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Claw-Free and Generalized Bull-Free Graphs of Large Diameter are Hamiltonian

Claw-free and generalized bull-free graphs of large diameter are hamiltonian

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Abstract

A generalized (i,j)-bull $B_{i,j}$ is a graph obtained by identifying each of some two distinct vertices of a triangle with an endvertex of one of two vertex-disjoint paths of lengths i,j. We prove that every 2-connected claw-free $B_{2,j}$-free graph of diameter at least $\max\{7,2j\}$ $(j \geq 2)$ is hamiltonian.

Keywords: hamiltonian graphs, forbidden subgraphs, claw-free graphs

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1 Introduction

In this paper we consider finite simple undirected graphs $G = (V(G), E(G))$ and for definitions not defined here we refer to [2].

For a set $S \subset V(G)$ we denote by $N(S)$ the *neighborhood* of S, i.e. the set of all vertices of G which have a neighbor in S. If $S = \{x\}$, we simply write $N(x)$ for $N(\{x\})$. For any subset $M \subset V(G)$, we denote $N_M(S) = N(S) \cap M$. If H is a subgraph of G, we write $N_H(S)$ for $N_{V(H)}(S)$. The induced subgraph on a set $M \subset V(G)$ will be denoted by $\langle M \rangle$.

By $\mathrm{diam}(G)$ we denote the *diameter* of G, i.e. the largest distance of a pair of vertices $x, y \in V(G)$. A path with endvertices x, y will be referred sometimes to as an xy-path. If x, z are vertices at distance $\mathrm{diam}(G)$, then any shortest xz-path will be called a *diameter path* of G.

If $H_1, \ldots, H_k (k \geq 1)$ are graphs, then a graph G is said to be $H_1, \ldots, H_k$*-free* if G contains no copy of any of the graphs $H_1, \ldots, H_k$ as an induced subgraph; the graphs $H_1, \ldots, H_k$ will be also referred to in this context as *forbidden subgraphs*. Specifically, the four-vertex star $K_{1,3}$ will be also denoted by C and called the *claw* and in this case we say that G is *claw-free*. Whenever vertices of an induced claw are listed, its *center*, (i.e. its only vertex of degree 3) is always the first vertex of the list. Further graphs that will be considered as forbidden subgraphs are shown in Fig. 1.

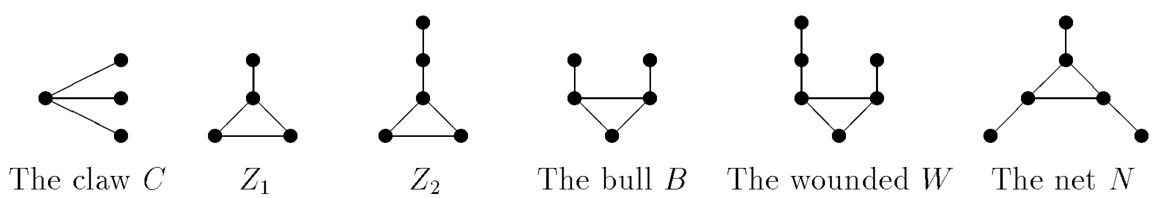

Figure 1

There are many results dealing with hamiltonian properties in classes of graphs defined in terms of forbidden induced subgraphs (see e.g. [9], [7], [10], [3], [4]). Bedrossian [1] (see also [8]) characterized all pairs X, Y of connected forbidden subgraphs implying hamiltonicity.

Theorem A [1]. *Let X and Y be connected graphs with X, $Y \neq P_3$, and let G be a 2-connected graph that is not a cycle. Then, G being XY-free implies G is hamiltonian if and only if (up to symmetry) $X = C$ and $Y = P_4, P_5, P_6, C_3, Z_1, Z_2, B, N$ or W.*

The results on hamiltonicity in CP_i-free, CZ_i-free and CN-free graphs were extended to larger classes (by characterizing the classes of nonhamiltonian exceptions) in [5] and

[6] by using the closure concept introduced in [11]. A similar extension is possible in the class of CB-free graphs by introducing the class of $CB_{i,j}$-free graphs, where by $B_{i,j}$ ($i, j \geq 1$) we denote the *generalized bull*, i.e. the graph obtained by identifying each of some two distinct vertices of a triangle with an endvertex of one of two vertex-disjoint paths of lengths i, j (see Fig. 2).

