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# A new method to analyse mosaics based on Symmetry Group theory applied to Islamic Geometric Patterns

F. Albert<sup>a</sup>, J.M. Gómis<sup>a</sup>, J. Blasco<sup>b</sup>, J.M. Valiente<sup>c</sup>, N. Aleixos<sup>a\*</sup>

<sup>a</sup> Inter-University Research Institute for Bioengineering and Human Centered Technology, Universitat Politècnica de València, 46022-Valencia, Spain, {fraalgi1, jmgomis, naleixos}@dig.upv.es

<sup>b</sup> Instituto Valenciano de Investigaciones Agrarias - IVIA, 46113-Moncada, Valencia, Spain, blasco\_josiva@gva.es

<sup>c</sup> Computer Engineering Department, Universitat Politècnica de València, 46022-Valencia, Spain, jvalient@disca.upv.es

**Abstract**— This article presents a new method for analysing mosaics based on the mathematical principles of Symmetry Groups. This method has been developed to get the understanding present in patterns by extracting the objects that form them, their lattice, and the Wallpaper Group. The main novelty of this method resides in the creation of a higher level of knowledge based on objects, which makes it possible to classify the objects, to extract their main features (Point Group, principal axes, etc.), and the relationships between them. In order to validate the method, several tests were carried out on a set of Islamic Geometric Patterns from different sources, for which the Wallpaper Group has been successfully obtained in 85% of the cases. This method can be applied to any kind of pattern that presents a Wallpaper Group. Possible applications of this computational method include pattern classification, cataloguing of ceramic coatings, creating databases of decorative patterns, creating pattern designs, pattern comparison between different cultures, tile cataloguing, and so on.

**Index Terms**—Mosaics, Symmetry Groups, Wallpaper Groups, Point Groups, image understanding, tiles, Islamic Geometric Patterns, pattern classification.

## 1 Introduction

This paper proposes a method to get the understanding generally present in any kind of pattern that presents a Wallpaper Group (WG), which describes the arrangement of a set of objects repeated from a set of isometries. Unlike other existing works, which only identify the WG (the so-called “structure”), this method obtains the objects present in the pattern, allowing them to be classified and their symmetries obtained. Moreover, the information extracted from the objects is used to obtain the isometries that constitute the WG. An important contribution is the possibility of approaching the pattern analysis in two

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\* Corresponding author:  
Tel.: +34 96 387 95 14  
Fax.: +34 96 387 74 19

33 ways: by discarding the colour of each object as a discriminating feature, which allows a different WG to  
34 be obtained when there are objects with the same shape and different colour, and second by obtaining the  
35 WG of each object class, apart from the WG of the entire pattern, thus obtaining all the WG present in the  
36 pattern. This method has been implemented through an application, using the Image Processing and Pattern  
37 Recognition techniques and the mathematical principles of Symmetry Groups, and has been tested and  
38 validated on Islamic Geometric Patterns (IGP) and a number of alternative datasets (mainly from  
39 [http://en.wikipedia.org/wiki/Wallpaper\\_group](http://en.wikipedia.org/wiki/Wallpaper_group), and other paving slabs and fabrics).

### 40 *1.1 Definitions*

41 The main terms used in this article are defined as follows:

- 42 • Region: set of neighbouring pixels in the image with similar colour, which are perceived as a unit.
- 43 • Object: geometric figure matching a region with a closed contour and a series of properties (colour,  
44 area, perimeter, etc.).
- 45 • Shape: description of the object depending on the points or parts making up the outer contour and  
46 how they are interrelated in terms of positions, orientations and relative sizes [1]. Shape is the main  
47 feature of an object and in this work shape and contour are used interchangeably.
- 48 • Object class: set of objects with the same shape, size and colour.
- 49 • Isometries: geometric transformations which, when applied to an object, do not change its shape.  
50 The isometries are: translations, rotations and reflections (symmetry axes). A glide reflection is the  
51 combination of a reflection relative to a symmetry axis and a translation in the direction of the  
52 same axis.
- 53 • Pattern: a repeated set of elements. In this work the elements are the objects, and they are  
54 duplicated and combined by means of isometries without overlap, covering a flat surface.
- 55 • Primitive Cell (PC) or Unit Lattice: the minimum part of the pattern that can generate the whole  
56 pattern by making copies and displacements. The Primitive Cell may be an object or a set of  
57 objects.
- 58 • Piece or Tile: fragment of thin ceramic material, with the outer face glazed, used to cover floors  
59 and walls.
- 60 • Islamic Geometric Patterns (IGP) or Islamic Mosaics: basic decorative elements of Islamic  
61 architecture consisting of a pattern formed by a set of small pieces with simple shapes (straight  
62 lines and arcs). In this work, for the purposes of the image processing of IGPs, the pieces are  
63 considered as objects.

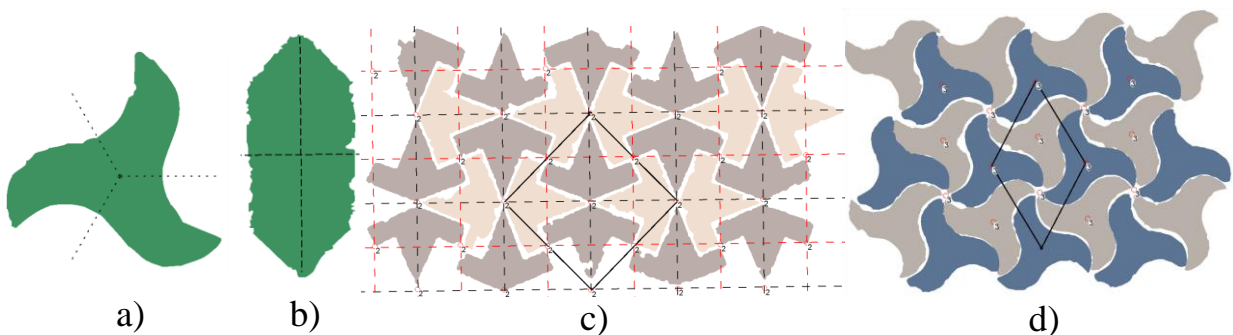
## 64 1.2 Symmetry Groups

65 A Symmetry Group (SG) is the set of all isometries  $\sigma$ , which transforms a set of objects  $S$  into itself  
 66 ( $\sigma S=S$ ). There are four types of Symmetry Groups: Point Groups, Frieze Groups, Wallpaper Groups and  
 67 Crystallographic Groups, depending on whether they have 0, 1, 2 or 3 independent translations [2] [3] [4].  
 68 This paper deals with Point Groups and Wallpaper Groups.

69 The Point Group (PG) describes the group of all isometries under which an object is invariant. The  
 70 classes of PG are the following:

- 71 • Rotational symmetry (also called  $n$ -fold rotational symmetry or cyclic symmetry): cyclic groups  
 72  $C_1, C_2, \dots, C_n$ , where  $C_n$  (of order  $n$ ) consists of all rotations about a fixed point (the centroid) by  
 73 multiples of  $360^\circ/n$  (Figure 1a Cyclic Group  $C_3$  with  $120^\circ$  rotations).
- 74 • Reflection symmetry (also called mirror symmetry or dihedral symmetry): dihedral groups  $D_1, D_2,$   
 75  $\dots, D_n$ , where  $D_n$  (of order  $2n$ ) consists of the rotations in  $C_n$  together with reflections in  $n$   
 76 symmetry axes that pass through the centroid (Figure 1b Dihedral Group  $D_2$  with  $180^\circ$  rotation and  
 77 two symmetry axes).

78 The WG describes the arrangement of a set of objects which are repeated to form a pattern, from a set of  
 79 isometries that always include two translations. There are 17 different WG [3]. The two smallest  
 80 independent translations in the pattern define the lattice. The PC is the smallest part of the pattern repeated  
 81 by lattice translations. Figure 1 shows the output of the application developed for two different objects and  
 82 two patterns. Figure 1c shows a WG CMM with glide and non-glide symmetry axes and  $180^\circ$  rotations, and  
 83 Figure 1d shows a WG P3 with  $120^\circ$  rotations and without any kind of symmetry axes. The two patterns  
 84 are shown with the Primitive Cell.



85  
 86 Figure 1. Output of the application for some samples: a) Object with Point Group  $C_3$ . The three sectors of  
 87 the object appear separated with dotted lines; b) Object with Point Group  $D_2$ . The symmetry axes are  
 88 represented with dashed lines; c) and d) Patterns with WG. The solid line is the Primitive Cell. Symmetry  
 89 axes are represented with dashed lines: red for glide and black for non-glide symmetry axes. Rotation

90 centres are represented with small circles with a number (two for 180° rotations and three for 120°  
91 rotations)

### 92 *1.3 State of the art*

93 The scientific literature contains research studies that generate or analyse IGP based on SG, and some  
94 others based on different geometric approaches. Among the SG-based works, there are many examples of  
95 pattern generation [5] [6] and the development of computer-aided design allowed the creation of software  
96 for generating patterns [7] [8], but this is straightforward, since it just consists in geometric transformations  
97 applied repetitively. As for analysing IGP, special mention should be made of the theoretical approach by  
98 [9], who roughly described the steps to be followed in pattern analysis, i.e. identifying the PC and  
99 obtaining the WG. In [10], the authors presented an application which obtains the translations, symmetry  
100 axes and centres of rotation of an image via image correlations with itself translated, reflected or rotated,  
101 respectively, to determine the PC and WG using heuristic methods. This computer application is one of the  
102 most comprehensive contributions in the area of image analysis based on SG, although precisely due to its  
103 simplicity (it does not deal with objects), no information can be obtained beyond that deduced from just the  
104 colour of pixels. Instead of correlations, [11] extracted the lattice of patterns using Distance Matching  
105 Functions (DMFs). Much like previous works, [12] [13] also obtained the PC and the WG, but using  
106 Wavelets and a Nearest Neighbour classifier. In [14], the authors used Liu's method described in [10] to  
107 obtain the PC, and the WG in order to detect defects in patterns. As they pointed out, the WG cannot be  
108 obtained if the number of defects is relevant. On the other hand, [15] analysed IGP obtaining their Point  
109 Group (for rosettes), Frieze Groups (for friezes) or Wallpaper Groups, and in order to obtain the isometries  
110 of patterns of the image itself, correlations between translations, reflections or rotations were performed.  
111 The final description of an IGP was a feature vector including the Symmetry Group of the pattern and a  
112 colour histogram of the Primitive Cell (patterns with Frieze or Wallpaper Groups) or Fold (patterns with  
113 Point Group) but did not consider objects, the only contributions being the use of rosettes, friezes and the  
114 colour histogram.

115 With respect to the non SG-based works, there are many different methods for generating patterns, for  
116 example from rosettes [16] or from two of the most common pieces used in IGP called Zohreh and Sili  
117 [17]. A review of the different methods can be found in [18]. For analysing Islamic Geometric Patterns [19]  
118 proposed a theoretical method based on two features that were defined as Minimum Number of Grids  
119 (MNG) and Lowest Geometric Shape (LGS). [20] also proposed a method for analysing Islamic Patterns,  
120 which presented a Point Group (rosettes), based on the decomposition of a square image into a quadtree  
121 structure searching for symmetries at different levels of abstraction; this method resembled the work of the

122 ancient artisans consisting in drawing the motif on a folded paper and then unfolding it.

