategies used after errors and learning outcomes. Some results that conflicted with the existing literature might ignite more thinking and future studies on learning strategies. For example, students exerted more varied patterns of strategy use when they were more competent, solved easy topics, or made careless errors. Students tended to request worked examples when making careless errors. Those results may inspire researchers to adopt refined methods to investigate learning strategies in various learning settings.

Future directions

This study was based purely on observations of a limited number of students' learning behaviors in ALEKS. In the future, work could be done to deepen and broaden the current understanding of the strategies students use to recover from errors. First, clustering students in terms of learning strategies could be a useful extension to clarify the characteristics of different learners and their effective learning strategies. Second, conducting experiments to compare different learning strategies could provide more solid and causal findings to interpret students' learning. Third, looking at specific topics, such as algebra and geometry, could promote an understanding of the differences between students' learning strategies in different disciplines. Fourth, investigating strategies in more distinctive learner groups and learning settings could help to generalize the findings and broaden the present understanding of learning strategies.

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Appendix A

Tables for the patterns of strategies utilized to learn from errors

Table 1

The strategies' likelihood of occurrence after an error, base rate, and conditional percentage after an error in the low-skill phase (6th graders)

	Help	Example-practice		Practice-example		Practice			
	EE	EW	EC	WE	CE	WW	WC	CW	CC
Base rate	0.07	0.15	0.10	0.21	0.005	0.18	0.12	0.08	0.09
Conditional percentage after an error	0.06	0.19	0.12	0.14	0.005	0.20	0.08	0.09	0.12
Likelihood of occurrence after an error (compared to base rate)	-0.01 ***	0.05 ***	0.03 ***	-0.09 ***	-0.0003	0.03 ***	-0.05 ***	0.01 ***	0.03 ***

Note: A one-sample t test (μ=0) was used to examine whether likelihood of occurrence after an error was different from base rate. If the likelihood was significantly below the base rate, then the value would be negative and marked with *. If the likelihood was significantly beyond the base rate, then the value would be positive and marked with *. If the likelihood was not significantly different than the base rate, then the value would NOT be marked with *. *, $p<0.006$, **, $p<0.001$, ***, $p<0.0001$

Table 2

The strategy's likelihood of occurrence after an error, base rate, and conditional percentage after an error in the low-skill phase (college students)

	Help	Example-practice		Practice-example		Practice			
	EE	EW	EC	WE	CE	WW	WC	CW	CC
Base rate	0.07	0.14	0.19	0.24	0.009	0.10	0.08	0.06	0.11

Conditional percentage after an error	0.06	0.13	0.19	0.31	0.01	0.02	0.007	0.08	0.18
Likelihood of occurrence after an error (compared to base rate)	-0.01***	-0.004	0.008	0.09***	0.002	-0.09***	-0.09***	0.03***	0.08***

Note: A one-sample t test (μ=0) was used to examine whether likelihood of occurrence after an error was different from base rate. If the likelihood was significantly below the base rate, then the value would be negative and marked with *. If the likelihood was significantly beyond the base rate, then the value would be positive and marked with *. If the likelihood was not significantly different than the base rate, then the value would NOT be marked with *. *, p<0.006, **, p<0.001, ***, p<0.0001

Table 3

The strategy's likelihood of occurrence after an error, base rate, and conditional percentage after an error in the medium-skill phase (6^{th} *graders)*

	Help	Example-practice		Practice-example		Practice			
	EE	EW	EC	WE	CE	WW	WC	CW	CC
Base rate	0.03	0.03	0.04	0.06	0.02	0.08	0.11	0.23	0.39
Conditional percentage after an error	0.08	0.09	0.11	0.11	-	0.14	0.11	0.12	0.24
Likelihood of occurrence after an error (compared to base rate)	0.05***	0.06***	0.07***	0.06***	-	0.07***	-0.006	-0.17***	-0.28***

Note: A one-sample t test (μ=0) was used to examine whether likelihood of occurrence after an error was different from base rate. If the likelihood was significantly below the base rate, then the value would be negative and marked with *. If the likelihood was significantly beyond the base rate, then the value would be positive and marked with *. If the likelihood was not significantly different than the base rate, then the value would NOT be marked with *. *, p<0.006, **, p<0.001, ***, p<0.0001

Table 4

The strategy's likelihood of occurrence after an error, base rate, and conditional percentage after an error in the medium-skill phase (college students)

	Help	Example-practice		Practice-example		Practice			
	EE	EW	EC	WE	CE	WW	WC	CW	CC
Base rate	0.03	0.04	0.08	0.10	0.06	0.04	0.07	0.17	0.41
Conditional percentage after an error	0.07	0.09	0.17	0.21	0.06	-	-	0.10	0.29
Likelihood of occurrence after an error (compared to base rate)	0.05 ***	0.06 ***	0.10 ***	0.11 ***	0.005	-	-	-0.08 ***	-0.22 ***

Note: A one-sample t test (μ=0) was used to examine whether likelihood of occurrence after an error was different from base rate. If the likelihood was significantly below the base rate, then the value would be negative and marked with *. If the likelihood was significantly beyond the base rate, then the value would be positive and marked with *. If the likelihood was not significantly different than the base rate, then the value would NOT be marked with *. *, $p<0.006$, **, $p<0.001$, ***, $p<0.0001$

Table 5

The strategy's likelihood of occurrence after an error, base rate, and conditional percentage after an error in the high-skill phase (6th graders)

	Help	Example-practice		Practice-example		Practice			
	EE	EW	EC	WE	CE	WW	WC	CW	CC
Base rate	0.01	0.10	0.02	0.02	0.02	0.03	0.06	0.19	0.63
Conditional percentage after an error	0.08	0.09	0.11	0.11	-	0.12	0.12	0.11	0.25
Likelihood of occurrence after an error (compared to base rate)	0.07 ***	0.07 ***	0.10 ***	0.09 ***	-	0.10 ***	0.07 ***	-0.10 ***	-1.16 ***

Note: A one-sample t test (μ=0) was used to examine whether likelihood of occurrence after an error was different from base rate. If the likelihood was significantly below the base rate, then the value would be negative and marked with *. If the likelihood was significantly beyond the base rate, then the value would be positive and marked with *. If the likelihood was not significantly different than the base rate, then the value would NOT be marked with *. *, p<0.006, **, p<0.001, ***, p<0.0001

Table 6

The strategy's likelihood of occurrence after an error, base rate, and conditional percentage after an error in the high-skill phase (college students)

	Help	Example-practice		Practice-example		Practice			
	EE	EW	EC	WE	CE	WW	WC	CW	CC
Base rate	0.01	0.01	0.03	0.03	0.04	0.01	0.04	0.14	0.69
Conditional percentage after an error	0.07	0.08	0.16	0.15	0.07	-	-	0.11	0.34
Likelihood of occurrence after an error (compared to base rate)	0.06 ***	0.07 ***	0.14 ***	0.13 ***	0.04 ***	-	-	-0.04 ***	-1.22 ***

Note: A one-sample t test (μ=0) was used to examine whether likelihood of occurrence after an error was different from base rate. If the likelihood was significantly below the base rate, then the value would be negative and marked with *. If the likelihood was significantly beyond the base rate, then the value would be positive and marked with *. If the likelihood was not significantly different than the base rate, then the value would NOT be marked with *. *, p<0.006, **, p<0.001, ***, p<0.0001

Table 7

The conditional probabilities of strategy shifts (6th graders)

n / n-1		Help (EE)	Example-Practice (EP)		Practice-example (PE)		Practice			
			EW	EC	WE	CE	CC	CW	WW	WC
Base rate of n strategy		0.08	0.15	0.12	0.13	0.05	0.12	0.10	0.16	0.09
Help (EE)		0.12	0.15	0.12	0.13	0.07	0.10	0.09	0.13	0.08
Example-practice (EP)	EC	0.07	0.12	0.12	0.13	0.05	0.15	0.10	0.15	0.10
	EW	0.08	0.16	0.11	0.13	0.03	0.10	0.10	0.19	0.09
Practice-example (PE)	WE	0.08	0.15	0.10	0.15	0.04	0.12	0.10	0.18	0.09
	CE	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11
Practice	CC	0.07	0.09	0.09	0.11	0.07	0.18	0.11	0.16	0.12
	CW	0.06	0.08	0.08	0.12	0.05	0.18	0.08	0.22	0.13
	WW	0.08	0.30	0.19	0.12	0.03	0.05	0.05	0.13	0.04
	WC	0.06	0.08	0.08	0.10	0.06	0.16	0.14	0.19	0.13

Note the conditional probabilities of transitions from one strategy added up to 1.

Table 8

The likelihood of strategy shifts (6th graders)

n	Help (EE)	Example-practice (EP)	Practice-example (PE)	Practice

n-1			EW	EC	WE	CE	CC	CW	WW	WC
Help (EE)		0.04 ***	0.003	0.004	0.002	0.02 ***	-0.03 ***	-0.005	-0.03 ***	-0.007
Example-practice (EP)	EC	-0.01 ***	-0.04 ***	0.008	0.002	0.002	0.04 ***	0.01	-0.01	0.008
	EW	-0.004	0.01	-0.01	0.008	-0.02 ***	-0.02 ***	0.001	0.04 ***	-0.002
Practice-example (PE)	WE	-0.005	-0.01	-0.02 ***	0.02 ***	-0.01 ***	-0.001	- 0.0002	0.03 ***	-0.003
	CE	0.03 ***	-0.05 ***	-0.01 *	-0.02 ***	0.06 ***	-0.01	0.01 ***	-0.06 ***	0.02 ***
Practice	CC	-0.01 ***	-0.08 ***	-0.03 ***	-0.02 ***	0.02 ***	0.07 ***	0.01	-0.01	-0.04 ***
	CW	-0.03 ***	-0.08 ***	-0.04 ***	-0.01	-0.003	0.08 ***	-0.03 ***	0.07 ***	0.05 ***
	WW	-0.002	0.18 ***	0.09 ***	-0.008	-0.03 ***	-0.08 ***	-0.05 ***	-0.03 ***	-0.05 ***
	WC	-0.02 ***	-0.08 ***	-0.04 ***	-0.03 ***	0.01 ***	0.04 ***	0.04 ***	0.04 ***	0.05 ***

Note: A one-sample t test was used to examine whether likelihood of occurrence after an error was different from base rate. If the likelihood was significantly below the base rate, then the value would be negative and marked with *. If the likelihood was significantly beyond the base rate, then the value would be positive and marked with *. If the likelihood was not significantly different than the base rate, then the value would NOT be marked with *. *, $p<0.0006$, **, $p<0.0001$, ***, $p<0.00001$

Table 9
The conditional probabilities of strategy shifts (college students)

n-1 \ n		Help (EE)	Example-practice (EP)		Practice-example (PE)		Practice			
			EW	EC	WE	CE	CC	CW	WW	WC
Base rate of n strategy		0.08	0.16	0.20	0.18	0.05	0.14	0.09	0.06	0.05
Help (EE)		0.12	0.14	0.17	0.16	0.07	0.11	0.09	0.08	0.07
Example-practice (EP)	EC	0.09	0.13	0.21	0.17	0.06	0.15	0.09	0.06	0.06
	EW	0.10	0.15	0.19	0.21	0.05	0.12	0.08	0.05	0.05
Practice-example (PE)	WE	0.07	0.10	0.12	0.28	0.04	0.18	0.11	0.06	0.05
	CE	0.11	0.11	0.12	0.13	0.11	0.12	0.11	0.10	0.10
Practice	CC	0.08	0.10	0.11	0.17	0.08	0.19	0.11	0.08	0.08
	CW	0.06	0.10	0.12	0.26	0.06	0.23	0.06	0.05	0.05
	WW	0.10	0.18	0.14	0.12	0.09	0.10	0.09	0.09	0.09
	WC	0.11	0.11	0.11	0.13	0.10	0.12	0.11	0.10	0.10

Note the conditional probabilities of transitions from one strategy added up to 1.

