THE STRONG COUPLING CONSTANTS OF EXCITED POSITIVE PARITY HEAVY MESONS IN LIGHT CONE QCD

T. M. ALIEV *, N. K. PAK and M. SAVCI Physics Department, Middle East Technical University 06531 Ankara, Turkey

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Abstract

We calculate the strong coupling constants $g_{P^{**}P^*\pi}$, where P^{**} (D^{**}, B^{**}) is the 1^+ p-wave state, in the framework of the light cone QCD sum rules, and using these values of $g_{P^{**}P^*\pi}$, we compute the hadronic decay widths for $D^{**} \to D^* \pi$ and $B^{**} \to B^* \pi$.

^{*}Permanent address: Institute of Physics, Azerbaijanian Academy of Sciences, Baku, Azerbaijan

1 Introduction

The main goal of the future $c - \tau$ and B - meson factories is a deeper and more comprehensive investigation of the properties of the heavy mesons, containing charm and beauty quarks. In particular, search for the excited states of D and B mesons and their decay modes constitute one of the main research program of the above mentioned factories, and can play an essential role for understanding the dynamics of the excited states.

In general, for the interpretation of the experimental data from heavy meson physics, we need to know the large distance, i.e., nonperturbative effects. For example, for the exclusive decays which can easily be measured experimentally, we need a more accurate estimation of the form factors and other hadronic matrix elements that are described by the long distance effects, thus one needs a method which takes into account the nonperturbative (long distance) effects. Among the different approaches in estimating the large distance effects, the QCD sum rule method [1] occupies a special place, since this method is based on the first principles of QCD and the fundamental QCD Lagrangian.

In this work we use a version of the QCD sum rule, namely the light cone QCD sum rule. This method is based on the Wilson Expansion of the T-product of currents near light cone, in terms of different non-local operators. These operators are characterized by their twists rather than their dimensions. Matrix elements of the non-local operators in the variable external field are identified with a set of the wave functions of increasing twist, and replace the vacuum expectation value of local operators that appear in the traditional sum rule method. The form of wave functions are restricted by the conformal invariance of QCD. More about the details of this method and its applications can be found elsewhere in the literature [2]-[19].

In this article this method is used for the calculation of the strong coupling constants $g_{P^{**}P^*\pi}$ of the excited positive parity meson decays, where $P^{**}(P^*)$ is the $1^+(1^-)$ meson state. In sect.2 we derive the sum rule for the $g_{P^{**}P^*\pi}$ coupling constants. Sect.3 is devoted to the numerical analysis, where we also compute the widths of the 1^+ meson decay to $P^*\pi$ state.

2 Light Cone Sum Rule for $g_{P^{**}P^*\pi}$ Coupling Constant

According to the general strategy of QCD sum rule method, the coupling constant $g_{P^{**}P^*\pi}$ can be calculated by equating the representations of a suitable correlator in hadronic and quark-gluon languages. For this aim we consider the following correlator:

$$\Pi_{\mu\nu} = i \int d^4x \ e^{ip_1x} \ \langle \pi(q) | T\left\{ \bar{d}(x)\gamma_{\mu}Q(x)\bar{Q}(0)\gamma_{\nu}\gamma_5 u(0) \right\} | 0 \rangle \ . \tag{1}$$

Here $\bar{d}\gamma_{\mu}Q(\bar{Q}\gamma_{\nu}\gamma_{5}u)$ is the interpolating current for $1^{-}(1^{+})$ meson state, Q is a heavy quark (charm quark for D meson, and beauty quark for B meson case), p_{1} is the momentum of the 1^{-} meson. When the pion is on the mass shell $q^{2} = m_{\pi}^{2}$, the correlator function (1) depends on two invariants, p^{2} and p_{1}^{2} . In what follows we set $m_{\pi} = 0$.

First consider the physical (hadronic) representation of (1). Physical part of it can be expressed in terms of the contribution of the lowest lying resonances P^{**} and P^* in the corresponding channels

$$\Pi_{\mu\nu} = \langle \pi(q) P^*(p_1) | P^{**}(p) \rangle \; \frac{\langle P^* | \bar{d}\gamma_\mu Q | 0 \rangle}{p_1^2 - m_{P^*}^2} \; \frac{\langle 0 | \bar{Q}\gamma_\nu \gamma_5 u | P^{**} \rangle}{p^2 - m_{P^{**}}^2} \; . \tag{2}$$

