CP violation in the inclusive $b \to sg$ decay in the framework of multi-Higgs doublet models

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Abstract

We study the decay width and CP asymmetry of the inclusive process $b \to sg$ (g denotes gluon) in the multi Higgs doublet models with complex Yukawa couplings, including next to leading QCD corrections. We analyse the dependencies of the decay width and CP asymmetry on the scale μ and CP violating parameter θ. We observe that there exist an enhancement in the decay width and CP asymmetry is at the order of 10^{-2}.

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1 Introduction

Rare B decays are induced by flavor changing neutral currents (FCNC) at loop level. Therefore they are phenomenologically rich and provide a comprehensive information about the theoretical models and the existing free parameters. The forthcoming experiments at SLAC, KEK B-factories, HERA-B and possible future accelerators stimulate the study of such decays since the large number of events can take place and various branching ratios, CP-violating asymmetries, polarization effects, etc., can be measured [1, 2].

Among B decay modes, inclusive $b \to sg$ is interesting since it is theoretically clean and sensitive to new physics beyond the SM, like two Higgs doublet model (2HDM) [3], minimal supersymmetric Standard model (MSSM) [4, 5], etc.

There are various studies on this process in the literature. The Branching ratio (Br) of $b \to sg$ decay in the SM is $Br(b \to sg) \sim 0.2\%$ for on-shell gluon [6]. This ratio can be enhanced with the addition of QCD corrections or by taking into account the extensions of the SM. The enhanced $Br(b \to sg)$ is among the possible explanations for the semileptonic branching ratio B_{SL} and the average charm multiplicity . The theoretical predictions of B_{SL} [7] are slightly different than the experimental measurements obtained at the $\Upsilon(4S)$ and Z^0 resonance [8]. Further the measured charm multiplicity η_c is smaller than the theoretical result. The enhancement of $Br(B \to X_{no\,charm})$ and therefore $Br(b \to sg)$ rate would explain the missing charm and B_{SL} problem [9]. Further, $Br(B \to \eta' X_s)$ reported by CLEO [10] stimulates to study on the enhancement of $Br(b \to sg)$.

In [11, 12], the enhancement of $Br\,(b \to sg)$ was obtained less than one order compared to the SM case in the framework of the 2HDM (Model I and II) for $m_{H^\pm} \sim 200\,GeV$ and $tan\,\beta \sim 5$. The possibility of large Br in the supersymmetric models was studied in [13]. In [14] Br was calculated in the model III and the prediction of the enhancement, at least one order larger compared to the SM one, makes it possible to describe the results coming from experiments [9]. In the case of time-like gluon, namely $b \to sg^*$ decay, Br should be consistent with the CLEO data [15]

$$Br\,(b \to sg^*) < 6.8\,\% \tag{1}$$

and in [14], it was showed that the model III enhancement was not contradict with this data for light-like gluon case. The calculation of $Br\,(b \to sg)$ with the addition of next to leading logarithmic (NLL) QCD corrections was done in [16] and it was observed that this ratio enhanced by more than a factor of 2.

CP violating asymmetry (A_{CP}) is another physical parameter which can give strong clues for the physics beyond the SM. The source of CP violating effects in the SM are complex Cabbibo-Cobayashi-Maskawa (CKM) matrix elements. A_{CP} for the inclusive $b \rightarrow sg$ decay vanishes in the SM and this forces one to go beyond the SM to check if a measurable A_{CP} is obtained.

In this work, we study the decay width Γ and A_{CP} of $b \rightarrow sg$ decay in the 3HDM and model III version of 2HDM. In these models, it is possible to enhance Γ and to get a measurable A_{CP}. Since the Yukawa couplings for new physics can be chosen complex and the addition of NLL corrections [16] brings additional complex quantities into the amplitude, theoretically, it is possible to get a considerable A_{CP}, at the order of the magnitude $2\,\%$. This effect is due to new physics beyond the SM, 3HDM and model III in our case.

The paper is organized as follows: In Section 2, we give a brief summary of the model III and $3HDM(O_2)$ and present the expressions appearing in the calculation of the decay width of the inclusive $b \rightarrow sg$ decay. Further we calculate the CP asymmetry A_{CP} of the process. Section 3 is devoted to discussion and our conclusions.

2 The inclusive process $b \rightarrow sg$ in the framework of the multi Higgs doublet models

In this section, we study NLL corrected $b \rightarrow sg$ decay width and the CP violating effects in the framework of the multi Higgs doublet models (model III version of 2HDM and 3HDM)

In the SM and model I and II 2HDM, the flavour changing neutral current at tree level is forbidden. However, they are permitted in the general 2HDM, so called model III with new parameters, i.e. Yukawa couplings. The Yukawa interaction in this general case reads as

$$\mathcal{L}_{Y} = \eta^{U}_{ij} \bar{Q}_{iL} \tilde{\phi}_{1} U_{jR} + \eta^{D}_{ij} \bar{Q}_{iL} \phi_{1} D_{jR} + \xi^{U\,\dagger}_{ij} \bar{Q}_{iL} \tilde{\phi}_{2} U_{jR} + \xi^{D}_{ij} \bar{Q}_{iL} \phi_{2} D_{jR} + h.c. \;, \qquad (2)$$

where L and R denote chiral projections $L(R) = 1/2(1 \mp \gamma_5)$, ϕ_k, for $k = 1, 2$, are the two scalar doublets, Q_{iL} are quark doublets, U_{jR} and D_{jR} are quark singlets, $\eta^{U,D}_{ij}$ and $\xi^{U,D}_{ij}$ are the matrices of the Yukawa couplings. The Flavor changing (FC) part of the interaction is given by

$$\mathcal{L}_{Y,FC} = \xi^{U\,\dagger}_{ij} \bar{Q}_{iL} \tilde{\phi}_{2} U_{jR} + \xi^{D}_{ij} \bar{Q}_{iL} \phi_{2} D_{jR} + h.c. \;. \qquad (3)$$

The choice of ϕ_1 and ϕ_2

$$\phi_1 = \frac{1}{\sqrt{2}} \left[\left(\begin{array}{c} 0 \\ v + H_0 \end{array} \right) + \left(\begin{array}{c} \sqrt{2} \chi^{+} \\ i \chi^{0} \end{array} \right) \right] ; \phi_2 = \frac{1}{\sqrt{2}} \left(\begin{array}{c} \sqrt{2} H^{+} \\ H_1 + i H_2 \end{array} \right) \;, \qquad (4)$$

and the vacuum expectation values,

$$< \phi_1 >= \frac{1}{\sqrt{2}} \left(\begin{array}{c} 0 \\ v \end{array} \right) \; ; < \phi_2 >= 0 \; , \tag{5}$$

allows us to carry the information about new physics in the doublet ϕ_2. Further, we take H_1, H_2 as the mass eigenstates h_0, A_0 respectively. Note that, at tree level, there is no mixing among CP even neutral Higgs particles, namely the SM one, H_0, and beyond, h_0.

