The $b \to sgg$ decay in the two and three Higgs doublet models with CP violating effects

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Abstract

We study the decay width and CP-asymmetry of the inclusive process $b \to sgg$ (g denotes gluon) in the three and two Higgs doublet models with complex Yukawa couplings. We analyse the dependencies of the differential decay width and CP-asymmetry to the s-quark energy E_s and CP violating parameter θ. We observe that there exist a considerable enhancement in the decay width and CP asymmetry is at the order of 10^{-2}. Further, it is possible to predict the sign of C_7^{eff} using the CP asymmetry.

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1 Introduction

Rare B-meson decays, induced by flavor changing neutral currents (FCNC), involve at the loop level in the standard model (SM), therefore they are phenomenologically rich. The measurements of the branching ratio (Br), CP asymmetry (A_{CP}), forward-backward asymmetry, polarization effects, etc., provide restrictions on the SM parameters, such as the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, leptonic decay constants, etc. In addition, the possibility of replacing the SM particles by non-standard ones results in the sensitivity of these decays beyond the SM, like multi-Higgs doublet models, minimal supersymmetric extension of the SM (MSSM) [1], etc. The experimental effort at SLAC, KEK B-factories, HERA-B and possible future accelerators [2, 3] stimulate the theoretical studies on these rare decays.

Among B-meson decay modes, inclusive $b \rightarrow sg$ decay becomes attractive since it is theoretically clean and affected by loop contributions due to new physics beyond the SM. In the literature, there are various theoretical calculations on the Br of this decay. In the SM, the Branching ratio of $b \rightarrow sg$ decay was calculated as $Br(b \rightarrow sg) \sim 0.2\%$ for on-shell gluon [4]. However, to decrease the averaged cham multiplicity η_c [5] and to increase kaon yields [6] the enhancement of $Br(b \rightarrow sg)$ is helpful. The possibilities for the enhancement is the addition of the QCD corrections and non-standard effects coming from the new physics. In [7, 8], $Br(b \rightarrow sg)$ was calculated in the 2HDM (Model I and II) for $m_{H^{\pm}} \sim 200\,GeV$ and $tan\,\beta \sim 5$ and it was found that there was an enhancement less than one order of magnitude. Further, this decay was studied in the supersymmetric models [9] and in the framework of model III 2HDM [10]. In the model III, the enhancement was found at least one order of magnitude larger compared to the SM one and it was observed that there was no contradiction with the CLEO data [11]

$$Br\,(b \rightarrow sg^*) < 6.8\,\% \tag{1}$$

for light-like gluon case. Recently, $O(\alpha_s)$ virtiual corrections and additional $O(\alpha_s)$ bremsstrahlung effects to the decay width of $b \rightarrow sg$ was calculated in the SM [12] and the enhancement in the Br was obtained as more than a factor of two larger of the previous SM results.

The inclusive process $b \rightarrow sgg$ is another decay which has Br at the same order as $Br\,(b \rightarrow sg)$ according to the studies in the literature [13, 14, 15]. This process becomes not only from the chain decay $b \rightarrow sg^*$ followed by $g^* \rightarrow gg$ but also from the emission of on-shell gluons from the quark lines to obey gauge invariance. In [14], the complete calculation was done in the

collinear and infrared singularity free region, in the SM and Br ratio was found at the order of magnitude 10^{-3}. In [10, 15] the additional contribution of gluon penguins in the model III was estimated as negligible. Recently $b \to sgg$ was studied in the model III with real Yukawa couplings [16] and a considerable enhancement was observed for the Br of the process even 2 orders of magnitude larger compared to the SM case.

In this work, we study the decay width Γ and the CP asymmetry A_{CP} of $b \to sgg$ decay in the model III and the $3HDM(O_2)$. The reason to study the $b \to sgg$ process is the possible considerable enhancement of Γ compared to the one in the SM and the measurable A_{CP} in the framework of the models underconsideration. In our theoretical calculations we choose the collinear and infrared singularity free kinematical region, following the procedure given in [14]. Here we take the source of CP violation as the complex Yukawa couplings appearing in both models.

The paper is organized as follows: In Section 2, we give a brief summary of the $3HDM(O_2)$ and present the calculation of the decay width of the inclusive $b \to sgg$ decay in the framework of the $3HDM(O_2)$ and the model III. Further we calculate the (differential) CP-asymmetry $(A_{CP}(E_s))$ A_{CP} of the process. Section 3 is devoted to discussion and our conclusions. In Appendixes, we give the form factors appearing in the matrix element of the decay underconsideration and summarize the theoretical results for the $3HDM(O_2)$.

2 The inclusive process $b \to sgg$ in the framework of the model III and $3HDM(O_2)$

The general Yukawa interaction in $3HDM$ is

$$
\begin{aligned}
\mathcal{L}_Y &= \eta_{ij}^U \bar{Q}_{iL} \tilde{\phi}_1 U_{jR} + \eta_{ij}^D \bar{Q}_{iL} \phi_1 D_{jR} + \xi_{ij}^U \bar{Q}_{iL} \tilde{\phi}_2 U_{jR} + \xi_{ij}^D \bar{Q}_{iL} \phi_2 D_{jR} \\
&+ \rho_{ij}^U \bar{Q}_{iL} \tilde{\phi}_3 U_{jR} + \rho_{ij}^D \bar{Q}_{iL} \phi_3 D_{jR} + h.c. \ ,
\end{aligned}
\tag{2}
$$

where L and R denote chiral projections $L(R) = 1/2(1 \mp \gamma_5)$, ϕ_i for $i = 1, 2, 3$, are three scalar doublets and $\eta_{ij}^{U,D}$, $\xi_{ij}^{U,D}$, $\rho_{ij}^{U,D}$ are the Yukawa matrices having complex entries, in general. With the choice of scalar Higgs doublets

$$
\phi_1 = \frac{1}{\sqrt{2}} \left[\begin{pmatrix} 0 \\ v + H^0 \end{pmatrix} + \begin{pmatrix} \sqrt{2}\chi^+ \\ i\chi^0 \end{pmatrix} \right] ,
$$

$$
\phi_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}H^+ \\ H^1 + iH^2 \end{pmatrix} , \quad \phi_3 = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}F^+ \\ H^3 + iH^4 \end{pmatrix} ,
$$

$$
\tag{3}
$$

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and the vacuum expectation values,