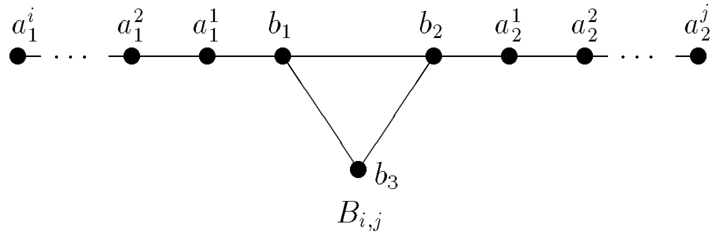

Figure 2

However, as shown in [5], the closure method is not applicable in this class since there are $CB_{i,j}$-free graphs such that their closure [11] is not $CB_{i,j}$-free.

It is easy to see that there are $CB_{i,j}$-free graphs of arbitrarily large diameter (a simple example can be obtained by taking $d+1$ vertex-disjoint cliques $K_0, K_1, \ldots, K_d$ and by adding all of the edges between consecutive cliques, namely $\{xy|\ x \in K_i, y \in K_{i+1}, i = 0, 1, \ldots, d-1\}$).

In the main result of this paper we show that, for any $j \geq 2$, all 2-connected non-hamiltonian $CB_{2,j}$-free graphs have small diameter.

2 Main result

Before we prove the main result of the paper, Theorem 2, we first make some preliminary observations on shortest paths and their neighborhoods.

Let G be a claw-free graph, let $x, y \in V(G)$ and let $P:\ x = v_0 v_1 v_2 \ldots v_k = y$ ($k \geq 3$) be a shortest xy-path in G. Let $z \in V(G) \setminus V(P)$.

1. If $|N_P(z)| = 1$, then, since G is claw-free, z is adjacent to x or to y.
2. If $|N_P(z)| \geq 2$ and $\{v_i, v_j\} \subset N_P(z)$, then, since P is a shortest path, $|i - j| \leq 2$.
3. By (1) and (2), $|N_P(z)| \leq 3$ for every vertex $z \in V(G) \setminus V(P)$. Moreover, if $2 \leq |N_P(z)| \leq 3$, then the vertices of $N_P(z)$ are consecutive on P.

This motivates the following notation:

$N_i := \{z \in V(G) \setminus V(P)|\ N_P(z) = \{v_{i-1}, v_i, v_{i+1}\}\}$ for $1 \leq i \leq k-1$,

$M_i := \{z \in V(G) \setminus V(P) |\ N_P(z) = \{v_{i-1}, v_i\}\}$ for $1 \leq i \leq k$,

$M_0 := \{z \in V(G) \setminus V(P) |\ N_P(z) = \{v_0\}\}$,

$M_{k+1} := \{z \in V(G) \setminus V(P) |\ N_P(z) = \{v_k\}\}$.

Thus, by (1), (2) and (3), $N(P) \cup V(P) = (\bigcup_{i=1}^{k} N_i) \cup (\bigcup_{i=0}^{k+1} M_i) \cup V(P)$.

We further denote $S = V(P) \cup N(P)$ and $R = V(G) \setminus S$.

Lemma 1. *Let $j \geq 2$, let G be a $CB_{2,j}$-free graph of diameter at least $\max\{7, 2j\}$ and let $P:\ v_0v_1v_2 \ldots v_d$ be a diameter path in G. Then*

(i) $\langle N_i \rangle$ is complete for $1 \leq i \leq d-1$ and $\langle M_j \rangle$ is complete for $0 \leq j \leq d+1$,

(ii) $M_i = \emptyset$ for $3 \leq i \leq d-2$,

(iii) $\langle N_i \cup N_{i+1} \rangle$ is complete for $1 \leq i \leq d-2$,

(iv) for every vertex $z \in R$ we have $N_P(z) = \emptyset$ and $N_S(z) \subseteq M_0 \cup M_1 \cup M_2 \cup M_{d-1} \cup M_d \cup M_{d+1}$.

