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Bribery and its welfare effects in first-price sealed-bid auctions

Şevket Alper Koç*
Kocaeli University, Department of Economics, 77843 Kocaeli, İzmit

Abstract

I study an auction in which the auctioneer, an agent of the seller, approaches all the bidders and tells them that if they pay a bribe and if they submit the highest bid, he will change their bid so that they only have to pay the second-highest bid. In equilibrium, only bidders who have valuations higher than some critical value pay the bribe, and they bid their valuations. Corruption has no effect on either the efficiency of the auction or the expected payoffs of the bidders. However, bribery results in a transfer of wealth from the seller to the auctioneer.

1. Introduction

In many cases, but not all, a sealed-bid auction has an auctioneer. Sometimes the auctioneer is a third party in the transaction, and sometimes it is an individual who works for the firm awarding the prize and who is given the task of collecting the bids from the bidders. The existence of an agent coming between the seller and the bidders raises the possibility of corruption in two ways. First, the auctioneer could look at the submitted bids and then solicit a bribe from the winner *after* the bids are submitted in exchange for changing the bid in a way that is favorable to the winner. In a standard high-bid auction, this would entail soliciting a bribe in exchange for lowering the winner's bid down to the second-highest bid. Second, the auctioneer could solicit bribes from the bidders *before* the bids are

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submitted[1], in exchange for a promise to reduce the bidder's bid should that bidder be the winner. Several existing papers address *ex post* bribery that occurs after all of the bids are submitted.[2] This paper analyzes *ex ante* bribery that occurs before the bids are submitted.[3]

This is not simply an academic exercise, because *ex ante* bribery has been documented in actual auctions. In their bids for corporate waste-disposal contracts in New York City, Mafia families would sometimes pay bribes for an "undertaker's look" at the bids of the other bidders before making their own bids.[4] In 1997 a Covington, Kentucky, developer was shown the bids of two competing developers for a $37 million dollar courthouse construction project.[5] In Chelsea, Massachusetts, in the 1980s, the city's auctioneer was accused of accepting bribes to rig auctions in favor of certain bidders, one time serving as a bidder's agent in an auction he was running.[6] Lengwiler and Wolfstetter (2000) relay two examples involving German firms which they claim provide evidence of *ex post* bribery, but I think provide better evidence of *ex ante* bribery. In one incident, one bidder illegally acquired the application documents of a rival bidder for the Berlin airport construction contract, and in a second incident, Siemens was barred from bidding in public procurement auctions in Singapore

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1 Burguet and Che (2004) accept that the bribery competition occurs simultaneously with the contract bidding. The authors argue that their model is equivalent to scenarios in which bribery occurs before and after contract bidding. First, they suppose that the bidders compete in contract bids and then compete in bribes. As a second game they examine the case when the bidders first pay bribes and then compete in price bids.

2 Lengwiler and Wolfstetter (2000) analyze auctions in which the winning bidder can bribe the auctioneer to change the bid after the auction has ended. Their results are similar to ours, although the results depend on the possibility of the corruption being detected and punished. Menezes and Monteiro (2001) consider a scenario in which there are two bidders and the auctioneer approaches one of them to solicit a bribe in return for changing the bid. The auctioneer can approach either the winner or the loser. Burguet and Perry (2002) study an auction in which one bidder is honest but one is corrupt. Burguet and Che (2004) and Celentani and Ganuza (2002) study a procurement auction in which the awarding of the contract is based on both the price and the quality of the project, and a corrupt auctioneer can manipulate the quality component in exchange for a bribe.

3 Corruption can also arise through bidding rings, in which the bidders collude to increase their surplus from the seller. See, for example, Graham and Marshall (1987), McAfee and McMillan (1992), and Marshall and Marx (2002). Comte et al. (2000) link the bidding ring literature and the bribery literature with a model of *ex post* bribery in which the bidders use corruption to enforce collusive behavior.

4 Cowan and Century (2002, pp. 223-231).

5 Crowley, Patrick, "Bid Scandal Bill in Trouble," *Cincinnati Enquirer*, January 21, 2000.

6 Murphy, Sean P., "Chelsea Businessman is Said to Allege Attempted Bribery," *Boston Globe*, September 22, 1993.

for five years because they had bribed an official for information about rival bids. Since the rival bids could be obtained and used before the bribers made their own bids, these could be instances of *ex ante* bribery.

