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Sliding contact on the interface of elastic body and rigid surface using a single block Burridge-Knopoff model

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Abstract: Burridge and Knopoff proposed a mass-spring model to explore interface dynamics along a fault during an earthquake. The Burridge and Knopoff (BK) model is composed of a series of blocks of equal mass connected to each other by springs of same stiffness. The blocks also are attached to a rigid driver via another set of springs that pulls them at a constant velocity against a rigid substrate. They studied dynamics of interface for an especial case with ten blocks and a specific set of fault properties. In our study effects of Coulomb and rate-state dependent friction laws on the dynamics of a single block BK model is investigated. The model dynamics is formulated as a system of coupled nonlinear ordinary differential equations in state-space form which lends itself to numerical integration methods, e.g. Runge-Kutta procedure for solution. The results show that the rate and state dependent friction law has the potential of triggering dynamic patterns that are different from those under Coulomb law.

Keywords: Burridge and Knopoff model, Sliding contact, Rate and state dependent friction

1. Introduction

Burridge and Knopoff proposed a mass-spring model to explore interface dynamics along a fault during an earthquake [1]. The Burridge and Knopoff (BK) model is composed of a series of blocks of equal mass connected to each other by springs of same stiffness. The blocks also are attached to a rigid driver via another set of springs that pulls them at a constant velocity against a rigid substrate. In their study of interface dynamics an especial case with ten blocks and specific proper-ties is studied. A numerical model of the system is constructed to investigate statistics of the shocks on the fault. Energy of the system is also of interest as a measure of shock magnitude. The applied velocity weakening friction law, considers both seismic radiation and viscosity of the fault.

Stability of steady state slipping of a single block BK model is investigated within small perturbation theory by Rice and Ruina [2]. Using a one-state variable friction law, an analytic expression for minimum spring stiffness for stick-slip instability is obtained.

A detailed analysis of dynamics of a symmetric BK model with two mass blocks is performed in [3]. Later an asymmetric BK model with two mass blocks (with a different frictional force for each block due to velocity-weakening friction) was investigated in [4] also a cellular automata model is applied to investigate the symmetric BK model with two mass blocks [5].

Carlson and Langer [6] simulated a BK model including up to 200 mass blocks with a velocity weakening friction law by a coupled system of differential equations and noticed manifestation of the purely empirical Gutenberg-Richter law (known from 1950s). They proposed their continuum dynamical system as an alternative to the cellular automaton models for representing self-organized criticality (SOC) in earthquakes [7,8,9,10]. The concept of SOC argues that the statistically stationary state of dynamical many body systems is scale free [8], i.e. these systems drive themselves to a critical state in which no characteristic length, time or energy scale does exist [7] and avalanches occur in all sizes and scales. An analysis of criticality in BK models with velocity weakening friction law is also done in [11]. A study of dynamics of BK models (both single block and multi-block) is done in [12]. Transition from small scale events to large scale events is investigated for both velocity weakening and rate and state friction laws. Another study of dynamics of BK model using both discrete equations (ODEs) and continuum formulation (PDE) is given in [13].

2. Problem definition and the model

In our study a single block BK model is considered (Figure 1). Mass of the model is assumed to be 1 kg and the stiffness is 10 kN. Note that the spring with stiffness "k" does not have a physical length, i.e. it cannot be compressed. Therefore the spring experiences tension only and compression is not possible. Then, when the mass is right below the attachment point of the spring and the driver (AP), no spring force is exerted on it. Also the spring is not capable of pushing the block away. Often instead of a coil spring, the spring connecting each mass to the driver is shown as a leaf spring in the literature.
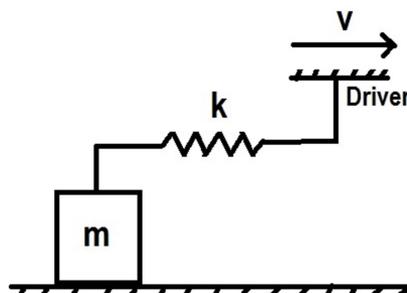

Figure 1: The single block BK model, mass of the block is represented by "m". "k" stands for stiffness of the coil spring and the driver moves with a speed "v" to the right.