Proof. (i) If some N_i or M_i is not complete, then some v_j, $j \in \{i-1, i, i+1\}$, is a center of an induced claw, a contradiction.

(ii) Suppose $M_i \neq \emptyset$ for some i, $3 \leq i \leq d-2$. Then, since $d \geq 2j$, for any vertex $x \in M_i$ we have $\langle \{v_{i-3}, v_{i-2}, v_{i-1}, x, v_i, v_{i+1}, \ldots, v_{i+j}\} \rangle \simeq B_{2,j}$ or $\langle \{v_{i-j-1}, v_{i-j}, \ldots, v_{i-1}, x, v_i, v_{i+1}, v_{i+2}\} \rangle \simeq B_{2,j}$, a contradiction.

(iii) Suppose $xy \notin E(G)$ for some i with $1 \leq i \leq d-2$ and two vertices $x \in N_i$, $y \in N_{i+1}$. Then $\langle \{v_{i-1}, x, v_{i+1}, y, v_{i+2}, v_{i+3}, \ldots, v_{i+2+j}\} \rangle \simeq B_{2,j}$ or $\langle \{v_{i+2}, y, v_i, x, v_{i-1}, v_{i-2}, \ldots, v_{i-1-j}\} \rangle \simeq B_{2,j}$, a contradiction.

(iv) By the definition of P and R, we have $N_P(z) = \emptyset$ for every vertex $z \in R$. Since G is claw-free, we have also $N_{N_i}(z) = \emptyset$ for $1 \leq i \leq d-1$. ■

We can now state the main result of the paper.

Theorem 2. *Let $j \geq 2$ be an integer and let G be a 2-connected $CB_{2,j}$-free graph of diameter $d \geq \max\{7, 2j\}$. Then G is hamiltonian.*

Remark. From [1] we know that every 2-connected $CB_{1,1}$-free or $CB_{2,1}$-free graph is hamiltonian. The graph in Fig. 3 indicates that there are 2-connected nonhamiltonian graphs of diameter $d = 6$ that are $CB_{2,j}$-free for any $j \geq 2$. The example in Fig. 4 shows that there are 2-connected nonhamiltonian graphs which are $CB_{2,j}$-free and have diameter $d = 2j - 1$ for any $j \geq 3$. Hence the requirement $d \geq \max\{7, 2j\}$ in Theorem 2 is sharp.

Moreover, the example in Figure 5 indicates that there are 2-connected nonhamiltonian graphs of arbitrary diameter $d \geq 3$ which are $CB_{i,j}$-free for any pair i, j such that $i \geq 3$, $j \geq i$. Hence the requirement $i = 2$ in Theorem 2 is also best possible.

It is easy to see that, in fact, each of the examples in Figures 3 – 5 yields an infinite family, since each of the vertical edges (marked in the figure by K_i) can be blown up to a clique of arbitrary order.