Finally, I have also been told that auctioneers solicit *ex ante* bribes for some types of procurement contracts in Turkey. The contracts are auctioned using a standard first-price sealed-bid auction, with the bidder who offers to supply the good at the lowest price winning the auction and supplying the good at that price. Before the bidding starts, the corrupt auctioneer approaches certain bidders with whom he has worked before, and offers to raise their bids to the second-best bid if they win in exchange for a bribe.[7]

The key feature of these examples is that in every case the bidders pay a bribe to secure some action that would allow them to earn higher profits if they win, but the bribe is paid before the bidders know whether or not they will win. I construct a model to fit this feature. The auction is a first-price sealed bid auction with no reserve price, with the high bidder winning. Before the bidding, the auctioneer announces the size of the bribe he demands. As many bidders as want to can pay the bribe, and if a bidder who pays the bribe submits the highest bid, the auctioneer lowers the winning bid to the second-highest bid.[8] The high bidder then wins the auction and pays the second-highest bid.

I show that in the case where all bidders draw their valuations independently from a single distribution, bidders who have valuations higher than some critical value pay a bribe to the auctioneer, and bidders with low valuations do not. Bidders who pay the bribe bid their own valuations as if they were in a second-price sealed-bid auction, and bidders who do not pay the bribe bid according to the standard equilibrium bid function from the first-price auction. The resulting bid function for all bidders is increasing, and therefore the bidder with the highest value wins the auction, whether he pays the bribe or not, and the auction is efficient. The bidders' expected

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7 Ingraham (2000) uses empirical methods to study bidder-auctioneer cheating in sealed-bid auctions. Based on statistical properties of the bids, he develops a regression method for analyzing potential cheating of this type. He applies this regression specification to data from the New York City School Construction Authority auctions, and finds evidence that there is cheating between the auctioneer and the bidders.

8 We ignore issues related to the credibility of the auctioneer's promise, assuming instead that the promise is enforceable. Credibility might occur, for example, if the auctioneer makes this promise repeatedly in auctions over time, so that reputational concerns cause the auctioneer to keep the promise.

equilibrium payoffs are unaffected by corruption. They are neither worse off nor better off in terms of the equilibrium expected payoffs. However, there is a transfer of wealth from the seller to the auctioneer.

I proceed as follows. Section 2 presents the game and the notation. Section 3 examines the behavior of bidders, determining who pays the bribe and how they bid. Section 4 examines the auctioneer's behavior, characterizing the optimal bribe. Section 5 explores the welfare properties of the game in comparison to a first-price auction without corruption. Finally, Section 6 summarizes the results.

2. Structure of the game

There is a seller of a single good who faces n risk neutral potential buyers. The seller has hired an auctioneer to run a sealed-bid first-price auction, and pays the auctioneer a fixed wage (as opposed to a commission) in exchange for his services.[9] In contrast to the standard first-price auction, the game is supplemented by corruption between the auctioneer and the bidders. The auctioneer approaches every bidder before the auction is held and tells them that if the bidder agrees to pay a bribe of α, and is the highest bidder, he pays the second-highest bid. If the highest bidder did not pay the bribe, he pays his bid. Consequently, the game is a 3-stage game. In the first stage the auctioneer sets α, in the second stage the bidders decide whether to pay α independently and simultaneously, and in the third stage the bidders choose their bids.

The bidders' valuations $v_1,...,v_n$ are independently and identically drawn from the distribution F with support $[0,1]$, with a density *f*, as in the standard symmetric private values model. I assume that the value of the object to the seller is zero and the reserve price is zero. There is no entry fee, making it optimal for all bidders to bid.

In a standard first-price or second-price sealed-bid auction a bidding strategy is a map $\beta^i : v_i \to b_i$. An equilibrium is a profile of strategies $(\beta^1,...,\beta^n)$ such that β^i is a best reply for i given the strategies of all other bidders. An equilibrium is symmetric if all bidders use the same strategy, $\beta^1 = ... = \beta^n$. I denote the equilibrium

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[9] In the U.S., at least, many auctioneers are paid a commission based on the sales price. Such a payment scheme may reduce the auctioneer's incentives to solicit bribes, but that issue is left to future research.

strategy of the symmetric equilibrium of the first price and second price auctions with β_1 and β_2, respectively.