In eq.(3) the couplings $\xi^{U,D}$ for the FC charged interactions are

$$\begin{aligned} \xi^{U}_{ch} &= \xi_{neutral} \, V_{CKM} \; , \\ \xi^{D}_{ch} &= V_{CKM} \, \xi_{neutral} \; , \end{aligned} \tag{6}$$

where $\xi^{U,D}_{neutral}$ is defined by the expression

$$\xi^{U(D)}_{N} = (V^{U(D)}_{R(L)})^{-1} \xi^{U,(D)} V^{U(D)}_{L(R)} \; . \tag{7}$$

where $\xi^{U,D}_{neutral}$ is denoted as $\xi^{U,D}_{N}$. Here the charged couplings are the linear combinations of neutral couplings multiplied by V_{CKM} matrix elements (see [20] for details). In the case of the general 3HDM, there is an additional Higgs doublet, ϕ_3, and the Yukawa interaction can be written as

$$\begin{aligned} \mathcal{L}_Y &= \eta^{U}_{ij} \bar{Q}_{iL} \tilde{\phi}_1 U_{jR} + \eta^{D}_{ij} \bar{Q}_{iL} \phi_1 D_{jR} + \xi^{U\,\dagger}_{ij} \bar{Q}_{iL} \tilde{\phi}_2 U_{jR} + \xi^{D}_{ij} \bar{Q}_{iL} \phi_2 D_{jR} \\ &+ \rho^{U\,\dagger}_{ij} \bar{Q}_{iL} \tilde{\phi}_3 U_{jR} + \rho^{D}_{ij} \bar{Q}_{iL} \phi_3 D_{jR} + h.c. \; , \end{aligned} \tag{8}$$

where $\rho^{U,D}_{ij}$ is the new Yukawa matrix having complex entries, in general. The similar choice of Higgs doublets

$$\begin{aligned} \phi_1 &= \frac{1}{\sqrt{2}} \left[\left(\begin{array}{c} 0 \\ v + H^0 \end{array} \right) + \left(\begin{array}{c} \sqrt{2} \chi^{+} \\ i \chi^0 \end{array} \right) \right] \; , \\ \phi_2 &= \frac{1}{\sqrt{2}} \left(\begin{array}{c} \sqrt{2} H^{+} \\ H^1 + i H^2 \end{array} \right) \; , \; \phi_3 = \frac{1}{\sqrt{2}} \left(\begin{array}{c} \sqrt{2} F^{+} \\ H^3 + i H^4 \end{array} \right) \; , \end{aligned} \tag{9}$$

with the vacuum expectation values,

$$< \phi_1 >= \frac{1}{\sqrt{2}} \left(\begin{array}{c} 0 \\ v \end{array} \right) \; ; < \phi_2 >= 0 \; ; < \phi_3 >= 0 \tag{10}$$

can be done and the information about new physics is carried beyond the SM in the last two doublets, ϕ_2 and ϕ_3. Further, we take H_1, H_2, H_3 and H_4 as the mass eigenstates h_0, A_0, h'_0, A'_0

where h_0', A_0' are new neutral Higgs bosons due to the additional Higgs doublet in the 3HDM (see [17]).

The Yukawa interaction for the Flavor Changing (FC) part is

$$\mathcal{L}_{Y,FC} = \xi^{U\,\dagger}_{ij} \bar{Q}_{iL} \tilde{\phi}_2 U_{jR} + \xi^{D}_{ij} \bar{Q}_{iL} \phi_2 D_{jR} + \rho^{U\,\dagger}_{ij} \bar{Q}_{iL} \tilde{\phi}_3 U_{jR} + \rho^{D}_{ij} \bar{Q}_{iL} \phi_3 D_{jR} + h.c. \;, \tag{11}$$

where the charged couplings $\xi^{U,D}_{ch}$ and $\rho^{U,D}_{ch}$ are

$$\begin{aligned}
\xi^{U}_{ch} &= \xi_N \; V_{CKM} \;, \\
\xi^{D}_{ch} &= V_{CKM} \; \xi_N \;, \\
\rho^{U}_{ch} &= \rho_N \; V_{CKM} \;, \\
\rho^{D}_{ch} &= V_{CKM} \; \rho_N \;,
\end{aligned} \tag{12}$$

and

$$\begin{aligned}
\xi^{U(D)}_{N} &= (V^{U(D)}_{R(L)})^{-1} \xi^{U,(D)} V^{U(D)}_{L(R)} \;, \\
\rho^{U(D)}_{N} &= (V^{U(D)}_{R(L)})^{-1} \rho^{U,(D)} V^{U(D)}_{L(R)} \;.
\end{aligned} \tag{13}$$

Since there exist additional charged Higgs particles, $F^{\pm}$, and neutral Higgs bosons h'^{0}, A'^{0} in the 3HDM, we introduce a new global $O(2)$ symmetry in the Higgs sector, considering three Higgs scalars as orthogonal vectors in a new space, which we call Higgs flavor space and we denote the Higgs flavor index by "m", where $m = 1, 2, 3$. The transformation reads

$$\begin{aligned}
\phi_1' &= \phi_1 \;, \\
\phi_2' &= cos\,\alpha \; \phi_2 + sin\,\alpha \; \phi_3 \;, \\
\phi_3' &= -sin\,\alpha \; \phi_2 + cos\,\alpha \; \phi_3 \;,
\end{aligned} \tag{14}$$

where α is the global parameter, which represents a rotation of the vectors ϕ_2 and ϕ_3 along the axis that ϕ_1 lies, in the Higgs flavor space. This symmetry ensures that the new particles are mass degenerate with their counterparts existing in model III (see [17] for details). Further the Yukawa Lagrangian (eq.(8)) is invariant under this transformation if the Yukawa matrices satisfy the expressions

$$\begin{aligned}
\bar{\xi}'^{U(D)}_{ij} &= \bar{\xi}^{U(D)}_{ij} cos\,\alpha + \bar{\rho}^{U(D)}_{ij} sin\,\alpha \;, \\
\bar{\rho}'^{U(D)}_{ij} &= -\bar{\xi}^{U(D)}_{ij} sin\,\alpha + \bar{\rho}^{U(D)}_{ij} cos\,\alpha \;.
\end{aligned} \tag{15}$$

and we get

$$(\bar{\xi}'^{U(D)})^{+} \bar{\xi}'^{U(D)} + (\bar{\rho}'^{U(D)})^{+} \bar{\rho}'^{U(D)} = (\bar{\xi}^{U(D)})^{+} \bar{\xi}^{U(D)} + (\bar{\rho}^{U(D)})^{+} \bar{\rho}^{U(D)} \;. \tag{16}$$

