Quantum effects in the diffusion process to form a heavy nucleus in heavy-ion fusion reactions

Kouhei Washiyama[*], Bülent Yilmaz[†,**], Sakir Ayik[‡] and
Noboru Takigawa[*]

[*]*Department of Physics, Tohoku University, 980-8578 Sendai, Japan*
[†]*Physics Department, Ankara University, TR-06100 Ankara, Turkey*
[**]*Physics Department, Middle East Technical University, TR-06531 Ankara, Turkey*
[‡]*Physics Department, Tennessee Technological University, Cookeville, Tennessee 38505, USA*

Abstract. We discuss quantum effects in the diffusion process which is used to describe the shape evolution from the touching configuration of fusing two nuclei to a compound nucleus. Applying the theory with quantum effects to the case where the potential field, the mass and friction parameters are adapted to realistic values of heavy-ion collisions, we show that the quantum effects play significant roles at low temperatures which are relevant to the synthesis of superheavy elements.

Keywords: Quantum diffusion, Non-Markovian effect, Langevin equation, Superheavy elements
PACS: 25.70.Jj, 02.50.Ga, 05.40.-a, 05.60.Gg

INTRODUCTION

It is now well accepted that it is not sufficient for the two nuclei in heavy-ion collisions to overcome the Coulomb barrier to form a heavy compound nucleus such as superheavy elements. This is because the conditional saddle, which should be overcome for two nuclei to fuse, is located inside the Coulomb barrier for collisions between two heavy nuclei. This provides an origin of the so-called fusion hindrance phenomena [1]. We thus need to describe the shape evolution from the touching configuration of fusing two nuclei to a more compact spherical-like compound nucleus by overcoming a potential barrier near the conditional saddle point.

A diffusion model has been applied to describing this process, especially to describing the formation of superheavy elements [2, 3, 4]. In these studies, so far the standard fluctuation-dissipation relation which holds at high temperatures has been postulated to relate the diffusion coefficients to the friction coefficients. Although these studies provide some illuminating information and look to be successful to some extent in the data analysis, one needs to carefully examine the validity of the standard fluctuation-dissipation relation in order to apply to the diffusion process at low temperatures which are relevant to the synthesis of superheavy elements. Since superheavy elements are stabilized by shell correction energies, one has to synthesize them at reasonably low excitation energies, that is, at low temperatures as low as 1 MeV or below. On the other hand, the barrier curvature around the conditional saddle point is also of the order of 1 MeV. It is thus likely that quantum effects play an important role in the compound nucleus formation process, especially in the synthesis of superheavy elements.

One can find a diffusion theory with quantum effects in some literatures. However,

most of them handle the quantum diffusion process in a potential well. To the contrary, our problem is the quantum diffusion along a potential barrier. In order to adapt to this situation, i.e., to the diffusion process along a potential barrier, especially at low temperatures, we have developed a quantum diffusion theory that takes the quantum fluctuation due to the finite curvature of the potential barrier into account [5]. Our theory incorporates also a memory effect. In Ref. [5], using a simplified model for the potential barrier, mass and friction parameters, we reported that the quantum effects, especially memory effects, enhance the probability of overcoming the barrier to form a compound nucleus compared with that calculated by assuming the standard fluctuation-dissipation theorem at low temperatures and for the potential curvature relevant to the synthesis of superheavy elements. In Ref. [6], we developed a Langevin equation version of the quantum diffusion theory. Also, we reformulated so as to introduce the dissipation effect in a way more suitable to nuclear processes than the Caldeira-Leggett model adopted in Ref. [5].

In this contribution we discuss quantum effects in the compound nucleus formation process with more realistic parameters of the potential, mass, and friction and show that these effects enhance the compound nucleus formation probability at low temperatures compared with that by the classical diffusion theory using the standard fluctuation-dissipation relation.

QUANTUM DIFFUSION THEORY

In this section, we explain two aspects of the quantum effects.

The first is that the connection between the diffusion and friction coefficients is modified from the well known fluctuation-dissipation theorem at high temperatures due to the quantum fluctuation originating from the finite barrier curvature. For the diffusion process in a potential well, it is known that the ratio of the diffusion to the friction coefficients is given by

$$\frac{D}{\gamma} = \frac{1}{2}\hbar\Omega\coth(\frac{\hbar\Omega}{2T}) \tag{1}$$

if the quantum fluctuation is taken into account. In Eq. (1) D, γ, and T are the diffusion and friction coefficients and the temperature, respectively. The Ω is defined by

$$\Omega = \sqrt{\frac{V''(R_b)}{M}}, \tag{2}$$

where $V''(R_b)$ and M are the second derivative of the potential well at the bottom position of the potential R_b and the mass parameter, respectively. The relevant formula for the diffusion along a potential barrier such as the diffusion process around the conditional saddle point can be obtained by analytic continuation of Eq. (1) with respect to the frequency parameter Ω [5, 7, 8]. The result reads,

