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Strong decay constants of heavy tensor mesons in light cone QCD sum rules

H.A. Alhendi,[a] T.M. Aliev[b,1] and M. Savcı[b]

[a] *Physics and Astronomy Department, King Saud University, Saudi Arabia*

[b] *Physics Department, Middle East Technical University, 06531 Ankara, Turkey*

E-mail: alhendi@ksu.edu.sa, taliev@metu.edu.tr, savci@metu.edu.tr

Abstract: Strong decay constants of the heavy tensor to heavy pseudoscalar (vector) and light pseudoscalar mesons are estimated within the light cone QCD sum rules. It is observed that the values of these coupling constants show a significant dependence on the choice of the Lorentz structure. Additionally, the decay widths of these mesons are calculated and discussed within the light of experimental data. A comparison of our results on these coupling constants with the predictions from the 3-point sum rules is performed.

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[1] Permanent address:Institute of Physics,Baku,Azerbaijan.

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Contents

1 Introduction

There has been significant progress on heavy hadron physics spectroscopy during the last decade or so. Many new particles were discovered (see [1–9] and the references therein) part of which are successfully described in framework of the quark model, while interpretation of the remaining ones require to go beyond the quark picture. Among them the masses and decay widths of some excited mesons such as $\mathcal{D}_2^*(2400)$, $\mathcal{D}_{S_2}^*(2573)$, $\mathcal{B}_2^*(5747)$ and $\mathcal{B}_{S_2}^*(5840)$ with the quantum numbers $J^P = 2^+$ are measured [8–10]. The properties of $\mathcal{D}$-wave and radially excited heavy light meson systems will also be examined in detail in the planned experiments at LHC-b and KEK-B.

The strong decays of $\mathcal{D}$-wave mesons, such as $\mathcal{D}_2(2460) \to \mathcal{D}^{*+}\pi^-$, $\mathcal{D}^+\pi^-$, $\mathcal{D}_2^+(2460) \to \mathcal{D}^0\pi^+$ [6, 12–14], $\mathcal{D}_{S_2}^+(2573) \to \mathcal{D}^0 K^+$ [6], $\mathcal{B}_2(5747) \to \mathcal{B}^{*+}\pi^-$, $\mathcal{B}^+\pi^-$ [9, 10], and $\mathcal{B}_{S_2}^+(5840) \to \mathcal{B}^+K^-$ [9, 10] have already been observed. The decay constants of the tensor mesons $\mathcal{D}_2^*(2460)$ and $\mathcal{D}_{S_2}^*(2573)$ have been studied within the three-point QCD sum rules method in [15]. In the same framework the strong constants of $\mathcal{D}_2^*(2460) \to \mathcal{D}\pi$, $\mathcal{D}_{S_2}^*(2573) \to \mathcal{D}K$, $\mathcal{B}_2^*(5747) \to \mathcal{B}\pi$, and $\mathcal{B}_{S_2}^*(5840) \to \mathcal{B}K$ transitions have also been calculated in [16]. Recently, the strong decay constants of the $\mathcal{D}_2^*(2460) \to \mathcal{D}^*\pi$, $\mathcal{D}_{S_2}^*(2573) \to \mathcal{D}^*K$, $\mathcal{B}_2^*(5747) \to \mathcal{B}^*\pi$, and $\mathcal{B}_{S_2}^*(5840) \to \mathcal{B}^*K$ transitions have been studied in [17] in framework of the three-point QCD sum rules as well as the local QCD sum rules methods.

In this paper, we will calculate the strong coupling constants of the aforementioned decays of the tensor mesons $\mathcal{D}_2^*(2460)$, $\mathcal{D}_{S_2}^*(2573)$, $\mathcal{B}_2^*(5747)$, and $\mathcal{B}_{S_2}^*(5840)$ in framework of the light cone QCD sum rules method. Light cone QCD sum rules are based on the expansion over twist near the light cone $x^2 \simeq 0$, in contrast to the traditional QCD sum rules where the operator product expansion is performed over dimensions of the operators. The light cone QCD sum rules had been introduced to solve (or, at least partially) the following problems of the traditional QCD sum rules: a) Operator product expansion breaks down at large momentum transfers (as an example, one can refer the analysis of the pion electromagnetic form factor in the traditional QCD sum rules method). Similar problem also exists for heavy-to-light meson decays at large recoil. b) The pollution of the sum rule by the so called "non-diagonal" transitions from the ground state to excited states. Therefore predictions of the light cone QCD sum rules seem to be more reliable compared to that of the 3-point traditional QCD sum rules method. The details of the light cone QCD sum rules can be found in [18].

The paper is organized as follows. In section 2, the light cone QCD sum rules derived for the coupling constants g_1 and g_2 of the $\mathcal{D}_2^* \to \mathcal{D}^{(*)}\pi$, $\mathcal{D}_{S_2}^* \to \mathcal{D}^{(*)}K$, $\mathcal{B}_2^* \to \mathcal{B}^{(*)}\pi$, and $\mathcal{B}_{S_2}^* \to \mathcal{B}^{(*)}K$ transitions are given. Section 3 is devoted to the numerical analysis of the aforementioned coupling constants, and present the values of the corresponding two-body strong decays. In the rest of the section, comparison of our results with the ones available in literature are presented. The section ends with our concluding remarks.

2 Light cone QCD sum rules for the strong coupling constants g_1 and g_2

In this section we derive the light cone QCD sum rules for the strong coupling constants g_1 and g_2 of heavy tensor to heavy pseudoscalar and vector mesons with the participation of the light pseudoscalar mesons. To achieve this we first consider the following correlation function,

$$\Pi_{\mu\nu\ (\tau)}(p,q) = i \int d^4x e^{ipx} \left\langle \mathcal{P}(q) \left| J_{\mu\nu}(x) J^\dagger_{5(\tau)}(0) \right| 0 \right\rangle , \tag{2.1}$$

where

$$J_{\mu\nu}(x) = \frac{1}{2}\left[\bar{q}(x)\gamma_\mu \overleftrightarrow{\mathcal{D}}_\nu Q(x) + \bar{q}(x)\gamma_\nu \overleftrightarrow{\mathcal{D}}_\mu Q(x)\right],$$

is the interpolating current of the heavy tensor meson, $J_5 = \bar{q} i\gamma_5 Q$, and $J_\tau = \bar{q}\gamma_\tau Q$ are the interpolating currents of the heavy pseudoscalar and vector mesons, respectively. $\mathcal{P}$ denotes the light pseudoscalar meson, and $q(Q)$ describe the light(heavy) quark. The covariant derivative $\overleftrightarrow{\mathcal{D}}_\mu$ is defined in the following way,

$$\begin{aligned}
\overleftrightarrow{\mathcal{D}}_\mu (x) &= \frac{1}{2}\left[\overrightarrow{\mathcal{D}}_\mu (x) - \overleftarrow{\mathcal{D}}_\mu (x)\right], \\
\overrightarrow{\mathcal{D}}_\mu (x) &= \overrightarrow{\partial}_\mu (x) - i\frac{g}{2}\lambda^a A^a_\mu(x), \\
\overleftarrow{\mathcal{D}}_\mu (x) &= \overleftarrow{\partial}_\mu (x) + i\frac{g}{2}\lambda^a A^a_\mu(x),
\end{aligned} \tag{2.2}$$

where λ^a are the Gell-Mann matrices, and $A^a_\mu(x)$ is the external gluon field, and in our calculations we shall use the Fock-Schwinger gauge, i.e., $x_\mu A^\mu(x) = 0$. Since in this gauge the gluon field can be expressed in terms of gluon field strength tensor

$$\begin{aligned}
A^a_\mu &= \int_0^1 dt\, G_{\nu\mu}(tx)x^\nu \\
&= \frac{1}{2}x^\nu G_{\nu\mu}(0) + \frac{1}{3}x^\alpha x^\nu \mathcal{D}_\alpha G_{\nu\mu} + \cdots .
\end{aligned} \tag{2.3}$$

