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Identifying pre-service physics teachers' misconceptions and conceptual difficulties about geometrical optics

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Abstract

This study investigated pre-service physics teachers' (PSPTs) misconceptions and conceptual difficulties about geometrical optics in the contexts of plane mirrors (single and hinged), spherical mirrors (concave and convex) and lenses (converging and diverging) using prolonged interviews and an open-ended test. The interviews were conducted in five sessions with a divergent sample of 16 PSPTs, and the open-ended geometrical optics test (OEGOT) was administered to a sample of 52 PSPTs in Turkey. The interview guide and the OEGOT included mainly qualitative questions, which were specifically designed to elicit and examine PSPTs' misconceptions about geometrical optics. Several misconceptions held by PSPTs were identified and discussed. We found that PSPTs mainly had difficulties with the ray model, the function of the observer in the real and virtual image formation and observation processes, and the function of the screen in image formation and observation processes. Also the misconceptions in plane mirrors were found to be more coherent and experience-based while the ones in spherical mirrors and lenses were fragmented and loosely held in nature and mostly based on instruction.

Keywords: physics education, pre-service training, misconceptions, geometrical optics

(Some figures may appear in colour only in the online journal)

1. Introduction

Student misconceptions have their origins in a diverse set of personal experiences. Possible sources of student misconceptions are listed by several researchers as: physical experience, direct observation, intuition, school teaching, outside of school teaching, social environment, peer culture, language, textbooks or other instructional materials, and teachers [1–5]. Teachers are thought to be the key to improving student performance [6–8]. However, they often subscribe to the same misconceptions as their students [5, 6, 9, 10]. There is substantial evidence that teachers are one of the main sources of student misconceptions [5, 6, 11, 12]. Therefore, before studying student conceptions, teachers' conceptions should be determined and, if necessary, modified in order to improve students' conceptions. In this respect exploring pre-service and in-service teachers' existing misconceptions on several physics concepts becomes an important concern to be investigated.

There are several studies in the literature on understanding students' conceptions in science and there has been more research in the domain of physics than in any other [5, 13–16]. However, these studies have focused on certain concepts in mechanics, electricity, and heat and temperature. Duit *et al* [14] documented a bibliography of the publications of individuals' ideas in science and reported that mechanics (especially the force concept) and electricity (especially simple electric circuits) are over-researched domains in physics. The authors mentioned the need for further attention to the other domains. While most high school and introductory university physics instructions and textbooks provide quantitative and algorithmic manipulations related to geometrical optics, the development of a coherent conceptual understanding might not result in most cases. Student understanding in geometrical optics has attracted the interest of some researchers in different countries from early childhood to university level and beyond [9, 17–39]. Even though vision is one of our most important senses and our experience with light starts with birth, all of these studies showed that individuals have serious problems in understanding the nature of light, light propagation, vision and optical imaging, regardless of their background, age, ability, culture or other characteristics. As Langley, Ronen and Eylon [30] claimed, students' ideas in geometrical optics are not simply attributable to the age of the subjects. Although maturation may change some of the ideas in geometrical optics [31, 40], these differences among ages might be due to everyday experiences and their effect on development [31]. Studies of college or university students according to age, or prospective teachers' knowledge [9, 18, 26–28, 38, 41] about light, seeing, shadows, mirror and lens images revealed very similar results to those found for younger subjects [17, 21, 22, 30–32, 40, 42]. The results of the studies also showed that the conceptions of individuals about geometrical optics vary only little, or not at all, from one culture to another [22, 30, 31]. Although studies in geometrical optics vary, there exist a limited number of studies on teachers and certain contexts in geometrical optics studied comparatively less. For instance, hardly any studies were found in the contexts of hinged plane mirrors, convex mirrors, and diverging lenses.

The main aim of this study is to explore Turkish pre-service physics teachers' (PSPTs) misconceptions and conceptual difficulties in geometrical optics. In order to explore PSPTs' conceptions, there is a need for a valid and reliable diagnostic tool which serves its purpose. Although there are a variety of methods to diagnose misconceptions in science (such as interviews, concept maps, open-ended tests, word associations, multiple-choice tests, multiple-tier tests with two, three or four-tiers), researchers have not reached a consensus regarding the best method for this purpose since each method has its own strengths and limitations [43]. It depends on the context of the topic to be investigated, the characteristics of the intended subjects to be investigated, and the ability and resources of the researcher. However, a combination of the strengths of several methods is proposed as an ideal course of action for testing understanding. With this aim in the scope of the dissertation study of the first author [44], a four-tier multiple-choice misconception test was developed to assess the misconceptions of PSPTs in geometrical optics. A four-tier test is a multiple-tier diagnostic test. The first tier is an ordinary multiple-choice test with its distractors addressing specific misconceptions. The second tier of the test asks for the confidence of the answer in the first tier. The third tier of the test asks for the reasoning for the answer in the first tier. The fourth tier of the test asks for the confidence of the answer in the third (reasoning) tier. In the four-tier test development process, the researchers benefited from prolonged interviews, open-ended test administration, and multiple-tier multiple-choice test administration. The diversity in the data collection methods enabled the researchers to gain valuable information about the PSPTs' misconceptions as well as providing a good foundation for developing a valid and reliable diagnostic assessment tool. In the present study, the qualitative part of the research, which includes the analysis of interviews and open-ended test, are going to be discussed. The purposes of conducting interviews were to identify the PSPTs' misconceptions and difficulties in geometrical optics, which have or have not been mentioned in the literature before, and to detect the most resourceful contexts to elicit the PSPTs' misconceptions in geometrical optics for the construction of an open-ended test. The purpose of administering the open-ended test was to identify and explore the misconceptions and difficulties in geometrical optics with a larger sample. Hence, the main research questions to be investigated in this study are as follows.

1. What contexts are more resourceful in order to elicit PSPTs' misconceptions and conceptual difficulties in geometrical optics as investigated with in-depth interviews?
2. What are the misconceptions and conceptual difficulties in geometrical optics of PSPTs who completed optics and optics laboratory courses as investigated with in-depth interviews and with open-ended questions?

2. Context for the study and methodology

Before describing our findings, it is useful to provide a brief background context to the educational environment in which the study was conducted. In Turkey, there exist a total of 13 state universities that have five-year undergraduate physics teacher education programs. Students are selected and placed into those universities according to their score in a national university entrance exam. Each university has its own education program in which PSPTs are expected to took several physics courses (e.g. mechanics, electricity and magnetism, electronics, modern physics, atomic physics, quantum mechanics), pedagogical courses (e.g. educational philosophy, classroom management, guidance) and courses that integrate subject matter and pedagogy (e.g. methods of physics teaching, instructional technology and material development in physics teaching, laboratory experiments in physics education, practice teaching in physics education). After successful completion of the five-year undergraduate teacher education programs, the PSPTs could be employed in high schools as physics teachers. Turkish PSPTs take geometrical optics courses during their elementary school, high school educations and in their undergraduate physics teacher education programs at their universities. Although the curricula for the elementary and high schools are the same for all, the curricula for the universities may differ. In some of the universities geometrical optics is discussed in the course of optics and waves together with physical optics, whereas in some other universities courses dedicated only to geometrical optics and the related laboratory are offered. Since each university has its own teaching program, geometrical optics and its laboratory are discussed in different grade levels as a compulsory or an elective course in the physics teacher education programs with different course credits.

PSPTs who completed their geometrical optics and geometrical optics laboratory courses in their programs are expected to conceptualize some basic scientific conceptions about geometrical optics. Those scientific conceptions that are expected to be known by the PSPTs for the present study are listed below and conceptions that are incompatible with those currently accepted scientific conceptions are called 'misconceptions' in the present study.

- Light propagates in a rectilinear path.
- An object is a collection of object points and an image is a collection of image points.
- When the light rays themselves converge to an image point it is called a real image. When the light rays do not converge to an image point, but the extensions of reflected or refracted rays converge to an image point it is called a virtual image.
- Each point on a real image emits (or reflects) an infinite number of rays in all directions. Each point on a virtual image appears to emit (or reflect) an infinite number of rays in all directions.
- When the light strikes a mirror surface, the rule of reflection holds true.
- When the light meets with a lens, the rule of refraction holds true.
- Light diverging from a real image point must enter an observer's eye for the image to be observed.
- Light diverging from a virtual image point must enter an observer's eye for the virtual image to be simultaneously formed and observed.
- The position of an image depends only on the position of the object relative to the mirror and independent of the observer's position.
- The number of images formed and observed in hinged plane mirrors (two mirrors case) depends on the angle between the mirrors, the position of the object located, and the position of the observer.
- A screen is a convenience for observation of real image points in space (i.e. aerial image) and there is a particular image position for a sharp image of an object to be observed on a screen.
- The light ray is a representational tool to show the direction of light prorogation. The special rays serve as an algorithm to locate the position of an image. Any combination of two of these is sufficient to locate the position of an image. However, special rays are sufficient but not necessary in order to form an image point.

For the interviews, specific courses were selected from the three universities in Ankara (the capital city of Turkey) on the basis of ease of access to the individuals for prolonged interviews. These specific courses were selected among the courses one semester after geometrical optics and/or geometrical optics laboratory course were discussed in their educational programs. Then a divergent sample of 16 participants was selected through maximum variation sampling [45] from these conveniently selected universities. The maximum variation was established according to a standardized score of the students in each university calculated as the arithmetic average of cumulative grade point average (CGPA) and the weighted average of geometrical optics and geometrical optics laboratory course grades with regarding their course credits.