$$\frac{D}{\gamma} = \frac{1}{2}\hbar\Omega\cot(\frac{\hbar\Omega}{2T}), \tag{3}$$
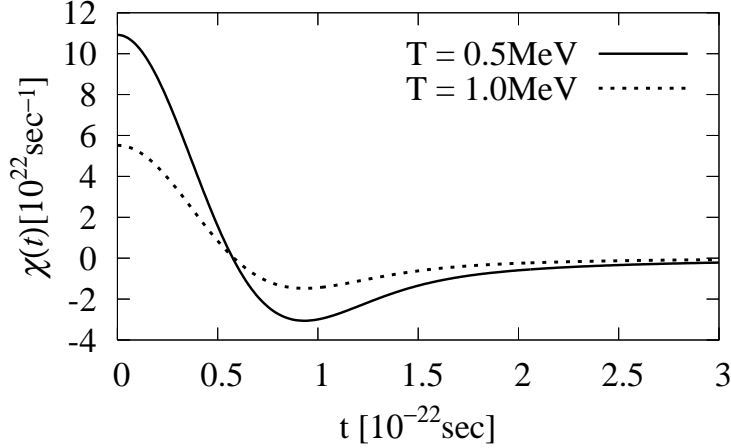
FIGURE 1.   Correlation function as a function of time.

with

$$\Omega = \sqrt{\frac{|V''(R_B)|}{M}}, \tag{4}$$

where $V''(R_B)$ is the second derivative of the potential at the barrier top position R_B.

One can easily confirm that both formulas, Eq. (1) and Eq. (3), reduce to the classical fluctuation-dissipation theorem, $D/\gamma = T$, in the high temperature limit where the thermal fluctuation far dominates the quantum fluctuation.

The second quantum effect is the non-Markovian effect, which leads to a colored noise problem. The time correlation function is given by [5, 6, 9]

$$\langle R(t)R(t') \rangle = 2\gamma T \cdot \chi(t - t'), \tag{5}$$

$$\chi(t - t') = \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} e^{-i\omega(t - t')} \frac{\hbar\omega}{2T} \coth \frac{\hbar\omega}{2T} \cdot C(\omega), \tag{6}$$

to define the property of the random force $R(t)$. The $C(\omega)$ is the cutoff function in the spectral density of the heat bath which corresponds to the subspace of nuclear intrinsic degrees of freedom in heavy-ion collisions. We employ the Gaussian form for the cutoff function, $C(\omega) = \exp\left[-(\hbar\omega)^2/2\Delta^2\right]$. We can see from Eq. (6) that the correlation function reduces to its classical Markovian form, $\chi(t - t') \rightarrow \delta(t - t')$, if the temperature is higher than the cutoff energy Δ and if the cutoff energy is sufficiently high. To the contrary, at low temperatures, the quantum colored noise property of the random force needs to be seriously considered.

In Fig. 1, we show the correlation function as a function of time for two temperatures, $T = 0.5$ MeV (the solid line) and 1.0 MeV (the dotted line). The cutoff energy has been fixed to be $\Delta = 15$ MeV as in Ref. [6]. At these temperatures, the non-Markovian effect is significant.
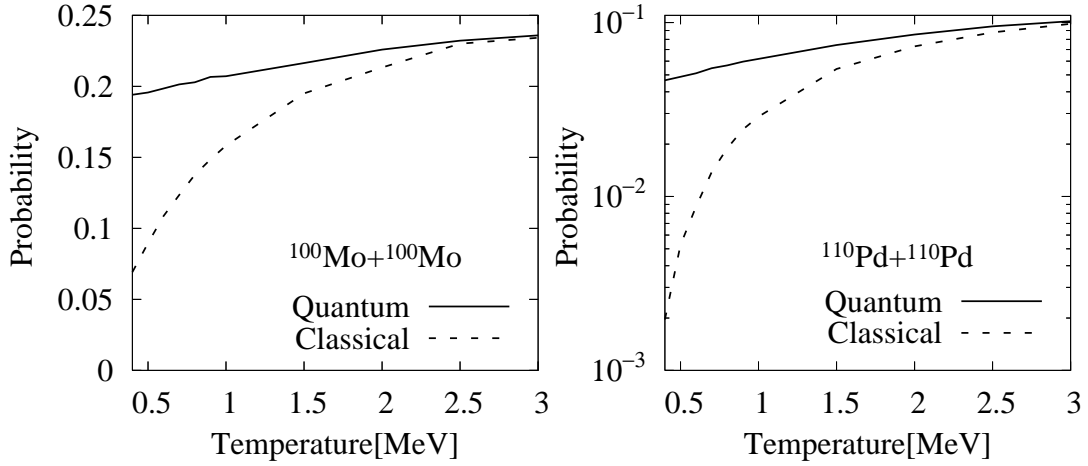
FIGURE 2. The compound nucleus formation probability as a function of nuclear temperature. The systems are ^{100}Mo $+^{100}$Mo (the left panel) and ^{110}Pd $+^{110}$Pd (the right panel) reactions. Note that different scales are used for the ordinates in the right and left figures.

