

Analysis of Various Polarization Asymmetries In The Inclusive $b \rightarrow s\ell^+\ell^-$ Decay In The Fourth-Generation Standard Model

V. Bashiry^{1*}; M. Bayar^{2†},

¹ Engineering Faculty, Cyprus International University,
Via Mersin 10, Turkey

² Physics Department, Middle East Technical University,
06531 Ankara, Turkey

Abstract

In this study a systematical analysis of various polarization asymmetries in inclusive $b \rightarrow s\ell^+\ell^-$ decay in the standard model (SM) with four generation of quarks is carried out. We found that the various asymmetries are sensitive to the new mixing and quark masses for both of the μ and τ channels. Sizeable deviations from the SM values are obtained. Hence, $b \rightarrow s\ell^+\ell^-$ decay is a valuable tool for searching physics beyond the SM, especially in the indirect searches for the fourth-generation of quarks (t' , b').

PACS numbers: 12.60.-i, 13.30.-a, 14.20.Mr

*e-mail: bashiry@ciu.edu.tr

†e-mail: mbayar@metu.edu.tr

1 Introduction

Despite incredible successes of the Standard Model with three generations of quarks (SM3) in explaining the experimental data, it is believed that SM3 of electroweak interaction is a low energy manifestation of some other more fundamental theory. Therefore, investigations of new physics beyond the standard model (SM) is now being performed in particle physics. A possible new physics is the existence of new quarks and leptons beyond the known ones. Whether or not there exist new generations has been investigated by many theoretical and experimental studies (for the most recent studies see [1, 2] and the references therein). These new generations might soon be detected by LHC, where the new generation is expected to be produced by gluon fusion. The cross section of the production of $t'\bar{t}'$ at LHC is about: $\sigma = 10(0.25)\text{pb}$ for $m_{t'} = 400(800)$ GeV, which is similar to the $t\bar{t}$ production [3].

The status of four generations has arisen many discussions from the experimental point of view. On the other hand, from theoretical point of view, it is favored because of two reasons; first, it might help in bringing the $SU(3) \otimes SU(2)_L \otimes U(1)_Y$ couplings close to each other at the unification point $\sim 10^{16}$ GeV [4]; second, new generations with new weak phases might bring better solution to baryogenesis [5].

The experimental constraints on the 4th generation are imposed by the ρ , S , T parameters of the SM and the measurement of the Z boson decay width. LEP and CDF [6] experiments provide constraints on the mass of 4th generation lepton (neutrinos heavier than half of the Z mass) and quarks (4th generation quarks heavier than 255 GeV), respectively. The LEP results exclude the possibility of the 4th generation of leptons with the mass ~ 1 eV [7]. These results are mostly interpreted as the exact value of the generation number, i.e, $N = 3$, since one assumes that the neutrinos must have very small masses. If we disregard this incorrect assumption, the LEP data does not exclude the existence of extra SM families with heavy neutrinos. Note that, the existence of the consequential 4th generation leptons besides the 4th generation quarks is indispensable to cancel the contributions of the 4th generation quarks in the gauge anomalies at loop level.

Flavor changing neutral current (FCNC) transition is in forefront of indirect investigation for the 4th generation via $b \rightarrow s(d)$ transition. These transitions, which lead to the so called rare decays, appear at quantum level, since, they are forbidden in tree level in the SM. The 4th consequential SM family of quarks, i.e, t' , like u , c , t quarks, can contribute to the loop. Such consequential extension of the SM has been formulated by many authors, i.e. [8, 9, 10]. Note that, the fourth generation effects have been widely studied in baryonic and semileptonic exclusive B decays [11]–[21]. Note also that, the calculation of the inclusive $b \rightarrow s\ell^+\ell^-$ decays is cleaner than the exclusive decays because the exclusive decays suffer from the hadronic uncertainties. In this study, we investigate the possibility of searching for new physics when looking at various asymmetries in the inclusive $b \rightarrow s\ell^+\ell^-$ decay using the SM with fourth generation of quarks (b' , t'). Note that, branching ratio, CP asymmetry and FB asymmetry for this decay in the fourth generation standard model (SM4) have been studied in [8].

The paper includes 6 sections: In section 2, we modify the effective Hamiltonian in the presence of 4th generation. In section 3 and 4, double lepton polarization and polarized FB asymmetries are derived, respectively. In section 5, we examine the sensitivity of these physical observable on the new parameters ($m_{t'}$, $V_{t'b}V_{t's}^*$). Section 6 is devoted to the

conclusions.