According to the ranking of that calculated scores, high–medium–low achieving PSPT categories were determined and an effort was made to select the participants for the interviews equally from those three categories. PSPTs that are one standard deviation above the score of the class average or more were assigned as high achieving, between one standard deviation above and one standard deviation below of the class average as medium achieving, and one standard deviation below the class average or less as low achieving in each three universities.

Table 1. Demographic information about the 16 participants selected for interviews.

University	Participant code	Registered semester of participants	Gender	Calculated score of participants	Calculated score of class
Middle East Technical University	M1	6	M	3.76	Avr: 2.57 StDv: 0.66
	M2		F	3.51	
	M3		M	2.55	
	M4		F	2.13	
	M5		M	1.81	
	M6		M	1.58	
Gazi University	G1	6	F	3.22	Avr: 2.62 StDv: 0.48
	G2		F	3.15	
	G3		F	3.11	
	G4		F	2.84	
	G5		F	2.65	
	G6		F	2.11	
Hacettepe University	H1	8	F	2.74	Avr: 2.28 StDv: 0.41
	H2		F	2.61	
	H3		M	2.16	
	H4		M	2.09	

Table 1 gives some demographic information about the participants selected for the interviews.

For open-ended test administration, the 13 universities were divided into three groups according to their rankings on the university placement scores. One university from each of these groups was selected purposefully. In this way, it was possible to get a divergent range of sample from different universities. Then the courses were selected by convenience sampling [45] by considering the ease of accessibility. There were a total of 82 registered PSPTs in the selected courses and the open-ended test was administered to 52 PSPTs (25 male and 27 female) in those three different universities located in different regions. The interviews and the open-ended test were conducted during the spring semester of the 2011–2012 academic year.

The Interview-About-Instances technique [46] was used in the interviewing part. During this process, the participants were provided with some diagrams related to the questions in the interview guide that was designed specifically for this study. The interviewer asked the questions in order to elicit the ideas held by the PSPTs. To establish the content validity, the interview guide was given to three instructors from the Department of Secondary Science and Mathematics Education with physics education majors at Middle East Technical University together with an expert opinion form. The instructors investigated the guide and made some suggestions to modify the items in the guide for better understanding, as well as some suggestions to the administration of the interviews. As a result, the 35-item Semi-Structured interview guide was prepared. A two-dimensional table with contexts in optical imaging (plane mirrors, spherical mirrors, lenses) in one dimension, and the cases in optical imaging (‘seeing/forming an image of oneself’ and ‘seeing/forming an image of others’) in the other dimension was prepared and approximately equal importance were given to each context-case combinations (see table 2). Some of the questions in the interview guide (~40%) were modified or inspired from the questions from the literature [27, 28, 33, 47–49], while some

Table 2. Distribution of items to the contexts and cases in the interview guide and open-ended test.

			Cases			
					Total	
			Observing oneself	Observing others	I	**O**
	Plane mirrors	Plane mirror	I1, I2, I3, I4, I5 (**O5**), I6, I7, I9, I10 (**O7**), I12, I13	I1(**O1**), I2, I3(**O2**), I4 (**O3**), I5(**O4**), I6(**O6**), I7, I8, I11(**O11**), I14	14	**8**
		Hinged mirrors	I28(**O18**), I29	I27(**O17**), I28, I29(**O19**)	3	**3**
Contexts	Spherical mirrors	Concave mirror	I15, I19(**O12**), I21, I24(**O14**)	I15(**O9**), I17, I19 (**O11**), I23(**O16**)	6	**5**
		Convex mirror	I16, I20(**O13**), I22, I25(**O15**)	I16(**O10**), I18, I20, I26	6	**3**
	Lenses	Converging lens	I32(**O23**)	I30, I31(**O20**, **O21**), I35	4	**3**
		Diverging lens	I33(**O24**)	I34(**O22**)	2	**2**
	Total	I	23	25	35	
		O	**9**	**15**		**24**

Note. 'I' represents the interview guide items and 'O' represents the open-ended test items. The open-ended test items shown in parentheses were produced from the corresponding items in the interview guide.

others (~60%) were constructed by the researchers. Most of the questions in the guide required students to draw diagrams in the explanation and description of their answers to the questions. The questions were open-ended in nature with some additional follow-up questions and probes. Special attention was given to the participants explaining their reasoning for the answers to the questions. The interviews were conducted individually within five sessions and one interview was done in a week with each single participant. Hence, the completion of all interviews with all participants took five weeks, with an average of 35–75 min interviews with each participant. The first session of the interviews was dedicated to building rapport between the interviewee and the participants, informing participants about the aim of the study and the contexts to be discussed, asking general questions about geometrical optics in order to determine the contexts in which the participants had difficulty. The remaining four interview sessions were in single plane mirrors, hinged plane mirrors, spherical mirrors, and lenses contexts, respectively. The interview sessions were video-recorded as a primary source of data analysis. The drawings of each participant on the questions were collected as a secondary data. The analyses of the interview data took three weeks. The results of the analysis were primarily used to identify misconceptions and to develop the open-ended test for this study.

After conducting and analyzing the interviews, the open-ended test was developed. The main aim of administering the open-ended test was to get a greater generalizability and to construct the alternatives of the multiple-choice multiple-tier test in geometrical optics, which is not the scope of the present study. Almost all of the questions in the open-ended test were selected from the interview guide. However, some small modifications were made in order to enhance the quality of some of the questions. The test was given to the three instructors for evaluation in order to establish content validity. The two-dimensional table with the contexts and the cases in its dimensions was regarded and approximately equal importance was given to each context-case combinations (see table 2). As a result, the 24-item open-ended geometrical optics test (OEGOT) was developed. In the first tier of each item, a question related to geometrical optics was asked; while in the second tier of the item, reasoning about a given answer was expected. Figure 1 provides the context of the question at the top, a sample two-tier item from the OEGOT on the right, and its corresponding form appeared in the interview guide on the left.

The OEGOT was administered to 52 PSPTs in approximately 90 min. In the analysis of this part, the researcher documented and grouped the responses of PSPTs to each question and then categorized the meaningful responses with their frequencies. Administration of the OEGOT and the analysis of the results took a total of five weeks. PSPTs' responses to the OEGOT were evaluated by two independent raters. Similarities and differences between the ratings of the two independent raters were discussed until a consensus was reached and a relatively high inter-rater reliability 0.88 was obtained.

3. Results

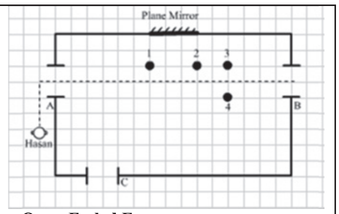
In this section, we present common misconceptions of the PSPTs in geometrical optics identified with the interviews and the OEGOT. The aim of the first session of the interviews was to detect the most resourceful contexts to elicit the misconceptions about the topic. As a result, the items in the interviews, so in the OEGOT, were modified by including the most triggering types of questions in the items. In section 3.1, the preliminary findings to determine the resourceful contexts are discussed. In sections 3.2 and 3.3, the findings of the interviews and the open-ended test about the misconceptions related to plane mirrors, spherical mirrors and lenses are presented. The references given in the parenthesis next to a misconception category represents that the misconception was discussed in the given studies and also found in the present study. A misconception category without any references next to it means no specific study was found in the literature related to that misconception. Yet, the present study detected that specific misconception.

3.1. Identification of resourceful contexts for eliciting common misconceptions

In the first session of the interviews, the participants were asked some general questions related to image formation and the observation process for plane mirrors (single and hinged plane mirrors), spherical mirrors (concave and convex mirrors), and lenses (converging and diverging lenses). The questions are mainly about the properties of the images formed in a certain optical element, the role of the observer in image formation process, the role of observer in the image observation process, the effect of a moving observer on the image formation, the effect of a moving object on image formation, the difference between observing oneself and observing other objects, and the effect of covering some parts of the optical element.

The interview results of the first session revealed that the PSPTs may use some key terms in the topic arbitrarily or mistakenly. When they say 'image' they may only refer to the reflection of light rays from an optical element such as mirrors, or they may refer to anything that is seen by the eye. Additionally, their definitions of real and virtual images often differ from one individual to another. Some participants made the distinction between real and virtual images as follows.

1. On the right you can see the top-view of a homogenously illuminated room. Hasan (*the observer*) enters the room from the left door and as he walks along the path shown with dashed line, he tries to observe images of the cats numbered by 1, 2, 3 and 4 in the plane mirror on the wall. So;



Interview form

A. Which of the cats' images will be formed in the plane mirror? Where? Explain your answer.
B. Which of the cats' image does Hasan see <u>first</u> in the plane mirror? Locate Hasan's and the image of cat's position at that time. Explain your answer.
C. In which order does Hasan see images of cats in the plane mirror as he walks along the path? Explain your answer.
D. Do the position and size of the images of the cats change as Hasan walks along the path? Explain your answer.
E. As Hasan walks along the path where he can first see his own image in the mirror? Locate Hasan's and his image's position at that time. Explain your answer.
F. Does the position and size of Hasan's own image change as he walks along the path? Explain your answer.
G. As Hasan walks along the path, he cannot see the image of a cat at certain places. Does it affect the image formation of that cat? Explain your answer.

Open-Ended Form

A.1. Which of the cats' images will be formed in the plane mirror? Draw the image places on the figure.
A2. Explain your reasoning for your answer.
B.1. In which order does Hasan see images of cats in the plane mirror as he walks along the path?
B.2. Explain your reasoning for your answer.

Figure 1. A sample item from the interview guide (I1) and the OEGOT (O1). Note: some of the other items are similar to the above item with minor differences. For instance, in I2 the observer enters the room from the right door (door B) and in I3 from the back door (door C). In I15 the plane mirror is replaced with a concave mirror and in I16 with a convex mirror.

