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Neutrino Masses in Effective Rank-5 Subgroups of E_6 I: Non-Supersymmetric Case

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Abstract

The neutral fermion sectors of E_6-inspired low energy models, in particular the Alternative Left-Right and Inert models, are considered in detail within the non-supersymmetric scenario. We show that in their simplest form, these models always predict, for each generation, the lightest neutrino to be an $SU(2)_L$ singlet, as well as two extra neutrinos with masses of the order of the up-quark mass. In order to recover Standard Model phenomenology, additional assumptions in the form of discrete symmetries and/or new interactions are needed. These are classified as the Discrete Symmetry (DS), Higher Dimensional Operators (HDO), and Additional Neutral Fermion (ANF) methods. The DS method can solve the problem, but requires additional Higgs doublets that do not get vacuum expectation values. The HDO method predicts no sterile neutrino, and that the active neutrinos mix with a heavy isodoublet neutrino, thus slightly suppressing the couplings of active neutrinos, with interesting phenomenological implications. The ANF method also predicts this suppression, and also naturally includes one or more "sterile" neutrinos. This scenario allows the existence of sterile neutrino(s) in either a $\mathbf{3+1}$ or $\mathbf{2+2}$ structure at low energies, which are favored by the LSND result.

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I. INTRODUCTION

The discovery of solar [1] and atmospheric [2] neutrino oscillations has provided the first confirmed scenario of physics beyond the Standard Model. The combined results from solar, atmospheric and long baseline neutrino experiments are well described by oscillations of three active neutrinos ν_e, ν_μ and ν_τ, with mass squared splittings estimated to be $5.4\times10^{-5} < \Delta m^2_{sol} < 9.5\times10^{-5}$ eV^2 and $1.2\times10^{-3} < \Delta m^2_{atm} < 4.8\times10^{-3}$ eV^2 [3]. However, the Los Alamos Liquid Scintillation Detector (LSND) requires $10 > \Delta m^2 > 0.2$ eV^2 [4], a serious disagreement with the other results. The MiniBooNE experiment at Fermilab [5] is in the process of checking the validity of the LSND experiment. Taking at face value the LSND results, a minimum of four neutrinos seems to be required to explain all available neutrino data. LEP-SLC measurements of the Z decay width restrict the number of active neutrinos to three; thus one or more of the neutrinos must be "sterile" [6]. Such scenarios have been studied extensively [7, 8, 9, 10, 11, 12, 13, 14, 15, 16]. Mixing of sterile and active neutrinos affects directly the present neutrino experiments and limits have been set on such mixings. A valid question remains: how natural is it, in a beyond the Standard Model scenario, to obtain physically acceptable mixings between sterile and active neutrinos, while maintaining the constraints from weak scale phenomenology.

Several extensions of the Standard Model predict the existence of exotic fermions. Of these, superstring theories represent the most promising scenario for a unified theory of all fundamental interactions. One set of superstring theories are anomaly-free ten dimensional theories based on $E_8 \times E_8$ heterotic strings coupled to N=1 gravity [17], with matter belonging to the **27** representation of E_6. Previous interest in the E_6 GUTs dates as far as 1970's [18] when it was noted that E_6 was the next anomaly-free choice group after $SO(10)$, and that each generation of fermions can be placed in the **27**-plet representation.

The E_6 spectrum contains several neutral exotic fermions, some which could be interpreted as sterile neutrinos. The precise details of mass generation and mixing with the active neutrinos would depend on which subgroup of E_6 is considered. There are many phenomenologically acceptable low energy models which arise from E_6. In this work we concentrate on rank-5 subgroups, which can always break to $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_\eta$ [19, 20].

We analyze neutrino masses and mixings, as well as active-sterile neutrino assignments and mixing in group decompositions of E_6 under the maximal subgroup $SU(3)_C \times SU(3)_L \times$

$SU(3)_H$ to the Standard Model. These intermediate subgroups can include extra $SU(2)$ groups, which give rise to the usual Left-Right symmetric model (LR) [21], the Alternative Left-Right symmetric model (ALR) [22] and the Inert model [19, 23]. Though there are small differences among these groups with regards to neutrino masses and mixing, we shall be able to present a study applicable to all. We keep this discussion valid for the non-supersymmetric case, leaving the details for the supersymmetric scenario to another work [24].

Our paper is organized as follows. We discuss these models in Section 2. In sections 3 and 4 we analyze neutrino masses and mixings in the Alternative Left-Right and Inert models, respectively. Both of these models suffer from predicting too large a Dirac mass for the active neutrinos. We suggests mechanisms to rectify this problem in Section 5. We discuss the implications of our results and conclude in Section 6.

II. THE MODELS

The fundamental representation of E_6, the **27**-plet, branches into

$$\begin{aligned} \mathbf{27} &= (3^c, 3, 1) + (\bar{3}^c, 1, \bar{3}) + (1^c, \bar{3}, 3) \\ &= \quad q \quad + \quad \bar{q} \quad + \quad l \end{aligned} \tag{2.1}$$

under the maximal subgroup, $SU(3)_C \otimes SU(3)_L \otimes SU(3)_H$. The particle content of the **27**-plet for one family under this decomposition can be written as

$$q = \begin{pmatrix} u \\ d \\ h \end{pmatrix}_L , \quad \bar{q} = (u^c \; d^c \; h^c)_L , \quad l = \begin{pmatrix} E^c & N & \nu \\ N^c & E & e \\ e^c & \nu^c & S^c \end{pmatrix}_L . \tag{2.2}$$

Here we have used the notation that $SU(3)_L (SU(3)_H)$ operates vertically (horizontally) and the minus signs in front of the fields are suppressed.[1] There are three ways to

[1] We write fields as left-chiral Dirac spinors and throughout the rest of the paper we use f_L^c for a fermion field f as a shorthand notation for $(f^c)_L$, as we know that the chiral projection and conjugation do not commute. Thus, $f_L^c \equiv (f_R)^c = C\gamma^0 f_R^*$ where $C = \begin{pmatrix} -\epsilon & 0 \\ 0 & \epsilon \end{pmatrix}$. Here we adopt the chiral representation and $\epsilon \equiv i\sigma_2$.

embed an $SU(2)_H$ into the $SU(3)_H$, just as I-spin, U-spin and V-spin can be embedded in the $SU(3)$ flavor group. The best-known breaking is when the first and the second columns form a $SU(2)_H$ doublet; this corresponds to the LR symmetric model ($H = R$). An alternative version is when the first and the third columns form an $SU(2)_H$ doublet; this corresponds to the ALR symmetric model ($H = R'$). Finally, the second and the third columns can form an $SU(2)_H$ doublet; this corresponds to the Inert model ($H = I$). In LR, $(u^c\, d^c)_L\, ((e^c\, \nu^c)_L)$, $\begin{pmatrix} E^c & N \\ N^c & E \end{pmatrix}_L$, and h^c_L (and the third column of l) become $SU(2)_R$ doublets, a bi-doublet, and singlets, respectively. For the ALR case, $(h^c\, u^c)_L\, ((e^c\, S^c)_L)$, $\begin{pmatrix} E^c & \nu \\ N^c & e \end{pmatrix}_L$, and d^c_L (and the particles in the second column of l) are the corresponding ones under $SU(2)_{R'}$. Finally in the Inert model, $(h^c\, d^c)_L\, ((\nu^c\, S^c)_L)$, $\begin{pmatrix} N & \nu \\ E & e \end{pmatrix}_L$, and u^c_L (and the particles in the first column of l) are the corresponding multiplets under $SU(2)_I$.

To determine the $U(1)$ quantum numbers, we need to look at the electromagnetic charge operator. If we consider the case where only $SU(3)_L$ is broken down to $SU(2)_L \otimes U(1)_{Y_L}$, the electromagnetic charge $Q_{em} = I_{3L} + Y/2$ for all $\bar{q}$ becomes zero. Therefore, $SU(3)_H \to SU(2)_H \otimes U(1)_{Y_H}$ is needed such that $SU(2)_H$ and/or $U(1)_{Y_H}$ can contribute to Q_{em}. When both $SU(2)_H$ and $U(1)_{Y_H}$ contribute to Q_{em}, we end up with the LR [2] and ALR symmetric models. The "Inert" model, is obtained when the $SU(2)_H$ does not contribute to Q_{em}. We will use the notation $H = R, R', I; Y_H = Y_{R,R',I}$ for the LR, ALR and Inert groups, respectively. The gauge groups are at this level $SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_L \otimes U(1)_R$, $SU(3)_C \otimes SU(2)_L \otimes SU(2)_{R'} \otimes U(1)_L \otimes U(1)_{R'}$, and $SU(3)_C \otimes SU(2)_L \otimes SU(2)_I \otimes U(1)_Y \otimes U(1)'$ for LR, ALR and Inert cases, respectively [19, 25]. It is further possible to break them into some effective rank-5 forms by reducing $U(1)_L \otimes U(1)_{R(R')} \to U(1)_{V=L+R(R')}$ for the LR (ALR) case and $SU(2)_I \otimes U(1)' \to SU(2)_I$ for the Inert case. The quantum numbers of the particles in ALR and Inert models are given in Table I.

