The effects of non-standard Z couplings on the lepton polarizations in $B_s \to \gamma\, \ell^+ \ell^-$ decays

Gürsevil Turan
Physics Department, Middle East Technical University
06531 Ankara, Turkey

April 5, 2018

Abstract

The fact that the measured $B \to \pi\,\pi$, $\pi\,K$ branching ratios exhibit puzzling patterns has received a lot of attention in the literature and motivated the formulation of a series of new physics scenarios with enhanced Z-penguins. In this work, we analyze the effect of such an enhancement on the lepton polarization asymmetries of $B_s \to \gamma \ell^+ \ell^-$ decays by applying the results of a specific scenario for the solution of "$B \to \pi K$ puzzle".

In the Standard Model (SM), CP violation originates from the the Cabibbo–Kobayashi–Maskawa (CKM) three generation quark mixing matrix [1], and this picture successfully explains the observed CP violation in the Kaon sector . As for the B-sector, in the near future, more number of experimental tests will be possible at the the B-factories providing stringent testing of the SM mechanism of CP violation. In the mean time, some of the current B-factory data seems to be at variance with the SM description of CP violation for a number of processes. These processes can be grouped into two systems: $B \to \pi\pi$, $B \to \pi K$ and $B \to \psi K$, $B \to \phi K$ and have received a lot of attention in the literature. In particular;

- The BaBar and Belle collaborations have recently reported the branching ratios of $B_d \to \pi^0\pi^0$ decays [2, 3], which turn out to be as large as six times the value given in a recent calculation [4] within QCD factorization [5], whereas the calculation of $B_d \to \pi^+\pi^-$ gives a branching ratio about two times larger than the current experimental average. On the other hand, the calculation of $B^+ \to \pi^+\pi^0$ agrees with the data quite well. As was recently pointed out [6], this "$B \to \pi\pi$ hierarchy" can be conveniently accommodated in the SM through non-factorizable hadronic interference effects.

  The CLEO, BaBar and Belle collaborations have also measured the ratios of charged $R_{\rm c}$ and neutral $R_{\rm n}$ branching ratios for $B \to \pi K$ [7] that are affected significantly by colour-allowed electroweak (EW) penguins, and reported a pattern of $R_{\rm c} > 1$ and $R_{\rm n} < 1$. As noted in [8], this pattern contradicts with the one obtained for decays $B \to \pi K$ as the extension of the results for $B \to \pi\pi$ decays using SU(3) symmetry. It has been pointed out that the quantities that may only get contributions from colour-suppressed forms do not show any anomalous behavior like $R_{\rm c}$ and $R_{\rm n}$ do, so that this "$B \to \pi K$ puzzle" may be a manifestation of new physics (NP) in the EW penguin sector [6, 8, 9].

- The decay $B_d \to \phi K_{\rm s}$ provides another suggestive contrast with the SM expectation. There is a very good agreement between the SM value of $\sin 2\beta_{\phi K_s}$, which is the most precise information on CP violation in the quark sector, and the most well established B-factory result determined from the decay $B \to \psi K_s$. However, for the decay $B \to \phi K_s$, whose time dependent CP violation is predicted to be the same as in $B \to \psi K_s$ by the SM, experimental results seem to be providing a contrast between $\sin 2\beta_{\psi K_s}$ and $\sin 2\beta_{\phi K_s}$. Since within the SM, this transition is governed by QCD penguins [10] and receives sizeable EW penguin contributions [11, 12], with more data, this would suggest definite evidence for NP beyond the SM.

It has been argued that a promising class of models that can explain the discrepancies summarized above involve flavor changing couplings of the Z-boson to $\bar{b}s$. The possibility of NP with the dominant Z^0-penguin contributions was first considered in [13]–[15], where correlations between rare K decays and ε'/ε were studied in model-independent analyses and also within particular supersymmetric scenarios. It was generalized to rare B decays in [16]. Recently, in [17], the effects of enhanced Z penguins on the lepton polarization asymmetries of $b \to s\ell^+\ell^-$ have been considered.

