# Lepton flavor violating $Z \to l_1^+ l_2^-$ decay in the split fermion scenario in the two Higgs Doublet model

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**Abstract**

We predict the branching ratios of $Z \to e^{\pm}\mu^{\pm}$, $Z \to e^{\pm}\tau^{\pm}$ and $Z \to \mu^{\pm}\tau^{\pm}$ decays in the framework of the 2HDM, in the split fermion scenario. We observe that the branching ratios are not sensitive to a single extra dimension, however, this sensitivity is considerably large for two extra dimensions.

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# 1 Introduction

The lepton flavor violating (LFV) Z decays are clean from the theoretical point of view since they are free from the long distance effects. On the other hand, they are rich in the sense that they exist at least at one loop level and carry a considerable information about the free parameters of the model used. Therefore, it is worthwhile to analyze these decays and there is an extensive work related to them in the literature [1]-[13].

Since the lepton flavor is conserved in the SM, for the flavor violation in the lepton sector, one needs to extend the SM. The so called $\nu$SM model, which is constructed by taking neutrinos massive and permitting the lepton mixing mechanism [14], is one of the candidate. However, the theoretical predictions for the branching ratios (BRs) of the LFV Z decays in this model are extremely small in the case of internal light neutrinos [1, 2]

$$
\begin{aligned}
BR(Z \to e^{\pm}\mu^{\pm}) \sim BR(Z \to e^{\pm}\tau^{\pm}) &\sim 10^{-54}\,, \\
BR(Z \to \mu^{\pm}\tau^{\pm}) &< 4 \times 10^{-60}\,.
\end{aligned}
\tag{1}
$$

and they are far from the experimental limits obtained at LEP 1 [3]:

$$
\begin{aligned}
BR(Z \to e^{\pm}\mu^{\pm}) &< 1.7 \times 10^{-6}\ [4]\,, \\
BR(Z \to e^{\pm}\tau^{\pm}) &< 9.8 \times 10^{-6}\ [4,\ 5]\,, \\
BR(Z \to \mu^{\pm}\tau^{\pm}) &< 1.2 \times 10^{-5}\ [4,\ 6]
\end{aligned}
\tag{2}
$$

and from the improved ones at Giga-Z [7]:

$$
\begin{aligned}
BR(Z \to e^{\pm}\mu^{\pm}) &< 2 \times 10^{-9}\,, \\
BR(Z \to e^{\pm}\tau^{\pm}) &< f \times 6.5 \times 10^{-8}\,, \\
BR(Z \to \mu^{\pm}\tau^{\pm}) &< f \times 2.2 \times 10^{-8}
\end{aligned}
\tag{3}
$$

with $f = 0.2 - 1.0$. Notice that these numbers are obtained for the decays $Z \to \bar{l}_1 l_2 + \bar{l}_2 l_1$, namely

$$
BR(Z \to l_1^{\pm} l_2^{\pm}) = \frac{\Gamma(Z \to \bar{l}_1 l_2 + \bar{l}_2 l_1)}{\Gamma_Z}\,. \tag{4}
$$

Other possible scenarios to enhance the BRs of the corresponding LFV Z decays are the extension of $\nu$SM with one heavy ordinary Dirac neutrino [2], the extension of $\nu$SM with two heavy right-handed singlet Majorana neutrinos [2], the Zee model [8], the model III version of the two Higgs doublet model (2HDM), which is the minimal extension of the SM [9], the supersymmetric models [10, 11], top-color assisted technicolor model [12].

In this work, we study the LFV processes $Z \to e^{\pm}\mu^{\pm}$, $Z \to e^{\pm}\tau^{\pm}$ and $Z \to \mu^{\pm}\tau^{\pm}$ in the framework of the 2HDM in the split fermion scenario [15]. This scenario is based on the idea that the hierarchy of fermion masses are coming from the overlap of the fermion Gaussian profiles in the extra dimensions and reached great interest in the literature [15]-[25]. In [16] the explicit positions of left and right handed components of fermions in the extra dimensions have been predicted. [17] is devoted to the restrictions on the split fermions in the extra dimensions using the leptonic W decays and the lepton violating processes. The CP violation in the quark sector has been analyzed in [18] and to find stringent bounds on the size of the compactification scale 1/R, the physics of kaon, neutron and B/D mesons has been studied in [19]. In [20] the rare processes in the split fermion scenario and in [21] the shapes and overlaps of the fermion wave functions in the split fermion model have been considered. In [22] the electric dipole moments of charged leptons have been predicted, and in [23] the radiative LFV decays have been examined, in the framework of this scenario. Recently, the Higgs localization in the split fermion models has been analyzed in [25].

In our calculations we estimate the sensitivities of BRs of the LFV Z decays to the compactification scale and the Gaussian widths of the charged leptons in the extra dimensions. We make the analysis including a single extra dimension and observe that the enhancement in the BRs of these decays are small. However, in the case of two extra dimensions, especially the one that charged leptons are restricted to the fifth extra dimension, with non-zero Gaussian profiles, there is a considerable enhancement in the BRs of the decays under consideration, even more than one order for the compactification scale interval considered.

The paper is organized as follows: In Section 2, we present the effective vertex and the BRs of LFV Z decays in the split fermion scenario. Section 3 is devoted to discussion and our conclusions. In appendix section, we give the explicit expressions of the form factors appearing in the effective vertex.

# 2 $Z \to l_1^- l_2^+$ decay in the split fermion scenario in the two Higgs doublet model.

The extremely small theoretical values of the BRs of the LFV Z boson decays forces one to go beyond and search a new mechanism to enhance these numerical values near to the experimental limits. From the theoretical point of view, the existence of the flavor changing neutral currents (FCNCs) is essential to create the LFV processes and, the multi Higgs doublet models, which are constructed by extending the Higgs sector of the SM, are among the possible models which permits the FCNC currents at tree level. The 2HDM is one of the candidate for the multi Higgs doublet model and, in general, it permits the FCNC at tree level. The LFV Z decay $Z \to l_1^- l_2^+$ can be induced at least in the one loop level in the framework of the 2HDM and new Higgs scalars play the main role for the large BRs of these decays. Furthermore, the inclusion of the spatial extra dimensions brings additional contributions to the physical quantities of the decays under consideration. Here, we respect the idea that the hierarchy of lepton masses are coming from the lepton Gaussian profiles in the extra dimensions, so called split fermion scenario.

The LFV Z decay $Z \to l_1^- l_2^+$ exist with the help of the Yukawa interactions and, in a single extra dimension, respecting the split fermion scenario, it reads

$$\mathcal{L}_Y = \xi^E_{5\,ij} \bar{\hat{l}}_{iL} \phi_2 \hat{E}_{jR} + h.c. \ , \tag{5}$$

where $L$ and $R$ denote chiral projections $L(R) = 1/2(1 \mp \gamma_5)$, $\phi_2$ is the new scalar doublet and $\xi^E_{5\,ij}$ are the flavor violating Yukawa couplings in five dimensions and they are complex in general. The lepton fields $\hat{l}_{iL}$ ($\hat{E}_{jR}$) are the zero mode [1] lepton doublets (singlets) with Gaussian profiles in the extra dimension which is represented by the coordinate $y$ and they can be defined as

$$\begin{aligned} \hat{l}_{iL} &= N\, e^{-(y-y_{iL})^2/2\sigma^2}\, l_{iL}, \\ \hat{E}_{jR} &= N\, e^{-(y-y_{jR})^2/2\sigma^2}\, E_{jR} \,. \end{aligned} \tag{6}$$

where $l_{iL}$ ($E_{jR}$) are the lepton doublets (singlets) in four dimensions with family indices $i$ and $j$, $\sigma$, satisfying the property $\sigma << R$, is the parameter representing the Gaussian width of the leptons in five dimensions and N is the normalization factor, $N = \frac{1}{\pi^{1/4}\sigma^{1/2}}$. Here the coordinates $y_{iL}$ and $y_{iR}$ represent the positions of the peaks of left and right handed parts of $i^{th}$ lepton in the fifth dimension and they are obtained by assuming that the mass hierarchy of leptons are coming from the relative positions of the Gaussian peaks of the wave functions located in the extra dimension [15, 16]. The observed lepton masses are the sources to calculate these coordinates and in [16] one possible set of locations of the lepton fields in the fifth dimension has been estimated as

$$P_{l_i} = \sqrt{2}\,\sigma \begin{pmatrix} 11.075 \\ 1.0 \\ 0.0 \end{pmatrix} \ , \quad P_{e_i} = \sqrt{2}\,\sigma \begin{pmatrix} 5.9475 \\ 4.9475 \\ -3.1498 \end{pmatrix} \ . \tag{7}$$

Now, we would like the present the Higgs sector of the model under consideration. The

[1]Notice that we take only the zero mode lepton fields in our calculations.