Therefore, it possible to parametrize the Yukawa matrices $\bar{\xi}^{U(D)}$ and $\bar{\rho}^{U(D)}$ as :

$$
\begin{aligned}
\bar{\xi}^{U(D)} &= \bar{\epsilon}^{U(D)} cos\,\theta \;, \\
\bar{\rho}^{U} &= \bar{\epsilon}^{U} sin\,\theta \;, \\
\bar{\rho}^{D} &= i\bar{\epsilon}^{D} sin\,\theta \;,
\end{aligned} \tag{17}
$$

where $\bar{\epsilon}^{U(D)}$ are real matrices satisfy the equation

$$
(\bar{\xi}'^{U(D)})^{+}\bar{\xi}'^{U(D)} + (\bar{\rho}'^{U(D)})^{+}\bar{\rho}'^{U(D)} = (\bar{\epsilon}^{U(D)})^{T}\bar{\epsilon}^{U(D)} \tag{18}
$$

and the angle θ is the source of CP violation. Here $X^{U(D)} = \sqrt{\frac{4G_F}{\sqrt{2}}}\,\bar{X}^{U(D)}$ with $X = \xi,\, \rho,\, \epsilon$ and T denotes transpose operation. In eq. (17), we take $\bar{\rho}^{D}$ complex to carry all CP violating effects in the third Higgs scalar.

Now, we would like to continue the study of the inclusive process $b \to sg$. Our starting point is the recent calculation of NLL corrected decay width [16]

$$
\Gamma(b \to sg) = \Gamma^{D} + \Gamma^{brems} \,, \tag{19}
$$

where

$$
\Gamma^{D} = c_1\, |D|^2 \,, \tag{20}
$$

with

$$
\begin{aligned}
D &= C_8^{0,eff} + \frac{\alpha_s}{4\,\pi}\{C_8^{1,eff} - \frac{16}{3} C_8^{0,eff} + C_1^{0}\,(l_1\, ln\frac{m_b}{\mu} + r_1) \\
&+ C_2^{0}\,(l_2\, ln\frac{m_b}{\mu} + r_2) + C_8^{0,eff}((l_8 + 8 + \beta_0)\, ln\frac{m_b}{\mu} + r_8)\} \,,
\end{aligned} \tag{21}
$$

and Γ^{brems} is the result for the finite part of bremsstrahlung corrections

$$
\Gamma^{brems} = c_2 \int dE_q dE_r\, (\tau_{11}^{+} + \tau_{22}^{+} + \tau_{22}^{-} + \tau_{12}^{+} + \tau_{18}^{+} + \tau_{28}^{+} + \tau_{28}^{-}) \,, \tag{22}
$$

where

$$
\begin{aligned}
\tau_{11}^{+} &= 48\, \hat{C}_1^2\, |\bar{\Delta} i_{23}|^2\, m_b^2\, (m_b^2 - 2\, E_q\, E_r) \;, \\
\tau_{22}^{+} &= \frac{56}{3}\, \hat{C}_2^2\, |\bar{\Delta} i_{23}|^2\, m_b^2\, (m_b^2 - 2\, E_q\, E_r) \;, \\
\tau_{22}^{-} &= 24\, \hat{C}_2^2\, |\bar{\Delta} i_{17}|^2\, m_b\, (16\, m_b\, E_q^2 - 16\, E_q^2\, E_r - 8 m_b^2\, E_q + 6\, m_b\, E_q\, E_r + m_b^3) \;, \\
\tau_{12}^{+} &= 32\, \hat{C}_1\, \hat{C}_2\, |\bar{\Delta} i_{23}|^2\, m_b^2\, (m_b^2 - 2\, E_q\, E_r) \;, \\
\tau_{18}^{+} &= 256\, \hat{C}_1\, Re[C_8^{0\, eff\, *}\, \bar{\Delta} i_{23}]\, m_b^2\, E_q\, E_r \;, \\
\tau_{28}^{+} &= \frac{16\;56}{3}\, \hat{C}_2\, Re[C_8^{0\, eff\, *}\, \bar{\Delta} i_{23}]\, m_b^2\, E_q\, E_r \;, \\
\tau_{28}^{-} &= -96\, \hat{C}_2\, Re[C_8^{0,eff\, *}\, \bar{\Delta} i_{17}]\, m_b^4\, (m_b\, (E_q + E_r) - 2\, (E_q^2 + E_r^2 + E_q\, E_r) \\
&+ 4\, \frac{E_q\, E_r\, (E_q + E_r)}{m_b}]/(E_q\, E_r)
\end{aligned} \tag{23}
$$

Here $\hat{C}_1 = \frac{1}{2} C_1^0$ and $\hat{C}_2 = C_2^0 - \frac{1}{6} C_1^0$, and $c_1 = \frac{\alpha_s\, m_b^5}{24\,\pi^4} |G_F\, V_{tb} V_{ts}^*|^2$ and $c_2 = \frac{|G_F\, V_{tb} V_{ts}^*|^2\, \alpha_s^2}{96\; 64\,\pi^2}$. (see [16] for details). In eqs. (21) and (23) the Wilson coefficients $C_8^{0,eff}$ and $C_{1(2)}^0$ (eq. (34)) includes LL corrections and new physics effects enter into the expressions through the coefficients $C_8^{0,eff}$ and $C_8^{1,eff}$ (see eq. (30)). The symbol η is defined as $\eta = \frac{\alpha_s(m_W)}{\alpha_s(\mu)}$ and $\beta_0 = 23/3$. The vectors a_i, h_i', e_i', f_i', k_i', l_i', a_i', appearing during QCD corrections, and the Wilson coefficients $C_4^{1,\mathrm{eff}}(m_W)$, $C_1^{1,\mathrm{eff}}(m_W)$ and $C_8^{1,eff}(m_W)$, the functions $\bar{\Delta} i_{17}$ and $\bar{\Delta} i_{23}$ in eqs. (23), r_1, r_2, r_8 and the numbers l_1, l_2, l_8 in eq. (21) are given in [16].

Now, we would like to start with the calculation of CP asymmetry for the inclusive decay underconsideration. The possible sources of CP violation in the model III (3HDM) are the complex Yukawa couplings. Our procedure is to neglect all Yukawa couplings except $\bar{\xi}^U_{N,tt}$ and $\bar{\xi}^D_{N,bb}$ ($\bar{\epsilon}^U_{N,tt}$ and $\bar{\epsilon}^D_{N,bb}$) (see eqs. (17, 18) and Discussion section) in the model III ($3HDM(O_2)$). Therefore, in the model III ($3HDM(O_2)$), only the combination $\bar{\xi}^U_{N,tt} \bar{\xi}^D_{N,bb}$ ($\bar{\epsilon}^U_{N,tt} \bar{\epsilon}^D_{N,bb}$) is responsible for A_{CP}. Using the definition of A_{CP}

$$A_{CP} = \frac{\Gamma(b \to s g) - \Gamma(\bar{b} \to \bar{s} g)}{\Gamma(b \to s g) + \Gamma(\bar{b} \to \bar{s} g)}, \tag{24}$$