'Image is anything around us that we see with our eyes. Real images do not require any optical element. For instance, when I directly look at this pencil, I do not need any instrument, this is a real image. However, virtual images are the ones formed by optical elements such as mirrors.' (Participants H1, G5, and M6)
'My real images are the ones that are directly the same with me in orientation and size. In plane mirrors, the images are real since they are directly the same with the object. When I say virtual, however, the upside-down images are coming to my mind.' (Participant M4)
'For instance lenses make the object bigger or smaller, they do not show you the real images of the objects.' (Participant H3)
'Real images are formed at the same side of the mirror or lens with the object. Virtual images, however, are formed at the opposite side.' (Participants H2 and M5)
'Real images are the ones that can only be seen on a screen, without a screen those images cannot be seen. However, the virtual images do not require a screen.' (Participants G2 and G4)
'Virtual images are the erect; real images are the upside-down images of the objects.' (Participants G2 and M3)
'Real image is the one the light rays themselves converge to an image point. The virtual image, is the one in which the light rays do not converge to an image point, but the extensions of reflected or refracted rays converge to that point or appear to emanate from that point.' (Participants M2 and H4)

This diversity in their conceptions of real and virtual images made the PSPTs' reasoning about different contexts in geometrical optics even more difficult to interpret. However, this diversity also signifies that, even after instruction, conceptions in geometrical optics may not be totally scientific, and needs to be investigated. The participants also used some other key terms, such as '*field of view*', arbitrarily. They used this term especially in their explanations related to the observation of images in phrases such as '*field of view of the mirror*', '*field of view of image*', '*point of view of observer*', '*angle of view of observer*' or '*field of view of observer*' interchangeably.

The participants also had some other difficulties in determining the image location, image type (real or virtual), or image orientation for a particular object located at a certain position relative to an optical element, such as a mirror or lens. However, those difficulties seem to be highly related to the context of the problems at hand. For instance, when they were asked about whether covering some parts of an optical element affects image formation and/or observation process, some participants claimed that it depends on which part of the instrument is covered. According to them, when the outer parts of an instrument are covered, it does not make any difference for image formation. However, when the inner sides of a spherical mirror or lens are covered, or the parts of the plane mirrors where the object is in the front is covered, it causes the image to disappear. Hence in the following interview sessions, questions were modified and asked for different coverings of optical elements. Extract from the interview with one of the participants (participant M3) is given below as an example.

> Participant M3: *If we cover some parts of the plane mirror, it does not affect the image formation. It is like decreasing the mirror size. Does it make any difference when a small object is in front of a* $10\,\mathrm{m}^2$ *or* $1\,\mathrm{m}^2$ *plane mirror? But, if the size of the object is bigger than the mirror, some part of the image is not formed.*
>
> ….
>
> Participant M3: *In a hinged plane mirror… Hmm… Let's say I cover the portion where two plane mirrors intersect with* $90°$ *originally. They will no longer intersect with* $90°$; *the angle would become smaller in that case.*
>
> ….
>
> Participant M3: *While covering the spherical mirror, if I am just decreasing the size of the mirror it is not a problem. But, if I am covering the mirror from the middle, it does.*
>
> Interviewer: *What do you mean? Could you explain more?*
>
> Participant M3: *Covering the contour of the mirror, it is just like cutting the sides. It makes the mirror smaller in size. Look at this* [participant sketched the drawing shown in figure 2(a) on the paper]. *However, if the middle of the mirror is covered like this* [participant sketched the drawing shown in figure 2(b) on the paper]*, the cardboard absorbs the light, and does not allow the light coming from the focal point to reflect. No image is formed in this case. Therefore, where you are covering of a spherical mirror is important.*
>
> ….
>
> Participant M3: *Similarly, if I cover the lens from its contour it does not make any difference, but covering one side does. For instance, if I cover right side of a lens, it is no more a converging lens* [participant sketched the drawing shown in figure 2(c) on the paper]. *One side behaves as if converging lens, the*

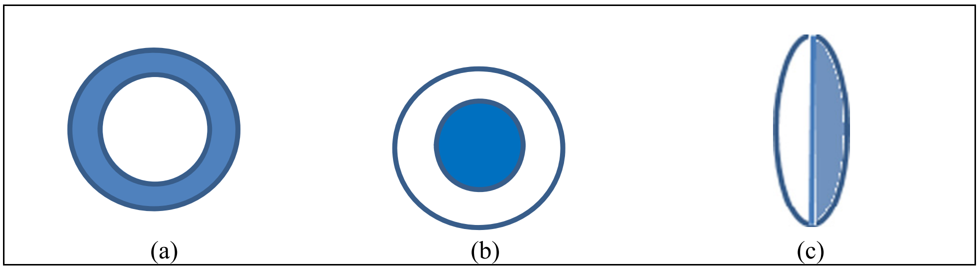


Figure 2. Redrawings of participant M3 for explaining how he covers (a) outer parts of a spherical mirror, (b) inner parts of a spherical mirror and (c) one side of a converging lens where shaded areas show covered parts.

> *other side behaves as if a plane mirror. Therefore, the refraction of the light lessens and the image if formed at a farther point from the lens.*

While explaining their reasoning, participants used several of their daily life experiences with spoons, security mirrors, mirrors at the hair dresser, mirrors in the car or bus, mirrors at the shopping malls, eye-glasses, magnifying glasses, etc. Some of the interview items were already in those contexts (at a hair dresser, in a bus, in a shop, etc). Even though a spoon was mentioned by many of the participants for explaining their conceptions about the convex and concave mirrors, this context was intentionally not used in items. The reason was the focal length of a tablespoon is less than 2 cm and for an individual to look at a spoon at a distance less than the focal length is usually uncommon or even impossible.

To sum up, those contexts and topics that are found to be resourceful for eliciting misconceptions or difficulties in geometrical optics are listed below.

- Formation of a virtual image is often thought to be independent of the observer's presence, such that a virtual image is formed whether an observer sees it or not. Hence, image formation and observation are thought as separate events and that a moving observer or moving object only makes a difference in the image observation process, while, image formation is not affected by them. Therefore, the items in which either the object or the observer is moving are resourceful contexts for inclusion in the study.
- Image size in a plane mirror is thought to depend on mirror size, distance between the object and the mirror, and distance of the observer to the mirror. Hence, in appropriate items, these situations were investigated.
- For the hinged plane mirrors, most of the participants thought that only the angle between the mirrors affects the number of images formed or observed in the plane mirrors, not the location of the object or the observer. So, interview items were designed to investigate all these variables.
- Covering some parts of an optical instrument may or may not make a difference in image formation and in the observation of an object depending on which part of the instrument is covered is also found to be a resourceful context.
- Some participants often thought that a real image can only be seen on a screen and without a screen real images cannot be observed. Other participants thought, however, that without a screen the observer should place his/her eye at the image location to see it. Moving the screen to a different location makes the image size bigger or smaller was another common belief.

- The inability to discriminate between observing oneself and observing others with an optical instrument was also pervasive among participants. Some of the participants claimed that it is possible to see oneself from a lens depending on one's position. Others claimed that it is only possible to see the virtual images of oneself if it is formed on the same side of the optical instrument as the observer.

In order to explore how PSPTs applied their knowledge in the topic within complex optical situations, some questions were modified to the forms which required careful consideration of position of the observer, object and image in moving observer and/or moving object contexts as a result of the first interview session. In the following sub-sections, the analysis of the interview results for each of the contexts (plane mirrors, spherical mirrors, and lenses) is going to be presented, considering the common misconceptions detected on the topic.

3.2 Misconceptions related to plane mirrors

Analysis of the interviews and the open-ended test in the context of plane mirrors revealed that participants had certain misconceptions. These misconceptions are presented in the following sections denoted by the roman numbers. Some of those have already been discussed in the previous studies on this topic (given with the references in parenthesis). The section sentence with references in parenthesis means that this misconception has already been discussed in the literature. The section with no references indicates that this misconception is new to the literature.

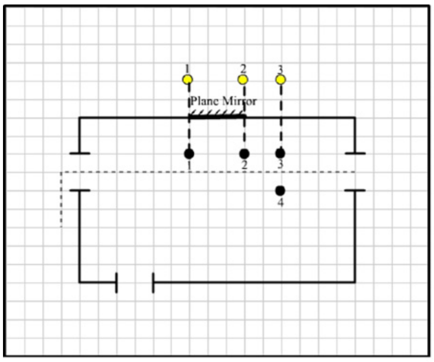
i. Image formation has no relation to the observer's presence or activity and image formation is a separate event from image observation [24, 25, 33, 42]:

Items 1, 2, 3, and 5 in the interview guide include a moving observer and stationary objects in a single plane mirror context (see figure 1 for item 1). Item 4, however, includes moving objects and a stationary observer. When the participants were asked about whether the observer's seeing the objects affects the image formation in a plane mirror or not, most of the participants claimed that image formation has no relation to the observer's presence. They thought image formation and observation as separate events. Participants usually claimed that there is an order: image is formed first and depending on the position of the observer this formed image may or may not be seen by the observer. As one of the participants remarked:

> [Referring to the figures in items 1, 2, and 3] *'The images of the cats are formed in the plane mirror, but depending on the position of Hasan (the moving observer), he may not see those formed images.'* (Participant H2)

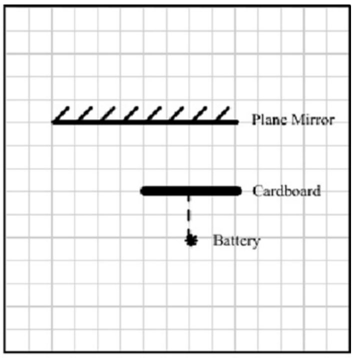
Participants with those ideas usually lacked the scientific idea that in the case of a virtual image, the image is formed only within the observer's eye and the observer constitutes an integral part of the optical system.