Higgs doublets $\phi_1$ and $\phi_2$ are chosen as

$$\phi_1 = \frac{1}{\sqrt{2}}\left[\left(\begin{array}{c} 0 \\ v+H^0 \end{array}\right) + \left(\begin{array}{c} \sqrt{2}\chi^+ \\ i\chi^0 \end{array}\right)\right] ; \phi_2 = \frac{1}{\sqrt{2}}\left(\begin{array}{c} \sqrt{2}H^+ \\ H_1+iH_2 \end{array}\right) , \tag{8}$$

and the vacuum expectation values are,

$$<\phi_1>= \frac{1}{\sqrt{2}}\left(\begin{array}{c} 0 \\ v \end{array}\right) ; <\phi_2>= 0 . \tag{9}$$

This choice brings the possibility that the SM (new) particles are collected in the first (second) doublet and the Higgs fields $H_1$ and $H_2$ become the mass eigenstates $h^0$ and $A^0$ respectively since no mixing occurs between two CP-even neutral bosons $H^0$ and $h^0$ at tree level in this case. As it is seen in eq. (5), the new Higgs field $\phi_2$ is responsible for the LFV interaction at tree level. With the addition of extra dimensions, after the compactification on the orbifold $S^1/Z_2$, the new Higgs field $\phi_2$ can be expanded as

$$\phi_2(x,y) = \frac{1}{\sqrt{2\pi R}}\left\{\phi_2^{(0)}(x) + \sqrt{2}\sum_{n=1}^{\infty}\phi_2^{(n)}(x)\cos(ny/R)\right\} , \tag{10}$$

where $\phi_2^{(0)}(x)$ ($\phi_2^{(n)}(x)$) is the Higgs doublet in the four dimensions (the KK modes) including the charged Higgs boson $H^+$ ($H^{(n)+}$), the neutral CP even-odd Higgs bosons $h^0$- $A^0$ ($h^{0(n)}$- $A^{0(n)}$). The non-zero $n^{th}$ KK mode of the charged Higgs mass is $\sqrt{m_{H^\pm}^2+m_n^2}$, and the neutral CP even (odd) Higgs mass is $\sqrt{m_{h^0}^2+m_n^2}$, ($\sqrt{m_{A^0}^2+m_n^2}$ ), with the $n$'th level KK particle mass $m_n = n/R$.

The $Z \to l_1^- l_2^+$ decay exist at least at one loop level in the 2HDM with the help of the internal neutral Higgs particles $h^0$ and $A^0$. In Fig. 1 the necessary 1-loop diagrams, the self energy and vertex diagrams, are given. With the inclusion of extra dimensions, there exists the additional contributions due to the KK modes of neutral Higgs particles. At this stage one needs the lepton-lepton-$S$ ($S = h^0, A^0$) interaction, which is modified in the case of the split fermion scenario and these vertex factors $V^n_{LR\,(RL)\,ij}$ in the vertices $\bar{\hat{f}}_{iL\,(R)}\,S^{(n)}(x)\,\cos(ny/R)\,\hat{f}_{jR\,(L)}$, with the right (left) handed $i^{th}$ flavor lepton fields $\hat{f}_{jR\,(L)}$ in five dimensions (see eq. (6)), are obtained by the integration over the fifth dimension. Finally, the vertex factor for $n^{th}$ KK mode Higgs fields S read

$$V^n_{LR\,(RL)\,ij} = e^{-n^2\,\sigma^2/4\,R^2}\, e^{-(y_{iL\,(R)}-y_{jR\,(L)})^2/4\sigma^2}\,\cos\left[\frac{n\,(y_{iL\,(R)}+y_{jR\,(L)})}{2\,R}\right] . \tag{11}$$

For $n = 0$, this factor becomes $V^0_{LR\,(RL)ij} = e^{-(y_{iL\,(R)}-y_{jR\,(L)})^2/4\sigma^2}$ and we define the Yukawa couplings in four dimensions as

$$\xi^E_{ij}\left((\xi^{E\dagger}_{ij})^\dagger\right) = V^0_{LR\,(RL)\,ij}\,\xi^E_{5\,ij}\left((\xi^E_{5\,ij})^\dagger\right)/\sqrt{2\pi R} . \tag{12}$$

Now, we would like to present the general effective vertex for the interaction of on-shell Z-boson with a fermionic current:

$$\Gamma_\mu = \gamma_\mu (f_V - f_A\, \gamma_5) + \frac{i}{m_W}\, (f_M + f_E\, \gamma_5)\, \sigma_{\mu\,\nu}\, q^\nu \tag{13}$$

where $q$ is the momentum transfer, $q^2 = (p - p')^2$, $f_V$ ($f_A$) is vector (axial-vector) coupling, $f_M$ ($f_E$) magnetic (electric) transitions of unlike fermions. Here $p$ ($-p'$) is the four momentum vector of lepton (anti-lepton). The vector (axial-vector) $f_V$ ($f_A$) couplings and the magnetic (electric) transitions $f_M$ ($f_E$) including the contributions coming from a single extra dimension can be obtained as

$$\begin{aligned}
f_V &= \sum_{i=1}^{3} \Big( f_{iV}^{(0)} + 2 \sum_{n=1}^{\infty} f_{iV}^{(n)} \Big)\,, \\
f_A &= \sum_{i=1}^{3} \Big( f_{iA}^{(0)} + 2 \sum_{n=1}^{\infty} f_{iA}^{(n)} \Big)\,, \\
f_M &= \sum_{i=1}^{3} \Big( f_{iM}^{(0)} + 2 \sum_{n=1}^{\infty} f_{iM}^{(n)} \Big)\,, \\
f_E &= \sum_{i=1}^{3} \Big( f_{iE}^{(0)} + 2 \sum_{n=1}^{\infty} f_{iE}^{(n)} \Big)\,,
\end{aligned} \tag{14}$$

where $f_{i(V,A,M,E)}^{(0)}$ are the couplings without scalar boson $S = h^0, A^0$ KK mode contributions and they can be obtained by taking $n = 0$ in eq. (18). On the other hand the couplings $f_{i(V,A,M,E)}^{(n)}$ are the ones due to the KK modes of the scalar bosons $S = h^0, A^0$ (see eq. (18)). Here the summation over the index $i$ represents the sum due to the internal lepton flavors, namely, $e, \mu, \tau$. We present $f_{i(V,A,M,E)}^{(n)}$ in the appendix, by taking into account all the masses of internal leptons and external lepton (anti-lepton). If we consider two extra dimensions where all the particles are accessible, the couplings $f_{i(V,A,M,E)}^{(n)}$ appearing in eq. (14) should be replaced by $f_{i(V,A,M,E)}^{(n,s)}$ and they read

$$\begin{aligned}
f_V &= \sum_{i=1}^{3} \Big( f_{iV}^{(0,0)} + 4 \sum_{n,s}^{\infty} f_{iV}^{(n,s)} \Big)\,, \\
f_A &= \sum_{i=1}^{3} \Big( f_{iA}^{(0,0)} + 4 \sum_{n,s}^{\infty} f_{iA}^{(n,s)} \Big)\,, \\
f_M &= \sum_{i=1}^{3} \Big( f_{iM}^{(0,0)} + 4 \sum_{n,s}^{\infty} f_i M^{(n,s)} \Big)\,, \\
f_E &= \sum_{i=1}^{3} \Big( f_{iE}^{(0,0)} + 4 \sum_{n,s}^{\infty} f_{iE}^{(n,s)} \Big)\,,
\end{aligned} \tag{15}$$

where the summation would be done over $n, s = 0, 1, 2...$ except $n = s = 0$. (See the appendix for their explicit forms).