The Higgs sector of the model is sometimes found by assuming, in the spirit of SUSY models, that the allowed representations also come from a **27**-plet. However, since we are not considering SUSY models, we do not assume that all of the states in the **27**-plet are

[2] This is the rank-6 version of the familiar LR symmetric model.

TABLE I: The quantum numbers of fermions in **27** of E_6 at $SU(3)_C \otimes SU(2)_L \otimes SU(2)_{R'} \otimes U(1)_{V=Y_L+Y_{R'}}$ and $SU(3)_C \otimes SU(2)_L \otimes SU(2)_I \otimes U(1)_Y$ levels.

state	I_{3L}	$I_{3R'}$	I_{3I}	$V/2$	$Y/2$	Q_{em}
u_L	1/2	0	0	1/6	1/6	2/3
u_L^c	0	-1/2	0	-1/6	-2/3	-2/3
d_L	-1/2	0	0	1/6	1/6	-1/3
d_L^c	0	0	-1/2	1/3	1/3	1/3
h_L	0	0	0	-1/3	-1/3	-1/3
h_L^c	0	1/2	1/2	-1/6	1/3	1/3
e_L	-1/2	-1/2	-1/2	0	-1/2	-1
e_L^c	0	1/2	0	1/2	1	1
E_L	-1/2	0	1/2	-1/2	-1/2	-1
E_L^c	1/2	1/2	0	0	1/2	1
ν_L	1/2	-1/2	-1/2	0	-1/2	0
ν_L^c	0	0	1/2	0	0	0
N_L	1/2	0	1/2	-1/2	-1/2	0
N_L^c	-1/2	1/2	0	0	1/2	0
S_L^c	0	-1/2	-1/2	1/2	0	0

present (so colored scalars will not be introduced, for example). For the ALR model, we can have H_S, singlet under both $SU(2)$ groups, H_1 doublet under $SU(2)_{R'}$ and singlet under $SU(2)_L$, H_2 doublet under $SU(2)_L$ and singlet under $SU(2)_{R'}$, and a bi-doublet H_3. The neutral components of H_S, H_1, H_2, and H_3 are scalars with the same quantum numbers as ν_L^c, S_L^c, N_L, and (N_L^c, ν_L) and they are from $(\mathbf{16}, \mathbf{1}), (\mathbf{1}, \mathbf{1}), (\mathbf{10}, \overline{\mathbf{5}})$, and $((\mathbf{10}, \overline{\mathbf{5}}), (\mathbf{16}, \overline{\mathbf{5}}))$ representations under $(SU(10), SU(5))$, respectively. In the case of the Inert model, however, the representations are slightly different [23]. There is no singlet scalar field (H_S) under $SU(2)_I$ but an additional neutral $SU(2)_I$ doublet H_D is needed. This doublet corresponds to the components ν_L^c and S_L^c of the fermion doublet. We parametrize these Higgs doublets vev's as

$$\langle H_1 \rangle = (0 \ N_1) \,, \ \ \langle H_2 \rangle = \begin{pmatrix} v_1 \\ 0 \end{pmatrix}, \ \ \langle H_3 \rangle = \begin{pmatrix} 0 & v_2 \\ v_3 & 0 \end{pmatrix}, \ \ \langle H_S \rangle = N_2, \tag{2.3}$$

in the ALR model and

$$\langle H_D \rangle = (N_2 \ N_1) \,, \ \ \langle H_2 \rangle = \begin{pmatrix} 0 \\ v_3 \end{pmatrix}, \ \ \langle H_3 \rangle = \begin{pmatrix} v_1 & v_2 \\ 0 & 0 \end{pmatrix}, \tag{2.4}$$

in the Inert model. The quantum numbers and vev's of the color-singlet, neutral Higgs fields in **27** of E_6 are given in Table II. We assume that the $SU(2)_L$ doublets acquire vev's $v_i \sim 10^2$

TABLE II: The quantum numbers of fermions in **27** of E_6 at $SU(3)_C \otimes SU(2)_L \otimes SU(2)_{R'} \otimes U(1)_{V=Y_L+Y_{R'}}$ and $SU(3)_C \otimes SU(2)_L \otimes SU(2)_I \otimes U(1)_Y$ levels.

vev	I_{3L}	$I_{3R'}$	I_{3I}	$V/2$	$Y/2$
v_1	1/2	0	1/2	-1/2	-1/2
v_2	1/2	-1/2	-1/2	0	-1/2
v_3	-1/2	1/2	0	0	1/2
N_1	0	-1/2	-1/2	1/2	0
N_2	0	0	1/2	0	0

GeV, the symmetry breaking scale of the electroweak gauge group, while the $SU(2)_L$ Higgs singlets get vev's N_i much larger than the scale of the electroweak symmetry breaking (that is, $N_i \gg v_i$). This hierarchy is needed from the fact that no experimental signal for the exotic quarks and leptons has been observed. The mass terms for the fermions can be obtained from the dimension-4 Yukawa interactions of the form $\mathcal{L}_Y = \lambda\psi(\mathbf{27})\psi(\mathbf{27})H(\mathbf{27})$. Here $\psi(\mathbf{27})$ is the **27**-plet of E_6 involving leptons and quarks, and $H(\mathbf{27})$ is the one involving Higgs scalars. The coefficient λ represents the corresponding generation dependent Yukawa coupling, where generation indices are suppressed. The explicit mass terms in the above Lagrangian $\mathcal{L}_Y$ can be written using the fact that each term has total hypercharge Y zero and is invariant under the gauge group of the model under consideration (that is, terms invariant under the $SU(3)_C \otimes SU(2)_L \otimes SU(2)_{R'} \otimes U(1)_V$ gauge group for the ALR model and under the $SU(3)_C \otimes SU(2)_L \otimes SU(2)_I \otimes U(1)_Y$ gauge group for the Inert model).

Therefore, all the allowed Yukawa terms can be written with the use of the Tables I and II. We consider the neutral sector of the **27**-plet of E_6 in the rest of the paper for the ALR and Inert models. Similar results can be obtained for LR models.

III. NEUTRINOS IN THE ALR SYMMETRIC MODEL

We now look at the allowed Yukawa couplings in the ALR model. For convenience, we use the following notation:

$$
\begin{aligned}
Q &= \begin{pmatrix} u \\ d \end{pmatrix}_L (3,2,1,1/6)\,,\ X^c = (h^c\ u^c)_L\,(\overline{3},1,2,-1/6)\,,\ L' = \begin{pmatrix} N \\ E \end{pmatrix}_L (1,2,1,0)\,,\\
F &= \begin{pmatrix} E^c & \nu \\ N^c & e \end{pmatrix}_L (1,2,2,0)\,,\ L^c = (e^c\ S^c)_L\,(1,1,2,1/2)\,. \qquad (3.1)
\end{aligned}
$$

Then, all possible Yukawa terms which are $SU(3)_C \otimes SU(2)_L \otimes SU(2)_{R'} \otimes U(1)$ invariant can be written using of the Higgs fields in Eq. (2.3). The Yukawa Lagrangian is

$$
\begin{aligned}
\mathcal{L}_{\mathcal{Y}} &= -\lambda_1\left[L^c F H_2 + L^c H_3 L' + H_1 F L'\right] + \frac{\lambda_2}{2}\left[F F H_S + F H_3 \nu_L^c\right] + \lambda_3 Q H_3 X^c \\
&\quad +\lambda_4 d_L^c Q H_2 + \lambda_5 h_L X^c H_1 + \lambda_6 h_L d_L^c H_S\,, \qquad (3.2)
\end{aligned}
$$

where we suppress all generation indices and use a shorthand notation for each term. So, for example, $L^c F H_2$ should be read as $(L^c)^T\,\epsilon\,F\,\epsilon\,H_2$ with $\epsilon = i\sigma_2$. The part of the Lagrangian relevant to our discussion here is (when the Higgs fields get vev's)

$$
\begin{aligned}
\mathcal{L}^0_{\mathcal{Y}} &= \lambda_1\left[v_1(e_L e_L^c - N_L^c S_L^c) - v_2 e_L^c E_L - v_3 N_L S_L^c + N_1(E_L E_L^c - N_L N_L^c)\right] \\
&\quad +\lambda_2\left[v_2 \nu_L^c N_L^c + v_3 \nu_L \nu_L^c + N_2(-e_L E_L^c + \nu_L N_L^c)\right] + \lambda_3 v_3 u_L u_L^c\,, \qquad (3.3)
\end{aligned}
$$

where we have suppressed family indices and include charged lepton terms and part of the λ_3 term for later convenience.[3] Here it should be understood that the $e_L e_L^c$ term, for example, stands for $(e^c)_L^T\,C\,e_L \equiv \overline{e_R} e_L$.

