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Degree Sums, k-Factors and Hamilton Cycles in Graphs

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Degree Sums, k-Factors and Hamilton Cycles in Graphs

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Abstract. We prove (a generalization of) the following conjecture of R. Häggkvist: Let G be a 2-connected graph on n vertices where every pair of nonadjacent vertices has degree sum at least $n - k$ and assume that G has a k-factor; then G is hamiltonian. This result is a common generalization of well-known theorems of Ore and Jackson, respectively.

1. Results

We use Bondy and Murty [5] for terminology and notation not defined here and consider finite, simple graphs only.

In Häggkvist [7] the following conjecture, among many others, appears.

Conjecture 1. *Let G be a 2-connected graph on n vertices where every pair of non-adjacent vertices has degree sum at least $n - k$ and assume furthermore that G has a k-factor. Then G is hamiltonian.*

The main goal of this paper it to show that Conjecture 1 is true. In fact, we will prove a more general result. For a graph G and integer $k \geq 1$, define $\sigma_k(G)$ by

$$\sigma_k(G) = \min\left\{\sum_{v \in S} d_G(v) \mid S \subseteq V(G) \text{ is an independent set of size } k\right\}$$

Now we can state our main result, the proof of which will be given in Section 2.

Theorem 2. *Let G be a 2-connected graph on n vertices that contains a k-factor and satisfies $\sigma_3(G) \geq \frac{3}{2}(n - k)$. Then either G is hamiltonian or $k = 2$ and $G \in \mathscr{F}_6$.*

Here $\mathscr{F}_6$ is a family of six graphs defined as follows. Let

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$$\mathscr{F}_6^- = \{K_1 + K_1 + K_3, K_1 + K_1 + K_4, K_1 + K_2 + K_3\},$$

where "+" denotes the disjoint union of two graphs. Then $\mathscr{F}_6$ is the family of all graphs that can be obtained as the join of K_2 or $\overline{K_2}$ and a graph in $\mathscr{F}_6^-$.

From Theorem 2 it is easy to deduce the truth of Conjecture 1.

Corollary 3. *Let G be a 2-connected graph on n vertices that contains a k-factor and satisfies $\sigma_2(G) \geq n - k$. Then G is hamiltonian.*

Proof. For any graph G and any three independent vertices u, v, w in $V(G)$ we have

$$d(u) + d(v) + d(w) = \tfrac{1}{2}[(d(u) + d(v)) + (d(u) + d(w)) + (d(v) + d(w))]$$
$$\geq \tfrac{1}{2} \cdot 3\sigma_2(G)$$

So for any graph G, $\sigma_3(G) \geq \frac{3}{2}\sigma_2(G)$. Hence if $\sigma_2(G) \geq n - k$, then $\sigma_3(G) \geq \frac{3}{2}(n - k)$. For $k = 2$, the graphs in $\mathscr{F}_6$ do not satisfy $\sigma_2 \geq n - 2$. Hence Corollary 3 follows from Theorem 2. □

Corollary 3 is best possible. This is shown by the graphs $G_{k,l} = K_l \vee (K_{k+1} + lK_1)$ ("$\vee$" denotes the join of two graphs). For any k, l with $l \geq k \geq 0$ and $l \geq 2$, $G_{k,l}$ is a 2-connected graph on $n = 2l + k + 1$ vertices that contains a k-factor, has $\sigma_2(G_{k,l}) = 2l = n - k - 1$ and does not contain a Hamilton cycle. The Petersen graph P is another example showing that Corollary 3 is best possible. (P has 10 vertices, contains a 3-factor and $\sigma_2(P) = 6 = 10 - 3 - 1$.) Theorem 2 is almost best possible. At the end of this section we state a best possible improvement.

Corollary 3 is a common generalization of two well-known results in hamiltonian graph theory. Every graph contains a 0-factor, so substituting $k = 0$ in Corollary 3 gives Ore's Theorem.