As is well known, the unique symmetric equilibrium of the first-price auction is the profile of strategies $(\beta^1,...,\beta^n)$ such that all β^i's are equal and all β^i's are best responses for i given the strategies of all other bidders. This unique symmetric equilibrium strategy is given by,

$$\beta_1 = b_1(v_i) = v_i - \frac{1}{F^{n-1}(v_i)} \int_0^{v_i} F^{n-1}(y)dy. \qquad (1)$$

Finally, the seller is passive in this game and we ignore issues related to the detection and punishment of corruption.

3. Bidder behavior

In this section I analyze the behavior of bidders given the size of the bribe, α, set by the auctioneer. Specifically, we characterize the equilibrium of the subgame that follows the auctioneer's choice of α. To accomplish this, I look for an equilibrium in which bidders with high valuations pay the bribe, and bidders with low valuations do not.

The first task is to find the bids of bidders who do and do not pay the bribe. If a bidder pays the bribe and is the highest bidder, he pays the second highest bid. Therefore, after paying the bribe the bidder essentially participates in a second price auction, and his dominant strategy is to bid his valuation.

Proposition 1: Any bidder who pays the bribe bids his valuation, v_i.

If a bidder does not pay the bribe, if he wins he must pay his own bid. Consequently, and for the standard reasons, he bids less than his valuation. How much less depends on the behavior of other bidders. An immediate result follows if all bidders with lower valuations also decline the bribe.

Proposition 2: If bidder i does not pay the bribe and all the bidders with valuations below v_i do not pay the bribe, bidder i bids according to the function $b_1(v_i)$.

Proof: Let $b(v)$ denote the equilibrium bid function for bidders who choose not to pay the bribe. For the standard reasons, b is assumed to be increasing. By Proposition 1, all bidders who do pay the bribe bid their valuations, and $v \geq b(v)$ for all v. If bidder i does not pay the bribe, and all bidders with valuations below v_i also do not pay the bribe, bidder i only wins when his is the highest valuation. The theory of first price auctions then implies that, conditional on his own valuation being the highest, bidder i's optimal bid is then $b_1(v_i)$.

Let v^* denote the threshold valuation such that a bidder with valuation v^* is indifferent about paying the bribe, and, by hypothesis, all bidders with valuations above v^* pay the bribe and all those with valuations below v^* do not. A bidder with valuation v^* who pays the bribe only beats bidders with lower valuations, and earns expected surplus of $\int_0^{v^*}[v^* - b_1(v)]dF^{n-1}(v) - \alpha$. A bidder with valuation v^* who does not pay the bribe earns expected surplus of $\int_0^{v^*}[v^* - b_1(v^*)]dF^{n-1}(v)$. The fact that the bidder is indifferent reduces to

$$\int_0^{v^*}\left[b_1(v^*) - b_1(v)\right]dF^{n-1}(v) = \alpha. \tag{2}$$

Thus, given α, the cutoff value v^* must satisfy the above equation.

A second interpretation of v^* arises from noticing that the first price bid function, $b_1(v)$, is the expected second-highest valuation conditional on v being the highest valuation. Consequently,[10]

$$\int_0^{v^*} b_1(v^*)dF^{n-1}(v) = \int_0^{v^*} v dF^{n-1}(v). \tag{3}$$

[10] Note that

$$\int_0^{v^*} b_1(v^*)dF^{n-1}(v) = F^{n-1}(v^*)\left[v^* - \frac{1}{F^{n-1}(v^*)}\int_0^{v^*} F^{n-1}(v)dv\right] =$$

$$v^*F^{n-1}(v^*) - \int_0^{v^*} F^{n-1}(v)dv = \int_0^{v^*} v dF^{n-1}(v),$$

where the last equality holds through integration by parts.

Using this fact, (2) can be re-written as

$$\int_0^{v^*} [v - b_1(v)]\, dF^{n-1}(v) = \alpha . \qquad (4)$$

Equation (4) has a straightforward interpretation. Suppose that a bidder with valuation v_1 pays the bribe, bids v_1, and wins the auction, and that the second-highest bidder has valuation v. If the second-highest bidder paid the bribe, he bids v, and the winning bidder's surplus is $v_1 - v$. If the second-highest bidder did not pay the bribe, he bids $b_1(v)$, and the winning bidder's surplus is $v_1 - b_1(v) > v_1 - v$. There is a clear benefit when the second highest bidder does not pay the bribe. Now, note that revenue equivalence implies that a bidder's expected surplus from a second-price auction is identical to his expected surplus from a first-price auction. So, his expected surplus (gross of the bribe) is the same if he and everyone else pay the bribe or if he and everyone else do not pay the bribe. The benefit from the bribe, then, must come from the additional surplus from facing people who do not pay the bribe. This additional surplus is $\int_0^{v^*} [v - b_1(v)]\, dF^{n-1}(v)$, which is the quantity on the left-hand side of (4). The equation says that enough people must choose not to pay the bribe so that the additional surplus from paying the bribe exactly offsets the cost of the bribe.