Table 10
The likelihood of strategy shifts (college students)

n	Help (EE)	Example-practice (EP)	Practice-example (PE)	Practice

n-1			EW	EC	WE	CE	CC	CW	WW	WC
Help (EE)		0.04***	-0.03***	-0.04***	-0.02	0.02***	-0.04***	0.004	0.02***	0.02***
Example-practice (EP)	EC	0.003	-0.04***	0.01	-0.01	0.01***	0.006	0.002	0.003	0.005***
	EW	0.01***	-0.01	-0.002	0.04***	-0.01***	-0.03***	-0.004	-0.001	-0.0002
Practice-example (PE)	WE	-0.02***	-0.07***	-0.10***	0.13***	-0.01***	0.04***	0.02***	0.01***	-0.01***
	CE	0.02***	-0.06***	-0.10***	-0.06***	0.05***	-0.03***	0.03***	0.05***	0.05***
Practice	CC	-0.0002	-0.07***	-0.11***	-0.01	0.03***	0.06***	0.02***	0.02***	0.03***
	CW	-0.02***	-0.07***	-0.10***	0.11***	0.01***	0.11***	-0.03***	-0.002	0.002
	WW	0.02***	0.03***	-0.08***	-0.08***	0.04***	-0.05***	0.01***	0.04***	0.04***
	WC	0.02***	-0.06***	-0.11***	-0.06***	0.05***	-0.03***	0.03***	0.05***	0.05***

Note: A one-sample t test was used to examine whether likelihood of occurrence after an error was different from base rate. If the likelihood was significantly below the base rate, then the value would be negative and marked with *. If the likelihood was significantly beyond the base rate, then the value would be positive and marked with *. If the likelihood was not significantly different than the base rate, then the value would NOT be marked with *. *, $p<0.0006$, **, $p<0.0001$, ***, $p<0.00001$

Table 11

The conditional probabilities of strategy shifts in the low-skill phase (6th graders)

n		Help (EE)	Example-practice (EP)		Practice-example (PE)		Practice			
n-1			EW	EC	WE	CE	CC	CW	WW	WC
Base rate of n strategy		0.08	0.16	0.12	0.13	0.05	0.11	0.10	0.16	0.09
Help (EE)		0.12	0.16	0.12	0.13	0.07	0.10	0.10	0.13	0.08
Example-practice (EP)	EC	0.07	0.12	0.12	0.13	0.05	0.15	0.11	0.15	0.10
	EW	0.08	0.16	0.11	0.13	0.03	0.10	0.10	0.19	0.09
Practice-example (PE)	WE	0.08	0.15	0.10	0.15	0.04	0.12	0.10	0.18	0.09
	CE	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11
Practice	CC	0.07	0.09	0.09	0.11	0.07	0.18	0.11	0.16	0.12
	CW	0.06	0.08	0.08	0.12	0.05	0.18	0.07	0.22	0.13
	WW	0.08	0.30	0.19	0.12	0.03	0.05	0.05	0.13	0.04
	WC	0.06	0.08	0.08	0.10	0.06	0.15	0.14	0.19	0.13

Note: The conditional probabilities of transitions from one strategy added up to 1.

Table 12

The likelihood of strategy shifts in the low-skill phase (6th graders)

n	Help (EE)	Example-practice (EP)	Practice-example (PE)	Practice

n-1		EW	EC	WE	CE	CC	CW	WW	WC
Help (EE)		0.04 ***	-0.002	0.004	0.005	0.02 ***	-0.02 ***	-0.004	-0.04 ***	-0.006
Example-practice (EP)	EC	-0.02 ***	-0.05 ***	0.007	0.0004	0.0005	0.05 ***	0.01 *	-0.01	0.009
	EW	-0.005	0.007	-0.008	0.005	-0.02 ***	-0.01	0.0002	0.04 ***	-0.002
Practice-example (PE)	WE	-0.006	-0.01	-0.02 ***	0.02 ***	-0.01 ***	0.008	0.0006	0.03 ***	-0.004
	CE	0.03 ***	-0.06 ***	-0.01 *	-0.02 ***	0.06 ***	0.002	0.01 ***	-0.06 ***	0.02 ***
Practice	CC	-0.01 ***	-0.08 ***	-0.03 ***	-0.02 ***	0.02 ***	0.07 ***	0.04 ***	-0.006	0.009
	CW	-0.03 ***	-0.09 ***	-0.04 ***	-0.01	-0.002	0.07 ***	-0.02 ***	0.07 ***	0.05 ***
	WW	-0.003	0.17 ***	0.08 ***	-0.009	-0.03 ***	-0.07 ***	-0.05 ***	-0.03 ***	-0.05 ***
	WC	-0.02 ***	-0.09 ***	-0.04 ***	-0.03 ***	0.01 ***	0.04 ***	0.04 ***	0.04 ***	0.04 ***

Note: *, p<0.0006, **, p<0.0001, ***, p<0.00001

Table 13

The conditional probabilities of strategy shifts in the low-skill phase (college students)

n	Help (EE)	Example-practice (EP)		Practice-example (PE)		Practice			
n-1		EW	EC	WE	CE	CC	CW	WW	WC
Base rate of n strategy	0.10	0.13	0.15	0.21	0.06	0.14	0.10	0.06	0.06
Help (EE)	0.12	0.07	0.17	0.16	0.07	0.10	0.09	0.08	0.07

Example-practice (EP)	EC	0.09	0.14	0.21	0.18	0.07	0.15	0.10	0.07	-
	EW	0.10	0.16	0.20	0.22	0.05	0.12	0.09	-	0.06
Practice-example (PE)	WE	0.07	0.10	0.12	0.28	0.04	0.16	0.11	0.07	0.05
	CE	0.13	0.14	0.15	0.16	0.13	0.14	0.14	-	-
Practice	CC	0.12	-	-	0.24	0.12	0.25	0.15	0.12	-
	CW	0.07	0.10	0.12	0.27	0.07	0.21	0.06	0.06	0.06
	WW	0.11	0.20	0.15	0.13	-	0.11	0.10	0.10	0.10
	WC	0.11	0.11	0.11	0.13	0.10	0.12	0.11	0.10	0.10

Note: The conditional probabilities of transitions from one strategy added up to 1.

Table 14
The likelihood of strategy shifts in the low-skill phase (college students)

n		Help (EE)	Example-practice (EP)		Practice-example (PE)		Practice			
n-1			EW	EC	WE	CE	CC	CW	WW	WC
Help (EE)		0.03 ***	0.01	0.01	-0.08 ***	0.02 ***	-0.04 ***	-0.02 ***	0.02 ***	0.02 ***
Example-practice (EP)	EC	-0.001	0.01	0.08 ***	-0.05 ***	0.01 ***	0.01	-0.01	0.005	-
	EW	0.01	0.04 ***	0.06 ***	0.01	-0.005	-0.02 ***	-0.02 ***	-	0.002

Practice-example (PE)	WE	-0.03 ***	-0.03 ***	-0.04 ***	0.08 ***	-0.01	0.03 ***	0.006	0.008	-0.008 ***
	CE	0.04 ***	0.01	-0.007	-0.08 ***	0.08 ***	0.008	0.04 ***	-	-
Practice	CC	0.03 ***	-	-	0.02	0.06 ***	0.13 ***	0.06 ***	0.06 ***	-
	CW	-0.03 ***	-0.03 ***	-0.04 ***	0.07 ***	0.01 ***	0.08 ***	-0.06 ***	-0.002	0.003
	WW	0.02 ***	0.08 ***	-0.01	-0.12 ***	-	-0.03 ***	-0.0004	0.04 ***	0.05 ***
	WC	0.01 ***	-0.02 ***	-0.05 ***	-0.11 ***	0.05 ***	-0.02 ***	0.01 *	0.05 ***	0.05 ***

Note: *, $p<0.0006$, **, $p<0.0001$, ***, $p<0.00001$

Table 15

The conditional probabilities of strategy shifts in the medium-skill phase (6th graders)

n		Help (EE)	Example-practice (EP)		Practice-example (PE)		Practice			
n-1			EW	EC	WE	CE	CC	CW	WW	WC
Base rate of n strategy		0.11	0.12	0.13	0.13	-	0.14	0.13	0.13	0.10
Help (EE)		0.12	0.13	0.13	0.13	-	0.12	0.12	0.13	0.13
Example-practice (EP)	EC	0.12	0.13	0.13	0.12	-	0.13	0.12	0.13	0.12

	EW	0.12	0.13	0.13	0.13	-	0.13	0.13	0.12	0.12
Practice-example (PE)	WE	0.12	0.13	0.13	0.13	-	0.13	0.13	0.12	0.12
	CE	-	-	-	-	-	-	-	-	-
Practice	CC	0.12	0.12	0.12	0.12	-	0.13	0.12	0.13	0.12
	CW	0.12	0.12	0.12	0.13	-	0.14	0.12	0.14	0.13
	WW	0.12	0.15	0.15	0.12	-	0.11	0.11	0.12	0.11
	WC	-	-	0.16	0.16	-	0.17	0.16	0.17	0.17

Note: The conditional probabilities of transitions from one strategy added up to 1.

