

ANALYSIS OF THE RARE $B_c \rightarrow D_{s,d}^* l^+ l^-$ DECAYS IN QCD

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The rare $B_c \rightarrow D_{s,d}^* l^+ l^-$ decays are investigated in the framework of the three point QCD sum rules approach. Considering the gluon condensate corrections to the correlation function, the form factors relevant to these transitions are calculated. The total decay width and branching ratio for these decays are also evaluated. The results for the branching ratios are in good agreement with the quark models.

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I. INTRODUCTION

The rare $B_c \rightarrow D_{s,d}^* l^+ l^-$ decays are proceeded by flavor changing neutral current (FCNC) transitions of $b \rightarrow s, d$. In the standard model (SM), these transitions occur at loop level and are not allowed in the tree level. This provides the most crucial framework to test the SM [1, 2]. Among the B mesons, the B_c decays have received great attention for the following reasons:

a) This meson constitutes a very rich laboratory for studying various decay channels, which are essential from both theoretical and experimental aspects. At LHC when it begins operation with the luminosity values of $\mathcal{L} = 10^{34} \text{cm}^{-2} \text{s}^{-1}$ and $\sqrt{s} = 14 \text{TeV}$, the number of B_c^\pm mesons is expected to be about $10^8 \sim 10^{10}$ per year [3, 4], so there are basic facilities to study not only some rare B_c decays, but also CP violation, T violation and polarization asymmetries.

b) B_c decays could be used for a determination of the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements V_{tq} ($q = d, s, b$).

c) It is the lowest bound state of two heavy quarks (b and c) with open flavor. This is also the reason why B_c

decays weakly and not strongly or electromagnetically.

d) It attracts the interest of physicists for checking predictions of the perturbative QCD in the laboratory.

e) The $b \rightarrow s, d$ transitions are very sensitive to the physics beyond the SM since some new particles might have contributions in the loops diagrams.

Some possible channels of B_c decays are $B_c \rightarrow l\bar{\nu}\gamma$, $B_c \rightarrow \rho^+\gamma$, $B_c \rightarrow K^{*+}\gamma$, $B_c \rightarrow B_u^*l^+l^-$, $B_c \rightarrow B_u^*\gamma$ and $B_c \rightarrow D_{s,d}^*\gamma$ which have been studied in the framework of light-cone and three point QCD sum rules [5, 6, 7, 8, 9]. Larger sets of exclusive non-leptonic and semi-leptonic decays of the B_c meson have been studied within a relativistic constituent quark model in Ref- [10]. This study describes the rare $B_c \rightarrow D_{s,d}^* l^+l^-$ decays in the framework of the three point QCD sum rules approach. Here, $l = e, \mu, \tau$ and $D_{s,d}^*$ are vector mesons. Analyzing these transitions could give valuable information about the nature of the vector D_s^* meson.

In $B_c \rightarrow D_{s,d}^* l^+l^-$ decays, the long distance dynamics are parameterized by transition form factors, calculation of which is a central problem for these decays. To calculate the form factors, we use the three point QCD sum rules approach (for details about this method see [11, 12]). This method has been successfully applied for various problems [13, 14, 15, 16, 17, 18, 19, 20]. Note that, these transitions have been analyzed in the framework of the relativistic constituent quark models (RCQM) [21], light front quark model (LFQM) and constituent quark model (CQM) [22].

The paper encompasses three sections: In section II, we calculate the expressions for transition form factors, where the light quark condensates don't contribute but the gluon correction contributions are considered. The numerical analysis, discussion and comparison of our results with the predictions of other quark models are presented in section III.

II. SUM RULES FOR THE $B_c \rightarrow D_{q'}^* l^+ l^-$ TRANSITION FORM FACTORS

In the SM, the effective Hamiltonian for $B_c \rightarrow D_{q'}^* l^+ l^-$ ($q' = s, d$) decays which occur via $b \rightarrow q' l^+ l^-$ loop transition can be written as:

$$H_{eff} = \frac{G_F \alpha}{2\sqrt{2}\pi} V_{tb} V_{tq'}^* \left[C_9^{eff} \bar{q}' \gamma_\mu (1 - \gamma_5) b \bar{l} \gamma_\mu l + C_{10} \bar{q}' \gamma_\mu (1 - \gamma_5) b \bar{l} \gamma_\mu \gamma_5 l - 2C_7^{eff} \frac{m_b}{q^2} \bar{q}' i \sigma_{\mu\nu} q^\nu (1 + \gamma_5) b \bar{l} \gamma_\mu l \right], \quad (1)$$

where C_7^{eff} , C_9^{eff} and C_{10} are Wilson coefficients related to the Z and photon penguin (see Fig. 1) and box diagrams. For more about the Wilson coefficients see [7, 21] and references therein. The transition amplitude of the $B_c \rightarrow D_{q'}^* l^+ l^-$ is obtained by sandwiching of the effective Hamiltonian between the initial and final states

$$M = \frac{G_F \alpha}{2\sqrt{2}\pi} V_{tb} V_{tq'}^* \left[C_9^{eff} \langle D_{q'}^*(p', \varepsilon) | \bar{q}' \gamma_\mu (1 - \gamma_5) b | B_c(p) \rangle \bar{l} \gamma_\mu l + C_{10} \langle D_{q'}^*(p', \varepsilon) | \bar{q}' \gamma_\mu (1 - \gamma_5) b | B_c(p) \rangle \bar{l} \gamma_\mu \gamma_5 l \right. \\ \left. - 2C_7^{eff} \frac{m_b}{q^2} \langle D_{q'}^*(p', \varepsilon) | \bar{q}' i \sigma_{\mu\nu} q^\nu (1 + \gamma_5) b | B_c(p) \rangle \bar{l} \gamma_\mu l \right], \quad (2)$$

where p and p' are the initial and final meson states, respectively, and ε is the polarization vector of $D_{q'}^*$ meson. Our aim is to calculate the matrix elements appearing in Eq. (2). From Lorentz invariance and parity conservation point

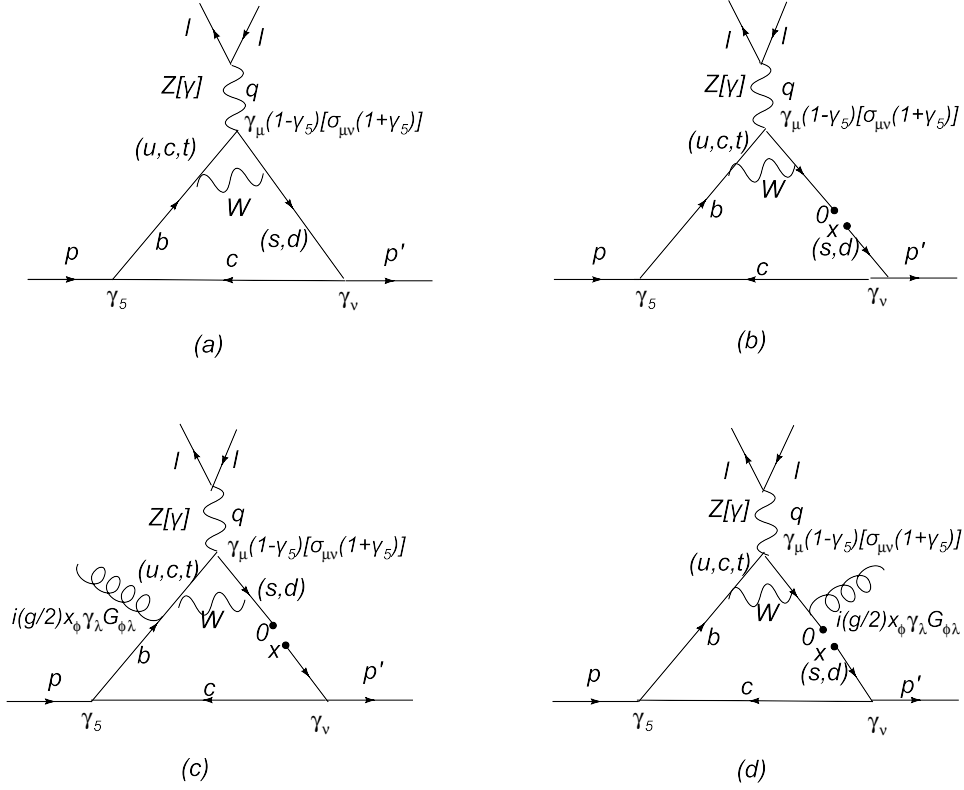


FIG. 1: Loop diagrams for $B_c \rightarrow D_{s,d}^* l^+ l^-$ transitions, bare -loop (diagram a) and light quark condensates (without any gluon diagram b and with one gluon emission diagrams c, d)

of view, these matrix elements can be parameterized in terms of the form factors in the following way:

$$\begin{aligned}
 \langle D_{q'}^*(p', \varepsilon) | \bar{q}' \gamma_\mu (1 - \gamma_5) b | B_c(p) \rangle &= \frac{2A_V(q^2)}{(m_{B_c} + m_{D_{q'}^*})} \varepsilon_{\mu\nu\alpha\beta} \varepsilon^{*\nu} p^\alpha p'^\beta - iA_0(q^2)(m_{B_c} + m_{D_{q'}^*}) \varepsilon_\mu^* \\
 &+ i \frac{A_+(q^2)}{(m_{B_c} + m_{D_{q'}^*})} (\varepsilon^* p) P_\mu + i \frac{A_-(q^2)}{(m_{B_c} + m_{D_{q'}^*})} (\varepsilon^* p) q_\mu,
 \end{aligned} \quad (3)$$

$$\begin{aligned}
 \langle D_{q'}^*(p', \varepsilon) | \bar{q}' \sigma_{\mu\nu} q^\nu (1 + \gamma_5) b | B_c(p) \rangle &= 2T_1(q^2) i \varepsilon_{\mu\nu\alpha\beta} \varepsilon^{*\nu} p^\alpha p'^\beta + T_2(q^2) \left\{ \varepsilon_\mu^* (m_{B_c}^2 - m_{D_{q'}^*}^2) \right. \\
 &\left. - (\varepsilon^* p) P_\mu \right\} + T_3(q^2) (\varepsilon^* p) \left\{ q_\mu - \frac{q^2}{m_{B_c}^2 - m_{D_{q'}^*}^2} P_\mu \right\},
 \end{aligned} \quad (4)$$

where $A_V(q^2)$, $A_0(q^2)$, $A_+(q^2)$, $A_-(q^2)$, $T_1(q^2)$, $T_2(q^2)$ and $T_3(q^2)$ are the transition form factors. $P_\mu = (p + p')_\mu$ and $q_\mu = (p - p')_\mu$, here, q is the momentum of the Z boson (photon). Also, for simplicity, we redefine those as:

$$\begin{aligned}
 A'_V(q^2) &= \frac{2A_V(q^2)}{(m_{B_c} + m_{D_{q'}^*})}, & A'_0(q^2) &= A_0(q^2)(m_{B_c} + m_{D_{q'}^*}), \\
 A'_+(q^2) &= -\frac{A_+(q^2)}{(m_{B_c} + m_{D_{q'}^*})}, & A'_-(q^2) &= -\frac{A_-(q^2)}{(m_{B_c} + m_{D_{q'}^*})}, \\
 T'_V(q^2) &= -2T_1(q^2), & T'_0(q^2) &= -T_2(q^2)(m_{B_c}^2 - m_{D_{q'}^*}^2), \\
 T'_-(q^2) &= -T_3(q^2).
 \end{aligned} \quad (5)$$