123 Not all the research papers cited that present proposals for the classification of IGP come with software  
 124 tools, and those that do include them only operate at pixel level, but do not retrieve the objects that form  
 125 the pattern. Therefore, it is impossible to obtain their symmetries, and image cataloguing is performed  
 126 using only colour information. Hence, they are not able to find alternative WG in patterns when there are  
 127 identical objects that differ in colour, which frequently occurs. In fact, in [19], the authors criticised the  
 128 methods based on Symmetry Groups because they did not reflect the way of thinking of the artisans (how  
 129 to combine the pieces) and were a rather general approximation that failed to explore edition capabilities  
 130 (to generate new patterns). Such criticism can be applied to the methods proposed in [10] [11] [12].

131 Thus, in this paper we propose a new method for analysing images of Islamic Geometric Patterns, and  
 132 its key feature is the incorporation of a higher level of abstraction: the objects forming the pattern. This  
 133 method has led to the development of an application. The information found can be used to generate new  
 134 patterns. Earlier versions of the method have been reported in [21] [22] [23]. The main contributions of this  
 135 research are the following:

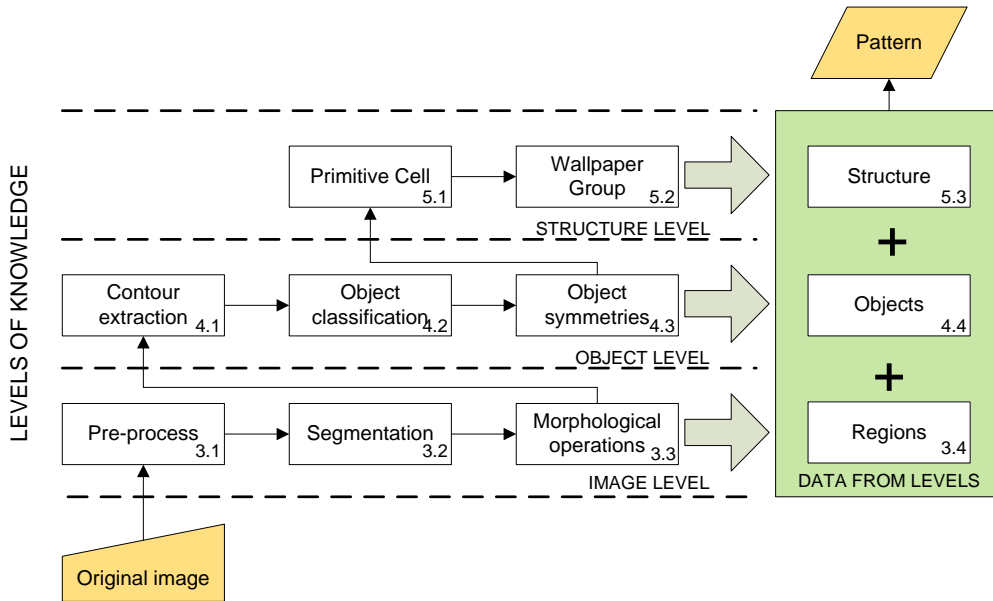
- 136 • Pattern analysis is more comprehensive: the objects are obtained and classified, and their Point  
 137 Group is also obtained.
- 138 • Obtaining the isometries, and therefore the WG, not only from the colours of the pixels, but also  
 139 from features extracted from the objects (considering colour, shape, Point Group and orientation).  
 140 Thus, the colour can be discarded when classifying objects, so if there are objects with the same  
 141 shape and different colours, patterns with more than one WG can be found.
- 142 • The WG is obtained for each class of objects. So, if there are objects with different WG, they can  
 143 be found.

144 The paper is organised as follows. In section 2 a brief description of the proposed method is presented.  
 145 Sections 3 to 5 offer a detailed description of the different levels of the method: Image Level, Object Level  
 146 and Structure Level. Section 6 presents the results of the application of the proposed method on a set of  
 147 IGP samples, the parameters of each stage and the computational cost, and the results on other sets  
 148 containing Wikipedia WG samples and some images of paving slabs from public constructions and fabrics,  
 149 in order to show the flexibility of the method. Finally, section 7 reports the conclusions and further work.

## 150 **2 Description of the Method**

151 The method is based on stages according to three levels of knowledge that they include: a) Image or  
 152 pixel-oriented level, in which the image is segmented to obtain the different regions of interest; b) Object-  
 153 oriented level, in which the regions obtained correspond to objects and their features extracted including

154 the contour and Point Group; and c) Structure Level, in which the translations, rotations and symmetry axes  
 155 existing in the pattern, that is, the structure or Wallpaper Group, are found. Figure 2 shows the scheme of  
 156 the methodology proposed. All the process that supports this methodology is entirely automatic, except the  
 157 tuning of the parameters used and the configuration of the Image Level (i.e. the segmentation method  
 158 applied in 3.2).



159

160 Figure 2. Scheme of the method including the stages and information obtained for each level. The numbers  
 161 of the sections where the stages are described are stated at the bottom right of each box

162 The initial values of the parameters have been obtained using an implementation of the optimization  
 163 algorithm Simulated Annealing described in [24], using a set of patterns with different Wallpaper Groups to  
 164 determine a set of optimal values for the parameters. The final adjustments to obtain successfully the  
 165 complete results (all classes of objects with their Point Group) require in some specific cases fine manual  
 166 adjustment after outcome. The computational cost, the parameters used and the intermediate graphical  
 167 results of the algorithm at each stage are described in section 6.2.

### 168 3 Image Level

169 This level is a pixel-oriented level, and its purpose is to extract the different regions of interest in the  
 170 image, which is achieved by means of pre-processing, segmentation and morphological operations.

#### 171 3.1 Pre-processing

172 A pre-processing stage is performed to reduce noise present in the image. This stage is performed using  
 173 two different low-pass filters: Gaussian blur or Median.

## 174 **3.2 Segmentation**

175 The next stages of the application depend greatly on the quality of the segmentation. A wide variety of  
176 image segmentation approaches have been reported in the literature [25] [26] [27].

177 Two different ways to segment the images have been implemented in this work. The first is through a  
178 clustering technique based on the Mean-Shift algorithm [28] [29], which is appropriate when the edges  
179 between regions are well defined. The second method consists in an edge-based technique, like the  
180 Watershed algorithm [30], which is more appropriate for images with weak edges between the regions  
181 because it emphasises the edges.

## 182 **3.3 Morphological operations**

183 Later, operations like Erosion, Dilation, Opening and Closing have been applied in order to remove small  
184 regions and holes or to separate linked regions [31], so the remaining regions will be considered the objects  
185 of the image. Finally, Hu moments [32] were used to calculate the properties of the objects like area,  
186 centroid, angle and size of principal axes that will be used in the following stages.

## 187 **3.4 Description of regions**

188 The result after applying the operations included in the Image Level consists of  $m$  regions. At the end of  
189 this level, an object  $O_k$  contains only the properties of its corresponding region:  $O_k, k \in [0, m-1]$  is described  
190 using a tuple  $O_k = (R^k, colour^k, p_c^k, A^k, \varphi_{min}^k, I_{min}^k, I_{max}^k)$ , where:

- 191 •  $R^k = \{p_i\}, i \in [0, n-1]$ : set of  $n$  pixels belonging to the region.
- 192 •  $colour^k$ : average colour of pixels of  $R^k$ .
- 193 •  $p_c^k = (x_c, y_c)$ : centroid of the region.
- 194 •  $A^k$ : area (number of pixels of  $R^k$ ).
- 195 •  $\varphi_{min}^k$ : angle of major axis.
- 196 •  $I_{min}^k, I_{max}^k$ : length of principal axes.

## 197 **4 Object Level**

198 The goal of this level is to complete the objects by extracting the shape (contour) of the regions found on  
199 the previous level, to classify them, and to obtain the Point Group for each class of objects.

### 200 **4.1 Contour extraction**

201 In order to extract the contour of each region, a boundary-following algorithm [33] has been used, thus the  
202 contour  $C$  is a closed sequence of 8-connected points defined as following:  $C = \{p_i\}, i \in [0, m-1]$ , where  $p_i$  are



203 the contour points and  $m$  is the number of points of such a contour.

#### 204 **4.2 Object classification**

205 Unlike industrially manufactured objects, in IGP, the objects found in the images are repeated with slight  
 206 differences between them due to fact that they are handmade. These differences can affect both the shape  
 207 and the orientation, and hence in order to obtain a classification of the objects these two features must be  
 208 taken into account. In this article, the classification methodology is based on an all-against-all comparison  
 209 process (each object is compared to all other objects) using a coarse-to-fine strategy (first a quick  
 210 comparison to discard obvious cases and later a precise comparison to detect smaller differences) in order  
 211 to optimise the computational cost. Firstly, each pair of objects is compared using the area and the length of  
 212 the principal axes. If the two objects are geometrically similar in this coarse comparison, they are  
 213 compared using a ‘fine’ process to obtain a measurement of their dissimilarity [34].

214 The works presented in [35] and [36] also perform a process of fine comparison using the Canny edge  
 215 detection algorithm to extract objects and then to obtain features like relevant points, type of sides, relative  
 216 lengths, orientation and concavity/convexity, colour information and also the topological relationships  
 217 among them. Dissimilarity was calculated by comparing one-by-one the types of sides between the relevant  
 218 points that were found, and also by using the relative information about size and orientation, in a similar  
 219 way to how [37] compared contours looking for tangencies, concentricity, parallelism, etc. The problem  
 220 with this technique is the noise typically present in the images of IGP, which makes it difficult to obtain all  
 221 of these characteristics with accuracy.

222 To deal with this problem, in this work, the fine comparison has been performed by using the contour-  
 223 based method described in [38]. In particular, by assuming the contour of the objects to be a discrete and  
 224 periodic function  $f[n] = (x_n, y_n)$ ,  $n \in [0, N-1]$ :

- 225 1. The contours were filtered by means of a discrete convolution using a 1D Gaussian filter.
- 226 2. The contours were re-sampled to 256 equidistant points because they must have the same number  
 227 of points to be compared properly.
- 228 3. The coordinates of the points in each contour were converted so as to be relative to the centroid of  
 229 its region.

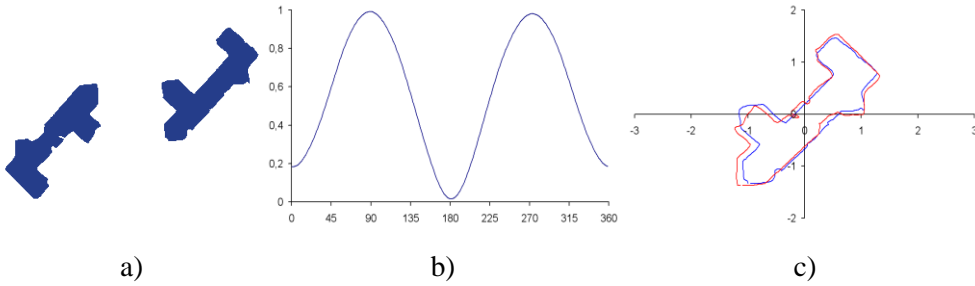
230 Then, the dissimilarity  $d(f_1, f_2)$  between the contours of two objects  $O_1$  and  $O_2$  was defined as:

$$231 \quad d(f_1, f_2) = \min_{\alpha, q} \sqrt{\frac{1}{N} \sum_{n=0}^{N-1} (f_1[n] - R_\alpha \cdot f_2[n+q])^2} \quad (1)$$

232 where  $N$  is the number of points on the two contours (256 after resampling),  $q$  is the starting point of

233 contour  $f_2$  to compare to  $f_1$ ,  $q \in [0, N-1]$ , and  $R_\alpha$  is a rotation of angle  $\alpha$ . Except in some cases, it is enough to  
 234 consider the multiples of  $30^\circ$  since in all cases the angles of rotation in Wallpaper Groups are  $180^\circ$ ,  $120^\circ$ ,  
 235  $90^\circ$  and  $60^\circ$ . The above expression is minimised with respect to  $q$  (starting from any point) and  $\alpha$   
 236 (orientation).