The matrix elements entering in eq.(2) are defined in the standard manner:

$$\langle P^* | \bar{d} \gamma_{\mu} Q | 0 \rangle = m_{P^*} f_{P^*} \epsilon_{\mu}(p_1) ,$$

$$\langle 0 | \bar{Q} \gamma_{\nu} \gamma_5 u | P^{**} \rangle = m_{P^{**}} f_{P^{**}} \epsilon_{\nu}^{(1)}(p) ,$$
 (3)

where m_{P^*} $(m_{P^{**}})$, f_{P^*} $(f_{P^{**}})$ and ϵ_{μ} $(\epsilon_{\nu}^{(1)})$ are the mass, leptonic decay constant, and the polarization vector of the vector $1^ (1^+)$ meson state. In general, the matrix element $\langle \pi P^* | P^{**} \rangle$ can be written as

$$\langle \pi(q)P^*(p_1)|P^{**}(p)\rangle = F_0(\epsilon^*\epsilon^{(1)}) + F_1(q\epsilon)(q\epsilon^{(1)})$$
 (4)

In obtaining eq.(4) we have used the transversality condition, $p_1 \epsilon = p \epsilon^{(1)} = 0$, and $p_1 = p - q$. The second term in eq.(4) gives negligible contribution to the decay width $P^{**} \to P^* \pi$, in comparison to the first term, since it is proportional to

$$\left(m_{\pi}^2 + \frac{\Delta^4}{m_{P^{**}}^2 m_{P^*}^2}\right) \;,$$

where $\Delta = m_{P^{**}} - m_{P^*}$. Therefore we shall neglect the second term in (4) and set $g_{P^{**}P^*\pi} \equiv F_0$. Using eqs.(2), (3), and (4) for the physical part, we get

$$\Pi_{\mu\nu} = \frac{m_{P^*}m_{P^{**}}f_{P^*}f_{P^{**}}g_{P^{**}P^*\pi}}{(p^2 - m_{P^{**}}^2)(p_1^2 - m_{P^*}^2)} \left\{ g_{\mu\nu} - \frac{p_{\mu}p_{\nu}}{m_{P^{**}}^2} - \frac{p_{1\mu}p_{1\nu}}{m_{P^*}^2} - \frac{(p_1p)p_{1\mu}p_{\nu}}{m_{P^*}^2} \right\} .$$
(5)

At this point we would like to make the following comment. Since the vector current $\bar{q}\gamma_{\mu}Q$ is not conserved, it also couples to $J^P = 0^+$ scalar mesons P_0 as well as the $J^P = 1^-$ vector mesons. Therefore the P_0 contribution should be taken into account in the sum rule and this addition introduces further uncertainities. In order to avoid the 0^+ meson contributions we must choose a structure that does not contain its contribution.

Noting that the corresponding transition matrix element is given as

$$\langle 0|\bar{q}\gamma_{\mu}Q|P_{0}\rangle = f_{D^{0}}m_{D^{0}}p_{1\mu} ,$$

that is, only the structure $\sim p_{1\mu}$ contains the 0⁺ meson contribution, it follows from eq.(5) that we have only two structures, namely $\sim g_{\mu\nu}$ and $p_{\mu}p_{\nu}$, which do not contain the $J^P = 0^+$ meson contribution. In our analysis we choose the structure $\sim g_{\mu\nu}$. We also perform calculations for the structure $p_{\mu}p_{\nu}$ and the final results of both structures in predicting of $g_{P^{**}P^*\pi}$ are practically the same.

After performing double Borel transformation over variables $-p^2$ and $-p_1^2$ (see eq.(5)) for the physical part of the sum rule for $g_{P^{**}P\pi}$, for the structure of $g_{\mu\nu}$, we get :

$$\Pi^{phys} = \frac{1}{M_1^2 M_2^2} g_{P^{**}P^*\pi} f_{P^{**}} f_{P^*} m_{P^{**}} m_{P^*} e^{\left(-\frac{m_{P^{**}}^2 + m_{P^*}^2}{2M^2}\right)} .$$
(6)