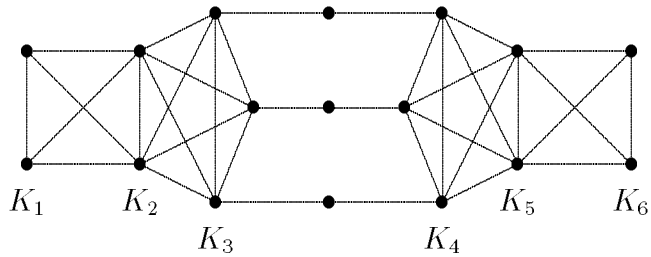

Figure 3

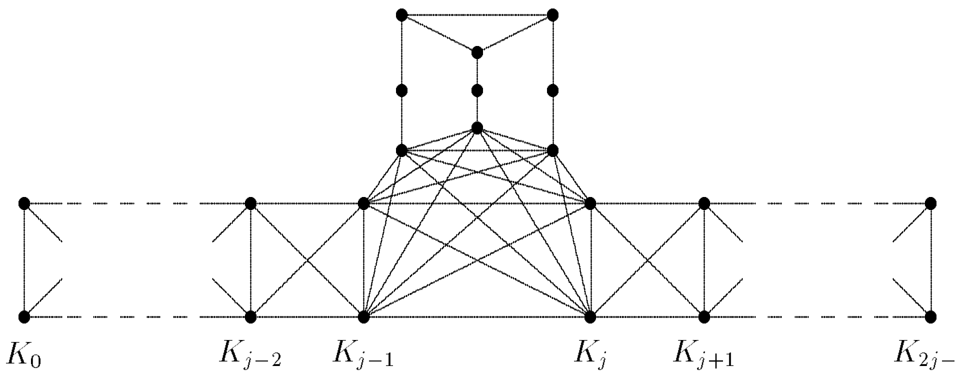

Figure 4

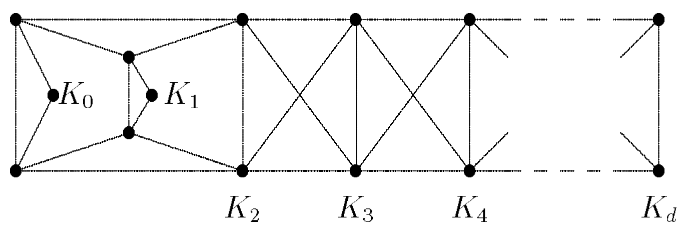

Figure 5

Proof of Theorem 2. Let G be a 2-connected $CB_{2,j}$-free graph of diameter $d \geq \max\{7, 2j\}$, $j \geq 2$, and let $P:\ v_0v_1v_2\ldots v_d$ be a diameter path in G. Let M_i, N_i, S, R be as in Lemma 1. For $i \in \{0,1,2,d-1,d,d+1\}$ further denote $M_i^* = N_R(M_i)$.

We first make the following observation concerning the structure of G "close" to the ends of P. Denote

$$S_0 = \cup_{i=0}^{2}(N_i \cup M_i \cup M_i^* \cup \{v_i\}),\ \ R_0 = V(G) \setminus (S \cup S_0)$$

and

$$S_d = \cup_{i=d-2}^{d}(N_i \cup M_{i+1} \cup M_{i+1}^* \cup \{v_i\}), \quad R_d = V(G) \setminus (S \cup S_d)$$

(where we set $N_0 = N_d = \emptyset$). Then we have the following claims.

Claim 1a. The subgraph $\langle S_0 \rangle$ satisfies one of the following:

(*i*) $N_{R_0}(S_0) = \emptyset$, $N_2 \neq \emptyset$, and for any $x_2 \in N_2$ there is a hamiltonian x_2v_2-path P_0 in $\langle S_0 \rangle$,

(*ii*) $N_{R_0}(S_0) \neq \emptyset$, $M_1^* = M_2 = M_2^* = \emptyset$, and $N_{S_0}(R_0) \subset M_0^*$.

Claim 1b. The subgraph $\langle S_d \rangle$ satisfies one of the following:

(*i*) $N_{R_d}(S_d) = \emptyset$, $N_{d-2} \neq \emptyset$, and for any $x_{d-2} \in N_{d-2}$ there is a hamiltonian $x_{d-2}v_{d-2}$-path P_d in $\langle S_d \rangle$,

(*ii*) $N_{R_d}(S_d) \neq \emptyset$, $M_d^* = M_{d-1} = M_{d-1}^* = \emptyset$, and $N_{S_d}(R_d) \subset M_{d+1}^*$.

By symmetry, it is sufficient to prove Claim 1a. We distinguish two cases.

Case 1: $M_2 \neq \emptyset$.