In the context of item 6 (see figure 3(b)), however, the observer is intentionally not included. Here participants were asked about whether the image of the battery is or is not formed in the plane mirror in the presence of an opaque obstacle between the object and the mirror. Some participants thought that image of the battery would be formed from the uncovered part of the mirror no matter if an observer sees it or not.



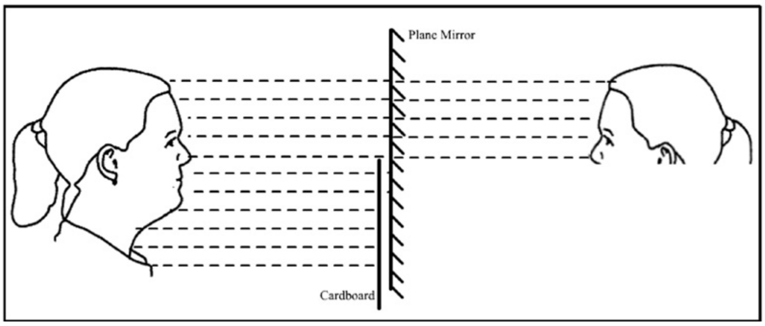
(a) Item 1

asks which of the cats' images would be formed in plane mirror



(b) Item 6

asks whether or not the battery's image would be formed in the plane mirror



(c) Item 13

asks which parts of her own face would the girl see from the plane mirror

Figure 3. Common redrawings of participants who claimed that any obstacle between an object and a plane mirror hinders the image formation and/or observation process.

For items 27, 28, and 29 (see figure 5 for item 28) in the hinged plane mirror context, similar claims were presented by the participants. In this context, they thought that multiple images of an object are formed in the plane mirrors by multiple reflections, but the observer could see some or all of the images depending on his/her location. In other words, they thought that images are first formed and then observed, instead of simultaneously formed and observed.

Among all 16 interview participants of the present study, only one of them correctly explained the simultaneity of virtual image formation and observation, as well as the observer's eye acting as an integral part of the process. Participants' ideas about the observer's passive role in virtual image formation but the active role in virtual image observation process were found to be independent of item context. In all related items in plane mirrors, the participants advocated their faulty conception. In explaining their answers to the questions on the OEGOT, 25% ($n = 13$) of the subjects used this misconception as a reasoning for their answers.

ii. Any obstacle between an object and a plane mirror hinders the image formation and/or observation process [24, 42].

For items 1, 2, 3, 6, and 13 (see figure 3(b) for item 6 and figure 3(c) for item 13) with obstacles or some other objects between the object and the plane mirror, participants claimed that the image of the object is not formed or observed in the plane mirror. As an illustration an excerpt of a participant's response is provided below.

> [Referring to the figures in items 1, 2, and 3] *'The image of cat* 4 *is not formed in the plane mirror since cat* 3 *acts as an obstacle between cat* 4 *and the plane mirror. For this reason only the images of cat* 1, 2, *and* 3 *are formed.'* (Participant G2)

Similarly in item 6, the participants who hold this faulty idea claimed that no image of the battery would be formed in the mirror because of the cardboard. There are two types of reasoning related to it: the parallel rays from the battery would not reach to the mirror due to the cardboard or the image of the cardboard hinders the image of the battery. In item 13, the participants proposed that the observer looking at her face in a plane mirror would only see the portion of her face which corresponds to the uncovered part of the plane mirror. Their reasoning with the presence of an obstacle for those items usually came along with some common type of ray diagram.

Participants who used this reasoning usually drew perpendicular rays from each object-point to the mirror surface. If the drawn rays would meet the mirror surface, the image of that object-point was thought to be formed or seen. Figure 3 illustrates the common ray diagrams used by the participants related to the misconception. Also, 31% ($n = 16$) of the subjects used this misconception as a reasoning for their answers to the questions on the OEGOT.

iii. An image of an object lying outside the borders of a mirror is not formed and/or observed.

In explaining their reasoning with image formation and observation in plane mirrors, some of the participants claimed that the object should be in line with the mirror for the image of it to be formed or seen. Participants arguing this idea stated that:

> [Referring to the figures in items 1, 2, and 3] *'Only the images of cats* 1 *and* 2 *are formed/seen in the plane mirror, because only those cats are within the borders of the mirror.'* (Participant H1)
> [Referring to the figure in item 4] *'The images of cat* 1 *and* 2 *are formed when the cats are in line with the mirror borders. Therefore, the observer can see both of the cats at the same time.'* (Participant H2)

In item 5, the participants were asked to assign the place where the climbing girl would see her whole body along her way for the first time in the plane mirror. Some participants falling in the present misconception category claimed that the whole body of the girl should be in line with the borders of the mirror for her to see her whole body. Similar reasoning was held for items 8, 9, 10, 11, and 12. The perpendicular rays were used in order to explain their reasoning like in the previous misconception category. Those participants usually thought that the image size equals to the mirror size when the plane mirror is smaller in size than the object. The present and the previous misconception category have something in common. One of the participant excerpts given below supports this:

> [Referring to the figure in item 13] *'Covering some part of a mirror is like decreasing the mirror size, so I could not see bottom part of my face from this small mirror.'* (Participant M5)

On the OEGOT, 17% ($n = 9$) of the subjects claimed that the images of objects would be formed or seen only when the object is lying within the borders of the mirror.

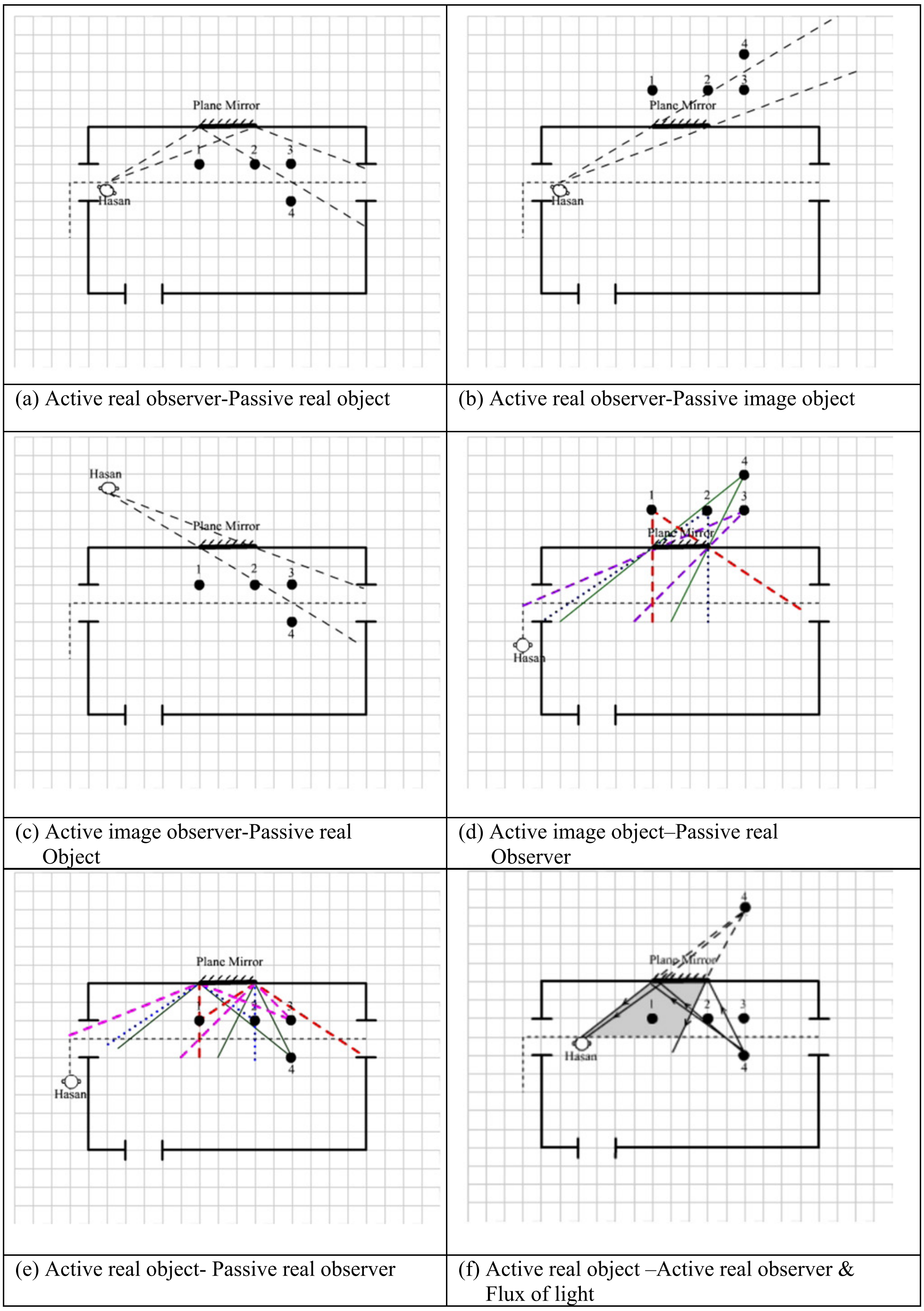
iv. Moving closer to/farther from a plane mirror affects the extent of the image seen in the plane mirror [27].