For the calculation of these form factors, the QCD sum rules method is applied. Following the general philosophy of the QCD sum rules, we start by considering the following correlators:

$$\begin{aligned}\Pi_{\nu\mu}^{V-A}(p^2, p'^2, q^2) &= i^2 \int d^4x d^4y e^{-ipx} e^{ip'y} \langle 0 | T[J_{\nu D_{q'}^*}(y) J_{\mu}^{V-A}(0) J_{B_c}(x)] | 0 \rangle, \\ \Pi_{\nu\mu}^{T-PT}(p^2, p'^2, q^2) &= i^2 \int d^4x d^4y e^{-ipx} e^{ip'y} \langle 0 | T[J_{\nu D_{q'}^*}(y) J_{\mu}^{T-PT}(0) J_{B_c}(x)] | 0 \rangle,\end{aligned}\quad (6)$$

where $J_{\nu D_{q'}^*}(y) = \bar{c}\gamma_{\nu}q'$ and $J_{B_c}(x) = \bar{b}\gamma_5 c$ are interpolating currents of the initial and final meson states, respectively. $J_{\mu}^{V-A} = \bar{q}'\gamma_{\mu}(1 - \gamma_5)b$ and $J_{\mu}^{T-PT} = \bar{q}'\sigma_{\mu\nu}q'(1 + \gamma_5)b$ are the vector, axial vector, tensor and pseudo tensor parts of the transition currents. To calculate the phenomenological part of the correlators given in Eq. (6), two complete sets of intermediate states with the same quantum numbers as the currents $J_{D_{q'}^*}$ and J_{B_c} are inserted. As a result of this procedure, we get the following representation of the above-mentioned correlators:

$$\begin{aligned}\Pi_{\nu\mu}^{V-A}(p^2, p'^2, q^2) &= - \\ &\frac{\langle 0 | J_{D_{q'}^*}' | D_{q'}^*(p', \varepsilon) \rangle \langle D_{q'}^*(p', \varepsilon) | J_{\mu}^{V-A} | B_c(p) \rangle \langle B_c(p) | J_{B_c} | 0 \rangle}{(p'^2 - m_{D_{q'}^*}^2)(p^2 - m_{B_c}^2)} + \dots, \\ \Pi_{\nu\mu}^{T-PT}(p^2, p'^2, q^2) &= - \\ &\frac{\langle 0 | J_{D_{q'}^*}' | D_{q'}^*(p', \varepsilon) \rangle \langle D_{q'}^*(p', \varepsilon) | J_{\mu}^{T-PT} | B_c(p) \rangle \langle B_c(p) | J_{B_c} | 0 \rangle}{(p'^2 - m_{D_{q'}^*}^2)(p^2 - m_{B_c}^2)} + \dots,\end{aligned}\quad (7)$$

where \dots represents contributions coming from higher states and continuum. The matrix elements $\langle 0 | J_{D_{q'}^*}' | D_{q'}^*(p', \varepsilon) \rangle$ and $\langle B_c(p) | J_{B_c} | 0 \rangle$ are defined in the standard way as:

$$\langle 0 | J_{D_{q'}^*}' | D_{q'}^*(p', \varepsilon) \rangle = f_{D_{q'}^*} m_{D_{q'}^*} \varepsilon^{\nu}, \quad \langle B_c(p) | J_{B_c} | 0 \rangle = -i \frac{f_{B_c} m_{B_c}^2}{m_b + m_c}, \quad (8)$$

where $f_{D_{q'}^*}$ and f_{B_c} are the leptonic decay constants of $D_{q'}^*$ and B_c mesons, respectively. Using Eq. (3), Eq. (4) and Eq. (8) and performing summation over the polarization of the $D_{q'}^*$ meson, for the phenomenological (physical) part of the correlation function we obtain

$$\begin{aligned}\Pi_{\nu\mu}^{V-A}(p^2, p'^2, q^2) &= -\frac{f_{B_c} m_{B_c}^2}{(m_b + m_c)} \frac{f_{D_{q'}^*} m_{D_{q'}^*}}{(p'^2 - m_{D_{q'}^*}^2)(p^2 - m_{B_c}^2)} \times \left[A'_0(q^2) g_{\mu\nu} + A'_+(q^2) P_{\mu} p_{\nu} \right. \\ &\quad \left. + A'_-(q^2) q_{\mu} p_{\nu} + i \varepsilon_{\mu\nu\alpha\beta} p^{\alpha} p'^{\beta} A'_V(q^2) \right] + \text{excited states}, \\ \Pi_{\nu\mu}^{T-PT}(p^2, p'^2, q^2) &= -\frac{f_{B_c} m_{B_c}^2}{(m_b + m_c)} \frac{f_{D_{q'}^*} m_{D_{q'}^*}}{(p'^2 - m_{D_{q'}^*}^2)(p^2 - m_{B_c}^2)} \times \left[-i T'_0(q^2) g_{\mu\nu} \right. \\ &\quad \left. - i T'_-(q^2) q_{\mu} p_{\nu} + \varepsilon_{\mu\nu\alpha\beta} p^{\alpha} p'^{\beta} T'_V(q^2) \right] + \text{excited states}.\end{aligned}\quad (9)$$

Next, we calculate the correlation function in the quark and gluon languages via the operator product expansion (OPE) which is called the QCD or theoretical side of the correlator. For this reason the correlation function is written as:

$$\begin{aligned}\Pi_{\nu\mu}^{V-A}(p^2, p'^2, q^2) &= \Pi_0^{V-A} g_{\mu\nu} + \Pi_+^{V-A} P_{\mu} p_{\nu} + \Pi_-^{V-A} q_{\mu} p_{\nu} + i \Pi_V^{V-A} \varepsilon_{\mu\nu\alpha\beta} p^{\alpha} p'^{\beta}, \\ \Pi_{\nu\mu}^{T-PT}(p^2, p'^2, q^2) &= -i \Pi_0^{T-PT} g_{\mu\nu} - i \Pi_-^{T-PT} q_{\mu} p_{\nu} + \Pi_V^{T-PT} \varepsilon_{\mu\nu\alpha\beta} p^{\alpha} p'^{\beta},\end{aligned}\quad (10)$$

where each Π_i with $i = 0, \pm$ and V is defined in terms of the perturbative and nonperturbative parts as follows

$$\Pi_i = \Pi_i^{pert} + \Pi_i^{nonpert}. \quad (11)$$

For calculating the perturbative part of the correlator, we consider the bare -loop diagram (Fig. 1 (a)), as for the nonperturbative part (Fig1 (b, c, d)), the light quark condensates diagrams up to operators having dimension $d = 5$ i.e., operators $d = 3, \langle \bar{q}q \rangle$, $d = 4, m_s \langle \bar{q}q \rangle$, $d = 5, m_0^2 \langle \bar{q}q \rangle$ are assumed. Contributions coming from the light quark condensates diagrams are eliminated by applying the double Borel transformations with respect to the initial and final momentums p and p' , so as first correction in the nonperturbative part of the correlation function in the QCD side, the two gluon condensates are calculated (see Fig. 2 (a, b, c, d, e, f)). In calculating the bare-loop contribution, we first write the double dispersion representation for the coefficients of the corresponding Lorentz structures, appearing in the correlation function, as:

$$\Pi_i^{per} = -\frac{1}{(2\pi)^2} \int ds' \int ds \frac{\rho_i(s, s', q^2)}{(s-p^2)(s'-p'^2)} + \text{subtraction terms}. \quad (12)$$

The integration region for the perturbative contribution in Eq. (12) is determined from the fact that arguments of the three δ functions must vanish simultaneously. The physical region in the s and s' plane is described by the following inequalities:

$$-1 \leq \frac{2ss' + (s + s' - q^2)(m_b^2 - s - m_c^2) + (m_c^2 - m_{q'}^2)2s}{\lambda^{1/2}(m_b^2, s, m_c^2)\lambda^{1/2}(s, s', q^2)} \leq +1. \quad (13)$$

The spectral densities $\rho_i(s, s', q^2)$ can be calculated from the usual Feynman integral with the help of Cutkosky rules, i.e., by replacing the quark propagators with Dirac delta functions: $\frac{1}{p^2 - m^2} \rightarrow -2\pi i \delta(p^2 - m^2)$, which implies that all quarks are real. After standard calculations for the corresponding spectral densities we obtain:

$$\begin{aligned} \rho_V^{V-A}(s, s', q^2) &= -N_c I_0(s, s', q^2) [-4m_c + 4(m_b - m_c)B_1 + 4(m_{q'} - m_c)B_2], \\ \rho_0^{V-A}(s, s', q^2) &= -N_c I_0(s, s', q^2) [8(-m_b + m_c)A_1 - 4m_b m_c m_{q'} \\ &\quad + 4(m_{q'} + m_b - m_c)m_c^2 - 2(m_{q'} - m_c)\Delta \\ &\quad - 2(m_b - m_c)\Delta' - 2m_c u], \\ \rho_+^{V-A}(s, s', q^2) &= N_c I_0(s, s', q^2) [4(m_b - m_c)(A_2 + A_3) + 2(m_b - 3m_c)B_1 \\ &\quad - 2(m_c - m_{q'})B_2 - 2m_c], \\ \rho_-^{V-A}(s, s', q^2) &= N_c I_0(s, s', q^2) [4(m_b - m_c)(A_2 - A_3) - 2(m_b + m_c)B_1 \\ &\quad + 2(m_c - m_{q'})B_2 + 2m_c], \\ \rho_V^{T-PT}(s, s', q^2) &= N_c I_0(s, s', q^2) [B_1(m_c^2 - m_b m_c + m_b m_{q'} - m_c m_{q'} + s - \Delta) \\ &\quad - B_2(m_c^2 - m_b m_c + m_b m_{q'} - m_c m_{q'} + s' - \Delta') \\ &\quad - (m_{q'} m_c + m_b m_c)], \end{aligned}$$

$$\begin{aligned}
\rho_0^{T-PT}(s, s', q^2) &= -N_c I_0(s, s', q^2) [2\Delta'(-m_c^2 + m_b m_c - m_b m_{q'} + m_c m_{q'} + s) \\
&\quad - 2\Delta(-m_c^2 + m_b m_c - m_b m_{q'} + m_c m_{q'} + s') \\
&\quad + 4s m_c(-m_c + m_{q'}) + 4s' m_c(m_c - m_b) + 2u m_c(-m_{q'} + m_b) \\
&\quad + 8A_1(s - u/2)], \\
\rho_-^{T-PT}(s, s', q^2) &= -N_c I_0(s, s', q^2) [2B_1(2m_c^2 - \Delta' + \Delta - m_b m_c + m_b m_{q'} - m_c m_{q'} - s) \\
&\quad - 2B_2(2m_c^2 - m_b m_c + m_b m_{q'} - m_c m_{q'} - s') \\
&\quad + 4(s - u/2)(A_2 - A_3) + 2m_c(m_b - m_{q'})], \tag{14}
\end{aligned}$$

where

$$\begin{aligned}
I_0(s, s', q^2) &= \frac{1}{4\lambda^{1/2}(s, s', q^2)}, \\
\lambda(a, b, c) &= a^2 + b^2 + c^2 - 2ac - 2bc - 2ab, \\
\Delta' &= (s' + m_c^2 - m_{q'}^2), \\
\Delta &= (s + m_c^2 - m_b^2), \\
u &= s + s' - q^2, \\
B_1 &= \frac{1}{\lambda(s, s', q^2)} [2s' \Delta - \Delta' u], \\
B_2 &= \frac{1}{\lambda(s, s', q^2)} [2s \Delta' - \Delta u], \\
A_1 &= -\frac{1}{2\lambda(s, s', q^2)} [(4ss' m_c^2 - s\Delta'^2 - s'\Delta^2 - u^2 m_c^2 + u\Delta\Delta')], \\
A_2 &= -\frac{1}{\lambda^2(s, s', q^2)} [8ss'^2 m_c^2 - 2ss'\Delta'^2 - 6s'^2 \Delta^2 - 2u^2 s' m_c^2 \\
&\quad + 6s' u \Delta \Delta' - u^2 \Delta'^2], \\
A_3 &= \frac{1}{\lambda^2(s, s', q^2)} [4ss' u m_c^2 + 4ss' \Delta \Delta' - 3s u \Delta'^2 - 3u \Delta^2 s' - u^3 m_c^2 + 2u^2 \Delta \Delta']. \tag{15}
\end{aligned}$$