Finally, the BR for $Z \to l_1^- \, l_2^+$ can be written in terms of the couplings $f_V$, $f_A$, $f_M$ and $f_E$ as

$$BR(Z \to l_1^- \, l_2^+) = \frac{1}{48\,\pi} \, \frac{m_Z}{\Gamma_Z} \, \{|f_V|^2 + |f_A|^2 + \frac{1}{2\,cos^2\,\theta_W} (|f_M|^2 + |f_E|^2)\} \tag{16}$$

where $\alpha_W = \frac{g^2}{4\pi}$ and $\Gamma_Z$ is the total decay width of Z boson. In our numerical analysis we consider the BR due to the production of sum of charged states, namely

$$BR(Z \to l_1^{\pm} \, l_2^{\pm}) = \frac{\Gamma(Z \to (\bar{l}_1 \, l_2 + \bar{l}_2 \, l_1)}{\Gamma_Z} \, . \tag{17}$$

# 3 Discussion

Since the LFV Z decays $Z \to l_1^{\pm} l_2^{\pm}$, $l_1 \neq l_2$, exist at least in the one loop level in the 2HDM, the internal leptons and new scalar bosons drive the interaction and the physical quantities of these decays are sensitive to the Yukawa couplings $\bar{\xi}^E_{N,ij}$, $i, j = e, \mu, \tau$, which are among the free parameters of the model [2] . Furthermore, respecting the split fermion scenario, which is based on the idea that the hierarchy of fermion masses are due to the fermion Gaussian profiles in the extra dimensions, there arise new parameters, namely, the compactification radius, the fermion widths and their locations in the new dimesion(s). In this scenario, the Yukawa couplings in four dimensions appear with a multiplicative exponential suppression factors after the integration of the extra dimension (see eq. (12)). This factor is coming from the different locations of various flavors and their left and right handed parts of lepton fields, in the Yukawa part of the lagrangian. Here, we consider that the couplings $\bar{\xi}^E_{N,ij}$, $i, j = e, \mu$ are smaller compared to $\bar{\xi}^E_{N,\tau\, i}$ $i = e, \mu, \tau$, respecting the Sher scenario [26], since latter ones contain heavy flavors. Furthermore, we assume that, in four dimensions, the couplings $\bar{\xi}^E_{N,ij}$ is symmetric with respect to the indices $i$ and $j$ and choose the appropriate numerical values for the Yukawa couplings, by respecting the current experimental measurements. The upper limit of $\bar{\xi}^E_{N,\tau\mu}$ is predicted as $30\, GeV$ (see [27] and references therein) by using the experimental uncertainty, $10^{-9}$, in the measurement of the muon anomalous magnetic moment and assuming that the new physics effects can not exceed this uncertainty. Using this upper limit and the experimental upper bound of BR of $\mu \to e\gamma$ decay, BR $\leq 1.2 \times 10^{-11}$ [28], the coupling $\bar{\xi}^E_{N,\tau e}$ can be restricted in the range, $10^{-3} - 10^{-2}\, GeV$ (see [29]). For the Yukawa coupling $\bar{\xi}^E_{N,\tau\tau}$, we have no explicit restriction region and we use the numerical values which are greater than $\bar{\xi}^E_{N,\tau\mu}$.

[2]Here, we use the dimensionful coupling $\bar{\xi}^E_{N,ij}$ with the definition $\xi^E_{N,ij} = \sqrt{\frac{4\, G_F}{\sqrt{2}}}\, \bar{\xi}^E_{N,ij}$ where N denotes the word "neutral".

Our study is devoted to the prediction of the effects of the extra dimensions on the BR of the LFV processes $Z \to l_1^{\pm} l_2^{\pm}$, in the split fermion scenario, in the framework of the 2HDM. The compactification scale $1/R$, which is one of the free parameter of the model, should be restricted. In the literature, there exist numerous constraints for this scale, in the case of the single extra dimension, in the split fermion scenario:

- $1/R > 800\ GeV$ due to the direct limits from searching for KK gauge bosons.
- $1/R > 1.0\ TeV$ from $B \to \phi\, K_S$, $1/R > 500\ GeV$ from $B \to \psi\, K_S$ and $1/R > 800\ GeV$ from the upper limit of the $BR$, $BR\,(B_s \to \mu^+ \mu^-) < 2.6\ \times 10^{-6}$ [20].
- A far more stringent limit $1/R > 3.0\ TeV$ [30] coming from the precision electro weak bounds on higher dimensional operators generated by KK exchange

In our numerical analysis, we choose an appropriate range for the compactification scale $1/R$, by respecting these limits in the case of a single extra dimension. For two extra dimensions we used the same broad range for $1/R$. Throughout our calculations we use the input values given in Table (1).

| Parameter | Value |
|---|---|
| $m_\mu$ | 0.106 (GeV) |
| $m_\tau$ | 1.78 (GeV) |
| $m_W$ | 80.26 (GeV) |
| $m_Z$ | 91.19 (GeV) |
| $m_{h^0}$ | 100 (GeV) |
| $m_{A^0}$ | 200 (GeV) |
| $G_F$ | $1.1663710^{-5} (GeV^{-2})$ |
| $\Gamma_Z$ | $2.490\,(GeV)$ |
| $sin\,\theta_W$ | $\sqrt{0.2325}$ |

Table 1: The values of the input parameters used in the numerical calculations.

In the present work, we estimate the BRs of the LFV Z decays in a single and two extra dimensions, by considering split leptons with a possible set of locations. For a single extra dimension (two extra dimensions) we use the estimated location of the leptons given in eq. (7) (eq. (23)) to calculate the lepton-lepton-Higgs scalar KK mode vertices. In the case of two extra dimensions we study two possibilities: the leptons are restricted to the fifth extra dimension, with non-zero Gaussian profiles and the leptons have non-zero Gaussian profiles also in the sixth dimension. In the former one, the enhancements in the BRs of the present decays are relatively large due to the well known KK mode abundance of Higgs fields. However, in the latter the additional exponential factor appearing in the second summation further suppresses the BRs.

Fig. 2 is devoted to the compactification scale $1/R$ dependence of the BR $(Z \to \mu^{\pm} e^{\pm})$ for $\bar{\xi}^{D}_{N,\tau e} = 0.01\, GeV$, $\bar{\xi}^{D}_{N,\tau\mu} = 1\, GeV$ and $\rho = \sigma/R = 0.01$. Here the solid (dashed, small dashed, dotted) line represents the BR without extra dimension (with a single extra dimension, with two extra dimensions where the leptons have non-zero Gaussian profiles in the fifth extra dimension, with two extra dimensions where the leptons have non-zero Gaussian profiles in both extra dimensions). It is observed that BR is at the order of the magnitude of $10^{-14}$ without extra dimensions and it is the weakly sensitive to the parameter $1/R$, for the $1/R > 500\, GeV$, for a single extra dimension. In the case of two extra dimensions, the enhancement of the BR is relatively larger, near one order of magnitude, for $1/R \sim 0.6\, TeV$, due to the Higgs scalar KK mode abundances. However, these contributions do not increase extremely due to the suppression exponential factor appearing in the summations. The enhancement in the BR becomes weak for $1/R > 3.0\, TeV$. Furthermore, the numerical values of BRs are slightly greater in the case that the leptons have non-zero Gaussian profiles only in the fifth extra dimension.

In Fig. 3, we present the compactification scale $1/R$ dependence of the BR $(Z \to \tau^{\pm} e^{\pm})$ for $\bar{\xi}^{D}_{N,\tau e} = 0.01\, GeV$, $\bar{\xi}^{D}_{N,\tau\tau} = 10\, GeV$ and $\rho = 0.01$. Here the solid (dashed, small dashed, dotted) line represents the BR without extra dimension (with a single extra dimension, with two extra dimensions where the leptons have non-zero Gaussian profiles in the fifth extra dimension, with two extra dimensions where the leptons have non-zero Gaussian profiles in both extra dimensions). This figure shows that the BR is at the order of the magnitude of $10^{-12}$ without extra dimensions. Similar to the previous decay, the BR is weakly sensitive to the parameter $1/R$, for the $1/R > 500\, GeV$, for a single extra dimension. In the case of two extra dimensions, the enhancement of the BR is almost one order larger than the one with a single extra dimension, for $1/R \sim 0.6\, TeV$ and this enhancement becomes weak for $1/R > 3.0\, TeV$.

Fig. 4 represents the compactification scale $1/R$ dependence of the BR $(Z \to \tau^{\pm} \mu^{\pm})$ for $\bar{\xi}^{D}_{N,\tau\mu} = 1\, GeV$, $\bar{\xi}^{D}_{N,\tau\tau} = 10\, GeV$ and $\rho = 0.01$. Here the solid (dashed, small dashed, dotted) line represents the BR without extra dimension (with a single extra dimension, with two extra dimensions where the leptons have non-zero Gaussian profiles in the fifth extra dimension, with two extra dimensions where the leptons have non-zero Gaussian profiles in both extra dimensions). For this decay, the BR is observed at the order of the magnitude of $10^{-8}$ without extra dimensions. The BR is weakly sensitive to the parameter $1/R$, for the $1/R > 500\, GeV$, for a single extra dimension. In the case of two extra dimensions, the enhancement of the BR is more than one order larger than the one with a single extra dimension, for $1/R \sim 0.6\,TeV$ and this enhancement becomes also weak for $1/R > 3.0\,TeV$.