$$< \phi_1 > = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \; ; < \phi_2 > = 0 \; ; < \phi_3 > = 0 \; , \tag{4}$$

the SM particles carried by the first doublet and the information about the new physics by the others. The Yukawa interaction

$$\mathcal{L}_{Y,FC} = \xi_{ij}^U \bar{Q}_{iL} \tilde{\phi}_2 U_{jR} + \xi_{ij}^D \bar{Q}_{iL} \phi_2 D_{jR} + \rho_{ij}^U \bar{Q}_{iL} \tilde{\phi}_3 U_{jR} + \rho_{ij}^D \bar{Q}_{iL} \phi_3 D_{jR} + h.c. \; . \tag{5}$$

describes the Flavor Changing (FC) one beyond the SM. Here, the couplings $\xi^{U,D}$ and $\rho^{U,D}$ for the charged FC interactions are

$$\begin{aligned}
\xi_{ch}^U &= \xi_N \, V_{CKM} \; , \\
\xi_{ch}^D &= V_{CKM} \, \xi_N \; , \\
\rho_{ch}^U &= \rho_N \, V_{CKM} \; , \\
\rho_{ch}^D &= V_{CKM} \, \rho_N \; ,
\end{aligned} \tag{6}$$

and

$$\begin{aligned}
\xi_N^{U,D} &= (V_L^{U,D})^{-1} \xi^{U,D} \, V_R^{U,D} \; , \\
\rho_N^{U,D} &= (V_L^{U,D})^{-1} \rho^{U,D} \, V_R^{U,D} \; ,
\end{aligned} \tag{7}$$

where the index "N" in $\xi_N^{U,D}$ denotes the word "neutral". Note that the Yukawa interaction for the model III can be obtained by taking into account only two doublets ϕ_1, ϕ_2 and Yukawa couplings η_{ij}^U, ξ_{ij}^U.

The decay amplitude of the process $b \rightarrow sgg$ is given by

$$M(b \rightarrow sgg) = i \frac{\alpha_s \, G_F}{\sqrt{2}\pi} \epsilon_a^\mu(k_1) \epsilon_b^\nu(k_2) \bar{s}(p') T_{\mu\nu}^{ab} \, b(p) \; , \tag{8}$$

where $\epsilon_a^\mu(k)$ are polarization vectors of the gluons with color a and momentum k,

$$T_{\mu\nu}^{ab} = T_{\mu\nu} \frac{\lambda^b}{2} \frac{\lambda^a}{2} + T_{\mu\nu}^E \frac{\lambda^a}{2} \frac{\lambda^b}{2} \; . \tag{9}$$

Here $\frac{\lambda^a}{2}$ are the Gell-Mann matrices and $T_{\mu\nu}^E$ can be obtained by the replacements $k_1 \leftrightarrow k_2$, $\mu \leftrightarrow \nu$ in the function $T_{\mu\nu}$.

Since the process occurs at least at one-loop level in the SM, the function $T_{\mu\nu}$ have contributions coming from light and heavy internal quarks, namely, u, c, t. Internal quarks d, s, b can also give contribution to the process beyond the SM. In the case of heavy internal quark,

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t-quark, the terms $k^2_{external}/m^2_i$ $(m^2_W, m^2_{H^\pm}, m^2_{F^\pm})$ are neglected. However, for light internal quarks, $k^2_{external}/m^2_i$ terms can give considerable contribution. This forces us to parametrize the function $T_{\mu\nu}$ as

$$T_{\mu\nu} = T^{heavy}_{\mu\nu} + T^{light}_{\mu\nu} \ , \tag{10}$$

(for the explicit forms of $T^{heavy}_{\mu\nu}$ and $T^{light}_{\mu\nu}$ see Appendix A). On the other hand, $T^{ab}_{\mu\nu}$ can be divided into color symmetric and antisymmetric parts as [14]

$$T^{ab}_{\mu\nu} = T^+_{\mu\nu}\{\frac{\lambda^b}{2}, \frac{\lambda^a}{2}\} + T^-_{\mu\nu}[\frac{\lambda^b}{2}, \frac{\lambda^a}{2}] \ , \tag{11}$$

with

$$
\begin{aligned}
T^+_{\mu\nu} &= \frac{1}{2}(T_{\mu\nu} + T^E_{\mu\nu}) \ , \\
T^-_{\mu\nu} &= \frac{1}{2}(T_{\mu\nu} - T^E_{\mu\nu}) \ .
\end{aligned}
\tag{12}
$$

Finally, using the expression

$$\Gamma^{Sym\,(Asym)} \sim Tr(C_+(-)T^{+\,(-)}_{\mu\nu}\,(\not{p} + m_b))\,\bar{T}^{+\,(-)}_{\mu'\nu'}\ \not{p}')\,P^{\mu\mu'}\,P^{\nu\nu'} \ , \tag{13}$$

with the color factors $C_+ = \frac{(N_c^2-1)(N_c^2-2)}{2\,N_c}$ and $C_- = \frac{N_c\,(N_c^2-1)}{2}$ and the polarization sum of the on-shell gluons

$$P^{\mu\mu'} = -g^{\mu\mu'} + \frac{k_1^\mu\,k_2^{\mu'} + k_2^\mu\,k_1^{\mu'}}{k_1.k_2} \ ,$$

we get the differential decay width of the process

$$\frac{d^2\,\Gamma}{dE_s\,dE_1} = \frac{1}{2\pi^3}\frac{1}{8\,m_b}|\bar{M}|^2 \ . \tag{14}$$

Here E_s is the s-quark energy and E_1 is the energy of gluon with polarization $\epsilon^a_\mu(k_1)$. $\bar{M}$ is the average decay amplitude, $\bar{M} = \frac{1}{2\,J+1}\frac{1}{N_c}\,M$, and $J = \frac{1}{2}$, $N_c = 3$.

In the expressions, the symmetric and antisymmetric parts do not mix each other. Further, the decay width can be divided into three sectors (see Appendix A):left (Γ^L), right (Γ^R) and left-right (Γ^{LR}). Left one is created by the nonvanishing $k^2_{external}/m^2_{light}$ terms, however right sector contains the forms factors with parameters m^2_i/m^2_W and m^2_i/m^2_H where $i = u, c, t$ and H is one of the Higgs bosons. Left-right sector contains mixed terms.