[3] Since this paper is concentrating on neutrinos, we will not discuss mixing between light and heavy fields in the charged lepton or quark sectors. Such mixing can have a wide range of interesting phenomenological effects, see Ref. [26] for a detailed discussion and a list of references.

From the above Yukawa interactions, the Majorana mass matrix for the neutral fields in the $(\nu_L, N_L, N_L^c, \nu_L^c, S_L^c)$ basis becomes (for one generation)

$$\mathcal{M}_{\text{neutral}} = \begin{pmatrix} 0 & 0 & \lambda_2 N_2 & \lambda_2 v_3 & 0 \\ 0 & 0 & -\lambda_1 N_1 & 0 & -\lambda_1 v_3 \\ \lambda_2 N_2 & -\lambda_1 N_1 & 0 & \lambda_2 v_2 & -\lambda_1 v_1 \\ \lambda_2 v_3 & 0 & \lambda_2 v_2 & 0 & 0 \\ 0 & -\lambda_1 v_3 & -\lambda_1 v_1 & 0 & 0 \end{pmatrix} . \tag{3.4}$$

Further we define $\lambda_1 v_1 \equiv m_{ee^c}, \lambda_1 N_1 \equiv m_{EE^c}$, and $\lambda_2 v_3 \equiv m_{\nu\nu^c}$ since it is clear from Eq. (3.3) that m_{ee^c}, m_{EE^c}, and $m_{\nu\nu^c}$ are the Dirac mass terms for the electron e_L, the exotic charged lepton E_L, and the ordinary (active) neutrino ν_L. Note that the SM (active) neutrino gets Dirac mass from the same source as the up quark. Thus, at the first stage, there appears to be a large Dirac mass problem for the neutrinos unless there is an (unnatural) hierarchy $\lambda_2 \ll \lambda_3$. Unlike the "conventional" see-saw model, we do not have a large Majorana mass term for the right-handed neutrino, so other techniques must be used to deal with this large mass. This problem is also severe in both the Inert and the ordinary LR symmetric models where the active neutrinos and up quark (the electron for LR case) get their Dirac masses from the same source. We will discuss the Inert model case in the next section. For the ordinary LR symmetric model, see [27, 28] for further details.

The secular equation for the eigenvalues cannot be solved exactly, and so we expand in powers of v_i/N_i. In this approximation (neglecting $O(v_i/N_i)$ terms), there are two roots of the secular equation which correspond to states with mass eigenvalue $\pm m_{\nu\nu^c}$. The other three mass eigenvalues can also be determined, again under the assumption that $\lambda_2 v_2 \sim m_{\nu\nu^c} \sim m_{ee^c} \ll \lambda_2 N_2 \sim m_{EE^c}$

$$\begin{aligned} R_1 &\simeq -\frac{2m_{\nu\nu^c}\left(m_{ee^c} m_{EE^c} + \lambda_2^2 v_2 N_2\right)}{m_{EE^c}^2 + \lambda_2^2 N_2^2} , \\ R_{2,3} &\simeq \pm\sqrt{m_{EE^c}^2 + \lambda_2^2 N_2^2} , \end{aligned} \tag{3.5}$$

where we neglect the terms of the order v_i/N_i. The associated eigenvectors with R_2 and R_3 form a Dirac spinor with mass $\sqrt{m_{EE^c}^2 + \lambda_2^2 N_2^2}$. R_1 is the lightest mass eigenvalue ($\ll m_{\nu\nu^c}$) which represents the lightest mass eigenstate. The corresponding eigenvectors can be found in a straightforward manner under the same assumption that we have used to get the eigenvalues and the transformation from the mass eigenstates to the flavor eigenstates becomes

$$
\begin{pmatrix} |\nu_L\rangle \\ |N_L\rangle \\ |N_L^c\rangle \\ |\nu_L^c\rangle \\ |S_L^c\rangle \end{pmatrix} = \begin{pmatrix} 0 & \frac{\lambda_2 N_2}{R} & -\frac{\lambda_2 N_2}{R} & -\frac{m_{EE^c}}{R} & -\frac{m_{EE^c}}{R} \\ 0 & \frac{m_{EE^c}}{R} & -\frac{m_{EE^c}}{R} & \frac{\lambda_2 N_2}{R} & \frac{\lambda_2 N_2}{R} \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{\lambda_2 N_2}{R} & 0 & 0 & \frac{m_{EE^c}}{R} & -\frac{m_{EE^c}}{R} \\ -\frac{m_{EE^c}}{R} & 0 & 0 & \frac{\lambda_2 N_2}{R} & -\frac{\lambda_2 N_2}{R} \end{pmatrix} \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \\ |\nu_3\rangle \\ |\nu_4\rangle \\ |\nu_5\rangle \end{pmatrix}, \tag{3.6}
$$

where $R \equiv \sqrt{2(m_{EE^c}^2 + \lambda_2^2 N_2^2)}$.

At this stage there appears another potential problem in that the lightest mass eigenstate is $|\nu_1\rangle = \frac{1}{\sqrt{m_{EE^c}^2+\lambda_2^2 N_2^2}}\left[\lambda_2 N_2 |\nu_L^c\rangle - m_{EE^c}|S_L^c\rangle\right]$. Both ν_L^c and S_L^c transform as singlets under the weak interaction gauge group $SU(2)_L$. This presumed physical neutrino state does not couple with the left handed SM particles at the low energy scale where the neutrinos are relevant.[4] The mass is of the order of magnitude of $m_{\nu\nu^c}^2/m_{EE^c}$, which is the expected order of magnitude for neutrinos. We thus have two problems: the active neutrinos have a mass of the same order of magnitude as the up quark mass, and the lightest neutrino is composed of $SU(2)_L$ singlets. After considering neutrinos in the Inert model, we will address the above issues and discuss the possible solutions.

IV. NEUTRINOS IN THE INERT MODEL

The neutral fermion mass matrix has similarities with that of the ALR model. The Yukawa interactions are invariant under the $SU(2)_I$ group which transforms $(N_L\ E_L) \Leftrightarrow (\nu_L\ e_L)$, $d_L^c \Leftrightarrow h_L^c$, and $\nu_L^c \Leftrightarrow S_L^c$. By following the same procedure as for the ALR symmetric model, one can obtain the Yukawa Lagrangian for the Inert group and the relevant part of it reads

$$
\begin{aligned}
\mathcal{L}_{\mathcal{Y}}'^{0} \;=\;& \lambda_1' \left[v_1 N_L^c S_L^c + v_2 \nu_L^c N_L^c + v_3 \left(\nu_L \nu_L^c + N_L S_L^c\right) + N_1 \left(N_L N_L^c + E_L E_L^c\right)\right. \\
& \left. + N_2 \left(\nu_L N_L^c + e_L E_L^c\right)\right] + \lambda_2' \left[v_1 e_L e_L^c + v_2 e_L^c E_L\right] + \lambda_3' v_3 u_L u_L^c\,,
\end{aligned} \tag{4.1}
$$

[4] Even though S_L^c is a part of $SU(2)_{R'}$ doublet and it is possible to consider its interaction with left handed SM leptons through Higgs bi-doublet at the scales where ALR gauge group is not broken.