In items 9, 10, 12, and 13, the participants were asked about the ways to see more of their body in the plane mirror. Many of the participants said that they would see more of themselves by moving back. Those participants believed that when they move farther away from the plane mirror, the image size becomes smaller and more of their body parts fit into the mirror. Some other PSPTs, however, thought that they would see more of themselves by moving toward the mirror. According to them, by moving closer to the plane mirror, their field of view would increase and they would see more of their body. These participants did not realize that the size or the orientation of the mirror on the wall, not distance from it, determines how much of themselves they can see.

In item 11, however, the participants were asked about the ways in which they see more of another person's body who is standing next to them. In this case, moving toward the mirror would enable the person to see more of his twin's body. However, moving away from the mirror, as some of the participants claimed, would not enable him to see more of his twin's body. 33% ($n = 17$) of the subjects used the similar reasoning for their answers on the OEGOT.

v. The closeness of an object to the observer determines the seeing order in plane mirrors.

Most of the participants' answers were correct for items 1, 2, 3, 4, and 5 in terms of stating the order of seeing objects in a plane mirror, but the ray diagrams they used to explain their reasoning differed a lot. To illustrate the difference, figure 4 can be examined. In the figure, different ray diagrams drawn by the participants were shown. These diagrams were drawn by the participants to explain their reasoning for the order of seeing the objects in item 1 as the observer (Hasan) moves along the dashed line, but similar drawings were also used in the other similar items as well. The participants, who used the drawings that are shown, managed to state the order correctly (cats 3, 2, 4, 1). However, the drawings gave some clues that the participants may or may not have understood the seeing process scientifically. In drawing (a), the observer has an active role, whereas the cats have passive roles. Two rays are drawn from the observer's eye to the edges of the mirror and reflect. The cats present in the region between these two reflected rays would be seen by the observer. Although this drawing helped the participants to order the cats correctly, it does not include the simultaneous image formation and observation process. The rays directed from the observer's eye to the mirror edges may be due to the participants' mistaken belief that light originates from the eye. In drawing (b), the observer's eye has also an active role. However, in that drawing the reflection of the light rays is neglected. The rays drawn from the observer's eye to the mirror edges are extended to the back of the mirror and the images of cats located between these two extended rays were thought to be seen. Similarly, in the third drawing (c), the reflection of light rays is not represented in the drawing. In that case, the light rays originate from the image of the observer to the edges of the mirror. The rays are extended to the front of the mirror and the cats located in the region between these two rays were thought to be visible. In the drawings (d) and (e), the objects (i.e., cats) have an active role while the observer has a passive one. In drawing (d), the images of the objects are already formed in the absence of the observer. In drawing (e), however, the observer's seeing and the formation of the image were not mentioned simultaneously. The last drawing (f), in which the image formation and observation for one of the objects (cat 4) is depicted correctly, was drawn by only a limited number of



(a) Active real observer-Passive real object
(b) Active real observer-Passive image object
(c) Active image observer-Passive real Object
(d) Active image object–Passive real Observer
(e) Active real object- Passive real observer
(f) Active real object –Active real observer & Flux of light

Figure 4. Examples of participants' redrawings for item 1.

participants. So the participants' correct answer to the question does not guarantee that they conceptualize the image formation and /or observation process scientifically.

Some of the participants, however, presumed that an object that is close to the observer can be seen earlier compared to the other objects. With this reasoning they claimed the seeing order to be cats 1, 2, 3, and 4 for item 1. In item 3, the observer moves along an axis which is perpendicular to the mirror surface and beyond the mirror edges. For this reason, that item produces more varied answers and reasoning. A total of 23% ($n = 12$) of the subjects used same reasoning for their answers on the OEGOT.

vi. Only the angle between the mirrors determines the number of images formed/seen in hinged plane mirrors [50, 51].

Many participants believed that the number of images formed/seen in hinged plane mirrors depends only on the angle between the mirrors. They thought the number being independent of the position of the object or the position of the observer. In fact, that is only true for the angles which are submultiples of 180°. Iona [48] and Kulkarni [49] discussed the physics in their articles. The participants of the present study were interviewed with three items (items 27, 28, and 29) in order to investigate their answers and related reasonings. Most of the participants held that misconception. They did not consider the simultaneity of the formation and observation of virtual images by multiple reflections. On the OEGOT, 60% ($n = 31$) of the PSPTs used that misconception to explain their reasoning. The prevalence of that misconception among the sample is obvious.

In item 28, the participants were provided with hinged plane mirrors with a 105° angle. In addition to the above-mentioned difficulty related to the position of object and observer, some other interesting answers were given. Many participants claimed that, for angles other than 90°, 60°, 45°, etc (which do not result in a whole number from the equation $(360°/\alpha) - 1$) in addition to the full images, some other partial images would be formed/seen. Some of the other participants, however, claimed that it is not possible to see partial images, so the number obtained from the equation should be rounded to the whole number. Two student statements related to this mistaken idea are provided below.

> '*Hmm… From the equation, I obtained* 2.46 *which is not a whole number. I think it is not possible to see a partial image in the mirrors. Therefore the boy would probably see only two images of the ball in the mirrors.*' (Participant G3)
>
> '*I used the equation* $(360°/\alpha) - 1$ *and it results about* 2.46. *So, the boy would see a half image of the ball in addition to two complete images of it. No matter where the object or the observer is located as long as between the two mirrors, the number would not change.*' (Participant H1)

One of the participants said that for angles other than 90°, 60°, 45°, etc, no images would be formed at all. Participants who used the ray tracing method instead of the equation, however, came out with a different answer. Figure 5 shows a drawing of a participant who claimed that four images of the ball would be formed and seen in the plane mirrors.

In fact, images of an object placed between hinged plane mirrors formed by multiple reflections by two plane mirrors as long as the light rays reflecting from the mirrors reach to an observer's eye. In other words, the multiple images would be formed simultaneously as they are seen. Therefore, in item 28, depending on the angle between the plane mirrors, the position of the observer and the position of the object (ball) only two images of the ball would be formed and seen simultaneously.

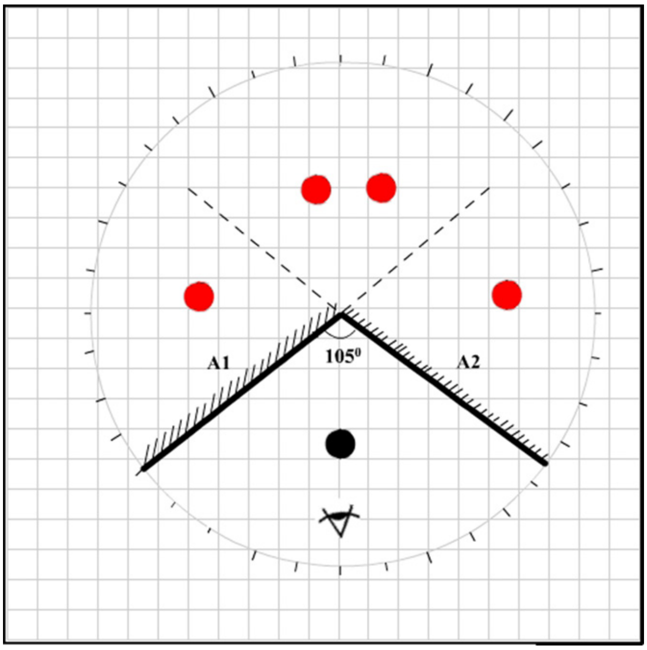
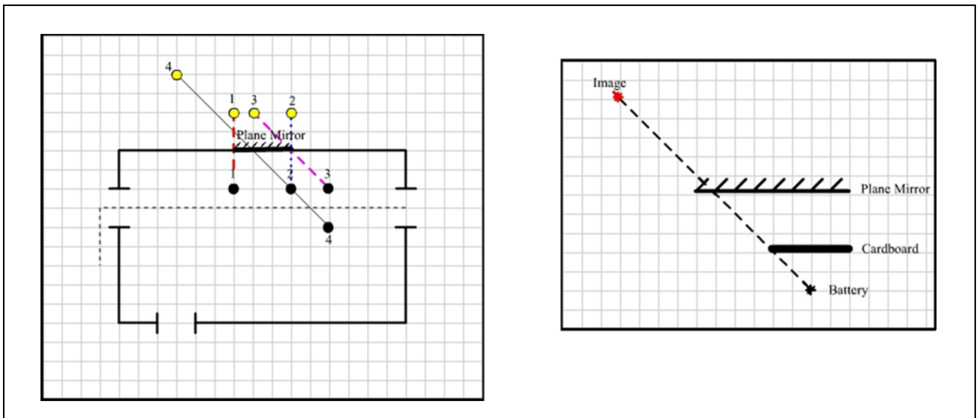


Figure 5. A participant's redrawing for item 28. The item asks for the number of images of the ball (shown in black) formed in the hinged plane mirror.

During the interviews, some other misconceptions were also detected. However, those misconceptions were not considered to be very common among the participants. For instance, few participants located the image of an object on the plane mirror surface (especially in the back silvered part) or along the incoming light ray. A participant who thought the image is located along the incoming light ray used the drawings shown in figure 6 to answer items 1 and 6. In items 27, 28, and 29, one of the participants stated that only the number of mirrors determines the number of images formed in hinged plane mirrors.

Another interesting, but not common, misconception is that any small part of a mirror is enough to form/see the image of an object. This is true only when light from the object could reach to the mirror. This misconception was expressed by one of the participants in the context of item 6. The following excerpt from the interview is illustrative.