Theorem 4 (Ore [9]). *If G is a graph on $n \geq 3$ vertices that satisfies $\sigma_2(G) \geq n$, then G is hamiltonian.*

If G is a k-regular graph, then obviously it contains a k-factor and satisfies $\sigma_2(G) \geq 2k$. So, if G is a k-regular graph on n vertices with $n \leq 3k$, then $\sigma_2(G) \geq 2k \geq n - k$. This shows that Corollary 3 generalizes the following result.

Theorem 5 (Jackson [8]). *A 2-connected, k-regular graph on at most 3k vertices is hamiltonian.*

It is possible to improve Theorem 2 to the following result. The proof of this result, which we will omit, uses the same techniques that will be used in Section 2.

Theorem 6. *Let G be a 2-connected graph on n vertices that contains a k-factor and satisfies $\sigma_3(G) > \frac{3}{2}(n - k - 1)$. Then G is hamiltonian, or G is a spanning subgraph of a graph in $\mathscr{F}_7$ (if $k = 2$), a spanning subgraph of a graph in $\mathscr{H}_k$ (if $k \geq 2$), or a spanning subgraph of a graph in $\mathscr{J}_k$ (if $k \geq 1$).*

Here $\mathscr{F}_7$, $\mathscr{H}_k$ and $\mathscr{J}_k$ are families of graphs defined as follows. Let

$$\mathscr{F}_7^- = \{K_1 + K_1 + K_m \mid m \in \{3,4,5,6\}\} \cup \{K_1 + K_2 + K_m \mid m \in \{3,4,5\}\}$$

$$\mathscr{H}_k^- = \{K_{k-1} + K_{k+1} + K_{k+1}, K_{k-1} + K_{k+1} + K_{k+2}\} \qquad (k \geq 2)$$

$$\mathscr{J}_k^- = \{K_k + K_k + K_{k+1}, K_k + K_k + K_{k+2}, K_k + K_{k+1} + K_{k+1}\} \qquad (k \geq 1)$$

Then $\mathscr{F}_7$, $\mathscr{H}_k$, $\mathscr{J}_k$ are the families of all graphs that can be obtained as the join of K_2 and a graph in $\mathscr{F}_7^-$, $\mathscr{H}_k^-$, $\mathscr{J}_k^-$, respectively.

Note that all exceptional graphs in Theorem 6 have connectivity 2. The bound $\sigma_3(G) > \frac{3}{2}(n-k-1)$ in Theorem 6 cannot be lowered without introducing exceptional graphs of arbitrary connectivity for each $k \geq 1$. This is shown by the graphs $G_{k,l}$ we introduced earlier. For any k, l with $l \geq k \geq 0$ and $l \geq 3$, $G_{k,l}$ is a graph on $n = 2l + k + 1$ vertices that contains a k-factor, has $\sigma_3(G_{k,l}) = 3l = \frac{3}{2}(n-k-1)$ and does not contain a Hamilton cycle. And also the Petersen graph shows that lowering the bound on $\sigma_3(G)$ in Theorem 6 is not possible (for $k = 3$).

Theorem 6 not only generalizes the previous results in this section, but it also generalizes the case $m = 2$ of the following result. (It is easy to show that the exceptional graphs in Theorem 6 do not satisfy $\sigma_3 > \frac{3}{2}(n-1)$.)

Theorem 7 (Bondy [3]). *Let G be an m-connected graph on n vertices such that $\sigma_{m+1}(G) > \frac{1}{2}(m+1)(n-1)$. Then G is hamiltonian.*

2. Proof of Theorem 2

Following Chvátal [6], we call a graph G **1-tough** if $\omega(G-S) \leq |S|$ for every subset $S \subseteq V(G)$ with $\omega(G-S) > 1$, where $\omega(H)$ denotes the number of components of a graph H. For $A, B \subseteq V(G)$ we denote the number of edges with one end vertex in A and the other in B by $\varepsilon_G(A, B)$, the edges with both ends in $A \cap B$ being counted twice.

The following lemma is a first but essential step in the proof of Theorem 2.