Figure 1 shows this graphically. Bidders who pay the bribe bid according to the second-price auction bid function $b_2(v) = v$, and bidders who do not pay the bribe bid according to the first-price auction bid function $b_1(v)$. The left-hand term in equation (4) is the weighted area between these two functions over the interval $[0,v^*)$, which is shown by the shaded area in the figure.[11] The weights are not shown in the graph, but they are given by the distribution function $F^{n-1}(v)$.

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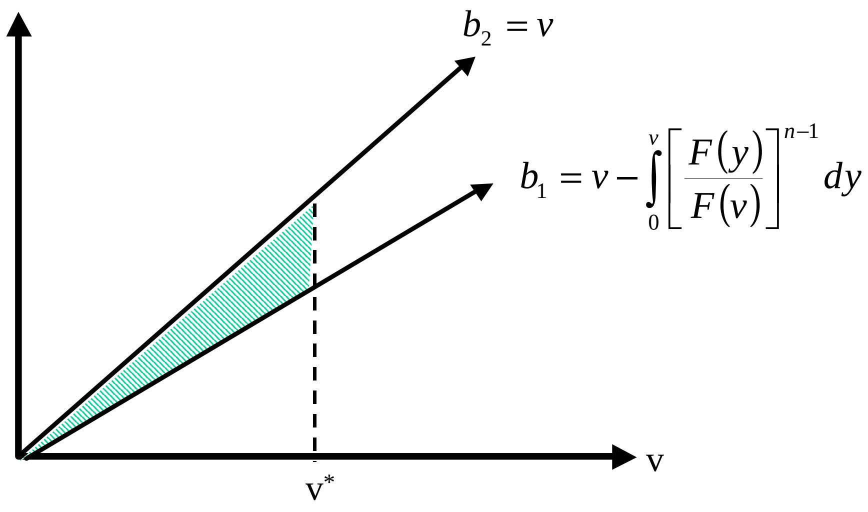
[11] The half-open interval $[0,v^*)$ is used because it is assumed that a bidder with valuation v^*, who is indifferent between paying the bribe and not paying it, elects to pay the bribe.

Figure 1
Illustration of the Bribe



Theorem 1: Given the amount of the bribe α, and given that $\alpha < \int_0^1 [b_1(1) - b_1(v)]\, dF^{n-1}(v)$, then there exists a unique Bayesian-Nash equilibrium in which bidders with values in [0,v^*) do not pay the bribe and bidders with values in [v^*, 1] do pay the bribe, where v^* solves

$$\int_0^{v^*} [b_1(v^*) - b_1(v)]\, dF^{n-1}(v) = \alpha \qquad (5)$$

and b_1 is the standard first-price auction bid function.

Proof: See the appendix.

Informally, a bidder who draws a value less than v^* prefers not to pay the bribe because if he pays the bribe his surplus rises only by a fraction of the shaded region in Figure 1 but he must pay an amount equal to the entire shaded region, so the bribe makes him worse off. A bidder who draws a value higher than v^* prefers to pay the bribe because the extra surplus in the shaded region is exactly offset by the bribe, but if he does not pay the bribe he loses the auction to people with valuations lower than his. Put another way, if a bidder with a valuation higher than v^* does not pay the bribe, he loses more than it costs to pay the bribe.

The uniqueness of v^* for a given α, together with the fact that the left-hand side of (5) is strictly increasing in v^*, implies that there exists a strictly increasing function $v^*(\alpha)$ that describes the equilibrium threshold valuation as a function of the bribe.