> Interviewer: [Referring to the figure in item 6] *Do you think the image of that battery would be formed in the plane mirror?*
> Participant M5: *Yes, it would. Suppose the part of the mirror behind the cardboard does not exist. The image of the battery would still be formed.*
> Interviewer: *Let's think the cardboard was one unit square longer. Would it affect the image formation?*
> Participant M5: *No I don't think so. Still image would be formed. Only a small part of a mirror is enough for the image to be formed.*
> Interviewer: *What about when the length of cardboard is equal to the length of the mirror?*



(a) Item 1 (b) Item 6

Figure 6. A participant's redrawings for locating an image along incoming light ray.

Participant M5: *Hmm… No change, the image would be formed… There is some distance between the cardboard and the mirror, isn't there?*
Interviewer: *Yes.*
Participant M5: *Yes, the image would be formed…But…Hmm…It seems stupid what I said, but…I changed my mind. The image is not formed.*
Interviewer: *Why? What is your reasoning for it?*
Participant M5: *The image of the cardboard would block the image of the battery, and nobody would see the image of the battery. If nobody can see the image of the battery, we cannot say the image of the battery exists.*

Light rays moves in straight lines. If the cardboard were one unit longer as the interviewer suggested during the interview, it would not be possible for the rays to reach the mirror and reflect to any of the observers. Hence, no image would be formed in that situation. However, as the participant stated, some may believe that only a small part of a mirror is enough to form an image.

In the interview guide, there were many items that were intended to elicit the participants' ideas about similar concepts. However, including all of them in the open-ended and multiple-tier tests is not possible in terms of feasibility (considering time constraints, length of the test, etc). Therefore, a critical analysis was done by the researchers to determine which items would be included in the paper-and-pencil tests and which ones would not be. Some of the less resourceful items to elicit misconceptions were not included in the open-ended test (items 7, 8, and 14). Some other items (items 2, 9, 12, and 13), however, were found to be answered in the same way as some other items in the instrument. Therefore, those items were not included in the open-ended test in order to shorten the length of the test.

3.3 Misconceptions related to spherical mirrors and lenses

Analysis of the interviews revealed that the participants have certain misconceptions about spherical mirrors and lenses, and those misconceptions share some similarities. For this reason, the misconceptions identified for these two contexts are given together. Some of the common misconceptions are presented below.

i. (All) Special rays are necessary to form/see an image [27].

A great many of the interview participants and 25% ($n = 11$) of the OEGOT subjects seemed to believe that the special rays are necessary to form an image in spherical mirrors or lenses. In fact, the special rays (at least two of them) are sufficient but not necessary in order to locate the position of the image. Since the special rays are convenient because of their simple rules for tracing the path of light rays; they are usually used in geometrical optics. It is apparent that the instruction itself may be a possible source of the misconception mentioned in this category. In items 15 to 20, the participants were asked about image formation and observation in spherical mirrors. Participants who believed that the special rays are necessary answered those items by stating that if the special rays from the object to the mirror surface would reach that surface, the image of that object would be formed or seen in the mirror. If not, no image would be formed. Participants who used this reasoning mostly used two of the special rays: the ray parallel to the principal axis and the ray from the focal point. The following excerpt shows a typical answer given by the participants for item 15 (see figure 1).

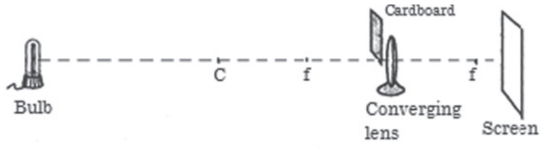
> *'The parallel rays drawn from cats* 3 *and* 4 *would not reach to the surface of the mirror. The other ray [referring to the special ray passing from the focal point and reflecting parallel to the principal axis] is not enough to form the image. There should be at least two rays intersecting at a point. Therefore only the images of cat* 1 *and cat* 2 *would be formed and seen.'* (Participant G5)

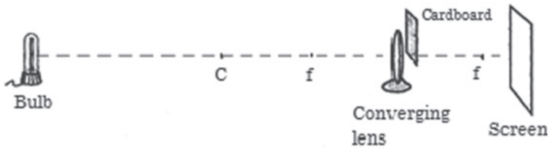
In items 21, 22, 23, 30, and 31E (see figure 7 for item 30), in which some part of the spherical mirror or lens is covered with opaque cardboard, the same misconception with special rays was activated. Some of the participants thought that if one or more of the special rays are blocked because of the cardboard, the image of the object would not be formed or seen at all or only partially. In item 30, the participants were asked a set of questions about image formation in a lens when different parts of the lens are covered with cardboard. Participants who thought that special rays are necessary could not understand that any portion of the lens would be sufficient to form a complete image as long as light from object can reach the lens, and covering some part of the lens only results in a dimmer image. Yet, their answers changed depending on which portion of the lens is blocked or the size of the object. Figure 7 illustrates a typical answer given by participant M3 to the related questions. Answers of the other participants were very similar to that one.

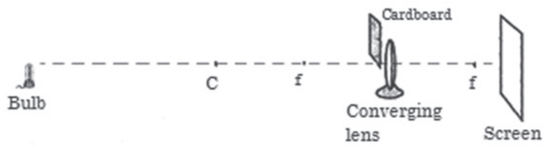
ii. No image is formed/seen of an object at the focal point in a convex mirror/diverging lens.

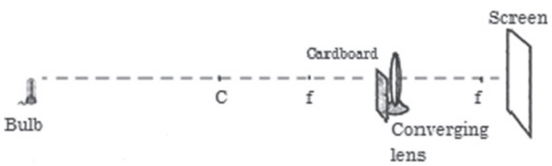
Many of the participants failed to answer the questions related to convex mirror or diverging lens because of their faulty transfers from the concave mirror and converging lens. As a striking example, participants' thought about an object located at the focal point can be given. Some of the participants claimed that no image of an object would be formed or seen which is located at the focal point. In fact, when the object is located at the focal point of a convex mirror or a diverging lens, a virtual image of the object would be formed between the focal point and the mirror or the lens. Some other participants claimed that the image of the objects in front of a convex mirror or diverging lens would be formed in the same regions as their counterparts in concave mirror or converging lens, but on the other side of the mirror or the lens. One of the student explanations below exemplifies this idea.

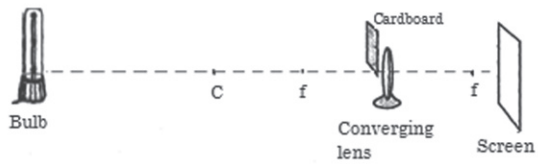
> [Referring to the figure in item 25] *'In a convex mirror, the image is always formed on the other side of the mirror. When Selin is standing beyond the radius of curvature; her image would be formed between the focal point and the center of the convex mirror on the other side. When she is at the center;*

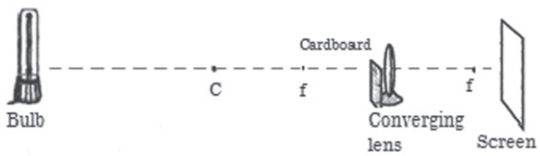
<table>
<tr><th>Item 30</th><th>Typical Answer</th></tr>
<tr><td>

(a)</td><td>• Only image of the lower part of the bulb would be formed on the screen.</td></tr>
<tr><td>

(b)</td><td>• Only image of the lower part of the bulb would be formed on the screen.</td></tr>
<tr><td>

(c)</td><td>• Image of the whole bulb would be formed on the screen.</td></tr>
<tr><td>

(d)</td><td>• No image of the bulb would be formed on the screen.</td></tr>
<tr><td>

(e)</td><td>• Only image of the bottom part of the bulb would be formed.</td></tr>
<tr><td>

(f)</td><td>• Only image of the middle part of the bulb would be formed.</td></tr>
</table>

Figure 7. Common participant answers to item 30. The item asks what would be seen on the screen in each situation.

> *her image would be at the center on the other side of the mirror. When she is between the focal point and the center; then her image would be beyond the radius of curvature on the other side. At the focal point, her image would be at the infinity and she would not see her image.'* (Participant H1)

That participant had fragmented knowledge about the convex mirrors such that she knows that an object's image in a convex mirror would be formed on the other side of the mirror. However, that participant transferred her knowledge about image positions in a concave mirror to the convex mirror context in a faulty way. In fact, in a convex mirror regardless of where the object is located, the image would be formed on the other side of the mirror and between the focal point and the mirror itself. Also, 17% ($n = 9$) of the subjects used this misconception as reasoning for their answers on the OEGOT.

iii. Real images can only be formed/seen on a screen [28].

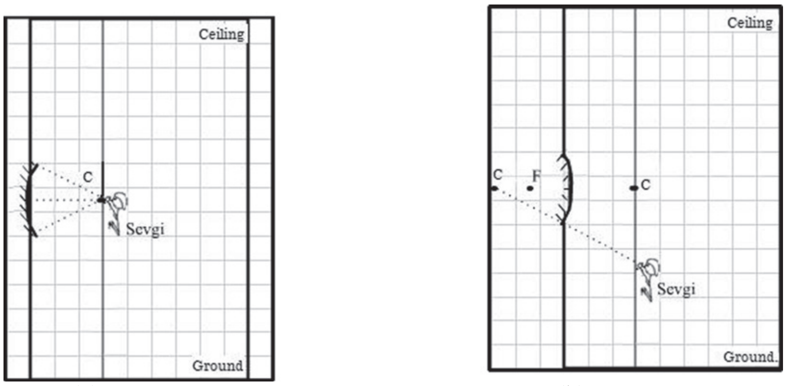
In the items related to concave mirrors and converging lens, some of the participants stated that the real images would only be formed or seen on a screen. Unfortunately, those participants could not conceive of a real image as existing in space, independent of a surface as an aerial image. In fact, the function of a screen is to reflect the light rays to the observer's eye so that the image may be seen from different places. In the absence of the screen, however, the observer's eye should be placed in a proper position such that the light rays diverging from the real image point can enter the eye. In the present study, however, the participants may or may not activate that misconception depending on the context of the question. Their experience with similar situations mentioned in the items may affect their answers and reasoning. Those students, who recognized that the screen was not necessary, may have a difficulty to determine where the observer should be placed to see the image. Most of them suggested placing the observer's eye to the point where the real image would be formed. Although a worth to mention number of participants used this misconception in their reasoning during the interviews, only 4% ($n = 2$) of the subjects used it in the OEGOT.

iv. An observer can see himself/herself outside of the region enclosed by the normal lines to the mirror borders.