237 Since an object can also appear reflected in the image, the same process is repeated but taking the  
 238 second contour reflected about the horizontal axis passing through the centroid of the object  $f^R[n] = (x_n, -y_n)$ ,  
 239  $n \in [0, N-1]$ , thus obtaining the dissimilarity of the reflected objects  $d^R(f_1, f_2) = d(f_1, f_2^R)$ . Finally, those pairs of  
 240 objects in the image are considered as belonging to the same class if  $d(f_1, f_2) < u$  or  $d^R(f_1, f_2) < u$ , where  $u$  is a  
 241 user-chosen threshold in the range  $[0.1..1]$ . If  $d^R(f_1, f_2) < d(f_1, f_2)$ , the objects are reflected. See Figure 3.

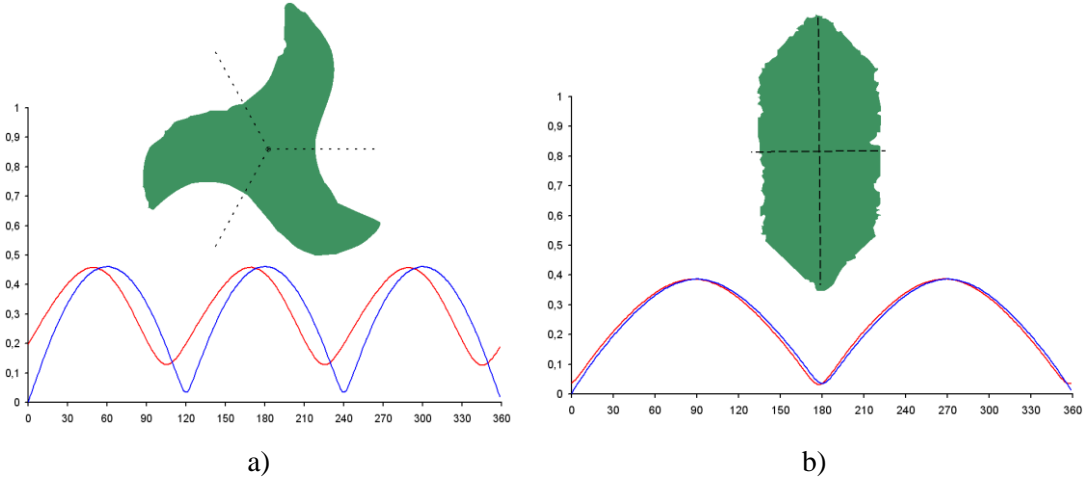


242 Figure 3. Fine comparison between a pair of objects: a) Original objects, b) Dissimilarity measurement  
 243  $d(f_1, f_2)$  for  $\alpha \in [0^\circ, 1^\circ, \dots, 359^\circ]$ , and c) The overlapped contours after rotating  $f_2$  the angle that minimises the  
 244 distance ( $180^\circ$ )

### 245 4.3 Object symmetries. Point Group

246 The fine comparison method described above is used to compute the PG of an object but, in this case, to  
 247 compare each object to itself the following aspects have been considered:

- 248 • The order of rotational symmetry  $C_n$  of an object  $O_i$  with contour  $f_i$  is the number of local minima  
 249 (under a threshold) of  $d(f_i, f_i)$ .
- 250 • The order of reflection symmetry  $D_n$  of an object  $O_i$  with contour  $f_i$  is the number of local minima  
 251 (under a threshold) of  $d^R(f_i, f_i)$ , where the angles of reflection are:  $\beta_i = (\alpha_i/2)$ ,  $i=0, 1, \dots, n-1$ , where  $\alpha_i$  is  
 252 each rotation angle which minimises  $d^R(f_i, f_i)$ . These angles are equally spaced according to the  
 253 order of the symmetry group:  $\beta_i = \beta_0 + i(180/n)$ ,  $i=0, 1, \dots, n-1$ .



254 Figure 4. Dissimilarity measurements  $d(f_i, f_i)$  (blue) and  $d^R(f_i, f_i)$  (red),  $\alpha \in [0^\circ, 1^\circ, \dots, 359^\circ]$ , for samples of  
 255 Figure 1a and Figure 1b respectively

256 The sample in Figure 4a  $d(f_i, f_i)$  has three minima below the threshold (0.05), but the minima of  $d^R(f_i, f_i)$   
 257 are always above the threshold. The minima are separated  $120^\circ$ , which means cyclic symmetry  $C_3$ . In  
 258 Figure 4b there are two local minima ( $0^\circ$  and  $180^\circ$ ) below the threshold, which means dihedral symmetry  
 259  $D_2$  with axes of  $0^\circ$  and  $90^\circ$ .

#### 260 4.4 Description of objects

261 For each object class, a representative object is randomly chosen as a relative reference, and the rotation  
 262 and/or reflection of the other objects in the class with respect to it are then stored.

263 In the same way that an object with symmetry  $C_n$  or symmetry  $D_n$  with order  $n$  has  $n$  rotations (being  
 264 invariant in all of them), there are  $n$  rotations that relate the representative of the class with any other object  
 265 in the class:  $\alpha_i = \alpha_0 + i(180/n)$ ,  $i=0, 1, \dots, n-1$ .

266 The result after applying the operations included on the Object Level is a description of each object as a  
 267 tuple  $O_k = (R^k, colour^k, p^k_c, A^k, \varphi^k_{min}, I^k_{min}, I^k_{max}, C^k, class^k, reflection^k, \alpha^k, pg^k, \beta^k)$ , where the new features  
 268 added are:

- 269 •  $C^k$ : contour (closed sequence of 8-connected points).
- 270 •  $class^k$ : class of the object.
- 271 •  $reflection^k$ : boolean value to indicate whether the object orientation is reflected with respect to the  
 272 object selected as a reference in the class.
- 273 •  $\alpha^k$ : orientation of the object with respect to the object selected as a reference in the class.
- 274 •  $pg^k$ : order  $n$  for Point Group (negative when  $D_n$  and positive when  $C_n$ ).

- $\beta^k$ : angle of one reflection axis, whether the object has reflection symmetry.

## 276 5 Structure Level

277 This is the highest level, and its purpose is to obtain the Primitive Cell and Wallpaper Group. For each pair  
 278 of objects of a class, there will be one or more isometries that relate them. If the same isometry is fulfilled  
 279 for several pairs of objects, it is an isometry of the Wallpaper Group. The isometries can be the following:

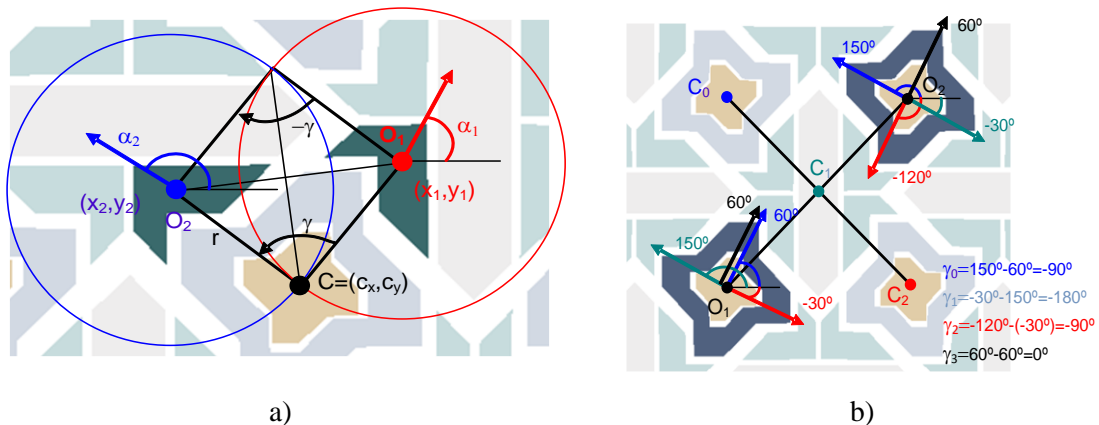
### 280 Rotation isometry

281 For each pair of objects  $O_1$  and  $O_2$  belonging to the same class, with centroids  $(x_1, y_1)$  and  $(x_2, y_2)$ ,  
 282  $reflection^1 = reflection^2$  and  $pg = C_1$  or  $D_1$ , there is a rotation of angle  $\gamma = \alpha_1 - \alpha_2$ , radius  $r$  and centre  $C$  which  
 283 relate them, being:

$$284 \quad r = \left| \frac{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} / 2}{\sin(\gamma/2)} \right| \quad (2)$$

$$285 \quad C \begin{cases} (c_x - x_1)^2 + (c_y - y_1)^2 = r^2 \\ (c_x - x_2)^2 + (c_y - y_2)^2 = r^2 \end{cases} \quad (3)$$

286 where the true centre  $C$  keeps the sign of the rotation angle, and when  $\gamma = 0^\circ$  and  $r = \infty$ , then the isometry is a  
 287 translation. Thus, for two objects of the same class with  $pg = C_n$  or  $D_n$ , the number of angles that relate both  
 288 objects through rotation should be  $n^2$ , yet the angles are repeated, so there are  $n$  rotations of angle  
 289  $\gamma = (\alpha_1 - \alpha_2) + l(360/n)$ , radius  $r_l$  and centre  $C_l$ , which relate them (see Figure 5a). In the case of Figure 5b,  
 290  $\gamma_3 = 0^\circ$  so there is a rotation of radius  $r_3 = \infty$  which corresponds to a translation.



291 Figure 5. Hand-drawn graphical information superimposed upon a WG sample to illustrate: a) Radius and  
 292 rotation centres between two objects with  $pg = C_1$ , and b) Rotation centres between two objects with  $pg = C_4$ .

293 Different colours have been used to represent centroids and orientations

### 294 Translation isometry

295 For two objects  $O_1$  and  $O_2$  of the same class, with centroids  $(x_1, y_1)$  and  $(x_2, y_2)$ ,  $reflection^1 = reflection^2$  and

296  $pg=C_n$  or  $D_n$ , there is a translation which relates them if there is a rotation angle  $\gamma=(\alpha_1-\alpha_2)+l(360/n)=0^\circ$ .