In this work we analyze the effects of such a non-standard Z coupling on the lepton polarization asymmetries of $B_s \to \gamma\ell^+\ell^-$ decays by applying the results of a specific scenario

C_1	C_2	C_3	C_4	C_5	C_6	$C_7^{\rm eff}$	C_9	C_{10}
-0.248	$+1.107$	$+0.011$	-0.026	$+0.007$	-0.031	-0.313	$+4.344$	-4.624

Table 1: Values of the SM Wilson coefficients at $\mu \sim m_b$ scale.

considered in [6] for the solution of "$B \to \pi K$ puzzle". This basically involves considering the decays $B \to \pi\pi$ and $B \to \pi K$ simultaneously within the SM and its simplest extension in which NP enters dominantly through enhanced EW penguins with new weak phases. Since $B \to \pi\pi$ decays are only insignificantly effected by EW penguins, this system can be described as in the SM and allows the extraction of the relevant hadronic parameters by assuming only the isospin symmetry. Using SU(3) flavor symmetry it is then determined the hadronic $B \to \pi K$ parameters through their $B \to \pi\pi$ counterparts. Here there are two parameters, $C(x_t)$ with $x_t = m_t^2/M_W^2$, which is the Z- penguin function and θ, its complex phase. The relevant EW penguin parameters for $B \to \pi K$ decays are q and ϕ, whose SM values are $q = 0.69$ and $\phi = 0$. It has been found that pattern of $R_c > 1$ and $R_n < 1$ can not be described properly for these SM values; however, treating them as free parameters and using the hadronic $B \to \pi K$ parameters, it is possible to convert the experimental results for R_c and R_n into the values of q and ϕ,

$$q = 1.78^{+1.24}_{-0.97} \ , \ \phi = -(85^{+11}_{-13})^\circ \ , \tag{1}$$

which describe all currently available data.

For the radiative $B_s \to \gamma \, \ell^+\ell^-$decay, the basic quark level process is $b \to s\ell^+\ell^-$, which can be written as

$$\mathcal{H}_{eff} \ = \ \frac{\alpha G_F}{\sqrt{2}\,\pi} V_{tb} V_{ts}^* \bigg\{ C_9^{eff} (\bar{s}\gamma_\mu P_L b) \, \bar{\ell}\gamma^\mu \ell + C_{10} (\bar{s}\gamma_\mu P_L b) \, \bar{\ell}\gamma^\mu\gamma_5 \ell - 2 C_7 \frac{m_b}{q^2} (\bar{s} i \sigma_{\mu\nu} q^\nu P_R b) \bar{\ell}\gamma^\mu \ell \bigg\} \ , \tag{2}$$

where $P_{L,R} = (1 \mp \gamma_5)/2$, q is the momentum transfer and V_{ij}'s are the corresponding elements of the CKM matrix. The values of the individual Wilson coefficients that appear in the SM are listed in Table (1). As for the C_9, its value is given as [18, 19]

$$C_9^{eff}(\mu) = C_9(\mu) + Y(\mu) \ , \tag{3}$$

where the function $Y(\mu)$ contains a part which arises from the one loop contributions of the four quark operators O_1,...,O_6 whose explicit forms can be found in [18, 20], and also a part which estimates the long-distance contributions from intermediate states $J/\psi, \psi', ...$[19, 21].

In the SM, the Wilson coefficient C_{10} is given by

$$C_{10} \ = \ \frac{-1}{\sin^2\theta_w} Y \ , \tag{4}$$

with

$$Y \ = \ C(x_t) - B(x_t) \ , \tag{5}$$

where explicit formulae of $C(x_t)$ and $B(x_t)$ can be found in [20, 22]. For the rare B decays with $\ell^+\ell^-$ in the final state the short distance function Y can be parametrized as

$$Y = |C|\, e^{i\theta} + 0.18\,, \tag{6}$$

and the connection between the rare decays and the $B \to \pi K$ system is established by relating the parameters (C, θ) to the EW penguin parameters (q, ϕ) by means of a renormalization group analysis that yields [6]

$$|C|\, e^{i\theta} = 2.35\bar{q}e^{i\phi} - 0.82\ ,\ \bar{q} = q\left[\frac{|V_{ub}/V_{cb}|}{0.086}\right]. \tag{7}$$

Furthermore, Y is rewritten as

$$Y = |Y|\, e^{i\theta_Y}\,, \tag{8}$$

where the constraint that $|Y| \leq 2.2$ follows from the BaBar and Belle data on $B \to X_s\mu^+\mu^-$ decay [23]. The central value for Y resulting from (1) violates the upper bound of $|Y|$, however considering only the subset of those values of (q, ϕ) that satisfies $|Y| = 2.2$ gives $\theta_Y = -(103 \pm 12)^\circ$. Now, with the contributions from the enhanced and complex value of the bsZ vertex, C_{10} becomes

$$C_{10} = \frac{-2.2}{\sin^2\theta_w}\, e^{i(103/180)\pi}\,, \tag{9}$$

and has a magnitude twice the SM one.

