Effect of the Geometric Inaccuracy in Multivariate Adaptive Regression Spline-based Inverse ECG Solution Approach

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Abstract

Spline-based approaches have been applied to inverse problems in several areas. If proper spline bases are chosen, dimension of the problem can be significantly reduced while increasing estimation accuracy and robustness of the inverse procedure. We proposed Multivariate Adaptive Regression Splines (MARS) based methods for the solution of the inverse electrocardiography (ECG) problem considering the temporal and spatial evolution of the epicardial potentials. Our model defines the spline functions in terms of spatial parameters based on the given epicardial surface geometry. Thus, any change in geometry can alter the constructed model for the purpose of obtaining an accurate estimate. In this study, we focused on the effects of the geometric model inaccuracies on the proposed MARS-based approach.

1. Introduction

Inverse electrocardiography (ECG), also known as electrocardiographic imaging (ECGI), is a non-invasive method in which the electrical activity of the heart is estimated using a large number of electrodes placed on the anterior and posterior torso surfaces, and a mathematical model that defines the relationship between torso measurements and heart's bioelectrical source model. Different from the standard 12-lead ECG, which produces scalar data changing over time, ECGI characterises the cardiac electrical activity by means of equipotential contours propagating over the heart surface, their shape and compactness, propagation patterns, magnitudes, spatial distributions and location of the extrema. Therefore, ECGI provides more comprehensive and quantitative information than the current non-invasive clinical practice and has a potential to be considered as a non-invasive, inexpensive examination technique for assessing patients with suspected cardiac abnormalities [1].

Spline-based approaches have been applied to inverse problems in several areas [2–5]. If proper spline bases are chosen, dimension of the problem can be significantly reduced while increasing estimation accuracy and robustness of the inverse procedure [5]. However, except the recently published studies [6–8], there are not many spline-based approaches proposed in the literature to solve the inverse ECG problems. Our proposed model differs from the studies presented in [6–8] such that; the underlying functional relationship between dependent and independent variables (i.e. number of spline functions in the model) do not need to be determined in advance to estimate the unknown function to be reconstructed. In our case, dependent and independent variables refer to the epicardial potentials and spatial coordinate variables respectively. The spline functions are defined in terms of spatial parameters based on the given epicardial surface geometry. We supply the maximum allowed model size, epicardial surface geometry, torso measurements and lack-of-fit criteria to the MARS algorithm as inputs, and it constructs the initial model. Then, the complexity of the model is decreased in the pruning step to obtain the best possible functional representation. The construction of spline basis collection is dependent on the given epicardial surface geometry, but the selection of those splines for the model from this collection is dependent on the defined lack-of-fit criteria. Thus, any change in geometry or measurements can alter the constructed model for the purpose of obtaining an accurate estimate. Consequently, the constructed epicardial potential distribution model may possess different functional relationships at each time instant in order to obtain a non-over-fitting model that yields minimum lack-of-fit error.

Computational electrocardiographic models based on realistic human torso include many model parameters. Because of the assumptions and simplification of the geometries, errors are associated to those model parameter [9]. In this study, we carried out simulations with modelling errors in the heart-torso geometry and evaluated the effects of these errors on the proposed MARS-based method. Estimation results were compared with the zero-order Tikhonov regularisation solutions.

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2. MARS

Multivariate Adaptive Regression Splines (MARS) is a non-parametric regression procedure that makes no specific assumption about the underlying functional relationship between the dependent and independent variables to estimate general functions of high-dimensional arguments, given sparse data [10, 11].