We first show that $M_0^* \subset M_2^*$. If $M_0 = \emptyset$, then obviously $M_0^* = \emptyset \subset M_2^*$. Hence we may assume that $M_0 \neq \emptyset$. Then $\langle M_0 \cup M_2 \rangle$ is complete, since otherwise, for some two vertices $x \in M_0$, $y \in M_2$ such that $xy \notin E(G)$ we have $\langle \{x, v_0, v_1, y, v_2, v_3, \dots, v_{2+j}\} \rangle \simeq B_{2,j}$, a contradiction (note that both $\langle M_0 \rangle$ and $\langle M_2 \rangle$ are complete by Lemma 1 (*i*)). Suppose now that $yz \notin E(G)$ for two vertices $y \in M_2$, $z \in M_0^*$. Then $\langle \{x, z, y, v_0\} \rangle \simeq C$ for a vertex $x \in M_0$, a contradiction. This implies that $yz \in E(G)$ for every $y \in M_2$, $z \in M_0^*$. But then every vertex in M_0^* has a neighbor in M_2, i.e. $M_0^* \subset M_2^*$, as required.

Next we show that $M_1^* \subset M_2^*$. We may assume that $M_1^* \neq \emptyset$. Let $z \in M_1^*$, i.e. $xz \in E(G)$ for some $x \in M_1$, and suppose that $zy \notin E(G)$ for some $y \in M_2$. If $xy \notin E(G)$, then $\langle \{z, x, v_1, y, v_2, v_3, \dots, v_{2+j}\} \rangle \simeq B_{2,j}$, and if $xy \in E(G)$, then $\langle \{x, z, y, v_0\} \rangle \simeq C$. Hence $zy \in E(G)$, implying $M_1^* \subset M_2^*$.

Thus, we conclude that $(M_0^* \cup M_1^*) \subset M_2^*$. Now, if $N_{R_0}(S_0) \neq \emptyset$, then $yz \in E(G)$ for some two vertices $y \in M_2^*$ and $z \in R_0$, but then, for a vertex $x \in M_2$, $\langle \{z, y, x, v_1, v_2, v_3, \dots, v_{2+j}\} \rangle \simeq B_{2,j}$, a contradiction. Hence $N_{R_0}(S_0) = \emptyset$.

There is also no edge from M_2^* to any of M_i, $i \geq 3$, since $M_i = \emptyset$ for $3 \leq i \leq d-2$ by Lemma 1(*ii*), and an edge from M_2^* to any of M_{d-1}, M_d, M_{d+1} yields a v_0v_d-path of length at most 6, contradicting the fact that P is a diameter path and $d \geq 7$. Consequently, $N_2 \cup \{v_2\}$ is a cutset of G. Since G is 2-connected, $N_2 \neq \emptyset$.

Summarizing, we already know that $\langle M_0 \cup M_2 \rangle$ is complete, $(M_0^* \cup M_1^*) \subset M_2^*$ and, by Lemma 1(*iii*), $\langle N_1 \cup N_2 \rangle$ is complete. Moreover, it is easy to see that $N(x) \cap M_2^*$ is complete or empty for all $x \in M_2$, and if $M_0 \neq \emptyset$, then $M_0^* = M_2^*$ (otherwise we have a claw with center in M_2). But then it is straightforward to check that in each of the possible cases (according to whether M_0, M_1, N_1 and M_2^* are empty or nonempty) there is a hamiltonian x_2v_2-path in $\langle S_0 \rangle$ for any $x_2 \in N_2$. Thus, we are in situation (i) of Claim 1a.

Case 2: $M_2 = \emptyset$.

We first consider the subcase when $M_0^* = M_1^* = \emptyset$. This immediately implies that $N_{R_0}(S_0) = \emptyset$ and, since G is 2-connected, $N_2 \neq \emptyset$. Moreover, if $M_0 \neq \emptyset$, then, since v_0 cannot be a cutvertex, there is an edge xy with $x \in M_0$ and $y \in M_1 \cup N_1$. In all these cases, it is easy to find a hamiltonian x_2v_2-path in $\langle S_0 \rangle$ for any $x_2 \in N_2$, i.e. we are again in situation (i) of Claim 1a.