The subscripts V, 0 and \pm for ρ^{V-A} correspond to the coefficients of the structures proportional to $i\varepsilon_{\mu\nu\alpha\beta} p^\alpha p'^\beta$, $g_{\mu\nu}$ and $\frac{1}{2}(p_\mu p_\nu \pm p'_\mu p'_\nu)$, respectively and the subscripts V, 0 and $-$ for ρ^{T-PT} correspond to the coefficients of the structures proportional to $\varepsilon_{\mu\nu\alpha\beta} p^\alpha p'^\beta$, $g_{\mu\nu}$ and $(p_\mu p_\nu - p'_\mu p'_\nu)$. In Eq. (14) $N_c = 3$ is the number of colors. Then, we consider the nonperturbative part of the correlator. As we mentioned before, the light quark condensate contribution to the nonperturbative part of the correlation function is zero, so we calculate the gluon condensates diagrams shown in Fig. 2. The calculations of these diagrams are performed in the Fock–Schwinger fixed–point gauge [23, 24, 25]

$$x^\mu A_\mu^a = 0,$$

where A_μ^a is the gluon field. In calculating the gluon condensate contributions, the following types of integrals are appeared [7, 26]:

$$\begin{aligned}
I_0[a, b, c] &= \int \frac{d^4 k}{(2\pi)^4} \frac{1}{[k^2 - m_b^2]^a [(p+k)^2 - m_c^2]^b [(p'+k)^2 - m_{q'}^2]^c}, \\
I_\mu[a, b, c] &= \int \frac{d^4 k}{(2\pi)^4} \frac{k_\mu}{[k^2 - m_b^2]^a [(p+k)^2 - m_c^2]^b [(p'+k)^2 - m_{q'}^2]^c}, \\
I_{\mu\nu}[a, b, c] &= \int \frac{d^4 k}{(2\pi)^4} \frac{k_\mu k_\nu}{[k^2 - m_b^2]^a [(p+k)^2 - m_c^2]^b [(p'+k)^2 - m_{q'}^2]^c}, \tag{16}
\end{aligned}$$

where k is the momentum of the spectator quark c . These integrals can be calculated by continuing to Euclidean space-time and using Schwinger representation for the Euclidean propagator

$$\frac{1}{k^2 + m^2} = \frac{1}{\Gamma(\alpha)} \int_0^\infty d\alpha \alpha^{n-1} e^{-\alpha(k^2 + m^2)}, \tag{17}$$

which is very suitable for the Borel transformation since

$$\mathcal{B}_{p^2}(M^2) e^{-\alpha p^2} = \delta(1/M^2 - \alpha). \tag{18}$$

Performing integration over loop momentum and over the two parameters which we have used in the exponential representation of propagators [24], and applying double Borel transformations over p^2 and p'^2 , we get the Borel transformed form of the integrals in Eq. (16) (see also [24]):

$$\begin{aligned}
\hat{I}_0(a, b, c) &= i \frac{(-1)^{a+b+c}}{16\pi^2 \Gamma(a)\Gamma(b)\Gamma(c)} (M_1^2)^{2-a-b} (M_2^2)^{2-a-c} \mathcal{U}_0(a+b+c-4, 1-c-b), \\
\hat{I}_\mu(a, b, c) &= \hat{I}_1(a, b, c) p_\mu + \hat{I}_2(a, b, c) p'_\mu, \\
\hat{I}_{\mu\nu}(a, b, c) &= \hat{I}_3(a, b, c) g_{\mu\nu} + \hat{I}_4(a, b, c) p_\mu p_\nu + \hat{I}_5(a, b, c) p'_\mu p'_\nu \\
&\quad + \hat{I}_6(a, b, c) p_\mu p'_\nu + \hat{I}_7(a, b, c) p_\nu p'_\mu, \tag{19}
\end{aligned}$$

where

$$\begin{aligned}
\hat{I}_1(a, b, c) &= i \frac{(-1)^{a+b+c+1}}{16\pi^2 \Gamma(a)\Gamma(b)\Gamma(c)} (M_1^2)^{2-a-b} (M_2^2)^{3-a-c} \mathcal{U}_0(a+b+c-5, 1-c-b), \\
\hat{I}_2(a, b, c) &= i \frac{(-1)^{a+b+c+1}}{16\pi^2 \Gamma(a)\Gamma(b)\Gamma(c)} (M_1^2)^{3-a-b} (M_2^2)^{2-a-c} \mathcal{U}_0(a+b+c-5, 1-c-b), \\
\hat{I}_3(a, b, c) &= i \frac{(-1)^{a+b+c+1}}{32\pi^2 \Gamma(a)\Gamma(b)\Gamma(c)} (M_1^2)^{3-a-b} (M_2^2)^{3-a-c} \mathcal{U}_0(a+b+c-6, 2-c-b),
\end{aligned}$$

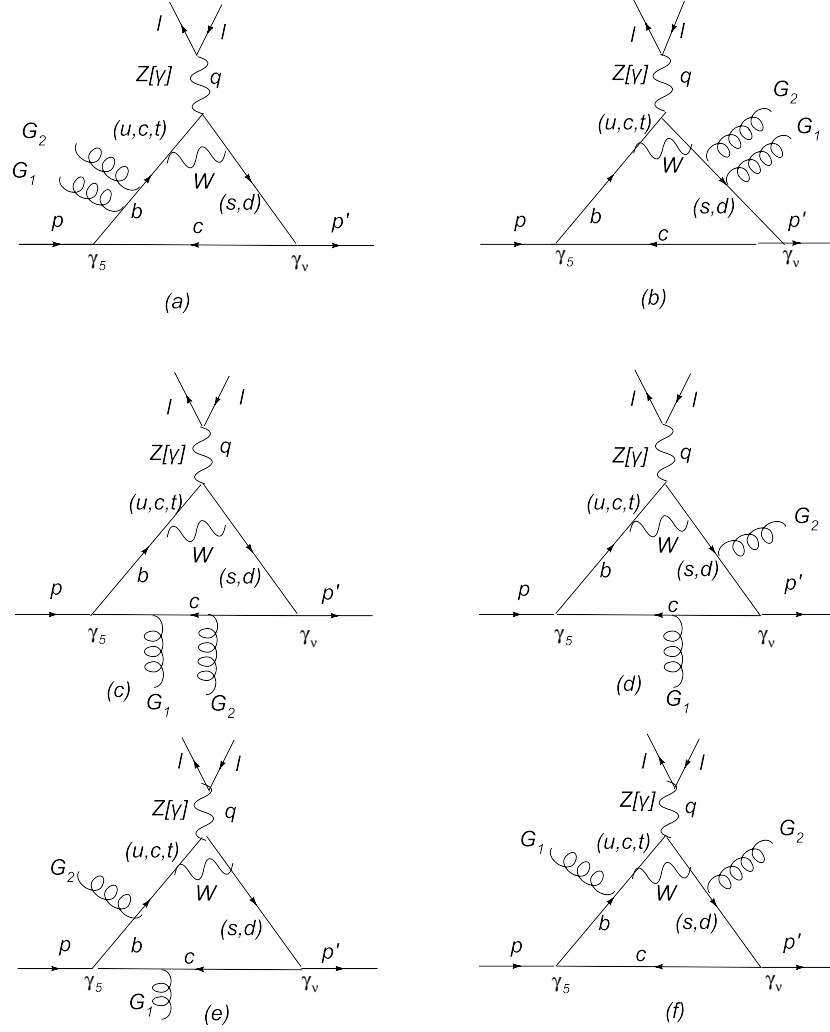


FIG. 2: Gluon condensate contributions to $B_c \rightarrow D_{s,d}^* l^+ l^-$ transitions

$$\hat{I}_4(a, b, c) = i \frac{(-1)^{a+b+c}}{16\pi^2 \Gamma(a)\Gamma(b)\Gamma(c)} (M_1^2)^{2-a-b} (M_2^2)^{4-a-c} \mathcal{U}_0(a+b+c-6, 1-c-b),$$

$$\hat{I}_5(a, b, c) = i \frac{(-1)^{a+b+c}}{16\pi^2 \Gamma(a)\Gamma(b)\Gamma(c)} (M_1^2)^{4-a-b} (M_2^2)^{2-a-c} \mathcal{U}_0(a+b+c-6, 1-c-b),$$

$$\hat{I}_6(a, b, c) = i \frac{(-1)^{a+b+c}}{16\pi^2 \Gamma(a)\Gamma(b)\Gamma(c)} (M_1^2)^{3-a-b} (M_2^2)^{3-a-c} \mathcal{U}_0(a+b+c-6, 1-c-b),$$

$$\hat{I}_7(a, b, c) = \hat{I}_6(a, b, c), \quad (20)$$

and M_1^2 and M_2^2 are the Borel parameters. The function $\mathcal{U}_0(\alpha, \beta)$ is defined as

$$\mathcal{U}_0(a, b) = \int_0^\infty dy (y + M_1^2 + M_2^2)^a y^b \exp\left[-\frac{B_{-1}}{y} - B_0 - B_1 y\right],$$

where

$$\begin{aligned}
B_{-1} &= \frac{1}{M_1^2 M_2^2} [m_{q'}^2 M_1^4 + m_b^2 M_2^4 + M_2^2 M_1^2 (m_b^2 + m_{q'}^2 - q^2)] , \\
B_0 &= \frac{1}{M_1^2 M_2^2} [(m_{q'}^2 + m_c^2) M_1^2 + M_2^2 (m_b^2 + m_c^2)] , \\
B_1 &= \frac{m_c^2}{M_1^2 M_2^2} .
\end{aligned} \tag{21}$$

Hat in above equations denotes the double Borel transformed form of integrals.

The QCD sum rules for the form factors A'_V , A'_0 , A'_+ , A'_- , T'_V , T'_0 and T'_- are obtained by equating the phenomenological expression given in Eq. (9) and QCD side in Eq. (10) and applying double Borel transformations with respect to the variables p^2 and p'^2 ($p^2 \rightarrow M_1^2, p'^2 \rightarrow M_2^2$) in order to suppress the contributions of higher states and continuum:

$$\begin{aligned}
A'_i(q^2) &= \frac{(m_b + m_c)}{f_{B_c} m_{B_c}^2} \frac{1}{f_{D_{q'}^*} m_{D_{q'}^*}} e^{m_{B_c}^2/M_1^2} e^{m_{D_{q'}^*}^2/M_2^2} \left[\frac{1}{(2\pi)^2} \int_{(m_c+m_{q'})^2}^{s'_0} ds' \int_{f_-(s')}^{\min(s_0, f_+(s'))} ds \rho_i^{V-A}(s, s', q^2) e^{-s/M_1^2 - s'/M_2^2} \right. \\
&\quad \left. + i \frac{1}{24\pi^2} C^{A'_i} \langle \frac{\alpha_s}{\pi} G^2 \rangle \right] , \\
T'_i(q^2) &= \frac{(m_b + m_c)}{f_{B_c} m_{B_c}^2} \frac{1}{f_{D_{q'}^*} m_{D_{q'}^*}} e^{m_{B_c}^2/M_1^2} e^{m_{D_{q'}^*}^2/M_2^2} \left[\frac{1}{(2\pi)^2} \int_{(m_c+m_{q'})^2}^{s'_0} ds' \int_{f_-(s')}^{\min(s_0, f_+(s'))} ds \rho_i^{T-PT}(s, s', q^2) e^{-s/M_1^2 - s'/M_2^2} \right. \\
&\quad \left. + i \frac{1}{24\pi^2} C^{T'_i} \langle \frac{\alpha_s}{\pi} G^2 \rangle \right] ,
\end{aligned} \tag{22}$$

where coefficients $C^{A'_i}$ and $C^{T'_i}$ come from the gluon condensates contributions and their explicit expressions are given in the appendix. The s_0 and s'_0 are the continuum thresholds in s and s' channels, respectively and $f_{\pm}(s')$ in the lower and upper limit in the integration over s are calculated from inequality (13) with respect to s , i.e., $s = f_{\pm}(s')$. By $\min(s_0, f_+(s'))$, for each value of the q^2 , we select the smaller one between s_0 and f_+ . In Eq. (22), in order to subtract the contributions of the higher states and the continuum the quark-hadron duality assumption is also used, i.e., it is assumed that

$$\rho^{higherstates}(s, s') = \rho^{OPE}(s, s') \theta(s - s_0) \theta(s - s'_0). \tag{23}$$

In three point QCD sum rules in the case of double dispersion integrals, for $q^2 > 0$ values, their could be a deviation between double dispersion integrals in Eq. (22) and corresponding coefficients of the structures in the feynman amplitudes in the bare loop diagram (see Fig. 1a), i.e., for our case

$$\int \frac{d^4 k}{(2\pi)^4} \frac{Tr\{(\not{k} + m_c)\gamma_\nu(\not{p}' + \not{k} + m_{q'})\gamma_\mu(1 - \gamma_5)(\not{p} + \not{k} + m_b)\}}{[k^2 - m_c^2][(p' + k)^2 - m_{q'}^2][(p + k)^2 - m_b^2]}, \tag{24}$$

where k is the momentum of the spectator quark c . In those cases, the double spectral density receives contributions beyond the contributions due to Landau-type singularities. This problem has been studied in details in [27]. In our case, we have studied such contributions and with the above continuum subtraction and the selecting integration

region, they turn out to be small. Here, we neglect such contributions, so our calculations are trustworthy in the above integration region.