Now we turn out attention to the theoretical part of (1). In this calculation we will use the notation of the work [16]. After a lengthy calculation we get (after double Borel transformation over $-p^2$ and $-p_1^2$)

$$\Pi^{theor} = f_{\pi} \frac{1}{M_{1}^{2} M_{2}^{2}} e^{-\frac{m^{2}}{M^{2}}} \left[\frac{m_{\pi}^{2}}{m_{u} + m_{d}} m_{Q} M^{2} \varphi_{P}(u_{0}) + 2m_{Q}^{2} g_{2}(u_{0}) - \frac{1}{2} M^{4} \varphi_{\pi}'(u_{0}) + 2(M^{2} + m_{Q}^{2})(g_{1}'(u_{0}) + G_{2}'(u_{0})) + \frac{1}{2} M^{2} \left[2 \int_{0}^{u_{0}} d\alpha_{1} \int_{u_{0} - \alpha_{1}}^{1 - \alpha_{1}} d\alpha_{3} \frac{\varphi_{\parallel}(\alpha_{1}, 1 - \alpha_{1} - \alpha_{3}, \alpha_{3})}{\alpha_{3}^{2}} + \int_{0}^{u_{0}} \frac{d\alpha_{3}}{\alpha_{3}} \left\{ \tilde{\varphi}_{\parallel}(u_{0} - \alpha_{3}, 1 - u_{0}, \alpha_{3}) - \varphi_{\parallel}(u_{0} - \alpha_{3}, 1 - u_{0}, \alpha_{3}) \right\} - \int_{0}^{1} \frac{d\alpha_{3}}{\alpha_{3}} \left\{ \tilde{\varphi}_{\parallel}(u_{0}, 1 - u_{0}, \alpha_{3}) + \varphi_{\parallel}(u_{0}, 1 - u_{0} - \alpha_{3}, \alpha_{3}) \right\} \right] + (\text{continuum contribution}) .$$

$$(7)$$

The pion wave functions $\varphi_{\pi}(u)$, $\varphi_{P}(u)$, $g_{1}(u)$ and $G_{2}(u)$, have the twists $\tau = 2$, $\tau = 3$, $\tau = 4$ and $\tau = 4$ respectively, and they appear in the matrix elements of nonlocal quark operators as shown below (see [6] and [16]):

$$\langle \pi(q) | \bar{d} \gamma_{\mu} \gamma_{5} u(0) | 0 \rangle = -i f_{\pi} q_{\mu} \int_{0}^{1} du \ e^{iqux} \Big[\varphi_{\pi}(u) + x^{2} g_{1}(u) + \mathcal{O}(x^{4}) \Big]$$

$$+ f_{\pi} \left(x_{\mu} - \frac{x^{2} q_{\mu}}{qx} \right) \int_{0}^{1} du \ e^{iqux} g_{2}(u) ,$$
 (8)

$$\langle \pi(q) | \bar{d}i\gamma_5 u(0) | 0 \rangle = \frac{f_\pi m_\pi^2}{m_u + m_d} \int_0^1 du \ e^{iqux} \varphi_P(u) \ , \tag{9}$$

$$G_2(u) = -\int_0^u g_2(v)dv \ . \tag{10}$$

The functions $\varphi(\alpha_i)$ and $\tilde{\varphi}(\alpha_i)$ are the twist-4 wave functions and are defined in the following way:

$$\langle \pi(q) | \bar{d}(x) \gamma_{\mu} \gamma_{5} g_{s} G_{\alpha\beta}(ux) u(0) | 0 \rangle = f_{\pi} \left[q_{\beta} \left(g_{\alpha\mu} - \frac{x_{\alpha} q_{\mu}}{qx} \right) - q_{\alpha} \left(g_{\beta\mu} \frac{x_{\beta} q_{\mu}}{qx} \right) \right] \int \mathcal{D}\alpha_{i} \varphi_{\perp}(\alpha_{i}) e^{iqx(\alpha_{1} + u\alpha_{3})} + f_{\pi} \frac{q_{\mu}}{qx} \left(q_{\alpha} x_{\beta} - q_{\beta} x_{\alpha} \right) \int \mathcal{D}\alpha_{i} \varphi_{\parallel}(\alpha_{i}) e^{iqx(\alpha_{1} + u\alpha_{3})} ,$$

$$(11)$$

$$\begin{aligned} &\langle \pi(q) | \bar{d}(x) \gamma_{\mu} g_{s} \tilde{G}_{\alpha\beta}(ux) u(0) | 0 \rangle = \\ &i f_{\pi} \left[q_{\beta} \left(g_{\alpha\mu} - \frac{x_{\alpha} q_{\mu}}{qx} \right) - q_{\alpha} \left(g_{\beta\mu} \frac{x_{\beta} q_{\mu}}{qx} \right) \right] \int \mathcal{D} \alpha_{i} \tilde{\varphi}_{\perp}(\alpha_{i}) e^{i q x (\alpha_{1} + u \alpha_{3})} \\ &+ i f_{\pi} \frac{q_{\mu}}{qx} \left(q_{\alpha} x_{\beta} - q_{\beta} x_{\alpha} \right) \int \mathcal{D} \alpha_{i} \tilde{\varphi}_{\parallel}(\alpha_{i}) e^{i q x (\alpha_{1} + u \alpha_{3})} , \end{aligned}$$
(12)

and

$$\widetilde{G}_{\alpha\beta} = \frac{1}{2} \epsilon_{\alpha\beta\sigma\lambda} G^{\sigma\lambda}, \text{ and}$$