we get

$$A_{CP} = Im[\bar{\xi}^D_{N,bb}]\, \frac{\Omega^D + \Omega^{br}}{\Lambda^D + \Lambda^{br}}\ , \tag{25}$$

in the model III where $\Omega^{D(br)}$ and $\Lambda^{D(br)}$ are the contributions coming from D-part (bremsstrahlung-part) and they read as

$$\begin{aligned}
\Omega^D &= \frac{\alpha_s}{\pi}\, c_1\, A_7\, Im[A_5]\ , \\
\Omega^{br} &= 2\, c_2 \int dE_q dE_r (B_5\, Im[\bar{\Delta}_{23}] + B_6\, Im[\bar{\Delta} i_{17}])\ , \\
\Lambda^D &= 2\, c_1 \{|A_6|^2 + |\bar{\xi}^D_{N,bb}|^2\, |A_7|^2 + 2\, A_7\, Re[\xi^D_{N,bb}]\, Re[A_6]\}\ , \\
\Lambda^{br} &= 2\, c_2 \int dE_q\, dE_r \{B_4 + Re[\bar{\xi}^D_{N,bb}]\, (B_5\, Re[\bar{\Delta} i_{23}] + B_6\, Re[\bar{\Delta} i_{17}])\}\, .
\end{aligned} \tag{26}$$

The functions $A_{5,6,7}$ and $B_{4,5,6}$ are defined as

$$\begin{aligned}
A_5 &= (C_1^0(\mu)\, [l_1 + ln[\frac{m_b}{\mu}] + r_1] + C_2^0(\mu)\, [l_2 + ln[\frac{m_b}{\mu}] + r_2])\ , \\
A_6 &= (\eta^{14/23} A_1 + A_3) + \frac{\alpha_s(\mu)}{4\pi}\, [A_4 + \chi A_1 - \frac{16}{3} \eta^{14/23}\, A_1 + A_3 \\
&+ (\eta^{14/23}\, A_1 + A_3)\, [(l_8 + 8 + \beta_0)\, ln[\frac{m_b}{\mu}] + r_8] + A_5]\ , \\
A_7 &= \eta^{14/23}\, A_2\, \{1 + \frac{\alpha_s(\mu)}{4\pi}\, [\eta^{-14/23} \chi - \frac{16}{3} + (l_8 + 8 + \beta_0)\, ln[\frac{m_b}{\mu}] + r_8]\}\, ,
\end{aligned} \tag{27}$$

and

$$\begin{aligned}
B_4 &= B_1 + B_2\,(\eta^{14/23}\,A_1 + A_3)\,Re[\bar{\Delta}\,i_{23}] + B_3\,(\eta^{14/23}A_1 + A_3)\,Re[\bar{\Delta}i_{17}]\ , \\
B_5 &= B_2\,\eta^{14/23}A_2\ , \\
B_6 &= B_3\,\eta^{14/23}A_2\ , \qquad (28)
\end{aligned}$$

$B_{1,2,3}$ appearing in eq. (28) read

$$\begin{aligned}
B_1 &= [\tau_{11}^{+} + \tau_{22}^{+} + \tau_{22}^{-} + \tau_{12}^{+}]\ , \\
B_2 &= 32\,m_b^2\,E_q\,E_r\,[8\,\hat{C}_1 + \frac{28}{3}\,\hat{C}_2]\ , \\
B_3 &= \frac{\tau_{28}^{-}}{Re[C_8^{0,eff\,*}\,\bar{\Delta}_{i17}]}\ . \qquad (29)
\end{aligned}$$

Here we use the parametrizations

$$\begin{aligned}
C_8^{0,eff}(m_W) &= A_1 + \bar{\xi}^{D}_{N,bb}\,A_2\ , \\
C_8^{0,eff}(\mu) &= \eta^{14/23}\,C_8^{0,eff}(m_W) + A_3\ , \\
C_8^{1,eff}(\mu) &= A_4 + \chi\,(A_1 + \bar{\xi}^{D}_{N,bb}A_2)\ , \qquad (30)
\end{aligned}$$

with

$$\begin{aligned}
A_1 &= C_8^{SM}(m_W) + C_8^{H(1)}(m_W)\ , \\
A_2 &= C_8^{H(2)}(m_W)\ , \\
A_3 &= \sum_{i=1}^{5} h_i'\,\eta^{a_i'}\,C_2^0(m_W)\ , \\
A_4 &= \eta^{37/23}C_8^{1,eff}(m_W) + \sum_{i=1}^{8}(e_i'\,\eta\,C_4^{1,\,\mathrm{eff}}(m_W) + (f_i' + k_i'\eta)\,C_2^0(m_W) \\
&+ \ l_i'\,\eta\,C_1^{1,\,\mathrm{eff}}(m_W))\eta^{a_i}\ , \qquad (31)
\end{aligned}$$

and the Wilson coefficients

$$\begin{aligned}
C_8^{SM}(m_W) &= -\frac{3x^2}{4(x-1)^4}\ln x + \frac{-x^3+5x^2+2x}{8(x-1)^3}\ , \\
C_8^{H(1)}(m_W) &= \frac{1}{m_t^2}\,|\bar{\xi}^{U}_{N,tt}|^2\,G_1(y_t)\ , \\
C_8^{H(2)}(m_W) &= \frac{1}{m_t m_b}\,(\bar{\xi}^{*U}_{N,tt})\,G_2(y_t)\,, \qquad (32)
\end{aligned}$$

with

$$\begin{aligned}
G_1(y_t) &= \frac{y_t(-y_t^2+5y_t+2)}{24(y_t-1)^3} + \frac{-y_t^2}{4(y_t-1)^4}\,\ln y_t\ , \\
G_2(y_t) &= \frac{y_t(y_t-3)}{4(y_t-1)^2} + \frac{y_t}{2(y_t-1)^3}\,\ln y_t \qquad (33)
\end{aligned}$$

The LL corrected Wilson coefficients C_1^0 and C_2^0 are

$$\begin{aligned} C_1^0(\mu) &= (\eta^{6/23} - \eta^{-12/23})\, C_2^0(m_W)\ , \\ C_2^0(\mu) &= (\frac{2}{3}\eta^{6/23} + \frac{1}{3}\eta^{-12/23}) C_2^0(m_W)\,, \end{aligned} \tag{34}$$

and

$$\begin{aligned} C_2^0(m_W) &= 1\ , \\ C_1^0(m_W) &= 0\,. \end{aligned} \tag{35}$$