where the λ'_3-term is especially included to show that, as in the ALR case, the ν_L neutrinos get Dirac masses from the same Higgs scalar as the up quark. Without fine tuning between λ'_1 and λ'_3, the Inert model has the same Dirac mass problem for active neutrinos as ALR. The mass matrix for one generation in the basis $(\nu_L, N_L, N_L^c, \nu_L^c, S_L^c)$

$$\mathcal{M}'_{\rm neutral} = \begin{pmatrix} 0 & 0 & \lambda'_1 N_2 & \lambda'_1 v_3 & 0 \\ 0 & 0 & \lambda'_1 N_1 & 0 & \lambda'_1 v_3 \\ \lambda'_1 N_2 & \lambda'_1 N_1 & 0 & \lambda'_1 v_2 & \lambda'_1 v_1 \\ \lambda'_1 v_3 & 0 & \lambda'_1 v_2 & 0 & 0 \\ 0 & \lambda'_1 v_3 & \lambda'_1 v_1 & 0 & 0 \end{pmatrix} . \tag{4.2}$$

Here we recall $\lambda'_1 v_1 \equiv m'_{ee^c}$, $\lambda'_1 N_1 \equiv m'_{EE^c}$, and $\lambda'_1 v_3 \equiv m'_{\nu\nu^c}$. The secular equation becomes

$$\begin{aligned} (R' - m'_{\nu\nu^c})\,(R' + m'_{\nu\nu^c})\,\big(R'^3 - R'\left(m'^2_{EE^c} + \lambda'^2_1(N_2^2 + v_1^2 + v_2^2) + m'^2_{\nu\nu^c}\right) \\ +2m'_{\nu\nu^c}\left(\lambda'^2_1 v_2 N_2 + \lambda'_1 v_1 m'_{EE^c}\right)\big) = 0\,, \end{aligned} \tag{4.3}$$

where there are two eigenvalues $\pm m_{\nu\nu^c}$ which are exact (unlike the ALR model). Diagonalization of the mass matrix gives the following eigenvalues, under the assumption $v_i \ll N_i$

$$\begin{aligned} R'_1 &\simeq -\frac{2m'_{\nu\nu^c}\,(\lambda' v_1 m'_{EE^c} + \lambda'^2_1 v_2 N_2)}{m^2_{EE^c} + \lambda'^2_1 N_2^2}\,, \\ R'_{2,3} &\simeq \pm\sqrt{m^2_{EE^c} + \lambda'^2_1 N_2^2}\,, \\ R'_{4,5} &= \pm m'_{\nu\nu^c}\,, \end{aligned} \tag{4.4}$$

It is clear from the ALR symmetric model results that there will be two very heavy neutrinos, one very light neutrino, and two neutrinos with masses of the scale of up quark mass. The lightest neutrino is $|\nu'_1\rangle = \frac{1}{\sqrt{m'^2_{EE^c}+\lambda'^2_1 N_2^2}}\,(\lambda'_1 N_2 |\nu_L^c\rangle - m'_{EE^c}|S_L^c\rangle)$ and suffers from the same problem that the ALR symmetric model neutrino does. We will discuss possible remedies these problems for both models in the next section.

V. SOLUTIONS TO THE NEUTRINO MASS PROBLEM

As shown in the last two sections, both ALR and Inert models (as well as the conventional LR symmetric model) have a Dirac neutrino mass problem at the first stage. This seems to be a general feature of string-inspired low-energy E_6 models. Both models under consideration predict that the lightest neutrino state, while having a reasonable mass, is composed of $SU(2)_L$ singlets. Furthermore, in their neutral fermion spectrum, there are neutrino eigenstates having masses of the order of the up quark mass (or the electron mass for the conventional LR model). There are three methods discussed in the literature to rectify this latter neutrino mass problem. The smallness of the neutrino masses can be achieved by introducing a discrete symmetry (the DS method) [29, 30, 31, 32, 33, 34], or by including a non-renormalizable higher-order dimensional operators (the HDO method) [28, 35, 36, 37], or using light E_6 singlet fields (the additional neutral fermion (ANF) method) [38, 39, 40]. We discuss the features of the models under consideration for each of these three methods. As we will see, the predictions are quite different. The DS method is the most attractive method among them as it doesn't require any further particles or the existence of some intermediate scale. However, it does not help in non-SUSY framework (at least for the simplest discrete symmetry) without introducing many additional particles. The HDO method will offer a partial solution but *does not predict any light sterile neutrino(s)* and requires new Higgs fields from $\overline{\mathbf{27}}$ representation of E_6, and the existence of some intermediate scale, which further breaks the gauge groups of the model. The ANF method works well for predicting the lightest state with sterile neutrino(s) mixing and can explain the LSND result. However, the method requires a discrete symmetry as well as new neutral E_6 fermion fields, and a pair of $\mathbf{27}+\overline{\mathbf{27}}$ split Higgs multiplets whose vev's do not require hierarchical separation.

A. The Discrete Symmetry Method

Following the above discussion, the Discrete Symmetry (DS) method is the most economical. The symmetry transformation which is introduced should restrict the existence of the Dirac mass term $v_3\nu_L\nu_L^c$ at tree level in the Lagrangian (Eqs. (3.3) and (4.1)) while allowing couplings so that one-loop radiative corrections can be used to generate naturally small Dirac masses for neutrinos (although it may be necessary to put some upper limits for products of some Yukawa couplings). The symmetry should also avoid rapid proton decay.

In the supersymmetric versions of both the ALR symmetric and the Inert model, there exist leptoquark couplings mediated by h_L and h_L^c particles and these couplings are needed to induce nonzero one-loop neutrino mass. Since we do not consider the existence of the

Higgs fields carrying $SU(3)$ color, there is no direct analogy in non-SUSY scenarios coming from the supersymmetrized versions of the models. It should be noted that the rapid proton decay is not an issue.

An example of such a symmetry, which was considered within the SUSY framework of the general E_6 model [29, 30] is $Z_2 \otimes Z_3$. The Z_2 in that case was related to SUSY, and in this non-SUSY framework a simple Z_3 will suffice. It is not difficult to see that such symmetries must be able to differentiate between generations as long as a non-zero one-loop Dirac neutrino mass is generated while at the same time eliminating the tree level mass term (see [33, 34] for details).

In both models considered here, tree level masses of both the neutrinos and the up quark are obtained from the Higgs field with vev v_3. As we shall show shortly, eliminating the v_3-term will cause difficulty. Let us consider the ALR model. The Inert model has very similar features. For a one-loop Dirac neutrino mass, as depicted in Fig. 1 for a specific choice, the H_1^0 $SU(2)_L$ Higgs singlet, H_2 and H_3 $SU(2)_L$ Higgs doublets must all participate. Restating their particle content from Eq. (2.3)

$$H_1 = \begin{pmatrix} H_1^+ & H_1^0 \end{pmatrix}, \quad H_2 = \begin{pmatrix} H_2^0 \\ H_2^- \end{pmatrix}, \quad H_{3,1} = \begin{pmatrix} H_3^+ \\ H_3^0 \end{pmatrix}, \tag{5.1}$$

where $\langle H_2^0 \rangle = v_1, \langle H_3^0 \rangle = v_3$, and $\langle H_1^0 \rangle = N_1$. Here $H_{3,1}$ represents the first column of the H_3 bi-doublet. Then the relevant terms in the Yukawa Lagrangian Eq. (3.3), including the charged Higgs fields interactions are

$$\begin{aligned} \Delta\mathcal{L}_{ALR} &= \lambda_1 \left[H_2^0 e_L e_L^c - H_2^- \nu_L e_L^c - H_1^+ E_L \nu_L + H_1^0 E_L E_L^c \right] \\ &\quad + \lambda_2 \left[H_3^0 \nu_L \nu_L^c - H_3^+ \nu_L^c e_L - H_2^- E_L^c \nu_L^c \right] . \end{aligned} \tag{5.2}$$

We also need the trilinear Higgs interactions to compute the diagram given in Fig. 1. The allowed interactions are

$$\begin{aligned} \Delta\mathcal{L}_H &= -\lambda_H H_2^T \, \epsilon \, H_3 H_1^0 \\ &= \lambda_H H_2^- H_3^+ H_1^0 - \lambda_H H_2^0 H_3^0 H_1^0, \end{aligned} \tag{5.3}$$

where λ_H is a dimensionful constant.