The Coulomb friction law is very straightforward: when moving, dynamic coefficient of friction is constant; if not moving, there is a shear force threshold which when exceeded the block have to start moving. However, there are friction laws in the literature according to which the dynamic coefficient of friction is not a constant, but changes in rather complicated ways. The rate and state dependent friction law which is used here is adopted from the seminal works of Dieterich [14] and Rice and Ruina [2]. Dealing with test results for rocks sliding on each other, it has founded out that friction can be described more accurately by considering the effects of not only speed of sliding (rate) but also the history of evolution of this speed (state). It is assumed that the coefficient of friction is a multiplication of two functions $f = f(\dot{x})$ where $\dot{x}$ is velocity of the block (slip rate) and $g = g(\theta)$, where θ is called internal state variable:

$$\mu(\dot{x}, \theta) = f(\dot{x}).g(\theta) \tag{1}$$

where

$$f(\dot{x}) = \left(\frac{\dot{x}}{v_0} + 1\right)^{1/m} \tag{2}$$

and

$$g(\theta) = \frac{\mu_d + (\mu_s - \mu_d)\exp[-\left(\frac{L_0/\theta}{v_1}\right)^p]}{\left(\frac{L_0/\theta}{v_1} + 1\right)^{1/m}} \tag{3}$$

Note that the evolution of state variable is found from a first order ODE which is called evolution law and is given below:

$$\dot{\theta} = B\left(1 - \frac{\dot{x}\theta}{L_0}\right) \tag{4}$$

The evolution law can be analytically integrated to give an algebraic expression for the state variable to be substituted in the expression for $g(\theta)$:

$$\theta(t) = \left(\theta_n - \frac{L_0}{\dot{x}}\right)\exp\left[\left(\frac{\dot{x}}{L_0}\right)B(t_n - t)\right] + \frac{L_0}{\dot{x}} \tag{5}$$

The parameters of the above formula for sliding between rock plates are given in table 1.

Table 1. The adopted parameters of rate and state dependent friction law

Parameter	Value
μ_s	0.6
μ_d	0.5
v_0	100
v_1	0.001
p	1.2
m	0.001
L_0	2e-5
B	4.6

To have an idea about the coefficient of friction as it obtained from the above relations and how does it differ from Coulomb's coefficient of friction, the proposed model is plotted for an especial case of abrupt changes of sliding velocity. In figure 2, coefficient of friction is plotted as a function of slipped distance for two step changes in the speed of two sliding plates. Initially (in region A), the sliding speed is 1 mm/s then it suddenly jumps up to 10 mm/s (region B) and then again suddenly it drops down back to 1 mm/s (region C). The higher the sliding velocity is, the lower is the steady state dynamic friction coefficient. Note that the minimum steady state friction happens in region B (where the velocity is the highest among A, B and C). Note also that however after a jump to a higher

velocity, a reduction in dynamic coefficient of friction (COF) is expected (due to what is called velocity weakening nature of the friction), the expected reduction starts only after a sudden rise (a spike) in the value of dynamic COF. In addition, when it is expected to increase, it decreases initially. Both spikes happen at the very beginning of the abrupt change in sliding velocity of the plates. The spike can even exceed the static coefficient of friction, but all of the dynamic steady state friction values are below the static coefficient of friction.
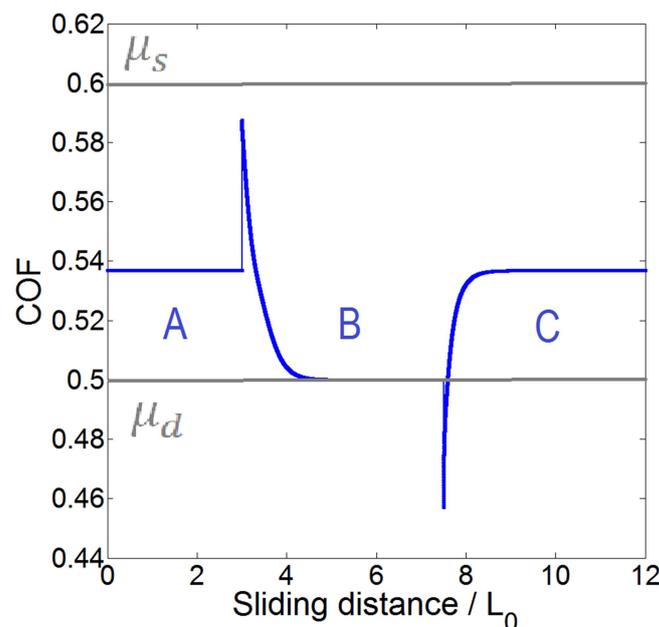

Figure 2: Coefficient of friction according to rate and state dependent friction law against nondimensionalized sliding distance. Sliding velocity of the plates are 1 mm/s in region A, 10 mm/s in region B and finally 1 mm/s again, in region C.