In eq. (27) the parameter χ is given by

$$\chi = 6.7441\,(\eta^{37/23} - \eta^{14/23})\,. \tag{36}$$

In our calculations we take only $\bar{\xi}^{D}_{N,bb}$ complex, $\bar{\xi}^{D}_{N,bb} = |\bar{\xi}^{D}_{N,bb}|\, e^{i\,\theta}$, where θ is the CP violating parameter which is restricted by the experimental upper limit of the neutron electric dipole moment eq. (41). For $3HDM(O_2)$, it is necessary to make the following replacements:

$$\begin{aligned} \bar{\xi}^{U}_{N,tt} &\rightarrow \bar{\epsilon}^{U}_{N,tt}\ , \\ Im[\bar{\xi}^{D}_{N,bb}] &\rightarrow \bar{\epsilon}^{D}_{N,bb}\, sin^2\,\theta\ , \\ Re[\bar{\xi}^{D}_{N,bb}] &\rightarrow \bar{\epsilon}^{D}_{N,bb}\, cos^2\,\theta\ , \\ |\bar{\xi}^{D}_{N,bb}|^2 &\rightarrow (\bar{\epsilon}^{D}_{N,bb})^2\,. \end{aligned} \tag{37}$$

3 Discussion

The general 3HDM model contains large number of free parameters, such as masses of charged and neutral Higgs bosons, complex Yukawa matrices, $\xi^{U,D}_{ij}$, $\rho^{U,D}_{ij}$ with quark family indices i, j. First, a new global $O(2)$ symmetry is introduced in the Higgs flavor space to connect the Yukawa matrices in the second and third doublet and to keep the masses of new charged (neutral) Higgs particles in the third doublet degenerate to the ones in the second doublet [17]. Second, the Yukawa couplings, which are entries of Yukawa matrices, is restricted using the experimental measurements, namely, $\Delta F = 2$ mixing, the ρ parameter [18] and the CLEO measurement [19],

$$Br(B \rightarrow X_s \gamma) = (3.15 \pm 0.35 \pm 0.32)\, 10^{-4}\ . \tag{38}$$

The constraints for the FC couplings from $\Delta F = 2$ processes and ρ parameter for the model III were investigated without QCD corrections [18] and the following predictions are reached:

$$\lambda_{uj} = \lambda_{dj} << 1\ ,\ i, j = 1, 2, 3\ ,$$

where $u(d)$ is up (down) quark and i, j are the generation numbers and further

$$\lambda_{bb} \ , \lambda_{sb} >> 1 \ \text{and} \ \lambda_{tt} \ , \lambda_{ct} << 1 \ . \tag{39}$$

In the analysis, the ansatz proposed by Cheng and Sher,

$$\xi^{UD}_{ij} = \lambda_{ij} \sqrt{\frac{m_i m_j}{v}} \ , \tag{40}$$

is used. Respecting these constraints and using the measurement by the CLEO [19] Collaboration we neglect all Yukawa couplings except $\bar{\xi}^{U}_{N,tt}$, $\bar{\xi}^{D}_{N,bb}$ in the model III. In $3HDM(O_2)$, the same restrictions are done by taking into account only the couplings $\bar{\epsilon}^{U}_{N,tt}$ and $\bar{\epsilon}^{D}_{N,bb}$.

This section is devoted to the study of the CP parameter $sin\theta$ and the scale μ dependencies of the decay width Γ and CP asymmetry of A_{CP} for the inclusive decay $b \rightarrow sg$, in the framework of the model III and $3HDM(O_2)$. In our analysis, we restrict the parameters θ, $\bar{\xi}^{U}_{N,tt}$ and $\bar{\xi}^{D}_{Nbb}$ ($\bar{\epsilon}^{U}_{N,tt}$ and $\bar{\epsilon}^{D}_{Nbb}$) in the model III ($3HDM(O_2)$), using the constraint for $|C_7^{eff}|$, $0.257 \leq |C_7^{eff}| \leq 0.439$, coming from the CLEO data eq. (38) (see [20]). Here C_7^{eff} is the effective magnetic dipole type Wilson coefficient for $b \rightarrow s\gamma$ vertex. The above restriction allows us to define a constraint region for the parameter $\bar{\xi}^{U}_{N,tt}$ ($\bar{\epsilon}^{U}_{N,tt}$) in terms of $\bar{\xi}^{D}_{N,bb}$ ($\bar{\epsilon}^{D}_{N,bb}$) and θ in the the model III, ($3HDM(O_2)$). Further, in our numerical calculations we respect the constraint for the angle θ, due to the experimental upper limit of neutron electric dipole moment, namely

$$d_n < 10^{-25} \text{e}\cdot\text{cm} \tag{41}$$

which places an upper bound on the couplings with the expression in the model III ($3HDM(O_2)$): $\frac{1}{m_t m_b} (\bar{\xi}^{U}_{N,tt} \, \bar{\xi}^{*D}_{N,bb}) \, sin\,\theta < 1.0$ ($\frac{1}{m_t m_b} (\bar{\epsilon}^{U}_{N,tt} \, \bar{\epsilon}^{*D}_{N,bb}) \, sin^2\,\theta < 1.0$) for $m_{H^\pm} \approx 200$ GeV [21].

Throughout these calculations, we take the charged Higgs mass $m_{H^\pm} = 400\, GeV$, and we use the input values given in Table (1).

Fig. 1 (2) is devoted to the $sin\,\theta$ dependence of Γ for $\mu = m_b$, $\bar{\xi}^{D}_{N,bb} = 40\, m_b$ ($\bar{\epsilon}^{D}_{N,bb} = 40\, m_b$) and $|r_{tb}| = |\frac{\bar{\xi}^{U}_{N,tt}}{\bar{\xi}^{D}_{N,bb}}| < 1$ ($|\frac{\bar{\epsilon}^{U}_{N,tt}}{\bar{\epsilon}^{D}_{N,bb}}| < 1$) in the model III ($3HDM(O_2)$). Here Γ is restricted between solid (dashed) lines for $C_7^{eff} > 0$ ($C_7^{eff} < 0$). As shown in Fig. 1, the decay width Γ can reach $(0.78 \pm 0.06) \times 10^{-14}$ in the region $0.2 \leq sin\,\theta \leq 0.7$ for $C_7^{eff} > 0$ and the possible enhancement, a factor of 4.2 compared to the SM one $(0.185 \pm 0.037) \times 10^{-14}\, GeV$ [16] can be reached. For $3HDM(O_2)$, the upper range for the decay width Γ is $(0.79 \pm 0.07) \times 10^{-14}$ in the region $0.2 \leq sin\,\theta \leq 0.7$ for $C_7^{eff} > 0$ and this leads to an enhancement, a factor of 4.3 compared to the SM one. Γ decreases with increasing $sin\theta$ for $C_7^{eff} > 0$ and it can get larger values compared to the $C_7^{eff} < 0$ case, in both models. The $sin\,\theta$ dependence of Γ is weak

Parameter	Value
m_c	1.4 (GeV)
m_b	4.8 (GeV)
$\lvert V_{tb} V_{ts}^*\rvert$	0.04
m_t	175 (GeV)
m_W	80.26 (GeV)
m_Z	91.19 (GeV)
Λ_{QCD}	0.214 (GeV)
$\alpha_s(m_Z)$	0.117

Table 1: The values of the input parameters used in the numerical calculations.

for $C_7^{eff} < 0$ and for this case, it takes slightly smaller values in the $3HDM(O_2)$ compared to the ones in the model III. In our numerical calculations, we observe that the contribution of bremsstrahlung corrections are almost one order smaller as a magnitude compared to the rest. Further, the restriction regions for $C_7^{eff} > 0$ and $C_7^{eff} < 0$ become more seperated with increasing values of the scale μ and this behaviour is strong in the $3HDM(O_2)$. The scale dependence of Γ is weak for the values $\mu > 2\,GeV$ and almost no dependence is observed for the large values of μ scale for both models. (see Figs. 3 and 4).