297 The translation vector is the difference between the centroids of the objects:  $D_{1-2}=(x_1-x_2, y_1-y_2)$ .

### 298 **Glide and non-glide reflection isometries**

299 For two objects  $O_1$  and  $O_2$  of the same class, with centroids  $(x_1, y_1)$  and  $(x_2, y_2)$ ,  $pg=C_n$  and  
 300  $reflection^1 \neq reflection^2$ , there are  $n$  symmetry axes of angle  $\delta_l$ , a point  $E$  (which is the midpoint between the  
 301 centroids and is the same for all axes) and a glide  $glide_l$  which relate them, being:

$$302 \quad \delta_l = (\alpha_1 + \alpha_2)/2 + l(180/n), \quad l \in [0, n-1] \quad (4)$$

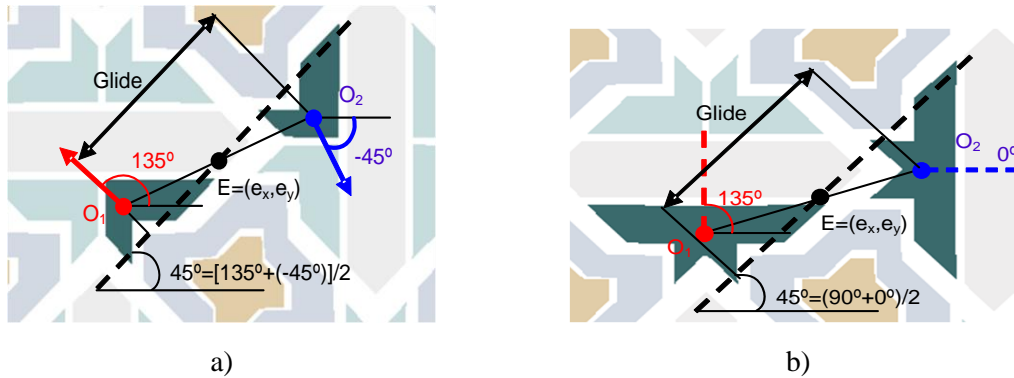
$$303 \quad E = (e_x, e_y) = \left( \frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2} \right) \quad (5)$$

$$304 \quad glide_l = (x_2 - x_1) \cdot \cos \delta_l + (y_2 - y_1) \cdot \sin \delta_l \quad (6)$$

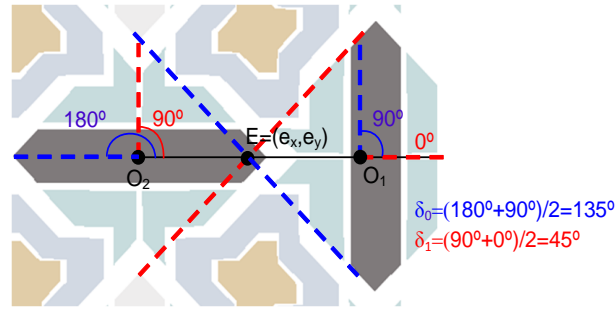
305 where (6) are the projections of the vector between the centroids over the axis. If  $glide_l \approx 0$  the symmetry  
 306 axis is non-glide, otherwise it is a glide symmetry axis.

307 Thus, for two objects  $O_1$  and  $O_2$  of the same class, with centroids  $(x_1, y_1)$  and  $(x_2, y_2)$ , with  $pg=D_n$ , there  
 308 are  $n$  symmetry axes of angle  $\delta_l = (\beta_1 - \beta_2)/2 + l(180/n)$ , point  $E$  and glide  $glide_l$  which relate them.

309 As happens with rotations, there are only  $n$  values for  $\delta_l$  that generate different angles, and therefore the  
 310 number of symmetry axes is  $n$ . The two examples in Figure 6a and Figure 6b show the symmetry axes  
 311 between two pairs of objects of the same class with  $pg=C_1$  (calculated from object rotations, Figure 6a) and  
 312  $pg=D_1$  (calculated from their internal symmetry axes, Figure 6b). On the other hand, Figure 7 shows the  
 313 two symmetry axes between two objects of the same class with  $pg=D_2$ . Each symmetry axis is calculated  
 314 from the combination of the internal axes represented with the same colour.



315 Figure 6. Hand-drawn graphical information superimposed upon a WG sample to illustrate Glide reflection  
 316 between two pairs of objects a) with  $pg=C_1$ , and b)  $pg=D_1$



317

318 Figure 7. Hand-drawn graphical information superimposed upon a WG sample to illustrate Symmetry axes  
 319 between two objects with  $pg=D_2$

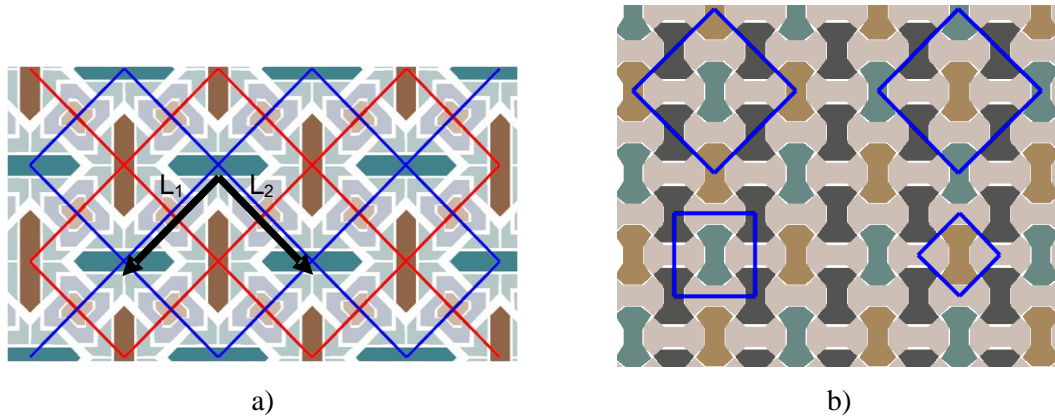
### 320 5.1. Primitive Cell

321 The lattice is defined by the translations in the pattern (see Figure 8) and thus, as defined in section 1.1, the  
 322 PC is the smallest part of the pattern that is repeated by translations of the lattice.

323 Two approaches can be used to obtain the lattice: from Image Level operations and from a set of  
 324 Scattered Elements. In the first case, some works have been carried out using techniques such as  
 325 autocorrelation [39] [40], Markov Random Field (MRF) and Mean Shift Belief Propagation (MSBP) [28]  
 326 to obtain the points forming the lattice, while others choose the directions which define the Primitive Cell  
 327 using techniques such as Hough Transform [10]. If the lattice is obtained from a set of Scattered Elements,  
 328 as for instance in [41] [42], some kind of visual similarity is performed in order to determine which  
 329 elements are equal and then an optimisation of the existing translations is performed. This latest approach  
 330 has been used in this work since it better suits the kind of objects found at the Object Level.

331 For each class of objects, the translation vectors between the objects with the same orientation are  
 332 obtained. All vectors that are a linear combination of any pair of smaller vectors are then removed. Finally,  
 333 only one pair of vectors remains ( $L_1$  and  $L_2$ ) as the basis of a vectorial space defined by the positions of  
 334 objects with the same rotation and reflection. Hence, the PC is defined by the vectors  $L_1$  and  $L_2$ .

335 When more than one lattice is found, they are not independent so that the objects do not overlap.  
 336 Therefore, the largest PC is chosen because it includes the others and it is the only one that is valid for the  
 337 entire pattern. In Figure 8b the PC including the brown and green objects is the only one valid for the entire  
 338 pattern and includes the other ones.



339 Figure 8. Hand-drawn graphical information superimposed upon WG samples to illustrate: a) Sample of  
 340 lattice among objects of a class with two different orientations represented with different colours and PC,  
 341 and b) Sample of different PC belonging to different lattices for different objects

## 342 5.2. Wallpaper Group

343 Table 1 shows the diagram of the 17 Wallpaper Groups that exist together with their isometries and the  
 344 classic nomenclature as well as the shape of their more generic PC [3]. The complexity of the isometries  
 345 increases from left to right (order of rotation centres) and from top to bottom (presence of more symmetry  
 346 axes). The solid black line of PC is not represented if it matches the symmetry axes, which are represented  
 347 using a blue dashed line for non-glide axes and a red dashed line for glide axes and rotation centres (order  
 348 2: circumference, order 3: triangle, order 4: square, order 6: hexagon).

349 Table 1. Isometries of each Wallpaper Group

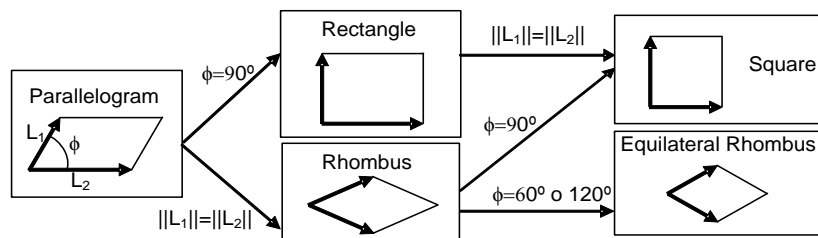
	Without centres	Order 2 centres	Order 3 centres	Order 4 centres	Order 6 centres
Without axes	 P1-P	 P2-P	 P3-ER	 P4-S	 P6-ER
Only non-glide axes	 PM-R	 PMM-R			
Only glide axes	 PG-R	 PGG-R			
Glide and non-glide axes	 CM-R	 CMM-R	 P3M1-ER	 P4M-S	 P6M-ER
		 PMG-R	 P31M-ER	 P4G-S	

350

351 Table 1 shows three characteristics (Primitive Cell, Rotation Centres and Symmetry axes) that allow the  
 352 WG to be detected for each class of objects in a pattern. These three characteristics and how to obtain them  
 353 are explained in the next three subsections.

354 **5.2.1. Shape of Primitive Cell**

355 The shape of a PC may be a generic parallelogram, rectangle, square, rhombus or equilateral rhombus.  
 356 The type of shape discards some Wallpaper Groups (e.g. if the PC is a generic parallelogram, the Wallpaper  
 357 Group can only be a P1 or P2). This shape is obtained from translations by comparing the size and the  
 358 angles of the sides of the PC (Figure 9). Due to the hierarchy among them, a WG may take the shape  
 359 indicated, and all those derived more restrictively.



360

361 Figure 9. Hierarchy and features of each PC shape

362 **5.2.2. Order of rotation centres**

363 Each rotation centre is obtained from the rotations between objects of the same class, by matching their  
 364 centres. The main feature is the order  $\lfloor 360^\circ/\gamma_{min} \rfloor$ ,  $\gamma_{min}$  being the smaller rotation angle  $\gamma_i$ .

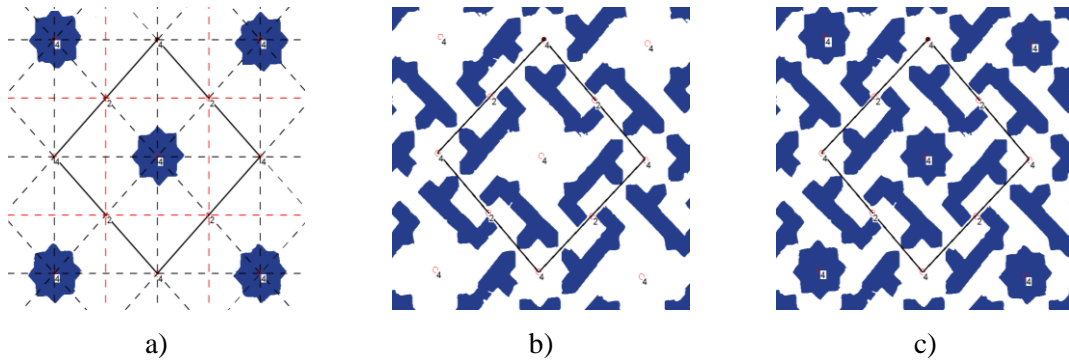
365 **5.2.3. Orientation of symmetry axes**



366 Each symmetry axis is obtained from the reflections, between objects of the same class, with the same  
 367 angle  $\delta_l$  and aligned; the symmetry axis glide is the lowest from all the symmetry axes between pairs of  
 368 objects (if  $glide \approx 0$ , the symmetry axis is non-glide). The main features are:

- 369 • If the angle  $\delta_l$  is parallel or orthogonal to the sides and diagonals of PC (see Figure 9).
- 370 • If it is a glide symmetry axis or non-glide symmetry axis.

371 As each object class can generate a different PC, it can generate different rotation centres and symmetry  
 372 axes and, consequently, different WG. Therefore, the WG for each object class is obtained separately, and  
 373 the least restrictive one is chosen, i.e. the one whose isometries are present in the others, which is the only  
 374 one valid for the entire pattern. In the pattern shown in Figure 10, WG P4 is the least restrictive one, its  
 375 rotation centres are present in P4M (Figure 10a), but the symmetry axes of P4M are not included in P4  
 376 (Figure 10b), so P4 is the only one that is valid for the entire pattern.