Having established the general form of the effective Hamiltonian, the next step is to calculate the matrix element of the $B_s \to \gamma\,\ell^+\ell^-$ decay, which is induced by the inclusive $b \to s\gamma\,\ell^+\ell^-$ one. Thus, the related matrix element can be found from the $b \to s\,\ell^+\ell^-$ decay by attaching a photon line to any charged internal or external line. However, contributions coming from the release of the free photon from any charged internal line will be suppressed by a factor of m_b^2/M_W^2 and can be neglected. When a photon is released from the initial quark lines it contributes to the so-called "structure dependent" (SD) part of the amplitude, whereas, "internal Bremsstrahlung" (IB) part of the amplitude arises when a photon is radiated from one of the final ℓ- leptons. Therefore, the total matrix element can be written as a sum of the SD and the IB contributions:

$$\mathcal{M} = \mathcal{M}_{SD} + \mathcal{M}_{IB}\,, \tag{10}$$

with

$$\mathcal{M}_{SD} = \langle\gamma(k)\,|\mathcal{H}_{eff}|\,B(p_B)\rangle\ ,\ \mathcal{M}_{IB} = \langle 0\,|\mathcal{H}_{eff}|\,B(p_B)\rangle\ . \tag{11}$$

The structures in Eq. (11) are parametrized in terms of the various form factors f, g, f_1, g_1, calculated in the framework of light-cone QCD sum rules [24, 25] and in the framework of the light front quark model [26]. In addition, it has been proposed another model in [27] for the $B \to \gamma$ form factors which obey all the restrictions obtained from the gauge invariance combined with the large energy effective theory.

By using the explicit expressions of the parametrizations for the above matrix elements in terms of the form factors, which can be found in [25], we calculate dilepton mass distribution for $B \to \gamma\, \ell^- \ell^+$ decay as [28, 29]

$$\frac{d\Gamma}{ds} = \frac{\alpha^3\, G_F^2\, |V_{tb} V_{ts}^*|^2}{2^{10}\pi^4}\, m_B \Delta \ , \tag{12}$$

where

$$\begin{aligned}
\Delta &= \Bigg\{ (1-s)^3 v \Bigg(4 m_B^2 r\, \mathrm{Re}[A_1 B_1^* + A_2 B_2^*] + \frac{2}{3} m_B^2 \Big(|A_1|^2 + |A_2|^2 + |B_1|^2 \\
&+ |B_2|^2 \Big)(s-r) \Bigg) + 4 f_B\, m_\ell \ln[u]\, \mathrm{Re}[(A_1 + B_1) F^*]\, (1-s)^2 \\
&- 4 f_B^2\, |F|^2 \Bigg(2v \frac{s}{(1-s)} + \ln[u] \Bigg[2 + \frac{2(2r-1)}{(1-s)} - (1-s) \Bigg] \Bigg) \Bigg\} \ .
\end{aligned} \tag{13}$$

Now, we would like to discuss the lepton polarizations in the rare $B_s \to \gamma\, \ell^+ \ell^-$decays. For this, it is introduced spin projection operator $N = (1 + \gamma_5 \not{S}_i)/2$ for ℓ^-, where $i = L,\ T,\ N$ correspond to longitudinal, transverse and normal polarizations, respectively. In the rest frame of ℓ^-, the orthogonal unit vectors S_i are defined as

$$\begin{aligned}
S_L^\mu &\equiv (0, \vec{e}_L) = \left(0, \frac{\vec{p}_1}{|\vec{p}_1|} \right), \\
S_N^\mu &\equiv (0, \vec{e}_N) = \left(0, \frac{\vec{k} \times \vec{p}_1}{|\vec{k} \times \vec{p}_1|} \right), \\
S_T^\mu &\equiv (0, \vec{e}_T) = \left(0, \vec{e}_N \times \vec{e}_L \right).
\end{aligned} \tag{14}$$

The longitudinal unit vector S_L is boosted to the CM frame of $\ell^+ \ell^-$ by Lorentz transformation:

$$S_{L,CM}^\mu = \left(\frac{|\vec{p}_1|}{m_\ell}, \frac{E_\ell\, \vec{p}_1}{m_\ell |\vec{p}_1|} \right), \tag{15}$$

while P_T and P_N are not changed by the boost since they lie in the perpendicular directions. For $i = L,\ T,\ N$, the polarization asymmetries P_i of the final ℓ^- lepton are defined as

$$P_i(s) = \frac{\frac{d\Gamma}{ds}(\vec{n} = \vec{e}_i) - \frac{d\Gamma}{ds}(\vec{n} = -\vec{e}_i)}{\frac{d\Gamma}{ds}(\vec{n} = \vec{e}_i) + \frac{d\Gamma}{ds}(\vec{n} = -\vec{e}_i)} \, . \tag{16}$$