Now we are ready to calculate the CP-violating asymmetry A_{CP} of the process $b \rightarrow sgg$. In the model III and $3HDM(O_2)$, the complex Yukawa couplings are possible sources for CP

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violation. Our procedure is to neglect neutral boson effects and all Yukawa couplings except $\bar{\xi}^U_{N,tt}$ and $\bar{\xi}^D_{N,bb}$ ($\bar{\epsilon}^U_{N,tt}$ and $\bar{\epsilon}^D_{N,bb}$) (see Discussion) in the model III ($3HDM(O_2)$) (see Appendix B for the definitions of $\bar{\epsilon}^U_{N,tt}$ and $\bar{\epsilon}^D_{N,bb}$). Therefore, in the model III ($3HDM(O_2)$), only the combination $\bar{\xi}^U_{N,tt}\bar{\xi}^D_{N,bb}$ ($\bar{\epsilon}^U_{N,tt}\bar{\epsilon}^D_{N,bb}$) is responsible for A_{CP}. Using the parametrization

$$\lambda_\theta = \begin{cases} \frac{1}{m_t m_b} \bar{\xi}^U_{N,tt} \bar{\xi}^D_{N,bb} e^{i\theta} & \text{(model III)}, \\[2ex] \frac{1}{m_t m_b} \bar{\epsilon}^U_{N,tt} \bar{\epsilon}^D_{N,bb} (cos^2\theta + isin^2\theta) & (3HDM(O_2)), \end{cases}$$

and the definition of differential CP asymmetry $A_{CP}(E_s)$

$$A_{CP}(E_s) = \frac{\frac{d^2\Gamma}{dE_s\,dE_1}(b \to sgg) - \frac{d^2\Gamma}{dE_s\,dE_1}(\bar{b} \to \bar{s}gg)}{\frac{d^2\Gamma}{dE_s\,dE_1}(b \to sgg) + \frac{d^2\Gamma}{dE_s\,dE_1}(\bar{b} \to \bar{s}gg)}, \tag{15}$$

we get

$$A_{CP}(E_s) = 2Im(\lambda_\theta)\,G_2(y_t)\,\frac{(Im(\Delta F_1 - \Delta i_2))\,\Omega}{\Lambda} \tag{16}$$

where

$$\begin{aligned} \Omega &= 18\,E_1\,m_b\,(2\,E1 - m_b)(-2\,E_s + m_b)\,((2\,E_s - m_b)\,m_b + 2\,E_1\,(2\,E_s + m_b)), \\ \Lambda &= -2\,|\tilde{F}_2|^2\,m_b\,(2496\,E_1^5 + 192\,E_1^4\,(20\,E_s - 23\,m_b) + m_b^3\,(-28\,E_1^2 + 44\,E_s\,m_b - 15\,m_b^2) \\ &+ 2\,E_1\,m_b^2\,(172\,E_s^2 - 208\,E_s\,m_b + 69\,m_b^2) - 4\,E_1^2\,m_b\,(316\,E_s^2 - 562\,E_s\,m_b + 213\,m_b^2) \\ &+ 8\,E_1^3\,(204\,E_s^2 - 638\,E_s\,m_b + 357\,m_b^2)) \\ &+ 2\,Re(\tilde{F}_2)\,Re(\Delta F_1 - \Delta i_2)\,\Omega + 8\,E_1^2\,(2\,E_1 - m_b)\,(-2\,E_s + m_b)(7\,|\Delta i_5|^2\,E_s\,m_b + 9\,|\Delta i_2|^2 \\ &(8\,E_1^2 + 8\,E_1\,(E_s - m_b) + m_b\,(-3\,E_s + 2\,m_b)) \\ &+ 9\,|\Delta F_1|^2\,(8\,E_1^2 + 8\,E_1\,(E_s - m_b) + m_b\,(-3\,E_s + 2\,m_b)) \\ &- 18\,Re(\Delta F_1^*\,\Delta i_2)\,(8\,(E_1^2 + E_1\,E_s - E_1\,m_b) - 3\,E_s\,m_b + 2\,m_b^2). \end{aligned} \tag{17}$$

Here θ is the CP violating parameter which is restricted by the experimental upper limit of the neutron electric dipole moment eq. (22) and $\tilde{F}_2 = F_2^{3HDM} - F_2^{SM}(0)$, ΔF_1, Δi_2, and Δi_5 are the Wilson coefficients (eqs.(24) and (30)). For the calculation of the CP asymmetry A_{CP}

$$A_{CP} = \frac{\Gamma(b \to sgg) - \Gamma(\bar{b} \to \bar{s}gg)}{\Gamma(b \to sgg) + \Gamma(\bar{b} \to \bar{s}gg)}, \tag{18}$$

the integration over E_1 and E_s should be done. However there are collinear divergences at the boundary of the kinematical region. To overcome these divercences we follow the procedure given in [14], namely taking a cutoff c in the integration over phase space as:

$$\frac{m_b}{2} - E_s \leq E_1 \leq \frac{m_b}{2}(1 - c), \tag{19}$$

and

$$c\frac{m_b}{2} \leq E_s \leq \frac{m_b}{2}(1-c) \, , \tag{20}$$

with $c = 0.1$. Note that left-right sector gives small contribution to Γ, however this part is responsible for the A_{CP}. Further A_{CP} contains only antisymmetric sector.

3 Discussion

In the general 3HDM model, there are many free parameters, such as masses of charged and neutral Higgs bosons, complex Yukawa matrices, $\xi_{ij}^{U,D}$, $\rho_{ij}^{U,D}$ where i,j are quark flavor indices. The additional global $O(2)$ symmetry in the Higgs flavor space connects the Yukawa matrices in the second and third doublet and also keeps the masses of new charged (neutral) Higgs particles in the third doublet to be the same as the ones in the second doublet (Appendix B). Further, the Yukawa couplings, which are entries of Yukawa matrices, can be restricted using the experimental measurements, $\Delta F = 2$ mixing, the ρ parameter [17] and the CLEO measurement [18],

$$Br(B \to X_s \gamma) = (3.15 \pm 0.35 \pm 0.32) \, 10^{-4} \, . \tag{21}$$

In our calculations, we neglect all Yukawa couplings except $\bar{\xi}_{N,tt}^{U}$, $\bar{\xi}_{N,bb}^{D}$, $\bar{\rho}_{N,tt}^{U}$ and $\bar{\rho}_{N,bb}^{D}$ by respecting these measurements. The same restrictions are done in the model III case and only $\bar{\xi}_{N,tt}^{U}$ and $\bar{\xi}_{N,bb}^{D}$ are taken into account.