In order to construct the sum rules for the strong coupling constants of the heavy tensor meson to heavy pseudoscalar (vector) and light pseudoscalar mesons the correlation function (2.1) has to be calculated in two different kinematical domains in accordance with the QCD sum rules philosophy. The idea is straightforward. On the one hand, the correlation function saturates at around $p^2 \simeq m^2_{P_Q}$ and $(p+q)^2 \simeq m^2_{T_Q}$ (the hadronic part). On the other hand, it can be evaluated in the deep Eucledian region where $p^2 \ll 0$, and

JHEP04(2016)050

$(p+q)^2 \ll 0$ by using the operator product expansion over twist (the theoretical part). By matching these two representations, we derive the sum rules for the aforementioned hadronic coupling constants.

We start our analysis by calculating the correlator function from the hadronic side. Inserting a complete set of hadrons, carrying the same quantum numbers as the interpolating currents $J_{\mu\nu}$ and $J_{5(\tau)}$, into the correlation function, and isolating the ground state contributions of the heavy tensor and heavy pseudoscalar (vector) mesons, puts the correlation function into the form

$$\Pi_{\mu\nu(\tau)} = \frac{\langle 0|J_{\mu\nu}|T_Q\rangle\langle \mathcal{P}(q)T_Q|P_Q(V_Q)\rangle\langle P_Q(V_Q)|J^\dagger_{5(\tau)}|0\rangle}{(p^2-m^2_{T_Q})(p'^2-m^2_{P_Q(V_Q)})} + \cdots\,, \tag{2.4}$$

where T_Q, P_Q, V_Q, and $\mathcal{P}$ stand for heavy tensor, heavy pseudoscalar, heavy vector and light pseudoscalar mesons, respectively, and higher states and continuum contributions are not shown explicitly.

The matrix elements in eq. (2.4) are defined as,

$$\begin{aligned}
\langle 0\,|J_{\mu\nu}|\,T_Q(p)\rangle &= f_T m^3_{T_Q}\epsilon_{\mu\nu}(s,p)\,, && (2.5)\\
\langle 0\,|J_5|\,P_Q(p)\rangle &= \frac{f_Q m^2_{P_Q}}{m_Q+m_q}\,, && (2.6)\\
\langle 0\,|J_\tau|\,V_Q(p)\rangle &= f_V m_{V_Q}\xi^*_\tau(p')\,, && (2.7)\\
\langle \mathcal{P}T_Q|P_Q\rangle &= g_1\epsilon^*_{\alpha\beta}p'^\alpha p'^\beta = g_1\epsilon^*_{\alpha\beta}q^\alpha q^\beta\,, && (2.8)\\
\langle \mathcal{P}T_Q|V_Q\rangle &= g_2\varepsilon^{\alpha\beta\eta\lambda}p_\alpha\epsilon_{\beta\varphi}q^\varphi p'_\eta\xi_\lambda\,, && (2.9)
\end{aligned}$$

where $\epsilon_{\mu\nu}$ is the polarization vector of the heavy tensor meson, f_T, f_Q, f_V are the decay constants of the tensor, heavy pseudoscalar, and heavy vector mesons; g_1 and g_2 are the hadronic coupling constants, and s is the polarization vector of the heavy vector mesons. By performing the sum over the spins of the heavy tensor and heavy vector mesons in eqs. (2.4)–(2.9), for the correlation functions describing interaction of the heavy tensor, heavy pseudoscalar (vector) and light pseudoscalar mesons we get,

$$\begin{aligned}
\Pi_{\mu\nu} &= g_1\frac{f_T m^3_{T_Q}}{p^2-m^2_{T_Q}}\frac{f_Q m^2_{P_Q}}{m_Q+m_q}\frac{q^\alpha q^\beta}{p'^2-m^2_{P_Q}}\left\{\frac{1}{2}\Big[\tilde{g}_{\mu\alpha}\tilde{g}_{\nu\beta}+\tilde{g}_{\mu\beta}\tilde{g}_{\nu\alpha}\Big]-\frac{1}{3}\tilde{g}_{\mu\nu}\tilde{g}_{\alpha\beta}\right\},\\
&= g_1\frac{f_T m^3_{T_Q}}{p^2-m^2_{T_Q}}\frac{f_Q m^2_{P_Q}}{m_Q+m_q}\frac{1}{p'^2-m^2_{P_Q}}\left\{g_{\mu\nu}\left(-\frac{1}{3}m^2_{\mathcal{P}}+\frac{\left(m^2_{T_Q}+m^2_{\mathcal{P}}-m^2_{P_Q}\right)^2}{12m^2_{T_Q}}\right)+q_\mu q_\nu\right.\\
&\quad-\frac{1}{2}\left(q_\mu p_\nu+q_\nu p_\mu\right)\frac{\left(m^2_{T_Q}+m^2_{\mathcal{P}}-m^2_{P_Q}\right)}{m^2_{T_Q}}\\
&\quad\left.+\frac{p_\mu p_\nu}{m^4_{T_Q}}\left[\frac{\left(m^2_{T_Q}+m^2_{\mathcal{P}}-m^2_{P_Q}\right)^2}{6}+\frac{m^2_{\mathcal{P}}m^2_{T_Q}}{3}\right]\right\}+\cdots\,,
\end{aligned} \tag{2.10}$$

JHEP04(2016)050

$$\Pi_{\mu\nu\tau} = g_2 f_V m_{V_Q} \frac{f_T m^3_{T_Q}}{p^2 - m^2_{T_Q}} \frac{1}{p'^2 - m^2_{V_Q}} \varepsilon^{\alpha\beta\eta\lambda} p_\alpha q^\varphi p'_\eta \left\{ \frac{1}{2} \left[\tilde{g}_{\mu\beta}\tilde{g}_{\nu\varphi} + \tilde{g}_{\mu\varphi}\tilde{g}_{\nu\beta} \right] - \frac{1}{3}\tilde{g}_{\mu\nu}\tilde{g}_{\beta\varphi} \right\}$$

$$\times \left\{ - g_{\lambda\tau} + \frac{p'_\lambda p'_\tau}{m^2_{V_Q}} \right\}$$

$$= \frac{1}{2} g_2 f_T f_V m^3_{T_Q} m_{V_Q} \frac{1}{p^2 - m^2_{T_Q}} \frac{1}{p'^2 - m^2_{V_Q}} \left\{ \varepsilon^{\mu\alpha\eta\tau} \left[p_\alpha q_\nu q_\eta + p_\alpha p_\nu q_\eta \frac{m^2_{T_Q} + m^2_{\mathcal{P}} - m^2_{V_Q}}{2m^2_{T_Q}} \right] \right.$$

$$\left. + (\mu \leftrightarrow \nu) \right\} + \cdots, \qquad (2.11)$$

where

$$\tilde{g}_{\mu\nu} = -g_{\mu\nu} + \frac{p_\mu p_\nu}{m^2_{T_Q}}.$$

We now turn our attention to the calculation of the correlation function from the theoretical side, i.e., in terms of the quark-gluon degrees of freedom. After contracting the heavy quark fields, the correlation function can be written as,