After some algebra, we obtain the following expressions for the differential polarization components of the ℓ^- lepton in $B_s \to \gamma\, \ell^+ \ell^-$decays:

$$\begin{aligned}
P_L(s) &= \frac{1}{6v\Delta} \Bigg\{ 4 m_B^2 v^3 s (1-s)^3 (|A_1|^2 + |A_2|^2 - |B_1|^2 - |B_2|^2) \\
&+ 24 f_B m_\ell (s-1) \Bigg[(s-1) \mathrm{Re}[(A_1 - B_1) F^*] \Big(v + \frac{(2r-s)}{s} \ln[u] \Big) \\
&+ (1+s) \mathrm{Re}[(A_2 + B_2) F^*] \Big(v(1-s) - \frac{2r}{s} \ln[u] \Big) \Bigg] \Bigg\} ,
\end{aligned} \tag{17}$$

$$
\begin{aligned}
P_T(s) &= \frac{1}{\Delta}\Bigg\{\frac{(2\sqrt{r}-\sqrt{s})}{sv}(1-s)f_B m_B \pi\Bigg(\pm s v^2(1+s)\mathrm{Re}[(A_1-B_1)F^*] \\
&+ (s-1)(4r+s)\mathrm{Re}[(A_2+B_2)F^*]\Bigg) \\
&- \frac{\pi v}{4\sqrt{s}}(s-1)^2 2m_B^2\sqrt{r}(s-1)s\,\mathrm{Re}[(A_1+B_1)(A_2+B_2)^*]\Bigg\}, \qquad (18)
\end{aligned}
$$

$$
\begin{aligned}
P_N(s) &= \frac{\pi}{4\Delta}(1-s)m_B\Bigg\{2m_B(1-s)^2\sqrt{rs}\,v^2(\mathrm{Im}[A_1B_2^*]+\mathrm{Im}[A_2B_1^*]) \\
&- 4(2\sqrt{r}-\sqrt{s})f_B\Big((1+s)\mathrm{Im}[(A_1+B_1)F^*]+(1-s)\mathrm{Im}[(A_2-B_2)F^*]\Big)\Bigg\}. \qquad (19)
\end{aligned}
$$

We present now our numerical analysis about the differential polarization asymmetries $P_L(s)$, $P_T(s)$ and $P_N(s)$ of ℓ^- for the $B_s \to \gamma\ell^+\ell^-$ decays with $\ell = \mu, \tau$, as well as the averaged polarization asymmetries $< P_L >$, $< P_T >$ and $< P_N >$. The input parameters used in our numerical analysis are as follows:

$$
\begin{aligned}
&m_B = 5.28\,GeV\,,\ m_b = 4.8\,GeV\,,\ m_\mu = 0.105\,GeV\,,\ m_\tau = 1.78\,GeV\,, \\
&f_B = 0.2\,GeV\,,\ |V_{tb}V_{ts}^*| = 0.045\,,\ \alpha^{-1} = 137\,, G_F = 1.17\times 10^{-5}\,GeV^{-2} \\
&\tau_{B_s} = 1.54\times 10^{-12}\,s\,. \qquad (20)
\end{aligned}
$$

To make some numerical predictions, we also need the explicit forms of the form factors g, f, g_1 and f_1. In this work we have used the values calculated in the framework of light-cone QCD sum rules given in [25].

We also like to note a technical point about calculations of averaged polarization asymmetries. The averaging procedure which we have adopted is

$$
\langle P_i \rangle = \frac{\int_{(2m_\ell/m_B)^2}^{1-\delta} P_i(s)\,\frac{d\Gamma}{ds}ds}{\int_{(2m_\ell/m_B)^2}^{1-\delta} \frac{d\Gamma}{ds}ds}\,. \qquad (21)
$$

We note that the part of $d\Gamma/ds$ in (12) which receives contribution from the $|\mathcal{M}_{IB}|^2$ term has infrared singularity due to the emission of soft photon. To obtain a finite result from these integrations, we follow the approach described in [25] and impose a cut on the photon energy, i.e., we require $E_\gamma \geq 25$ MeV, which corresponds to detect only hard photons experimentally. This cut implies that $E_\gamma \geq \delta\, m_B/2$ with $\delta = 0.01$.