  In this section, we study the s quark energy E_s dependency of the differential decay width $\frac{d\Gamma}{d E_s}$, differential CP-asymmetry $A_{CP}(E_s)$ and the parameter $sin\theta$ dependency of the decay width Γ, CP-asymmetry A_{CP} for the inclusive decay $b \to sgg$ in the framework of the model III and $3HDM(O_2)$. In our analysis, we restrict the parameters θ, $\bar{\epsilon}_{N,tt}^{U}$ and $\bar{\epsilon}_{Nbb}^{D}$ ($\bar{\xi}_{N,tt}^{U}$ and $\bar{\xi}_{Nbb}^{D}$ in the model III) using the constraint for $|C_7^{eff}|$, $0.257 \leq |C_7^{eff}| \leq 0.439$ where the upper and lower limits were calculated in [19] following the procedure given in [20]. Here C_7^{eff} is the effective magnetic dipole type Wilson coefficient for $b \to s\gamma$ vertex (see [19]). The above restriction allows us to define a constraint region for the parameter $\bar{\epsilon}_{N,tt}^{U}$ ($\bar{\xi}_{N,tt}^{U}$) in terms of $\bar{\epsilon}_{N,bb}^{D}$ ($\bar{\xi}_{N,bb}^{D}$) and θ in the $3HDM(O_2)$ (the model III). Our numerical calculations based on this restriction and the constraint for the angle θ, due to the experimental upper limit of neutron electric dipole moment, namely

$$d_n < 10^{-25} \text{e·cm} \tag{22}$$

which places an upper bound on the couplings with the expression in $3HDM(O_2)$ (*model III*): $\frac{1}{m_t m_b}(\bar{\epsilon}^U_{N,tt} \bar{\epsilon}^{*D}_{N,bb})sin^2\theta < 1.0$ $(\frac{1}{m_t m_b}(\bar{\xi}^U_{N,tt} \bar{\xi}^{*D}_{N,bb})sin\theta < 1.0)$ for $m_{H^\pm} \approx 200$ GeV [21]. Throughout these calculations, we take the charged Higgs mass $m_{H^\pm} = 400\,GeV$, and we use the input values given in Table (1).

Parameter	Value
m_c	1.4 (GeV)
m_b	4.8 (GeV)
λ_t	0.04
m_t	175 (GeV)
m_W	80.26 (GeV)
m_Z	91.19 (GeV)
Λ_{QCD}	0.214 (GeV)
$\alpha_s(m_Z)$	0.117
c	0.1

Table 1: The values of the input parameters used in the numerical calculations.

In Fig. 1 (2) we plot $\frac{d\Gamma}{dE_s}$ with respect to the s quark energy E_s, for $sin\theta = 0.5$, $\bar{\xi}^D_{N,bb} = 40\,m_b$ and $|r_{tb}| = |\frac{\bar{\xi}^U_{N,tt}}{\bar{\xi}^D_{N,bb}}| < 1$. $\frac{d\Gamma}{dE_s}$ is restricted in the region bounded by solid (dashed) lines for $C_7^{eff} > 0$ ($C_7^{eff} < 0$). Dotted line represents the SM contribution. There is a large enhancement in the differential decay width for $C_7^{eff} > 0$ and $C_7^{eff} < 0$ in both models. (see [16] for the model III with real Yukawa couplings). In the $3HDM(O_2)$, the enhancement is smaller and the restriction regions are broader compared to the ones in the model III.

Fig. 3 is devoted to the $sin\theta$ dependence of Γ for $\bar{\xi}^D_{N,bb} = 40\,m_b$ and $C_7^{eff} < 0$ in the region $|r_{tb}| < 1$. Here Γ in the model III ($3HDM(O_2)$) is restricted in between solid (dashed) lines. As shown in the figure, the decay width Γ increases with increasing $sin\theta$. Further, the upper and lower bounds for Γ in the model III exceed the ones in the $3HDM(O_2)$ especially for the intermediate values of the parameter $sin\theta$. Further Γ can reach the value $10^{-3}\,GeV$ in both models and this is a considerable enhancement compared to the SM one, which is at the order of magnitude $10^{-5}\,GeV$ (see [14, 16]).

Fig. 4 (5) shows the E_s dependence of $A_{CP}(E_s)$ in the model III ($3HDM(O_2)$). Here solid (dashed) lines are the boundaries of the allowed regions of $A_{CP}(E_s)$ for $C_7^{eff} > 0$ ($C_7^{eff} < 0$). In model III, the restriction region for $A_{CP}(E_s)$ is narrow for $C_7^{eff} > 0$ and it has only negative values at the order of magnitude 10^{-3}. However, for $C_7^{eff} < 0$, this region is broader and contains both negative and positive values. The possible values of $|A_{CP}(E_s)|$ reaches $\sim 4\%$ for $0.8\,GeV \leq E_s \leq 1.0\,GeV$. In the $3HDM(O_2)$, upper and lower boundaries of the allowed

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region for $A_{CP}(E_s)$ are almost coincides for $C_7^{eff} > 0$ and this region becomes narrower for $C_7^{eff} < 0$ compared to the one in the model III. In this model $|A_{CP}(E_s)|$ can be $\sim 2.5\%$ as a maximum value.

In Fig. 6 and Fig. 7, we represent the $sin\theta$ dependence of A_{CP} in the model III and $3HDM(O_2)$. A_{CP} is restricted in the narrow region bounded by solid lines for $C_7^{eff} > 0$ and it reaches -0.8% for $sin\theta = 0.7$ in both models. All possible values of A_{CP} are negative in this case. However, for $C_7^{eff} < 0$, allowed region becomes broader and A_{CP} can take positive and negative values. For this case, $|A_{CP}|$ can reach 3.4%. Note that the restricted regions are broader in the model III compared to the ones in $3HDM(O_2)$.