$$\Pi_{\mu\nu(\tau)} = -\frac{1}{2} \int d^4x e^{ipx} \left\langle \mathcal{P}(q) \left| \bar{q}(x)\gamma_\mu \overset{\leftrightarrow}{\mathcal{D}}_\nu (x) S_Q(x) \gamma_{5(\tau)} q(0) + (\mu \leftrightarrow \nu) \right| 0 \right\rangle, \qquad (2.12)$$

where $S_Q(x)$ is the heavy quark propagator whose form in the coordinate representation is given as,

$$S_Q(x) = \frac{m^2_Q}{4\pi^2} \left\{ \frac{K_1\left(m_Q\sqrt{-x^2}\right)}{\sqrt{-x^2}} + i\frac{\not{x}}{-x^2} K_2\left(m_Q\sqrt{-x^2}\right) \right\} \qquad (2.13)$$

$$- \frac{g_s m_Q}{16\pi^2} \int_0^1 du \left[G_{\mu\nu}(ux) \left(\sigma^{\mu\nu}\not{x} + \not{x}\sigma^{\mu\nu}\right) \frac{K_1\left(m_Q\sqrt{-x^2}\right)}{\sqrt{-x^2}} + 2\sigma^{\mu\nu} K_0\left(m_Q\sqrt{-x^2}\right) \right],$$

where K_n is the modified Bessel function of degree n. The following relations for the derivatives of the Bessel functions will be used:

$$\frac{d}{dx}K_n(x) = -\frac{1}{2}\Big[K_{n-1}(x) + K_{n+1}(x)\Big],$$
$$\frac{d}{dx}(x^n K_n) = -x^n K_{n-1}(x),$$
$$\frac{d}{dx}(x^{-n} K_n) = -x^{-n} K_{n+1}(x). \qquad (2.14)$$

It should be noted here that the expansion of the quark operator up to twist four terms is given in [19], which gets contributions from $\bar{q}Gq$, $\bar{q}GGq$, and $\bar{q}q\bar{q}q$ nonlocal operators. In the present work we consider operators with one gluon field, and neglect the contributions of the operators $\bar{q}GGq$ and $\bar{q}q\bar{q}q$. Indeed taking into account of higher Fock-space component requires simultaneous calculations of the corrections (with conformal spin $j = 5$) to both two– and three-particle distribution amplitudes (DAs). The contribution of higher conformal spin terms should be small. Therefore, neglecting the contributions of the $\bar{q}GGq$ and $\bar{q}q\bar{q}q$ operators can be justified on the basis of an expansion in conformal spin.

Having the expression of the heavy quark operator at hand, we now calculate the correlation function from the QCD side. The expression of the correlation function in deep Eucledian domain, $p^2 \to -\infty$ $(p+q)^2 \to -\infty$, can be obtained by using the operator product expansion. It follows from eq. (2.12) that, in order to calculate the correlation function from the QCD side, the matrix elements of the nonlocal operators between vacuum and one-particle light pseudoscalar meson states are needed. The matrix elements $\langle \mathcal{P}(q)\, |\bar{q}(x)\Gamma_i q'(0)|\, 0\rangle$ are parametrized in terms of the (DAs) [20, 21] and they are determined as,

$$
\begin{aligned}
\langle \mathcal{P}(p)\,|\bar{q}_1(x)\gamma_\mu\gamma_5 q_1(0)|\,0\rangle &= -if_{\mathcal{P}}q_\mu \int_0^1 du e^{i\bar{u}qx}\left(\varphi_{\mathcal{P}}(u)+\frac{1}{16}m_{\mathcal{P}}^2x^2\mathbb{A}(u)\right)\\
&\quad -\frac{i}{2}f_{\mathcal{P}}m_{\mathcal{P}}^2\frac{x_\mu}{qx}\int_0^1 du e^{i\bar{u}qx}\mathbb{B}(u)\,,\\
\langle \mathcal{P}(p)\,|\bar{q}_1(x)i\gamma_5 q_2(0)|\,0\rangle &= \mu_{\mathcal{P}}\int_0^1 du e^{i\bar{u}qx}\phi_P(u)\,,\\
\langle \mathcal{P}(p)\,|\bar{q}_1(x)\sigma_{\alpha\beta}\gamma_5 q_2(0)|\,0\rangle &= \frac{i}{6}\mu_{\mathcal{P}}\left(1-\widetilde{\mu}_{\mathcal{P}}^2\right)(q_\alpha x_\beta - q_\beta x_\alpha)\int_0^1 du e^{i\bar{u}qx}\phi_\sigma(u)\,,\\
\langle \mathcal{P}(p)\,|\bar{q}_1(x)\sigma_{\mu\nu}\gamma_5 g_s G_{\alpha\beta}(vx)q_2(0)|\,0\rangle &= i\mu_{\mathcal{P}}\left[q_\alpha q_\mu\left(g_{\nu\beta}-\frac{1}{qx}(q_\nu x_\beta+q_\beta x_\nu)\right)\right.\\
&\quad -q_\alpha q_\nu\left(g_{\mu\beta}-\frac{1}{qx}(q_\mu x_\beta+q_\beta x_\mu)\right)\\
&\quad -q_\beta q_\mu\left(g_{\nu\alpha}-\frac{1}{qx}(q_\nu x_\alpha+q_\alpha x_\nu)\right)\\
&\quad \left.+q_\beta q_\nu\left(g_{\mu\alpha}-\frac{1}{qx}(q_\mu x_\alpha+q_\alpha x_\mu)\right)\right]\\
&\quad \times\int \mathcal{D}\alpha e^{i(\alpha_{\bar{q}}+v\alpha_g)qx}\mathcal{T}(\alpha_i)\,,\\
\langle \mathcal{P}(p)\,|\bar{q}_1(x)\gamma_\mu\gamma_5 g_s G_{\alpha\beta}(vx)q_2(0)|\,0\rangle &= q_\mu(q_\alpha x_\beta - q_\beta x_\alpha)\frac{1}{qx}f_{\mathcal{P}}m_{\mathcal{P}}^2\int \mathcal{D}\alpha e^{i(\alpha_{\bar{q}}+v\alpha_g)qx}\mathcal{A}_\parallel(\alpha_i)\\
&\quad +\left[q_\beta\left(g_{\mu\alpha}-\frac{1}{qx}(q_\mu x_\alpha+q_\alpha x_\mu)\right)\right.\\
&\quad \left.-q_\alpha\left(g_{\mu\beta}-\frac{1}{qx}(q_\mu x_\beta+q_\beta x_\mu)\right)\right]f_{\mathcal{P}}m_{\mathcal{P}}^2\\
&\quad \times\int \mathcal{D}\alpha e^{i(\alpha_{\bar{q}}+v\alpha_g)qx}\mathcal{A}_\perp(\alpha_i)\,,\\
\langle \mathcal{P}(p)\,|\bar{q}_1(x)\gamma_\mu i g_s G_{\alpha\beta}(vx)q_2(0)|\,0\rangle &= q_\mu(q_\alpha x_\beta - q_\beta x_\alpha)\frac{1}{qx}f_{\mathcal{P}}m_{\mathcal{P}}^2\int \mathcal{D}\alpha e^{i(\alpha_{\bar{q}}+v\alpha_g)qx}\mathcal{V}_\parallel(\alpha_i)\\
&\quad +\left[q_\beta\left(g_{\mu\alpha}-\frac{1}{qx}(q_\mu x_\alpha+q_\alpha x_\mu)\right)\right.\\
&\quad \left.-q_\alpha\left(g_{\mu\beta}-\frac{1}{qx}(q_\mu x_\beta+q_\beta x_\mu)\right)\right]f_{\mathcal{P}}m_{\mathcal{P}}^2\\
&\quad \times\int \mathcal{D}\alpha e^{i(\alpha_{\bar{q}}+v\alpha_g)qx}\mathcal{V}_\perp(\alpha_i)\,,
\end{aligned}
\tag{2.15}
$$