In Figs. (1)-(6) we present our results for the various differential polarization asymmetries within the SM and also with the new value of the coefficient C_{10} given by (9), which results from the enhanced values of the Z penguins. In table (2), we have given the averaged values of these asymmetries. As can be seen, the new value of C_{10} can give substantial changes in the SM results. We note that due to the complex and enhanced value of the bsZ vertex, the value of longitudinal polarizations decrease as compared to their SM values for both decay modes, however, the transverse and normal asymmetries show a substantial increase from their respective SM values. This increase is especially manifest for P_N, which changes its sign too as compared to its SM value and its magnitude increases by one order of magnitude for μ mode, and by almost 200 % for τ mode.

Therefore, future measurements of the enhanced normal and transverse polarization asymmetry would be a suitable testing ground for the validity of a complex bsZ vertex and the model in ref. [6].

Decay Mode	$B_s \to \gamma\,\mu^+\mu^-$				$B_s \to \gamma\,\tau^+\tau^-$			
	BR $\times 10^8$	$\mathcal{P}_L^-$	$\mathcal{P}_T^-$	$\mathcal{P}_N^-$	BR $\times 10^8$	$\mathcal{P}_L^-$	$\mathcal{P}_T^-$	$\mathcal{P}_N^-$
SM	1.51	- 0.85	- 0.07	- 0.01	1.14	- 0.23	- 0.19	- 0.07
enhanced bsZ	3.70	0.08	-0.15	0.10	4.68	0.07	-0.25	0.22

Table 2: Predictions of the observables.

References

[1] N. Cabibbo, *Phys. Rev. Lett.* **10**, (1963) 531; M. Kobayashi and T. Maskawa, *Prog. Theor. Phys.* **49**, (1973) 652.

[2] B. Aubert *et al.* [BaBar Collaboration], *Phys. Rev. Lett.* **91**, (2003) 241801.

[3] K. Abe *et al.* [Belle Collaboration], *Phys. Rev. Lett.* **91**, (2003) 261801.

[4] M. Beneke and M. Neubert, *Nucl. Phys.* **B675**, (2003) 333.

[5] M. Beneke, G. Buchalla, M. Neubert and C.T. Sachrajda, *Phys. Rev. Lett.* **83**, (1999) 1914.

[6] A.J. Buras, R. Fleischer, S. Recksiegel and F. Schwab, *Phys. Rev. Lett.* **92**, (2004) 101804; A.J. Buras, R. Fleischer, S. Recksiegel and F. Schwab, hep-ph/0402112.

[7] A.J. Buras and R. Fleischer, *Eur. Phys. J.* **C11** (1999) 93.

[8] A.J. Buras and R. Fleischer, *Eur. Phys. J.* **C16**, (2000) 97.

[9] A.J. Buras, R. Fleischer, S. Recksiegel and F. Schwab, *Eur. Phys. J.* **C32**, (2003) 45.

[10] D. London and R. Peccei, *Phys. Lett.* **B223**, (1989) 257;
N.G. Deshpande and J. Trampetic, *Phys. Rev.* **D41**, (1990) 895 and 2926;
J.-M. Gérard and W.-S. Hou, *Phys. Rev.* **D43**, (1991) 2909.

[11] R. Fleischer, *Z. Phys.* **C62**, (1994) 81.

[12] N.G. Deshpande and X.-G. He, *Phys. Lett.* **B336**, (1994) 471.

[13] A.J. Buras and L. Silvestrini, *Nucl. Phys.* **B546**, (1999) 299.

[14] A.J. Buras, A. Romanino and L. Silvestrini, *Nucl. Phys.* **B520**, (1998) 3.

[15] A.J. Buras, G. Colangelo, G. Isidori, A. Romanino and L. Silvestrini, *Nucl. Phys.* **B566**, (2000) 3.

[16] G. Buchalla, G. Hiller and G. Isidori, *Phys. Rev.* **D63**, (2001) 014015;
D. Atwood and G. Hiller, LMU-09-03 [hep-ph/0307251].

[17] S. Rai Choudhury and N. Gaur, hep-ph/0402273.

[18] B. Grinstein, M. J. Savage and M. B. Wise, *Nucl. Phys.* **B319**, (1989) 271; A. J. Buras and M. Münz, *Phys. Rev.* **D52**, (1995) 186.

[19] A. Ali, T. Mannel and T. Morozumi, *Phys. Lett.* **B273**, (1991) 505; C. S. Lim, T. Morozumi and A. I. Sanda, *Phys. Lett.* **B218**, (1989) 343; N. G. Deshpande, J. Trampetic and K. Panose, *Phys. Rev.* **D39**, (1989) 1461; P. J. O'Donnell and H. K. Tung, *Phys. Rev.* **D43**, (1991) 2067 .