As a conclusion, we get an enhancement in the decay width of the process $b \to sgg$ in both models, model III and $3HDM(O_2)$. This enhancement is too large to respect the total decay width $\Gamma^{tot}(b \to sX) = 3.50 \pm 1.50 \, 10^{-3} \, GeV$ for $C_7^{eff} > 0$. For $C_7^{eff} < 0$, Γ can reach the values more than two orders of magnitude larger compared to the SM case. Further, we study A_{CP} of the process $b \to sgg$ in both models. In the SM, the only source for the CP-violation is the complex Cabbibo-Kobayashi-Maskawa matrix elements and almost there is no violation for this process. However in the model III and the $3HDM(O_2)$, the absolute value of A_{CP} can reach to $3 - 4\%$, which is a measurable quantity. In addition, we observe that C_7^{eff} is necessarily negative if A_{CP} has positive values. Therefore the experimental study of the decay width Γ and A_{CP} of the process $b \to sgg$ can give important clues for the physics beyond the SM and also the sign of C_7^{eff}.

Appendix

A   The form factors appearing in the $b \to sgg$ decay

The function $T_{\mu\nu}$ can be divided into two parts:

$$T_{\mu\nu} = T_{\mu\nu}^{heavy} + T_{\mu\nu}^{light} \ .$$

Here $T_{\mu\nu}^{heavy}$ is the contribution due to the heavy internal quark and neglecting s-quark mass, it is given as

$$
\begin{aligned}
T_{\mu\nu}^{heavy} &= -i\,\lambda_t\,F_2^{3HDM}\,\{(\frac{2\,p'_\nu + \gamma_\nu\,\not{k}_2}{2\,p'.k_2}\,\sigma_{\mu\alpha}k_1^\alpha + \sigma_{\nu\alpha}k_2^\alpha\,\frac{2p_\mu - \not{k}_1\gamma_\mu}{-2p.k_1}) \\
&+ \frac{1}{q^2}(2\,\sigma_{\alpha\beta}k_1^\alpha\,k_2^\beta g_{\mu\nu} + 2\,\sigma_{\nu\alpha}k_{2\,\mu}\,q^\alpha - 2\,\sigma_{\mu\alpha}k_{1\,\nu}\,q^\alpha + \sigma_{\mu\nu}q^2)\}\,m_b\,R \ .
\end{aligned}
\tag{23}
$$

where q is the momentum transfer, $q = k_1 + k_2$, λ_t is the CKM matrix combination $\lambda_t = V_{tb}V_{ts}^*$ and F_2^{3HDM} is the form factor

$$F_2^{3HDM} = F_2^{SM}\,(x_t) + F_2^{Beyond}\,(y_t, y'_t). \tag{24}$$

In eq. (24), $F_2^{SM}\,(x_t)$ is the magnetic dipole form factor of $b \to sg^*$ vertex

$$F_2^{SM}(x_t) = \frac{-8 + 38\,x_t - 39\,x_t^2 + 14\,x_t^3 - 5\,x_t^4 + 18\,x_t^2\,ln\,x_t}{12\,(-1 + x_t)^4} \ , \tag{25}$$

and $F_2^{Beyond}\,(y_t, y'_t)$ is the contribution coming from the charged Higgs bosons in $3HDM(O)_2$:

$$
\begin{aligned}
F_2^{Beyond}\,(y_t, y'_t) &= \frac{1}{m_t^2}\,(\bar{\xi}_{N,tt}^{*U} + \bar{\xi}_{N,tc}^{*U}\frac{V_{cs}^*}{V_{ts}^*})\,(\bar{\xi}_{N,tt}^{U} + \bar{\xi}_{N,tc}^{U}\frac{V_{cb}}{V_{tb}})G_1(y_t) \\
&+ \frac{1}{m_t m_b}\,(\bar{\xi}_{N,tt}^{*U} + \bar{\xi}_{N,tc}^{*U}\frac{V_{cs}^*}{V_{ts}^*})\,(\bar{\xi}_{N,bb}^{D} + \bar{\xi}_{N,sb}^{D}\frac{V_{ts}}{V_{tb}})G_2(y_t) \\
&+ \frac{1}{m_t^2}\,(\bar{\rho}_{N,tt}^{*U} + \bar{\rho}_{N,tc}^{*U}\frac{V_{cs}^*}{V_{ts}^*})\,(\bar{\rho}_{N,tt}^{U} + \bar{\rho}_{N,tc}^{U}\frac{V_{cb}}{V_{tb}})G_1(y'_t) \\
&+ \frac{1}{m_t m_b}\,(\bar{\rho}_{N,tt}^{*U} + \bar{\rho}_{N,tc}^{*U}\frac{V_{cs}^*}{V_{ts}^*})\,(\bar{\rho}_{N,bb}^{D} + \bar{\rho}_{N,sb}^{D}\frac{V_{ts}}{V_{tb}})G_2(y'_t)
\end{aligned}
\tag{26}
$$

with

$$
\begin{aligned}
G_1(y) &= \frac{y}{12\,(-1 + y)^4}((-1 + y)\,(-2 - 5\,y + y^2) + 6\,y\,ln\,y)) \ , \\
G_2(y) &= \frac{1}{2\,(-1 + y)^4}\,(y\,(3 - 4\,y + y^2) + 2\,(-1 + y)\,y\,ln\,y) \ .
\end{aligned}
\tag{27}
$$

where $y_t = \frac{m_t^2}{m_{H^\pm}}$ and $y_t' = \frac{m_t^2}{m_{F^\pm}}$ (see appendix B). In eq. (26) we used the redefinition

$$\xi\,(\rho)^{U,D} = \sqrt{\frac{4G_F}{\sqrt{2}}}\,\bar{\xi}(\bar{\rho})^{U,D}\,. \tag{28}$$

In eq. (10), $T_{\mu\nu}^{light}$ contains two different parts related to the light internal quarks. The first one is obtained from $T_{\mu\nu}^{heavy}$ with the replacement $F_2^{3HDM} \to -F_2^{SM}(0)$ and the second one $T_{2\,\mu\nu}^{light}$ is the contribution due to the non-vanishing $k_{external}^2/m_{light}^2$ terms

$$
\begin{aligned}
T_{2\,\mu\nu}^{light} &= -\lambda_t\,\{(\Delta\,i_2 - \Delta\,F_1)(\not{k}_1 - \not{k}_2)\,g_{\mu\nu}\,L + \Delta\,i_5\,i\,\epsilon_{\alpha\mu\nu\beta}\gamma^\beta(k_1^\alpha - k_2^\alpha)\,L \\
&\quad -\ 2\Delta\,F_1\,(\gamma_\nu\,k_{2\,\mu} - \gamma_\mu\,k_{1\,\nu})\,L\}
\end{aligned}
\tag{29}
$$

where

$$
\begin{aligned}
\Delta F_1 &= -\frac{2}{9} - \frac{4}{3}\frac{Q_0(z)}{z} - \frac{2}{3}Q_0(z)\,, \\
\Delta i_2 &= -\frac{5}{9} - 2\frac{Q_-(z)}{z} + \frac{8}{3}\frac{Q_0(z)}{z} - \frac{2}{3}Q_0(z)\,, \\
\Delta i_5 &= -1 - 2\frac{Q_-(z)}{z}\,,
\end{aligned}
\tag{30}
$$

with

$$
\begin{aligned}
Q_0(z) &= -2 - (u_+ - u_-)(ln\frac{u_-}{u_+} + i\pi)\,, \\
Q_-(z) &= \frac{1}{2}(ln\frac{u_-}{u_+} + i\pi)^2\,.
\end{aligned}
\tag{31}
$$