Now we would like to estimate the sensitivity of the BRs of the Z decays under consideration to the Gaussian widths, $\sigma = \rho\, R$ of leptons, where $\rho$ is the free parameter which regulates the amount of width in the extra dimension.

Fig. 5 (6 ; 7) shows the parameter $\rho$ dependence of the BR of the decay $Z \to \mu^\pm\, e^\pm$ ( $(Z \to \tau^\pm\, e^\pm)$ ; $(Z \to \tau^\pm\, \mu^\pm)$) for $1/R = 500\, GeV$, and the real couplings $\bar{\xi}^E_{N,\tau\mu} = 1\, GeV$, $\bar{\xi}^E_{N,\tau e} = 0.01\, GeV$ ($\bar{\xi}^E_{N,\tau\tau} = 10\, GeV$, $\bar{\xi}^E_{N,\tau e} = 0.01\, GeV$ ; $\bar{\xi}^E_{N,\tau\tau} = 10\, GeV$, $\bar{\xi}^E_{N,\tau\mu} = 1\, GeV$). Here the solid (dashed, small dashed, dotted) line represents the BR without extra dimension (with a single extra dimension, with two extra dimensions where the leptons have non-zero Gaussian profiles in the fifth extra dimension, with two extra dimensions where the leptons have non-zero Gaussian profiles in both extra dimensions). It is observed that the BR of the decay $Z \to \mu^\pm\, e^\pm$ ( $(Z \to \tau^\pm\, e^\pm)$ ; $(Z \to \tau^\pm\, \mu^\pm)$) increases almost 50% (50% ; 50%) for $\rho > 0.03$ and reaches 70% (74% ; 74%) for $\rho \sim 0.001$, in the case of a single extra dimension. This shows that the sensitivities of the BRs of the present decays to the parameter $\rho$ are not weak, especially for its small values. For two extra dimensions, these sensitivities increases considerably. For the decay $Z \to \mu^\pm\, e^\pm$ ( $(Z \to \tau^\pm\, e^\pm)$ ; $(Z \to \tau^\pm\, \mu^\pm)$), the BR enhances more than one order ( more than one order ; $\sim$ 2000%) for the intermediate values of the parameter $\rho$, in the case of two extra dimensions where the leptons have non-zero Gaussian profiles in the fifth extra dimension. This enhancement reaches $\sim$ 4000% ($\sim$ 4000% ; $\sim$ 4000%) for $\rho \sim 0.001$. In the case that the leptons have non-zero Gaussian profiles in both extra dimensions, the BR enhances less than one order (less than one order ; more than one order), for the intermediate values of the parameter $\rho$, for the decay $Z \to \mu^\pm\, e^\pm$ ( $(Z \to \tau^\pm\, e^\pm)$ ; $(Z \to \tau^\pm\, \mu^\pm)$). It is shown that the sensitivities of the BRs of studied LFV decays are considerably large for two extra dimensions and the BRs enhances more than one order, especially for the LFV Z decays with heavy lepton flavors.

As a summary, the BR is weakly sensitive to the parameter $1/R$ for $1/R > 500\, GeV$ for a single extra dimension, however, there is an enhancement in the BR, even more than one order for the scale $1/R \sim 600\, GeV$. This enhancement decreases with the increasing values of the scale $1/R$. Furthermore, the BR is sensitive to the parameter $\rho$ especially for two extra dimensions case. Therefore, the LFV Z decays are worthwhile to study and with the help of the forthcoming more accurate experimental measurements of the these decays, especially the

$Z \to \tau^{\pm}\, \mu^{\pm}$ one, the valuable information can be obtained to detect the effects due to the extra dimensions in the case of split fermion scenario.

# 4 Acknowledgement

This work has been supported by the Turkish Academy of Sciences in the framework of the Young Scientist Award Program. (EOI-TUBA-GEBIP/2001-1-8)

# 5 The explicit expressions appearing in the text

Here we present the explicit expressions for $f^{(n)}_{iV}$, $f^{(n)}_{iA}$, $f^{(n)}_{iM}$ and $f^{(n)}_{iE}$ [9] (see eq. (14)):

$$
\begin{aligned}
f^{(n)}_{iV} &= \frac{g}{64\,\pi^2\,cos\,\theta_W} \int_0^1 dx\, \frac{1}{m^2_{l_2^+}-m^2_{l_1^-}} \Bigg\{ c_V\,(m_{l_2^+}+m_{l_1^-}) \\
\Bigg( & \quad (-m_i\,\eta_i^+ + m_{l_1^-}(-1+x)\,\eta_i^V)\,ln\,\frac{L^{self}_{1,\,h^0}}{\mu^2} + (m_i\,\eta_i^+ - m_{l_2^+}(-1+x)\,\eta_i^V)\,ln\,\frac{L^{self}_{2,\,h^0}}{\mu^2} \\
&+ \quad (m_i\,\eta_i^+ + m_{l_1^-}(-1+x)\,\eta_i^V)\,ln\,\frac{L^{self}_{1,\,A^0}}{\mu^2} - (m_i\,\eta_i^+ + m_{l_2^+}(-1+x)\,\eta_i^V)\,ln\,\frac{L^{self}_{2,\,A^0}}{\mu^2}\Bigg) \\
&+ \quad c_A\,(m_{l_2^+}-m_{l_1^-}) \\
\Bigg( & \quad (-m_i\,\eta_i^- + m_{l_1^-}(-1+x)\,\eta_i^A)\,ln\,\frac{L^{self}_{1,\,h^0}}{\mu^2} + (m_i\,\eta_i^- + m_{l_2^+}(-1+x)\,\eta_i^A)\,ln\,\frac{L^{self}_{2,\,h^0}}{\mu^2} \\
&+ \quad (m_i\,\eta_i^- + m_{l_1^-}(-1+x)\,\eta_i^A)\,ln\,\frac{L^{self}_{1,\,A^0}}{\mu^2} + (-m_i\,\eta_i^- + m_{l_2^+}(-1+x)\,\eta_i^A)\,ln\,\frac{L^{self}_{2,\,A^0}}{\mu^2}\Bigg)\Bigg\} \\
&- \quad \frac{g}{64\,\pi^2\,cos\,\theta_W} \int_0^1 dx \int_0^{1-x} dy\,\Bigg\{ m_i^2\,(c_A\,\eta_i^A - c_V\,\eta_i^V)\,(\frac{1}{L^{ver}_{A^0}}+\frac{1}{L^{ver}_{h^0}}) \\
&- \quad (1-x-y)\,m_i\,\Bigg( c_A\,(m_{l_2^+}-m_{l_1^-})\,\eta_i^-\,(\frac{1}{L^{ver}_{h^0}}-\frac{1}{L^{ver}_{A^0}}) + c_V\,(m_{l_2^+}+m_{l_1^-})\,\eta_i^+\,(\frac{1}{L^{ver}_{h^0}}+\frac{1}{L^{ver}_{A^0}})\Bigg) \\
&- \quad (c_A\,\eta_i^A + c_V\,\eta_i^V)\Bigg( -2 + (q^2\,x\,y + m_{l_1^-}\,m_{l_2^+}\,(-1+x+y)^2)\,(\frac{1}{L^{ver}_{h^0}}+\frac{1}{L^{ver}_{A^0}}) - ln\,\frac{L^{ver}_{h^0}}{\mu^2}\,\frac{L^{ver}_{A^0}}{\mu^2}\Bigg) \\
&- \quad (m_{l_2^+}+m_{l_1^-})\,(1-x-y)\,\Bigg( \frac{\eta_i^A\,(x\,m_{l_1^-}+y\,m_{l_2^+}) + m_i\,\eta_i^-}{2\,L^{ver}_{A^0\,h^0}} + \frac{\eta_i^A\,(x\,m_{l_1^-}+y\,m_{l_2^+}) - m_i\,\eta_i^-}{2\,L^{ver}_{h^0\,A^0}}\Bigg) \\
&+ \quad \frac{1}{2}\eta_i^A\,ln\,\frac{L^{ver}_{A^0\,h^0}}{\mu^2}\,\frac{L^{ver}_{h^0\,A^0}}{\mu^2}\Bigg\}\,, \\
f^{(n)}_{iA} &= \frac{-g}{64\,\pi^2\,cos\,\theta_W} \int_0^1 dx\, \frac{1}{m^2_{l_2^+}-m^2_{l_1^-}} \Bigg\{ c_V\,(m_{l_2^+}-m_{l_1^-}) \\
\Bigg( & \quad (m_i\,\eta_i^- + m_{l_1^-}(-1+x)\,\eta_i^A)\,ln\,\frac{L^{self}_{1,\,A^0}}{\mu^2} + (-m_i\,\eta_i^- + m_{l_2^+}(-1+x)\,\eta_i^A)\,ln\,\frac{L^{self}_{2,\,A^0}}{\mu^2}
\end{aligned}
$$