377 Figure 10. Output of the application for a pattern sample with two WG: a) WG P4M for the stars; and b)  
 378 WG P4 for the objects of the other class; c) WG P4 is chosen for the entire pattern

### 379 5.3. Description of the structure

380 The pattern structure contains all the isometries, and it is contained in a tuple  $E=(O,V_1,V_2,PC,WP,\theta)$ ,  
 381 where:

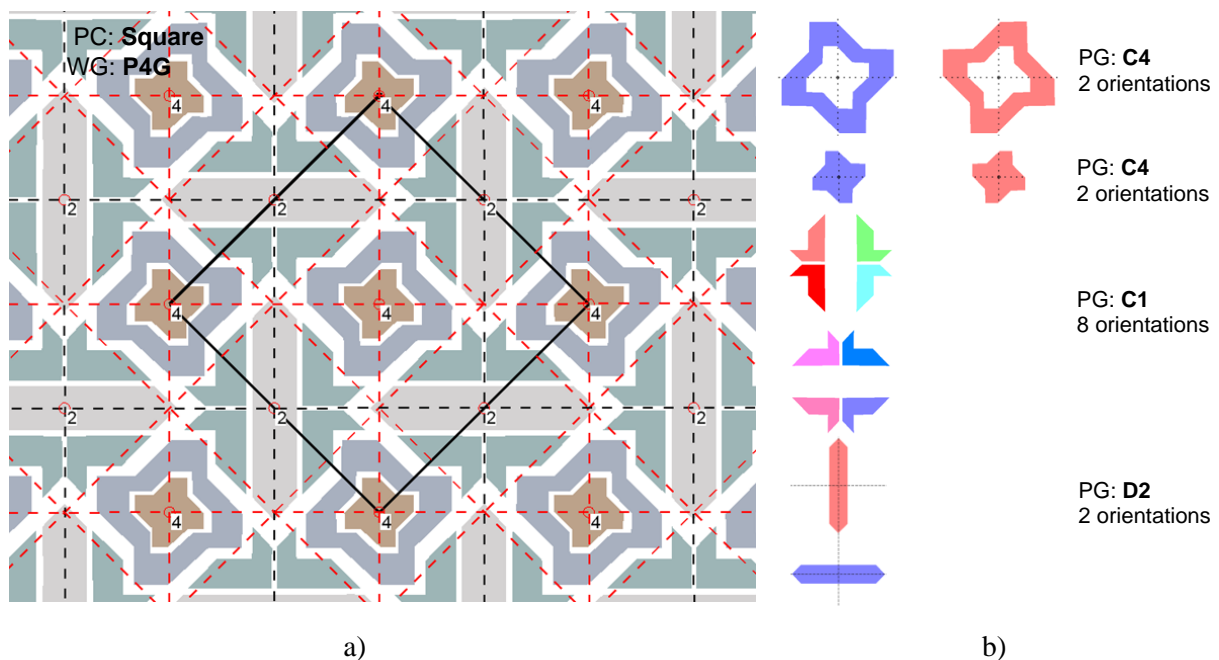
- 382 •  $O$ : origin of Primitive Cell.
- 383 •  $V_1, V_2$ : vectors of Primitive Cell sides.
- 384 •  $PC$ : shape of Primitive Cell (generic parallelogram, rectangle, square, rhombus, equilateral  
 385 rhombus).
- 386 •  $WP$ : Wallpaper Group (P1, PM, PG, CM, P2, PMM, PMG, PGG, CMM, P3, P31M, P3M1, P4, P4M,  
 387 P4G, P6, P6M).
- 388 •  $\theta$ : angle of an axis (with respect to PC elements: sides or diagonals) for Wallpaper Groups PM, PG,  
 389 CM and PMG. This is necessary because in the PM and PG groups the symmetry axes may be

390 parallel to one side or another, in the CM group the symmetry axes may be parallel to one diagonal  
 391 or another, and in the PMG group non-glide symmetry axes may be parallel to one side or another.

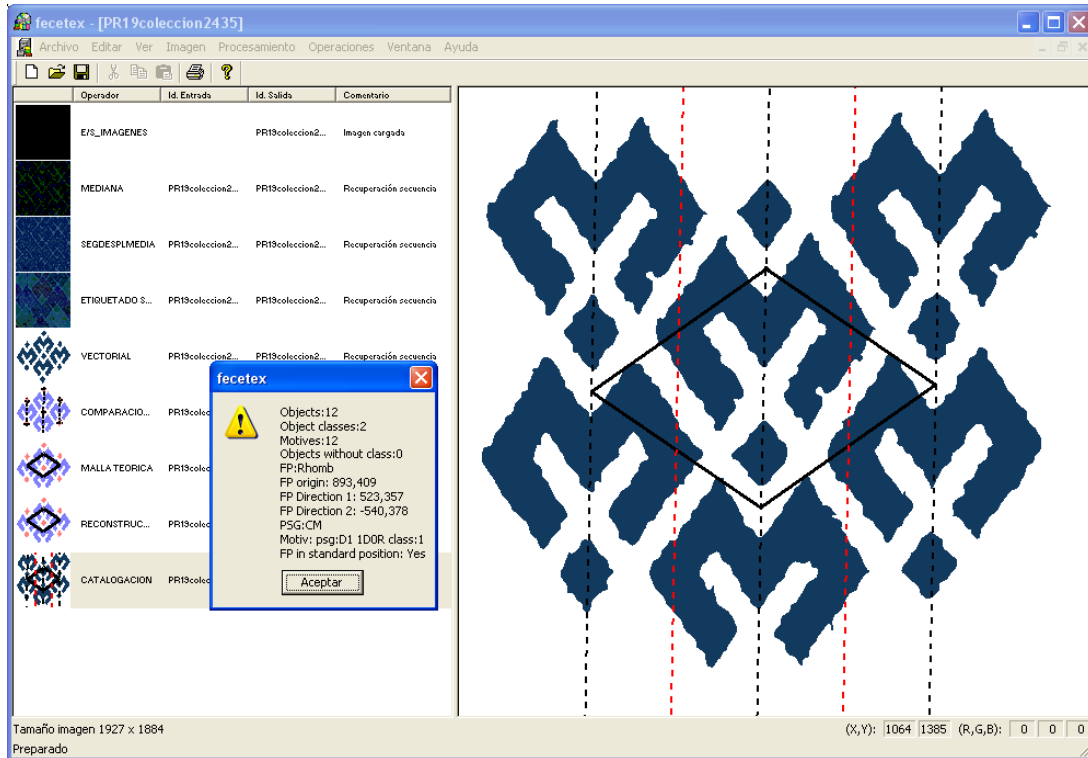
## 392 6 Results and Discussion

393 Figure 11 shows the output of the application for a sample with a synthetic pattern. The pattern has  
 394 different orders of rotation centres and different symmetry axes, which have allowed the accuracy of the  
 395 method developed to be validated. Figure 11a shows the pattern Wallpaper Group, which is P4G (with a PC  
 396 Square). Figure 11b shows the objects obtained by applying the proposed method with the different  
 397 orientations in which they appear in the pattern, containing their internal symmetries and Point Group.  
 398 Figure 12 shows a screenshot of the interface of the application developed. The different stages of the  
 399 method levels and a message box with the information obtained about the WG for the entire pattern are  
 400 shown on the left.

401



402 Figure 11. Output of the application for a sample with a synthetic pattern: a) Synthetic pattern analysis with  
 403 its structure, and b) The object classes contained (different colour for the different orientations for each  
 404 class)



405

406

Figure 12. Interface of the application software that was developed

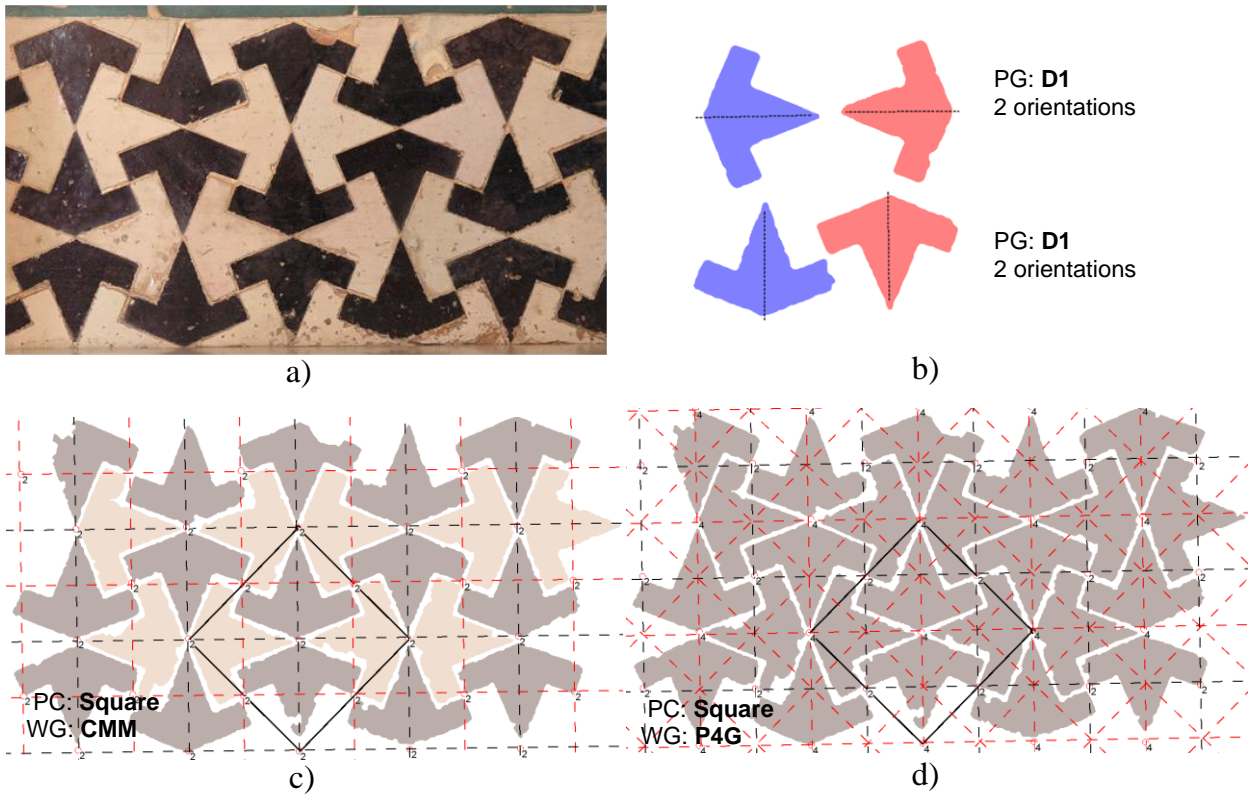
### 407 6.1. Results on IGP

408 The proposed method was tested over a set of 96 images containing Islamic Geometric Patterns from the  
 409 Alhambra of Granada (Spain) (42), the Alcázar of Seville (Spain) (23), Morocco (26, mostly from [43]),  
 410 and the Great Mosque of Damascus (Syria) (5).

411 Figure 13 to Figure 15 show the analysis of three real IGP with the application and its output. Figure 13  
 412 and Figure 14 show the object classes found with their PG and the different orientations in which the  
 413 objects appear. Particularly, for the IGP in Figure 13 the analysis found a different WG when considering  
 414 only the shape of objects and discarding their colour. In the case of Figure 14 the analysis found the WG  
 415 considering only the stars, which has more isometries than the global WG. In the case of the IGP in Figure  
 416 15, only two colours were considered for the foreground and for the background, WG P3 being found as  
 417 the valid WG for the entire pattern. These three examples show the flexibility provided by the Object Level  
 418 and why, in most cases, the WG perceived does not always coincide with the WG of the pattern, as [43]  
 419 pointed out. Compared to the analysis from the methods presented in the state of the art, none of them can  
 420 obtain information about objects like shapes, classes, orientations and Point Groups (rotational and  
 421 reflection symmetry). For instance, for the sample in Figure 13 the pattern analysis would be performed  
 422 always taking into account the colour of the objects, so the WG in Figure 13d would never be found. For

423 the sample in Figure 14 the pattern analysis would only obtain the solution in Figure 14c.