3. The equations of motion

Presence of the friction force makes the equation of motion (EOM) nonlinear. The first manifestation of this non-linearity is that direction of the friction must be opposite to that of the block velocity. Note that the velocity is obtained by solving the equation of motion and is not known a priori. Thus the solution for velocity is to be monitored continuously to switch the friction sign in opposition. It must happen any time the velocity passes through zero. The second manifestation is that when the block stops (sticks) the static friction (which is exactly equal and opposite to the spring force) starts to accumulate. Then the next stage of slipping is to be solved by considering motion of the driver. The block can stick and slip intermittently.

Using d'Alambert's principle:

$$m\ddot{x} = k(vt - x) - sign(\dot{x}) * f$$

or:

$$\ddot{x} = \frac{1}{m}[k(vt - x) - sign(\dot{x}) * f]$$

gives a rather incomplete picture of the dynamics of the mass under dry friction. It can also be misunderstood easily since according to the definition of the sign function: $\text{sign}(0) = 0$, while it is not obviously the case for the stick state. Therefore the equation of motion is split to two cases; namely:

Case (1): motion towards right (positive velocity direction)

$$\ddot{x} = \frac{1}{m}[k(vt - x) - f]$$

Case (2): motion towards left (negative velocity direction)

$$\ddot{x} = \frac{1}{m}[k(vt - x) + f]$$

Putting into state space form by defining:

$$x_1 = x$$
$$x_2 = \dot{x}$$

(Which implies $\dot{x}_1 = x_2$)

In case (1):

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\dfrac{k}{m} & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{1}{m}(kvt - f) \end{bmatrix}$$

In case (2):

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\dfrac{k}{m} & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{1}{m}(kvt + f) \end{bmatrix}$$

Thus the strategy for developing the code is to make the algorithm choose between the different cases that may occur according to the outcome of the previous step and advance in a piecewise manner.

4. On the algorithm and the code

The mass can be either in stick state or in slip state (and slipping can occur in the right or left directions). The algorithm is to recognize whether the mass is stuck or it is slipping in every time step. It seems simple: to look at the velocity and if it zero then it is in stick state. Since it is hardly possible to detect the exact zero (if any) in the numerical solution of velocity, "zero velocity" is accepted to happen right at the beginning of the time increment in which the block changes sign

The loop syntax selected to perform this kind of piecewise solution of the EOM was at first written using a "for" command. But, the MATLAB ode23 function (which applies 4th order Runge-Kutta scheme for integration) automatically refines the given time step of the loop to even finer increments, it is quite possible that "the passing through zero" happen within the specified time step of the loop (and therefore missed). However, a "for" loop does not allow alternation of the loop variable (here, time) within itself. This fact makes it impossible to capture the "zero velocity" moment by a "for"
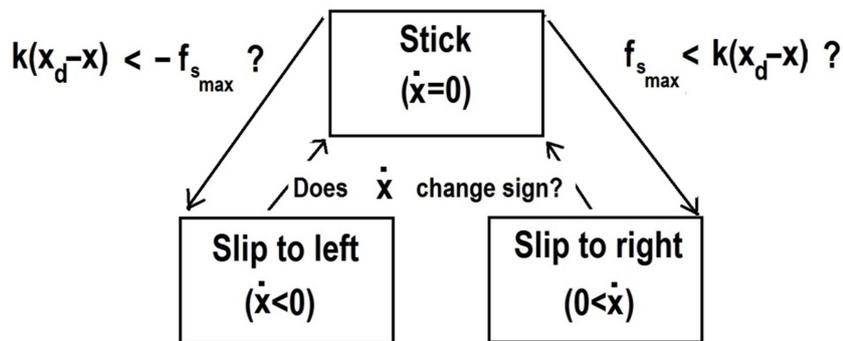
Figure 3: The part of the flowchart which determines the state of the block in each time step: Is it stuck? Does it continue being stuck? Will it slip to the right or to the left? Does it continue slipping?

loop. Then the proper loop command specified as "while". As a subsequence, the conditions for stick and slip became more straightforward. The flowchart for the code is shown in Figure 3.