 $\mathcal{D}\alpha_i = d\alpha_1 d\alpha_2 d\alpha_3 \delta(1 - \alpha_1 - \alpha_2 - \alpha_3).$

In (7) we set

$$M^2 = \frac{M_1^2 M_2^2}{M_1^2 + M_2^2}$$
, $u_0 = \frac{M_1^2}{M_1^2 + M_2^2}$ and $\varphi' = \frac{d\varphi}{du}|_{u=u_0}$.

We omitted the path-ordered factor $Pe^{[ig_s \int_0^1 du \ x^{\mu} A_{\mu}(ux)]}$, since in the Fock-Schwinger gauge $x^{\mu}A_{\mu} = 0$, it is trivial. In numerical calculations, we choose the symmetric point $u_0 = \frac{1}{2}$ which means that quark and antiquark have equal momenta inside the pion. At this point the subtraction of the continuum can be done by substituting

$$e^{-\frac{m_Q^2}{M^2}} \to e^{-\frac{m_Q^2}{M^2}} - e^{-\frac{s_0^2}{M^2}}$$
, (13)

at least for the twist-3 contribution [16]. However, we use this substitution everywhere in (7) since higher twist contributions are negligible. Equating eqs. (6) and (7) and using (13) we finally obtain the following sum rule for the strong coupling constant $g_{P^{**}P\pi}$:

,

$$g_{P^{**}P\pi}f_{P^{**}}f_{P^{*}} = \left(\frac{1}{m_{P^{**}}m_{P^{*}}}\right)e^{\left(\frac{m_{P^{**}}^{2}+m_{P^{*}}^{2}}{2M^{2}}\right)}f_{\pi}M^{2}\left(e^{-\frac{m_{Q}^{2}}{M^{2}}}-e^{-\frac{s_{0}^{2}}{M^{2}}}\right)\times \left[\frac{m_{\pi}^{2}}{m_{u}+m_{d}}m_{Q}\varphi_{P}(u_{0})+\frac{2m_{Q}^{2}}{M^{2}}g_{2}(u_{0})-\frac{1}{2}M^{2}\varphi_{\pi}'(u_{0})\right. \\ \left.+2\left(1+\frac{m_{Q}^{2}}{M^{2}}\right)\left(g_{1}'(u_{0})+G_{2}'(u_{0})\right)\right. \\ \left.+\frac{1}{2}\left[2\int_{0}^{u_{0}}d\alpha_{1}\int_{u_{0}-\alpha_{1}}^{1-\alpha_{1}}d\alpha_{3}\frac{\varphi_{\parallel}(\alpha_{1},1-\alpha_{1}-\alpha_{3},\alpha_{3})}{\alpha_{3}^{2}}\right. \\ \left.+\int_{0}^{u_{0}}\frac{d\alpha_{3}}{\alpha_{3}}\left\{\tilde{\varphi}_{\parallel}(u_{0}-\alpha_{3},1-u_{0},\alpha_{3})-\varphi_{\parallel}(u_{0}-\alpha_{3},1-u_{0},\alpha_{3})\right\}\right] \right].$$
(14)

From (14) it follows that, for the calculation of the value of the strong coupling constant $g_{P^{**}P^*\pi}$, we need to know the leptonic decay constants of the P^{**} and P^* , $f_{P^{**}}$, and f_{P^*} . The decay constant f_{P**} can be obtained from the two point sum rules:

$$f_{P^{**}}^2 m_{P^{**}}^2 = \frac{1}{8\pi^2} \int_{m_Q^2}^{s_0} dse^{\left(\frac{m_{P^{**}}^2 - s}{M^2}\right)} \frac{(s - m_Q^2)^2}{s} \left(2 + \frac{m_Q^2}{s}\right) + m_Q \langle \bar{q}q \rangle e^{\left(\frac{m_{P^{**}}^2 - m_Q^2}{M^2}\right)} \left(1 - \frac{m_0^2 m_Q^2}{4M^2}\right) , \qquad (15)$$

where

$$m_0^2 = \frac{\langle \bar{q} \sigma_{\alpha\beta} G^{\alpha\beta} q \rangle}{\langle \bar{q} q \rangle} = (0.8 \pm 0.2) \ GeV^2 \ .$$

To obtain f_{P^*} it is necessary to make the following replacements: $m_{P^{**}} \to m_{P^*}$ and change the sign in front of the second term in (15) (in this case, of course the value of the continuum threshold must also change). Note that we do not take into account the perturbative $\mathcal{O}(\alpha_s)$ corrections in (15), as they are not included in (14) either. Note that the values of the decay constants f_{B^*} and f_{D^*} we use in our calculations are given in [16].