Table 16

The likelihood of strategy shifts in the medium-skill phase (6th graders)

n		Help (EE)	Example-practice (EP)		Practice-example (PE)		Practice			
n-1			EW	EC	WE	CE	CC	CW	WW	WC
Help (EE)		0.01 ***	0.01 ***	-0.003	-0.003	-	-0.03 ***	-0.006 ***	-0.007 ***	0.02 ***
Example-practice (EP)	EC	0.008 ***	0.01 ***	-0.002	-0.004	-	-0.02 ***	-0.005	-0.006	0.02 ***
	EW	0.005 ***	0.01 ***	0.0004	-0.003	-	-0.02 ***	-0.005	-0.01 ***	0.02 ***
Practice-example (PE)	WE	0.007 ***	0.009 ***	-0.001	-0.001	-	-0.02 ***	-0.004	-0.01 ***	0.02 ***

	CE	-	-	-	-	-	-	-	-	-
Practice	CC	0.008***	0.007***	-0.005***	-0.005***	-	-0.01***	-0.007***	-0.007***	0.02***
	CW	0.001	0.0003	-0.009***	-0.003	-	-0.008	-0.01***	0.006	0.03***
	WW	0.002	0.04***	0.03***	-0.01	-	-0.04***	-0.02***	-0.02***	0.01***
	WC	-	-	0.04***	0.04***	-	0.03***	0.04***	0.04***	0.07***

Note: *, $p<0.0006$, **, $p<0.0001$, ***, $p<0.00001$

Table 17

The conditional probabilities of strategy shifts in the medium-skill phase (college students)

n		Help (EE)	Example-practice (EP)		Practice-example (PE)		Practice			
n-1			EW	EC	WE	CE	CC	CW	WW	WC
Base rate of n strategy		0.12	0.13	0.17	0.16	0.09	0.20	0.14	-	-
Help (EE)		0.17	0.17	0.17	0.17	-	0.17	0.17	-	-
Example-practice (EP)	EC	0.14	0.14	0.15	0.15	0.13	0.16	0.13	-	-
	EW	0.16	0.16	0.18	0.17	-	0.18	0.15	-	-
Practice-example (PE)	WE	-	0.18	0.18	0.23	-	0.23	0.18	-	-
	CE	-	0.20	0.20	0.20	0.20	0.20	-	-	-

Practice	CC	0.16	-	-	0.17	0.16	0.18	0.17	-	-
	CW	0.15	0.15	0.16	0.19	0.15	0.21	-	-	-
	WW	-	-	-	-	-	-	-	-	-
	WC	-	-	-	-	-	-	-	-	-

Note: The conditional probabilities of transitions from one strategy added up to 1.

Table 18

The likelihood of strategy shifts in the medium-skill phase (college students)

n		Help (EE)	Example-practice (EP)		Practice-example (PE)		Practice			
n-1			EW	EC	WE	CE	CC	CW	WW	WC
Help (EE)		0.05***	0.04***	0.002	0.005	-	-0.04***	0.02***	-	-
Example-practice (EP)	EC	0.02***	0.003	-0.02***	-0.02***	0.05***	-0.05***	-0.01***	-	-
	EW	0.04***	0.03***	0.02***	0.003	-	-0.02***	0.01	-	-
Practice-example (PE)	WE	-	0.05***	0.02***	0.08***	-	0.05***	0.04***	-	-
	CE	-	0.07***	0.04***	0.04***	0.12***	0.006	-	-	-
Practice	CC	0.05***	-	-	0.002	0.08***	-0.02***	0.02***	-	-

	CW	0.03***	0.02***	-0.01	0.03***	0.06***	0.02	-	-	-
	WW	-	-	-	-	-	-	-	-	-
	WC	-	-	-	-	-	-	-	-	-

Note: *, $p<0.0006$, **, $p<0.0001$, ***, $p<0.00001$

Table 19
The conditional probabilities of strategy shifts in the high-skill phase (6th graders)

n		Help (EE)	Example-practice (EP)		Practice-example (PE)		Practice			
n-1			EW	EC	WE	CE	CC	CW	WW	WC
Base rate of n strategy		0.08	0.13	0.10	0.14	-	0.15	0.12	0.15	0.05
Help (EE)		0.20	-	0.20	0.20	-	0.20	-	0.20	-
Example-practice (EP)	EC	-	0.25	0.25	-	-	0.25	-	0.25	-
	EW	0.12	0.13	0.13	0.13	-	0.13	0.12	0.12	0.12
Practice-example (PE)	WE	0.12	0.12	0.13	0.13	-	0.13	0.12	0.13	0.12
	CE	-	-	-	-	-	-	-	-	-
Practice	CC	-	-	0.17	0.17	-	0.17	0.17	0.17	0.17
	CW	-	0.14	0.14	0.14	-	0.16	0.14	0.14	0.15
	WW	0.12	0.14	0.15	0.12	-	0.12	0.12	0.12	0.12

WC	-	0.16	-	0.16	-	0.17	0.17	0.17	0.17

Note: The conditional probabilities of transitions from one strategy added up to 1.

Table 20

The likelihood of strategy shifts in the high-skill phase (6th graders)

n		Help (EE)	Example-practice (EP)		Practice-example (PE)		Practice			
n-1			EW	EC	WE	CE	CC	CW	WW	WC
Help (EE)		0.13***	-	0.11***	0.07***	-	0.06***	-	0.06***	-
Example-practice (EP)	EC	-	0.14***	0.17***	-	-	0.12***	-	0.12***	-
	EW	0.05***	0.001	0.03***	-0.01***	-	-0.03***	0.003	-0.03***	0.08***
Practice-example (PE)	WE	0.05***	-0.002	0.03***	-0.01***	-	-0.03***	0.005**	-0.03***	0.08***
	CE	-	-	-	-	-	-	-	-	-
Practice	CC	-	-	0.07***	0.03***	-	0.02***	0.05***	0.02***	0.12***
	CW	-	0.02***	0.04***	0.001	-	0.009	0.02***	-0.006	0.10***
	WW	0.04***	0.01**	0.06***	-0.02***	-	-0.04***	-0.005***	-0.03***	0.07***

WC	-	0.04 ***	-	0.03 ***	-	0.02 ***	0.05 ***	0.02 ***	0.12 ***

Note: *, $p<0.0006$, **, $p<0.0001$, ***, $p<0.00001$

Table 21

The conditional probabilities of strategy shifts in the high-skill phase (college students)

n		Help (EE)	Example-practice (EP)		Practice-example (PE)		Practice			
n-1			EW	EC	WE	CE	CC	CW	WW	WC
Base rate of n strategy		0.13	0.14	0.16	0.17	0.06	0.21	0.12	-	-
Help (EE)		0.11	0.11	0.11	0.11	0.11	0.11	-	-	-
Example-practice (EP)	EC	0.16	0.16	0.18	0.16	-	0.17	0.16	-	-
	EW	0.14	0.14	0.15	0.14	0.14	0.15	0.14	-	-
Practice-example (PE)	WE	0.13	0.13	0.14	0.16	0.13	0.17	0.14	-	-
	CE	-	-	0.50	-	-	0.50	-	-	-
Practice	CC	-	-	-	0.25	0.25	0.24	0.26	-	-
	CW	0.18	0.18	0.18	0.20	-	0.26	-	-	-
	WW	-	-	-	-	-	-	-	-	-
	WC	-	-	-	-	-	-	-	-	-

Note: The conditional probabilities of transitions from one strategy added up to 1.

Table 22

The likelihood of strategy shifts in the high-skill phase (college students)

n		Help (EE)	Example-practice (EP)		Practice-example (PE)		Practice			
n-1			EW	EC	WE	CE	CC	CW	WW	WC
Help (EE)		-0.02***	-0.04***	-0.06***	-0.07***	0.05***	-0.12***	-	-	-
Example-practice (EP)	EC	0.04***	0.02***	0.01***	-0.01***	-	-0.05***	0.04***	-	-
	EW	0.02***	-0.002	-0.01	-0.04***	0.08***	-0.08***	0.02***	-	-
Practice-example (PE)	WE	0.004	-0.008	-0.03***	-0.02***	0.07***	-0.05***	0.01***	-	-
	CE	-	-	0.40***	-	-	0.37***	-	-	-
Practice	CC	-	-	-	0.09***	0.19***	0.07***	0.14***	-	-
	CW	0.06***	0.04***	0.02***	0.03***	-	0.07***	-	-	-
	WW	-	-	-	-	-	-	-	-	-
	WC	-	-	-	-	-	-	-	-	-

Note: *, $p<0.0006$, **, $p<0.0001$, ***, $p<0.00001$

Appendix B

Tables for the factors impacting strategies after errors

Table 1

The strategy's likelihood of occurrence after an error, base rate, and conditional percentage after an error for easy topics (6th graders)

	Help	Example-practice		Practice-example		Practice			
	EE	EW	EC	WE	CE	WW	WC	CW	CC
Base rate	0.04	0.08	0.08	0.14	0.01	0.15	0.10	0.10	0.30
Conditional percentage after an error	0.05	0.15	0.15	0.13	0.004	0.16	0.08	0.08	0.20
Likelihood of occurrence after an error (compared to base rate)	0.01	0.08***	0.08***	-0.005	-0.003***	0.01**	-0.03***	-0.02***	-0.16***

Note: A One-sample t test was used to examine whether likelihood of occurrence after an error was different from base rate. If the likelihood was significantly below the base rate, then the value would be negative and marked with *. If the likelihood was significantly beyond the base rate, then the value would be positive and marked with *. If the likelihood was not significantly different than the base rate, then the value would NOT be marked with *. *, p<0.006, **, p<0.001, ***, p<0.0001

Table 23

The strategy's likelihood of occurrence after an error, base rate, and conditional percentage after an error for difficult topics (6th graders)

	Help	Example-practice		Practice-example		Practice			
	EE	EW	EC	WE	CE	WW	WC	CW	CC
Base rate	0.08	0.13	0.07	0.18	0.01	0.19	0.08	0.09	0.17

Conditional percentage after an error	0.08	0.21	0.12	0.15	0.01	0.19	0.06	0.07	0.10
Likelihood of occurrence after an error (compared to base rate)	0.005	0.10 ***	0.06 ***	-0.04 ***	-0.004 ***	- 0.003	-0.02 ***	-0.02 ***	-0.09 ***