$$
\begin{aligned}
&+ \quad (-m_i\,\eta_i^- + m_{l_1^-}(-1+x)\,\eta_i^A)\,ln\,\frac{L^{self}_{1,\,h^0}}{\mu^2} + (m_i\,\eta_i^- + m_{l_2^+}(-1+x)\,\eta_i^A)\,ln\,\frac{L^{self}_{2,\,h^0}}{\mu^2}\Big) \\
&+ \quad c_A\,(m_{l_2^+} + m_{l_1^-}) \\
&\Big( \quad (m_i\,\eta_i^+ + m_{l_1^-}(-1+x)\,\eta_i^V)\,ln\,\frac{L^{self}_{1,\,A^0}}{\mu^2} - (m_i\,\eta_i^+ + m_{l_2^+}(-1+x)\,\eta_i^V)\,ln\,\frac{L^{self}_{2,\,A^0}}{\mu^2} \\
&+ \quad (-m_i\,\eta_i^+ + m_{l_1^-}(-1+x)\,\eta_i^V)\,ln\,\frac{L^{self}_{1,\,h^0}}{\mu^2} + (m_i\,\eta_i^+ - m_{l_2^+}(-1+x)\,\eta_i^V)\,\frac{ln\,L^{self}_{2,\,h^0}}{\mu^2}\Big)\Big\} \\
&+ \quad \frac{g}{64\,\pi^2\,cos\,\theta_W}\int_0^1 dx\int_0^{1-x} dy\Big\{m_i^2\,(c_V\,\eta_i^A - c_A\,\eta_i^V)\,(\frac{1}{L^{ver}_{A^0}} + \frac{1}{L^{ver}_{h^0}}) \\
&- \quad m_i\,(1-x-y)\Big(c_V\,(m_{l_2^+} - m_{l_1^-})\,\eta_i^- + c_A\,(m_{l_2^+} + m_{l_1^-})\,\eta_i^+\Big)\,(\frac{1}{L^{ver}_{h^0}} - \frac{1}{L^{ver}_{A^0}}) \\
&+ \quad (c_V\,\eta_i^A + c_A\,\eta_i^V)\Big(-2 + (q^2\,x\,y - m_{l_1^-}\,m_{l_2^+}\,(-1+x+y)^2)(\frac{1}{L^{ver}_{h^0}} + \frac{1}{L^{ver}_{A^0}}) - ln\,\frac{L^{ver}_{h^0}}{\mu^2}\,\frac{L^{ver}_{A^0}}{\mu^2}\Big) \\
&- \quad (m_{l_2^+} - m_{l_1^-})\,(1-x-y)\,\Big(\frac{\eta_i^V\,(x\,m_{l_1^-} - y\,m_{l_2^+}) + m_i\,\eta_i^+}{2\,L^{ver}_{A^0\,h^0}} + \frac{\eta_i^V\,(x\,m_{l_1^-} - y\,m_{l_2^+}) - m_i\,\eta_i^+}{2\,L^{ver}_{h^0\,A^0}}\Big) \\
&- \quad \frac{1}{2}\eta_i^V\,ln\,\frac{L^{ver}_{A^0\,h^0}}{\mu^2}\,\frac{L^{ver}_{h^0\,A^0}}{\mu^2}\Big\}\,, \\
f^{(n)}_{iM} \quad &= \quad -\frac{g\,m_W}{64\,\pi^2\,cos\,\theta_W}\int_0^1 dx\int_0^{1-x} dy\Big\{\Big((1-x-y)\,(c_V\,\eta_i^V + c_A\,\eta_i^A)\,(x\,m_{l_1^-} + y\,m_{l_2^+}) \\
&+ \quad m_i\,(c_A\,(x-y)\,\eta_i^- + c_V\,\eta_i^+\,(x+y))\Big)\,\frac{1}{L^{ver}_{h^0}} \\
&+ \quad \Big((1-x-y)\,(c_V\,\eta_i^V + c_A\,\eta_i^A)\,(x\,m_{l_1^-} + y\,m_{l_2^+}) - m_i\,(c_A\,(x-y)\,\eta_i^- + c_V\,\eta_i^+\,(x+y))\Big)\,\frac{1}{L^{ver}_{A^0}} \\
&- \quad (1-x-y)\Big(\frac{\eta_i^A\,(x\,m_{l_1^-} + y\,m_{l_2^+})}{2}\,\big(\frac{1}{L^{ver}_{A^0\,h^0}} + \frac{1}{L^{ver}_{h^0\,A^0}}\big) + \frac{m_i\,\eta_i^-}{2}\,\big(\frac{1}{L^{ver}_{h^0\,A^0}} - \frac{1}{L^{ver}_{A^0\,h^0}}\big)\Big)\Big\}\,, \\
f^{(n)}_{iE} \quad &= \quad -\frac{g\,m_W}{64\,\pi^2\,cos\,\theta_W}\int_0^1 dx\int_0^{1-x} dy\Big\{\Big((1-x-y)\,\big(-(c_V\,\eta_i^A + c_A\,\eta_i^V)\,(x\,m_{l_1^-} - y\,m_{l_2^+})\big) \\
&- \quad m_i\,(c_A\,(x-y)\,\eta_i^+ + c_V\,\eta_i^-\,(x+y))\Big)\,\frac{1}{L^{ver}_{h^0}} \\
&+ \quad \Big((1-x-y)\,\big(-(c_V\,\eta_i^A + c_A\,\eta_i^V)\,(x\,m_{l_1^-} - y\,m_{l_2^+})\big) + m_i\,(c_A\,(x-y)\,\eta_i^+ + c_V\,\eta_i^-\,(x+y))\Big)\,\frac{1}{L^{ver}_{A^0}} \\
&+ \quad (1-x-y)\,\Big(\frac{\eta_i^V}{2}\,(m_{l_1^-}\,x - m_{l_2^+}\,y)\,\big(\frac{1}{L^{ver}_{A^0\,h^0}} + \frac{1}{L^{ver}_{h^0\,A^0}}\big) + \frac{m_i\,\eta_i^+}{2}\,\big(\frac{1}{L^{ver}_{A^0\,h^0}} - \frac{1}{L^{ver}_{h^0\,A^0}}\big)\Big)\Big\}\,, \qquad (18)
\end{aligned}
$$

where

$$
\begin{aligned}
L^{self}_{1,\,h^0} &= m^{(n)2}_{h^0}\,(1-x) + (m_i^2 - m^2_{l_1^-}\,(1-x))\,x\,, \\
L^{self}_{1,\,A^0} &= L^{self}_{1,\,h^0}(m^{(n)}_{h^0} \to m^{(n)}_{A^0})\,, \\
L^{self}_{2,\,h^0} &= L^{self}_{1,\,h^0}(m_{l_1^-} \to m_{l_2^+})\,,
\end{aligned}
$$

$$
\begin{aligned}
L^{self}_{2,\,A^0} &= L^{self}_{1,\,A^0}(m_{l_1^-} \to m_{l_2^+})\,, \\
L^{ver}_{h^0} &= m_{h^0}^{(n)2}\,(1-x-y) + m_i^2\,(x+y) - q^2\,x\,y\,, \\
L^{ver}_{h^0\,A^0} &= m_{A^0}^{(n)2}\,x + m_i^2\,(1-x-y) + (m_{h^0}^{(n)2} - q^2\,x)\,y\,, \\
L^{ver}_{A^0} &= L^{ver}_{h^0}(m_{h^0}^{(n)} \to m_{A^0}^{(n)})\,, \\
L^{ver}_{A^0\,h^0} &= L^{ver}_{h^0\,A^0}(m_{h^0}^{(n)} \to m_{A^0}^{(n)})\,,
\end{aligned} \tag{19}
$$