where

$$\mu_{\mathcal{P}} = f_{\mathcal{P}} \frac{m_{\mathcal{P}}^2}{m_{q_1} + m_{q_2}}\,, \qquad \widetilde{\mu}_{\mathcal{P}} = \frac{m_{q_1} + m_{q_2}}{m_{\mathcal{P}}}\,,$$

and q_1 and q_2 are the quarks in the meson $\mathcal{P}$, $\mathcal{D}\alpha = d\alpha_{\bar{q}} d\alpha_q d\alpha_g \delta(1-\alpha_{\bar{q}}-\alpha_q-\alpha_g)$. Here $\varphi_{\mathcal{P}}(u)$ is the leading twist-two, $\phi_P(u)$, $\phi_\sigma(u)$, $\mathcal{T}(\alpha_i)$ are the twist-three, and $\mathbb{A}(u)$, $\mathbb{B}(u)$, $\mathcal{A}_\perp(\alpha_i)$, $\mathcal{A}_\parallel(\alpha_i)$, $\mathcal{V}_\perp(\alpha_i)$ and $\mathcal{V}_\parallel(\alpha_i)$ are the twist-four DAs, respectively. Their explicit expressions are given in the next section.

Using eqs. (2.12), (2.13) and (2.15) the theoretical part of the correlation function can be calculated in a straightforward manner. By equating the coefficients of the respective Lorentz structures calculated from the hadronic and QCD sides of the correlation function, and performing Borel transformation for the variables $-p^2$ and $-p'^2 = -(p+q)^2$, to suppress the continuum and higher states contribution, we obtain the sum rules for the strong coupling constants g_1 and g_2,

$$\begin{aligned}
\frac{f_{P_Q} m_{P_Q}^2}{m_Q + m_q} f_T m_{T_Q}^3 g_1 A_i e^{-m_{T_Q}^2/M_1^2} e^{-m_{P_Q}^2/M_2^2} + \int ds_1 ds_2 \rho_1^h(s_1,s_2) e^{-s_1/M_1^2 - s_2/M_2^2} &= \widetilde{\Pi}_i^{th.(P_Q)}\,, \\
f_{V_Q} m_{V_Q} f_T m_{T_Q}^3 g_2 B_i e^{-m_{T_Q}^2/M_1^2} e^{-m_{V_Q}^2/M_2^2} + \int ds_1 ds_2 \rho_2^h(s_1,s_2) e^{-s_1/M_1^2 - s_2/M_2^2} &= \widetilde{\Pi}_i^{th.(V_Q)}\,,
\end{aligned} \tag{2.16}$$

where tilde in (2.16) means Borel transformed invariant function. The second term on the left hand side of eq. (2.16) corresponds to the contributions of the higher states, as well as the continuum. In order to calculate this contribution, the hadron-quark duality ansatz is employed, i.e., above some threshold in the (s_1, s_2) plane, and then the hadronic spectral density is equal to the spectral density obtained from QCD side. Using this ansatz, the continuum subtraction can be performed by using the procedure given in [22]. In performing the continuum subtraction procedure we have used the fact that, the initial and final heavy baryon masses are close to each other, so that we can set $M_1^2 = M_2^2 = 2M^2$, and in this limit we get $u_0 = 1/2$, where we shall use further numerical analysis. Omitting the technical details of the subtraction procedure we note that, in the case $M_1^2 = M_2^2 = 2M^2$ and at the point $u_0 = 1/2$, the subtraction procedure can be performed by using the following formula,

$$M^{2n} e^{-m_Q^2/M^2} \to \frac{1}{\Gamma(n)} \int_{m_Q^2}^{s_0} ds e^{-s/M^2} (s - m_Q^2)^{n-1}\,, \quad (n \geq 1)\,,$$

from which we obtain

$$M^2 e^{-m_Q^2/M^2} \to M^2 \left[e^{-m_Q^2/M^2 - s_0/M^2} \right]\,.$$

For the higher twist terms which are proportional to the zeroth or to the negative powers of M^2, continuum subtraction is not performed, since their contribution is expected to be small (for more details, see [22]). The expressions of A_i and B_i in the above-expression

JHEP04(2016)050

are as follows,

$$A_i = \begin{cases} 1, \text{ for the } q_\mu q_\nu \text{ structure}\,, \\ -\frac{1}{2m_T^2}\left(m_{T_Q}^2+m_{\mathcal{P}}^2-m_{P_Q}^2\right), \text{ for the } q_\mu p_\nu+q_\nu p_\mu \text{ structure}\,, \\ -\frac{1}{3}m_{\mathcal{P}}^2+\frac{\left(m_{T_Q}^2+m_{\mathcal{P}}^2-m_{P_Q}^2\right)^2}{12m_{T_Q}^2}, \text{ for the } g_{\mu\nu} \text{ structure}\,, \\ \frac{1}{m_{T_Q}^4}\left[\frac{1}{6}\left(m_{T_Q}^2+m_{\mathcal{P}}^2-m_{P_Q}^2\right)^2+\frac{m_{\mathcal{P}}^2 m_{T_Q}^2}{3}\right], \text{ for the } p_\mu p_\nu \text{ structure}\,. \end{cases}$$

$$B_i = \begin{cases} 1, \text{ for the } \varepsilon^{\mu\alpha\eta\tau}p_\alpha q_\nu q_\eta \ (\varepsilon^{\nu\alpha\eta\tau}p_\alpha q_\mu q_\eta) \text{ structure}\,, \\ \frac{1}{4m_{T_Q}^2}\left(m_{T_Q}^2+m_{\mathcal{P}}^2-m_{P_Q}^2\right), \text{ for the } \varepsilon^{\mu\alpha\eta\tau}p_\alpha p_\nu q_\eta \ (\varepsilon^{\nu\alpha\eta\tau}p_\alpha p_\mu q_\eta) \text{ structure}\,. \end{cases} \tag{2.17}$$

The expressions of the $\widetilde{\Pi}_i^{th.(P_Q)}$ and $\widetilde{\Pi}_i^{th.(V_Q)}$ are listed below:

1) Coefficient of the $q_\mu q_\nu$ structure

$$\begin{aligned} \widetilde{\Pi}_1^{th.(P)} &= e^{m_{\mathcal{P}}^2/4M^2}e^{-m_Q^2/M^2}\bigg\{\frac{1}{48M^2}f_{\mathcal{P}}m_{\mathcal{P}}^2 m_Q^3\mathbb{A}(u_0)-\frac{1}{144}M^2\Big\{12f_{\mathcal{P}}m_Q\varphi_{\mathcal{P}}(u_0) \\ &\quad+\mu_{\mathcal{P}}\Big[6\phi_{\mathcal{P}}(u_0)-(1-\widetilde{\mu}_{\mathcal{P}}^2)\Big(8\phi_\sigma(u_0)+\phi_\sigma'(u_0)\Big)\Big]\Big\}+\frac{1}{24}f_{\mathcal{P}}m_{\mathcal{P}}^2 m_Q\widetilde{j}_1(\mathbb{B}) \\ &\quad+\frac{1}{432m_Q}\Big[9f_{\mathcal{P}}m_{\mathcal{P}}^2 m_Q^2\mathbb{A}(u_0)-6m_Q(m_{\mathcal{P}}^2-2m_Q^2)\mu_{\mathcal{P}}(1-\widetilde{\mu}_{\mathcal{P}}^2)\phi_\sigma(u_0)\Big]\bigg\}. \end{aligned}$$

2) Coefficient of the $p_\mu q_\nu + p_\nu q_\mu$ structure

$$\begin{aligned} \widetilde{\Pi}_2^{th.(P)} &= e^{m_{\mathcal{P}}^2/4M^2}e^{-m_Q^2/M^2}\bigg\{\frac{1}{48M^2}f_{\mathcal{P}}m_{\mathcal{P}}^2 m_Q^3\mathbb{A}(u_0)-\frac{1}{72}M^2\Big\{6f_{\mathcal{P}}m_Q\varphi_{\mathcal{P}}(u_0) \\ &\quad+\mu_{\mathcal{P}}\Big[6\phi_{\mathcal{P}}(u_0)-(1-\widetilde{\mu}_{\mathcal{P}}^2)(4\phi_\sigma(u_0)+\phi_\sigma'(u_0)\Big)\Big]\Big\}+\frac{1}{12}f_{\mathcal{P}}m_{\mathcal{P}}^2 m_Q\widetilde{j}_1(\mathbb{B}) \\ &\quad+\frac{1}{432m_Q}\Big[9f_{\mathcal{P}}m_{\mathcal{P}}^2 m_Q^2\mathbb{A}(u_0)-12m_Q(m_{\mathcal{P}}^2-m_Q^2)\mu_{\mathcal{P}}(1-\widetilde{\mu}_{\mathcal{P}}^2)\phi_\sigma(u_0)\Big]\bigg\}. \end{aligned}$$

3) Coefficient of the $g_{\mu\nu}$ structure

$$\begin{aligned} \widetilde{\Pi}_3^{th.(P)} &= e^{m_{\mathcal{P}}^2/4M^2}e^{-m_Q^2/M^2}\bigg\{-\frac{1}{72}M^4\mu_{\mathcal{P}}\Big[12\phi_{\mathcal{P}}(u_0)-(1-\widetilde{\mu}_{\mathcal{P}}^2)\phi_\sigma'(u_0)\Big] \\ &\quad+\frac{1}{12}M^2 f_{\mathcal{P}}m_{\mathcal{P}}^2 m_Q\widetilde{j}_1(\mathbb{B})-\frac{1}{36}M^2 m_{\mathcal{P}}^2\mu_{\mathcal{P}}(1-\widetilde{\mu}_{\mathcal{P}}^2)\phi_\sigma(u_0)\bigg\}. \end{aligned}$$

4) Coefficient of the $p_\mu p_\nu$ structure

$$\widetilde{\Pi}_4^{th.(P)} = e^{m_{\mathcal{P}}^2/4M^2} e^{-m_Q^2/M^2} \left\{ -\frac{1}{36} M^2 \mu_{\mathcal{P}} \left[6\phi_{\mathcal{P}}(u_0) - (1-\widetilde{\mu}_{\mathcal{P}}^2)\phi_\sigma'(u_0) \right] \right.$$
$$\left. + \frac{1}{6} f_{\mathcal{P}} m_{\mathcal{P}}^2 m_Q \widetilde{j}_1(\mathbb{B}) - \frac{1}{18} m_{\mathcal{P}}^2 \mu_{\mathcal{P}} (1-\widetilde{\mu}_{\mathcal{P}}^2)\phi_\sigma(u_0) \right\}.$$

5) Coefficient of the $\varepsilon^{\mu\alpha\eta\tau} p_\alpha q_\nu q_\eta$ and $\varepsilon^{\nu\alpha\eta\tau} p_\alpha q_\mu q_\eta$ structures

$$\widetilde{\Pi}_1^{th.(V)} = e^{m_{\mathcal{P}}^2/4M^2} e^{-m_Q^2/M^2} \left\{ -\frac{1}{96 m_Q M^2} f_{\mathcal{P}} m_{\mathcal{P}}^2 m_Q^3 \mathbb{A}(u_0) \right.$$
$$\left. + \frac{1}{24} f_{\mathcal{P}} M^2 \varphi_{\mathcal{P}}(u_0) - \frac{1}{144} \left[3 f_{\mathcal{P}} m_{\mathcal{P}}^2 \mathbb{A}(u_0) + 2 m_Q \mu_{\mathcal{P}} (1-\widetilde{\mu}_{\mathcal{P}}^2)\phi_\sigma(u_0) \right] \right\}.$$

6) Coefficient of the $\varepsilon^{\mu\alpha\eta\tau} p_\alpha p_\nu q_\eta$ and $\varepsilon^{\nu\alpha\eta\tau} p_\alpha p_\mu q_\eta$ structures

$$\widetilde{\Pi}_2^{th.(V)} = e^{m_{\mathcal{P}}^2/4M^2} e^{-m_Q^2/M^2} \left\{ -\frac{1}{48 m_Q M^2} f_{\mathcal{P}} m_{\mathcal{P}}^2 m_Q^3 \mathbb{A}(u_0) \right.$$
$$\left. + \frac{1}{12} f_{\mathcal{P}} M^2 \varphi_{\mathcal{P}}(u_0) - \frac{1}{72} \left[3 f_{\mathcal{P}} m_{\mathcal{P}}^2 \mathbb{A}(u_0) + 2 m_Q \mu_{\mathcal{P}} (1-\widetilde{\mu}_{\mathcal{P}}^2)\phi_\sigma(u_0) \right] \right\},$$

where

$$u_0 = \frac{M_1^2}{M_1^2+M_2^2}, \qquad M^2 = \frac{M_1^2 M_2^2}{M_1^2+M_2^2}.$$

The function $\widetilde{j}_1(f(u))$ appearing in the expressions above is defined as:

$$\widetilde{j}_1(f(u)) = \int_{u_0}^1 du f(u).$$

It should be noted here that, in the above expressions the light quark masses m_u, m_d and m_s are all set to zero, while in the numerical calculations the mass m_s of the strange quark is taken into account.

3 Numerical analysis

This section is devoted to the numerical analysis of the sum rules for the strong coupling constants g_1 and g_2.