Note: A one-sample t test was used to examine whether likelihood of occurrence after an error was different from base rate. If the likelihood was significantly below the base rate, then the value would be negative and marked with *. If the likelihood was significantly beyond the base rate, then the value would be positive and marked with *. If the likelihood was not significantly different than the base rate, then the value would NOT be marked with *. *, $p<0.006$, **, $p<0.001$, ***, $p<0.0001$

Table 3

The strategy's likelihood of occurrence after an error, base rate, and conditional percentage after an error for easy topics (college students)

	Help	Example-practice		Practice-example		Practice			
	EE	EW	EC	WE	CE	WW	WC	CW	CC
Base rate	0.03	0.06	0.11	0.13	0.02	0.06	0.07	0.09	0.44
Conditional percentage after an error	0.04	0.10	0.21	0.28	0.01	0.01	0.003	0.07	0.28
Likelihood of occurrence after an error (compared to base rate)	0.01 ***	0.04 ***	0.11 ***	0.18 ***	-0.01 ***	-0.06 ***	-0.07 ***	-0.02 ***	-0.35 ***

Note: If the likelihood is significantly lower than the base rate, then the value will be negative and marked with *. If the likelihood is significantly beyond the base rate, then the value will be positive and marked with *. If the likelihood is not significantly different than the base rate, then the value will NOT be marked with *. *, $p<0.006$, **, $p<0.001$, ***, $p<0.0001$

Table 4

The strategy's likelihood of occurrence after an error, base rate, and conditional percentage after an error for difficult topics (college students)

	Help	Example-practice		Practice-example		Practice			
	EE	EW	EC	WE	CE	WW	WC	CW	CC
Base rate	0.05	0.10	0.11	0.16	0.02	0.08	0.07	0.09	0.32
Conditional percentage after an error	0.06	0.13	0.19	0.31	0.01	0.02	0.01	0.07	0.20
Likelihood of occurrence after an error (compared to base rate)	0.01**	0.05***	0.09***	0.18***	-0.01***	-0.07***	-0.07***	-0.02***	-0.24***

Note: If the likelihood is significantly lower than the base rate, then the value will be negative and marked with *. If the likelihood is significantly beyond the base rate, then the value will be positive and marked with *. If the likelihood is not significantly different than the base rate, then the value will NOT be marked with *. *, $p<0.006$, **, $p<0.001$, ***, $p<0.0001$

Table 5

The conditional probabilities of strategy shifts for easy topics (6th graders)

n		Help (EE)	Example-practice (EP)		Practice-example (PE)		Practice			
n-1			EW	EC	WE	CE	CC	CW	WW	WC
Base rate of n strategy		0.08	0.14	0.13	0.12	0.05	0.15	0.10	0.14	0.09
Help (EE)		0.11	0.14	0.12	0.13	0.07	0.11	0.10	0.12	0.09
Example-practice (EP)	EC	0.07	0.11	0.13	0.12	0.05	0.17	0.11	0.14	0.10

	EW	0.08	0.16	0.13	0.13	0.04	0.13	0.10	0.16	0.08
Practice-example (PE)	WE	0.07	0.13	0.12	0.14	0.05	0.15	0.10	0.15	0.08
	CE	0.11	0.11	0.11	0.11	0.11	0.12	0.11	0.11	0.11
Practice	CC	0.08	0.08	0.09	0.11	0.07	0.21	0.10	0.14	0.11
	CW	0.06	0.08	0.09	0.12	0.05	0.21	0.08	0.17	0.13
	WW	0.08	0.25	0.21	0.12	0.04	0.06	0.05	0.13	0.06
	WC	0.07	0.08	0.08	0.10	0.07	0.19	0.13	0.16	0.12

Note: The conditional probabilities of transitions from one strategy added up to 1.

Table 6

The likelihood of strategy shifts for easy topics (6th graders)

n		Help (EE)	Example-practice (EP)		Practice-example (PE)		Practice			
n-1			EW	EC	WE	CE	CC	CW	WW	WC
Help (EE)		0.04***	0.003	-0.006	0.008	0.02***	-0.04***	-0.0004	-0.03*	-0.002
Example-practice (EP)	EC	-0.01	-0.04***	0.006	0.002	0.0002	0.03	0.01	-0.007	0.004
	EW	-0.001	0.03	0.009	0.008	-0.02***	-0.03***	-0.003	0.02	-0.01
Practice-example	WE	-0.005	-0.003	-0.006	0.02	-0.006*	-0.002	-0.003	0.01	-0.009

(PE)	CE	0.03 ***	-0.03 ***	-0.02 **	-0.02 ***	0.05 ***	-0.04 ***	0.01	-0.03 ***	0.02 ***
Practice	CC	-0.003	-0.06 ***	-0.04 ***	-0.02	0.02 ***	0.08 ***	0.001	-0.001	0.02 ***
	CW	-0.02 ***	-0.07 ***	-0.04 ***	-0.005	0.001	0.08 ***	-0.02 ***	0.04 *	0.04 ***
	WW	0.003	0.13 ***	0.09 ***	-0.002	-0.02 ***	-0.10 ***	-0.05 ***	-0.02	-0.04 ***
	WC	-0.01 ***	-0.07 ***	-0.05 ***	-0.02***	0.01 ***	0.04 ***	0.04 ***	0.02	0.03 ***

Note: *, P<0.0006, **, P<0.0001, ***, P<0.00001

Table 7

The conditional probabilities of strategy shifts for difficult topics (6th graders)

n		Help (EE)	Example-practice (EP)		Practice-example (PE)		Practice			
n-1			EW	EC	WE	CE	CC	CW	WW	WC
Base rate of n strategy		0.09	0.17	0.12	0.14	0.05	0.10	0.09	0.16	0.08
Help (EE)		0.14	0.16	0.13	0.13	0.06	0.08	0.09	0.14	0.07
Example-practice (EP)	EC	0.08	0.13	0.13	0.12	0.05	0.15	0.10	0.15	0.09
	EW	0.08	0.21	0.11	0.15	0.03	0.083	0.09	0.18	0.08
Practice-example (PE)	WE	0.08	0.17	0.10	0.16	0.04	0.10	0.09	0.18	0.09

	CE	0.11	0.11	0.11	0.11	0.11	0.12	0.11	0.11	0.11
Practice	CC	0.08	0.10	0.09	0.12	0.07	0.15	0.11	0.16	0.12
	CW	0.07	0.09	0.09	0.14	0.05	0.15	0.07	0.21	0.12
	WW	0.09	0.31	0.18	0.12	0.05	0.05	0.06	0.14	0.04
	WC	0.07	0.10	0.09	0.12	0.06	0.12	0.12	0.19	0.12

Note: The conditional probabilities of transitions from one strategy added up to 1.

Table 8

The likelihood of strategy shifts for difficult topics (6th graders)

n		Help (EE)	Example-practice (EP)		Practice-example (PE)		Practice			
n-1			EW	EC	WE	CE	CC	CW	WW	WC
Help (EE)		0.05***	-0.01	0.01	-0.007	0.01***	-0.02	-0.005	-0.03	-0.01*
Example-practice (EP)	EC	-0.01	-0.05***	0.02	-0.01	0.002	0.05***	0.006	-0.01	0.007
	EW	-0.007	0.04*	-0.003	0.01	-0.02***	-0.03***	-0.0003	0.02	-0.006
Practice-example (PE)	WE	-0.009	-0.008	-0.02***	0.03***	-0.01***	-0.003	0.001	0.02	0.003
	CE	0.02***	-0.08***	-0.02	-0.02***	0.06***	-0.01	0.02***	-0.06***	0.03***

Practice	CC	-0.01	-0.10***	-0.03***	-0.02	0.04***	0.05***	0.02*	0.004	0.04***
	CW	-0.02***	-0.09***	-0.03***	-0.003	0.006	0.06***	-0.02	0.06***	0.04***
	WW	-0.002	0.16***	0.08***	-0.02	-0.02***	-0.05***	-0.05***	-0.03	-0.04***
	WC	-0.03***	-0.08***	-0.03***	-0.02	0.02***	0.03*	0.03***	0.04***	0.04***

Note: *, $p<0.0006$, **, $p<0.0001$, ***, $p<0.00001$

Table 9

The conditional probabilities of strategy shifts for easy topics (college students)

n		Help (EE)	Example-practice (EP)		Practice-example (PE)		Practice			
n-1			EW	EC	WE	CE	CC	CW	WW	WC
Base rate of n strategy		0.09	0.12	0.14	0.17	0.08	0.14	0.10	0.08	0.08
Help (EE)		0.11	0.12	0.14	0.13	0.10	0.11	0.10	0.10	0.10
Example-practice (EP)	EC	0.10	0.12	0.16	0.13	0.09	0.15	0.10	0.08	0.08
	EW	0.10	0.13	0.17	0.16	0.07	0.12	0.09	0.08	0.07
Practice-example (PE)	WE	0.08	0.09	0.11	0.22	0.06	0.18	0.11	0.08	0.07
	CE	0.11	0.11	0.11	0.12	0.11	0.11	0.11	0.11	0.11
Practice	CC	0.10	0.10	0.11	0.14	0.10	0.15	0.11	0.10	0.10

	CW	0.08	0.09	0.12	0.19	0.19	0.08	0.08	0.08	0.08
	WW	0.11	0.14	0.12	0.11	0.10	0.11	0.11	0.10	0.10
	WC	0.11	0.11	0.11	0.12	0.11	0.11	0.11	0.11	0.11

Note: The conditional probabilities of transitions from one strategy added up to 1.