MARS is an adaptive procedure because the selection of the basis functions (BFs) is data-based and specific to the given problem at hand. A special advantage of MARS lies in its ability to estimate the contributions of the basis functions so that both the additive and the interactive effects of the predictors are allowed to determine the response variable [12]. MARS uses expansions in piecewise linear one-dimensional basis functions of the form $(v-\tau)_+$ and $(\tau-v)_+$, where "$(\cdot)_+$"means the positive part:

$$(v-\tau)_+ = \begin{cases} v-\tau, & \text{if } v>\tau, \\ 0, & \text{otherwise,} \end{cases} \tag{1}$$

$$(\tau-v)_+ = \begin{cases} \tau-v, & \text{if } v<\tau, \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

The relation between the input and the response in the general model is expressed as:

$$T = f(\mathbf{V}) + \varepsilon, \tag{3}$$

where T is a response variable, $\mathbf{V} = (V_1, V_2, \ldots, V_p)^T$ is a vector of predictors and ε is the additive stochastic error term in the observation with zero mean and finite variance. MARS builds reflected pairs for each input V_j $(j = 1, 2, \ldots, p)$ with p-dimensional knots $\tau_{\mathbf{i}} = (\tau_{\mathbf{i1}}, \tau_{\mathbf{i2}}, \ldots, \tau_{\mathbf{ip}})^{\mathbf{T}}$ at, or just nearby, each input data vectors $\tilde{\mathbf{v}}_i = (\tilde{v}_{i1}, \tilde{v}_{i2}, \ldots, \tilde{v}_{ip})^T$ of that input $(i = 1, 2, \ldots, N)$. Then, the collection of BFs is:

$$\varphi := \{(V_j-\tau)_+, (\tau-V_j)_+ \quad | \quad \tau \in \{\tilde{v}_{1j}, \tilde{v}_{2j}, \ldots, \tilde{v}_{Nj}\}, \; j \in \{1, 2, \ldots, p\}\}. \tag{4}$$

The fundamental idea of MARS is to use products and, then, the combination of the linear truncated basis functions to approximate the model. Thus, the functions of MARS consist of single spline functions or the product of two or more of the truncated power functions to allow for the interactions.

3. Method

In this study, we modelled the potential distribution on the epicardial surface based on MARS, using the heart geometry and the body surface measurements. Epicardial potential distribution actually is a continuous function of time, therefore methods that only exploit spatial constraints without considering the temporal evolution of potentials are not ideal. Consequently, simultaneous use of spatial and temporal constraints could improve the estimation accuracy. The proposed approach will be called ST-MARS (spatio-temporal MARS) in the rest of the paper.

We treated and modelled the potential distribution on the epicardial surface as a function $f(\mathbf{p})$ defined over a 3-dimensional epicardial surface. Consequently, the epicardial potential vector $\mathbf{x} = [x_1, x_2, \ldots, x_N]^T$ can be expressed as a collection of function values $f(\mathbf{p})$ at predefined coordinates $\mathbf{p}_i$ $(i = 1, 2, ..., N)$:

$$x_i = f(\mathbf{p}_i) \quad (\mathbf{p}_i \in \Omega). \tag{5}$$

Here, $\Omega \in \mathbb{R}^3$ denotes the 3-dimensional epicardial surface and $\mathbf{p}$ stands for coordinate vector of any point on this surface. We then write the linear inverse ECG problem as follows:

$$\mathbf{y} = \mathbf{A}\mathbf{x} + \mathbf{n}, \tag{6}$$

If we treat y_i $(i = 1, 2, \ldots, M)$ as the responses, and $\mathbf{p}_j$ $(j = 1, 2, \ldots, N)$ as the predictor values, then MARS method can be applied to estimate the function $f(\mathbf{p})$. Thus, the MARS estimate $\hat{f}(\mathbf{p})$ of the unknown function $f(\mathbf{p})$ can be written in the following form:

$$\hat{f}(\mathbf{p}) = \theta_o + \sum_{\mathbf{l}=\mathbf{1}}^{\mathbf{L}} \theta_{\mathbf{l}} \psi_l(\mathbf{p}). \tag{7}$$