The main input parameters of the light cone QCD sum rules are the distribution amplitudes (DAs), whose expressions are given below [20, 21],

$$\varphi_{\mathcal{P}}(u) = 6u\bar{u}\left[1 + a_1^{\mathcal{P}} C_1(2u-1) + a_2^{\mathcal{P}} C_2^{3/2}(2u-1)\right],$$
$$\mathcal{T}(\alpha_i) = 360\eta_3 \alpha_{\bar{q}} \alpha_q \alpha_g^2 \left[1 + w_3 \frac{1}{2}(7\alpha_g - 3)\right],$$

$$
\begin{aligned}
\phi_P(u) &= 1+\left[30\eta_3-\frac{5}{2}\frac{1}{\mu_{\mathcal{P}}^2}\right]C_2^{1/2}(2u-1)\,,\\
&\quad+\left(-3\eta_3w_3-\frac{27}{20}\frac{1}{\mu_{\mathcal{P}}^2}-\frac{81}{10}\frac{1}{\mu_{\mathcal{P}}^2}a_2^{\mathcal{P}}\right)C_4^{1/2}(2u-1)\,,\\
\phi_\sigma(u) &= 6u\bar{u}\left[1+\left(5\eta_3-\frac{1}{2}\eta_3w_3-\frac{7}{20}\mu_{\mathcal{P}}^2-\frac{3}{5}\mu_{\mathcal{P}}^2a_2^{\mathcal{P}}\right)C_2^{3/2}(2u-1)\right]\,,\\
\mathcal{V}_{\parallel}(\alpha_i) &= 120\alpha_q\alpha_{\bar{q}}\alpha_g\left(v_{00}+v_{10}(3\alpha_g-1)\right)\,,\\
\mathcal{A}_{\parallel}(\alpha_i) &= 120\alpha_q\alpha_{\bar{q}}\alpha_g\left(0+a_{10}(\alpha_q-\alpha_{\bar{q}})\right)\,,\\
\mathcal{V}_{\perp}(\alpha_i) &= -30\alpha_g^2\left[h_{00}(1-\alpha_g)+h_{01}(\alpha_g(1-\alpha_g)-6\alpha_q\alpha_{\bar{q}})+h_{10}\left(\alpha_g(1-\alpha_g)-\frac{3}{2}(\alpha_{\bar{q}}^2+\alpha_q^2)\right)\right]\,,\\
\mathcal{A}_{\perp}(\alpha_i) &= 30\alpha_g^2(\alpha_{\bar{q}}-\alpha_q)\left[h_{00}+h_{01}\alpha_g+\frac{1}{2}h_{10}(5\alpha_g-3)\right]\,,\\
B(u) &= g_{\mathcal{P}}(u)-\varphi_{\mathcal{P}}(u)\,,\\
g_{\mathcal{P}}(u) &= g_0C_0^{1/2}(2u-1)+g_2C_2^{1/2}(2u-1)+g_4C_4^{1/2}(2u-1)\,,\\
\mathbb{A}(u) &= 6u\bar{u}\left[\frac{16}{15}+\frac{24}{35}a_2^{\mathcal{P}}+20\eta_3+\frac{20}{9}\eta_4+\left(-\frac{1}{15}+\frac{1}{16}-\frac{7}{27}\eta_3w_3-\frac{10}{27}\eta_4\right)C_2^{3/2}(2u-1)\right.\\
&\quad\left.+\left(-\frac{11}{210}a_2^{\mathcal{P}}-\frac{4}{135}\eta_3w_3\right)C_4^{3/2}(2u-1)\right]\,,\\
&\quad+\left(-\frac{18}{5}a_2^{\mathcal{P}}+21\eta_4w_4\right)\left[2u^3(10-15u+6u^2)\ln u\right.\\
&\quad\left.+2\bar{u}^3(10-15\bar{u}+6\bar{u}^2)\ln\bar{u}+u\bar{u}(2+13u\bar{u})\right]\,,
\end{aligned}
\tag{3.1}
$$

where $C_n^k(x)$ are the Gegenbauer polynomials, and

$$
\begin{aligned}
h_{00} &= v_{00} = -\frac{1}{3}\eta_4\,,\\
a_{10} &= \frac{21}{8}\eta_4w_4-\frac{9}{20}a_2^{\mathcal{P}}\,,\\
v_{10} &= \frac{21}{8}\eta_4w_4\,,\\
h_{01} &= \frac{7}{4}\eta_4w_4-\frac{3}{20}a_2^{\mathcal{P}}\,,\\
h_{10} &= \frac{7}{4}\eta_4w_4+\frac{3}{20}a_2^{\mathcal{P}}\,,\\
g_0 &= 1\,,\\
g_2 &= 1+\frac{18}{7}a_2^{\mathcal{P}}+60\eta_3+\frac{20}{3}\eta_4\,,\\
g_4 &= -\frac{9}{28}a_2^{\mathcal{P}}-6\eta_3w_3\,.
\end{aligned}
\tag{3.2}
$$

The values of the parameters $a_1^{\mathcal{P}}$, $a_2^{\mathcal{P}}$, η_3, η_4, w_3, and w_4 entering eq. (3.2) are listed in table 1 for the pseudoscalar π, K and η mesons.

In performing the numerical analysis, in addition to the above-mentioned parameters, the decay constants of the heavy tensor, heavy pseudoscalar (vector) and light pseudoscalar

	π	K
$a_1^{\mathcal{P}}$	0	0.050
$a_2^{\mathcal{P}}$ (set-1)	0.11	0.15
$a_2^{\mathcal{P}}$ (set-2)	0.25	0.27
η_3	0.015	0.015
η_4	10	0.6
w_3	-3	-3
w_4	0.2	0.2

Table 1. Parameters of the wave function calculated at the renormalization scale $\mu = 1\,\text{GeV}$.

mesons, and masses of the quarks are necessary. The decay constants of the heavy tensor mesons are calculated in [16, 17, 23] and their numerical values are $f_{\mathcal{D}_2^*} = (0.018 \pm 0.007)$, $f_{\mathcal{D}_{S_2}^*} = (0.023 \pm 0.011)$, $f_{\mathcal{B}_2^*} = 0.011$, $f_{\mathcal{B}_{S_2}^*} = 0.013$, $f_{\mathcal{D}^*} = 0.24\,\text{GeV}$, $f_{\mathcal{B}^*} = 0.16\,\text{GeV}$ [22].

In the present work we use the $\overline{MS}$ values of the quark masses given the Particle Data Group [8]: $\overline{m}_c(m_c) = (1.275 \pm 0.025)\,\text{GeV}$, $\overline{m}_b(m_b) = (4.18 \pm 0.03)\,\text{GeV}$, and $m_s(2\,\text{GeV}) = (0.095 \pm 0.005)\,\text{GeV}$. The masses of the heavy mesons are calculated in framework of the the QCD sum rules, having the values, $m_{\mathcal{D}_2^*} = (2.460 \pm 0.009)\,\text{GeV}$, $m_{\mathcal{B}_2^*} = (5.73 \pm 0.06)\,\text{GeV}$, $m_{\mathcal{B}_{S_2}^*} = (5.84 \pm 0.06)\,\text{GeV}$, which are all very close to their experimental values.

There are three extra parameters entering into the sum rules expressions, namely, the continuum threshold s_0 and the Borel mass parameters M_1^2 and M_2^2. In the present analysis we set $M_1^2 = M_2^2 = 2M^2$, and this choice corresponds to $u_0 = 1/2$. The continuum threshold s_0 is determined from an analysis of the two-point correlation function which leads to the following results: $s_{0,\mathcal{D}_2^*} = (8.5 \pm 0.5)\,\text{GeV}^2$, $s_{0,\mathcal{D}_{S_2}^*} = (9.5 \pm 0.5)\,\text{GeV}^2$, $s_{0,\mathcal{B}_2^*} = (39.0 \pm 1.0)\,\text{GeV}^2$, $s_{0,\mathcal{B}_{S_2}^*} = (41.0 \pm 1.0)\,\text{GeV}^2$ [16, 17].