Without specifying the charges of the fields under the discrete symmetry, let us consider the one-loop mass diagram. One can assign suitable charges to both Higgs and fermion fields

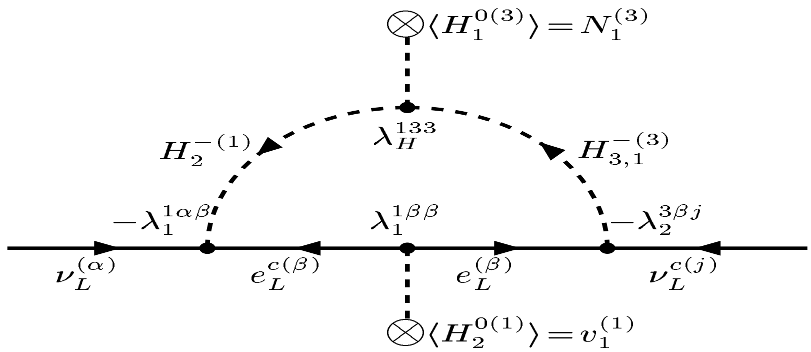

FIG. 1: The one-loop Dirac masses for $\nu_L^{(\alpha)}\nu_L^{c(j)}$ where α runs over only the second and the third generations.

such that the $H_3^0\nu_L\nu_L^c$ term, a tree level Dirac mass term for ν_L, is transformed to itself with a nonzero phase factor and one is then required to set λ_2 zero for all 3 generations. If the SM charged leptons and H_2^- and H_3^- fields are circulating in the loop, the $H_3^+\nu_L^c e_L$ interaction is also proportional to λ_2, thus this diagram vanishes. For the case when E_L, E_L^c are circulating in the loop instead of the SM charged leptons, it is still necessary to have a nonzero λ_2 (clear from Eq. (5.2)) to get a one loop Dirac mass for ν_L. Therefore, eliminating $H_3^0\nu_L\nu_L^c$ by the Z_3 symmetry also prevents one-loop mass generation. This fact remains true for higher order loops. The same conclusion applies for the Inert model as well.

One could consider the possibility that v_3 could be zero. Then λ_2 doesn't need to be zero and one-loop Dirac neutrino mass generation is possible. In that case, however, all the up quarks (u, c, t) become massless at tree level and generating the top quark mass from a loop diagram is very unlikely, within the context of perturbation theory.

It still is possible to generate a one-loop Dirac neutrino mass if many additional fields are introduced. For example, if one allows for "generations" of Higgs fields, then the λ parameters above are all third rank tensors. In such a case, one can arrange the potential so that some of the H_3 vev's vanish. Then the discrete symmetry can couple $\nu_L\nu_L^c$ to fields that do not get vev's, thus allowing a one-loop Dirac mass to be generated. To do that, let's assign the following charges for the matter fields under Z_3

$$
\begin{aligned}
Z_3: \quad & [Q, d_L^c, h_L, h_L^c, L, \nu_L^c]^{(i)} \to \eta\, [Q, d_L^c, h_L, h_L^c, L, \nu_L^c]^{(i)} \ , \\
& F_1^{(1)} \to \eta^{-1} F_1^{(1)}\ , \ F_1^{(2)} \to F_1^{(2)}\ , \ F_1^{(3)} \to \eta F_1^{(3)}\ , \\
& H^{(1)} \to \eta^{-1} H^{(1)}\ , \ H^{(2)} \to \eta H^{(2)}\ , \ H^{(3)} \to H^{(3)}\ ,
\end{aligned}
$$

$$S_L^{c(1)} \to \eta^{-1} S_L^{c(1)}\,,\ S_L^{c(2)} \to \eta S_L^{c(2)}\,,\ S_L^{c(3)} \to S_L^{c(3)}\,, \tag{5.4}$$

where F_1 is the first column of the bidoublet F, and similarly the Higgs fields as

$$\begin{aligned} Z_3:\quad & H_{3,1}^{(1)} \to \eta^{-1} H_{3,1}^{(1)}\,,\ H_{3,1}^{(2)} \to H_{3,1}^{(2)}\,,\ H_{3,1}^{(3)} \to \eta H_{3,1}^{(3)}\,, \\ & H_2^{(1)} \to \eta^{-1} H_2^{(1)}\,,\ H_2^{(2)} \to \eta H_2^{(2)}\,,\ H_2^{(3)} \to H_2^{(3)}\,, \\ & H_1^{0(1)} \to \eta^{-1} H_1^{0(1)}\,,\ H_1^{0(2)} \to \eta H_1^{0(2)}\,,\ H_1^{0(3)} \to H_1^{0(3)}\,, \\ & H_{3,2}^{(i)} \to \eta H_{3,2}^{(i)}\,,\ H_S^{(i)} \to \eta H_S^{(i)}\,, \end{aligned} \tag{5.5}$$

where the rest of the fields are assumed to be invariant under Z_3 and $\eta^3 = 1$. In this particular choice we take the vev of $H_{3,1}, v_3^{(3)}$, as zero. Then, the Lagrangian for the ALR symmetric model, given in Eq. (3.2) reduces to

$$\begin{aligned} \mathcal{L}_{\mathcal{Y}} = & -\lambda_1^{1\alpha\beta}\left[H_2^{(1)} L^{(\alpha)} e_L^{c(\beta)} + H^{(1)} H_{3,2}^{(\alpha)} e_L^{c(\beta)} + H^{(1)} L^{(\alpha)} H_1^{+(\beta)} + H_2^{(1)} F_1^{(\alpha)} S_L^{c(\beta)} \right. \\ & \left. + H^{(1)} H_{3,1}^{(\alpha)} S_L^{c(\beta)} + H^{(1)} F_1^{(\alpha)} H_1^{0(\beta)}\right] + \lambda_2^{3ij}\left[H_{3,1}^{(3)} L^{(i)} \nu_L^{c(j)} + F_1^{(3)} H_{3,2}^{(i)} \nu_L^{c(j)}\right. \\ & \left. + F_1^{(3)} L^{(i)} H_S^{(j)}\right] + \lambda_3^{1ij}\left[H_{3,1}^{(1)} u_L^{c(i)} Q^{(j)} + H_{3,2}^{(1)} h_L^{c(i)} Q^{(j)}\right] + \lambda_4^{2ij} H_2^{(2)} Q^{(i)} d_L^{c(j)} \\ & + \lambda_5^{2ij} H_1^{(2)} h_L^{(i)} h_L^{c(j)} + \lambda_6^{ijk} h_L^{(i)} d_L^{c(j)} H_S^{(k)}\,, \end{aligned} \tag{5.6}$$

where α and β run only over the second and third generations. Now, the only tree level Dirac mass term for ν_L, $\lambda_2^{3ij} H_{3,1}^{(3)} L^{(i)} \nu_L^{c(j)}$, vanishes if all the particles are neutral due to zero vev $v_3^{(3)}$. Note that writing the Lagrangian for the Inert model can be done easily by applying the following substitutions to Eq. (5.6); $F_1 \Leftrightarrow L'$, $u_L^c \Leftrightarrow d_L^c$, $e^c \Leftrightarrow \nu^c$, $H_{3,1} \Leftrightarrow H_2$, $H_1 \to H_D$, $H_S \to 0$. The grouping of the terms in Inert case will be slightly different. We will stick the ALR case in the rest of the subsection.

Due to the radiative corrections based on the remaining interactions given in Eq. (5.6), $\nu_L^{(i)} \nu_L^{c(j)}$ Dirac masses are induced through one-loop diagram shown in Fig. 1. If we assume that the product $\lambda_H N_1$ is of the same order as the charged Higgs masses, which are further assumed degenerate and much heavier than any fermion in the loop, the magnitudes of the Dirac masses are roughly estimated as

$$M_{\nu\nu^c}^{\alpha j} = \frac{m_{\tau\tau^c}}{16\pi^2} \lambda_1^{1\alpha 3} \lambda_2^{33j}. \tag{5.7}$$

In order for such radiative masses to be of the order of 10^{-1} eV, the product of the relevant Yukawa couplings $\lambda_1^{1\alpha 3}\lambda_2^{33j}$ should be less than $O(10^{-8})$. It is further possible to generate very light Majorana masses for both S_L^c and ν_L^c through one-loop.[5] Majorana masses for S_L^c are obtained by replacing the tau lepton in Fig. 1 with the E lepton, but are very supressed ($\sim \lambda_1^2 m_{H^-}^2/m_{EE^c}$) and similarly for ν_L^c. If we include these Majorana masses, this opens up the possibility of having so-called pseudo-Dirac neutrinos when $M_{S^cS^c}, M_{\nu^c\nu^c} \ll M_{\nu\nu^c}$ is satisfied [41].