As can be seen in figure 3, there is no bridge between slipping to the right and slipping to the left since the block has to pass through zero velocity point (i.e. stick state) and it is reasonable. In the case of zero initial conditions, the block is stuck at t=0. Figure 2 also specifies the very conditions for entering or exiting a stick or slip state. In each of these states the block is checked to see if it continues to remain in that state or it is to depart.

5. Results

Besides kinematics of the block motion, i.e. solution of the EOM (or what is called dynamic response of the block), kinetics of the motion (here we imply friction) is represented for a range of different μ_d values. Note that while for Coulomb law this quantity is the value of dynamic COF, it becomes a parameter in the rate and state dependent friction law. The other parameters of rate and state dependent friction law are as given in the table 1. The driver velocity is fixed to 1 mm/s. Mass of the block is sat to 1 kg, stiffness of the coil spring is 10 kN/m. The static COF is 0.6.

Effects of μ_d via the constitutive law of friction, eqn. (1), are investigated for three cases. In case (A) μ_d is sat to 0.1, in case (B) it is sat to 0.5 and in case (C) it is equal to 0.6, meaning that the dynamic COF is equal to the static threshold of friction.

The kinematics of case (A) is given in figure 4. The plots of position and velocity of the block (and the driver) against time are superimposed for both Coulomb and rate and state dependent friction laws are presented in figure 4. Note that the driver passes the block nearly at t=1s. Both of the friction laws trigger the stick-slip instability. Note that the driver passes the block nearly at t=1s. While the Coulomb law causes symmetric spikes of slip rate (the black line), under rate and state dependent law (the red line), those spikes are asymmetric, higher and sparser in time and start smoothly and end abruptly.

The kinetics of case (A) is represented in figure 5. The plots of shear divided by normal force (apparent COF) for both Coulomb and rate and state dependent friction laws are presented. While there is no transition from the stick state to the slip state under Coulomb law (the black line), there exist a smooth transition from stick to the slip for the rate and state dependent friction law (the red
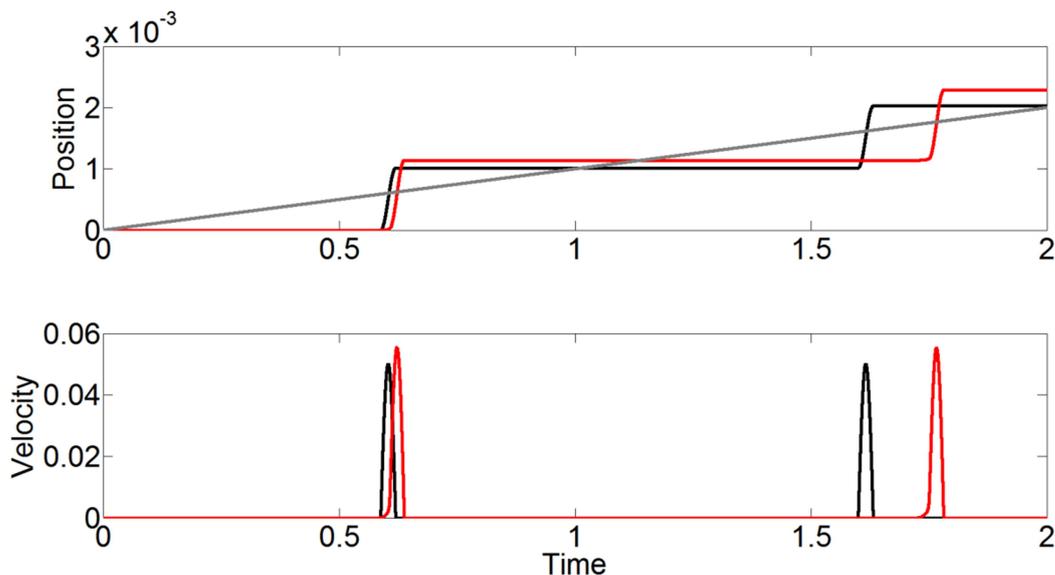
Figure 4: Case (A), for dynamic COF of 0.1, position (top) and velocity (bottom) of the mass block subjected to Coulomb law (in black), subjected to rate and state dependent friction law (in red) and the driver (in grey) against time for a constant driver velocity of 1 mm/s.
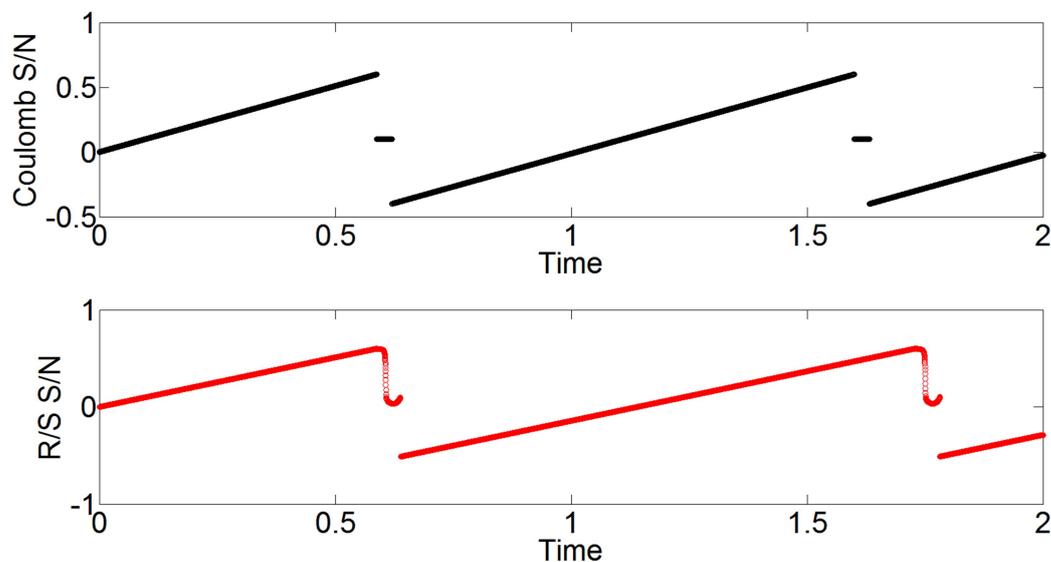