and

$$
\begin{aligned}
\eta_i^V &= e^{-n^2\,\sigma^2/2\,R^2}\,\{c_n\,(l_1,i)\,c_n\,(l_2,i)\,\xi^E_{il_1}\xi^{E*}_{il_2} + c'_n\,(l_1,i)\,c'_n\,(l_2,i)\,\xi^{E*}_{l_1 i}\xi^E_{l_2 i}\}\,, \\
\eta_i^A &= e^{-n^2\,\sigma^2/2\,R^2}\,\{c_n\,(l_1,i)\,c_n\,(l_2,i)\,\xi^E_{il_1}\xi^{E*}_{il_2} - c'_n\,(l_1,i)\,c'_n\,(l_2,i)\,\xi^{E*}_{l_1 i}\xi^E_{l_2 i}\}\,, \\
\eta_i^+ &= e^{-n^2\,\sigma^2/2\,R^2}\,\{c'_n\,(l_1,i)\,c_n\,(l_2,i)\,\xi^{E*}_{l_1 i}\xi^{E*}_{il_2} + c_n\,(l_1,i)\,c'_n\,(l_2,i)\,\xi^E_{il_1}\xi^E_{l_2 i}\,\}\,, \\
\eta_i^- &= e^{-n^2\,\sigma^2/2\,R^2}\,\{c'_n\,(l_1,i)\,c'_n\,(l_2,i)\,\xi^{E*}_{l_1 i}\xi^{E*}_{il_2} - c_n\,(l_1,i)\,c'_n\,(l_2,i)\,\xi^E_{il_1}\xi^E_{l_2 i}\}\,.
\end{aligned} \tag{20}
$$

The parameters $c_V$ and $c_A$ are $c_A = -\frac{1}{4}$ and $c_V = \frac{1}{4} - sin^2\,\theta_W$ and the masses $m_S^{(n)}$ read $m_S^{(n)} = \sqrt{m_S^2 + n^2/R^2}$, where $R$ is the compactification radius. In eq. (20) the flavor changing couplings $\xi^E_{il_j}$ represent the effective interaction between the internal lepton $i$, $(i = e, \mu, \tau)$ and outgoing (incoming) $j = 1\,(j = 2)$ one. The parameters $c_n\,(f,i)$, $c'_n\,(f,i)$ read

$$
\begin{aligned}
c_n\,(f,i) &= \cos[\frac{n\,(y_{fR} + y_{iL})}{2\,R}]\,, \\
c'_n\,(f,i) &= \cos[\frac{n\,(y_{fL} + y_{iR})}{2\,R}]\,.
\end{aligned} \tag{21}
$$

In the case of two extra dimensions which all the particles feel, the parameters $c_n\,(f,i)$ and $c'_n\,(f,i)$ are replaced by

$$
\begin{aligned}
c_{(n,s)}\,(f,i) &= \cos[\frac{n\,(y_{fR} + y_{iL}) + s\,(z_{fR} + z_{iL})}{2\,R}]\,, \\
c'_{(n,s)}(f,i) &= \cos[\frac{n\,(y_{fL} + y_{iR}) + s\,(z_{fL} + z_{iR})}{2\,R}]\,,
\end{aligned} \tag{22}
$$

and the exponential factor $e^{-n^2\,\sigma^2/2\,R^2}$ becomes $e^{-(n^2+s^2)\,\sigma^2/2\,R^2}$. Furthermore, the masses $m_S^{(n)}$ are replaced by $m_S^{(n,s)}$, $m_S^{(n,s)} = \sqrt{m_S^2 + m_n^2 + m_s^2}$, with $m_n = n/R$, $m_s = s/R$. Here we use a possible positions of left handed and right handed leptons in the two extra dimensions, by using the observed masses [3]. With the assumption that the lepton mass matrix is diagonal, one of the possible set of locations for the Gaussian peaks of the lepton fields in the two extra dimensions can be obtained as [22]

$$
P_{l_i} = \sqrt{2}\,\sigma \begin{pmatrix} (8.417, 8.417) \\ (1.0, 1.0) \\ (0.0, 0.0) \end{pmatrix}\,, \quad P_{e_i} = \sqrt{2}\,\sigma \begin{pmatrix} (4.7913, 4.7913) \\ (3.7913, 3.7913) \\ (-2.2272, -2.2272) \end{pmatrix}\,, \tag{23}
$$

[3]The calculation is similar to the one presented in [16] which is done for a single extra dimension.

where the numbers in the parenthesis denote the y and z coordinates of the location of the Gaussian peaks of lepton flavors in the extra dimensions. Here we choose the same numbers for the y and z locations of the Gaussian peaks.

Finally, the couplings $\xi^E_{l_j i}$ may be complex in general and they can be parametrized as

$$\xi^E_{l_i j} = |\xi^E_{l_i j}|\, e^{i\theta_{ij}} \ , \tag{24}$$

where $i, l_j$ denote the lepton flavors and $\theta_{ij}$ are CP violating parameters which are the possible sources of the lepton EDM. However, in the present work we take these couplings real.

# References


[1] T. Riemann and G. Mann, "Nondiagonal $Z$ decay: $Z \to e\mu$", in *Proc. of the Int. Conf. Neutrino'82, 14-19 June 1982, Balatonfüred, Hungary* (A. Frenkel and E. Jenik, eds.), vol. II, pp. 58-, Budapest, 1982, scanned copy at http://www.ifh.de/∼riemann, G. Mann and T. Riemann, *Annalen Phys.* **40** (1984) 334.

[2] J. I. Illana, M. Jack and T. Riemann, *hep-ph/0001273* (2000), J. I. Illana, and T. Riemann, *Phys. Rev.* **D63** 053004 (2001).

[3] Particle Data Group Collaboration, C. Caso *et al.*, *Eur. Phys. J.* **C3** (1998) 1.

[4] OPAL Collaboration, R. Akers *et al.*, *Z. Phys.* **C67** (1995) 555.

[5] L3 Collaboration, O. Adriani *et al.*, *Phys. Lett.* **B316** (1993) 427.

[6] DELPHI Collaboration, P. Abreu *et al.*, *Z. Phys.* **C73** (1997) 243.

[7] G. Wilson, "Neutrino oscillations: are lepton-flavor violating Z decays observable with the CDR detector?" and "Update on experimental aspects of lepton-flavour violation", talks at DESY-ECFA LC Workshops held at Frascati, Nov 1998 and at Oxford, March 1999, transparencies obtainable at
http://wwwsis.lnf.infn.it/talkshow/ and at
http://hepnts1.rl.ac.uk/ECFA_DESY_OXFORD/scans/0025_wilson.pdf.

[8] A. Ghosal, Y. Koide and H. Fusaoka, *Phys.Rev.* **D64** (2001) 053012.

[9] E. O. Iltan, I. Turan, *Phys.Rev.* **D65** (2002) 013001; E. O. Iltan, *Eur. Phys. J* **C41** (2005) 233.

[10] J. Illiana, M. Masip, *Phys. Rev.* **D67** (2003) 035004.

[11] J. Cao, Z. Xiong, J. Mi. Yang, *Eur. Phys. J.* **C32**(2004) 245.

[12] C. Yue, W. Wang, F. Zhang, *J. Phys.* **G30** (2004) 1065.

[13] M. A .Perez, G. T. Velasco, J. J. Toscano, *Int.J.Mod.Phys.* **A19** (2004) 159; A. Flores-Tlalpa, J.M. Hernandez, G. Tavares-Velasco, J.J. Toscano, *Phys. Rev.* **D65** (2002) 073010, R. Hawkings and K. Mönig, *Eur. Phys. J. direct* **C8** (1999) 1; V. Ganapathi, T. Weiler, E. Laermann, I. Schmitt, and P. Zerwas, *Phys. Rev.* **D27** (1983) 579, M. Clements, C. Footman, A. Kronfeld, S. Narasimhan, and D. Photiadis, *Phys. Rev.* **D27** (1983) 570.

[14] B. Pontecorvo, *Zh. Eksp. Teor. Fiz.* **33** (1957) 549, Z. Maki, M. Nakagawa, and S. Sakata, *Prog. Theor. Phys.* **28** (1962) 870, B. Pontecorvo, *Sov. Phys. JETP* **26** (1968) 984.