In our calculations the following Borel transformations are also used

$$\begin{aligned} \mathcal{B}_{p^2} \left\{ \frac{1}{m^2(s) - p^2} \right\} &= e^{-\frac{m^2(s)}{M_1^2}}, \\ \mathcal{B}_{p'^2} \left\{ \frac{1}{m^2(s') - p'^2} \right\} &= e^{-\frac{m^2(s')}{M_2^2}}. \end{aligned} \quad (25)$$

III. NUMERICAL ANALYSIS

From the explicit expressions for the form factors $A_V, A_0, A_+, A_-, T_1, T_2, T_3$ and effective hamiltonian, it is clear that the main input parameters entering to the expressions are gluon condensate, Wilson coefficients C_7^{eff}, C_9^{eff} and C_{10} , the CKM matrix elements V_{tb}, V_{ts} and V_{td} , leptonic decay constants f_{B_C} and $f_{D_s^*}$, Borel parameters M_1^2 and M_2^2 , as well as the continuum thresholds s_0 and s'_0 .

In further numerical analysis we choose the value of the gluon condensate $\langle \frac{\alpha_s}{\pi} G^2 \rangle = 0.012 \text{ GeV}^4$ [11], $C_7^{eff} = -0.313$, $C_9^{eff} = 4.344$, $C_{10} = -4.669$ [28, 29], $|V_{tb}| = 0.77_{-0.24}^{+0.18}$, $|V_{ts}| = (40.6 \pm 2.7) \times 10^{-3}$ | $V_{td}| = (7.4 \pm 0.8) \times 10^{-3}$ [30], $f_{D_s^*} = 266 \pm 32 \text{ MeV}$ [31], $f_{D^*} = 0.23 \pm 0.02 \text{ GeV}$ [32], $f_{B_c} = 350 \pm 10 \text{ MeV}$ [33, 34, 35], $m_c(\mu = m_c) = 1.275 \pm 0.015 \text{ GeV}$, $m_s(1 \text{ GeV}) \simeq 142 \text{ MeV}$ [36], $m_b = (4.7 \pm 0.1) \text{ GeV}$ [37], $m_d = (3 - 7) \text{ MeV}$, $m_{D_s^*} = 2.112 \text{ GeV}$, $m_{D^*} = 2.010 \text{ GeV}$ and $m_{B_C} = 6.258 \text{ GeV}$ [38].

The expressions for the form factors contain four auxiliary parameters: Borel mass squares M_1^2 and M_2^2 and continuum threshold s_0 and s'_0 . These are not physical quantities, hence the physical quantities, form factors, must be independent of these auxiliary parameters. In other words, we should find the "working regions" of these auxiliary parameters, where the form factors are independent of them. We try to find the working region of M_1^2 and M_2^2 by requiring that the upper bound of $M_{1,2}^2$ is fixed such that the continuum contribution should be less than the contribution of the first resonance. The lower bound of $M_{1,2}^2$ are determined by requiring that the highest power of $1/M_{1,2}^2$ is less than about 30%/ of the highest power of $M_{1,2}^2$. These two conditions are both satisfied in the following regions; $10 \text{ GeV}^2 \leq M_1^2 \leq 25 \text{ GeV}^2$ and $4 \text{ GeV}^2 \leq M_2^2 \leq 10 \text{ GeV}^2$. The continuum threshold parameters are also determined from the two-point QCD sum rules: $s_0 = 45 \text{ GeV}^2$ and $s'_0 = 8 \text{ GeV}^2$ [5, 11, 31].

In order to estimate the decay width of $B_c \rightarrow D_{s,d}^* l^+ l^-$ decays, it is necessary to know the q^2 dependency of the form factors $A_V, A_0, A_+, A_-, T_1, T_2$ and T_3 in the whole physical region, $4m_l^2 \leq q^2 \leq (m_{B_c} - m_{D_{s,d}^*})^2$, which is correspond to $4m_l^2 \leq q^2 \leq 17.2 \text{ GeV}^2$ and $4m_l^2 \leq q^2 \leq 18 \text{ GeV}^2$ for D_s^* and D_d^* , respectively. For extracting the q^2 dependencies of the form factors from QCD sum rules, we should consider a range of q^2 where the correlation function can reliably be calculated. Our sum rules for the form factors are truncated at $q^2 \simeq 14 \text{ GeV}^2$. In order to extend our results to the full physical region, we look for parametrization of the form factors in such a way that in the region $0 \leq q^2 \leq 14 \text{ GeV}^2$, this parametrization coincides with the sum rules prediction. The values of the form factors at

$q^2 = 0$ are shown in Table I.

	$B_c \rightarrow D_s^* l^+ l^-$	$B_c \rightarrow D_d^* l^+ l^-$
$A_V(0)$	0.54 ± 0.018	0.63 ± 0.055
$A_0(0)$	0.30 ± 0.017	0.34 ± 0.005
$A_+(0)$	0.36 ± 0.013	0.41 ± 0.037
$A_-(0)$	-0.57 ± 0.04	-0.68 ± 0.082
$T_1(0)$	0.31 ± 0.017	0.36 ± 0.039
$T_2(0)$	0.33 ± 0.016	0.37 ± 0.039
$T_3(0)$	0.29 ± 0.034	0.37 ± 0.058

TABLE I: The values of the form factors at $q^2 = 0$, for $M_1^2 = 17 \text{ GeV}^2$ and $M_2^2 = 7 \text{ GeV}^2$.

	$A_V(q^2)$	$A_0(q^2)$	$A_+(q^2)$	$A_-(q^2)$	$T_1(q^2)$	$T_2(q^2)$	$T_3(q^2)$
$F(0)$	0.54	0.30	0.36	-0.57	0.31	0.33	0.29
α	-1.28	-0.13	-0.67	-1.11	-1.28	-0.10	-0.91
β	-0.23	-0.18	-0.066	-0.14	-0.23	-0.097	0.007

TABLE II: Parameters appearing in the form factors of the $B_c \rightarrow D_s^* l^+ l^-$ decay in a four-parameter fit, for $M_1^2 = 17 \text{ GeV}^2$ and $M_2^2 = 7 \text{ GeV}^2$

	$A_V(q^2)$	$A_0(q^2)$	$A_+(q^2)$	$A_-(q^2)$	$T_1(q^2)$	$T_2(q^2)$	$T_3(q^2)$
$F(0)$	0.63	0.34	0.41	-0.68	0.36	0.37	0.37
α	-1.22	0.015	-0.58	-1.06	-1.22	0.025	-0.90
β	-0.23	-0.074	-0.022	-0.13	-0.23	0.002	0.014

TABLE III: Parameters appearing in the form factors of the $B_c \rightarrow D_d^* l^+ l^-$ decay in a four-parameter fit, for $M_1^2 = 17 \text{ GeV}^2$ and $M_2^2 = 7 \text{ GeV}^2$

Our numerical calculations shows that the best parameterization of the form factors with respect to q^2 are as follows:

$$F(q^2) = \frac{F(0)}{1 + \alpha \hat{q} + \beta \hat{q}^2} \quad (26)$$

where $\hat{q} = q^2/m_{B_c}^2$. The values of the parameters $F(0)$, α and β for $B_c \rightarrow D_s^* l^+ l^-$ and $B_c \rightarrow D_d^* l^+ l^-$ are given in Tables II and III, respectively.

The next step is to calculate the total decay width. After straight forward calculations, the differential decay width for these decays are obtained

$$\frac{d\Gamma}{d\hat{q}} = \frac{G_F^2 \alpha^2 m_{B_c}}{2^{14} \pi^5} |V_{tb} V_{tq'}^*|^2 \lambda^{1/2}(1, \hat{r}, \hat{q}) v \Delta(\hat{q}), \quad (27)$$

where

$$\Delta = \frac{2}{3 \hat{r} \hat{q}} m_{B_c}^2 \text{Re} \left[-12 m_{B_c}^2 \hat{m}_l^2 \hat{q} \lambda(1, \hat{r}, \hat{q}) \left\{ (E_3 - D_2 - D_3) E_1^* \right. \right.$$

$$\begin{aligned}
& - (E_2 + E_3 - D_3)D_1^* \Big\} + 12m_{B_c}^4 \hat{m}_l^2 \hat{q}(1 - \hat{r})\lambda(1, \hat{r}, \hat{q})(E_2 - D_2)(E_3^* - D_3^*) \\
& + 48\hat{m}_l^2 \hat{r} \hat{q} \Big\{ 3E_1 D_1^* + 2m_{B_c}^4 \lambda(1, \hat{r}, \hat{q}) E_0 D_0^* \Big\} - 16m_{B_c}^4 \hat{r} \hat{q} (\hat{m}_l^2 - \hat{q}) \lambda(1, \hat{r}, \hat{q}) \Big\{ |E_0|^2 + |D_0|^2 \Big\} \\
& - 6m_{B_c}^4 \hat{m}_l^2 \hat{q} \lambda(1, \hat{r}, \hat{q}) \Big\{ 2(2 + 2\hat{r} - \hat{q}) E_2 D_2^* - \hat{q} |(E_3 - D_3)|^2 \Big\} \\
& - 4m_{B_c}^2 \lambda(1, \hat{r}, \hat{q}) \Big\{ \hat{m}_l^2 (2 - 2\hat{r} + \hat{q}) + \hat{q} (1 - \hat{r} - \hat{q}) \Big\} (E_1 E_2^* + D_1 D_2^*) \\
& + \hat{q} \Big\{ 6\hat{r} \hat{q} (3 + v^2) + \lambda(1, \hat{r}, \hat{q}) (3 - v^2) \Big\} \Big\{ |E_1|^2 + |D_1|^2 \Big\} \\
& - 2m_{B_c}^4 \lambda(1, \hat{r}, \hat{q}) \Big\{ \hat{m}_l^2 [\lambda(1, \hat{r}, \hat{q}) - 3(1 - \hat{r})^2] - \lambda(1, \hat{r}, \hat{q}) \hat{q} \Big\} \Big\{ |E_2|^2 + |D_2|^2 \Big\} ,
\end{aligned} \tag{28}$$

and $\hat{q} = q^2/m_{B_c}^2$, $\hat{r} = m_{D_{q'}^*}^2/m_{B_c}^2$, $\hat{m}_l = m_l/m_{B_c}$ and $v = \sqrt{1 - 4\hat{m}_l^2/\hat{q}}$ is the final lepton velocity. We have also used the following definitions:

$$\begin{aligned}
D_0 &= (C_9^{eff} + C_{10}) \frac{A_V(q^2)}{m_{B_c} + m_{D_{q'}^*}} + (2m_b C_7^{eff}) \frac{T_1(q^2)}{q^2} , \\
D_1 &= (C_9^{eff} + C_{10})(m_{B_c} + m_{D_{q'}^*}) A_0(q^2) + (2m_b C_7^{eff})(m_{B_c}^2 - m_{D_{q'}^*}^2) \frac{T_2(q^2)}{q^2} , \\
D_2 &= \frac{C_9^{eff} + C_{10}}{m_{B_c} + m_{D_{q'}^*}} A_+(q^2) + (2m_b C_7^{eff}) \frac{1}{q^2} \left[T_2(q^2) + \frac{q^2}{m_{B_c}^2 - m_{D_{q'}^*}^2} T_3(q^2) \right] , \\
D_3 &= (C_9^{eff} + C_{10}) \frac{A_-(q^2)}{m_{B_c} + m_{D_{q'}^*}} - (2m_b C_7^{eff}) \frac{T_3(q^2)}{q^2} , \\
E_0 &= (C_9^{eff} - C_{10}) \frac{A_V(q^2)}{m_{B_c} + m_{D_{q'}^*}} + (2m_b C_7^{eff}) \frac{T_1(q^2)}{q^2} , \\
E_1 &= (C_9^{eff} - C_{10})(m_{B_c} + m_{D_{q'}^*}) A_0(q^2) + (2m_b C_7^{eff})(m_{B_c}^2 - m_{D_{q'}^*}^2) \frac{T_2(q^2)}{q^2} , \\
E_2 &= \frac{C_9^{eff} - C_{10}}{m_{B_c} + m_{D_{q'}^*}} A_+(q^2) + (2m_b C_7^{eff}) \frac{1}{q^2} \left[T_2(q^2) + \frac{q^2}{m_{B_c}^2 - m_{D_{q'}^*}^2} T_3(q^2) \right] , \\
E_3 &= (C_9^{eff} - C_{10}) \frac{A_-(q^2)}{m_{B_c} + m_{D_{q'}^*}} - (2m_b C_7^{eff}) \frac{T_3(q^2)}{q^2} .
\end{aligned} \tag{29}$$

At the end of this section, we present the value of the branching ratio for $B_c \rightarrow D_{s,d}^* l^+ l^-$ decays. Taking into account the q^2 dependence of the form factors and performing integration over q^2 in the limit $4m_l^2 < q^2 < (m_{B_c} - m_{D_{s,d}^*})^2$ and using the total life-time $\tau_{B_c} = 0.46 \times 10^{-12} s$ [39], we obtain the branching ratios for $B_c \rightarrow D_s^* l^+ l^-$ and $B_c \rightarrow D_d^* l^+ l^-$ presented in Tables IV and V, respectively. These Tables contain also comparison of our results with the predictions of the quark models. These Tables show a good agreement between our results and that of the quarks models especially when we consider the errors.