In the concave and convex mirror contexts, some of the participants thought that it is possible to see one's eye even if it is located at outside of the region enclosed by the normal lines to the mirror borders. In fact, for an observer to see his/her own eye from a mirror, the light rays from the eye must reflect back to the eye. So, it is only possible when the eye is along the normal line to the mirror surface. Because only when the eye is along the normal line, the light rays can reflect back to the eye (assuming the size of the eyes and the distance between the eyes are small). The normal line at any point on a spherical mirror can be obtained by connecting that point with the center of the mirror with a straight line. Items 19 and 20 (see figure 8) asked to the participants for the position on the rope where the girl first sees her eye on the concave and convex mirrors, respectively. Figure 8 illustrates those positions for the concave and convex mirrors for the girl to see her eye.

Some of the participants, however, claimed that the girl would see her eye out of the region enclosed by the normal lines to the mirror borders. One of the participant's statement was:

> *'The rope that she climbs passes through the center of the mirror. So anywhere along the rope she would see her eye from the mirror.'* (Participant G5)



(a) Item 19
asks where the girl first sees her eye from the concave mirror as she climbs the rope.

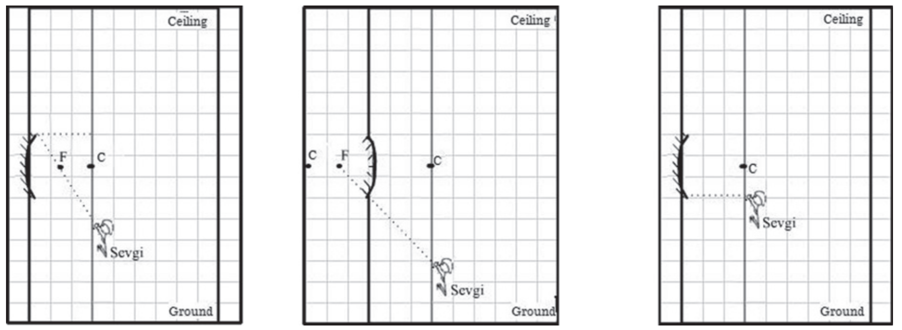
(b) Item 20
asks where the girl first sees her eye from the convex mirror as she climbs the rope.

Figure 8. Positions where the girl first sees her eye from the (a) concave and (b) convex mirrors.

Some others who thought that special rays are necessary to form or see an image also claimed that she would see her eye from some other positions. They thought that when her eyes along the first possible line drawn from the eye to the concave mirror that is passing from the focal point (figures 9(a) and (b)) or when her eye is along the bottom border of the mirror (figure 9(c)), she would see her eye. According to the OEGOT results, 8% (n = 4) of the subjects stated this misconception in their answers.

v. A real image can be seen with a different size/orientation if a screen is placed at a different location than the image point [27, 52].

Some participants believed that when the screen is moved back along the principal axis from the real image position, the size or orientation of the image would change. Those participants did not have an understanding that, unless the screen is placed at a particular image position, no image would be seen on the screen. Because for other positions of the screen the image becomes blurred and disappears altogether. Most of the participants, who thought the image size would change, claimed that the image gets bigger. According to their reasoning, when the screen moved back the light rays diverging from the image point produce a bigger image on the screen. Some others claimed that the orientation of the image would also change (i.e. become erect), since the rays diverging from the image point would invert. Participant M3 claimed that 'the image on the screen would be bigger, erect and virtual, because beyond the image point the light rays diverges and do not intersect at a point anymore'. The reasoning of the participants were very similar to the reasoning mentioned in the study of Goldberg and McDermott [27]. A small number of the participants who claimed that the image gets smaller, however, used a proportional reasoning which is different from the justifications mentioned in the previous studies. According to the participants in the present study, the converging lens, which produces an image in smaller size, would produce a



(a) Item 19 (b) Item 20 (c) Item 19

Figure 9. Participants' redrawings showing an observer can see herself out of the region enclosed by the normal lines to the mirror borders.

much smaller image when the screen is moved back. That is, the farther away the screen, the smaller the image on the screen. The failure of the participants to recognize that there is a single position for an image to be seen on a screen would result this misconception. A total of 25% ($n = 13$) of the subjects hold this misconceptions according to the OEGOT results.

There are also some other misconceptions detected from the interview with the participants. Those are listed and briefly explained below.

vi. Only the images formed on the same side as an observer can be seen.

Those participants who have this misconception believed that an observer can only see the images formed on the same side with the observer (i.e. for the spherical mirrors only real images, and for the lenses only virtual images of an object would be seen). This reasoning was mostly used by some of the participants for items 24, 25, 32, 33, 34, and 35. 21% ($n = 11$) of the subjects used this reasoning in their answers to the OEGOT questions.

vii. Unlike real images, virtual images cannot be seen in spherical mirrors/lenses.

As discussed before, some of the participants have difficulty in conceptualizing the real and virtual images. Those who thought virtual images cannot be seen, but only real images can, are included in this category. Depending on the context of the item, those participants were found to activate their faulty conception. 6% ($n = 3$) of the subjects used this reasoning in their answers to the OEGOT questions.

viii. An observer can directly see himself/herself from a lens.

In a lens the light rays from an object refract and pass to the other side of the lens. Therefore, there is no way for someone directly to see themself from a lens (ignoring the reflections), because the light rays cannot reach back to the observer's eye. In items 32 and 33, the participants were asked about the properties of their own image that they would see from the lens. Some of the participants of the present study claimed that someone would see themself in different positions depending on the distance from the lens. 27% ($n = 14$) of the subjects used this reasoning in their answers to the OEGOT questions.

4. Discussion of the results

Analysis of the interviews demonstrated that participants had serious difficulties even in the very basic concepts of geometrical optics. They used some of the key terms (such as 'image', 'reflection', 'refraction', 'field of vision' etc) to explain their reasoning arbitrarily or mistakenly. They might either have had difficulty in using the correct terminology for their explanations or had serious conceptual problems in geometrical optics. Participants' experience with optical elements, discussed in the context of the item, was found to influence their answers and reasoning either in a constructive or destructive way. From the interviews, it appeared that most of their experiences in geometrical optics were from either their daily usage of the optical elements or from their laboratory experiences at their universities. The experience of the participants with spherical mirrors or lenses was less than their experience with plane mirrors both in school and out of school. Experience as a possible source of student conceptions has been discussed in several studies [5, 27, 53–55]. In the study of Goldberg and McDermott [27], for example, the influence of experience with a plane mirror on student reasoning was discussed in detail. The authors suggested that the participants' strong conviction that by moving back they could see more of themselves in a plane mirror might have been due to invalid inferences drawn from their experience. People usually stand farther back to see their entire body with a minimum amount of eye movement. This common experience might have been a possible reason for the faulty reasoning that one can see more of oneself by standing farther away from a mirror.

Another important result that was deduced from analysis of both the interview and the open-ended test was that the PSPTs' knowledge in geometrical optics, especially about spherical mirrors and lenses, was fragmented in nature. An important outcome of studying geometrical optics is expected to be in the ability to relate general physical principles to everyday experience. However, the results of this study have shown that, even after formal instruction in schools from the elementary school to university, the subjects were unable to connect their formal knowledge to the specific contexts in geometrical optics which were presented through the items. Even though, they seemed to answer easily the standard textbook questions in geometrical optics by rote or by comprehension at a surface level, they failed to relate their knowledge to simple but different contexts that require a deep level of comprehension. The comparatively high level of deficient or not meaningful proportions of answers to the open-ended test items might be an indicator of their loosely organized, ill-defined and fragmented knowledge in this topic.

Both the oral and written discussions during the interviews and the written responses on the open-ended test items indicated that PSPTs had a loosely organized understanding of concepts in geometrical optics. Their misconceptions, if any, were mostly not coherent but context dependent. Some of the reasoning activated in one context to explain phenomena was found not to be used in another context that requires the same or similar knowledge. For example, the reasoning that 'any obstacle between an object and a mirror hinders the image formation and/or observation process' was activated by item 1, in which the observer was moving parallel to the plane mirror surface, whereas most of the same participants did not use the same reasoning for item 16 in which the observer was moving parallel to a convex mirror surface. These findings show similarity with the findings of previous studies [17, 24, 30, 56] that expressed the context dependence of misconceptions in geometrical optics.