[15] N. Arkani-Hamed and M. Schmaltz, *Phys. Rev.* **D61**, 033005 (2000); N. Arkani-Hamed, Y. Grossman and M. Schmaltz, *Phys. Rev.* **D61** (2000) 115004 .

[16] E. A. Mirabelli, Schmaltz, *Phys. Rev.* **D61** (2000) 113011.

[17] W. F. Chang, I. L. Ho and J. N. Ng, *Phys. Rev.* **D66** 076004 (2002).

[18] G. C. Branco,A. Gouvea, M. N. Rebelo, *Phys. Lett.* **B506** (2001) 115.

[19] W. F. Cang, J. N. Ng, *JHEP* **0212** (2002) 077.

[20] B. Lillie, J. L. Hewett, *Phys. Rev.* **68** (2003) 116002.

[21] Y. Grossman and G. Perez, *Phys. Rev.* **D67** (2003) 015011 ; *Pramana* **62** (2004) 733.

[22] E. O. Iltan, hep-ph/0503001 (2005).

[23] E. O. Iltan, hep-ph/0504013 (2005).

[24] A. Delgado, A. Pomarol, M. Quiros, *JHEP* **0001** (2000) 030; Y. Grossman, *Int. J. Mod. Phys.* **A15** (2000) 2419; D. E. Kaplan and T. M. Tait, *JHEP* **0111** (2001) 051; G. Barenboim, *et. al.*, *Phys. Rev.* **D64** (2001) 073005; W. F. Chang, I. L. Ho and J. N. Ng, *Phys. Rev.* **D66** (2002) 076004; W. Skiba and D. Smith, *Phys. Rev.* **D65** (2002) 095002; Y. Grossman, R. Harnik, G. Perez, M. D. Schwartz and Z. Surujon, *Phys. Rev.* **D71** (2005) 056007; P. Dey, G. Bhattacharya, *Phys. Rev.* **D70** (2004) 116012; Y. Nagatani, G. Perez, *JHEP* **052** (2005) 068; R. Harnik, G. Perez, M. D. Schwartz, Y. Shirman, *JHEP* **0503** (2005) 068); C. Yue, D. Yu, L. Liu, *Phys. Rev.* **D69** (2004) 095003; C. Yue, H. Li, Y. Zhang. *Phys. Lett.* **B536** (2002) 67.

[25] Z. Surujon, hep-ph/0507036.

[26] T. P. Cheng and M. Sher, *Phy. Rev.* **D35** (1987) 3383 .

[27] E. O. Iltan, H. Sundu, *Acta Phys.Slov.* **53** (2003) 17.

[28] M. L. Brooks et. al., MEGA Collaboration, *Phys. Rev. Lett.* **83**, 1521 (1999).

[29] E. O. Iltan, *Phys. Rev.* **D64**, 115005, (2001); *Phys. Rev.* **D64** 013013, (2001).

[30] T. G. Rizzo, J. D. Wells, *Phy. Rev.* **D61** (2000) 016007.

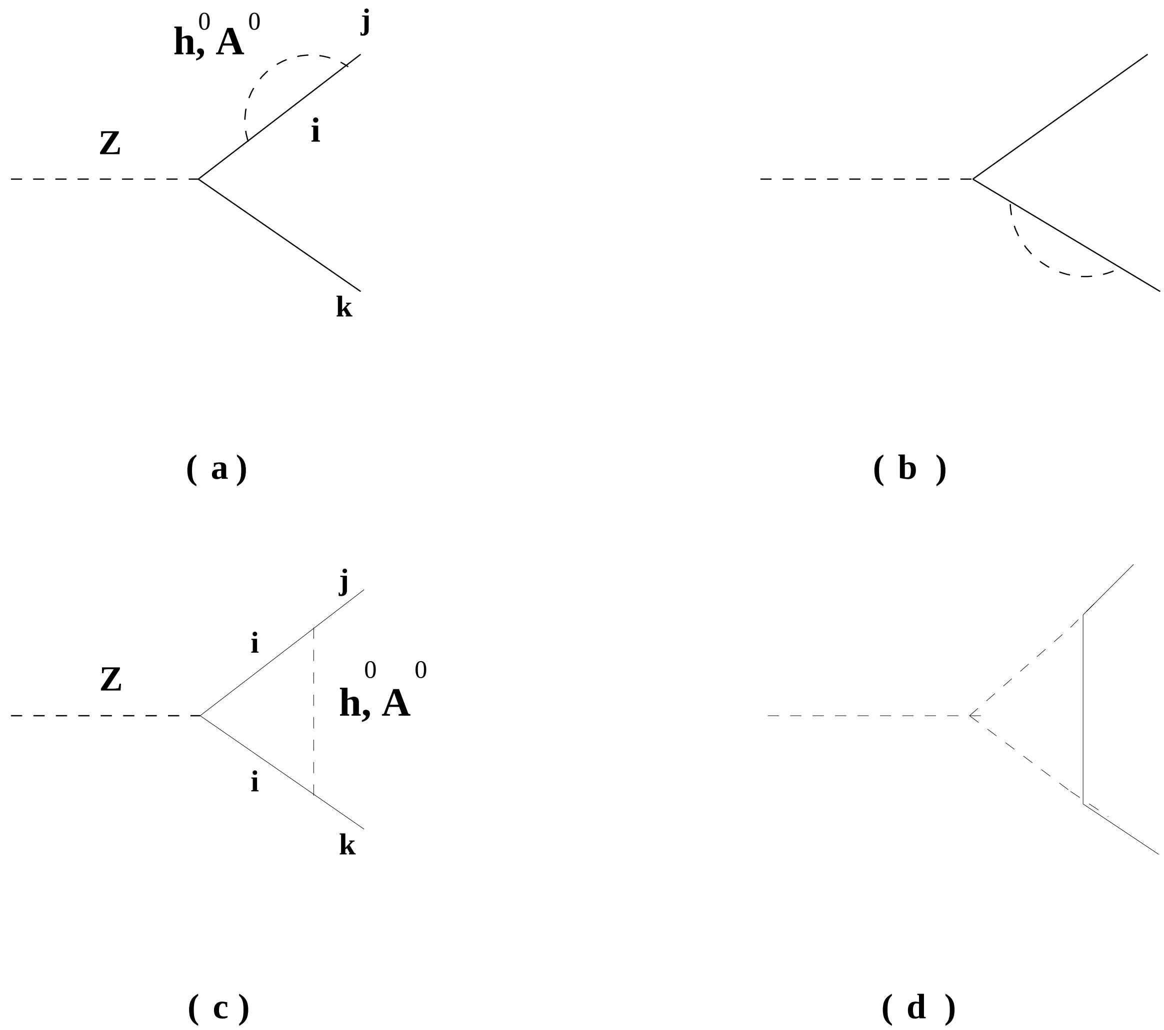


Figure 1: One loop diagrams contribute to $Z \to k^+ j^-$ decay due to the neutral Higgs bosons $h_0$ and $A_0$ in the 2HDM. $i$ represents the internal, $j$ ($k$) outgoing (incoming) lepton, dashed lines the vector field Z, $h_0$ and $A_0$ fields. In the case 5 (6) dimensions there exits also the KK modes of $h_0$ and $A_0$ fields.

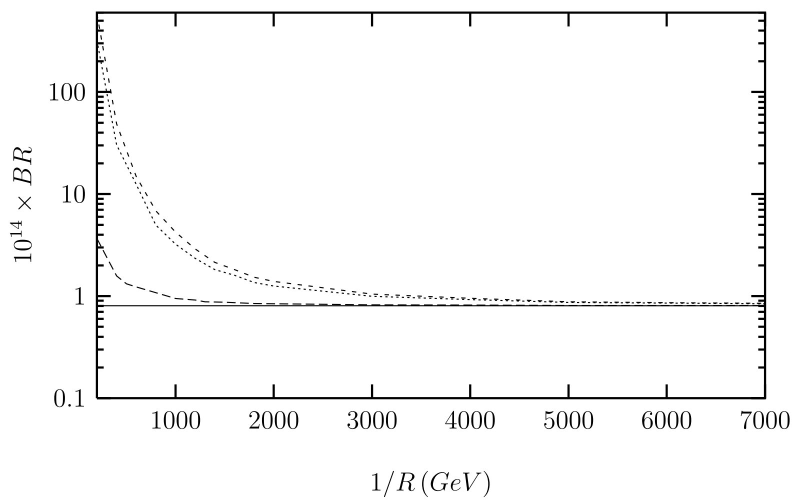


Figure 2: BR( $(Z \to \mu^{\pm} e^{\pm})$) with respect to the scale $1/R$ for $\rho = 0.01$, $\bar{\xi}^{E}_{N,\tau e} = 0.01\,GeV$, $\bar{\xi}^{E}_{N,\tau\mu} = 1\,GeV$. Here the solid (dashed, small dashed, dotted) line represents the BR without extra dimension (with a single extra dimension, with two extra dimensions where the leptons have non-zero Gaussian profiles in the fifth extra dimension, with two extra dimensions where the leptons have non-zero Gaussian profiles in both extra dimensions)