4. Auctioneer behavior

In the first period the auctioneer chooses the size of the bribe α that a bidder must pay in order to learn the second highest bid if he is the highest bidder. So, the auctioneer aims to maximize his expected revenue by choosing α. By Theorem 1, though, for any given α there is a unique threshold valuation v^* such that bidders with valuations above v^* pay the bribe and those with valuations below v^* do not. Because of the uniqueness, choosing α is the same as choosing v^*. Let

$$\alpha(v^*) = \int_0^{v^*} \left[b_1\left(v^*\right) - b_1(v)\right] dF^{n-1}(v) . \qquad (6)$$

The auctioneer's expected revenue is given by

$$R\left(v^*\right) = n\left(1 - F\left(v^*\right)\right)\alpha\left(v^*\right) \qquad (7)$$

where n is the number of bidders, $1 - F\left(v^*\right)$ is the probability that a given bidder pays the bribe, and $\alpha\left(v^*\right)$ is the size of the bribe. Since choosing α is the same as choosing v^*, the auctioneer's problem is to choose v^* to maximize expected revenue.

It is apparent from (6) that $\alpha\left(v^*\right)$ is continuous since it is differentiable. As long as the distribution F of bidders' valuations is continuous, it follows that $R\left(v^*\right)$ is continuous, which establishes the next result.

Proposition 3: There exists an α that maximizes expected revenue, and the corresponding v^* lies in the interval (0,1).

Proof. The problem of choosing α to maximize revenue is isomorphic to the problem of choosing $v^* \in$ [0,1] to maximize $R\left(v^*\right)$. Since the function is continuous on [0,1], it obtains a maximum. When $\alpha = 0, v^* = 0$ and $R(0) = 0$.

Also, when $v^* = 1, 1 - F(v^*) = 0$, and $R(1) = 0$. Finally, since $R(v^*) > 0$ when $v \in$ (0,1), the result holds.

For some distributions there is a unique α that maximizes the auctioneer's expected revenue. The uniform distribution is such an example. For the uniform distribution the revenue of the auctioneer is

$$R(v^*) = n(1 - v^*)\alpha(v^*)$$

where

$$\alpha(v^*) = (v^*)^n \left(\frac{n-1}{n^2} \right).$$

So, the auctioneer's problem becomes

$$\max_{v^*} \; n(1 - v^*)(v^*)^n \left(\frac{n-1}{n^2} \right).$$

The first order condition reduces to

$$v^* = \frac{n}{n+1}.$$

Surprisingly, this is the expected value of the highest value of the n bidders. Therefore, the auctioneer maximizes his bribe revenue by soliciting a bribe so large that only bidders whose valuations are above the expected highest valuation pay the bribe.

Another example is a triangular distribution with a density $f(v) = 2v$ and a cumulative distribution $F(v) = v^2$. For this distribution,

$$\alpha(v^*) = \frac{2n-2}{(2n-1)^2} (v^*)^{2n-1}$$

So, the auctioneer's problem becomes

$$\max_{\{v^*\}} \; n(1 - v^*)(v^*)^{2n-1} \left(\frac{2n-2}{(2n-1)^2} \right)$$

Therefore,

$$v^* = \frac{2n-1}{2n}$$

This is smaller than the expected highest value, which is $\frac{2n}{2n+1}$.

5. Welfare properties

I now turn to the welfare properties of the auction with bribery. I am interested in two issues. First, is the auction with bribery efficient; that is, does the bidder with the highest valuation get the object? Second, how do participants fare in comparison to a standard first-price auction without bribery? I begin with efficiency.

Proposition 4: The auction with bribery is efficient.

In general, an auction that awards the prize to the highest bidder is efficient if the bid function is increasing in the bidder's valuation. In the auction with bribery, the bid function can be written

$$b(v) = \begin{cases} b_1(v) & \text{if } v < v^* \\ v & \text{if } v \geq v^* \end{cases} \tag{8}$$

where $b_1(v)$ is the standard first-price auction bid function, which is increasing. Since $b_1(v^*) < v^*$, the bid function $b(v)$ is increasing, and consequently the auction is efficient.

The next issue is a comparison with a first-price auction without bribery. Suppose that the auctioneer sets the bribe at α, and so, by Theorem 1, there exists a threshold valuation v^* such that bidders with valuations higher than v^* pay the bribe and those with valuations below v^* do not. Of course, if a bidder who does not pay the bribe wins the auction, no one else has paid a bribe either, and the outcome of the game is exactly the same as the outcome of the standard first-price auction without bribery. Furthermore, since only bidders with high valuations pay the bribe in equilibrium, no bidder loses to anyone who would not have beaten him in the standard first-price auction without bribery.