Lemma 8. *Let G be a 2-connected graph on n vertices that contains a k-factor and satisfies $\sigma_3(G) \geq \frac{3}{2}(n-k)$. Then either G is 1-tough or $k = 2$ and $G \in \mathscr{F}_6$.*

Proof. Suppose G is a 2-connected graph on n vertices that contains a k-factor and satisfies $\sigma_3(G) \geq \frac{3}{2}(n-k)$. Let F be a k-factor in G. Assume that G is not 1-tough, hence there exists a vertex cut $S \subseteq V(G)$ with $\omega(G-S) > |S|$. Define $s = |S|$ and $t = \omega(G-S)$; hence $t \geq s+1$. G is 2-connected, so $s \geq 2$. Let $C_1, C_2, \ldots, C_t$ be the components of $G-S$ and let c_i be the number of vertices in component C_i $(1 \leq i \leq t)$, where we assume $c_1 \leq c_2 \leq \cdots \leq c_t$. First we make some observations.

A vertex $v \in V(G)$ has k neighbors in F, hence it has at least k neighbors in G. All neighbors of a vertex $v \in V(C_i)$ lie in $(V(C_i) - \{v\}) \cup S$, implying that $c_i - 1 + s \geq k$. Thus

$$c_i \geq k - s + 1, \qquad (1 \leq i \leq t) \tag{1}$$

Since F is a k-factor, $\varepsilon_F(V(C_i), V(G)) = kc_i$. Also, $\varepsilon_F(V(C_i), V(C_i)) \le (c_i - 1)c_i$, so

$$\begin{aligned}\varepsilon_F(V(C_i), V(G) - V(C_i)) &= \varepsilon_F(V(C_i), V(G)) - \varepsilon_F(V(C_i), V(C_i)) \\ &\ge kc_i - (c_i - 1)c_i \qquad (1 \le i \le t)\end{aligned} \tag{2}$$

Hence, whenever $c_i \le k$,

$$\varepsilon_F(V(C_i), V(G) - V(C_i)) \ge kc_i - (c_i - 1)c_i = k + (k - c_i)(c_i - 1) \ge k \tag{3}$$

Suppose that $c_i \le k$ for $1 \le i \le m$. Then from (3) we have

$$\begin{aligned}km &\le \sum_{i=1}^{m} \varepsilon_F(V(C_i), V(G) - V(C_i)) = \sum_{i=1}^{m} \varepsilon_F(V(C_i), S) \\ &= \varepsilon_F\left(\bigcup_{i=1}^{m} V(C_i), S\right) \le \varepsilon_F(V(G) - S, S) \le ks.\end{aligned} \tag{4}$$

From (4) it follows that $m \le s$, whence

$$c_i \ge k + 1, \qquad \text{for } s + 1 \le i \le t. \tag{5}$$

Let $v_i \in V(C_i)$, $1 \le i \le 3$. Then $d(v_i) \le c_i - 1 + s$. We distinguish two cases, depending on the value of s.

Case 1. $s \ge 3$
In this case we have $\frac{3}{2}(n - k) \le \sigma_3(G) \le d(v_1) + d(v_2) + d(v_3) \le 3s + c_1 + c_2 + c_3 - 3$, which gives

$$n \le k + 2s + \tfrac{2}{3}(c_1 + c_2 + c_3 - 3) \tag{6}$$

By definition of c_i we have $c_i \ge c_3$ for $3 \le i \le s$, and by (5) we have $c_i \ge k + 1$ for $s + 1 \le i \le t$. Together with $t \ge s + 1$ this gives

$$n \ge s + c_1 + c_2 + (s - 2)c_3 + (t - s)(k + 1) \ge s + c_1 + c_2 + (s - 2)c_3 + k + 1 \tag{7}$$

Combining (6) and (7) we obtain $s + c_1 + c_2 + (s - 2)c_3 + k + 1 \le k + 2s + \frac{2}{3}(c_1 + c_2 + c_3 - 3)$, which is equivalent to $(3s - 8)(c_3 - 1) + c_1 + c_2 + 1 \le 0$. But we assumed $s \ge 3$, so $(3s - 8)(c_3 - 1) + c_1 + c_2 + 1 \ge 1$, and we obtain a contradiction in this case.