In summary, we analyzed the $B_c \rightarrow D_{q'}^* l l$ transitions in the framework of the QCD sum rules. The q^2 dependent expressions for form factors were calculated. The quark condensates contributions to the correlation function were zero, so we considered the gluon corrections to the correlator. Finally, we calculated the total decay width and

branching ratio of these decays and compared our results with the predictions of the quark models. Our results are in good agreement with the quarks model.

	$B_c \rightarrow D_s^* \mu^+ \mu^-$	$B_c \rightarrow D_s^* \tau^+ \tau^-$	$B_c \rightarrow D_s^* e^+ e^-$
Present study	$(2.99 \pm 0.50) \times 10^{-7}$	$(0.205 \pm 0.076) \times 10^{-7}$	$(4.21 \pm 0.62) \times 10^{-7}$
[21]	$(1.41 - 1.76) \times 10^{-7}$	$(0.15 - 0.22) \times 10^{-7}$	–
[22]	$(3.14 - 4.09) \times 10^{-7}$	$(0.34 - 0.51) \times 10^{-7}$	–

TABLE IV: Values for the branching ratio of the $B_c \rightarrow D_s^* l^+ l^-$ decay and their comparison with the predictions of the RCQM [21] and LFQM (CQM) [22].

	$B_c \rightarrow D_d^* \mu^+ \mu^-$	$B_c \rightarrow D_d^* \tau^+ \tau^-$	$B_c \rightarrow D_d^* e^+ e^-$
Present study	$(1.58 \pm 0.20) \times 10^{-8}$	$(0.156 \pm 0.013) \times 10^{-8}$	$(1.97 \pm 0.20) \times 10^{-8}$
[21]	$(0.58 - 0.71) \times 10^{-8}$	$(0.08 - 0.11) \times 10^{-8}$	–
[22]	$(0.78 - 1.01) \times 10^{-8}$	$(0.13 - 0.18) \times 10^{-8}$	–

TABLE V: Values for the branching ratio of the $B_c \rightarrow D_d^* l^+ l^-$ decay and their comparison with the predictions of the RCQM [21] and LFQM (CQM) [22].

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Appendix

In this section, we present the explicit expressions of the coefficients $C^{A'_i}$ and $C^{T'_i}$ corresponding to the gluon condensates entering to the expressions for the form factors in Eq. (22). Here, we have ignored the s and d quark masses.

$$\begin{aligned}
C^{A'_V} = & -30 I_1(4, 1, 1)m_c^3 + 10 I_2(3, 2, 1)m_c^3 - 20 I_1^{0,1}(3, 1, 2)m_b + 20 I_1^{0,1}(3, 1, 2)m_c \\
& +10 I_1^{0,2}(3, 2, 2)m_b - 20 I_1(2, 3, 1)m_b^3 + 20 I_2^{0,1}(3, 2, 2)m_c^3 + 20 I_1^{0,1}(3, 2, 2)m_c^3 \\
& -20 I_1^{0,1}(3, 2, 1)m_b + 20 I_1^{0,1}(2, 2, 2)m_c - 20 I_1^{0,1}(2, 2, 2)m_b + 20 I_1(1, 2, 2)m_b \\
& -20 I_1(1, 2, 2)m_c - 10 I_1^{0,1}(3, 2, 2)m_b^3 - 20 I_1(2, 2, 2)m_c^3 - 10 I_0(3, 2, 2)m_c^5 \\
& +10 I_1(2, 2, 2)m_b^3 + 20 I_0(2, 2, 1)m_c - 10 I_1(3, 2, 2)m_c^5 - 10 m_c^3 I_0(3, 1, 2) \\
& +30 m_c I_0^{0,1}(3, 1, 2) - 10 m_c I_0^{0,2}(3, 2, 2) + 60 m_b I_2(1, 3, 1) + 10 m_c I_2^{0,1}(3, 2, 1) \\
& -20 m_c I_0(1, 2, 2) - 20 m_c^3 I_0(2, 2, 2) + 20 m_c I_0^{0,1}(2, 2, 2) - 60 I_1(1, 4, 1)m_b^3 \\
& -20 I_1(2, 1, 2)m_c + 30 I_1(2, 1, 2)m_b - 30 m_c^3 I_2(4, 1, 1) + 20 m_c I_2^{0,1}(2, 2, 2) \\
& -20 m_c^3 I_2(2, 2, 2) + 10 m_c I_2(3, 1, 1) + 100 m_b I_0(1, 3, 1) + 60 m_b^2 m_c I_0(1, 4, 1) \\
& +40 m_b m_c^2 I_0(2, 3, 1) + 60 m_b^2 m_c I_2(1, 4, 1) - 10 m_c I_1(3, 1, 1) - 10 m_c I_2^{0,2}(3, 2, 2) \\
& -30 m_c^3 I_0(4, 1, 1) + 20 m_b I_1(1, 3, 1) + 40 m_b I_2^{0,1}(2, 3, 1) - 10 m_c^3 I_2(3, 1, 2) \\
& -10 m_c I_0(3, 1, 1) + 20 m_b I_1^{0,1}(2, 3, 1) - 30 m_c I_0(2, 1, 2) - 30 m_c I_2(2, 1, 2) \\
& +20 m_c I_0^{0,1}(3, 2, 1) + 30 m_c I_2^{0,1}(3, 1, 2) - 20 m_c I_2(1, 2, 2) + 10 I_0^{0,1}(3, 2, 2)m_b^2 m_c \\
& +10 I_2(3, 2, 2)m_c^3 m_b^2 - 30 I_1(3, 2, 1)m_b^2 m_c + 10 I_1(3, 2, 1)m_b m_c^2 + 30 I_1(4, 1, 1)m_b m_c^2 \\
& -20 I_2(3, 2, 1)m_b^2 m_c + 10 I_1(3, 2, 2)m_c^3 m_b^2 + 10 I_1(3, 2, 2)m_b m_c^4 - 10 I_1(3, 2, 2)m_b^3 m_c^2
\end{aligned}$$

$$\begin{aligned}
& +60 I_1(1, 4, 1)m_b^2 m_c + 10 I_1^{0,1}(3, 2, 2)m_b^2 m_c - 20 I_1^{0,1}(3, 2, 2)m_b m_c^2 - 10 I_2(3, 2, 1)m_b m_c^2 \\
& +10 I_2^{0,1}(3, 2, 2)m_b^2 m_c + 20 I_1(2, 3, 1)m_b m_c^2 - 20 I_0(3, 2, 1)m_b^2 m_c + 10 I_0(3, 2, 1)m_b m_c^2 \\
& +10 I_0(3, 2, 2)m_c^3 m_b^2 + 20 I_1(2, 2, 2)m_b m_c^2 + 20 I_1(2, 2, 1)m_c - 50 I_1(2, 2, 1)m_b \\
& -20 I_0(2, 2, 1)m_b + 20 I_0^{0,1}(3, 2, 2)m_c^3 + 30 I_2(2, 2, 1)m_c - 10 I_2(3, 2, 2)m_c^5 \\
& -20 I_2(2, 2, 1)m_b + 20 I_1(3, 2, 1)m_b^3 - 10 I_1^{0,2}(3, 2, 2)m_c + 20 I_1^{0,1}(3, 2, 1)m_c
\end{aligned}$$

$$\begin{aligned}
C^{A_6} = & -5 I_0(2, 2, 1)m_b^3 - 5 I_0(3, 1, 1)m_c^3 - 40 I_3^{0,1}(3, 2, 2)m_c^3 + 20 I_3^{0,1}(3, 2, 2)m_b^3 \\
& -10 I_0(1, 3, 1)m_b m_c^2 - 15 I_0^{0,1}(3, 2, 2)m_b m_c^4 + 5 I_0^{0,1}(3, 2, 2)m_c m_b^4 + 15 I_0^{0,2}(3, 2, 2)m_b m_c^2 \\
& -5 I_0^{0,2}(3, 2, 2)m_c m_b^2 - 20 I_3(3, 2, 2)m_c^3 m_b^2 - 10 I_0(2, 3, 1)m_b^3 m_c^2 + 10 I_0(2, 3, 1)m_b m_c^4 \\
& -20 I_3(3, 1, 2)m_b m_c^2 - 20 I_3(2, 2, 2)m_b m_c^2 + 20 I_3(2, 2, 2)m_c m_b^2 + 40 I_3(2, 3, 1)m_b m_c^2 \\
& +30 I_0(1, 3, 1)m_c m_b^2 - 10 I_0(2, 2, 2)m_c^3 m_b^2 + 10 I_0(2, 2, 1)m_c m_b^2 - 15 I_0(2, 2, 1)m_b m_c^2 \\
& +10 I_0(3, 2, 1)m_b^3 m_c^2 - 20 I_3(3, 2, 2)m_b m_c^4 + 20 I_3(3, 2, 2)m_b^3 m_c^2 - 20 I_0^{0,1}(2, 3, 1)m_b m_c^2 \\
& +10 I_0^{0,1}(2, 2, 2)m_c m_b^2 - 30 I_0^{0,1}(2, 2, 2)m_b m_c^2 + 15 I_0(4, 1, 1)m_b m_c^4 - 15 I_0(4, 1, 1)m_c^3 m_b^2 \\
& -15 I_0(2, 1, 2)m_c m_b^2 + 15 I_0(2, 1, 2)m_b m_c^2 + 10 I_0(3, 1, 1)m_b m_c^2 + 10 I_0^{0,1}(3, 2, 2)m_c^3 m_b^2 \\
& -10 I_0(1, 2, 2)m_c m_b^2 + 20 I_0(1, 2, 2)m_b m_c^2 - 120 I_3(1, 4, 1)m_c m_b^2 - 40 I_3(3, 2, 1)m_b m_c^2 \\
& -5 I_0(3, 1, 1)m_c m_b^2 + 15 I_0(2, 2, 2)m_b m_c^4 + 20 I_3(3, 2, 1)m_c m_b^2 + 5 I_0^{0,1}(3, 2, 1)m_c m_b^2 \\
& -20 I_0^{0,1}(3, 2, 1)m_b m_c^2 - 10 I_0^{0,1}(3, 1, 2)m_b m_c^2 + 20 I_0^{0,1}(3, 1, 2)m_c m_b^2 - 5 I_0(3, 2, 1)m_c^3 m_b^2 \\
& +5 I_0(3, 2, 1)m_b m_c^4 - 10 I_0(3, 2, 1)m_c m_b^4 - 20 I_3^{0,1}(3, 2, 2)m_c m_b^2 + 40 I_3^{0,1}(3, 2, 2)m_b m_c^2 \\
& -60 I_3(4, 1, 1)m_b m_c^2 + 30 I_0(1, 4, 1)m_c m_b^4 - 30 I_0(1, 4, 1)m_b^3 m_c^2 - 5 I_0(3, 2, 2)m_b^3 m_c^4 \\
& +5 I_0(3, 2, 2)m_b m_c^6 - 5 I_0(3, 2, 2)m_c^5 m_b^2 + 5 I_0(3, 2, 2)m_c^3 m_b^4 - 20 m_b I_0^{0,1}(1, 2, 2) \\
& +10 m_b I_0^{0,2}(2, 3, 1) + 10 m_b I_0^{0,1}(1, 3, 1) - 5 m_b I_0^{0,3}(3, 2, 2) - 15 m_b I_0^{0,1}(2, 2, 1) \\
& +30 m_b^3 I_0^{0,1}(1, 4, 1) + 10 m_b I_0^{0,2}(3, 1, 2) - 15 m_b m_c^2 I_0^{0,1}(4, 1, 1) - 25 m_b I_0^{0,1}(2, 1, 2) \\
& +15 m_b I_0^{0,2}(2, 2, 2) + 15 m_b I_0^{0,2}(3, 2, 1) + 40 m_b I_3^{0,1}(2, 3, 1) + 40 m_b I_3(1, 3, 1) \\
& +60 I_3(4, 1, 1)m_c^3 + 40 I_3(3, 2, 1)m_c^3 + 120 I_3(1, 4, 1)m_b^3 + 5 I_0(1, 2, 2)m_b^3 + 20 I_3(3, 1, 2)m_c^3 \\
& -15 (I_{0,0,1})(3, 2, 1)m_b^3 - 20 I_3(3, 2, 1)m_b^3 + 40 I_3(2, 2, 2)m_c^3 - 20 I_3(2, 2, 2)m_b^3 \\
& -20 I_3(2, 2, 1)m_c + 40 I_3(2, 2, 1)m_b + 20 m_b I_3(1, 2, 2) + 20 I_3^{0,1}(2, 2, 2)m_b \\
& -15 I_0(2, 1, 1)m_b + 5 I_0(2, 1, 1)m_c - 40 I_3(3, 1, 1)m_c - 40 I_3^{0,1}(2, 2, 2)m_c \\
& +15 I_0(1, 2, 1)m_b + 40 I_3(3, 1, 1)m_b - 40 I_3(2, 3, 1)m_b^3 - 20 I_3^{0,1}(3, 1, 2)m_c \\
& +40 I_3^{0,1}(3, 1, 2)m_b - 5 I_0(1, 2, 1)m_c - 40 I_3^{0,1}(3, 2, 1)m_c + 20 I_3^{0,1}(3, 2, 1)m_b \\
& +20 I_3(2, 1, 2)m_b + 5 I_0^{0,2}(3, 2, 2)m_b^3 + 20 I_0(1, 3, 1)m_b^3 - 20 I_3(2, 1, 2)m_c
\end{aligned}$$