Hence we suppose that $M_0^* \cup M_1^* \neq \emptyset$. Now, if $M_1^* \neq \emptyset$, then for any vertex $v \in M_1^*$, any shortest vv_d-path through M_1 has length $d+1$ (note that there is no path $vu_1u_2v_3 \dots v_d$ of length d with $u_1 \in M_1$ and $u_2 \in N_2$, since otherwise we have $\langle \{u_1, v, v_0, u_2\} \rangle \simeq C$, a contradiction). Since G has diameter d, there must be another vv_d-path P' of length at most d. Let w be the successor of v on P'. If $w \in M_{d-1} \cup M_d \cup M_{d+1}$, then we get a v_0v_d-path of length at most 5; hence $w \in R_0$. But then, for a vertex $x \in M_1$, $\langle \{w, v, x, v_0, v_1, \dots, v_{1+j}\} \rangle \simeq B_{2,j}$, a contradiction. Hence $M_1^* = \emptyset$, implying $M_0^* \neq \emptyset$.

Summarizing, we now have $M_2 = \emptyset$, $M_1^* = \emptyset$ and $M_0^* \neq \emptyset$. By the definition of R_0 and M_i^* $(i = 0, 1, 2)$, there is no edge between R_0 and $M_0 \cup M_1 \cup M_2$, which implies that $N_{S_0}(R_0) \subset M_0^*$ (if nonempty). Thus, if $N_{R_0}(S_0) \neq \emptyset$, we are in situation (ii) of Claim 1a. Hence finally suppose that $N_{R_0}(S_0) = \emptyset$. Then $N(M_0^*) \subset M_0^* \cup M_0 \cup M_{d-1} \cup M_d \cup M_{d+1}$. If $N_{S_d}(M_0^*) \neq \emptyset$, we obtain a v_0v_d-path of length $\ell < 7$, a contradiction. Hence $N(M_0^*) \subset M_0^* \cup M_0$, but then any vertex $x \in M_0^*$ is at distance at least $d+2$ from v_d, contradicting the fact that P is a diameter path. Hence the claim follows. □

Suppose now that S_0 satisfies (i) of Claim 1a and S_d satisfies (i) of Claim 1b. Then every $\{v_i\} \cup N_i$ is a cutset of G, and since G is 2-connected, $N_i \neq \emptyset$ for $3 \leq i \leq d-3$. Let P_i be a hamiltonian path in $\langle N_i \rangle$, $3 \leq i \leq d-3$. Then $x_2P_0v_2P_3P_4 \dots P_{d-3}x_{d-2}P_dv_{d-2}v_{d-3} \dots v_3x_2$ is a hamiltonian cycle in G.

By symmetry, it remains to consider the case when $\langle S_0 \rangle$ satisfies (ii) of Claim 1a. Let $x \in M_0$. If $xy \in E(G)$ for some $y \in N_1$, then $\langle \{z, x, y, v_1, v_2, v_3, \dots, v_{2+j}\} \rangle \simeq B_{2,j}$ for a vertex $z \in M_0^*$, a contradiction. Hence $N_{N_1}(M_0) = \emptyset$. If $N(x) \cap (M_{d-1} \cup M_d \cup M_{d+1}) \neq \emptyset$, we get a v_0v_d-path of length at most $d-1$. Since $M_i = \emptyset$ for $2 \leq i \leq d-2$, there is no xv_d-path in $\langle S \rangle$ of length at most d. Hence there is a shortest v_dx-path P' of length ℓ such that $d-1 \leq \ell \leq d$ and $V(P') \cap R \neq \emptyset$. This immediately implies that $N_{R_d}(S_d) \neq \emptyset$, i.e. $\langle S_d \rangle$ satisfies (ii) of Claim 1b (specifically, the successor of v_d on P' is in M_{d+1}).