[20] M. Misiak, *Nucl. Phys.* **B393**, (1993) 23.

[21] F. Krüger and L. M. Sehgal, *Phys. Lett.* **B380**, (1996) 199; J. L. Hewett, *Phys. Rev.* **D53**, (1996)4964; S. Rai Choudhury, A. Gupta and N. Gaur, *Phys. Rev.* **D60**, (1999) 115004; S. Fukae, C. S. Kim and T. Yoshikawa, *Phys. Rev.* **D61**, (2000) 074015.

[22] G. Buchalla and A.J. Buras, *Phys. Lett.* **B333**, (1994) 221; *Phys. Rev.* **D54**, (1996) 6782.

[23] J. Kaneko *et al.* [Belle Collaboration], *Phys. Rev. Lett.* **90**, (2003) 021801; B. Aubert *et al.* [BaBar Collaboration], hep-ex/0308016.

[24] G. Eilam, I. Halperin and R. Mendel, *Phys. Lett.* **B 361**,(1995) 137.

[25] T. Aliev, A. Özpineci and M. Savci, *Phys. Rev.* **D55**, (1997) 7059 .

[26] C. Q. Geng, C. C. Lih and W. M. Zhang, *Phys. Rev.* **D62**, (2000) 074017.

[27] F. Krüger and D. Melikhov, *Phys. Rev.* **D67**, (2003) 034002.

[28] T. Aliev, N. K. Pak and M. Savci, *Phys. Lett.* **B424**, (1998) 175 .

[29] E. O. Iltan, and G. Turan , *Phys. Rev.* **D61**, (2000) 034010.

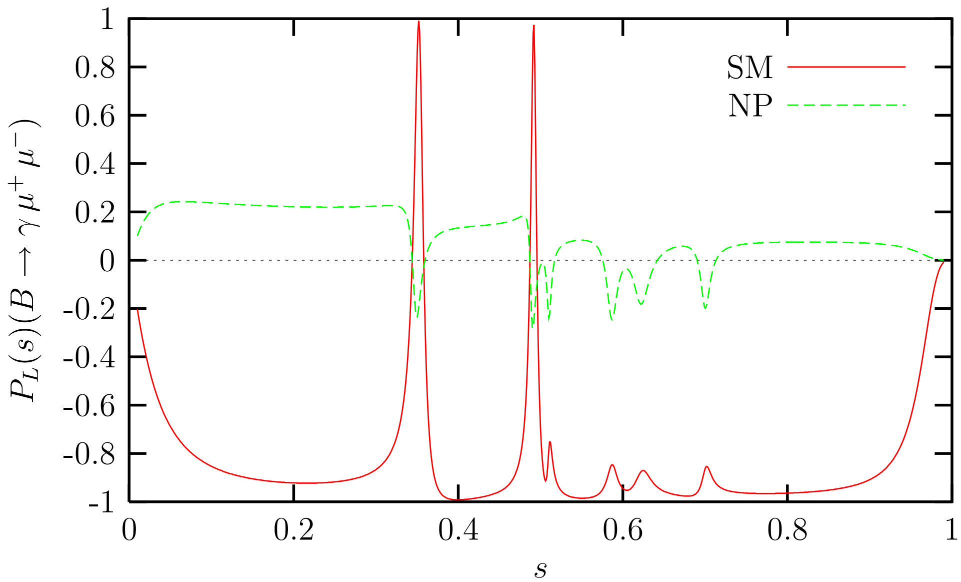

Figure 1: The dependence of the longitudinal polarization asymmetry $P_L(s)$ of ℓ^- for the $B_s \to \gamma\,\mu^+\mu^-$ decay on s .