$$\begin{aligned}
& -10 I_0^{0,1}(3, 1, 1)m_b - 5 I_0^{0,1}(3, 1, 1)m_c - 10 I_0^{0,1}(2, 3, 1)m_b^3 + 20 I_3(3, 2, 2)m_c^5 \\
& + 20 I_3^{0,2}(3, 2, 2)m_c - 20 I_3^{0,2}(3, 2, 2)m_b + 15 I_0(1, 1, 2)m_b - 5 I_0(1, 1, 2)m_c \\
& - 10 I_0^{0,1}(2, 2, 2)m_b^3
\end{aligned}$$

$$\begin{aligned}
C^{A_4} = & -20 I_7^{0,1}(3, 2, 1)m_c - 20 I_4(3, 1, 1)m_c + 10 I_7^{0,1}(3, 2, 1)m_b - 10 I_4(2, 2, 2)m_b^3 \\
& + 20 I_4(2, 2, 2)m_c^3 + 60 I_7(1, 4, 1)m_b^3 + 20 I_7(3, 1, 1)m_b - 20 I_7(3, 1, 1)m_c \\
& + 10 I_4(3, 2, 2)m_c^5 - 20 I_4(3, 2, 1)m_b m_c^2 - 10 I_4(3, 1, 2)m_b m_c^2 - 10 I_7(3, 1, 2)m_b m_c^2 \\
& + 20 I_4(2, 3, 1)m_b m_c^2 + 10 I_7(3, 2, 1)m_b^2 m_c - 20 I_7(3, 2, 1)m_b m_c^2 - 5 I_0(3, 2, 2)m_c^3 m_b^2 \\
& - 35 I_1(3, 2, 1)m_b m_c^2 - 60 I_4(1, 4, 1)m_b^2 m_c - 30 I_4(4, 1, 1)m_b m_c^2 - 10 I_4(3, 2, 2)m_b m_c^4 \\
& - 10 I_4(3, 2, 2)m_c^3 m_b^2 + 10 I_4(3, 2, 2)m_b^3 m_c^2 - 60 I_7(1, 4, 1)m_b^2 m_c - 10 I_7(2, 2, 2)m_b^3 \\
& + 10 I_4^{0,1}(3, 2, 2)m_b^3 - 20 I_4^{0,1}(3, 2, 2)m_c^3 - 10 I_0^{0,1}(3, 2, 2)m_c^3 + 20 I_7(2, 2, 2)m_c^3 \\
& - 10 I_4^{0,1}(3, 1, 2)m_c + 20 I_4^{0,1}(3, 1, 2)m_b - 10 I_1(3, 2, 1)m_b^3 + 10 I_7(3, 1, 2)m_c^3 \\
& + 20 I_4(2, 2, 1)m_b - 10 I_4(2, 2, 1)m_c + 5 I_1^{0,1}(3, 2, 2)m_b^3 + 20 I_1(2, 1, 2)m_b \\
& + 10 I_0(2, 2, 1)m_b + 30 I_1(1, 4, 1)m_b^3 + 20 I_7^{0,1}(3, 1, 2)m_b - 10 I_7^{0,1}(3, 1, 2)m_c \\
& - 20 I_7^{0,1}(2, 2, 2)m_c + 5 I_2(3, 2, 2)m_c^5 - 10 I_7^{0,2}(3, 2, 2)m_b + 10 I_7^{0,2}(3, 2, 2)m_c \\
& + 10 I_4^{0,2}(3, 2, 2)m_c - 10 I_4^{0,2}(3, 2, 2)m_b + 20 m_b I_4(1, 3, 1) + 20 m_b I_1(1, 2, 2) \\
& + 20 m_b I_4^{0,1}(2, 3, 1) + 5 m_c I_0(2, 2, 1) + 30 m_c^3 I_1(2, 2, 2) + 20 m_c^3 I_1(3, 1, 2) \\
& + 10 m_b I_2(2, 2, 1) + 15 m_c I_2(2, 1, 2) + 5 m_c^3 I_0(3, 1, 2) - 20 I_7(2, 3, 1)m_b^3 - 20 I_4(2, 3, 1)m_b^3 \\
& + 10 m_c^3 I_0(2, 2, 2) + 10 I_7(3, 2, 2)m_c^5 + 10 I_7(2, 1, 2)m_b - 10 I_7(2, 1, 2)m_c + 10 I_1^{0,1}(3, 2, 1)m_b \\
& + 10 I_4(2, 1, 2)m_b - 10 I_4(2, 1, 2)m_c + 5 I_0(3, 2, 2)m_c^5 - 5 I_1^{0,2}(3, 2, 2)m_b + 20 m_b I_7(1, 3, 1) \\
& - 5 m_c I_2(3, 1, 1) + 25 m_c^3 I_0(3, 2, 1) - 30 m_c I_1^{0,1}(3, 1, 2) + 15 m_c I_1^{0,2}(3, 2, 2) \\
& - 15 m_c I_1(3, 1, 1) + 10 m_c I_1(2, 1, 2) + 30 m_b I_1(2, 2, 1) - 35 m_c I_1^{0,1}(3, 2, 1) + 45 m_c^3 I_1(4, 1, 1) \\
& - 30 m_c I_1^{0,1}(2, 2, 2) + 10 m_c I_0(3, 1, 1) - 10 m_b I_2(1, 3, 1) - 15 m_c I_0^{0,1}(3, 2, 1) \\
& - 10 m_c I_2^{0,1}(3, 2, 1) + 10 m_b I_1^{0,1}(2, 3, 1) + 5 m_c (I_0^{0,2})(3, 2, 2) + 20 I_7(3, 2, 1)m_c^3 \\
& - 10 I_7(3, 2, 1)m_b^3 + 15 I_1(3, 2, 2)m_c^5 + 10 I_4^{0,1}(2, 2, 2)m_b + 15 m_c^3 I_0(4, 1, 1) \\
& + 5 m_c I_2^{0,2}(3, 2, 2) - 10 m_b I_0(1, 3, 1) - 15 m_c I_2^{0,1}(3, 1, 2) + 15 m_c I_0(2, 1, 2) \\
& + 20 m_b I_7^{0,1}(2, 3, 1) - 10 m_b I_1(1, 3, 1) + 15 m_c^3 I_2(4, 1, 1) + 5 m_c^3 I_2(3, 1, 2) \\
& - 10 m_c I_0^{0,1}(2, 2, 2) - 10 m_c I_2^{0,1}(2, 2, 2) - 30 m_c m_b^2 I_0(1, 4, 1) + 20 m_b m_c^2 I_2(2, 3, 1) \\
& - 30 m_c m_b^2 I_2(1, 4, 1) - 15 m_c I_0^{0,1}(3, 1, 2) - 20 I_4^{0,1}(2, 2, 2)m_c - 20 I_7^{0,1}(3, 2, 2)m_c^3 \\
& + 20 I_4(3, 1, 1)m_b - 5 I_1(2, 2, 1)m_c + 10 m_c^3 I_2(2, 2, 2) + 30 I_4(4, 1, 1)m_c^3 + 10 I_7^{0,1}(3, 2, 2)m_b^3
\end{aligned}$$

$$\begin{aligned}
& +10I_1^{0,1}(3,1,2)m_b + 10I_4^{0,1}(3,2,1)m_b - 20I_4^{0,1}(3,2,1)m_c - 30I_1^{0,1}(3,2,2)m_c^3 \\
& -10I_2^{0,1}(3,2,2)m_c^3 + 30I_7(4,1,1)m_c^3 + 20I_4(3,2,1)m_c^3 + 45I_1(3,2,1)m_c^3 \\
& +10I_1^{0,1}(3,2,2)m_b m_c^2 - 5I_2(3,2,2)m_c^3 m_b^2 - 5I_1(3,2,2)m_b m_c^4 + 5I_1(3,2,2)m_b^3 m_c^2 \\
& +10I_4(3,1,2)m_c^3 + 20I_7(2,2,1)m_b - 10I_4(3,2,1)m_b^3 + 20I_2(3,2,1)m_c^3 + 60I_4(1,4,1)m_b^3 \\
& -10I_7(2,2,1)m_c + 10I_7^{0,1}(2,2,2)m_b - 10I_1(2,3,1)m_b^3 + 10m_b I_7(1,2,2) \\
& +10m_b I_4(1,2,2) + 20m_b m_c^2 I_0(2,3,1) - 90m_b^2 m_c I_1(1,4,1) + 50m_b m_c^2 I_1(2,3,1) \\
& +10I_7(3,2,2)m_b^3 m_c^2 - 15I_1(3,2,2)m_c^3 m_b^2 + 10I_1(2,2,2)m_b^2 m_c - 15I_1^{0,1}(3,2,2)m_b^2 m_c \\
& -30I_7(4,1,1)m_b m_c^2 - 5I_1(3,1,2)m_b^2 m_c - 10I_4^{0,1}(3,2,2)m_b^2 m_c + 10I_7(2,2,2)m_b^2 m_c \\
& -10I_7(2,2,2)m_b m_c^2 - 15I_1(4,1,1)m_b m_c^2 + 20I_7^{0,1}(3,2,2)m_b m_c^2 - 10I_7^{0,1}(3,2,2)m_b^2 m_c \\
& -5I_2^{0,1}(3,2,2)m_b^2 m_c - 10I_1(3,1,2)m_b m_c^2 + 20I_7(2,3,1)m_b m_c^2 - 10I_7(3,2,2)m_b m_c^4 \\
& -10I_7(3,2,2)m_c^3 m_b^2 - 5I_0(3,2,1)m_b m_c^2 + 15I_0(3,2,1)m_b^2 m_c + 20I_4^{0,1}(3,2,2)m_b m_c^2 \\
& +10I_4(2,2,2)m_b^2 m_c - 10I_4(2,2,2)m_b m_c^2 - 5I_0^{0,1}(3,2,2)m_b^2 m_c + 30I_1(3,2,1)m_b^2 m_c \\
& -15I_2(3,2,1)m_b m_c^2 + 10I_2(3,2,1)m_b^2 m_c + 10I_4(3,2,1)m_b^2 m_c
\end{aligned}$$