Case 2. $s = 2$
In this case we have $d(v_i) \le c_i - 1 + s = c_i + 1$, hence

$$\tfrac{3}{2}(n - k) \le \sigma_3(G) \le c_1 + c_2 + c_3 + 3 \le n + 1 \tag{8}$$

By (1) and (5) we know $c_1, c_2 \ge k - 1$ and $c_3, \ldots, c_t \ge k + 1$.
We distinguish three subcases, depending on the values of c_1 and c_2.

Case 2.1. $c_1 = c_2 = k - 1$
By (2) we have, for $i = 1, 2$,

$$\varepsilon_F(V(C_i), S) = \varepsilon_F(V(C_i), V(G) - V(C_i)) \ge kc_i - (c_i - 1)c_i = 2(k - 1) \tag{9}$$

This gives $2 \cdot 2(k-1) \le \varepsilon_F(S, V(G)-S) \le 2k$, hence $k \le 2$. But since $k-1 = c_1 \ge 1$, we have $k = 2$. By (8) we have $\frac{3}{2}(n-2) \le n+1$, which gives $n \le 8$. We conclude that G is a spanning subgraph of $K_2 \vee (K_1 + K_1 + K_3)$ or $K_2 \vee (K_1 + K_1 + K_4)$. It is easy to check that the only spanning subgraphs of those two graphs that satisfy $\sigma_3 \ge \frac{3}{2}(n-2)$ are the graphs themselves and the graphs $\overline{K_2} \vee (K_1 + K_1 + K_3)$ and $\overline{K_2} \vee (K_1 + K_1 + K_4)$.

Case 2.2. $c_1 = k-1$, $c_2 = k$
By (2) we have

$$\begin{aligned} \varepsilon_F(V(C_1), S) &\ge kc_1 - (c_1 - 1)c_1 = 2(k-1) \\ \varepsilon_F(V(C_2), S) &\ge kc_2 - (c_2 - 1)c_2 = k \end{aligned} \tag{10}$$

which gives $k + 2(k-1) \le \varepsilon_F(S, V(G)-S) \le 2k$, hence $k \le 2$. As in Case 2.1, it follows that $k = 2$ and $n \le 8$. We conclude that G is a spanning subgraph of $K_2 \vee (K_1 + K_2 + K_3)$. And again, the only spanning subgraphs of this graph that have $\sigma_3 \ge \frac{3}{2}(n-2)$ are the graph itself and the graph $\overline{K_2} \vee (K_1 + K_2 + K_3)$.

Case 2.3. $c_1 + c_2 \ge 2k$
In this case we have $n \ge 2k + k + 1 + 2 = 3k + 3$. By (8) we know $\frac{3}{2}(n-k) \le n+1$, which is equivalent to $n \le 3k+2$. Combining these two inequalities gives an easy contradiction in this last case. □

For the remainder of this section we assume that G is a nonhamiltonian, 2-connected graph on n vertices that contains a k-factor F, satisfies $\sigma_3(G) \ge \frac{3}{2}(n-k)$, and is not one of the graphs in $\mathscr{F}_6$. Then we know by Lemma 8 that G is 1-tough. Moreover, if $k \le \frac{1}{3}n$, then $\sigma_3(G) \ge \frac{3}{2}(n - \frac{1}{3}n) = n$, and if $k \ge \frac{1}{3}n$, then $\sigma_3(G) \ge \sigma_3(F) = 3k \ge n$. Hence $\sigma_3(G) \ge n$ in both cases. This means that we can use the following lemma. The first part of Lemma 9 is Bauer, Veldman, Morgana and Schmeichel [2, Theorem 5]; the second part is implicit in the proof of [2, Theorem 9] (the full lemma appears as Lemma 3 in Bauer, Broersma and Veldman [1]).

Lemma 9 [1,2]. *Let G be a 1-tough graph on $n \ge 3$ vertices with $\sigma_3(G) \ge n$. Then every longest cycle of G has the property that $V(G) - V(C)$ is an independent set. Moreover, if G is nonhamiltonian, then G contains a longest cycle C such that* $\max\{d(v) | v \in V(G) - V(C)\} \ge \frac{1}{3}\sigma_3(G)$.