Here the parameter $u_\pm$ is

$$u_\pm = \frac{1}{2}(1 \pm \sqrt{1 - \frac{4}{z}})\,, \tag{32}$$

and

$$z = \frac{q^2}{m_i^2}\,,\quad i = u, c\,. \tag{33}$$

Finally the function $T_{\mu\nu}$ reads as

$$T_{\mu\nu} = \alpha_R\,(T_{\mu\nu}^{heavy} + T_{\mu\nu}^{heavy}(F_2^{3HDM} \to -F_2^{SM}(0))) + \alpha_L\,T_{2\,\mu\nu}^{light}\,. \tag{34}$$

Here α_R, α_L are real parameters to seperate the parts with factors R and L in the fuction $T_{\mu\nu}$. With this parametrization Γ can be written as

$$\Gamma = \alpha_R^2\Gamma^R + \alpha_L^2\Gamma^L + \alpha_R\alpha_L\Gamma^{LR}|_{\alpha_L \to 1, \alpha_R \to 1}\,. \tag{35}$$

Note that the expressions for the model III case can be obtained by disregarding the Yukawa couplings $\bar{\rho}_{N,ij}^{U,(D)}$ in eq. (26).

B $3HDM(O_2)$

In the multi-Higgs doublet $(n > 2)$ models, the Higgs sector is extended and therefore the number of free parameters, namely, masses of charged and neutral Higgs particles, Yukawa couplings, extremely increases. In our problem we choose $n = 3$ and to overcome the difficulty coming from the large number of free parameters we consider 3 Higgs scalars as orthogonal vectors in the Higgs flavor space, denoting by the index m", where $m = 1, 2, 3$. At this stage we introduce a new global $O(2)$ symmetry on the Higgs sector [22]

$$
\begin{aligned}
\phi'_1 &= \phi_1 , \\
\phi'_2 &= \cos\alpha\ \phi_2 + \sin\alpha\ \phi_3 , \\
\phi'_3 &= -\sin\alpha\ \phi_2 + \cos\alpha\ \phi_3 ,
\end{aligned}
\tag{36}
$$

where α is the global parameter, which represents a rotation of the vectors ϕ_2 and ϕ_3 along the axis that ϕ_1 lies, in the Higgs flavor space. This transformation keeps the kinetic term of 3HDM Lagrangian invariant:

$$
\begin{aligned}
\mathcal{L}_{Kinetic} &= (D_\mu \phi_i)^+ D^\mu \phi_i = \\
&\quad (\partial_\mu \phi_i^+ + i\frac{g'}{2} B_\mu \phi_i^+ + i\frac{g}{2}\phi_i^+ \frac{\vec{\tau}}{2}\vec{W}_\mu) \\
&\quad (\partial^\mu \phi_i - i\frac{g'}{2} B^\mu \phi_i - i\frac{g}{2}\phi_i \frac{\vec{\tau}}{2}\vec{W}^\mu)
\end{aligned}
\tag{37}
$$

where

$$
\phi_i = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \quad i = 1, 2, 3 .
\tag{38}
$$

The invariance of the potential term

$$
\begin{aligned}
V(\phi_1, \phi_2, \phi_3) &= c_1(\phi_1^+\phi_1 - v^2/2)^2 + c_2(\phi_2^+\phi_2)^2 \\
&+ c_3(\phi_3^+\phi_3)^2 + c_4[(\phi_1^+\phi_1 - v^2/2) + \phi_2^+\phi_2 + \phi_3^+\phi_3]^2 \\
&+ c_5[(\phi_1^+\phi_1)(\phi_2^+\phi_2) - (\phi_1^+\phi_2)(\phi_2^+\phi_1)] \\
&+ c_6[(\phi_1^+\phi_1)(\phi_3^+\phi_3) - (\phi_1^+\phi_3)(\phi_3^+\phi_1)] \\
&+ c_7[(\phi_2^+\phi_2)(\phi_3^+\phi_3) - (\phi_2^+\phi_3)(\phi_3^+\phi_2)] \\
&+ c_8[Re(\phi_1^+\phi_2)]^2 + c_9[Re(\phi_1^+\phi_3)]^2 + c_{10}[Re(\phi_2^+\phi_3)]^2 \\
&+ c_{11}[Im(\phi_1^+\phi_2)]^2 + c_{12}[Im(\phi_1^+\phi_3)]^2 + c_{13}[Im(\phi_2^+\phi_3)]^2 + c_{14}
\end{aligned}
\tag{39}
$$

can be obtained if the following conditions on the free parameters are satisfied:

$$c_5 = c_6 \ , c_8 = c_9 \ , c_{11} = c_{12} \ ,$$

$$c_2 = c_3 = c_7 = c_{10} = 0 \ . \tag{40}$$

This implies that the masses of new particles are the same as the older ones, namely,

$$m_{F^\pm} = m_{H^\pm} = c_5 \frac{v^2}{2} \ ,$$

$$m_{H^3} = m_{H^1} = c_8 \frac{v^2}{2} \ ,$$

$$m_{H^4} = m_{H^2} = c_{11} \frac{v^2}{2} \ ,$$

$$\tag{41}$$

Further, the application of this transformation to the Yukawa Lagrangian (eq.(2)) keeps it invariant if the transformed Yukawa matrices satisfy the expressions

$$\bar{\xi}'^{U(D)}_{ij} = \bar{\xi}^{U(D)}_{ij} \cos\alpha + \bar{\rho}^{U(D)}_{ij} \sin\alpha \ ,$$