$$\begin{aligned}
C^{A^-} = & -10I_7(3,2,2)m_c^5 + 10I_2^{0,1}(3,2,2)m_c^3 - 10I_7^{0,1}(3,2,2)m_b^3 + 20I_7^{0,1}(3,2,2)m_c^3 \\
& -10I_1^{0,1}(2,2,2)m_b - 10I_7(3,1,2)m_c^3 - 5I_2(3,2,2)m_c^5 + 10I_7^{0,2}(3,2,2)m_b \\
& -10I_7^{0,2}(3,2,2)m_c + 35I_1(3,2,1)m_c^3 + 10I_0^{0,1}(3,2,2)m_c^3 + 20I_7^{0,1}(2,2,2)m_c \\
& -10I_7^{0,1}(2,2,2)m_b - 20I_4^{0,1}(3,2,1)m_c + 10I_4^{0,1}(3,2,1)m_b + 10I_4^{0,1}(3,2,2)m_b^3 \\
& -20I_4^{0,1}(3,2,2)m_c^3 + 30I_4(4,1,1)m_c^3 - 20I_7(3,2,1)m_c^3 - 20I_4^{0,1}(2,2,2)m_c \\
& +10I_4^{0,1}(2,2,2)m_b + 10I_7(2,2,1)m_c - 20I_7(2,2,1)m_b - 10I_1^{0,1}(3,1,2)m_b \\
& -10I_4^{0,2}(3,2,2)m_b + 10I_4^{0,2}(3,2,2)m_c - 60I_7(1,4,1)m_b^3 + 20I_4(2,2,2)m_c^3 \\
& -5I_0(3,2,2)m_c^5 + 10I_1(3,2,1)m_b^3 + 20I_7^{0,1}(3,2,1)m_c - 10I_7^{0,1}(3,2,1)m_b \\
& -30I_1(1,4,1)m_b^3 + 10I_4(3,2,2)m_c^5 - 20I_1(2,1,2)m_b - 10I_1^{0,1}(3,2,1)m_b \\
& -20I_4(2,3,1)m_b^3 + 10I_7(3,2,1)m_b^3 + 10m_b I_4(1,2,2) + 10I_1(3,1,2)m_b m_c^2 \\
& +20I_4^{0,1}(3,2,2)m_b m_c^2 - 10I_4^{0,1}(3,2,2)m_c m_b^2 - 30I_4(4,1,1)m_b m_c^2 \\
& -10I_7(3,2,1)m_c m_b^2 + 20I_7(3,2,1)m_b m_c^2 - 15I_0(3,2,1)m_c m_b^2 + 5I_0(3,2,1)m_b m_c^2 \\
& -10I_4(3,2,2)m_c^3 m_b^2 - 10I_4(3,2,2)m_b m_c^4 + 10I_4(3,2,2)m_b^3 m_c^2 + 20I_4(2,3,1)m_b m_c^2 \\
& -60I_4(1,4,1)m_c m_b^2 - 10I_7(2,2,2)m_c m_b^2 + 60I_7(1,4,1)m_c m_b^2 + 60I_4(1,4,1)m_b^3 \\
& +10I_7(2,2,2)m_b^3 - 20I_7(2,2,2)m_c^3 - 20I_7(3,1,1)m_b + 20I_7(3,1,1)m_c \\
& +20I_7(2,3,1)m_b^3 - 10I_4(2,2,2)m_b^3 + 5I_1(2,2,1)m_c + 10I_7(2,2,2)m_b m_c^2
\end{aligned}$$

$$\begin{aligned}
& -5 I_1^{0,1}(3, 2, 2) m_c m_b^2 - 5 I_1(3, 2, 2) m_c^3 m_b^2 - 10 I_2(3, 2, 1) m_c m_b^2 + 15 I_2(3, 2, 1) m_b m_c^2 \\
& + 10 I_4(3, 2, 1) m_c m_b^2 - 20 I_4(3, 2, 1) m_b m_c^2 + 5 I_1(3, 2, 2) m_b m_c^4 - 20 I_7(2, 3, 1) m_b m_c^2 \\
& - 5 I_1(3, 2, 2) m_b^3 m_c^2 - 10 I_4(2, 2, 2) m_b m_c^2 + 5 I_0(3, 2, 2) m_c^3 m_b^2 + 15 I_1(4, 1, 1) m_b m_c^2 \\
& - 10 I_4(3, 1, 2) m_b m_c^2 - 10 I_1^{0,1}(3, 2, 2) m_b m_c^2 + 30 I_7(4, 1, 1) m_b m_c^2 + 10 I_7(3, 2, 2) m_c^3 m_b^2 \\
& + 10 I_7(3, 2, 2) m_b m_c^4 - 10 I_7(3, 2, 2) m_b^3 m_c^2 + 5 I_2^{0,1}(3, 2, 2) m_c m_b^2 - 20 I_7^{0,1}(3, 2, 2) m_b m_c^2 \\
& + 10 I_7^{0,1}(3, 2, 2) m_c m_b^2 + 10 I_7(3, 1, 2) m_b m_c^2 + 10 I_4(2, 2, 2) m_c m_b^2 + 10 I_7^{0,1}(3, 1, 2) m_c \\
& - 20 I_7^{0,1}(3, 1, 2) m_b + 20 I_4(3, 1, 1) m_b - 20 I_4(3, 1, 1) m_c + 10 I_1(2, 3, 1) m_b^3 \\
& - 10 I_4^{0,1}(3, 1, 2) m_c + 20 I_4^{0,1}(3, 1, 2) m_b - 10 I_4(2, 2, 1) m_c - 5 I_1^{0,1}(3, 2, 2) m_b^3 \\
& - 10 I_0(2, 2, 1) m_b - 10 I_7(2, 1, 2) m_b + 10 I_1(2, 2, 2) m_b m_c^2 - 10 I_1(2, 2, 2) m_c m_b^2 \\
& + 10 I_7(2, 1, 2) m_c - 30 I_7(4, 1, 1) m_c^3 - 10 I_1^{0,1}(3, 2, 2) m_c^3 + 5 I_1(3, 2, 2) m_c^5 \\
& - 20 I_2(3, 2, 1) m_c^3 + 20 I_4(3, 2, 1) m_c^3 - 10 I_4(3, 2, 1) m_b^3 + 10 I_4(2, 1, 2) m_b \\
& - 10 I_4(2, 1, 2) m_c + 5 I_1^{0,2}(3, 2, 2) m_b + 30 m_b m_c^2 I_1(2, 3, 1) - 20 m_b m_c^2 I_2(2, 3, 1) \\
& + 30 m_c m_b^2 I_0(1, 4, 1) + 30 m_c m_b^2 I_2(1, 4, 1) - 30 m_c m_b^2 I_1(1, 4, 1) - 20 m_b m_c^2 I_0(2, 3, 1) \\
& - 10 m_b I_7(1, 2, 2) + 10 I_4(3, 1, 2) m_c^3 + 5 I_2(3, 2, 2) m_c^3 m_b^2 + 10 I_1(3, 2, 1) m_c m_b^2 \\
& - 25 I_1(3, 2, 1) m_b m_c^2 + 5 I_0^{0,1}(3, 2, 2) m_c m_b^2 + 5 I_1(3, 1, 2) m_c m_b^2 - 10 m_c^3 I_1(3, 1, 2) \\
& - 10 m_c I_1^{0,1}(2, 2, 2) + 10 m_c^3 I_1(2, 2, 2) + 20 m_b I_4(1, 3, 1) + 20 m_b I_1(2, 2, 1) \\
& - 15 m_c I_1(3, 1, 1) + 20 m_b I_4^{0,1}(2, 3, 1) + 15 m_c I_0(2, 2, 1) + 15 m_c^3 I_0(3, 2, 1) \\
& - 20 m_c I_1^{0,1}(3, 1, 2) - 5 m_c I_1^{0,1}(3, 2, 1) + 15 m_c^3 I_1(4, 1, 1) + 5 m_c I_1^{0,2}(3, 2, 2) \\
& + 10 m_c I_2^{0,1}(3, 2, 1) - 15 m_c I_2(2, 1, 2) - 15 m_c^3 I_2(4, 1, 1) + 10 m_b I_2(1, 3, 1) \\
& + 10 m_c I_2^{0,1}(2, 2, 2) + 10 m_c I_0^{0,1}(2, 2, 2) - 10 m_c^3 I_2(2, 2, 2) + 10 m_b I_0(1, 3, 1) \\
& - 30 m_b I_1(1, 3, 1) - 5 m_c I_2^{0,2}(3, 2, 2) + 15 m_c I_2^{0,1}(3, 1, 2) + 5 m_c I_2(3, 1, 1) \\
& - 5 m_c^3 I_2(3, 1, 2) - 10 m_b I_1^{0,1}(2, 3, 1) - 20 m_b I_7^{0,1}(2, 3, 1) - 5 m_c I_0^{0,2}(3, 2, 2) \\
& - 15 m_c^3 I_0(4, 1, 1) - 10 m_c^3 I_0(2, 2, 2) - 15 m_c I_0(2, 1, 2) - 20 m_b I_7(1, 3, 1) \\
& + 15 m_c I_0^{0,1}(3, 2, 1) + 15 m_c I_0^{0,1}(3, 1, 2) - 5 m_c^3 I_0(3, 1, 2) - 10 m_b I_2(2, 2, 1) \\
& - 10 m_c I_0(3, 1, 1) - 10 m_b I_1(1, 2, 2) + 20 I_4(2, 2, 1) m_b
\end{aligned}$$

$$\begin{aligned}
C^{T_V} = & -10 I_0^{0,1}(2, 2, 1) - 5 I_0^{0,2}(3, 1, 2) - 10 I_2(1, 2, 1) - 10 I_1(1, 1, 2) - 5 m_b^2 I_0^{0,2}(3, 2, 2) \\
& - 10 m_b^2 I_1^{0,2}(3, 2, 2) - 10 m_c^4 I_1(3, 1, 2) + 20 m_b^2 I_1^{0,1}(2, 2, 2) + 10 m_b^2 I_2(1, 2, 2) \\
& + 60 I_0(2, 1, 1) - 15 m_c^2 m_b^2 I_0(4, 1, 1) + 10 m_b^2 I_0^{0,1}(2, 2, 2) + 20 m_b^3 m_c I_0(2, 3, 1) \\
& - 10 m_b^2 I_1(1, 2, 2) + 10 m_c^2 I_2(3, 1, 1) - 30 m_c^2 m_b^2 I_1(4, 1, 1) + 10 m_c^2 m_b^2 I_2(3, 2, 1)
\end{aligned}$$