Here, L is the number of basis functions in the model, ψ_l $(l = 1, 2, \ldots, L)$ are linearly independent BFs from φ or products of two or more such functions and θ_l are the unknown coefficients for the l^{th} basis function or for the constant 1 $(l = 0)$. In light of the equations given above, the i^{th} torso measurement y_i can be written as:

$$y_i = \sum_{j=1}^{N} a_{ij} \hat{f}(\mathbf{p}_j) + n_i, \tag{8}$$

If we substitute Eqn. (7) into Eqn. (8):

$$y_i = \sum_{j=1}^{N} a_{ij} \left(\theta_0 + \sum_{l=1}^{L} \theta_l \psi_l(\mathbf{p}_j) \right) + n_i. \tag{9}$$

Then, Eqn. (6) can be expressed based on spline functions and corresponding coefficients as:

$$\mathbf{y} = \mathbf{A}\mathbf{\Psi}\boldsymbol{\theta} + \mathbf{n}, \tag{10}$$

Here, $\mathbf{\Psi}$ is a matrix composed of spline bases, which are constructed based on epicardial surface geometry, $\boldsymbol{\theta}$ represents a corresponding coefficient vector, $\mathbf{A}$ is the forward transfer matrix, $\mathbf{y}$ is the torso measurements and $\mathbf{n}$ is the measurement noise. In other words, the $\mathbf{\Psi}\boldsymbol{\theta}$ term is the approximation of the unknown epicardial potential distribution over the epicardial surface.

Since the problem given in Eqn. (10) is ill-posed, the solution needs to be constrained. We proposed the following problem to estimate the spatio-temporal behaviour of the epicardial potentials.

$$\underset{\mathbf{x}}{\text{minimize}} \;\; \|\mathbf{y}-\mathbf{A}\mathbf{x}\|_{\mathbf{2}}^{\mathbf{2}}+\lambda_{\mathbf{1}}\|\mathbf{x}\|_{\mathbf{2}}^{\mathbf{2}}+\lambda_{\mathbf{2}}\|\mathbf{x}-\hat{\mathbf{x}}_{k-1}\|_{\mathbf{2}}^{\mathbf{2}}. \quad (11)$$

Here, $\hat{\mathbf{x}}_{k-1}$ is the estimated epicardial potential vector for the previous time instant, and the initial state is $\hat{\mathbf{x}}_{\text{o}} = \mathbf{0}$. $\lambda_1, \lambda_2 \geq 0$ are the corresponding regularisation parameters.

4. Results and Conclusion

The relationship between the body surface potential measurements (BSPM) and epicardial potentials given in Eq. 6 is defined by the solution of the forward ECG problem. Constructing a forward problem requires heart-torso model that contains inhomogeneities with their geometries and conductivity values. On the other hand, since heart and torso are irregular and complex surfaces, numerical methods are employed to calculate the forward problem. However, imaging modalities like CT and MRI are not perfect, so errors in the locations boundaries or size of the organs may occur. Consequently, it is important to test the robustness of the regularisation against the modelling errors.

In this part, we carried out simulations with modelling errors in the heart geometry to evaluate the performance of ST-MARS method under various circumstances. In all experiments BSPM were simulated using forward transfer matrix corresponding to exact heart-torso geometry, and by adding 30 dB SNR Gaussian noise to noise-free BSPMs. The results were compared with classical zero-order Tikhonov solutions. For comparison purposes we presented the estimation result obtained by error free model in Table 1.

In order to quantitatively compare accuracy of the inverse ECG solution, we utilised correlation coefficient (CC) that measure the similarity of potential patterns between true and estimated epicardial potentials. The correlation coefficient is defined as:

$$CC = \frac{\sum_{i=1}^{N}\left(x_i-\bar{x}_i\right)\left(\hat{x}_i-\bar{\hat{x}}_i\right)}{\sqrt{\sum_{i=1}^{N}\left(x_i-\bar{x}_i\right)^2\sum_{i=1}^{N}\left(\hat{x}_i-\bar{\hat{x}}_i\right)^2}}. \quad (12)$$

Here, x_i, $\hat{x}_i$ are the true and the estimated potentials and $\bar{x}_i$, $\bar{\hat{x}}_i$ refers to the mean potential values on the i^{th} node of the epicardial surface respectively. In this study, CC was calculated at each time instant, and then, mean and standard deviation values were obtained over time for comparison of results.