424

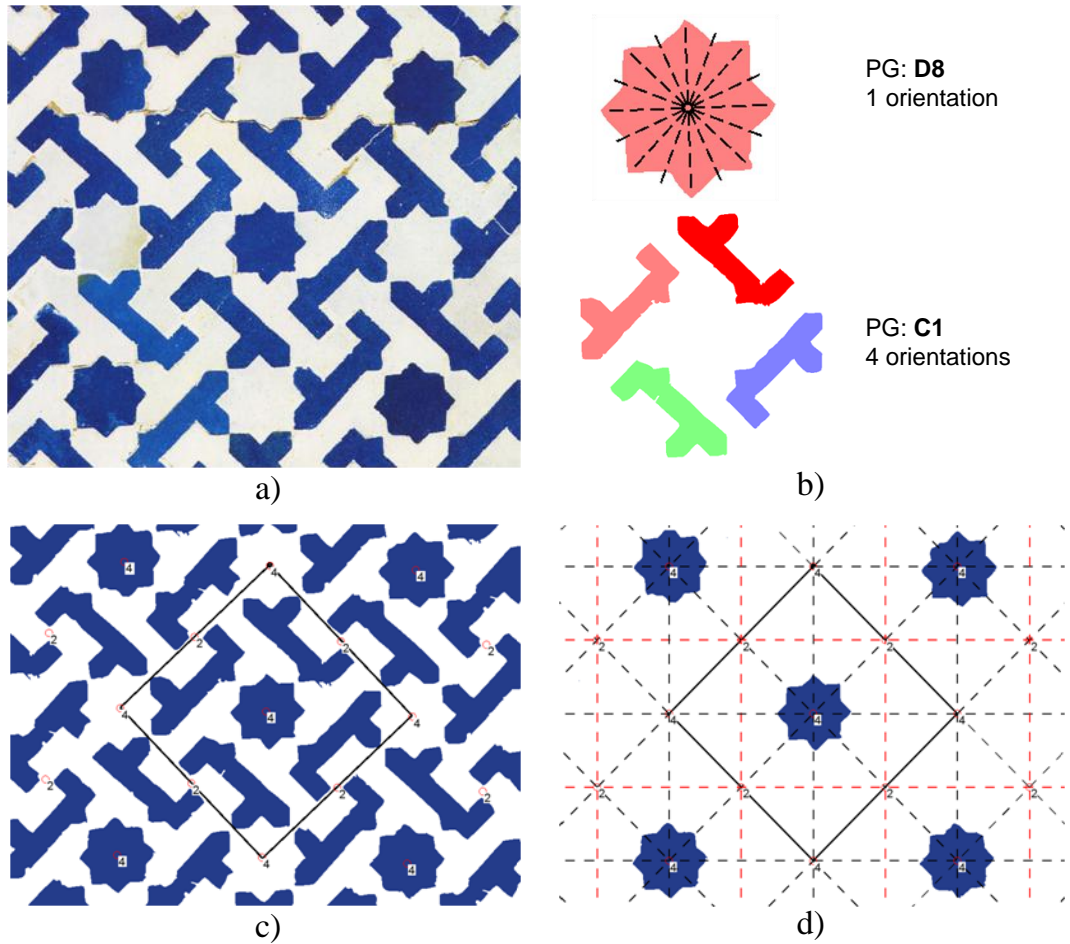


425

426 Figure 13. a) Islamic Geometric Pattern from the Alhambra of Granada; b) Object classes; c) WG CMM

427

after analysis; and d) WG P4G taking into account only the shape of the objects found

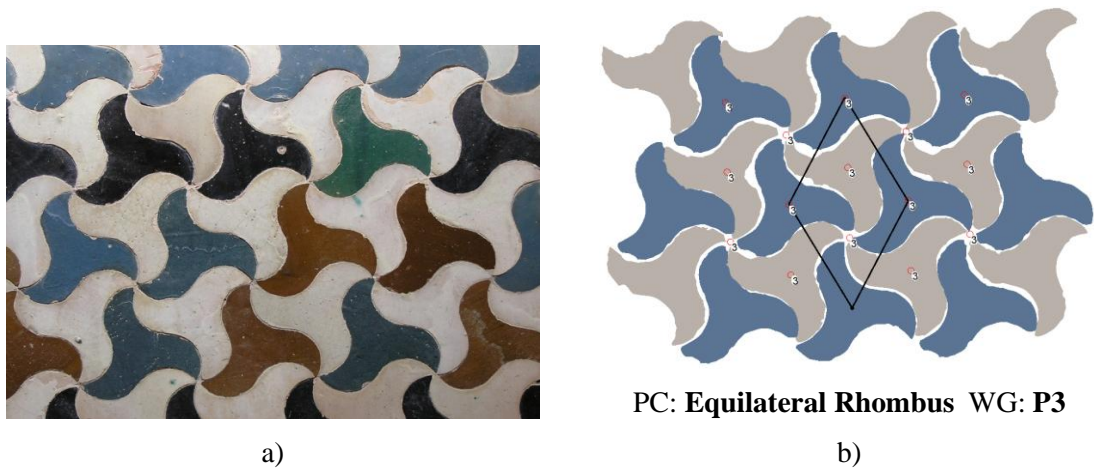


428

429

430

Figure 14. a) Islamic Geometric Pattern from the Palais Royal of Rabat (Morocco); b) Object classes; c) WG P4M found only for stars; and d) WG P4 found for the entire pattern (the stars and other objects)

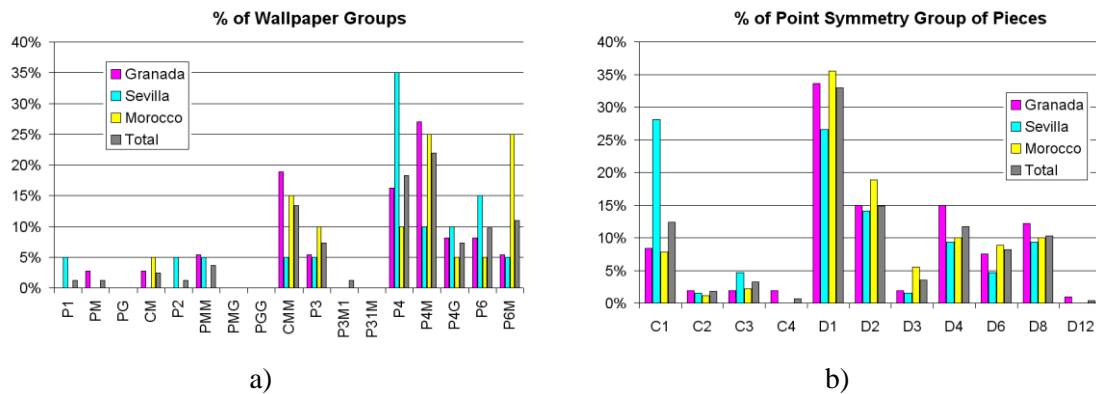


431

432

Figure 15. a) Islamic Geometric Pattern from the Alhambra of Granada; and b) WG P3 found for the entire pattern

433 Figure 16 shows the percentage of Wallpaper Groups and Point Groups found in the IGP analysed. There  
 434 are only five patterns from the Great Mosque of Damascus, so they are included in the total score. The WG  
 435 which appear most often (P4M, P4, CMM, P6M, P6, P3 and P4G) have dense structures, plenty of  
 436 isometries, and the existence of WG for only a particular object class, usually CMM combined with PMM  
 437 or P4G, is also very common. In contrast, some groups have few occurrences or never appear (PG, PMG,  
 438 PGG, P31M, P1, PM, P2, P3M1). It is also noteworthy that the predominant WG are different (P4M, CMM  
 439 and P4 in the Alhambra, P4 in the Alcázar, and P4M and P6M in Morocco).



440 Figure 16. a) Percentage of Wallpaper Groups found in the IGP analysed; and b) Percentage of Point  
 441 Groups found in the pieces of IGP analysed

442 Figure 17a shows the results of the correct identification of the WG in the IGP coming from different  
 443 locations. The WG was found for 85% of the patterns, while for 73% pieces of all classes, patterns were  
 444 also found with their Point Group successfully obtained. The patterns from the Alcázar of Sevilla are more  
 445 complex and contain a larger number of different pieces, and therefore the success rate was lower when  
 446 obtaining the pieces, although the identification of the WG was still good.

447 Figure 17b shows the results of the correct identification of the different WG. Dismissing the case of  
 448 PMM (of which there are only 5 patterns) the worst results are obtained for P4, because most of them are  
 449 patterns from the Alcázar of Sevilla, which are more complex and with a much wider range of different  
 450 pieces.



451 Figure 17. Success rate in the analysis carried out for the IGP: a) Success by location: two different rates  
 452 are shown, i.e. the patterns with the WG found successfully, and the patterns where all classes of pieces  
 453 were found, and b) Success rate by WG

454 In most of the cases where the application failed, this was due to a low performance of the segmentation  
 455 process (Image Level) caused by the poor quality of the patterns, since they belong to ancient damaged  
 456 patterns (Figure 18 left). The problems at the Image Level severely influenced the outcome of the further  
 457 levels, for example, by causing errors in the comparison of objects and in obtaining their symmetries.

458 Even in the cases where segmentation was good, another source of problems that was encountered came  
 459 from the inaccuracies introduced by the handmade nature of the different pieces: colour was not uniform,  
 460 the objects of the same class were not exactly equal, the shapes were not perfectly symmetrical and the  
 461 geometric transformations which relate objects of the same class were not exactly equal. This caused  
 462 mistakes when determining the morphological features and the relations among objects. Finally, some  
 463 patterns could not be analysed because of the high complexity and large number of different pieces they  
 464 contained, as shown in Figure 18 middle and right.



465  
 466 Figure 18. Images of some examples of IGP that failed

## 467 6.2. Parameters, computational cost and intermediate results of the algorithm

468 The table 2 shows the stages of the algorithm, their parameters and their computational cost. In the last  
 469 column it is shown the elapsed time in each stage for the sample of Islamic Geometric Pattern of Fig. 13. In

470 this sample, the full image size is 2000x1500 and the full pattern has 33 objects. The analysis was  
 471 performed with an Intel Core 2 Duo 3Ghz and the time of each stage includes the generation of the image.