Figure 5: Case (A), for dynamic COF of 0.1, friction force (shear) divided by normal force (the block weight), conventionally called apparent COF is presented for Coulomb law (top) and rate- state dependent law (bottom) against time for a constant driver velocity of 1 mm/s.

line). The velocity weakening property of rate and state dependent friction law shows up in the concave shape of the COF in the slipping regions.

The kinematics of case (B) is given in figure 6. The plots of position and velocity of the block (and the driver) against time are superimposed for both Coulomb and rate and state dependent friction laws are
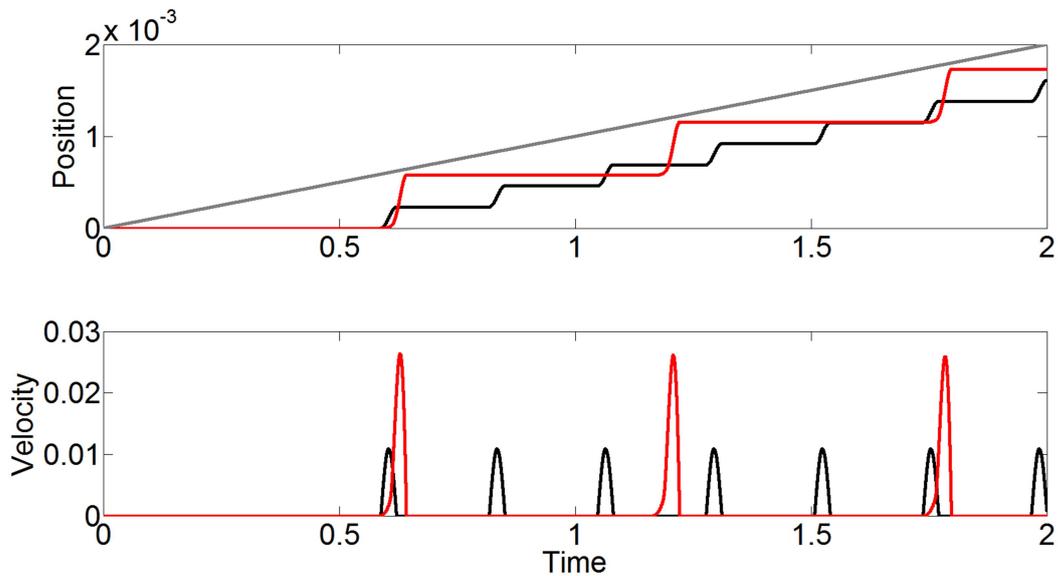
Figure 6: Case (B), for dynamic COF of 0.5, position (top) and velocity (bottom) of the mass block subjected to Coulomb law (in black), subjected to rate and state dependent friction law (in red) and the driver (in grey) against time for a constant driver velocity of 1 mm/s.
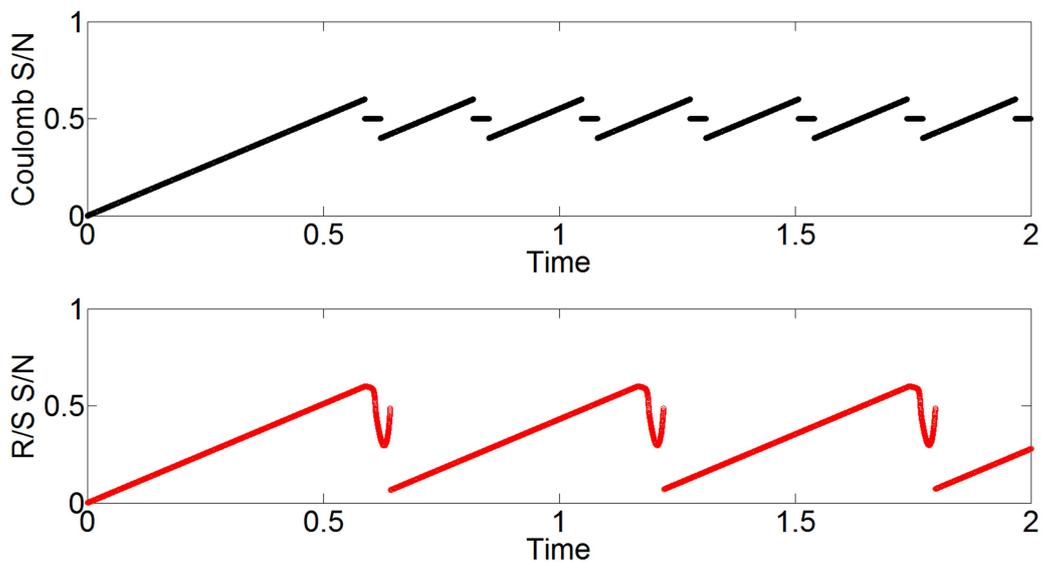