In dealing with plane mirrors (including a single plane mirror and hinged plane mirrors), more than half of the participants based at least one of their explanations on the belief that the image formation and observation are separate events. As discussed by Galili [24], Galili, Goldberg and Bendall [25], and Ronen and Eylon [33] for their post-instructional students, the participants of the present study also seemed to think that the image is formed in a plane mirror by rays from the object to the mirror and that after the image is formed, light rays transfer the image to the observer's eye. Whether or not the mirror image is observed, it always stays in the mirror. There is usually a lack of scientific understanding that image formation and observation are simultaneous events in plane mirrors, and that the image is formed only on the retina in the observer's eye. The PSPTs in the present study had already completed their courses in geometrical optics and geometrical optics laboratory. In these courses, they are taught about virtual image formation in plane mirrors. However, many of the current optics textbooks and traditional instructions do not include the function of the eye in the formation of a virtual image. Therefore, faulty inferences by the PSPTs seem nearly unavoidable.

Participants who hold the misconceptions 'any obstacle between an object and a plane mirror hinders the image formation/observation of an object' and 'only the images of the objects within the borders of the plane mirror are formed/seen' were found to use the same diagrams in explaining their answers for the items. They usually drew a single light ray from each object point preferentially in the direction of the optical element, in our case in the direction of the plane mirror (see figure 4(c)). Their inability to conceptualize the idea that all object points emit light equally in all directions isotropically, led them to faulty conceptions. Similar findings were discussed by Rice and Feher [32] in the context of the formation of images through pinholes by the use of extended light sources. Galili [24] labeled that kind of representation as the 'flashlight model' and found those representations as a serious barrier to the genuine understanding of optical phenomena.

For the items that asked for ways to increase the image size which is observable from a plane mirror, students' experience with plane mirrors was found to influence their reasoning. Instead of using any ray drawings to justify their answers, most of the participants answered the items intuitively depending on their everyday life experiences. Those findings were very similar to the findings of Goldberg and McDermott [27]. Bendall *et al* [18] and Galili [24] proposed that extensive experience with a plane mirror in everyday life naturally produces rich and rather well structured naïve knowledge to account for images that are observed in mirrors.

Misconceptions about hinged plane mirrors are a less often studied context in geometrical optics and are given little emphasis in classroom practices. Classroom discussion about hinged plane mirrors is mostly about the formula: $N = (360°/\alpha) - 1$, which gives the number of images obtained by multiple reflections in two hinged plane mirrors. Participants, however, could not conceptualize that number of the images are not independent of object position and observer position. Also, the same participants had the misconception that the full number of images for angles other than 90°, 60°, 45°, etc, possibly are not obtained because the formula would give a fraction. It is nearly impossible to find in a textbook the derivation of the aforementioned formula or the inequalities which enable one to calculate the number of images for any value of α, for the specific observer positions [50] and object positions [51].

Participants' reasoning for their answers on questions involving in the spherical mirrors and lenses were found to be similar. Mostly their reasoning for concave mirrors shared something in common with their reasoning for converging lenses, as did their reasoning for convex mirrors and diverging lenses. Therefore, the findings for those contexts were presented together. The most prominent difficulty in those contexts was found to be related to the use of light rays. Representations are powerful tools, especially in geometrical optics. However, the ray model of light was found to be a not fully developed concept for most of the participants. Similar to the discussions in the previous studies [18, 28, 31, 42], participants in the present study thought of light rays as physical entities that form an image rather than a geometrical representation of how light behaves under certain situations. Especially in the usage of special rays in the contexts of spherical mirrors and lenses, they had difficulty in understanding the idea that special rays are sufficient but not necessary for image formation.

Participants' knowledge of spherical mirrors and lenses was mostly fragmented. The situation for convex mirrors and the diverging lens was even worse. From their formal instruction, the participants had acquired certain pieces of knowledge related to the topic but those fragments do not comprise full conceptual understanding of the topic. In some cases, the participants were found to transfer their knowledge in one context incorrectly to another context. For instance, although most of the participants expressed that in a convex mirror the image would be formed in the back of the mirror regardless of where the object is located, they thought that an image of an object located at the focal point would not be formed in a convex mirror. Those kinds of transfers, which led to incorrect answers to the items, were quite common among the participants of the present study but were not mentioned in the literature before.

Another common misconception in the context of concave mirror and converging lens was related to the usage of the screen in the observation of a real image. Participants had difficulty in understanding the scientific conception that a screen is only a convenience for observation of real image points in space (i.e., aerial image) and there is a particular image position for an image of an object to be observed on a screen. Some participants defined real images in relation to the presence of a screen, and therefore thought that real images could not be present in space in the absence of a screen. Some others, however, thought that if a screen moved farther or closer to the optical element, the image of the object would be still seen on the screen with a different size or orientation. Those findings show similarities with those of Goldberg and McDermott [28] and Galili [24]. The representations and ray diagrams are usually responsible for that type of reasoning.

The analysis of the OEGOT results was also interesting. The proportions of deficient and not meaningful reasoning for the items on the test were relatively high for more than half of the items. This is another indication that the PSPTs' knowledge was fragmented. Their correct response rate for the items in familiar contexts from their daily life (such as plane mirrors) and from their instruction is higher than the less familiar contexts (such as spherical mirrors). This small rate of correct responses might be due to the lack of knowledge in those contexts. The OEGOT was beneficial in that more subjects than the participants in the interviews were tested in the topic of interest. Their written responses guided the researchers to construct the alternatives on the multiple-choice test from the common subject responses together with the interview results.

5. Conclusions and implications

The PSPTs, even after they have completed their courses in geometrical optics, have certain misconceptions about geometrical optics. The interview results revealed that the misconceptions in the context of plane mirrors were found to be usually context-independent, while the ones in the contexts of spherical mirrors and lenses are usually fragmented and context-dependent. This situation might be related to the subjects' having more experience with plane mirrors compared to spherical mirrors and lenses.

One of the implications of this study is that there is a need for conceptual teaching of physics topics. Complex problem solving or even advanced courses taken do not necessarily lead to scientific understanding, but there is a need for conceptual model development. Many student conceptions are not scientifically acceptable regardless of whether or not they receive instruction, and that these conceptions are difficult to change through regular instruction. Even though the misconceptions in geometrical optics discussed in this study for the PSPTs are in common with many high school or university students, the situation for pre-service or in-service teachers is more serious and needs more attention. Since teachers are the key to improve student performance [7–9, 18], the results of this study signify the need to deepen the conceptual understanding of pre-service and in-service teachers in the topics they are expected to teach, more specifically for this study in the topic of geometrical optics. In order to make physics meaningful to their students, teachers should have a higher content knowledge in specific physics topics, as well as having a strong pedagogical content knowledge about how to teach a specific topic more effectively. Including certain advanced physics courses in teacher education programs would be desirable but not sufficient. A more crucial attempt would be bringing mostly conceptual content teaching courses to the PSPT education programs. Within those courses teacher candidates not only develop more coherent models on the specific physics concepts, but also be aware of probable student difficulties in those concepts for their future teaching career.

The interview and OEGOT results all showed that the PSPTs mainly have difficulties with the ray model, the function of the observer, and the function of the screen in image formation and observation process. Although the ray model is a powerful tool to explain most optical phenomena, the misuse of the model may cause the surface understanding and wrong interpretations of the phenomena. Therefore, the ray model should be discussed in detail during geometrical optics instruction. Especially for the use of special rays, the emphasis should be given for their simplification, and not necessity. It is also necessary to emphasize the importance of the observer's eye in image formation and observation process, and intentionally use and emphasize the observer's eye in any optical phenomena. The screens, especially in a real image observation process, should be used, but at the same time students should be provided with an experience to see an aerial image in the absence of the screen. The inability to relate the theoretical knowledge to the practical context was one of the main weaknesses of many students as revealed from the interviews. Therefore, they should be provided with more inquiry-oriented laboratory experiences during their geometrical optics instruction.

Textbooks play a vital role in the teaching and learning of physics topics. Teachers as well as their students seem to use textbooks most of the time during the teaching and learning process [57] and textbooks have a very important influence on both teachers and their students. However, sometimes textbooks become a source of student misconceptions with the information they provide. Teachers often have the same alternative conceptions as their students and the poorly written textbooks may be responsible for the persistence of their alternative conceptions. Therefore, readers of the books and especially teachers should be aware of the explanations and representations in the textbooks and discuss them with their students to prevent construction of misconceptions in their minds. Also, textbook writers and textbook analyzers should be watchful for possible sources of student misconceptions. For instance, in many textbooks the role of the observer's eye is ignored or not specifically emphasized in the image formation and observation process. Similarly, special rays are discussed as if they are the only rays that form an image. Hence, teachers and students should be alert about the information presented in textbooks since they do not always contain the scientifically accepted knowledge, but may be incorrect or incomplete, vague or misleading.

Many PSPTs clearly need more preparation in geometrical optics than is usually provided. The implications for in-service teachers are obvious. In-service teachers of physics have completed the same kinds of courses as the PSPTs who participated in this study, and it is unlikely that teaching geometrical optics as they themselves taught this material has greatly increased their own understanding. All teachers of physics should have the opportunity to participate in an inquiry-oriented, laboratory-based course in geometrical optics in which they are guided in constructing a ray model for light from their own observations. By extension, this recommendation applies other topics in physics, as well as to other sciences.

The nature of the diagnostic methods used in the present study (i.e. interviews and an open-ended test) provided the reserachers with an in-depth investigation of misconceptions held by the PSPTs in geometrical optics. However, they require a large amount of time to obtain and analyze data from a larger sample. Therefore, there is a need for a time-efficient easy-to-administer and easy-score tool to assess the misconceptions of a larger sample in a valid and reliable way. Based on the findings of this study a four-tier multiple-choice misconception test [58] could be developed and administered to a larger group of PSPTs to diagnose the prevalence of the misconceptions identified in the present study. However, a discussion of the four-tier test development and administration is left to future studies.

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