472 Table 2. Stages of the algorithm with their corresponding parameters, their computational cost and the  
 473 elapsed time of each stage for a specific sample

STAGES	PARAMETERS	COMPUTATIONAL COST	TIME* (sec)
Pre-process (Gaussian blur)	<i>Radius</i> : size of Gaussian kernel	O(pixels)	3.88
Segmentation (Watershed)	<i>MinEdgeHeight</i> : minimum difference of grey level between an edge and a neighbouring valley, if the difference is lower, it is not an edge and the neighbouring valleys join <i>MaxValleyHeight</i> : maximum grey level of a valley, if the level is higher, it is not a valley, it is considered background	O(pixels)	8.94
Morphological operations	<i>Iterations</i>	O(objects)	7.69
Contour extraction	---	O(objects)	11.78
Object classification	<i>MaxCoarse</i> : maximum area ratio and axis ratio <i>MaxFine</i> : maximum contour distance	O(objects <sup>2</sup> )	2.14
Object symmetries	<i>MaxFine</i> : maximum contour distance	O(objects)	0.81
Primitive Cell	<i>SizeTolerance</i> : maximum size difference between vectors to be considered equal <i>AngleTolerance</i> : maximum angle difference between vectors to be considered equal <i>MinPercentage</i> : minimum ratio of occurrences of a vector with respect to the number of objects of its class and orientation	O(objects <sup>2</sup> )	0.25
Wallpaper Group	<i>DistanceTolerance</i> (% of PC lower side): maximum difference between glides or distance between axis or distance between rotation centres to be considered equal <i>AngleTolerance</i> : maximum angle difference between axis to be considered equal <i>MinPercentage</i> : minimum ratio of occurrences of an axis or rotation centre with respect to the number of objects of its class	O(objects <sup>2</sup> )	0.63

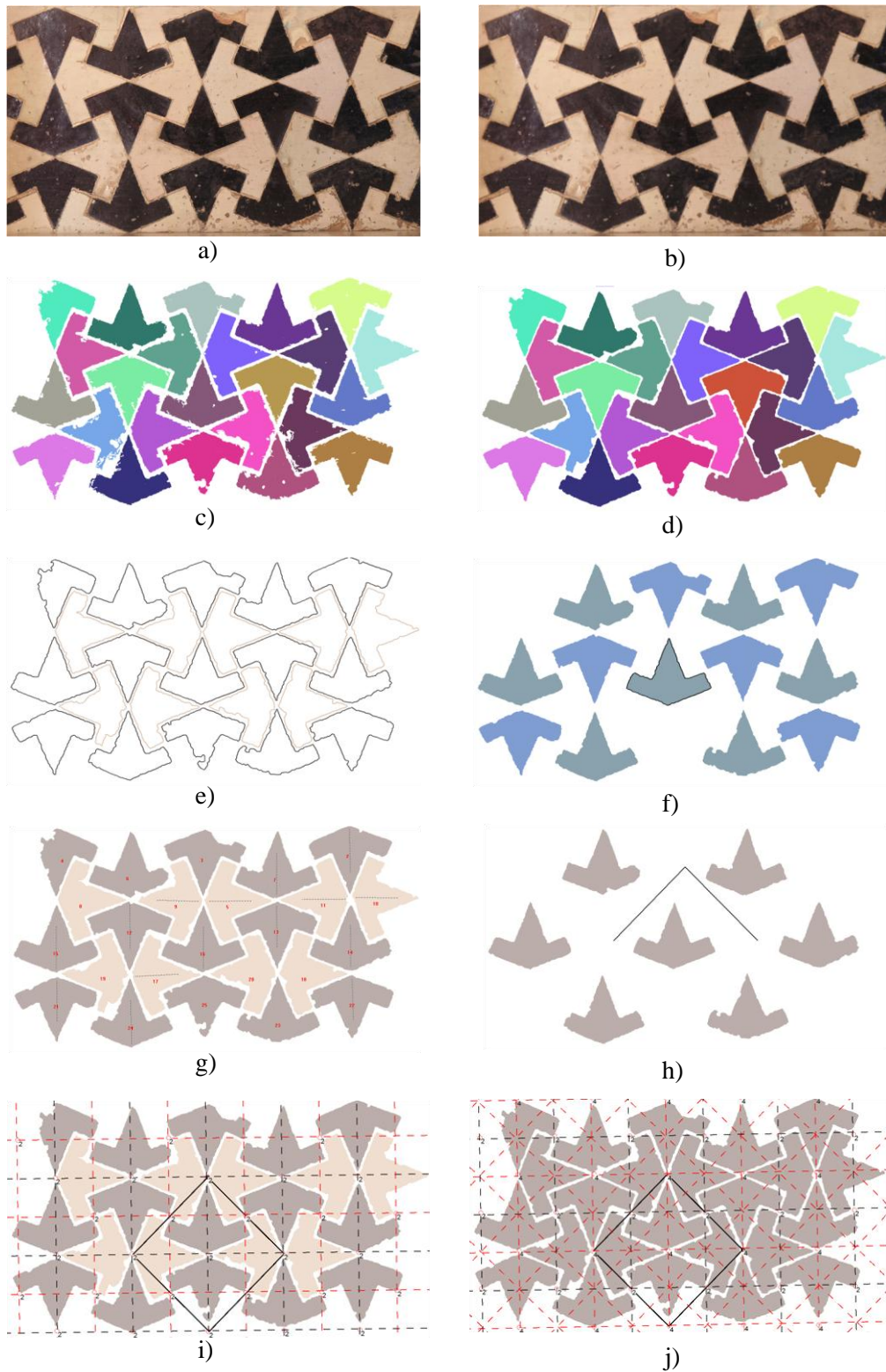
474 \* Time consumed at each stage for the IGP of the Fig. 13

475

476 Fig. 19 shows the intermediate graphical results of each stage of the algorithm for the Islamic Geometric  
 477 Pattern of Fig. 13. For the Object Classification stage and the Primitive Cell stage only one class is shown  
 478 (the class shown can be chosen by the user).

479





480

481 Figure 19. Intermediate results of each stage for the IGP of Fig. 13: a) Original image; b) Pre-process stage

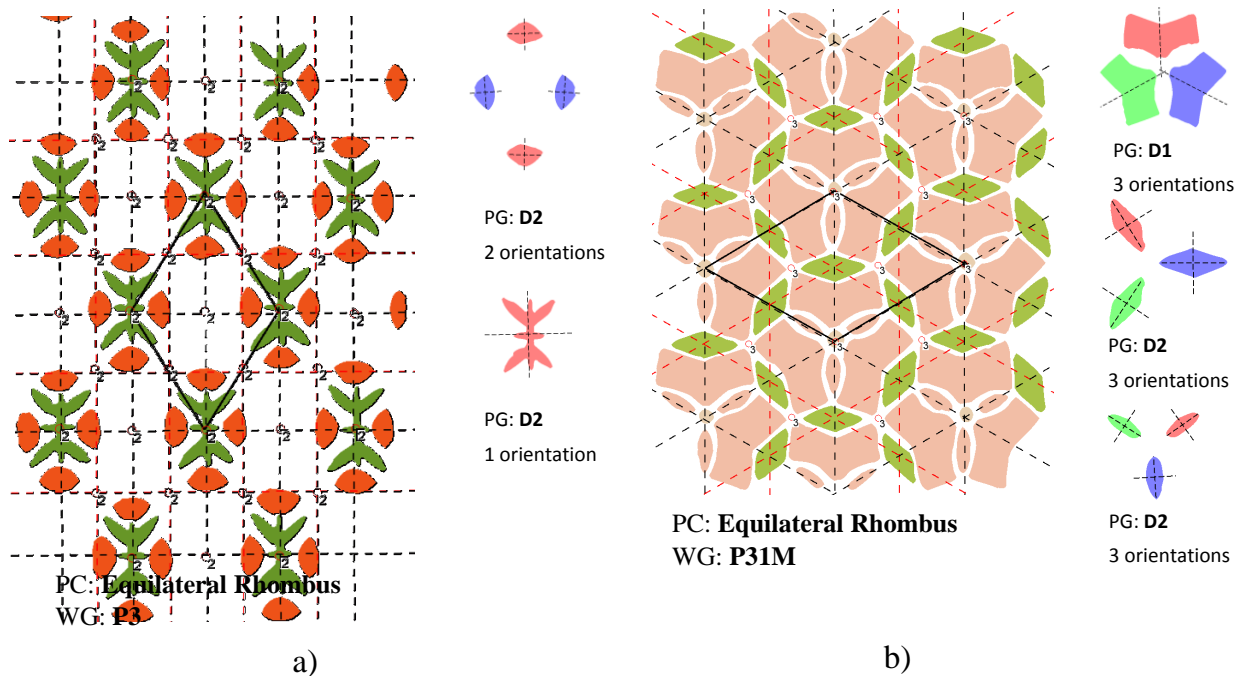
482 (Gaussian filter); c) Segmentation stage (Watershed); d) Morphological Operations stage (Closing); e)

483 Contour Extraction stage; f) Object Classification stage; g) Object Symmetries stage; h) Primitive Cell  
 484 stage; i) Wallpaper Group stage; and j) Wallpaper Group stage discarding colour

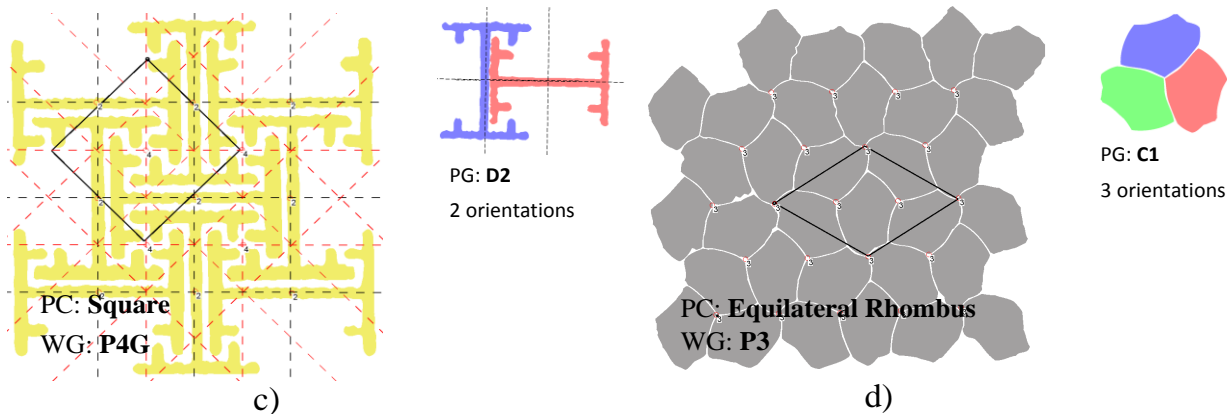
485 Describing Fig. 19 briefly, Fig. 19a shows the original image; Fig. 19b is the image after a Gaussian  
 486 filter; Fig. 19c is the segmented image where each object is represented with a random colour; Fig. 19d  
 487 shows the image after closing operations where each object is represented with a random colour; Fig. 19e  
 488 shows the contours extracted; Fig. 19f shows only one class of objects found, where the objects with the  
 489 same orientation are represented with the same colour and the representative object is outlined; Fig. 19g  
 490 shows each object represented with its Point Group; Fig. 19h shows the pair of vectors remaining between  
 491 objects with the same orientation of one class (two black lines); Fig. 19i shows the Wallpaper Group  
 492 resulting a WG CMM that has been obtained taking into account the colour of the objects; and finally, Fig.  
 493 19j shows the Wallpaper Group resulting a WG P4M that has been obtained discarding the colour of the  
 494 objects.

### 495 6.3. Application of the method to alternative datasets

496 In order to test the flexibility of the proposed method, some images containing mosaics from alternative  
 497 datasets have been analysed. Figure 20 shows some examples of WG from the Wikipedia  
 498 ([http://en.wikipedia.org/wiki/Wallpaper\\_group](http://en.wikipedia.org/wiki/Wallpaper_group)) and Figure 21 shows some examples of paving slabs from  
 499 public constructions and fabrics [23]. Table 2 shows the results of the application for all the mosaics from  
 500 the Wikipedia that were analysed.