Figure 7: Case (B), for dynamic COF of 0.5, friction force (shear) divided by normal force (the block weight), conventionally called apparent COF is presented for Coulomb law (top) and rate- state dependent law (bottom) against time for a constant driver velocity of 1 mm/s.

presented in figure 6. Note that the driver never passes the block in this case, however, both of the friction laws trigger the stick-slip instability again. The spikes of slip rate have become more frequent and shorter it height.

The kinetics of case (B) is represented in figure 7. The plots of shear divided by normal force (apparent COF) for both of the laws are presented. The concave shape of drop in apparent COF has become more apparent, and well below the value of dynamic COF, 0.5.
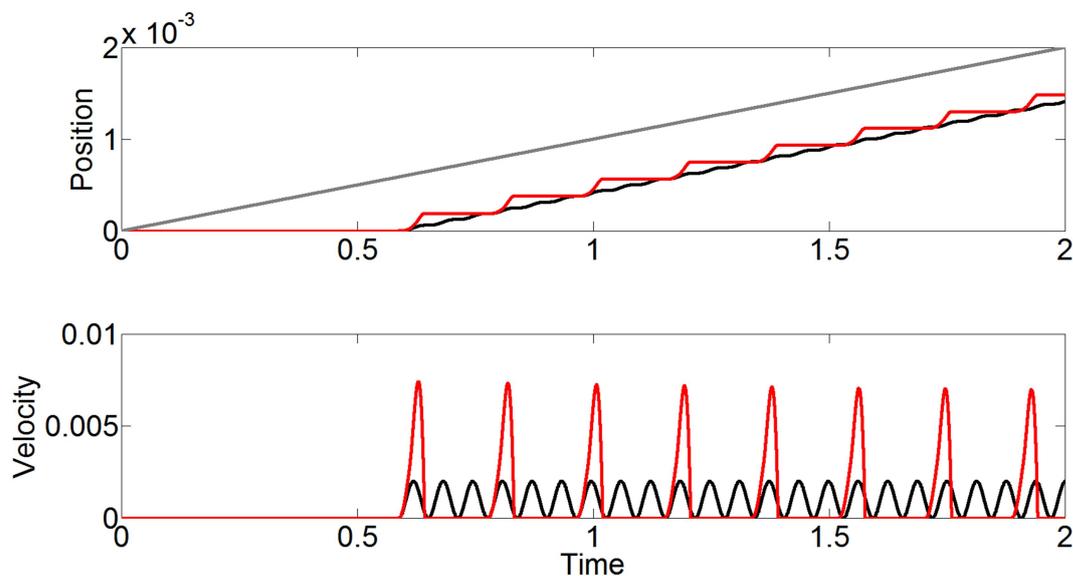
Figure 8: Case (C), for dynamic COF of 0.5999, position (top) and velocity (bottom) of the mass block subjected to Coulomb law (in black), subjected to rate and state dependent friction law (in red) and the driver (in grey) against time for a constant driver velocity of 1 mm/s.
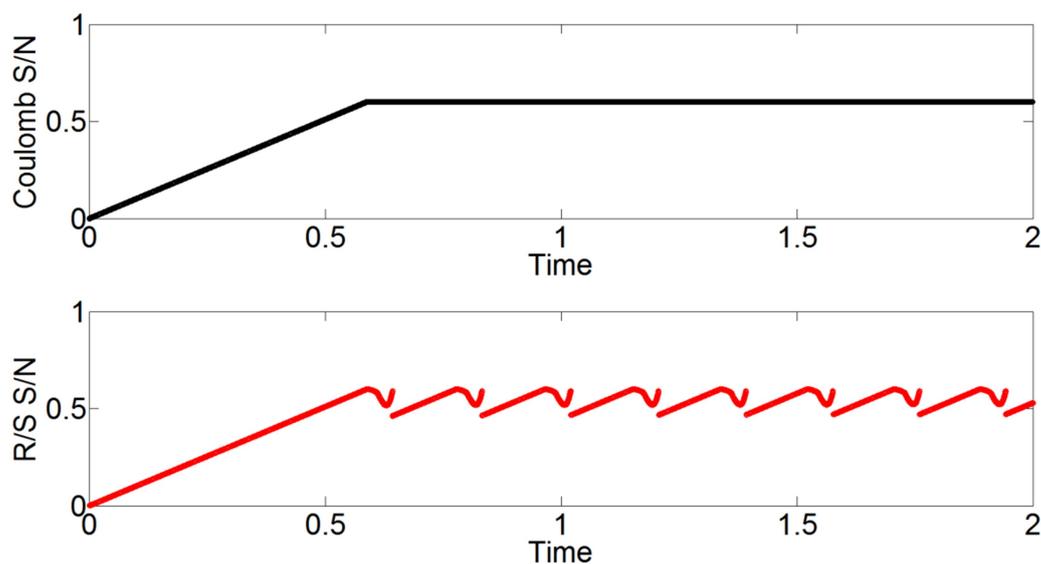

Figure 9: Case (C), for dynamic COF of 0.5999, friction force (shear) divided by normal force (the block weight), conventionally called apparent COF is presented for Coulomb law (top) and rate- state dependent law (bottom) against time for a constant driver velocity of 1 mm/s.