Let $v_d, v_{d+1}, \dots, v_{d+\ell} = x$ be the vertices of P'. If $\ell = d$, denote by P'' the path $v_{d+1}v_{d+2} \dots v_{2d}v_0$; otherwise set $P'' = P'$. It is apparent that P'' is also a diameter path.

Denote by $\vec{C}$ the cycle (with an orientation) $v_0v_1\dots v_dv_{d+1}\dots v_{2d}(v_0)$ (of length $2d+1$ or $2d$, respectively). We show that $\vec{C}$ has no chord.

Since P, P' and P'' are shortest paths, there is no chord xy with $x,y \in V(P)$ or $x,y \in V(P'')$. By Lemma 1(ii) (applied on P, P' and P''), and since all the paths P, P', P'' satisfy (ii) of Claim 1a, 1b, the only possible chords in $\vec{C}$ are the edges $v_{d-1}v_{d+1}$ and xv_1. But, if e.g. $v_{d-1}v_{d+1} \in E(G)$, then $v_{d+1} \in M_d$, implying $v_{d+2} \in M_d^*$, which contradicts Claim 1b (ii). Hence $v_{d-1}v_{d+1} \notin E(G)$ and, by symmetry, $xv_1 \notin E(G)$.

Now we observe that, for any $x \in V(G)\setminus V(\vec{C})$, $|N_{\vec{C}}(x)| \leq 3$. Immediately $|N_{\vec{C}}(x)| \leq 4$, since G is claw-free and $\vec{C}$ is chordless. If $|N_{\vec{C}}(x)| = 4$, then x has neighbors on both P and P''. Since $M_i = \emptyset$ for $2 \leq i \leq d-1$ and no vertex in any N_i can have a neighbor outside S, the only possibility (up to a symmetry) is $N_{\vec{C}}(x) = \{v_{d-2}, v_{d-1}, v_d, v_{d+1}\}$, but then $\langle\{v_{d+2}, v_{d+1}, x, v_{d-1}, v_{d-2}, \dots, v_{d-2-j}\}\rangle \simeq B_{2,j}$, a contradiction. Hence for every $x \in V(G)\setminus V(\vec{C})$ with $N_{\vec{C}}(x) \neq \emptyset$ we have $|N_{\vec{C}}(x)| \leq 3$ and, since $\vec{C}$ is chordless and G is claw-free, $|N_{\vec{C}}(x)| \geq 2$. We can thus define analogously as before:

$$N_i^C := \{z \in V(G)\setminus V(\vec{C}) |\ zv_{i-1}, zv_i, zv_{i+1} \in E(G)\},$$

$$M_i^C := \{z \in V(G)\setminus V(\vec{C}) |\ zv_{i-1}, zv_i \in E(G)\}$$

for $1 \leq i \leq |V(\vec{C})|$ (indices are considered modulo $|V(\vec{C})|$).

By Lemma 1(ii) (applied on P, P' and P''), and by Claim 1a, 1b(ii), we have $M_i = \emptyset$ for $2 \leq i \leq d-1$ and $d+2 \leq i \leq 2d-1$ (and also $i = 2d$ if $x = v_{2d}$). But this and the fact that $\vec{C}$ has no chords implies, together with Lemma 1(iv), that, for $t = \lceil \frac{d}{2} \rceil$, the path $P''' :\ v_tv_{t+1}\dots v_{t+d}$ is a shortest v_tv_{t+d}-path in G. Since P''' has length d, P''' is a diameter path, implying that, by Lemma 1 (iv) (applied to P'''), $M_d = M_{d+1} = \emptyset$. By symmetry, we also have $M_{2d-1} = M_{2d} = M_1 = \emptyset$. But then, by Lemma 1 ($iv$), $V(G) = \cup_{i=0}^{|V(\vec{C})|}(\{v_i\} \cup N_i)$. Let P_i be a hamiltonian path in $\langle N_i \rangle$, $i = 0, \dots, |V(\vec{C})|$. Then $v_0P_0v_1P_1\dots v_{2d-1}P_{2d-1}v_{2d}(P_{2d}v_0)$ is a hamiltonian cycle in G. ■

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