501

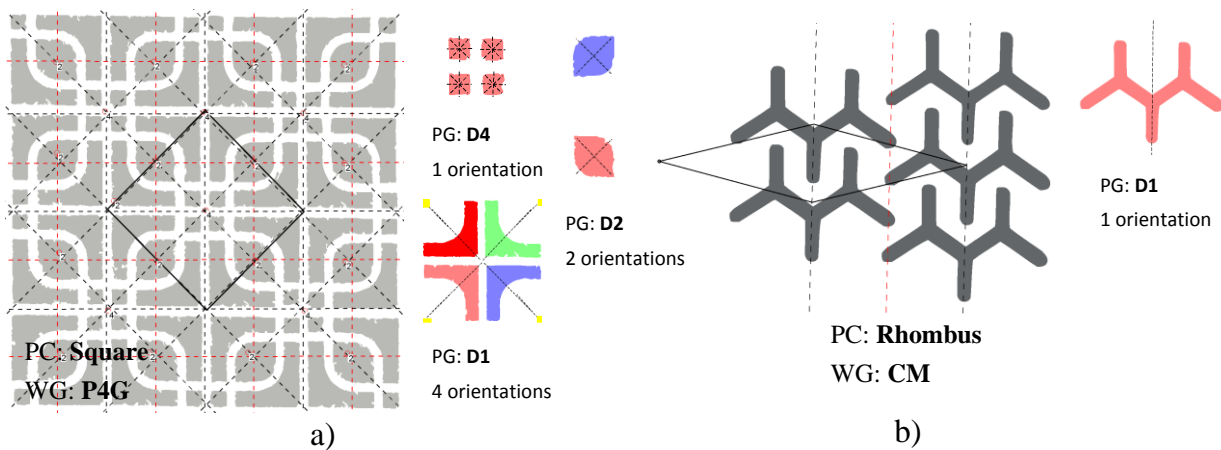


502

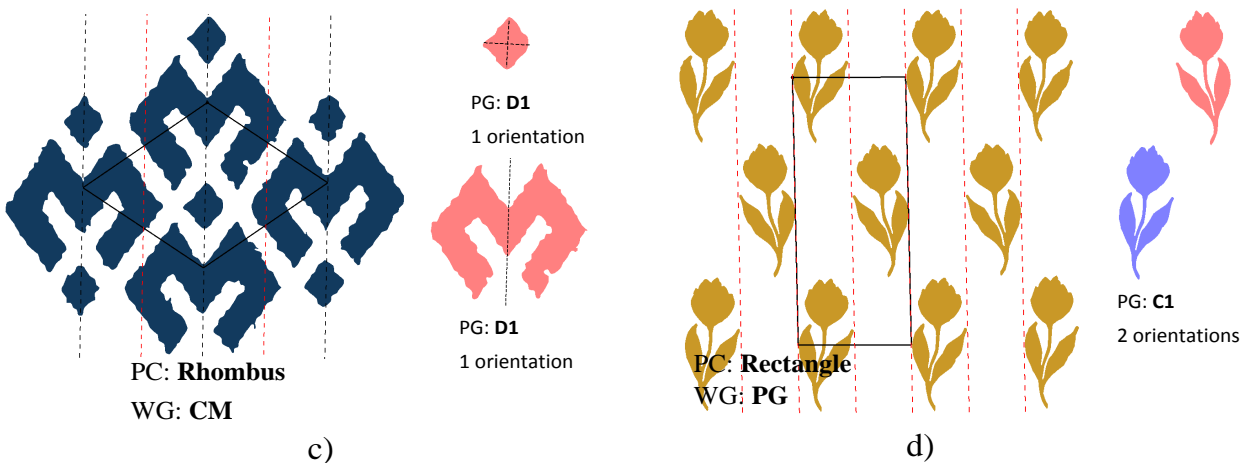
503 Figure 20. Application output for some examples from the Wikipedia: a) WG: CMM; b) WG: P31M; c)

504

WG: P4G; and d) WG: P3

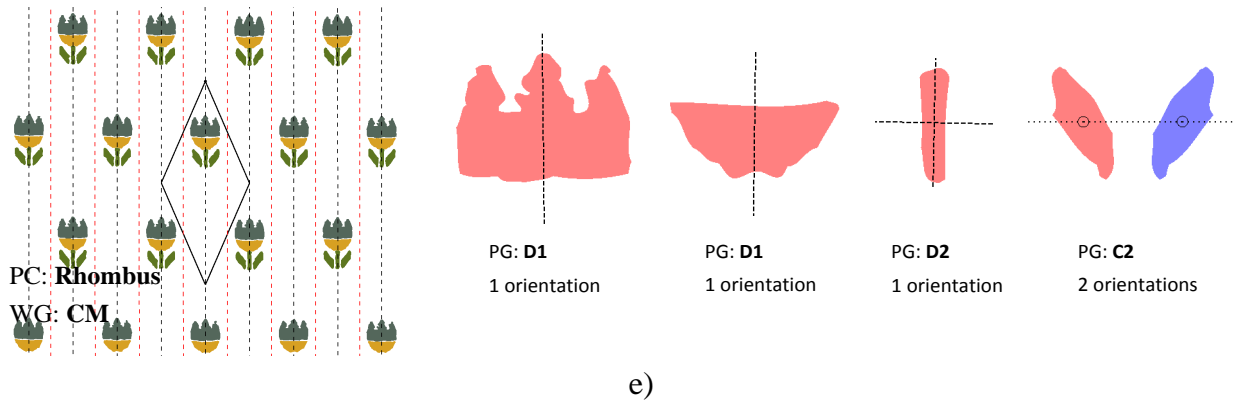


505



506

507



508

509 Figure 21. Application output for some examples of paving slabs from public constructions and fabrics.

510 Paving slabs: a) WG: P4M; and b) WG: CM; Fabrics; c) WG: CM; d) WG: PG; and e) WG: CM

511

Table 3. Results for all the samples from the Wikipedia that were analysed

Name	Fig. in this article	WG (Other WG)	Objects Point Group (Number of classes)	Comments
Wallpaper_group-cm-6 (Persian tapestry).jpg				WG not found
Wallpaper_group-cm-7 (Indian metal-work at the Great Exhibition in 1851).jpg		CM	C1 (2) D1 (3) D2 (2)	Some object class not found
Wallpaper_group-cmm-4 (Persian tapestry).jpg	Fig.20a	CMM	D2 (2)	
Wallpaper_group-p3-1 (Street pavement in Zakopane, Poland).jpg	Fig. 20d	P3	C1 (1)	
Wallpaper_group-p3m1-1 (Persian glazed tile (ignoring colours p6m)).jpg				WG not found
Wallpaper_group-p4-2 (Ceiling of Egyptian tomb).jpg		P4 (P4M all object classes except 1)	C2 (1) D1 (1) D2 (1)	Some object class not found
Wallpaper_group-p4g-1 (Bathroom linoleum, U.S.).jpg		P4G	D2 (1)	
Wallpaper_group-p4g-2 (Painted porcelain, China).jpg	Fig. 20c	P4G	D2 (1)	
Wallpaper_group-p4m-1 (Ornamental painting, Nineveh, Assyria).jpg		P4M	D2 (1) D4 (4)	
Wallpaper_group-p4m-4 (Egyptian tomb).jpg				WG not found
Wallpaper_group-p4m-6 (Persian glazed tile).jpg		P4M	D1 (3) D4 (1)	
Wallpaper_group-p31m-1 (Persian glazed tile).jpg	Fig. 20b	P31M	D1 (1) D2 (2) Dinf (circle) (1)	
Wallpaper_group-p31m-2 (Painted porcelain, China).jpg		P31M	D3 (2)	
Wallpaper_group-p31m-3 (Painting, China).jpg		P31M	D3 (3)	
Wallpaper_group-pg-2 (Pavement with				WG not found

herringbone pattern in Salzburg).jpg				
Wallpaper_group-pgg-2 (Pavement in Budapest, Hungary).jpg		PGG	D2 (1)	
Wallpaper_group-pm-3 (Dress of a figure in a tomb at Biban el Moluk, Egypt).jpg		PM (CM ignoring colours)	D1 (2)	
Wallpaper_group-pm-5 (Indian metalwork at the Great Exhibition in 1851. This is almost pm (ignoring short diagonal lines between ovals motifs, which make it p1)).jpg				WG not found
Wallpaper_group-pmm-2 (Mummy case stored in The Louvre).jpg		PMM (P4M ignoring colours)	D2 (1) D4 (2) Dinf (circle) (1)	

512

513 For the method developed to work properly, the contours of the objects must be well defined and closed,  
514 and the size should be large enough to be considered objects. However, we can see that the method has  
515 achieved good results and in some cases has even obtained other WG ignoring colours or ignoring some  
516 object classes.

517 All the images used in this article can be downloaded from [http://personales.upv.es/maalbor/Files-](http://personales.upv.es/maalbor/Files-WG/IGP.rar)  
518 [WG/IGP.rar](http://personales.upv.es/maalbor/Files-WG/IGP.rar) and <http://personales.upv.es/maalbor/Files-WG/Wiki-Others.rar>

## 519 7. Conclusions and Further Work

520 This work presents a method to gain an understanding of IGP patterns (and any kind of pattern presenting  
521 a Wallpaper Group) based on Image Processing, Pattern Recognition and Symmetry Groups, the main  
522 novelty of which is the addition of a new object-oriented level of knowledge. This method is structured on  
523 three main levels of knowledge: the lower level or Image Level, the intermediate level or Object Level,  
524 and the upper level or Structure Level.

525 The Image Level is pixel-oriented and its aim is to obtain the regions of interest in the image of the  
526 pattern, these regions constitute the objects that form the pattern. The Object Level is a higher level from  
527 which the features of the objects are extracted (including their Point Group), a classification of the objects  
528 is carried out, and the relationships among them are determined, which introduces a deeper analysis of the  
529 patterns and hence a deeper understanding can be found. Other important advantages introduced by the  
530 Object Level are the obtaining of the WG of each object class, apart from the WG of the entire pattern, and  
531 also the possibility of analysing the pattern taking into account the objects found regardless of their colour  
532 (considering only their shape). Finally, on the Structure Level, the pattern structure (Primitive Cell and  
533 Wallpaper Group) is obtained from the relationships, that is, the isometries (translations, symmetry axes  
534 and rotations) among objects of the same class, provided by all the understanding attained on the Object

535 Level.

536 The main contribution of this work resides precisely in these two levels. None of the methods presented  
537 in the state of the art is able to obtain this information.

538 The tests performed using a set of 96 Islamic Geometric Patterns show that this method is capable of  
539 obtaining Wallpaper Groups and all classes of pieces with their corresponding Point Group from different  
540 types of real IGP. In addition, the tests show differences between Wallpaper Group patterns and Point  
541 Group pieces, depending on their origin, as shown in Figure 16a. Moreover, the method has been tested on  
542 some images from alternative datasets (Wikipedia, paving slabs from public constructions and fabrics) and  
543 was seen to be capable of achieving the expected results in most of the samples analysed.

544 Possible applications of this computational method include pattern classification, cataloguing of ceramic  
545 coatings, creating databases of decorative patterns, creating pattern designs, pattern comparison between  
546 different cultures, tile cataloguing, and so on.

547 Plans have been made to carry out further work in order to:

- 548 • Use the information obtained to ‘reconstruct’ the pattern geometrically, that is, to retrieve missing  
549 objects, to make objects of the same class look exactly alike, to perform the geometric  
550 transformations which relate objects of the same class in exactly the same way, and to make the  
551 shapes perfectly symmetrical. This reconstruction can be performed by copying objects and  
552 applying geometric transformations of the Wallpaper Group (to recover missing objects of the  
553 same class), redoing the shape of objects according to their point group (making the shapes  
554 perfectly symmetrical), adjusting their position and orientation to the translations, centres of  
555 rotation and symmetry axes of the pattern, and any other key feature.
- 556 • Processing complex patterns successfully, which can be done, for instance, by grouping objects and  
557 also by classifying groups to obtain the isometries of the pattern from groups instead of objects.

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