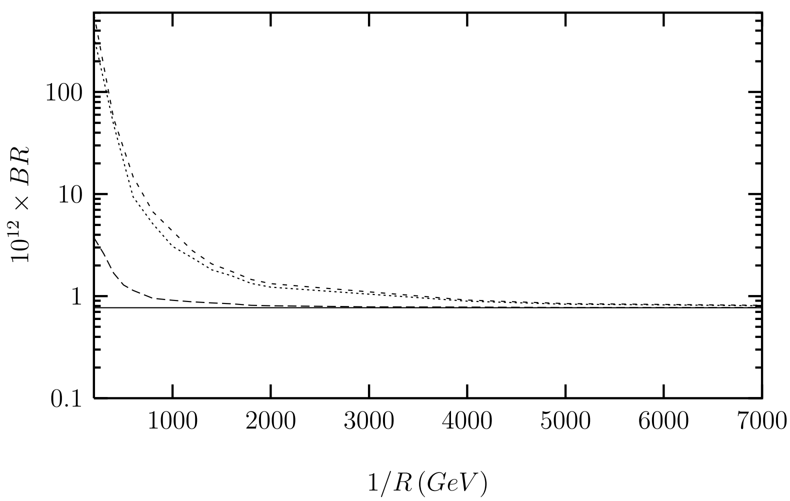


Figure 3: BR( $(Z \to \tau^{\pm}\, e^{\pm})$) with respect to the scale $1/R$ for $\rho = 0.01$, $\bar{\xi}^{E}_{N,\tau e} = 0.01\, GeV$, $\bar{\xi}^{E}_{N,\tau\tau} = 10\, GeV$. Here the solid (dashed, small dashed, dotted) line represents the BR without extra dimension (with a single extra dimension, with two extra dimensions where the leptons have non-zero Gaussian profiles in the fifth extra dimension, with two extra dimensions where the leptons have non-zero Gaussian profiles in both extra dimensions)

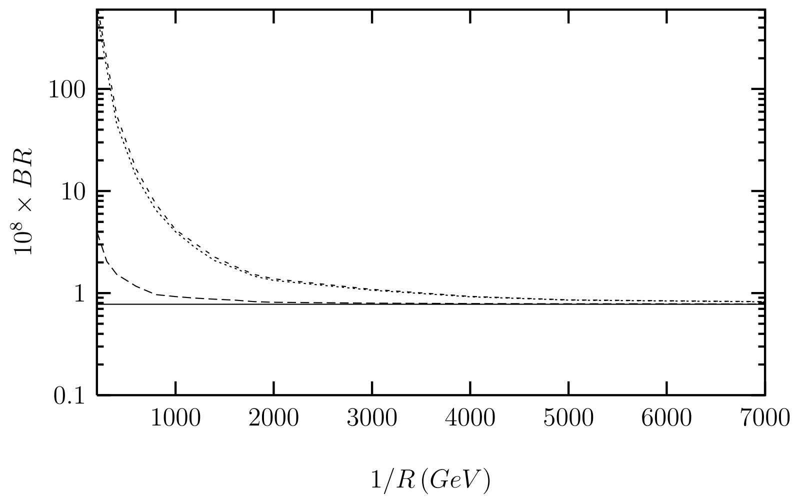


Figure 4: BR( $(Z \to \mu^{\pm}\, e^{\pm})$) with respect to the scale $1/R$ for $\rho = 0.01$, $\bar{\xi}^{E}_{N,\tau mu} = 1\,GeV$, $\bar{\xi}^{E}_{N,\tau\mu} = 1\,GeV$. Here the solid (dashed, small dashed, dotted) line represents the BR without extra dimension (with a single extra dimension, with two extra dimensions where the leptons have non-zero Gaussian profiles in the fifth extra dimension, with two extra dimensions where the leptons have non-zero Gaussian profiles in both extra dimensions)

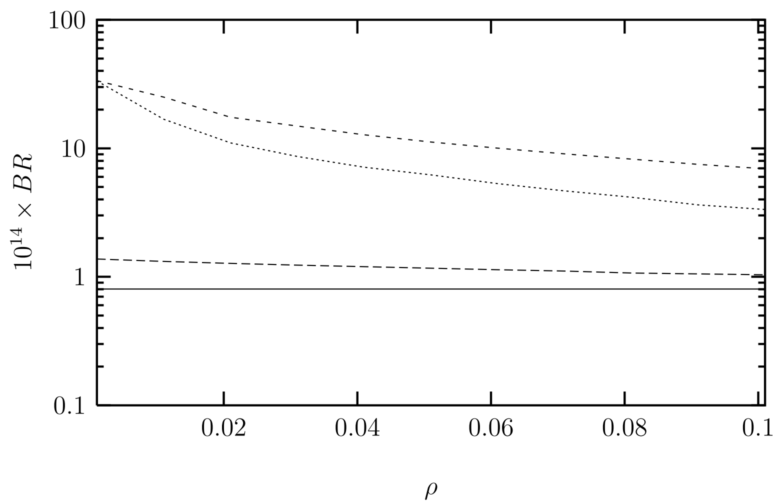


Figure 5: The same as Fig. 2 but with respect to parameter $\rho$ and for $1/R = 500\,GeV$.

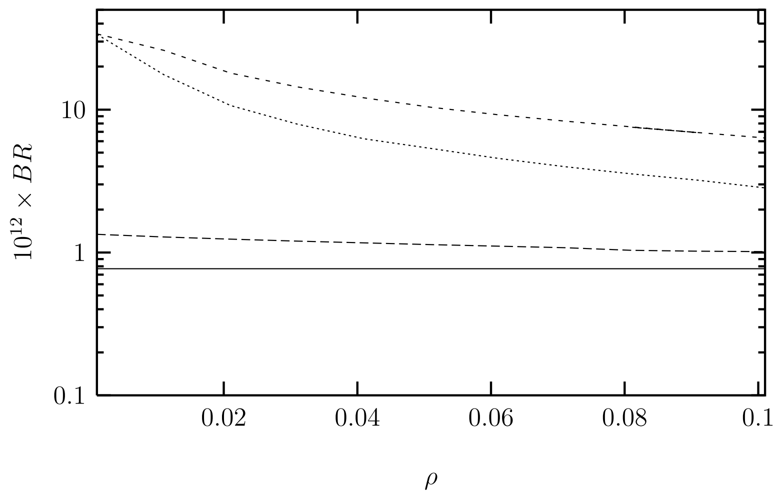


Figure 6: The same as Fig. 3 but with respect to parameter $\rho$ and for $1/R = 500\,GeV$.

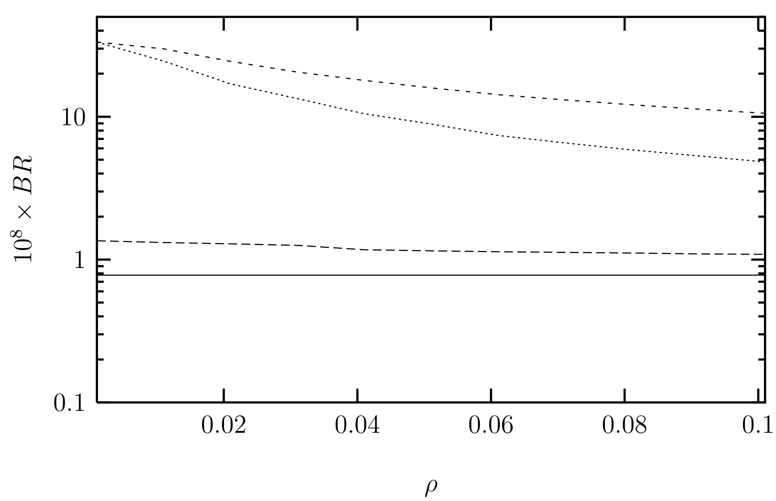


Figure 7: The same as Fig. 4 but with respect to parameter $\rho$ and for $1/R = 500\,GeV$.