$$\bar{\rho}'^{U(D)}_{ij} = -\bar{\xi}^{U(D)}_{ij} \sin\alpha + \bar{\rho}^{U(D)}_{ij} \cos\alpha \ . \tag{42}$$

and therefore

$$(\bar{\xi}'^{U(D)})^+ \bar{\xi}'^{U(D)} + (\bar{\rho}'^{U(D)})^+ \bar{\rho}'^{U(D)} = (\bar{\xi}^{U(D)})^+ \bar{\xi}^{U(D)} + (\bar{\rho}^{U(D)})^+ \bar{\rho}^{U(D)} \ , \tag{43}$$

which allows us the following possible parametrization of the Yukawa matrices $\bar{\xi}^{U(D)}$ and $\bar{\rho}^{U(D)}$:

$$\bar{\xi}^{U(D)} = \epsilon^{U(D)} \cos\theta \ ,$$

$$\bar{\rho}^U = \epsilon^U \sin\theta \ ,$$

$$\bar{\rho}^D = i\epsilon^D \sin\theta \ , \tag{44}$$

where $\epsilon^{U(D)}$ are real matrices satisfy the equation

$$(\bar{\xi}'^{U(D)})^+ \bar{\xi}'^{U(D)} + (\bar{\rho}'^{U(D)})^+ \bar{\rho}'^{U(D)} = (\epsilon^{U(D)})^T \epsilon^{U(D)} \tag{45}$$

Here T denotes transpose operation. In eq. (44), we take $\bar{\rho}^D$ complex to carry all CP violating effects in the third Higgs scalar.

Therefore we can reduce the number of free parameters taking the new charged and neutral boson masses as the same as the older ones and connecting the Yukawa matrices $\bar{\xi}^{U(D)}$ and $\bar{\rho}^{U(D)}$ using the expression eq. (45).

Note that, neglecting the off-dioganal Yukawa couplings, the expression for $F_2^{Beyond}(y_t, y_t')$ (eq. (26)) can be written as

$$F_2^{Beyond}(y_t) \quad = \quad \frac{1}{m_t^2} \left(\bar{\epsilon}_{N,tt}^U \right)^2 G_1(y_t) + \frac{1}{m_t m_b} \bar{\epsilon}_{N,tt}^U \bar{\epsilon}_{N,bb}^D G_2(y_t) \left(\cos^2 \theta + i \sin^2 \theta \right) \qquad (46)$$

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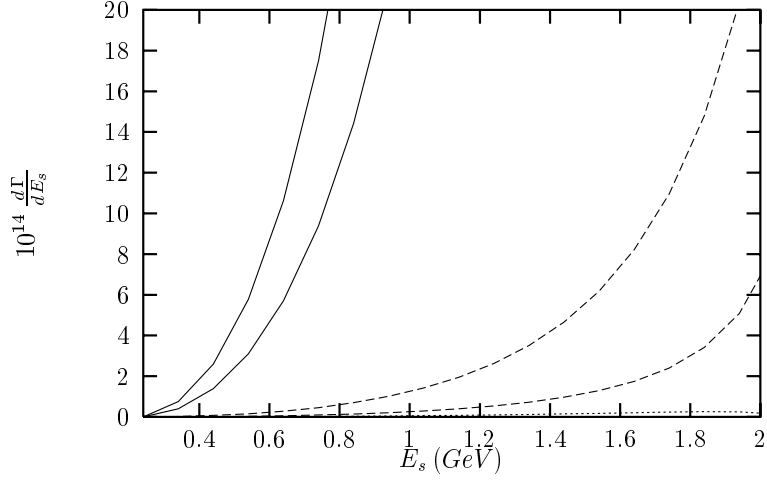
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Figure 1: $\frac{d\Gamma}{dE_s}$ as a function of E_s for fixed $\bar{\xi}^D_{N,bb} = 40\,m_b$, $sin\theta = 0.5$ and $|r_{tb}| = |\frac{\bar{\xi}^U_{N,tt}}{\bar{\xi}^D_{N,bb}}| < 1$ in the model III. Here $\frac{d\Gamma}{dE_s}$ is restricted in the region bounded by solid (dashed) lines for $C_7^{eff} > 0$ ($C_7^{eff} < 0$). Dotted line represents the SM contribution.
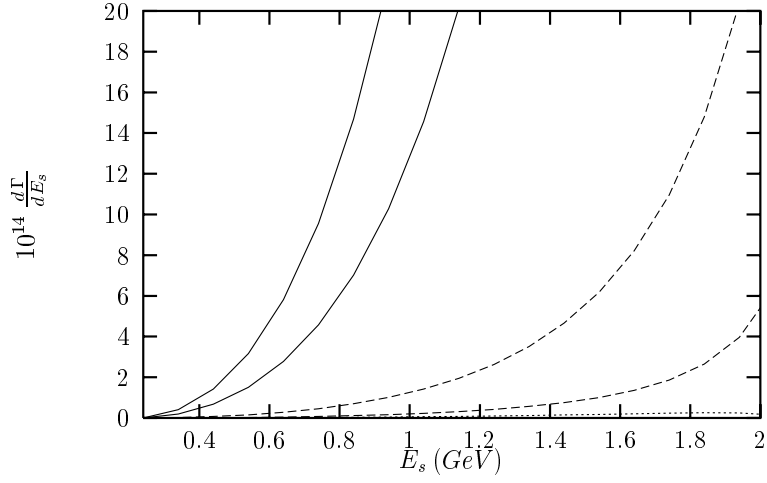
Figure 2: The same as Fig. 1 but for $3HDM(O_2)$.
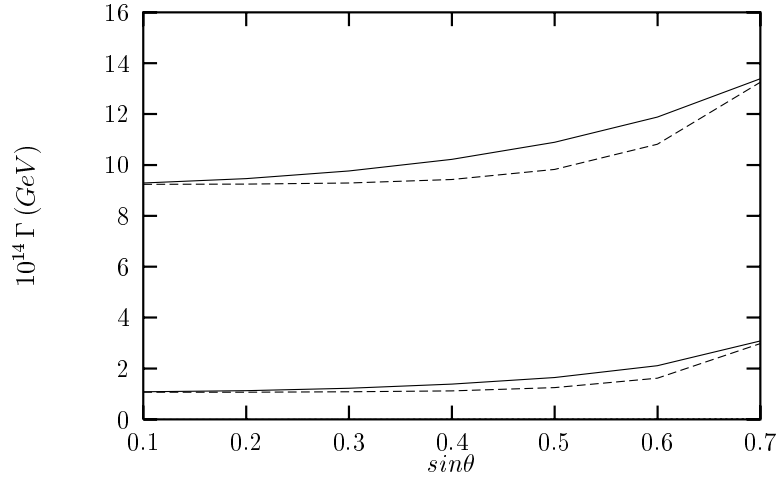
Figure 3: Γ as a function of $sin\,\theta$ for $C_7^{eff} < 0$, $\bar{\xi}_{N,bb}^D = 40\,m_b$, and $|r_{tb}| < 1$. Here Γ is restricted in the region bounded by solid (dashed) lines for the model III ($3HDM(O_2)$).
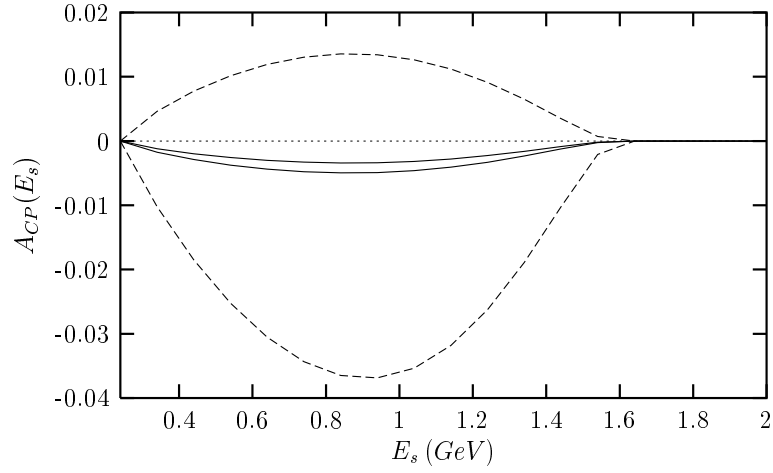
17