Table 10

The likelihood of strategy shifts for easy topics (college students)

n		Help (EE)	Example-practice (EP)		Practice-example (PE)		Practice			
n-1			EW	EC	WE	CE	CC	CW	WW	WC
Help (EE)		0.02***	0.004	0.002	-0.05***	0.02***	-0.04***	-0.002	0.02***	0.02***
Example-practice (EP)	EC	0.005	0.0007	0.02*	-0.05***	0.007	0.0005	-0.005	0.002	0.005
	EW	0.01	0.02	0.04***	-0.02	-0.006	-0.03***	-0.01*	-0.004	-0.003
Practice-example (PE)	WE	-0.02***	-0.02***	-0.03***	0.06***	-0.02***	0.05***	0.006	-0.004	-0.01***
	CE	0.02***	-0.005	-0.03***	-0.07***	0.03***	-0.04***	0.01***	0.03***	0.03***
Practice	CC	0.004	-0.01***	-0.03***	-0.05***	0.02***	0.008	0.007	0.02***	0.02***

	CW	-0.01***	-0.03***	-0.02***	0.02	0.003	0.05***	-0.02***	-0.0007	0.003
	WW	0.02***	0.02***	-0.02***	-0.07***	0.03***	-0.05***	0.003	0.03***	0.03***
	WC	0.02***	-0.008***	-0.03***	-0.07***	0.03***	-0.04***	0.01***	0.03***	0.03***

Note: *, P<0.0006, **, P<0.0001, ***, P<0.00001

Table 11

The conditional probabilities of strategy shifts for difficult topics (college students)

n		Help (EE)	Example-practice (EP)		Practice-example (PE)		Practice			
n-1			EW	EC	WE	CE	CC	CW	WW	WC
Base rate of n strategy		0.09	0.11	0.14	0.17	0.08	0.15	0.10	0.08	0.08
Help (EE)		0.12	0.14	0.15	0.15	0.08	0.10	0.09	0.08	0.08
Example-practice (EP)	EC	0.09	0.12	0.18	0.16	0.08	0.13	0.09	0.07	0.07
	EW	0.10	0.15	0.17	0.20	0.06	0.11	0.08	0.06	0.06
Practice-example (PE)	WE	0.07	0.11	0.11	0.28	0.05	0.14	0.10	0.08	0.06
	CE	0.11	0.11	0.12	0.13	0.11	0.12	0.11	0.10	0.10
Practice	CC	0.09	0.10	0.11	0.15	0.09	0.15	0.11	0.09	0.09
	CW	0.08	0.10	0.11	0.23	0.08	0.18	0.08	0.07	0.07

WW	0.11	0.17	0.13	0.11	0.09	0.10	0.10	0.10	0.09
WC	0.11	0.11	0.11	0.12	0.11	0.11	0.12	0.11	0.11

Note: The conditional probabilities of transitions from one strategy added up to 1.

Table 12

The likelihood of strategy shifts for difficult topics (college students)

n		Help (EE)	Example-practice (EP)		Practice-example (PE)		Practice			
n-1			EW	EC	WE	CE	CC	CW	WW	WC
Help (EE)		0.03 ***	0.03 *	0.01	-0.03	0.002	-0.05 ***	-0.009	0.002	0.004
Example-practice (EP)	EC	0.001	0.008	0.05 ***	-0.02	-0.003	-0.02	-0.01	-0.01 *	-0.008
	EW	0.01	0.04 ***	0.04 ***	0.03	-0.02 ***	-0.05 ***	-0.02 ***	-0.02 ***	-0.02 ***
Practice-example (PE)	WE	-0.02 ***	-0.005	-0.04 ***	0.12 ***	-0.03 ***	-0.002	-0.0005	0.002	-0.03
	CE	0.02 ***	-0.005	-0.02 ***	-0.06 ***	0.03 ***	-0.04 ***	0.008	0.02 ***	0.03 ***
Practice	CC	-0.0002	-0.01	-0.03 ***	-0.03	0.01 ***	0.009	0.008	0.01 ***	0.01 ***
	CW	-0.01 *	-0.01	-0.03 ***	0.06 ***	-0.002	0.04 ***	-0.03 ***	-0.01	-0.007
	W	0.01 *	0.06 ***	-0.01	-0.07	0.02 ***	-0.05	-0.008	0.02	0.02

W				***		***		***	***
WC	0.01***	-0.003	-0.03***	-0.06***	0.03***	-0.04***	0.01***	0.03***	0.03***

Note: *, p<0.0006, **, p<0.0001, ***, p<0.00001

Appendix C
Tables for learning outcomes and strategies after errors

Table 1

The results of multiple linear regressions on relationships between strategies and posttest (6th graders)

Strategy		Coefficient	VIF	Adjusted R^2
Help	Intercept	9.48***	-	0.52
	Pretest	0.69***	1.01	
	EE	-6.24		
Example-practice	Intercept	10.62***	-	0.52
	Pretest	0.68***	1.06	
	EW	-9.81		
	Intercept	8.32***		0.52
	Pretest	0.68***	1.02	
	EC	27.93		

Pratice-example	Intercept	10.01***	-	0.52
	Pretest	0.68***	1.07	
	WE	8.92		
	Intercept	9.14***	-	0.52
	Pretest	0.69***	1.00	
	CE	-121.71		
Practice	Intercept	9.41***	-	0.52
	Pretest	0.68***	1.00	
	WW	27.34		
	Intercept	10.52***	-	0.52
	Pretest	0.69***	1.01	
	WC	47.64		
	Intercept	9.43***	-	0.52
	Pretest	0.69***	1.00	
	CW	-6.00		
	Intercept	9.48***	-	0.52
	Pretest	0.69***	1.01	
	CC	-6.24		

Table 2

The results of simple linear regressions on relationships between strategies and posttest (6th graders)

	Coefficient
Pretest	0.69
EE	32.87
EW	-72.21
EC	81.12
WE	72.56
CE	-80.02
WW	49.32
WC	6.23
CW	2.25
CC	-72.24

Table 3

The results of multiple linear regressions on relationships between strategies and posttest (college students)

Strategy		Coefficient	VIF	Adjusted R^2
Help	Intercept	37.32***	-	0.34
	Pretest	0.87***	1.02	
	EE	261.10***		

Example-practice	Intercept	34.62***	-	0.28
	Pretest	0.99***	1.02	
	EW	69.64*		
	Intercept	32.73***	-	0.32
	Pretest	0.93***	1.00	
	EC	69.16***		
Pratice-example	Intercept	44.06***	-	0.30
	Pretest	0.96***	1.00	
	WE	-33.76**		
	Intercept	36.55***	-	0.27
	Pretest	0.96***	1.00	
	CE	-237.44		
Practice	Intercept	50.45***	-	0.32
	Pretest	0.97***	1.00	
	WW	171.60***		
	Intercept	47.34***	-	0.29
	Pretest	1.02***	1.05	
	WC	138.31*		

	Intercept	36.86***	-	0.28
	Pretest	0.97***	1.00	
	CW	-122.66*		
	Intercept	29.18***	-	0.34
	Pretest	0.91***	1.01	
	CC	-50.12***		

Table 4

The results of simple linear regressions on relationships between strategies and posttest (college students)

	Coefficient
Pretest	0.95
EE	329.17
EW	37.60
EC	74.93
WE	-31.86
CE	-204.74
WW	156.61
WC	31.61
CW	-94.36
CC	-56.96

Table 5

The results of multiple linear regression on relationship between randomness of strategies and posttest (6th graders)

	Coefficient	VIF	Adjusted R^2
Intercept	-37.70	-	0.52
Pretest	0.67^{***}	1.15	
Randomness (Entropy)	27.25		

Table 6

The results of simple linear regression on relationship between randomness of strategies and posttest (6th graders)

	Coefficient
Pretest	0.69
Randomness (Entropy)	176.87

Table 7

The results of multiple linear regression on relationship between randomness of strategies and posttest (college students)

	Coefficient	VIF	Adjusted R^2
Intercept	-392.81	-	0.28
Pretest	0.86	1.17	
Randomness (Entropy)	240.54		

Table 8

The results of simple linear regression on relationship between randomness of strategies and posttest (college students)

	Coefficient
Pretest	0.95
Randomness (Entropy)	568.3

Table 9

The results of multiple linear regressions on relationships between strategy shifts in the low-skill phase and delayed performance (6th graders)

Strategy transition		Coefficient	VIF	Adjusted R^2
Help → Practice-example	Intercept	10.00	-	0.53
	Pretest	0.68	1.01	
	EE→WE	31.07*	1.01	
Help → Practice	Intercept	9.78	-	0.53
	Pretest	0.67	1.01	
	EE→WC	-49.24*	1.01	
Example-practice → Example-practice	Intercept	9.74	-	0.52
	Pretest	0.68	1.00	
	EC→EC	-24.26*	1.00	
Practice→ Example-practice	Intercept	8.16	-	0.53
	Pretest	0.66	1.05	

	WW→EC	33.50**	1.05	
Practice → Practice	Intercept	8.35	-	0.53
	Pretest	0.68	1.00	
	CW→WC	28.04*	1.00	

Table 10

The results of simple linear regressions on relationships between strategy shifts and delayed performance in low-skill phase (6th graders)

	Coefficient
Pretest	0.68
EE→WE	50.30
EE→WC	-85.06
EC→EC	-26.80
WW→EC	71.11
CW→WC	34.35

Table 11

The results of multiple linear regressions on relationships between strategy shifts in low-skill phase and delayed performance (college students)

Strategy transition		Coefficient	VIF	Adjusted R^2
Help → Help	Intercept	34.99***	-	0.30

	Pretest	0.98***	1.00	
	EE→EE	107.34**	1.00	
Help → Practice	Intercept	35.39***	-	0.29
	Pretest	0.96***	1.00	
	EE→CC	-80.83**	1.00	
Example-practice → Example-practice	Intercept	33.17***	-	0.32
	Pretest	0.97***	1.00	
	EC→EC	66.09***	1.00	
Example-practice → Practice-example	Intercept	40.18***	-	0.30
	Pretest	1.00***	1.02	
	EC →CE	-216.34**	1.02	
Example-practice → Practice	Intercept	37.42***	-	0.29
	Pretest	0.93***	1.01	
	EW→CC	-73.04*	1.01	
	Intercept	38.11***	-	0.35
	Pretest	1.04***	1.03	
	EC→WW	-296.21***	1.03	
Practice-example → Example-practice	Intercept	36.41***	-	0.28

	Pretest	0.95***	1.00	
	WE→EC	-53.65*	1.00	
	Intercept	38.66***	-	0.34
	Pretest	0.92***	1.01	
	CE→EC	-98.38***	1.01	
Practice-example → Practice-example	Intercept	22.19***	-	0.31
	Pretest	0.94***	1.01	
	CE→CE	22.19***	1.01	
Practice-example → Practice	Intercept	41.72***	-	0.32
	Pretest	0.91***	1.01	
	WE→WC	305.68***	1.01	
	Intercept	33.87***	-	0.29
	Pretest	0.99***	1.01	
	CE→CW	107.43*	1.01	
Practice→ Help	Intercept	37.79***	-	0.28
	Pretest	0.91***	1.02	
	WC→EE	151.71*	1.02	
	Intercept	33.86***	-	0.28