Table 1. Estimation mean correlations for error free models.

Method	Test data 1	Test data 2
ST-MARS	0.70 ± 0.24	0.73 ± 0.15
Tikhonov	0.60 ± 0.30	0.63 ± 0.21

- ***Variation in Forward Transfer Matrix:*** This simulation mimics the inaccuracy in the model by adding 30 dB SNR Gaussian noise into the forward transfer matrix.

Table 2. Estimation mean correlations if the forward transfer matrix contaminated by 30 dB SNR noise.

Method	Test data 1	Test data 2
ST-MARS	0.70 ± 0.24	0.72 ± 0.16
Tikhonov	0.52 ± 0.30	0.60 ± 0.20

- ***Variation in Size of the Heart:*** In order to observe the sensitivity of the estimations to heart size in the model, scaled heart geometry and corresponding forward transfer matrix were utilised for the inverse solution procedure.

Table 3. Estimation mean correlations for ST-MARS method for different heart sizes.

Heart size	Test data 1	Test data 2
0.7	0.65 ± 0.24	0.69 ± 0.14
0.9	0.66 ± 0.22	0.68 ± 0.12
1.2	0.64 ± 0.23	0.66 ± 0.13
1.4	0.62 ± 0.23	0.64 ± 0.14

Table 4. Estimation mean correlations for Tikhonov method for different heart sizes.

Heart size	Test data 1	Test data 2
0.7	0.47 ± 0.27	0.57 ± 0.23
0.9	0.57 ± 0.27	0.61 ± 0.19
1.2	0.57 ± 0.27	0.62 ± 0.19
1.4	0.57 ± 0.27	0.62 ± 0.19

- ***Variation in Heart Position:*** Position of the heart inside the torso is another important factor that can alter the estimation accuracy. We tested its disruptive effect for the inverse solution using forward transfer matrices corresponding to shifted heart position in the three main spatial axes.

According to the results given in Tables 1–6, our observations are as follows:

- In all test cases, mean correlation coefficients of the ST-MARS estimations were higher or at least equal to the zero order Tikhonov reconstructions.
- One of the noticeable points about the results is, while ST-MARS estimations almost were not effected by the 30 dB SNR noise imposed forward transfer matrix, Tikhonov estimation accuracy degraded especially for test data 1.

Table 5. Estimation mean correlations for ST-MARS method if heart position is shifted inside the torso.

Shift	Test data 1	Test data 2
+10 in x direction	0.58 ± 0.21	0.65 ± 0.11
+10 in y direction	0.59 ± 0.23	0.62 ± 0.13
+10 in z direction	0.59 ± 0.18	0.62 ± 0.10

Table 6. Estimation mean correlations for Tikhonov methof if heart position is shifted inside the torso.

Shift	Test data 1	Test data 2
+10 in x direction	0.52 ± 0.23	0.59 ± 0.17
+10 in y direction	0.57 ± 0.27	0.62 ± 0.19
+10 in z direction	0.57 ± 0.27	0.62 ± 0.19

- Heart size caused degradation in both method's estimations accuracies as we expected. While the Tikhonov method was significantly affected by the smaller heart size, ST-MARS performance reduced because of extended heart size.
- The position of the heart more severely decreased the accuracy of the ST-MARS method compared to the Tikhonov method.

5. Discussions and Future Works

In this study we examined the effects of geometric uncertainties on the proposed ST-MARS methods. Except the disturbance on the heart position, ST-MARS method seems to be more robust against the geometric errors compared to the zero-order Tikhonov method. Our study is continuing using larger set of experimental data to comprehensively understand and investigate the effects of geometric uncertainties in our approach, such as wave-front construction and locating pacing site.

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