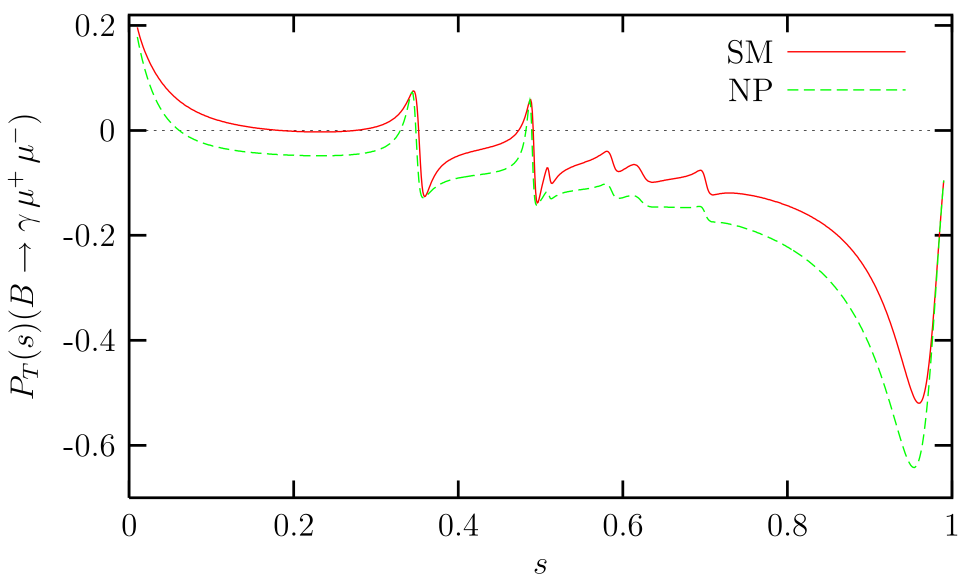

Figure 2: The dependence of the transverse polarization asymmetry $P_T(s)$ of ℓ^- for the $B_s \to \gamma\,\mu^+\mu^-$ decay on s .

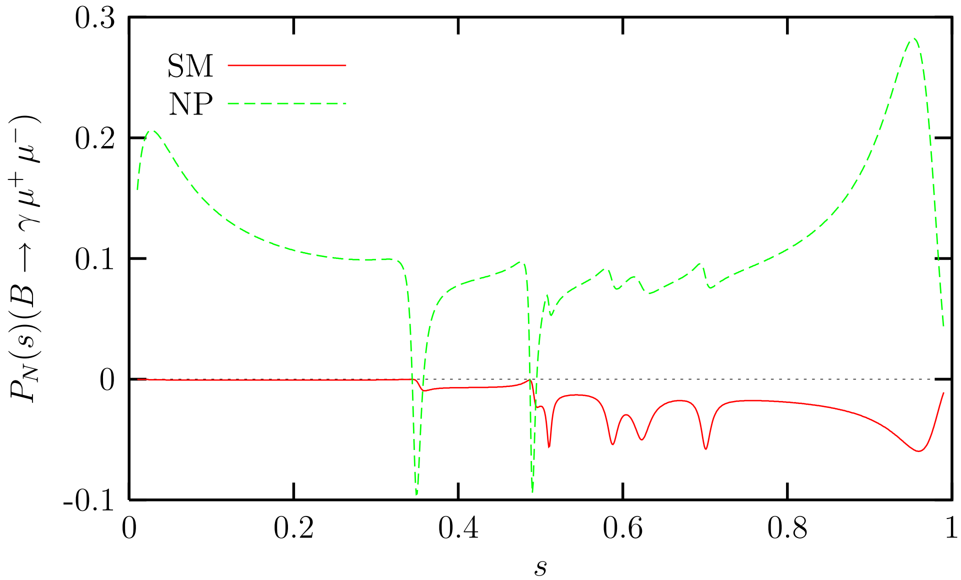

Figure 3: The dependence of the normal polarization asymmetry $P_N(s)$ of ℓ^- for the $B_s \to \gamma\,\mu^+\mu^-$ decay on s .

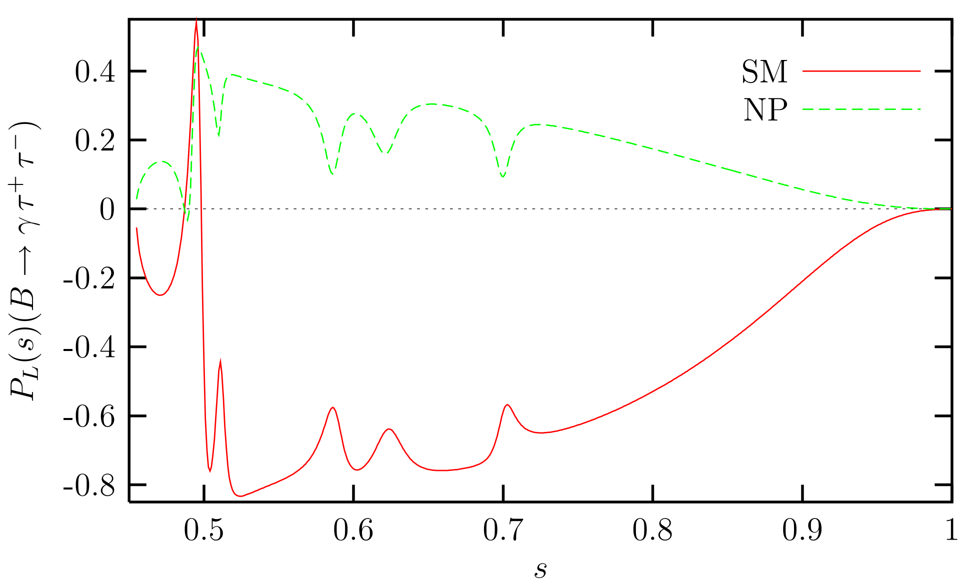

Figure 4: The same as Fig.(1), but for the $B_s \to \gamma\,\tau^+\tau^-$ decay.