Figure 4: $A_{CP}(E_s)$ as a function of E_s for fixed $\bar{\xi}^D_{N,bb} = 40\, m_b$, $sin\theta = 0.5$ and $|r_{tb}| < 1$ in the model III. Here $A_{CP}(E_s)$ is restricted in the region bounded by solid (dashed) lines for $C_7^{eff} > 0$ ($C_7^{eff} < 0$).
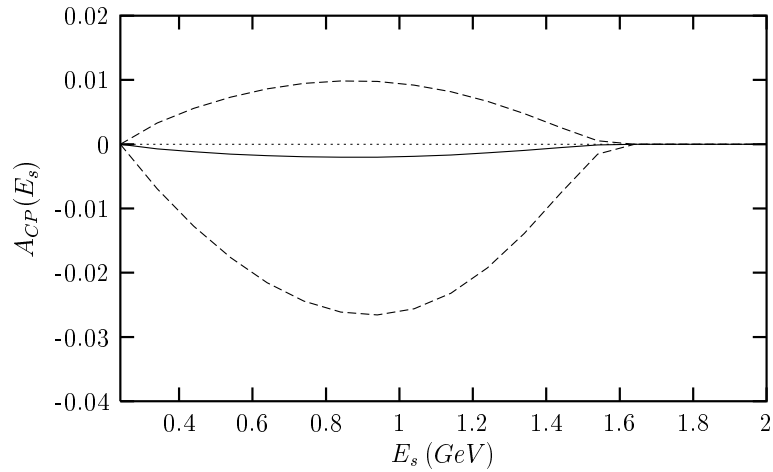

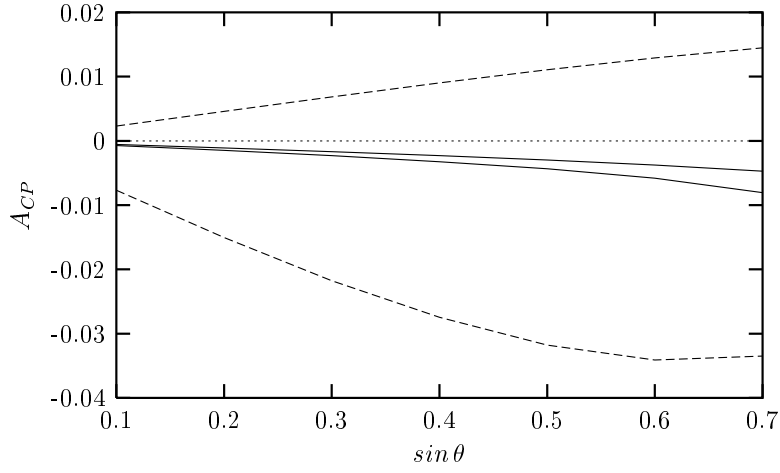Figure 5: The same as Fig. 4, but for $3HDM(O_2)$.

Figure 6: A_{CP} as a function of $sin\,\theta$ for $\bar{\xi}^{D}_{N,bb} = 40\,m_b$ and $|r_{tb}| < 1$ in the model III. Here A_{CP} is restricted in the region bounded by solid (dashed) lines for $C_7^{eff} > 0$ ($C_7^{eff} < 0$)
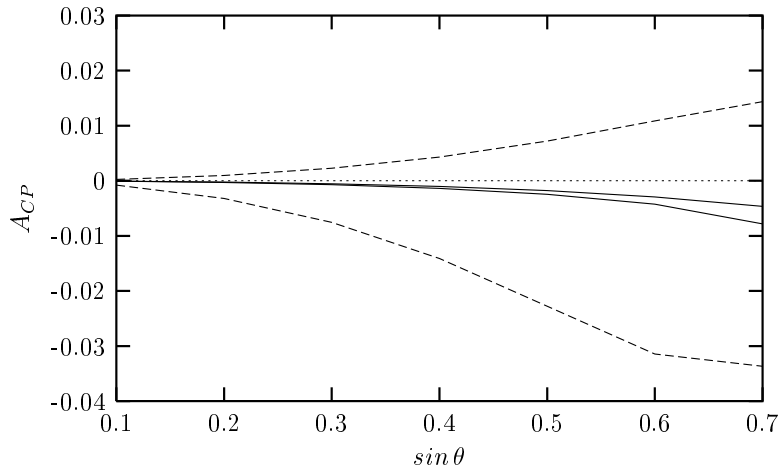
.

Figure 7: The same as Fig 6, but for $3HDM(O_2)$.