$$\begin{aligned}
& +40 m_b^3 m_c I_2(2, 3, 1) + 20 I_1^{0,1}(2, 1, 2) + 5 I_0^{0,2}(3, 2, 1) - 15 I_0(3, 2, 1) m_c^2 m_b^2 \\
& +10 I_0^{0,1}(3, 2, 2) m_c^2 m_b^2 + 40 I_2(2, 2, 1) m_b m_c - 20 I_1(3, 2, 1) m_c^2 m_b^2 + 10 I_1(1, 2, 1) \\
& -10 I_1(2, 2, 2) m_b^4 + 20 I_1(2, 2, 1) m_c^2 + 80 I_1(2, 2, 1) m_b^2 + 5 I_0(3, 2, 1) m_c^4 \\
& -10 I_0(3, 2, 1) m_b^4 - 5 I_0(2, 2, 2) m_b^4 + 20 I_1^{0,1}(3, 1, 2) m_c^2 + 5 I_0^{0,1}(3, 2, 2) m_b^4 \\
& +50 I_2(2, 2, 1) m_b^2 + 10 I_1(3, 2, 1) m_c^4 - 20 I_1(3, 2, 1) m_b^4 - 5 I_0(3, 2, 2) m_c^4 m_b^2 \\
& -5 I_0(3, 1, 1) m_b m_c + 20 I_0(2, 2, 1) m_b m_c + 5 I_0(3, 1, 2) m_c^2 m_b^2 + 20 I_1^{0,1}(3, 2, 2) m_c^2 m_b^2 \\
& -10 I_0(2, 2, 2) m_c^2 m_b^2 - 20 I_1(2, 2, 2) m_c^2 m_b^2 - 10 I_1(3, 2, 2) m_c^4 m_b^2 + 10 I_1(3, 2, 2) m_c^2 m_b^4 \\
& -5 I_0(3, 2, 1) m_b^3 m_c + 5 I_0(3, 2, 2) m_c^2 m_b^4 + 10 I_1^{0,1}(3, 1, 1) - 10 I_0(3, 1, 1) m_b^2 \\
& +20 I_1^{0,1}(3, 1, 2) m_b^2 + 10 I_0^{0,1}(3, 1, 2) m_c^2 - 10 I_0(2, 1, 2) m_c^2 - 20 I_0(2, 1, 2) m_b^2 \\
& -20 I_1(2, 1, 2) m_c^2 - 30 I_1(2, 1, 2) m_b^2 - 20 I_1^{0,1}(3, 2, 1) m_c^2 + 10 I_1^{0,1}(3, 2, 1) m_b^2 \\
& +10 I_0(2, 2, 1) m_c^2 + 50 I_0(2, 2, 1) m_b^2 - 5 I_0(3, 1, 2) m_c^4 + 15 I_0^{0,1}(3, 1, 2) m_b^2 \\
& -10 I_0^{0,1}(3, 2, 1) m_c^2 + 5 I_0^{0,1}(3, 2, 1) m_b^2 + 10 I_1^{0,1}(3, 2, 2) m_b^4 - 10 I_0(3, 1, 1) m_c^2 \\
& +50 I_2(2, 1, 1) - 10 I_1^{0,2}(3, 1, 2) + 10 I_1^{0,2}(3, 2, 1) + 90 I_1(2, 1, 1) + 10 I_0^{0,1}(2, 1, 2) \\
& +10 I_2(1, 1, 2) - 20 I_1^{0,1}(2, 2, 1)
\end{aligned}$$

$$\begin{aligned}
C^{T'_0} = & -15 m_b^2 I_0^{0,2}(2, 2, 2) - 5 I_0^{0,3}(3, 2, 1) + 5 m_b^2 I_0^{0,3}(3, 2, 2) - 20 I_0^{0,1}(3, 1, 2) m_b^3 m_c \\
& -50 I_0(1, 2, 1) m_b m_c + 20 I_0^{0,1}(2, 2, 1) m_b m_c + 10 I_0(2, 1, 2) m_b m_c^3 + 15 I_0(2, 1, 2) m_b^3 m_c \\
& -15 I_0(2, 1, 2) m_c^2 m_b^2 + 10 I_0(1, 2, 2) m_b^3 m_c + 5 I_0(3, 1, 2) m_b m_c^5 + 10 I_0^{0,1}(3, 1, 1) m_b m_c \\
& -5 I_0(3, 2, 2) m_c^6 m_b^2 - 5 I_0(3, 2, 2) m_b^5 m_c^3 + 5 I_0(3, 2, 2) m_b^3 m_c^5 + 5 I_0(3, 2, 2) m_c^4 m_b^4 \\
& +30 I_0(1, 3, 1) m_c^2 m_b^2 - 70 I_0(1, 3, 1) m_b^3 m_c - 30 I_0(2, 1, 1) m_b m_c - 10 I_0^{0,1}(2, 1, 2) m_b m_c \\
& +30 I_0(1, 4, 1) m_c^2 m_b^4 - 30 I_0(1, 4, 1) m_b^5 m_c + 15 I_0^{0,1}(3, 1, 2) m_c^2 m_b^2 - 10 I_0^{0,1}(3, 1, 2) m_b m_c^3 \\
& -10 I_0(2, 3, 1) m_b^3 m_c^3 - 15 I_0(4, 1, 1) m_c^4 m_b^2 + 15 I_0(4, 1, 1) m_b^3 m_c^3 - 15 I_0^{0,2}(3, 2, 2) m_c^2 m_b^2 \\
& +5 I_0^{0,2}(3, 2, 2) m_b^3 m_c + 15 I_0^{0,1}(3, 2, 2) m_c^4 m_b^2 - 10 I_0^{0,1}(3, 2, 2) m_b^3 m_c^3 - 5 I_0^{0,1}(3, 2, 2) m_b^5 m_c \\
& +10 I_0(3, 2, 1) m_b^3 m_c^3 - 5 I_0(3, 2, 1) m_b m_c^5 - 10 I_0(3, 2, 1) m_c^4 m_b^2 + 10 I_0(1, 1, 2) m_b m_c \\
& +10 I_0(2, 3, 1) m_b^5 m_c + 5 I_0^{0,2}(3, 1, 2) m_b m_c - 5 I_0(3, 1, 1) m_c^2 m_b^2 + 5 I_0(3, 1, 1) m_b^3 m_c \\
& -20 I_0(1, 2, 2) m_c^2 m_b^2 - 5 I_0^{0,2}(3, 2, 1) m_b m_c - 5 I_0(2, 2, 1) m_b^3 m_c - 20 I_0(2, 2, 1) m_b m_c^3 \\
& -10 I_0(3, 2, 1) m_c^2 m_b^4 + 10 I_0(3, 2, 1) m_b^5 m_c - 15 I_0(2, 2, 2) m_c^4 m_b^2 + 10 I_0(2, 2, 2) m_b^3 m_c^3 \\
& +30 I_0^{0,1}(2, 2, 2) m_c^2 m_b^2 - 10 I_0^{0,1}(2, 2, 2) m_b^3 m_c + 15 I_0^{0,1}(3, 2, 1) m_c^2 m_b^2 + 10 I_0^{0,1}(3, 2, 1) m_b m_c^3 \\
& -30 m_b^4 I_0^{0,1}(1, 4, 1) + 10 m_b^3 m_c I_0^{0,1}(2, 3, 1) + 20 I_0^{0,1}(1, 1, 2) + 15 m_c^2 m_b^2 I_0^{0,1}(4, 1, 1) \\
& -20 I_0(1, 1, 2) m_c^2 - 15 I_0^{0,1}(3, 2, 1) m_c^4 + 10 I_0^{0,1}(2, 2, 2) m_b^4 + 5 I_0(3, 2, 1) m_c^6
\end{aligned}$$

$$\begin{aligned}
& -5I_0^{0,2}(3,2,2)m_b^4 - 15I_0(1,1,2)m_b^2 - 5I_0^{0,2}(3,2,1)m_b^2 + 20m_b^2I_0^{0,1}(1,2,2) \\
& + 15I_0(2,2,1)m_c^4 + 10I_0(2,2,1)m_b^4 + 10I_0^{0,1}(3,2,1)m_b^4 + 15I_0^{0,2}(3,2,1)m_c^2 \\
& + 15I_0^{0,1}(3,1,2)m_c^4 + 20I_0(2,1,1)m_b^2 + 30I_0^{0,1}(2,1,2)m_c^2 + 30I_0^{0,1}(2,1,2)m_b^2 \\
& + 15I_0(2,1,1)m_c^2 - 15I_0(2,1,2)m_c^4 - 5I_0^{0,1}(2,2,1)m_b^2 - 30I_0^{0,1}(2,2,1)m_c^2 \\
& + 10I_0^{0,1}(3,1,1)m_b^2 - 5I_0(1,2,2)m_b^4 - 15I_0^{0,2}(3,1,2)m_b^2 + 20I_0(1,2,1)m_c^2 \\
& + 10I_0(1,2,1)m_b^2 - 15I_0^{0,2}(3,1,2)m_c^2 - 5I_0(3,1,2)m_c^6 - 30m_b^2I_0^{0,1}(1,3,1) \\
& + 5I_0^{0,3}(3,1,2) + 15I_0(1,1,1) - 20I_0^{0,1}(1,2,1) - 15I_0^{0,1}(2,1,1) \\
& - 15I_0^{0,2}(2,1,2) + 15I_0^{0,2}(2,2,1)
\end{aligned}$$

$$\begin{aligned}
C^{T'} &= 5/2 m_b^2 I_0^{0,2}(3,2,2) - 5I_0(2,1,1) - 5/2 I_0^{0,2}(3,2,1) - 5I_0(1,2,1) \\
& - 5I_1^{0,2}(3,1,2) - 20m_b^3 m_c I_1(2,3,1) - 15m_c^2 m_b^2 I_1(4,1,1) + 10I_1^{0,1}(2,1,2) \\
& + 15/2 m_c^2 m_b^2 I_0(4,1,1) + 10m_b^3 m_c I_0(2,3,1) - 15/2 m_c^2 m_b^2 I_0(3,2,1) - 15I_1(2,1,1) \\
& + 5I_0(3,1,1)m_b^2 + 5I_0^{0,1}(2,2,1) + 5/2 I_0^{0,2}(3,1,2) - 10m_c^2 I_1^{0,1}(3,2,1) \\
& - 5m_b^2 I_0^{0,1}(2,2,2) - 5m_b^2 I_1^{0,2}(3,2,2) + 5m_b^2 I_0(1,2,2) - 5m_c^4 I_1(3,1,2) \\
& - 10m_c^2 I_0(3,1,1) + 30m_b^4 I_1(1,4,1) + 10m_b^2 I_1^{0,1}(2,2,2) - 10I_1^{0,1}(2,2,1) \\
& - 15m_b^4 I_0(1,4,1) - 15m_b^2 I_0(1,3,1) + 30m_b^2 I_1(1,3,1) + 5I_0(2,2,2)m_c^2 m_b^2 \\
& + 10I_1(2,1,2)m_b m_c + 10I_0(2,2,1)m_b m_c - 30I_1(2,2,1)m_b m_c - 5I_0^{0,1}(3,2,2)m_c^2 m_b^2 \\
& + 5I_1(3,1,2)m_c^2 m_b^2 - 5I_1^{0,1}(3,1,1) + 5I_1^{0,2}(3,2,1) + 5/2 I_0(2,2,2)m_b^4 \\
& + 15I_1(2,1,2)m_b^2 - 5I_0(2,2,1)m_c^2 + 5I_1(3,1,1)m_b^2 + 10I_1(2,2,1)m_c^2 \\
& - 10I_1(2,1,2)m_c^2 - 5/2 I_0^{0,1}(3,2,2)m_b^4 + 10I_1^{0,1}(3,1,2)m_c^2 + 15I_1^{0,1}(3,1,2)m_b^2 \\
& - 5I_1(3,1,1)m_c^2 + 5/2 I_0(3,1,2)m_c^4 + 5I_1^{0,1}(3,2,2)m_b^4 + 10I_0(2,1,2)m_b^2 \\
& + 5I_0(2,1,2)m_c^2 - 5/2 I_0(3,2,1)m_c^4 - 5I_1(3,2,1)m_b^3 m_c + 5/2 I_0(3,2,2)m_c^4 m_b^2 \\
& - 5/2 I_0(3,2,2)m_c^2 m_b^4 + 10I_1^{0,1}(3,2,2)m_c^2 m_b^2 - 5I_1(3,1,2)m_b^3 m_c + 10I_1(2,2,2)m_b^3 m_c \\
& - 10I_1(2,2,2)m_c^2 m_b^2 - 5I_1(3,2,2)m_c^4 m_b^2 + 5I_1(3,2,2)m_c^2 m_b^4 - 5/2 I_0(3,1,2)m_c^2 m_b^2 \\
& + 5/2 I_0(3,2,1)m_b^3 m_c - 10I_1(3,2,1)m_c^2 m_b^2 + 5I_0(3,2,1)m_b^4 + 5I_1(3,2,1)m_c^4 \\
& - 5I_1(3,2,1)m_b^4 - 5I_0^{0,1}(3,1,2)m_c^2 - 15/2 I_0^{0,1}(3,1,2)m_b^2 + 5I_0^{0,1}(3,2,1)m_c^2 \\
& - 5/2 I_0^{0,1}(3,2,1)m_b^2 + 5/2 I_0(3,1,1)m_b m_c - 5I_0^{0,1}(2,1,2) + 5I_0(1,1,2)
\end{aligned}$$

where

$$\hat{I}_n^{[i,j]}(a,b,c) = (M_1^2)^i (M_2^2)^j \frac{d^i}{d(M_1^2)^i} \frac{d^j}{d(M_2^2)^j} \left[(M_1^2)^i (M_2^2)^j \hat{I}_n(a,b,c) \right].$$