	Pretest	1.00***	1.03	
	CW→EE	-125.85*	1.03	
	Intercept	42.51***	-	0.30
	Pretest	0.97***	1.00	
	CC→EE	-131.98**	1.00	
Practice → Example-practice	Intercept	38.09***	-	0.31
	Pretest	0.95***	1.00	
	WW→EC	-54.83***	1.00	
	Intercept	34.34***	-	0.31
	Pretest	1.03***	1.04	
	WC→EW	-122.48**	1.04	
	Intercept	32.62***	-	0.38
	Pretest	0.92***	1.00	
	WC→EC	-124.06***	1.00	
	Intercept	35.09***	-	0.30
	Pretest	0.95***	1.00	
	CW→EC	-83.56**	1.00	
Practice → Practice-example	Intercept	6.74	-	0.44
	Pretest	0.99***	1.00	

	WC→CE	621.80***	1.00	
	Intercept	46.11***	-	0.28
	Pretest	0.98***	1.01	
	CC→CE	-127.62*	1.01	
Practice → Practice	Intercept	33.72***	-	0.29
	Pretest	0.88***	1.05	
	WW→WW	144.12**	1.05	
	Intercept	30.37***	-	0.29
	Pretest	0.90***	1.03	
	WW→WC	192.30*	1.03	
	Intercept	34.94***	-	0.32
	Pretest	0.92***	1.01	
	WW→CC	-125.12***	1.01	
	Intercept	10.40*	-	0.45
	Pretest	0.89***	1.01	
	WC→WW	637.11***	1.01	
	Intercept	0.21	-	0.48
	Pretest	0.97***	1.00	
	WC→WC	748.98***	1.00	

	Intercept	37.46***	-	0.29
	Pretest	0.95***	1.00	
	WC→CW	113.28*	1.00	
	Intercept	28.08***	-	0.33
	Pretest	1.05***	1.05	
	CC→CC	61.78***	1.05	

Table 12

The results of simple linear regressions on relationships between strategy shifts in low-skill phase and delayed performance (college students)

	Coefficient
Pretest	0.95
EE→EE	88.89
EE→CC	-76.37
EW→CC	-90.67
EC→EC	58.62
WW→EC	-55.82
WW→WW	233.64
WW→WC	293.77
WW→CC	-144.92

WC→EE	232.45
WC→EW	-52.87
WC→EC	-132.84
WC→CE	583.44
WC→WW	691.81
WC→WC	733.00
WC→CW	121.85
CW→EE	-51.48
CW→EC	-84.70
CC→EE	-119.92
CC→CE	-75.60
CC→CC	33.49

Table 13

The results of multiple linear regressions on relationships between strategy shifts in the medium-skill phase and delayed performance (6th graders)

Strategy transition		Coefficient	VIF	Adjusted R^2
Help → Example-practice	Intercept	8.79^{***}	-	0.54
	Pretest	0.67^{***}	1.03	
	EE→EW	139.24^{**}	1.03	

Practice-example→ Practice	Intercept	9.69***	-	0.54
	Pretest	0.70***	1.00	
	WE→WW	86.15*	1.00	
Practice→ Example-practice	Intercept	8.21***	-	0.54
	Pretest	0.69***	1.00	
	CC→EW	133.53*	1.00	
Practice → Practice	Intercept	8.51***	-	0.53
	Pretest	0.69***	1.00	
	WW→WW	-45.44*	1.00	

Table 14
The results of simple linear regressions on relationships between strategies transition and delayed performance in the medium-skill phase (6th graders)

	Coefficient
Pretest	0.70
EE→EW	259.74
WE→WW	65.16
CC→EW	146.195
WW→WW	-60.66

Table 15

The results of multiple linear regressions on relationships between strategy shifts in the medium-skill phase and delayed performance (college students)

Strategy transition		Coefficient	VIF	Adjusted R^2
Help → Example-practice	Intercept	30.56^{***}	-	0.33
	Pretest	1.00^{***}	1.01	
	EE→EW	194.12^{**}	1.01	
Help → Practice-example	Intercept	38.42^{***}	-	0.29
	Pretest	1.00^{***}	1.01	
	EE→WE	-84.73^{*}	1.01	
Help→ Practice	Intercept	32.59^{***}	-	0.44
	Pretest	0.91^{***}	1.01	
	EE→CC	-185.35^{***}	1.01	
Example-practice → Example-practice	Intercept	35.00^{***}	-	0.30
	Pretest	1.00^{***}	1.01	
	EW→EW	84.08^{*}	1.01	
	Intercept	37.24^{***}	-	0.30
	Pretest	0.98^{***}	1.00	
	EW→EC	56.93^{*}	1.00	
Example-practice → Practice	Intercept	39.61^{***}	-	0.32

	Pretest	1.00***	1.00	
	EW→CW	-170.64***	1.00	
	Intercept	35.88***	-	0.34
	Pretest	1.00***	1.00	
	EW→CC	-87.18***	1.00	
	Intercept	36.27***	-	0.29
	Pretest	0.97***	1.00	
	EC→CW	-203.10*	1.00	
	Intercept	34.29***	-	0.32
	Pretest	0.96***	1.00	
	EC→CC	-91.64***	1.00	
Practice-example → Example-practice	Intercept	17.78**	-	0.32
	Pretest	0.93***	1.01	
	CE→EW	293.02***	1.01	
	Intercept	43.87***	-	0.30
	Pretest	0.96***	1.00	
	CE→EC	-133.84**	1.00	
Practice-example → Practice-example	Intercept	42.60***	-	0.30

	Pretest	0.99***	1.01	
	CE→WE	-104.26*	1.01	
	Intercept	-21.58*		0.41
	Pretest	0.92***	1.00	
	CE→CE	522.85***	1.00	
Practice-example → Practice	Intercept	40.87***	-	0.41
	Pretest	0.91***	1.01	
	CE→CC	-183.43**	1.01	
Practice→ Help	Intercept	21.78***	-	0.33
	Pretest	0.94***	1.01	
	CC→EE	346.62***	1.01	
Practice → Example-practice	Intercept	38.02***	-	0.32
	Pretest	0.93***	1.01	
	CW→EC	-108.57**	1.01	
	Intercept	8.21***	-	0.54
	Pretest	0.69***	1.00	
	CC→EW	133.53*	1.00	
Practice → Practice-example	Intercept	3.63	-	0.34

	Pretest	0.91^{***}	1.01	
	CC→CE	468.43^{***}	1.01	
Practice → Practice	Intercept	37.84^{***}	-	0.32
	Pretest	0.92^{***}	1.02	
	CC→CC	-98.51^{***}	1.02	

Table 16

The results of simple linear regressions on relationships between strategy shifts in medium-skill phase and delayed performance (college students)

	Coefficient
Pretest	0.97
EE→EW	159.25
EE→WE	-50.92
EE→CC	-203.73
EW→EW	49.91
EW→EC	48.46
EW→CW	-142.10
EW→CC	-76.29
EC→CW	-194.00
EC→CC	-94.70

WE→EC	-41.3
CE→EW	360.23
CE→EC	-142.48
CE→WE	-76.37
CE→CE	572.84
CE→CC	-205.30
CW→EC	-135.17
CC→EE	397.52
CC→EW	146.20
CC→CE	573.86
CC→CC	-127.80

Table 17

The results of multiple linear regressions on relationships between strategy shifts in the high-skill phase and delayed performance (6th graders)

Strategy transition		Coefficient	VIF	Adjusted R^2
Practice→ Example-practice	Intercept	14.12^{***}	-	0.53
	Pretest	0.69^{***}	1.00	
	WC→EW	-111.86^{*}	1.00	
Practice→ Practice	Intercept	3.18	-	0.53

	Pretest	0.70***	1.00
	CW→WC	58.53*	1.00

Table 18

The results of simple linear regressions on relationships between strategy shifts in high-skill phase and delayed performance (6th graders)

	Coefficient
Pretest	0.69
WC→EW	-131.87
CW→WC	45.56

Table 19

The results of multiple linear regressions on relationships between strategy shifts in the high-skill phase and delayed performance (college students)

Strategy transition		Coefficient	VIF	Adjusted R^2
Help → Help	Intercept	51.06***	-	0.52
	Pretest	1.04***	1.00	
	EE→EE	722.68***	1.00	
Help → Example-practice	Intercept	57.97***	-	0.47
	Pretest	1.09***	1.01	
	EE→EW	588.86***	1.01	

	Intercept	31.53***	-	0.29
	Pretest	0.97***	1.00	
	EE→EC	-126.98**	1.00	
Help → Practice-example	Intercept	50.61***	-	0.29
	Pretest	0.98***	1.01	
	EE→WE	158.02*	1.01	
	Intercept	-6.29	-	0.39
	Pretest	1.05***	1.01	
	EE→CE	868.21***	1.01	
Help→ Practice	Intercept	9.37*	-	0.50
	Pretest	1.06***	1.00	
	EE→CC	-225.63***	1.00	
Example-practice → Help	Intercept	32.92***	-	0.36
	Pretest	1.04***	1.01	
	EW→EE	298.70***	1.01	
	Intercept	28.14***	-	0.32
	Pretest	0.99***	1.00	
	EC→EE	259.46***	1.00	

Example-practice → Example-practice	Intercept	36.66^{***}	-	0.39
	Pretest	1.12^{***}	1.03	
	EW→EW	318.82^{***}	1.03	
	Intercept	30.61^{***}	-	0.33
	Pretest	1.06^{***}	1.01	
	EC→EW	280.16^{***}	1.01	
Example-practice → Practice	Intercept	22.03^{***}	-	0.44
	Pretest	1.09^{***}	1.01	
	EW→CC	-178.64^{***}	1.01	
	Intercept	32.83^{***}	-	0.36
	Pretest	0.99^{***}	1.00	
	EC→CC	-133.54^{***}	1.00	
Practice-example → Help	Intercept	37.66^{***}	-	0.28
	Pretest	1.01^{***}	1.00	
	WE→EE	183.87^{*}	1.00	
Practice-example → Example-practice	Intercept	39.64^{***}	-	0.29
	Pretest	1.01^{***}	1.00	
	WE→EW	156.52^{**}	1.00	

	Intercept	34.02***	-	0.32
	Pretest	1.03***	1.00	
	WE→EC	-137.75***	1.00	
	Intercept	90.57***	-	0.29
	Pretest	1.05***	1.02	
	CE→EC	-131.94**	1.02	
Practice-example → Practice-example	Intercept	40.38***	-	0.27
	Pretest	0.97***	1.01	
	WE→WE	58.62*	1.01	
Practice-example → Practice	Intercept	36.83***	-	0.28
	Pretest	0.97***	1.00	
	WE→CC	-47.51*	1.00	
	Intercept	107.30***	-	0.35
	Pretest	0.95***	1.01	
	CE→CC	-183.37***	1.01	
Practice → Example-practice	Intercept	42.33***	-	0.33
	Pretest	1.00***	1.00	
	CW→EC	-154.44**	1.00	