RESULT

In applying our theory to the compound nucleus formation process we use the liquid drop model [10] to calculate the potential energy surface in the space of nuclear deformation, the hydrodynamical mass [11] for the mass parameter, and the one-body dissipation [12] for the friction tenser. The colored noise random force is handled by the spectral method given in Ref. [13]. We choose the ^{100}Mo $+ ^{100}$Mo and ^{110}Pd $+ ^{110}$Pd systems, whose experimental fusion cross sections are available, to examine the quantum effects on the probability of overcoming the conditional saddle to form a compound nucleus. We use the separation distance between two fragments to describe the dynamics from inside the Coulomb barrier to inside the conditional saddle and determine its time evolution by solving the Langevin equation for a single macroscopic variable one hundred thousands times. The other macroscopic degrees of freedom in the two center shell model parametrization [14] are frozen during the compound nucleus formation process as; the mass partition parameter $\alpha = 0$, the deformation parameter $\delta_1 = \delta_2 = 0$, and the neck parameter $\varepsilon = 0.9$. We initiate each trajectory from the touching configuration of the two fusing nuclei with zero momentum. This corresponds to assuming that a strong energy dissipation from the macroscopic motion, i.e., the relative motion between the fusing two fragments, to nuclear intrinsic motions takes place inside the Coulomb barrier. For simplicity, we ignore the change of the temperature of nuclear intrinsic degrees of freedom during the time evolution of the system.

Figure 2 compares the compound nucleus formation probability as a function of nuclear temperature calculated by the quantum diffusion theory and by the classical diffusion theory which postulates the standard fluctuation-dissipation relation for the ^{100}Mo $+^{100}$Mo (the left panel) and for the ^{110}Pd $+^{110}$Pd (the right panel) reactions. The solid lines are the results of the quantum diffusion theory, while the dashed lines the classical diffusion theory. These figures show that quantum effects become significant at

low temperatures relevant to the experiments to synthesize superheavy elements. They increase the compound nucleus formation probability at low temperatures.

SUMMARY

We discussed quantum effects in the formation process of a heavy compound nucleus described as a diffusion process along a potential barrier. We have shown that the quantum effects increase the compound nucleus formation probability at low excitation energies, which are relevant to the synthesis of superheavy elements.

Further developments are needed in various aspects to apply the theory to more realistic problems. One of the essential developments is to generalize the present model which explicitly handles only one macroscopic variable, i.e., the relative distance between the colliding fragments, to the diffusion process in a multidimensional space by taking, for instance, the mass partition into account. This will be crucial to discuss the competition between the complete fusion and quasi-fission by including quantum effects.

ACKNOWLEDGMENTS

REFERENCES

1. W. J. Swiatecki, *Physica Scripta*, **24**, 113–122 (1981).
2. Y. Aritomo, T. Wada, M. Ohta, and Y. Abe, *Phys. Rev. C* **59**, 796–809 (1999).
3. C. Shen, G. Kosenko and Y. Abe, *Phys. Rev. C* **66**, 061602(R)1–5 (2002).
4. Y. Aritomo and M. Ohta, *Nucl. Phys.* **A744**, 3–14 (2004).
5. N. Takigawa, S. Ayik, K. Washiyama, and S. Kimura, *Phys. Rev. C* **69**, 054605–1–5 (2004).
6. S. Ayik, B. Yilmaz, A. Gokalp, O. Yilmaz, and N. Takigawa, *Phys. Rev. C* **71**, 054611–1–8 (2005).
7. H. Hofmann, *Phys. Rep.* **284**, 137–380 (1998).
8. C. Rummel and H. Hofmann, *Nucl. Phys.* **A727**, 24–40 (2003).
9. A. O. Caldeira and A. J. Leggett, *Physica* **121A**, 587–616 (1983).
10. H. J. Krappe, J. R. Nix, and A. J. Sierk, *Phys. Rev. C* **20**, 992–1013 (1979).
11. K. T. R. Davies, A. J. Sierk, and J. R. Nix, *Phys. Rev. C* **13**, 2385–2412 (1976).
12. J. Blocki, Y. Boneh, J.R. Nix, J. Randrup, M. Robel, A.J. Sierk and W.J. Swiatecki, *Ann. Phys.* **113**, 330–386 (1978).
13. A. H. Romero and J. M. Sancho, *J. Comp. Phys.*, **156**, 1–11 (1999).
14. J. Maruhn and W. Greiner, *Z. Phys.* **251**, 431–457 (1972).