3 Numerical Analysis

For the numerical analysis of the QCD sum rule (14) we first give the values of the input parameters:

$$\begin{split} f_{\pi} &= 132 \ MeV, \quad \frac{m_{\pi}^2}{m_u + m_d} (1 \ GeV) = 1.65 \ GeV, \quad m_c = 1.3 \ GeV, \\ m_b &= 4.7 \ GeV, \quad m_{D^{**}} = 2.420 \ GeV, \quad m_{D^*} = 2.01 \ GeV, \\ m_{B^*} &= 5.279 \ GeV, \quad m_{B^{**}} = 5.732 \ GeV, \\ (s_0)_D &= 6 \div 8 \ GeV^2, \quad (s_0)_B = 35 \div 40 \ GeV^2 \ . \end{split}$$

Using these parameters, from (15) for the leptonic decay constants $f_{D^{**}}$ and $f_{B^{**}}$ we get

$$f_{D^{**}} = (300 \pm 30) \ MeV \ , \tag{16}$$

$$f_{B^{**}} = (200 \pm 20) \ MeV \ . \tag{17}$$

Sum rule for $g_{P^{**}P^*\pi}$ contains the nonperturbative quantities, namely the wave functions. In our numerical analysis we use the wave functions proposed in [6] (see also [16]). Having the values of the input parameters, one must find the region of Borel parameter M^2 , for which the sum rule eq.(14) is reliable. The lowest value of M^2 is usually fixed by imposing the condition that the terms proportional to the $\frac{1}{M^2}$ are resonably small. The upper bound for M^2 is usually fixed by the condition that the continuum and higher states contributions constitute about (25 ÷ 30%) of the ground resonance contribution. Under these conditions the fiducial range of M^2 for B(D) case turns out to be $8 \ GeV^2 < M^2 < 20 \ GeV^2$ (2 $GeV^2 < M^2 < 6 \ GeV^2$). Using the values of the input parameters we get

$$f_{B^{**}}f_{B^*}g_{B^{**}B^*\pi} = (0.78 \pm 0.12) \ GeV^3 , \qquad (18)$$

$$f_{D^{**}} f_{D^*} g_{D^{**}D^*\pi} = (0.68 \pm 0.10) \ GeV^3 \ . \tag{19}$$

Dividing this product by the decay constants, we finally obtain for the $D^{**}D^*\pi$ ($B^{**}B^*\pi$) coupling constants :

$$g_{B^{**}B^*\pi} = 24 \pm 3 \ GeV \ , \tag{20}$$

$$g_{D^{**}D^*\pi} = 10 \pm 2 \ GeV \ . \tag{21}$$

Substituting these values in the expressions for the decay widths,

$$\Gamma(P^{**0} \to P^{*+} \pi^{-}) = \frac{g_{P^{**}P^{*}\pi}^{2}}{24\pi} \left(2 + \frac{(m_{P^{**}}^{2} + m_{P^{*}}^{2})^{2}}{4m_{P^{**}}^{2}m_{P^{*}}^{2}} \right) \times \frac{\left[\left\{ m_{P^{**}}^{2} - (m_{P^{*}} + m_{\pi})^{2} \right\} \left\{ m_{P^{**}}^{2} - (m_{P^{*}} - m_{\pi})^{2} \right\} \right]^{\frac{1}{2}}}{2m_{P^{**}}^{3}}$$
(22)

we get,

$$\Gamma(D^{**0} \to D^{*+} \pi^{-}) \simeq 249 \ MeV ,$$

 $\Gamma(B^{**0} \to B^{*+} \pi^{-}) \simeq 296 \ MeV .$
(23)

Strong coupling constants (and correspondingly the decay widths) of the decays $P^{**-} \rightarrow P^{*0} \pi^-$, $P^{**-} \rightarrow P^{*-} \pi^0$ and $P^{**0} \rightarrow P^{*0} \pi^0$ can easily be obtained from $P^{**0} \rightarrow P^{*+} \pi^-$ with the help of the isotopic invariance.

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