Such models have far too many parameters to be predictive and are very contrived. We thus turn to the HDO and ANF schemes, which are much more predictive.

B. The HDO Method in the ALR and the Inert Models

This method has been discussed in the framework of rank-6 version of the LR symmetric model [28] where it has been shown that the higher dimensional operators (HDO), specifically dimension-5 operators, give sizable contributions to the neutral sector of the fermion mass matrix. The method requires the existence of an intermediate scale at which the group is broken to the SM gauge group. Two of the Higgs fields (for our discussion, H_1 and H_S in the ALR case, and H_D in the Inert case) will acquire vev's of the order of the intermediate scale ($\sim 10^{11}$ GeV).

The leading HDO Yukawa interactions are the dimension-5 operators. If we neglect the contributions coming from operators with dim > 5,[6] the non-renormalizable dimension-5 operator is

$$\mathcal{L}_{\mathcal{Y}}^{(5)} = \frac{f}{M_c}\psi^T(\mathbf{27})\,\epsilon\, H(\overline{\mathbf{27}})\, C\, H^T(\overline{\mathbf{27}})\,\epsilon\,\psi(\mathbf{27})\,, \tag{5.8}$$

where the Higgs fields H are from the $\overline{\mathbf{27}}$ representation of E_6 and their quantum numbers are taken as the opposite of the ones listed in Table II. Here, M_c is the compactification scale, or 10^{18} GeV. The inclusion of the above dimension-5 interactions will modify all entries in the fermion sector (both the charged and the neutral fields). However, from Table I, it is possible to show that except the $\nu_L^c - S_L^c$ submatrix in the neutral sector all entries get contributions which are negligible compared to with their dimension-4 entries.[7]

[5] Neither N_L nor N_L^c can get such one-loop Majorana masses in this framework.

[6] It is safe to neglect them since they are suppressed by some quadratic, cubic or higher powers of the compactification scale, $M_c(\sim 10^{18}$ GeV).

[7] Negligible contributions are either 0, or $\frac{f v_i v_j}{M_c}$, or $\frac{f v_i N_j}{M_c}$ form to the appropriate entries, but not $\frac{f N_i N_j}{M_c}$.

The $\nu_L^c - S_L^c$ submatrix, a null 2×2 matrix at the dimension-4 level, becomes, in the ALR model

$$\mathcal{M}_{\nu^c - S^c} = \begin{pmatrix} K_1 & K_{12} \\ K_{12} & K_2 \end{pmatrix} , \tag{5.9}$$

where $K_{12} \equiv 2f\frac{N_1 N_2}{M_c}$ and $K_i \equiv f\frac{N_i^2}{M_c}$. Obviously, $K_i \sim K_{12} \simeq 10^4$ GeV for an intermediate scale 10^{11} GeV and the coupling constant f is of order of unity. The nonzero 2×2 submatrix with large entries gives a new "see-saw-like" structure to the 5×5 matrix. The submatrix in the (ν_L^c, S_L^c) basis will induce small but non-zero entries in the upper-left 2×2 submatrix spanned by (ν_L, N_L). The mass eigenvalues for the matrix in Eq. (3.4) with the above modification become

$$\begin{aligned} R_1 &\simeq \frac{(\lambda_1\lambda_2 v_3\sqrt{K_1}N_2 + m_{\nu\nu^c}M_{EE^c}\sqrt{K_2})^2 + \lambda_1\lambda_2 v_3 N_2 m_{\nu\nu^c} m_{EE^c} K_{12}}{(m_{EE^c}^2 + \lambda_2^2 N_2^2)(K_{12}^2 - K_1 K_2)} , \\ R_{2,3} &\simeq \pm\sqrt{m_{EE^c}^2 + \lambda_2^2 N_2^2} , \\ R_{4,5} &\simeq \frac{1}{2}\left[K_1 + K_2 \pm \sqrt{(K_1 - K_2)^2 + 4K_{12}^2}\right] , \end{aligned} \tag{5.10}$$

where we use the assumptions $v_i \ll K_i \sim K_{12} \ll m_{EE^c} \sim \lambda_2 N_2$ and neglect all the m_i^2 terms. The first apparent modification from the mass eigenvalues is that the states with masses $R_{4,5}$, which previously had masses of the order of the up quark mass, now get modified at the scale $K_{1,2,12} \sim 10^4$ GeV. After the diagonalization, the transformation matrix (the analogous to the dimension-4 case (Eq. (3.6)) in the dimension-5 level) is

$$\begin{pmatrix} |\nu_L\rangle \\ |N_L\rangle \\ |N_L^c\rangle \\ |\nu_L^c\rangle \\ |S_L^c\rangle \end{pmatrix} = \begin{pmatrix} a_1 m_{EE^c} & 0 & 0 & \frac{a_1\lambda_2 N_2}{\sqrt{2}} & \frac{a_1\lambda_2 N_2}{\sqrt{2}} \\ -a_1\lambda_2 N_2 & 0 & 0 & \frac{a_1 m_{EE^c}}{\sqrt{2}} & \frac{a_1 m_{EE^c}}{\sqrt{2}} \\ 0 & 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ 0 & a_2 K_{12} & a_3 K_{12} & 0 & 0 \\ 0 & a_2(R_4 - K_1) & a_3(R_5 - K_1) & 0 & 0 \end{pmatrix} \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \\ |\nu_3\rangle \\ |\nu_4\rangle \\ |\nu_5\rangle \end{pmatrix} , \tag{5.11}$$

where $a_1 \equiv \frac{1}{\sqrt{m_{EE^c}^2 + \lambda_2 N_2^2}}$, $a_2 \equiv \frac{1}{\sqrt{K_{12}^2 + (R_4 - K_1)^2}}$, $a_3 \equiv \frac{1}{\sqrt{K_{12}^2 + (R_5 - K_1)^2}}$. The above matrix elements are derived in the same limit as we used before to get the mass eigenvalues. Now, the spectrum consists of one light state, ν_1, and four heavy states, $\nu_{2,3,4,5}$. Moreover, the light state is formed by the flavor states ν_L and N_L of the form

$$\nu_1 \simeq \frac{1}{\sqrt{m_{EE^c}^2 + \lambda_2^2 N_2^2}} \left[m_{EE^c}|\nu_L\rangle - \lambda_2 N_2 |N_L\rangle\right] , \tag{5.12}$$

which is an acceptable physical state as both ν_L and N_L are members of two different $SU(2)_L$ doublets. Therefore our physical neutrino state can now couple with the electron and the other SM particles in a desired way. The mass of the state is still as light as $m^2_{\nu\nu^c}/K_{1,2,12}$ (or $(\lambda_1 v_3)^2/K_{1,2,12}$) ~ 0.02 eV when we take the $m_{\nu\nu^c}$ around the mass of the up quark.

One can repeat the same calculation for the Inert model. The features are very similar. Except the $\nu^c_L - S^c_L$ submatrix, all other entries get negligible contributions from Eq. (5.8) and in the submatrix, the corresponding $SU(2)_I$ Higgs doublet H_D from the $\overline{\mathbf{27}}$-plet of E_6 is involved and the submatrix will be the same as the one in Eq. (5.9). The eigenvalues are slightly different from the ones given in Eq. (5.10)

$$\begin{aligned} R'_1 &\simeq \frac{m'^2_{\nu\nu^c}(\lambda'^2_1 N_2 K_1 + m'^2_{EE^c}K_2 + 2\lambda'_1 N_2 m'_{EE^c}K_{12})}{(m'^2_{EE^c} + \lambda'^2_2 N^2_2)(K^2_{12} - K_1K_2)}\,, \\ R'_{2,3} &\simeq \pm\sqrt{m'^2_{EE^c} + \lambda'^2_2 N^2_2}\,, \\ R_{4,5} &\simeq \frac{1}{2}\left[K_1 + K_2 \pm \sqrt{(K_1 - K_2)^2 + 4K^2_{12}}\right]\,, \end{aligned} \tag{5.13}$$

under the same assumptions as previously stated. Then the transformation matrix can be formed by finding the corresponding mass eigenstates and it has the same form as the one in the ALR model given in Eq. (5.11). Note that the results differ from each other when we, for example, keep terms in the $O(v_i/N_i, v_i/K_{1,2,12})$ order. The lightest state ν'_1 is composed of ν_L and N_L of the form

$$\nu'_1 \simeq \frac{1}{\sqrt{m'^2_{EE^c} + \lambda'^2_1 N^2_2}}\left[m'_{EE^c}|\nu_L\rangle - \lambda'_1 N_2|N_L\rangle\right]\,, \tag{5.14}$$

where the flavor states ν_L and N_L mix, like in the ALR model. From these results we see that the HDO method solves the problems in both models, under the assumption that there exists an intermediate scale at the order of 10^{11} GeV and both N_1 and N_2 get vev's at that scale.