In Fig. 5 and 6, we present the $sin\,\theta$ dependence of A_{CP} for $\mu = m_b$, $\bar{\xi}^D_{N,bb} = 40\,m_b$ ($\bar{\epsilon}^D_{N,bb} = 40\,m_b$) and $|r_{tb}| < 1$ in the model III ($3HDM(O_2)$). Here A_{CP} is restricted in the region bounded by solid (dashed) lines for $C_7^{eff} > 0$ ($C_7^{eff} < 0$). As shown in figures, $|A_{CP}|$ reaches $2.5\,\%$ for $sin\,\theta = 0.7$ and all possible values of A_{CP} are negative. However, for $C_7^{eff} < 0$, the allowed region becomes broader and A_{CP} can take both signs, even can vanish. For this case, $|A_{CP}|$ reaches almost $1\,\%$ as an upper limit in both models. Further A_{CP} is more sensitive to $sin\,\theta$ in the $3HDM(O_2)$ compared to the model III.

Fig. 7 and 8 represent the scale μ dependence of A_{CP} for $sin\theta = 0.5$, $|\bar{\xi}^D_{N,bb}|$ ($\bar{\epsilon}^D_{N,bb}$) $= 40\,m_b$ and $|r_{tb}| < 1$ in both models underconsideration. The scale dependence of A_{CP} is also weak for the values $\mu > 2\,GeV$ similar to that of Γ. Here the increasing values of $sin\,\theta$ cause to increase the size of restriction region.

At this stage we give the numerical values of Γ and A_{CP} for $|\bar{\xi}^D_{N,bb}| = 40\,m_b$ ($\bar{\epsilon}^D_{N,bb} = 40\,m_b$) and $\mu = m_b$ in the range $0.2 \leq sin\theta \leq 0.7$, for model III ($3HDM(O_2)$):

$$
\begin{aligned}
0.72\,(0.72)\times 10^{-14}\,GeV \leq \;\; & \Gamma \;\; \leq 0.84\,(0.86)\times 10^{-14}\,GeV \;\; \text{(upper boundary)} \;\; for\, C_7^{eff} > 0 \;, \\
0.28\,(0.28)\times 10^{-14}\,GeV \leq \;\; & \Gamma \;\; \leq 0.40\,(0.42)\times 10^{-14}\,GeV \;\; \text{(lower boundary)} \;\; for\, C_7^{eff} > 0 \;, \\
& \Gamma \;\; = 0.50\,(0.48)\times 10^{-14}\,GeV \;\; \text{(upper boundary)} \;\; for\, C_7^{eff} < 0 \;,
\end{aligned}
$$

$$\Gamma \;\; = 0.20\,(0.20) \times 10^{-14}\, GeV \;\text{(lower boundary)}\; for\, C_7^{eff} < 0 \;, \tag{42}$$

and

$$\begin{aligned}
0.0080\,(0.0015) \leq \;\; & |A_{CP}| \;\; \leq 0.0250\,(0.0250)\;\text{(upper boundary)}\; for\, C_7^{eff} > 0 \;, \\
0.0050\,(0.0010) \leq \;\; & |A_{CP}| \;\; \leq 0.0170\,(0.0165)\;\text{(lower boundary)}\; for\, C_7^{eff} > 0 \;, \\
0.0020\,(0.0010) \leq \;\; & \;A_{CP} \;\; \leq 0.0060\,(0.0060)\;\text{(upper boundary)}\; for\, C_7^{eff} < 0 \;, \\
-0.0100\,(-0.0100) \leq \;\; & \;A_{CP} \;\; \leq -0.0020\,(-0.0010)\;\text{(lower boundary)}\; for\, C_7^{eff} < 0 \;.
\end{aligned} \tag{43}$$

Now we would like to present our conclusions:

- Γ can reach $0.84\,(0.86) \times 10^{-14}$ in the model III ($3HDM(O_2)$) and this is an enhancement a factor of 4 compared to the SM one.

- A measurable CP asymmetry A_{CP} exists with the addition of NLL QCD corrections and choice of complex Yukawa coupling $\bar{\xi}^D_{N,bb}$ ($\bar{\rho}^D_{N,bb}$ (see section 2)) in the model III ($3HDM(O_2)$). $|A_{CP}|$ can be obtained at the order of the magnitude of $\%\,2.5$. This physical parameter is coming from the new physics effects and it can give strong clues about the physics beyond the SM.

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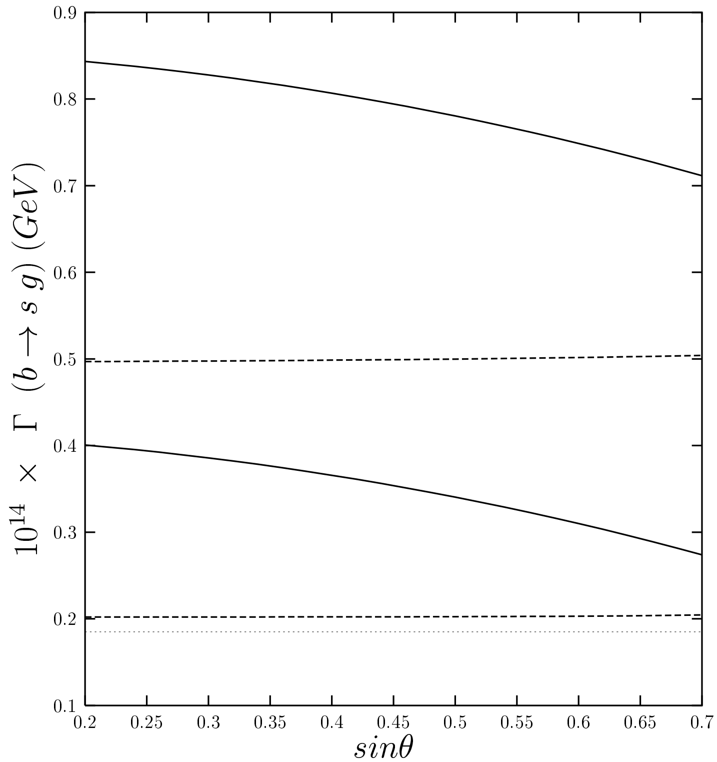

Figure 1: Γ as a function of $sin\theta$ for $|r_{tb}| = |\frac{\bar{\xi}^{U}_{N,tt}}{\bar{\xi}^{D}_{N,bb}}| < 1$, $\bar{\xi}^{D}_{N,bb} = 40\, m_b$ and $\mu = m_b$. Here Γ is restricted in the region bounded by solid (dashed) lines for $C_7^{eff} > 0$ ($C_7^{eff} < 0$), in the model III. Dotted line represents the SM contribution.