The kinematics of case (C) is given in figure 8. The plots of position and velocity of the block (and the driver) against time are superimposed for both Coulomb and rate and state dependent friction laws are presented in figure 8. Note that the driver does not pass the block. While the motion under Coulomb is oscillatory, stick-slip motion still happens for the rate and state- governed mass and the trend of increase in frequency and decrease in the height of slip rate spikes continues.

The kinetics of case (C) is represented in figure 9. The plots of shear divided by normal force (apparent COF) for both Coulomb and rate and state dependent friction laws are presented. While under Coulomb law (the black line), there exist a constant slip and therefore constant COF, rate and state dependent friction still causes decrease and increase of friction force during the slip period.

6. Conclusions

The rate and state dependent friction laws essentially proposed for modelling earthquakes and therefore rock plate sliding with abrupt change in their relative slip rate. Our study tried to detect differences in dynamics of a single block Burridge-Knopoff (BK) model under to Coulomb and rate and state dependent friction laws. During the slip period, instead of the step change of sliding velocity of the block an impulse of slip rate happen. This causes the apparent COF not to exactly imitate the behavior expected in figure 2. The velocity weakening nature of the law can be observed in the results. Increasing dynamic COF made slip spikes more frequent and less strong but could not eliminate stick-slip motion in the rate and state dependent law case. However, it caused a steady oscillatory motion in the Coulomb law - governed case.

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