The interesting issue pertaining to the welfare of bidders involves bidders who pay the bribe. To that end, suppose that bidder i has valuation $v_i \geq v^*$, so that bidder i pays the bribe. His expected surplus is

$$E[U_{ic}] = \int_0^{v^*}[v_i - b_1(v)]dF^{n-1}(v) + \int_{v^*}^{v_i}[v_i - v]dF^{n-1}(v) - \alpha.$$

Using equation (4), this can be rewritten

$$E[U_{ic}] = \int_0^{v^*}[v_i - v]dF^{n-1}(v) + \int_{v^*}^{v_i}[v_i - v]dF^{n-1}(v) = \int_0^{v_i}[v_i - v]dF^{n-1}(v),$$

which is a bidder's expected surplus in a second-price auction. From revenue equivalence, however, I know that the expected surplus in a first-price auction and the expected surplus in a second-price auction are identical, and so bidders are indifferent between the auction with bribery and the auction without bribery. By Proposition 3, however, the auctioneer earns positive expected revenue from bribes. Because the auctioneer gains from the bribery but the bidders have the same expected surplus with and without bribery, it must be the case that the seller loses what the auctioneer gains. This proves the final proposition.

Proposition 5: In equilibrium, bribes are a transfer from the seller to the auctioneer.

In summary, although bribery changes the bid functions of some bidders, namely those with sufficiently high valuations, it has no effect on the final allocation of the prize or the welfare of the bidders. The bribes generate expected revenue for the auctioneer, and because bidders are not affected, the bribes also generate an expected loss to the seller compared to a first-price auction without bribery.

This analysis suggests that since bidders are not hurt by the corruption, but the seller is, it should be the seller who takes measures to fight corruption. No policy need be enacted to "protect" bidders from "unscrupulous" auctioneers.

6. Conclusion

In this paper I analyzed a model of bribery in sealed-bid first-price auctions. The bribery involves the auctioneer, who acts as an agent on behalf of the seller, and the bidders. The results show that, given the size of the bribe set by the auctioneer, bidders with valuations above some threshold pay the bribe, while bidders with lower valuations do not. In equilibrium, bidders who pay the bribe bid their valuations while bidders who do not pay the bribe bid according to the standard first-price auction bid function. The auctioneer sets the bribe to trade off the amount collected from a bidder who pays the bribe and the number of bidders expected to pay it.

I also studied the welfare properties of the auction with bribery and show that it is efficient and, in equilibrium, bribes are a transfer from the seller to the auctioneer. Although bribery changes the bid functions of some bidders, namely those with sufficiently high valuations, it has no effect on the final allocation of the prize or the welfare of the bidders. The bribes generate expected revenue for the auctioneer, and because bidders are not affected, the bribes also generate an expected loss to the seller compared to a first-price auction without bribery.

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Appendix

Theorem 1: Given the amount of the bribe α, and given that $\alpha < \int_0^1 [b_1(1) - b_1(v)]\, dF^{n-1}(v)$, then there exists a unique Bayesian-Nash equilibrium in which bidders with values in $[0, v^*)$ do not pay the bribe and bidders with values in $[v^*, 1]$ do pay the bribe, where v^* solves

$$\int_0^{v^*} [b_1(v^*) - b_1(v)]\, dF^{n-1}(v) = \alpha$$

and b_1 is the standard first-price auction bid function.

Proof of theorem 1

Lemma: In any equilibrium every bidder uses a cutoff strategy.

Proof of Lemma: Fix any equilibrium and consider the (right-continuous) cdf, $G_i(b)$, of the highest bid of bidders $j \neq i$. Also let $x_i(b)$ denote the probability of i winning with bid b against the rival bidders employing their equilibrium strategies. (Note that $x_i(b)$ may not equal $G_i(b)$ since a tie may arise at a mass point b.) Let B_c be the set of b's for which G is continuous, and let B_m be the set of b's for which G jumps. Then

$$U_{ic}(v) = \int_{b \le v, b \in B_c} (v - b)\, dG_i(b) + \sum_{b \le v, b \in B_m} (v - b)[G_i(b_+) - G_i(b_-)] - \alpha.$$

$U_{ic}(\cdot)$ is absolutely continuous and can be rewritten as

$$U_{ic}(v) = \int_{v'}^{v} G_i(s)\, ds + U_{ic}(v'), \qquad \text{(A1)}$$

for any v'.

Now consider

$U_{in}(v) = \sup_b (v - b) x_i(b)$.