Table 20

The results of simple linear regressions on relationships between strategy shifts in the high-skill phase and delayed performance (college students)

	Coefficient
Pretest	0.99
EE→EE	688.81
EE→EW	509.05
EE→EC	-149.11
EE→WE	184.20
EE→CE	760.38
EE→CC	-209.27
EW→EE	258.80
EW→EW	232.71
EW→CC	-150.29
EC→EE	264.09
EC→EW	212.37
EC→CC	-136.31
WE→EE	145.66
WE→EW	131.19
WE→EC	-118.26

WE→WE	79.86
WE→CC	-59.21
CE→EC	-80.11
CE→CC	-208.10
CW→EC	-148.71

Appendix D

Tables for learning outcomes, strategies, and the factors impacting the strategies

Table 1

The correctness on the next attempt after using strategies to learn from errors in different levels of prior knowledge (6th graders)

Strategy		Low prior knowledge		High prior knowledge	
		Mean of correctness percentage	S. D. of correctness percentage	Mean of correctness percentage	S. D. of correctness percentage
Help	EE	0.21	0.14	0.23	0.17
Example-practice	EW	0.19	0.11	0.20	0.10
	EC	0.54	0.12	0.59	0.14
Practice-example	WE	0.31	0.10	0.35	0.12
	CE	0.34	0.32	0.37	0.35
Practice	WW	0.06	0.05	0.07	0.05
	WC	0.42	0.17	0.52	0.12

	CW	0.37	0.15	0.41	0.15
	CC	0.76	0.10	0.77	0.08

Table 2

The correctness on the next attempt after using strategies to learn from errors in different levels of prior knowledge (college students)

Strategy		Low prior knowledge		High prior knowledge	
		Mean of correctness percentage	S. D. of correctness percentage	Mean of correctness percentage	S. D. of correctness percentage
Help	EE	0.34	0.22	0.48	0.25
Example-practice	EW	0.21	0.19	0.22	0.17
	EC	0.68	0.15	0.72	0.16
Practice-example	WE	0.48	0.12	0.55	0.14
	CE	0.51	0.29	0.51	0.31
Practice	WW	0.07	0.15	0.05	0.12
	WC	0.52	0.28	0.52	0.28
	CW	0.43	0.18	0.47	0.18
	CC	0.85	0.07	0.90	0.06

Table 3

The results of multiple linear regressions on relationships between strategies and delayed performance in different levels of prior knowledge (6th graders)

Strategy		Low prior knowledge			High prior knowledge		
		Coefficient	VIF	Adjusted R^2	Coefficient	VIF	Adjusted R^2
Help	Intercept	19.42***	-	0.08	3.94	-	0.38
	Pretest	0.33**	1.04		0.81***	1.01	
	EE	6.54			11.09		
Example-practice	Intercept	24.81***	-	0.11	3.02	-	0.39
	Pretest	0.28**	1.09		0.81***	1.00	
	EW	-42.99			12.43		
	Intercept	18.85***	-	0.08	2.63	-	0.40
	Pretest	0.33**	1.02		0.77***	1.02	
	EC	11.27			54.56		
Practice-example	Intercept	20.94***	-	0.09	20.94***	-	0.08
	Pretest	0.31**	1.08		0.31**	1.01	
	WE	23.71			23.71		
	Intercept	20.03***	-	0.08	4.06	-	0.41
	Pretest	0.33**	1.00		0.80***	1.00	

	CE	211.95			-171.24		
Practice	Intercept	18.82***		0.09	4.14***	-	0.41
	Pretest	0.34***	1.00		-.80**	1.00	
	WW	39.49			-0.96		
	Intercept	20.28***		0.09	4.82	-	0.39
	Pretest	0.34***	1.01		0.81***	1.00	
	WC	51.32			34.02		
	Intercept	19.58***		0.08	4.10	-	0.40
	Pretest	0.34**	1.01		0.80***	1.00	
	CW	30.34			-44.69		
	Intercept	17.45***		0.09	2.68	-	0.39
	Pretest	0.32**	1.02		0.78***	1.02	
	CC	-24.68			-22.69		

Table 4
The results of simple linear regressions on relationships between strategies and delayed performance in different levels of prior knowledge (6th graders)

	Coefficient	
	Low prior knowledge	High prior knowledge
Pretest	0.33	0.80

EE	23.48	-30.16
EW	-62.39	0.90
EC	31.27	90.27
WE	42.09	18.08
CE	265.87	-187.87
WW	35.05	33.09
WC	35.26	15.75
CW	17.32	-57.84
CC	-33.84	-43.32

Table 5

The results of multiple linear regressions on relationships between strategies and posttest in different levels of prior knowledge (college students)

Strategy		Low prior knowledge			High prior knowledge		
		Coefficient	VIF	Adjusted R^2	Coefficient	VIF	Adjusted R^2
Help	Intercept	25.44^{***}	-	0.35	42.40^{***}	-	0.19
	Pretest	2.11^{***}	1.00		0.70^{***}	1.00	
	EE	303.18^{***}			189.22^{*}		
Example-practice	Intercept	20.73^{***}	-	0.26	41.67^{***}	-	0.25
	Pretest	2.35^{***}	1.09		0.67^{***}	1.00	

	EW	75.78			127.55***		
	Intercept	19.57***	-	1.01	40.01***	-	0.33
	Pretest	2.25***	1.01		0.65***	1.00	
	EC	71.44*			77.64***		
Practice-example	Intercept	30.71***	-		53.34***	-	0.27
	Pretest	2.24***	1.01		0.72***	1.01	
	WE	-28.59			-50.95***		
	Intercept	24.99***	-	0.24	46.49	-	0.17
	Pretest	2.157***	1.00		0.62	1.03	
	CE	-180.37			-330.16		
Practice	Intercept	36.44***	-	0.27	63.48***	-	0.35
	Pretest	2.30***	1.03		0.67 ***	1.00	
	WW	159.65*			227.35***		
	Intercept	36.45***	-	0.27	68.58***	-	0.26
	Pretest	2.51***	1.17		0.57***	1.03	
	WC	204.58*			222.03***		
	Intercept	22.25***	-	0.26	48.43***	-	0.24
	Pretest	2.36***	1.08		0.59***	1.03	
	CW	-165.22			-176.42***		

	Intercept	9.52	-	0.35	40.91***	-	0.27
	Pretest	2.33***	1.01		0.60***	1.02	
	CC	-77.77***			-40.47***		

Table 6

The results of simple linear regressions on relationships between strategies and posttest in different levels of prior knowledge (college students)

	Coefficient	
	Low prior knowledge	High prior knowledge
Pretest	2.16	0.68
EE	319.66	166.75
EW	2.50	129.40
EC	56.30	79.79
WW	95.24	229.91
WC	-16.44	258.46
CW	-14.11	-207.72
CC	-63.91	-45.68

Table 7

The results of multiple linear regressions on relationships between strategy shifts and delayed performance in low prior knowledge level (6th graders)

Strategy shift		Coefficient	VIF	Adjusted R^2
Help → practice	Intercept	19.21^{***}	-	0.14
	Pretest	0.33^{***}	1.00	
	EE→WC	-77.05^{**}		
Example-practice → Practice-example	Intercept	27.59^{***}	-	0.14
	Pretest	0.27^{***}	1.05	
	EW→CE	292.22^{**}		
Example-practice → Example-practice	Intercept	18.13^{***}	-	0.12
	Pretest	0.32^{**}	1.00	
	EC→ EW	-33.91^{*}		
	Intercept	19.98^{***}	-	0.13
	Pretest	0.32^{**}	1.00	
	EC→ EC	-41.08^{*}		
Practice-example → Practice-example	Intercept	18.52^{***}	-	0.13
	Pretest	0.33^{**}	1.00	
	WE →WE	42.93^{*}		
Practice → Example-practice	Intercept	17.71^{***}	-	0.13

	Pretest	0.29^{**}	1.04	
	WW→EC	39.80^{*}		
	Intercept	17.90^{***}	-	0.12
	Pretest	0.31^{**}	1.02	
	CW→EC	-57.43^{*}		
Practice→ Practice	Intercept	15.64^{***}	-	0.11
	Pretest	0.27^{*}	1.11	
	WW→CC	-71.10^{*}		
	Intercept	17.07^{***}	-	0.16
	Pretest	0.32^{**}	1.00	
	CW→WC	58.61^{**}		

Table 8

The results of simple linear regressions on relationships between strategies shifts and delayed performance in low prior knowledge level (6th graders)

	Coefficient
Pretest	0.33
EE→WC	-77.14
EW→CE	362.16
EC→EW	-36.45

EC→EC	-43.89
WE→WE	44.58
WW→EC	44.62
CW→EC	-66.79
WW→CC	-100.07
CW→WC	62.25

Table 9

The results of multiple linear regressions on relationships between strategies shifts and delayed performance in low prior knowledge level (college students)

Strategy shift		Coefficient	VIF	Adjusted R^2	Strategy shift	Coefficient	VIF	Adjusted R^2
Help → Practice	Intercept	28.40^{***}	-	0.27	-	-	-	-
	Pretest	2.36^{***}	1.06		-	-	-	
	EE→ WW	-251.36^{*}			-	-		
Example-practice → Example-practice	Intercept	20.36^{***}	-	0.27	Intercept	15.44^{**}	-	0.40
	Pretest	1.99^{***}	1.04		Pretest	1.87^{***}	1.03	
	WE→ EC	-84.52^{*}			CE→ EC	-159.18^{***}		
Practice-example→ Practice-example	Intercept	32.94^{***}	-	0.29	Intercept	11.13	-	0.36
	Pretest	2.05^{***}	1.01		Pretest	1.72^{***}	1.08	