By Lemma 9 we can choose a longest cycle C in G and a vertex $a \in V(G) - V(C)$ such that $N(a) \subseteq V(C)$ and $d_G(a) \ge \frac{1}{3}\sigma_3(G)$. We choose an orientation $\vec{C}$ of C. If $u \in V(C)$, then u^+ denotes the successor of u on $\vec{C}$ and u^- denotes its predecessor. If $A \subseteq V(C)$, then $A^+ = \{v^+ | v \in A\}$ and $A^- = \{v^- | v \in A\}$. For $u, v \in V(C)$, $u\vec{C}v$ denotes the set of consecutive vertices of C from u to v in the direction specified by $\vec{C}$.

In the remainder of our proof we use several ideas of the proof of the main result in Bondy and Kouider [4].

Set $Y_0 = \{a\}$ and define, for $i \ge 1$,

$$X_i = N(Y_{i-1}), \qquad Y_i = \{a\} \cup \{c \in V(C) | c^-, c^+ \in X_i\}$$

Then $N(a) = X_1 \subseteq X_2 \subseteq X_3 \subseteq \cdots$ and $\{a\} = Y_0 \subseteq Y_1 \subseteq Y_2 \subseteq \cdots$. Set $X = \bigcup_i X_i$ and $Y = \bigcup_i Y_i$. Since C is a longest cycle in G and there exists no cycle C' with the same length as C satisfying $\omega(G - V(C')) < \omega(G - V(C))$, we can use the "Hopping Lemma" from Woodall [10].

Lemma 10 (Hopping Lemma, [10, Lemma 12.3]). *Let C, X and Y be defined as above. Then X and Y have the following properties.*

(a) $N(Y) = X \subseteq V(C)$
(b) $X \cap X^+ = \varnothing$
(c) $X \cap Y = \varnothing$

Set $x = |X|$ and $y = |Y|$ and define $Z^+ = X^+ - Y$, $Z^- = X^- - Y$ and $Z = Z^+ \cup Z^-$. Then, using Lemma 10, $|Z^+| = |Z^-| = x - y + 1$.

The subgraph $C - X$ consists of segments of the cycle C. There are two types of segments: segments consisting of isolated vertices (the vertices in $Y - \{a\}$), and segments consisting of two or more vertices. The latter segments can be considered as paths with one end vertex in Z^+ and the other end vertex in Z^-. We denote these "long" segments by $C_0, \ldots, C_{x-y}$, the element of $V(C_i)$ in Z^+ by p_i, and the element of $V(C_i)$ in Z^- by q_i. Set, for $0 \le i \le x - y$, $S_i = V(C_i)$, $s_i = |S_i|$ and define $S = \bigcup_i S_i$, $R = V(G) - (Y \cup X \cup S)$ and $r = |R|$.

We will use the following two results.

Lemma 11 (Jackson [8, Corollary 1]). *Let C, Z^+, Z^- and R be defined as above. Then the following hold.*

(a) *Z^+ and Z^- are independent sets.*
(b) *If $u \in Z^+$ and $v \in Z^-$, then there exist no $x, y \in v^{++}\vec{C}u^{--}$ such that $x \in N(u)$, $y \in N(v)$ and $x = y^-$ or $x = y^+$.*
(c) *If $u, v \in Z^+$, $u \neq v$, then there exists no $x \in u^{++}\vec{C}v^-$ such that $x \in N(u)$ and $x^- \in N(v)$.*
(d) *If $u, v \in Z^-$, $u \neq v$, then there exists no $x \in u^{++}\vec{C}v^-$ such that $x \in N(u)$ and $x^- \in N(v)$.*
(e) *Every vertex of R has at most one vertex of Z^+ and at most one vertex of Z^- as a neighbor.*

Lemma 12 (Jackson [8, Lemma 2]). *Let S_i, p_i and q_i be defined as above. Then for all $i \neq j$ we have $\varepsilon_G(\{p_i, q_i\}, S_j) \le s_j - 1$.*

Recall that F is a k-factor in G. We will derive a lower and an upper bound for $\varepsilon_F(S, X)$.