Of course, we need to find such a region of M^2 where the strong coupling constants g_1 and g_2 are insensitive to the variation in M^2. The upper bound of M^2 is determined from the condition that the higher states and continuum contributions constitute, say, 30% of the total result. The lower bound of the Borel mass parameter M^2 is determined from the condition that the contribution of the highest term with the power $1/M^2$ remains less than 25% of the contribution coming from the highest power of M^2. These two conditions lead to the following working regions of M^2: $2\,\text{GeV}^2 \leq M^2 \leq 4\,\text{GeV}^2$ (for $\mathcal{D}_2^*$ and $\mathcal{D}_{S_2}^*$), $4\,\text{GeV}^2 \leq M^2 \leq 7\,\text{GeV}^2$ (for $\mathcal{B}_2^*$ and $\mathcal{B}_{S_2}^*$).

Having determined the working regions of M^2, we now calculate the strong coupling constants g_1 and g_2 for the $T_Q P_Q \mathcal{P}$ and $T_Q V_Q \mathcal{P}$ vertices, respectively. By performing numerical analysis we obtain the values of the coupling constants g_1 and g_2 which are presented in tables 2 and 3. In these calculations we use two different sets of parameters that appear in the expressions of the DAs.

In table 3 "no stability" means that there is no region of the Borel parameter M^2 where the results for the strong coupling constants are insensitive to its variation.

	[15]		[16]		present work							
	$g_{\mu\nu}$	$p_\mu p_\nu$	$g_{\mu\nu}$	$p_\mu p_\nu$	$g_{\mu\nu}$		$p_\mu p_\nu$		$q_\mu q_\nu$		$p_\mu q_\nu + p_\nu q_\mu$	
					set-1	set-2	set-1	set-2	set-1	set-2	set-1	set-2
$\mathcal{D} \to \mathcal{D}\pi^+$	15.3	4.63	16.5	12.3	39 ± 13	40 ± 3	11 ± 1	12 ± 1	0.10	0.08	17 ± 5	17 ± 6
$\mathcal{D}_{S_2} \to \mathcal{D}K^0$	18.3	5.76	12.2	9.9	78 ± 20	74 ± 5	10 ± 1	10 ± 1	0.10	0.13	19 ± 6	9 ± 3
$\mathcal{B} \to \mathcal{B}\pi^+$	—	—	39.3	17.1	90 ± 25	88 ± 20	70 ± 15	73 ± 20	0.30	0.15	40 ± 12	36 ± 12
$\mathcal{B}_{S_2} \to \mathcal{B}K^0$	—	—	26.3	12.9	400 ± 100	360 ± 50	58 ± 12	55 ± 12	0.25	0.30	24 ± 8	22 ± 8

Table 2. The values of the strong coupling constants for the $T_Q P_Q \mathcal{P}$ vertices for different Lorentz structures.

	[16]		present work			
	$\varepsilon_{\nu\tau\rho\sigma} p'^{\mu} p^{\rho} p'^{\sigma}$	$\varepsilon_{\nu\tau\rho\sigma} p^{\mu} p^{\rho} p'^{\sigma}$	$\varepsilon_{\rho\sigma\alpha\tau} p^{\beta} p^{\rho} q^{\sigma}$		$\varepsilon_{\rho\sigma\alpha\tau} q^{\beta} p^{\rho} q^{\sigma}$	
			set-1	set-2	set-1	set-2
$\mathcal{D}_2^0 \to \mathcal{D}^{*+}\pi^-$	4.00	0.73	4.4 ± 1.0	no stability	0.42 ± 0.10	no stability
$\mathcal{D}_{S_2}^0 \to \mathcal{D}^{*-}K^+$	2.98	0.79	4.4 ± 1.0	no stability	-0.37 ± 0.13	no stability
$\mathcal{B}_2^0 \to \mathcal{B}^{*+}\pi^-$	3.87	—	5.9 ± 2.0	no stability	0.22 ± 0.05	no stability
$\mathcal{B}_{S_2}^0 \to \mathcal{B}^{*+}K^-$	2.89	—	3.0 ± 1.0	no stability	0.28 ± 0.06	no stability

Table 3. Same as in table 2, but for the $T_Q V_Q \mathcal{P}$ vertices.

From these tables we deduce the following conclusions:

- The value of the strong coupling constant depends very strongly on the choice of the corresponding Lorentz structure, especially for the $T_Q \to P_Q \mathcal{P}$ transition.
- We also observe that the value of the strong coupling constant for the $T_Q \to P_Q \mathcal{P}$ transition ranges in a rather wide region from 0.3(0.15) to 290(400) for the $\mathcal{B}_2 \to \mathcal{B}^-\pi^+$ ($\mathcal{B}_{S_2}^* \to \mathcal{B}^0 K^0$). In the case of $\mathcal{D}_{2(S_2)} \to \mathcal{D}^+\pi^-$ ($\mathcal{D}^0 K^0$) transitions the values of the corresponding coupling constants vary in the range 0.15 to 78.0. These results also show that the strong coupling constants are very sensitive to the values of the input parameters appearing in the DAs.

The huge differences in the values of the strong coupling constants g_1 and g_2 can be understood in the following way. It is well known that the Lorentz structures having maximum number of momenta exhibit good convergence of the OPE compared to the structures containing less number of momenta. Therefore the prediction of the $g_{\mu\nu}$ structure on the coupling constant is not reliable compared to the other structures which contain maximum number of momenta. The origin of different values for the coupling constant predicted by the $\varepsilon_{\rho\sigma\alpha\tau} p^{\beta} p^{\rho} q^{\sigma}$ and $\varepsilon_{\rho\sigma\alpha\tau} q^{\beta} p^{\rho} q^{\sigma}$ structures (for the set 1) can be attributed to the fact that the ratio of the sum rules is crudely proportional to $m_{T_Q}/\Delta m$, where $\Delta m = m_{T_Q} - m_{V_Q}$.

As far as the coupling constant g_1 is concerned, the structures $g_{\mu\nu}$ and $p_\mu p_\nu$ do not contain leading twist term, and therefore predictions of these structures are not reliable.

JHEP04(2016)050

It follows from table 1 that the most reliable value for the $T_Q P_Q \mathcal{P}$ vertex follows from the $q_\mu p_\nu + q_\nu p_\mu$ structure, from which we get,

$$g_1 = \begin{cases} 17 \pm 5, & \mathcal{D}_2^0 \to \mathcal{D}^+ \pi^- \,, \\ 19 \pm 6, & \mathcal{D}_{S_2}^+ \to \mathcal{D}^0 K^+ \,, \\ 40 \pm 12, & \mathcal{B}_2^0 \to \mathcal{B}^- \pi^+ \,, \\ 24 \pm 8, & \mathcal{B}_{S_2}^0 \to \mathcal{B}^0 K^0 \,. \end{cases} \tag{3.3}$$

Using the set 1 values of the parameters of the wave functions, for the strong coupling constants of the $T_Q \to V_Q \mathcal{P}$ transitions (for the structure $\varepsilon_{\rho\sigma\alpha\tau} p^\beta p^\rho q^\sigma$) we obtain,

$$g_2 = \begin{cases} 4.4 \pm 1.0, & \mathcal{D}_2^0 \to \mathcal{D}^{*+} \pi^- \,, \\ 4.4 \pm 1.0, & \mathcal{D}_{S_2}^+ \to \mathcal{D}^{*0} K^+ \,, \\ 5.9 \pm 2.0, & \mathcal{B}_2^0 \to \mathcal{B}^{*-} \pi^+ \,, \\ 3.0 \pm 1.0, & \mathcal{B}_{S_2}^0 \to \mathcal{B}^{*+} K^- \,. \end{cases} \tag{3.4}$$

We now compare our results on the strong coupling constants with those predicted by the 3-point QCD sum rules method. For the structure $g_{\mu\nu}$ our prediction for the coupling constant $g_{\mathcal{D}_2^* \mathcal{D}^+ \pi^-}$ is approximately two times larger compared to the one predicted in [15] and [16]. For the $\mathcal{B}_2 \to \mathcal{B}^- \pi^+$ and $\mathcal{B}_{S_2} \to \mathcal{B}^0 K^0$ transition coupling constants our results are two to more than ten times larger than the ones predicted in [16]. Our predictions on the $T_Q V_Q \mathcal{P}$ coupling constants for the structures $\varepsilon_{\rho\sigma\alpha\tau} p^\beta p^\rho q^\sigma$ $(\varepsilon_{\rho\sigma\alpha\tau} q^\beta p^\rho q^\sigma)$ are approximately eight times larger(ten times smaller), respectively compared to the ones predicted by [16].