	WE→ CE	350.23**			CE→ CE	366.93***		
Practice-example→ Practice	Intercept	33.17***	-	0.32	Intercept	7.70	-	0.34
	Pretest	1.90***	1.04		Pretest	1.67***	1.13	
	WE→ WC	491.99 ***			CE→ WW	539.60 ***		
	Intercept	1.06	-	0.35	Intercept	20.66***	-	0.28
	Pretest	1.77***	1.07		Pretest	2.20***	1.01	
	CE→ WC	593.86 ***			CE→ CW	188.56 *		
Practice→ Help	Intercept	19.75***			-	-	-	-
	Pretest	2.30***			-	-	-	
	CW→ EE	-240.41 **			-	-		
Practice→ Example-practice	Intercept	16.83***	-	0.39	Intercept	19.44**	-	0.27
	Pretest	2.28***	1.01		Pretest	2.20***	1.00	
	WW→ EC	-116.92 ***			WC→EW	-93.93*		
	Intercept	8.83	-	0.47	Intercept	18.84***	-	0.32
	Pretest	1.72***	1.04		Pretest	1.80***	1.08	
	WC→ EC	-220.73 ***			CW→EC	-120.02 ***		
	Intercept	18.05	-	0.27	-	-	-	-
	Pretest	2.17	1.00		-	-	-	
	CC→ EW	-101.62*			-	-		

Practice→ Practice-example	Intercept	15.75*	-	0.30	Intercept	018	-	0.40
	Pretest	1.82***	1.01		Pretest	1.84***	1.03	
	WW→ CE	417.93**			WC→ CE	549.64 ***		
	Intercept	21.73***	-	0.28	-	-	-	-
	Pretest	2.35***	1.04		-	-	-	
	CW→ CE	218.24*						
Practice→ Practice	Intercept	19.81***	-	0.30	Intercept	14.95*	-	0.30
	Pretest	1.90***	1.06		Pretest	1.98***	1.03	
	WW→ WW	298.96**			WW→ WC	360.09***		
	Intercept	23.97***	-	0.30	Intercept	-11.01	-	0.47
	Pretest	1.66***	1.22		Pretest	1.77***	1.04	
	WW→ CC	-171.39 **			WC→ WC	772.26 ***		
	Intercept	-11.01	-	0.49	Intercept	20.09***	-	0.30
	Pretest	1.77 ***	1.08		Pretest	2.14***	1.00	
	WC→ WW	828.04 ***			CC→ CC	76.83 **		

Table 10

The results of simple linear regressions on relationships between strategy shifts and posttest in low prior knowledge level (college students)

		Coefficient		Coefficient
Pretest		2.16	-	-
Help→ Practice	EE→WW	-80.77	-	-
Practice-example→ Example-practice	WE→EC	-121.93	CE→EC	-187.23
Practice-example→ Practice-example	WE→CE	419.96	CE→CE	476.12
Practice-example→ Example	WE→WC	629.16	CE→WW	743.31
	CE→WC	767.98	CE→CW	171.76
Practice→ Help	CW→EE	-140.95	-	-
Practice→ Example-practice	WW→EC	-105.75	WC→EW	-80.66
	WC→EC	-257.44	CW→EC	-162.24
	CC→EW	-157.64	-	-
Practice→ Practice-example	WW→CE	617.91	WC→CE	648.63
	CW→CE	110.16		
Practice→ Practice	WW→ WW	413.32	WW→ WC	466.81
	WW→ CC	-273.75	WC→ WC	891.90
	WC→WW	981.49	CC→CC	79.74

Table 11

The results of multiple linear regressions on relationships between strategy shifts and delayed performance in high prior knowledge level (6th graders)

Strategy shift		Coefficient	VIF	Adjusted R^2
Example-practice → Example-practice	Intercept	5.92	-	0.41
	Pretest	0.78^{***}	1.02	
	EW→ EC	43.23^{*}		
Example-practice → Practice	Intercept	4.15	-	0.43
	Pretest	0.83^{***}	1.00	
	EW→WW	-36.89^{**}		
Practice-example→ Practice	Intercept	4.98	-	0.42
	Pretest	0.79^{***}	1.00	
	WE→CW	40.30^{*}		

Table 12

The results of simple linear regressions on relationships between strategy shifts and delayed performance in high prior knowledge level (6th graders)

	Coefficient
Pretest	0.80
EW→ EC	63.95
EW→WW	-26.18

WE→CW	49.45

Table 13

The results of multiple linear regressions on relationships between strategy shifts and delayed performance in high prior knowledge level (college students)

Strategy shift		Coefficient	VIF	Adjusted R^2	Strategy shift	Coefficient	VIF	Adjusted R^2
Help → Help	Intercept	40.73***	-	0.21	-	-	-	-
	Pretest	0.72***	1.01		-	-	-	
	EE→ EE	118.38**			-	-		
Example-practice → Help	Intercept	46.68***	-	0.18	-	-	-	-
	Pretest	0.70***	1.00		-	-	-	
	EC→ EE	-99.52*				-		
Example-practice→ Example-practice	Intercept	47.14***	-	0.25	-	-	-	-
	Pretest	0.65***	1.00		-	-	-	
	EC→ EC	67.10***			-	-		
Example-practice→ Practice	Intercept	46.45 ***	-	0.26	Intercept	47.50***	-	0.25
	Pretest	0.73 ***	1.01		Pretest	0.72***	1.01	
	EC→ WW	-377.30 ***			EC→ WC	-352.14 **		
	Intercept	6.11***	-	0.19	-	-	-	-

	Pretest	0.67***	1.00		-	-	-	
	EC→ CC	-57.51*			-	-		
Practice-example→ example-practice	Intercept	40.71***	-	0.21	Intercept	32.49***	-	0.40
	Pretest	0.69***	1.00		Pretest	0.68 ***	1.00	
	WE→ EC	-59.73**			CE→ EC	-136.66 ***		
Practice-example→ Practice-example	Intercept	49.54***	-	0.26	Intercept	31.34***	-	0.23
	Pretest	0.69***	1.00		Pretest	0.71***	1.00	
	WE→ CE	343.47 ***			CE→ CE	270.67 **		
Practice-example→ Practice	Intercept	46.71***	-	0.22	Intercept	31.26***	-	023
	Pretest	0.70***	1.00		Pretest	0.70***	1.00	
	WE→ WC	242.96 **			CE→ WW	307.78 **		
	Intercept	26.22***	-	0.25	Intercept	42.19***	-	0.21
	Pretest	0.71***	1.00		Pretest	0.70***	1.00	
	CE→ WC	386.49 ***			CE→ CW	190.64 **		
Practice→ Help	Intercept	46.17***			-	-	-	-
	Pretest	0.72***			-	-	-	
	CC→ EE	-210.77 ***			-	-		
Practice→ Example-	Intercept	42.07***	-	0.27	Intercept	26.57***	-	0.50

practice	Pretest	0.65***	1.00		Pretest	0.77***	1.01	
	WW→ EC	-64.20***			WC→EC	-147.59 ***		
	Intercept	6.75***	-	0.24	Intercept	38.04***	-	0.30
	Pretest	0.73***	1.01		Pretest	0.65***	1.00	
	CW→ EC	-83.07**			CC→EC	-86.05 ***		
Practice→ Practice-example	Intercept	31.35***	-	0.24	Intercept	5.72	-	0.40
	Pretest	0.72***	1.01		Pretest	0.85	1.04	
	WW→ CE	324.43 **			WC→ CE	673.39		
Practice→ Practice	Intercept	34.29***	-	0.23	Intercept	26.57**	-	0.26
	Pretest	0.73***	1.01		Pretest	0.77***	1.02	
	WW→ WW	237.33**			WW→ WC	369.44 ***		
	Intercept	38.67***	-	0.19	Intercept	0.18	-	0.45
	Pretest	0.76***	1.05		Pretest	0.90***	1.06	
	WW→ CC	-90.44 *			WC→ WC	721.46 ***	-	
	Intercept	8.36	-	0.38	Intercept	45.27***	-	0.23
	Pretest	0.85***	1.04		Pretest	0.59***	1.03	
	WC→ WW	580.88 ***			WC→ CW	158.89 **		
	Intercept	45.55***	-	0.20	-	-	-	-
	Pretest	0.68***	1.00		-	-		

CC→ CC	52.27*	-	-	

Table 14
The coefficients of simple linear regressions on relationships between strategy shifts and delayed performance in high prior knowledge level (college students)

		Coefficient		Coefficient
Pretest		0.68	-	-
Help→ Help	EE→EE	98.77	-	-
Example-practice→ Help	EC→EE	-87.45	-	-
Example-practice→ Example-practice	EC→ EC	70.79	-	-
Example-practice→ Practice	EC→WW	-333.65	EC→WC	-312.53
	EC→CC	-58.84	-	-
Practice-example→ Example-practice	WE→EC	-57.79	CE→EC	-136.41
Practice-example→ Practice-example	WE→CE	333.68	CE→CE	249.68
Practice-example→ Practice	WE→WC	222.20	CE→WW	286.91
	CE→WC	258.91	CE→CW	178.13
Practice→ Help	CC→EE	-183.32		
Practice→ Example-practice	WW→EC	-66.71	WC→EC	-138.01
	CW→EC	-72.28	CC→EC	-88.80
Practice→ Practice-example	WW→CE	290.65	WC→CE	547.27

Practice→ Practice	WW→ WW	204.13	WW→ WC	299,75
	WW→ CC	-51.85	WC→ WW	461.86
	WC→WC	568.03	WC→CW	189.88
	CC→CC	51.30		

Table 15

The correctness on the next attempt after using strategies to learn from errors in different levels of topic difficulty (6th students)

Strategy		Easy topics		Difficult topics	
		Mean of correctness percentage	S. D. of correctness percentage	Mean of correctness percentage	S. D. of correctness percentage
Help	EE	0.30	0.25	0.20	0.13
Example-practice	EW	0.22	0.12	0.15	0.08
	EC	0.63	0.16	0.54	0.17
Practice-example	WE	0.42	0.17	0.27	0.11
	CE	0.38	0.33	0.32	0.31
Practice	WW	0.07	0.07	0.05	0.05
	WC	0.60	0.19	0.44	0.17
	CW	0.44	0.20	0.32	0.15
	CC	0.82	0.09	0.73	0.14

Table 16

The correctness on the next attempt after using strategies to learn from errors in different levels of topic difficulty (college students)

Strategy		Easy topic		Difficult topic	
		Mean of correctness percentage	S. D. of correctness percentage	Mean of correctness percentage	S. D. of correctness percentage
Help	EE	0.46	0.29	0.43	0.26
Example-practice	EW	0.22	0.23	0.20	0.19
	EC	0.77	0.17	0.71	0.17
Practice-example	WE	0.60	0.19	0.49	0.19
	CE	0.53	0.28	0.48	0.30
Practice	WW	0.07	0.14	0.07	0.17
	WC	0.56	0.23	0.55	0.27
	CW	0.48	0.30	0.43	0.27
	CC	0.91	0.09	0.87	0.11