Since there is only one light state (per generation, of course), there is no sterile neutrino in the model. The N_L only couples to the E, and which is very heavy, the net effect of the mixing (in either the ALR or Inert model) will be to lower the coupling of the electron neutrino to the electron and W_L-boson. For the ALR case (the Inert case is basically the same), the coupling is reduced by a factor of $\frac{\lambda_1 N_1}{\sqrt{\lambda_1^2 N_1^2 + \lambda_2^2 N_2^2}}$. Since the mixing must be small, $\lambda_2 N_2 \ll \lambda_1 N_1$, and this factor then becomes $1 - \frac{\lambda_2^2 N_2^2}{2\lambda_1^2 N_1^2}$.

This reduction would give a very clear signature for the model. The electron neutrino would not oscillate into a sterile neutrino (ignoring inter-generational mixing), and yet its coupling is reduced relative to the standard model. Similar reductions would occur for the muon and tau neutrino interactions. The phenomenological implications of this reduction will be discussed in the next Section.

C. The Additional Neutral Fermion Method

In some E_6-based superstring-based models, such as those with Calabi-Yau compactification, in addition to the $\mathbf{27}$ and $\overline{\mathbf{27}}$ representations of E_6 for the matter multiplets, there typically exist split multiplets, parts of the $\mathbf{27}+\overline{\mathbf{27}}$ representations, as well as some E_6 singlets $\mathbf{1}$ [44, 45, 46]. We have already considered the existence of such Higgs multiplets by considering $\overline{\mathbf{27}}$ components of the above $\mathbf{27}+\overline{\mathbf{27}}$ representation inducing dimension-5 terms (of the form discussed in the previous subsection). In addition to the $(\mathbf{27})^{\mathbf{3}}$ and the higher-dimensional $(\mathbf{27}\cdot\overline{\mathbf{27}})^{\mathbf{2}}$ types of interactions, we may have $\mathbf{27}\cdot\overline{\mathbf{27}}\cdot\mathbf{1}$ type of interactions as well. The Additional Neutral Fermion (ANF) method follows this approach. The existence of E_6 singlets (and thus the $\mathbf{27}\cdot\overline{\mathbf{27}}\cdot\mathbf{1}$ interactions) has been discussed in different context of the superstring models [38, 39, 40] to tackle the rapid proton decay problem, large neutrino mass problem and others. In order to give light neutrino masses consistent with present experimental observations, the additional Higgs fields are required to have vev's chose in a strong hierarchical way, which seems unnatural. Such an odd pattern, however, is not necessary in the non-SUSY versions of the models discussed here. We discuss the method in the ALR symmetric model and later point out the difference with the Inert model.

In the ALR model, we consider one additional E_6 neutral fermion singlet[8] ϕ, and one pair of $\mathbf{27}+\overline{\mathbf{27}}$ Higgs multiplets $H+\overline{H}$ (the Betti-Hodge number $b_{1,1}=1$). We do not include a corresponding $\mathbf{27}+\overline{\mathbf{27}}$ chiral fermion multiplet relevant for supersymmetrized versions of the models considered in future studies.[9] Let us assume that both H and $\overline{H}$ have ν^c-like and S^c-like components $H_{\nu^c,S^c}, \overline{H}_{\nu^c,S^c}$. Since we don't want to alter the interactions in the

[8] For simplicity, we assume one additional field ϕ even when we extend our discussion to the three generation case later in this section.

[9] In principle, one can add such new fields and the corresponding interactions. We would like to be as conservative as possible as far as the number of new parameters are concerned.

$(\mathbf{27})^{\mathbf{3}}$ sector discussed earlier, we assume that only $\overline{H}_{\nu^c,S^c}$ get nonzero vev's and further, that there is a Z_2 discrete symmetry under which all fields except ϕ, H_{ν^c,S^c} and $\overline{H}_{\nu^c,S^c}$ have even charges. Therefore, two additional gauge invariant interactions for one generation survive of the form

$$\Delta\mathcal{L}^{\phi}_{ALR} = \lambda_S S^c_L \overline{H}_{S^c}\phi + \lambda_\nu \nu^c_L \overline{H}_{\nu^c}\phi \,. \tag{5.15}$$

Then, the mass matrix in the neutral fermion sector in the $(\nu_L, N_L, N^c_L, \nu^c_L, S^c_L, \phi)$ basis can be obtained directly by adding a column and a row for ϕ field to the one given in Eq. (3.4)

$$\mathcal{M}_{\rm neutral} = \begin{pmatrix} 0 & 0 & \lambda_2 N_2 & \lambda_2 v_3 & 0 & 0 \\ 0 & 0 & -\lambda_1 N_1 & 0 & -\lambda_1 v_3 & 0 \\ \lambda_2 N_2 & -\lambda_1 N_1 & 0 & \lambda_2 v_2 & -\lambda_1 v_1 & 0 \\ \lambda_2 v_3 & 0 & \lambda_2 v_2 & 0 & 0 & \lambda_\nu V \\ 0 & -\lambda_1 v_3 & -\lambda_1 v_1 & 0 & 0 & \lambda_S \mu \\ 0 & 0 & 0 & \lambda_\nu V & \lambda_S \mu & 0 \end{pmatrix}, \tag{5.16}$$

where we define $\langle \overline{H}_{S^c} \rangle \equiv \mu$ and $\langle \overline{H}_{\nu^c} \rangle \equiv V$.

The eigenvalues can be found by following the same methodology as before and under the assumption $v_i, m_{ee^c}, m_{\nu\nu^c} \ll N_1, N_2, \mu, V$ (we assume $N_i \sim \mu, V$) giving

$$\begin{aligned} R_{1,2} &\simeq \pm \frac{m_{\nu\nu^c} m_{ee^c} (\lambda_2 N_2)(\lambda_S \mu)(\lambda_\nu V)}{(\lambda_2^2 N_2^2 + m^2_{EE^c})\,(\lambda_S^2 \mu^2 + \lambda_\nu^2 V^2)}\,, \\ R_{3,4} &\simeq \pm\sqrt{\lambda_S^2 \mu^2 + \lambda_\nu^2 V^2}\,, \\ R_{5,6} &\simeq \pm\sqrt{\lambda_2^2 N_2^2 + m^2_{EE^c}}\,. \end{aligned} \tag{5.17}$$

Now, we have two light eigenvalues $R_{1,2}$. The masses of these states can be approximated as $(m_{\nu\nu^c} m_{ee^c})/m_{EE^c}$ and could possibly be in the experimentally favored region while obeying the the experimental bounds on $\nu_L - N_L$ mixing. It is straightforward to get the mass eigenstates corresponding to the above eigenvalues. The transformation matrix equation from mass to flavor eigenstates is given by

$$\begin{pmatrix} |\nu_L\rangle \\ |N_L\rangle \\ |N_L^c\rangle \\ |\nu_L^c\rangle \\ |S_L^c\rangle \\ |\phi\rangle \end{pmatrix} = \begin{pmatrix} \frac{m_{EE^c}\,\cos\theta}{R_5} & \frac{m_{EE^c}\,\sin\theta}{R_5} & 0 & 0 & \frac{1}{\sqrt{2}}\frac{\lambda_2 N_2}{R_5} & \frac{1}{\sqrt{2}}\frac{\lambda_2 N_2}{R_5} \\ \frac{-\lambda_2 N_2\,\cos\theta}{R_5} & \frac{-\lambda_2 N_2\,\sin\theta}{R_5} & 0 & 0 & \frac{1}{\sqrt{2}}\frac{m_{EE^c}}{R_5} & \frac{1}{\sqrt{2}}\frac{m_{EE^c}}{R_5} \\ 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ \frac{-\lambda_S\,\mu\,\sin\theta}{R_3} & \frac{\lambda_S\,\mu\,\cos\theta}{R_3} & \frac{1}{\sqrt{2}}\frac{\lambda_\nu V}{R_3} & \frac{1}{\sqrt{2}}\frac{\lambda_\nu V}{R_3} & 0 & 0 \\ \frac{\lambda_\nu V\,\sin\theta}{R_3} & \frac{-\lambda_\nu V\,\cos\theta}{R_3} & \frac{1}{\sqrt{2}}\frac{\lambda_S\,\mu}{R_3} & \frac{1}{\sqrt{2}}\frac{\lambda_S\,\mu}{R_3} & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 & 0 \end{pmatrix} \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \\ |\nu_3\rangle \\ |\nu_4\rangle \\ |\nu_5\rangle \\ |\nu_6\rangle \end{pmatrix} ,\tag{5.18}$$