It follows that

$U_{in}(v) = \max_b (v - b) G_i(b)$,

since $(v - b)G_i(b)$ is an upper envelope of $(v - b)x_i(b)$. One can check that $U_{in}(v)$ is absolutely continuous, that the maximum is well defined (since an upper envelope is upper semicontinuous and the choice can be bound to a compact set without loss of generality), and that $f(b,v) := (v - b)G_i(b)$ is differentiable in v for every b in the equilibrium support. Hence, one can invoke Theorem 2 of Milgrom and Segal to show that

$$U_{in}(v) = \int_{v'}^{v} G_i(b^*(s))ds + U_{in}(v'), \quad \text{(A2)}$$

for $b^*(s) \in \text{argmax}_b(v - b)x_i(b)$.

It follows from (A1) and (A2) that

$$U_{ic}(v) - U_{in}(v) = \int_{v'}^{v} [G_i(s) - G_i(b^*(s))]ds + [U_{ic}(v') - U_{in}(v')]. \quad \text{(A3)}$$

Since $b^*(s) < s$ for almost every s, it is clear from (A3) that, whenever $U_{ic}(v') - U_{in}(v') > 0$, it must be that $U_{ic}(v) - U_{in}(v) > 0$ for $v > v'$, which proves that the equilibrium strategy must involve a cutoff strategy with some threshold v_i^*. □

By Lemma 2 every bidder uses a cutoff strategy, so there exist values $v_1^*,\ldots,v_n^*$ such that bidder i pays the bribe if $v_i \geq v_i^*$ and does not pay the bribe if $v_i < v_i^*$. It remains to show that $v_1^* = \ldots = v_n^* = v^*$.

I first show that the lowest of the n cutoff points is v^*. Suppose that player i has the lowest threshold point, v_i^*, and draws the valuation v_i. If he pays the bribe he bids his valuation, but if he does not pay the bribe he bids the standard first-price equilibrium bid, since everyone below him also bids according to the standard first-price equilibrium bid function. Then, letting $H_i(\cdot)$ denote the cdf of the highest valuation of bidders $j \neq i$,

$$U_{ic}(v_i^*) = \int_0^{v_i^*} (v_i^* - b_1(v))dH_i(v) - \alpha$$

and

$$U_{in}(v_i^*) = (v_i^* - b_1(v_i^*))H_i(v_i^*).$$

Therefore

$$U_{ic}(v_i^*) - U_{in}(v_i^*) = \int_0^{v_i^*} (b_1(v_i^*) - b_1(v))dH_i(v) - \alpha. \quad \text{(A4)}$$

The right-hand side is obviously increasing in v_i^* (since the bid function increases) and it is equal to zero when $v_i^* = v^*$. Consequently, there is no equilibrium in which the lowest threshold point is below v^*.

I next show that the highest of the n cutoff points is also v^*. Suppose that i has the highest threshold value $v_i^* > v^*$, and choose $v_i \in (v^*, v_i^*)$. It follows from equation (A3) in the proof of Lemma 2 that

$$U_{ic}(v_i) - U_{in}(v_i) = \int_{v^*}^{v_i} [G_i(s) - G_i(b^*(s))]ds + [U_{ic}(v^*) - U_{in}(v^*)]. \quad \text{(A5)}$$

By (A4), $U_{ic}(v^*) - U_{in}(v^*) = 0$. The integral in (A5) is positive for $v_i > v^*$, since $b^*(s) < s$ for almost all s. Consequently, bidder i wants to pay the bribe when he draws the valuation $v_i < v_i^*$, which is a contradiction.

Özet

Birinci fiyat kapalı zarf açıkartırmalarında rüşvet ve bunun refah etkileri

Bu çalışma mal sahibinin aracısı olan müzayedecinin tüm teklif verenlere yaklaşıp onlara, eğer bir miktar rüşvet verirlerse ve en yüksek teklifi verirlerse tekliflerini değiştirip sadece ikinci en yüksek teklifi ödemelerini söz verdiği bir açık artırma üzerinedir. Dengede, belirli bir miktarın üzerindeki değere sahip teklif verenler rüşveti verecektir ve kendi değerlerini teklif vereceklerdir. Rüşvet ve ahlaki bozulma açık artırmanın verimliliği ve teklif verenlerin beklenen kazançları üzerine bir etki yaratmamıştır. Ancak, toplam rüşvet satıcıdan müzayedeciye bir servet transferi olarak karşımıza çıkar.