First we derive a lower bound. It is obvious that for all i,

$$\varepsilon_G(\{p_i, q_i\}, S_i) \le 2(s_i - 1) \tag{11}$$

Lemma 12 and (11) together give

$$\begin{aligned} \varepsilon_G(Z, S) &= \sum_i \sum_j \varepsilon_G(\{p_j, q_j\}, S_i) \le \sum_i (x - y + 2)(s_i - 1) \\ &= (x - y + 2) \sum_i (s_i - 1) = (x - y + 2)(|S| - (x - y + 1)) \end{aligned} \tag{12}$$

By definition we have $|S| = |V(G) - (X \cup Y \cup R)| = n - x - y - r$, so from (12) we obtain

$$\varepsilon_F(Z, S) \leq \varepsilon_G(Z, S) \leq (x - y + 2)(n - 2x - r - 1) \tag{13}$$

By Lemma 10(a),

$$\varepsilon_F(Z, Y) = \varepsilon_G(Z, Y) = 0 \tag{14}$$

and by Lemma 11(e),

$$\varepsilon_F(Z, R) \leq \varepsilon_G(Z, R) \leq 2r \tag{15}$$

And, since F is a k-factor,

$$\varepsilon_F(Z, V(G)) = k|Z| = 2k(x - y + 1) \tag{16}$$

Combining (13)–(16) and using $Z \subseteq S$ we obtain

$$\begin{aligned} \varepsilon_F(S, X) \geq \varepsilon_F(Z, X) &= \varepsilon_F(Z, V(G)) - \varepsilon_F(Z, Y) - \varepsilon_F(Z, S) - \varepsilon_F(Z, R) \\ &\geq 2k(x - y + 1) - (x - y + 2)(n - 2x - r - 1) - 2r \end{aligned} \tag{17}$$

Next we derive an upper bound for $\varepsilon_F(S, X)$. Since F is a k-factor, we immediately have

$$\begin{aligned} \varepsilon_F(V(G), X) &= kx \\ \varepsilon_F(R, X) &\geq 0 \\ \varepsilon_F(X, X) &\geq 0 \end{aligned} \tag{18}$$

Also, since $N_G(Y) \subseteq X$ and hence $N_F(Y) \subseteq X$,

$$\varepsilon_F(Y, X) = ky \tag{19}$$

Combining (18) and (19) gives

$$\varepsilon_F(S, X) = \varepsilon_F(V(G), X) - \varepsilon_F(Y, X) - \varepsilon_F(X, X) - \varepsilon_F(R, X) \leq kx - ky \tag{20}$$

The inequalities (17) and (20) together give

$$2k(x - y + 1) - (x - y + 2)(n - 2x - r - 1) - 2r \leq kx - ky \tag{21}$$

which is equivalent to

$$r(x - y) + (x - y + 2)(2x + k - n + 1) \leq 0 \tag{22}$$

By the definition of X and Y we know $y \leq x + 1$, so $x - y + 2 \geq 1$. Furthermore, $N(a) \subseteq X$, hence

$$x \geq d_G(a) \geq \tfrac{1}{3}\sigma_3(G) \geq \tfrac{1}{3} \cdot \tfrac{3}{2}(n - k) = \tfrac{1}{2}(n - k) \tag{23}$$

If $k = 0$, then $x \geq \frac{1}{2}n$ and we obtain a contradiction with (13). So we can assume $k \geq 1$. By (20) we have $k(x - y) \geq 0$, so $x - y \geq 0$. We conclude that

$$r(x - y) + (x - y + 2)(2x + k - n - 1) \geq 0 + 1 \cdot (2 \cdot \tfrac{1}{2}(n - k) + k - n + \cdot 1) \geq 1 \tag{24}$$

a contradiction with (22). This completes the proof of Theorem 2. □

Note. After the preparation of this paper, R. Häggkvist informed us that he also obtained a proof of Corollary 3.

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