where R_3 and R_5 are given in Eq. (5.17). The parameter θ is arbitrary in the model, but it would be fixed both by the requirement that the coupling of W_L to neutrinos and leptons must be in agreement with the experimental data and by the required mixing angle between active and sterile neutrinos. The mass eigenstates $|\nu_3\rangle, |\nu_4\rangle, |\nu_5\rangle$, and $|\nu_6\rangle$ corresponding to eigenvalues $R_{3,4,5,6}$ respectively are heavy and irrelevant to our discussion at low energies. There are two light mass eigenstates of the form

$$\begin{aligned} |\nu_1\rangle &= \cos\theta\left(\frac{m_{EE^c}}{R_5}|\nu_L\rangle - \frac{\lambda_2 N_2}{R_5}|N_L\rangle\right) + \sin\theta\left(\frac{\lambda_\nu V}{R_3}|S_L^c\rangle - \frac{\lambda_S\,\mu}{R_3}|\nu_L^c\rangle\right) , \\ |\nu_2\rangle &= \sin\theta\left(\frac{m_{EE^c}}{R_5}|\nu_L\rangle - \frac{\lambda_2 N_2}{R_5}|N_L\rangle\right) - \cos\theta\left(\frac{\lambda_\nu V}{R_3}|S_L^c\rangle - \frac{\lambda_S\,\mu}{R_3}|\nu_L^c\rangle\right) . \end{aligned}\tag{5.19}$$

The above results apply to the Inert group, with an additional constraint coming from $SU(2)_I$ symmetry. Since ν_L and S_L^c form an $SU(2)_I$ doublet, the couplings λ_ν and λ_S are required to be equal.

Thus, we have two interesting features of the model. The slight suppression of the coupling of the active neutrino discusssed in the last subsection is present. However, now we *also* have a sterile neutrino with an arbitrary mixing angle with the active neutrino. This model could then easily accommodate the LSND result (if confirmed by MiniBooNE).

With the addition of only one singlet, for simplicity, there are three active neutrinos. In this case, λ_S and λ_ν have generation indices. Each active neutrino has a light mass, and will mix with an arbitrary mixing angle with the sterile neutrino. Note that in the single-generation case, the two light mass eigenstates are, to leading order, identical. Thus, if the mixing angle is small for two of the three generations, we will have a $\mathbf{2}+\mathbf{2}$ structure, whereas if it is sizeable for all three generations, there will be a $\mathbf{3}+\mathbf{1}$ structure. Of course, one could introduce several singlet fields, giving more complicated structures.

VI. DISCUSSION OF THE RESULTS

If the LSND result is confirmed by MiniBooNE, the existence of sterile neutrino(s) at low energies might be unavoidable. Thus is it important to analyze extensions of the Standard Model which predict the existence of extra neutral fermions, and verify that they have the desired experimental features. Though we have explicitly considered here the E_6 subgroups, $SU(3)_C \otimes SU(2)_L \otimes SU(2)_{R'} \otimes U(1)_V$ (ALR) and $SU(3)_C \otimes SU(2)_L \otimes SU(2)_I \otimes U(1)_Y$ (Inert), and concentrated on the neutrino spectrum in non-SUSY framework, our work is valid for the $SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_V$ (LR) group as well.

These models predict several exotic neutral fermions. We have shown that both the ALR and Inert models generally predict neutrino sectors inconsistent with current observations. The lightest state turns out to contain only $SU(2)_L$ singlets (ν_L^c and S_L^c) which do not interact with SM particles. Additionally, in contradiction with present experimental observations, two more light neutrino states with masses around the up quark mass exist. The main reason for such a spectrum is the existence of tree level Dirac mass term in the Lagrangian. We have discussed three possible remedies to this problem.

The most attractive one is the Discrete Symmetry (DS) method which only requires imposing an extra symmetry. The aim is to eliminate the tree level Dirac mass term by assigning suitable charges to the fields under some discrete symmetries, and generate Dirac neutrino masses through radiative corrections. The discrete symmetry needs to distinguish generations. As discussed earlier, there is no way to induce a non-zero one-loop Dirac mass while eliminating the tree level term. The only way out is to have a $SU(2)_L$ Higgs doublet (a part of the bidoublet) with vanishing vev. For this, we considered the simplest symmetry, Z_3. It leads to Dirac masses from one-loop diagrams which are estimated around 10^{-1} eV, by imposing an upper bound to the product of the Yukawa couplings of the order of 10^{-8}. It is also possible to generate very light Majorana masses for S_L^c and ν_L^c. Since these masses are much smaller than the Dirac mass term for ν_L, a spectrum with pseudo-Dirac neutrinos is obtained.

The Higher Dimensional Operators (HDO), the second method, requires additional Higgs fields from **27**-plet of E_6 and the existence of some intermediate scale. We introduce interactions which are suppressed by one power of the compactification scale, through dimension-5 operators. The method solves the mass problems but does not predict any sterile neutrino component(s) in the lightest neutrino state, which is an admixture of ν_L and N_L. The effect of the mixing will be to lower the electron neutrino coupling to the electron and the W_L boson by a factor of $1 - \frac{1}{2}\Delta_e^2$, where $\Delta_e = \lambda_2 N_2/\lambda_1 N_1$. The reduction for the muon and tau neutrino interactions will be given by the same expression, with Δ_e being replaced by Δ_μ and Δ_τ (which depend on different λ_i and N_i). The phenomenological implications are interesting. If the Δ_i are different, then $e - \mu - \tau$ universality will be violated in neutrino interactions. By comparing the muon decay rate and the rate for leptonic tau decays, one finds [42, 43] that the reductions of $1 - \frac{1}{2}\Delta_i^2$ cannot differ by more than 0.005. Even if the Δ_i are all the same, however, one would still find a discrepancy in, for example, $\tau \to \pi\nu_\tau$ vs. $\tau \to \mu\overline{\nu}_\mu\nu_\tau$, which would depend on Δ_μ, with a similar dependence on the electronic decay. Comparing all of these bounds, we find that none of the reductions can exceed 0.005, leading to a bound, for each generation, of $\lambda_2 N_2/\lambda_1 N_1 < 0.1$, which is not particularly fine-tuned. A more detailed study comparing many hadronic decays with the leptonic decays of the τ could lead to a somewhat more precise bound (or, better yet, an indication of a discrepancy).

The last method we have discussed is the Additional Neutral Fermion (ANF), which requires the existence of both new particles and discrete symmetries. If one considers an E_6 singlet field, the additional interactions will be of the type $\mathbf{27} \cdot \overline{\mathbf{27}} \cdot \mathbf{1}$, which further require additional Higgs doublets from the $\mathbf{27} + \overline{\mathbf{27}}$ representation. In order not to alter already existing couplings, the vev's of the new fields need to be chosen suitably, together with an additional Z_2 symmetry. Under these circumstances we obtain two light states given in Eq. (5.19). The neutrino states have an active neutrino part of exactly the form predicted by the HDO method, but this time they mix with a sterile flavor state (formed by ν_L^c and S_L^c). The mixing is completely arbitrary. If we extend the picture to three generations, the model contains two structures, $\mathbf{2+2}$ and $\mathbf{3+1}$, which have been discussed extensively in the literature [47]. When the above mixing is sizable only for one generation, only the $\mathbf{2+2}$ structure arises naturally, since the states in Eq. (5.19) are degenerate in the leading order. Otherwise, $\mathbf{3+1}$ is possible. More realistically, when we include three generations of ν_L^c and S_L^c, we obtain a $\mathbf{3+3}$ structure.

Recent analyses show that neither $\mathbf{2+2}$ nor $\mathbf{3+1}$ provide a good description of the combined atmospheric, solar, reactor, and accelerator data even though it appears that $\mathbf{3+1}$ works better. However, there is no consensus about whether the scenarios with four neutrinos are ruled out or not [15, 47]. From our considerations, the ANF method allows both $\mathbf{3+2}$ or $\mathbf{3+3}$ structures, which enhance the effects in favor of LSND data [15].

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