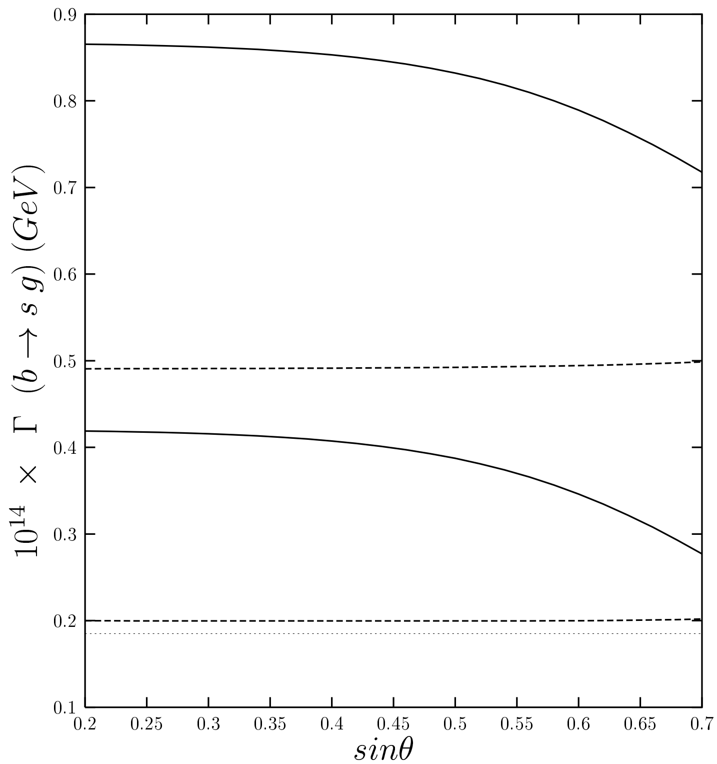

Figure 2: The same as Fig. 1 but for $3HDM(O_2)$.

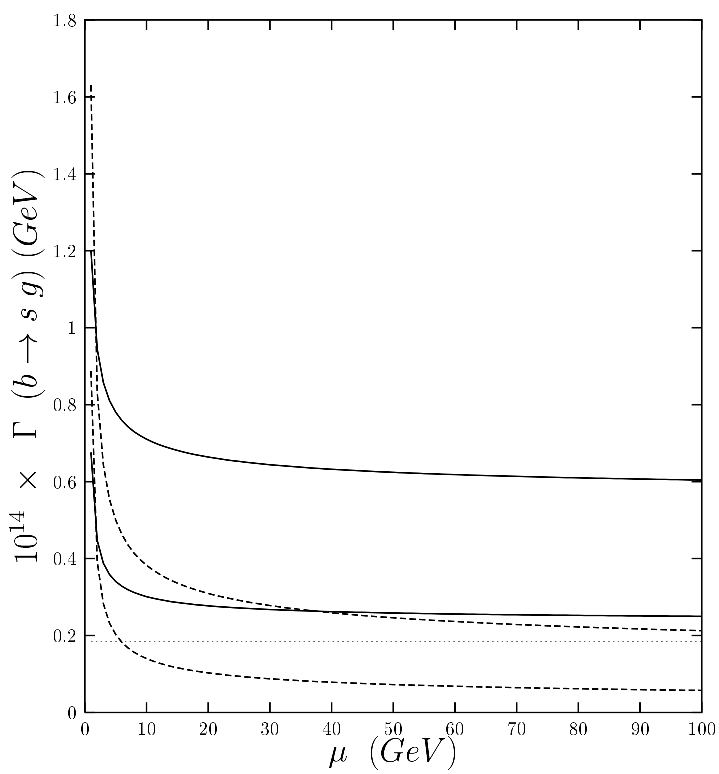

Figure 3: The same as Fig. 1 but Γ as a function of μ for $sin\theta = 0.5$.

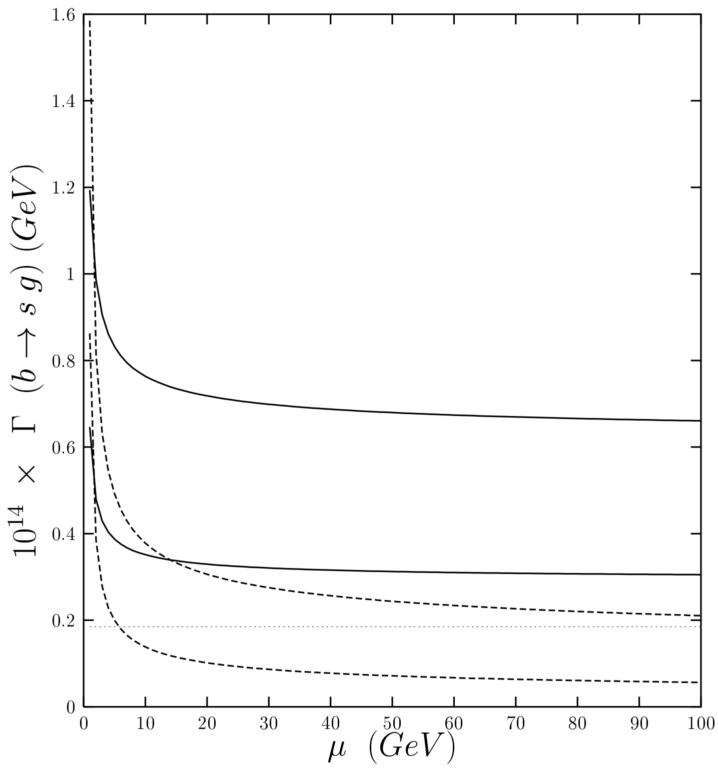

Figure 4: The same as Fig. 3 but for $3HDM(O_2)$.