At the end of this section we present the results for the decay widths of all transitions under consideration, whose expressions are gives as,

$$\begin{aligned} \Gamma_{T_Q P_Q \mathcal{P}} &= \frac{g_1^2 \, |\vec{p}|^5}{60 \pi m_{T_Q}^2} \,, \\ \Gamma_{T_Q V_Q \mathcal{P}} &= \frac{g_2^2 \, |\vec{p}|^5}{40 \pi} \,, \end{aligned} \tag{3.5}$$

where

$$|\vec{p}| = \frac{1}{2 m_{T_Q}} \left(m_{T_Q}^4 + m_{P_Q}^4 + m_{\mathcal{P}}^4 - 2 m_{T_Q}^2 m_{P_Q}^2 - 2 m_{T_Q}^2 m_{\mathcal{P}}^2 - 2 m_{P_Q}^2 m_{\mathcal{P}}^2 \right)^{1/2} . \tag{3.6}$$

Note that the $|\vec{p}|^5$ dependence is an indication of the fact that the decay takes place at the D-wave level. The results for the decay widths can be summarized as follows.

- In the case for the transitions of the heavy tensor mesons to heavy pseudoscalar and light pseudoscalar mesons (for the $p_\mu q_\nu + p_\nu q_\mu$ structure), the widths are,

$$\begin{aligned} \Gamma \left(\mathcal{D}_2^0(2460) \to \mathcal{D}^+ \pi^- \right) &= (8.6 \pm 5.2) \times 10^{-3} \, \text{GeV} \,, \\ \Gamma \left(\mathcal{D}_{S_2}^+(2573) \to \mathcal{D}^0 K^0 \right) &= (4.4 \pm 2.4) \times 10^{-3} \, \text{GeV} \,, \\ \Gamma \left(\mathcal{B}_2^0(5747) \to \mathcal{B}^- \pi^+ \right) &= (3.7 \pm 2.4) \times 10^{-3} \, \text{GeV} \,, \\ \Gamma \left(\mathcal{B}_{S_2}^0(5840) \to \mathcal{B}^0 K^0 \right) &= (8.6 \pm 5.7) \times 10^{-5} \, \text{GeV} \,. \end{aligned} \tag{3.7}$$

- In the case for the transitions of the heavy tensor mesons to heavy vector and light pseudoscalar mesons (for the $\varepsilon_{\rho\sigma\alpha\tau}p^{\beta}p^{\rho}q^{\sigma}$ structure), they are

$$\Gamma\left(\mathcal{D}_2^0(2460) \to \mathcal{D}^{*+}\pi^-\right) = (5.2 \pm 2.3) \times 10^{-3}\,\mathrm{GeV}\,,$$
$$\Gamma\left(\mathcal{D}_{S_2}^+(2573) \to \mathcal{D}^{*0}K^+\right) = (2.30 \pm 0.85) \times 10^{-3}\,\mathrm{GeV}\,,$$
$$\Gamma\left(\mathcal{B}_2^0(5747) \to \mathcal{B}^{*-}\pi^+\right) = (4.0 \pm 2.7) \times 10^{-3}\,\mathrm{GeV}\,,$$
$$\Gamma\left(\mathcal{B}_{S_2}^0(5840) \to \mathcal{B}^{*0}K^0\right] = (6.9 \pm 4.6) \times 10^{-5}\,\mathrm{GeV}\,.$$

It follows from the experimental data that [8], the ratios are

$$\frac{\Gamma(\mathcal{D}_2(2460) \to \mathcal{D}^+\pi^-)}{\Gamma(\mathcal{D}_2(2460) \to \mathcal{D}^{*+}\pi^-)} = 1.55\,,$$
$$\frac{\Gamma(\mathcal{B}_2(5747) \to \mathcal{B}^+\pi^-)}{\Gamma(\mathcal{B}_2(5747) \to \mathcal{B}^{*+}\pi^-)} = 0.91\,,$$

as well as from the BaBar Collaboration data [24]

$$\frac{\Gamma(\mathcal{D}_2(2460) \to \mathcal{D}^+\pi^-)}{\Gamma(\mathcal{D}_2(2460) \to \mathcal{D}^+\pi^-) + \Gamma(\mathcal{D}_2(2460) \to \mathcal{D}^{*+}\pi^-)} = 0.62 \pm 0.03 \pm 0.02\,.$$

When we calculate the same ratios from eqs. (3.6) and (3.7), we obtain that,

$$\frac{\Gamma(\mathcal{D}_2(2460) \to \mathcal{D}^+\pi^-)}{\Gamma(\mathcal{D}_2(2460) \to \mathcal{D}^{*+}\pi^-)} = 1.64 \pm 1.24\,,$$
$$\frac{\Gamma(\mathcal{B}_2(5747) \to \mathcal{B}^-\pi^+)}{\Gamma(\mathcal{B}_2(5747) \to \mathcal{B}^{*-}\pi^+)} = 0.93 \pm 0.85\,,$$

and

$$\frac{\Gamma(\mathcal{D}_2(2460) \to \mathcal{D}^+\pi^-)}{\Gamma(\mathcal{D}_2(2460) \to \mathcal{D}^+\pi^-) + \Gamma(\mathcal{D}_2(2460) \to \mathcal{D}^{*+}\pi^-)} = 0.67 \pm 0.50\,.$$

We see that our prediction on the ratio of the decay widths (for the central values of the considered decays) are in good agreement with the experimental results, as well as they are quite close to the results in [16] and [17].

Few words about the perturbative $\mathcal{O}(\alpha_s)$ corrections are in order. These corrections increase the correlation function of the coupling constant of the $\mathcal{B}^* \to \mathcal{B}\pi$ transition about 50% in the light cone QCD sum rules [25]. If we assume that this increase in the correlation function is correct, it is expected that the coupling constant increases with the same order, i.e., $g_i \to 1.5g_i$. This increase in the coupling constant doubles the values of the decay widths as well.

In conclusion, we calculate the strong coupling constant of the heavy tensor meson to heavy pseudoscalar (vector) and light pseudoscalar mesons in framework of the light cone QCD sum rules method. It is seen that the values of the hadronic decay constants depend very strongly on the choice of the Lorentz structure. Furthermore, using the determined values of the coupling constants we also estimate the corresponding decay widths. A comparison of our predictions on these hadronic coupling constants with the results of the 3-point sum rules is presented.

JHEP04(2016)050

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JHEP04(2016)050