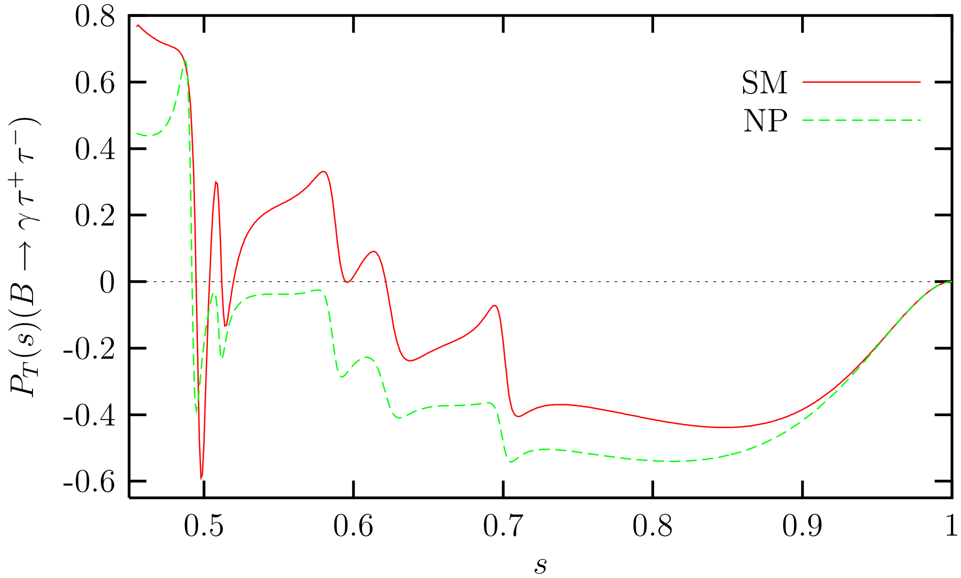

Figure 5: The same as Fig.(2), but for the $B_s \to \gamma\,\tau^+\tau^-$ decay .

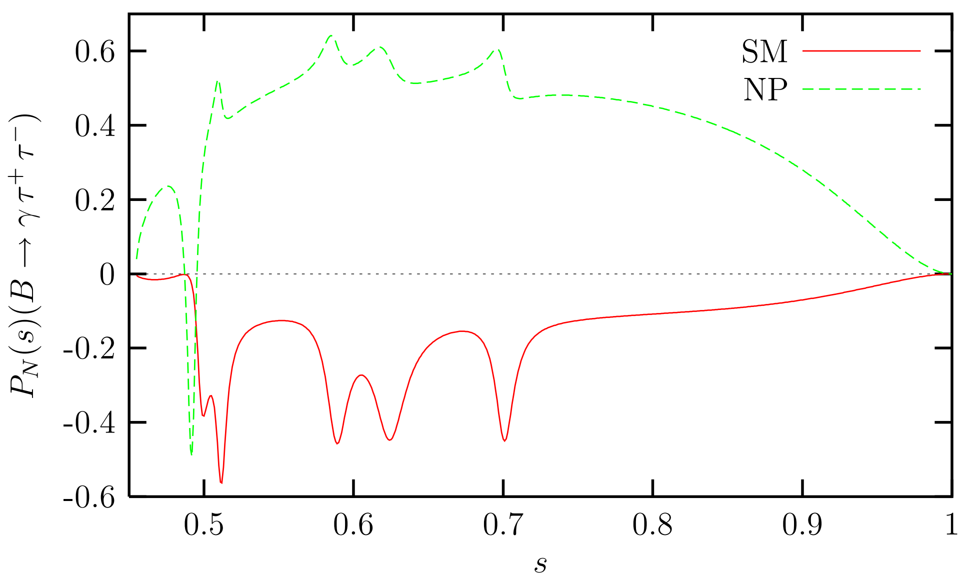

Figure 6: The same as Fig.(3), but for the $B_s \to \gamma\,\tau^+\tau^-$ decay.