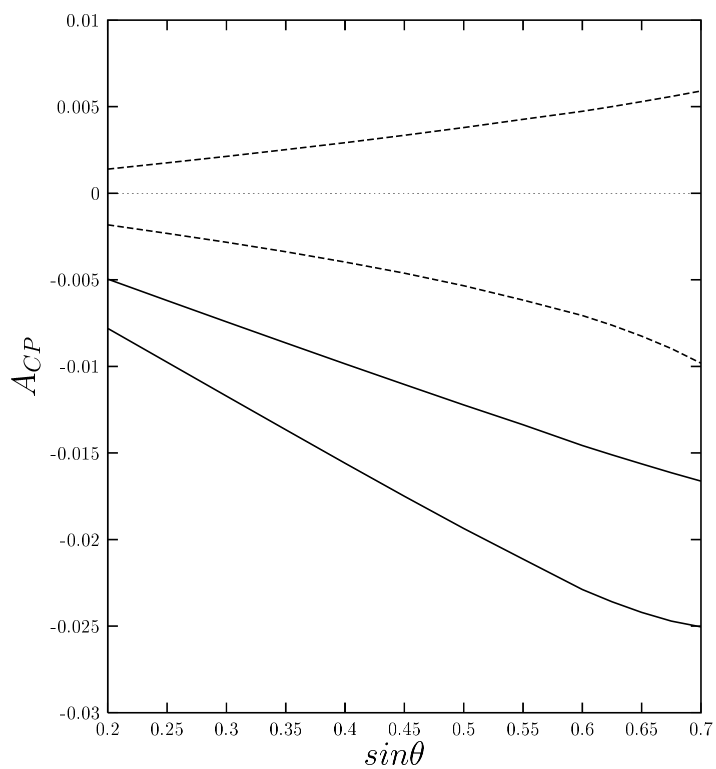

Figure 5: The same as Fig. 1 but A_{CP} as a function of $sin\theta$.

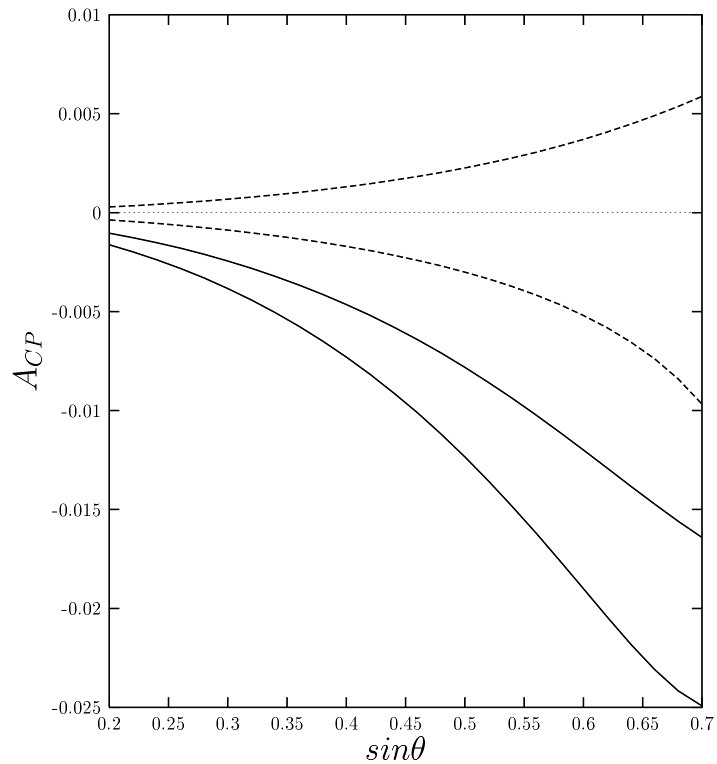

Figure 6: The same as Fig. 2 but A_{CP} as a function of $sin\,\theta$.

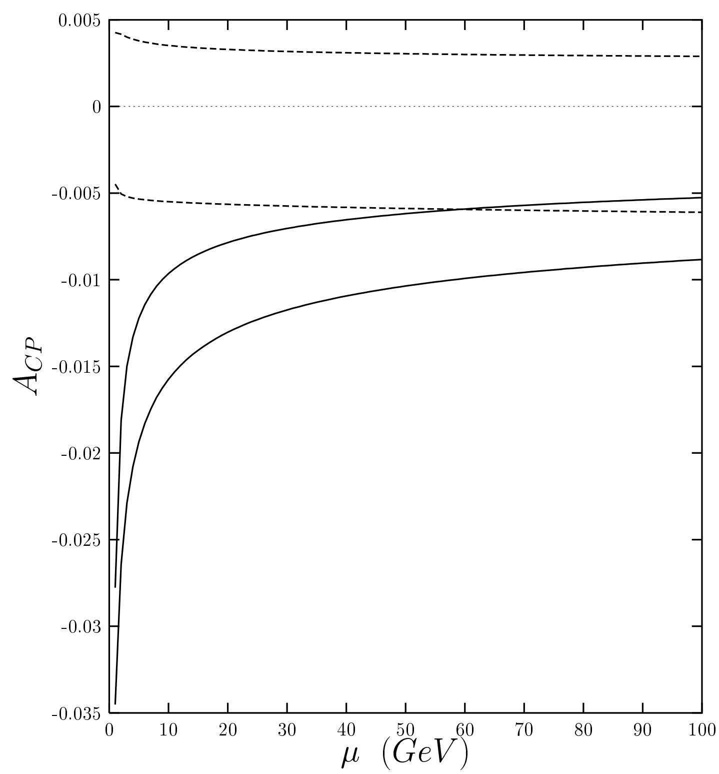

Figure 7: The same as Fig. 3 but A_{CP} as a function of μ.

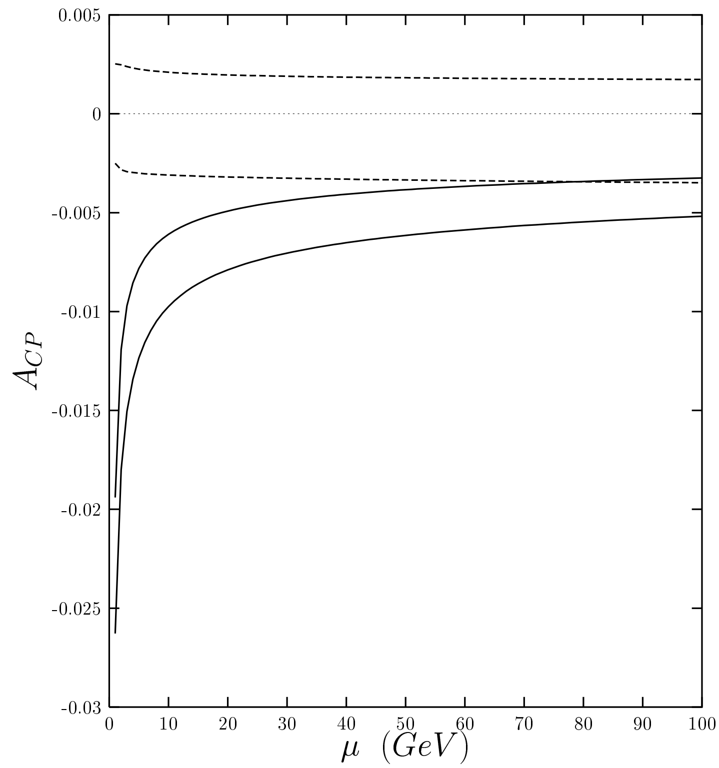

Figure 8: The same as Fig. 4 but A_{CP} as a function of μ.