2 Theoretical Framework

In this section, we present the theoretical expressions for the decay width. As we mentioned above, we extend the SM3 to the fourth-generation standard model (SM4) and as a result of this extension the Wilson coefficient of the SM3 is modified by the existence of the 4th generation quark t' . It is easy to see that if a fourth generation is introduced in the same way as the other three generations in the SM3, new operators do not appear. In other words, the full operator set for the SM4 is exactly the same as in the SM3.

The Wilson coefficients are modified as follows:

$$\lambda_t C_i \rightarrow \lambda_t C_i^{SM} + \lambda_{t'} C_i^{new}, \quad (1)$$

where $\lambda_f = V_{fb}^* V_{fs}$. The unitarity of the 4×4 CKM matrix leads to

$$\lambda_u + \lambda_c + \lambda_t + \lambda_{t'} = 0. \quad (2)$$

One can neglect $\lambda_u = V_{ub}^* V_{us}$ in Eq. 2 which is very small in strength compared to the others ($|\lambda_u| \sim 10^{-3}$). Then, $\lambda_t \approx -\lambda_c - \lambda_{t'}$.

Now, we can re-write Eq. 1 as:

$$\lambda_t C_i^{SM} + \lambda_{t'} C_i^{new} = -\lambda_c C_i^{SM} + \lambda_{t'} (C_i^{new} - C_i^{SM}). \quad (3)$$

It is clear that when $m_{t'} \rightarrow m_t$ or $\lambda_{t'} \rightarrow 0$, $\lambda_{t'} (C_i^{new} - C_i^{SM})$ vanishes, as is required by the GIM mechanism.

The most important operators for $B \rightarrow X_s \ell^+ \ell^-$ are

$$\begin{aligned} O_7 &= \frac{e}{16\pi^2} \bar{m}_b(\mu) (\bar{s}_L \sigma_{\mu\nu} b_R) F^{\mu\nu}, \\ O_9 &= \frac{\alpha_{em}}{4\pi} (\bar{s}_L \gamma_\mu b_L) (\bar{\ell} \gamma^\mu \ell), \\ O_{10} &= \frac{\alpha_{em}}{4\pi} (\bar{s}_L \gamma_\mu b_L) (\bar{\ell} \gamma^\mu \gamma_5 \ell). \end{aligned} \quad (4)$$

The large q^2 region is usually considered less favorable, because it has a smaller rate and suffers from large nonperturbative corrections. However, the experimental efficiency is better in that region [22]. The operator O_7 is dominant at small q^2 due to the $1/q^2$ pole from the photon propagator. At high q^2 region the O_7 contribution is rather small [23]. The rate in the high q^2 region has a smaller renormalization scale dependence and m_c dependence [24]. Despite the experimental advantages, the large q^2 region has been considered less favored, because it has a large hadronic uncertainty [25]. The $1/m_b^3$ corrections are not much smaller than the $1/m_b^2$ ones [26] when the operator product expansion becomes an expansion in $\Lambda_{QCD}/(m_b - \sqrt{q^2})$ [27] instead of Λ_{QCD}/m_b .

The QCD corrected effective Hamiltonian for the $b \rightarrow s \ell^+ \ell^-$ transitions leads to the following matrix element:

$$\begin{aligned} M = \frac{G_F V_{tb} V_{ts}^*}{\sqrt{2}\pi} \alpha_{em} [& C_9^{tot} (\bar{s} \gamma_\mu P_L b) \bar{\ell} \gamma_\mu \ell + C_{10}^{tot} (\bar{s} \gamma_\mu P_L b) \bar{\ell} \gamma_\mu \gamma_5 \ell \\ & - 2 C_7^{tot} \bar{s} i \sigma_{\mu\nu} \frac{q^\nu}{q^2} (m_b P_R + m_s P_L) b \bar{\ell} \gamma_\mu \ell], \end{aligned} \quad (5)$$

where q denotes the four momentum of the lepton pair and C_i^{tot} 's are as follows:

$$C_i^{tot}(\mu) = C_i^{eff}(\mu) + \frac{\lambda_{t'}}{\lambda_t} C_i^{new}(\mu), \quad (6)$$

where the last terms in these expressions describe the contributions of the t' quark to the Wilson coefficients. $\lambda_{t'}$ can be parameterized as:

$$\lambda_{t'} = V_{t'b}^* V_{t's} = r_{sb} e^{i\phi_{sb}}. \quad (7)$$

Neglecting the terms of $O(m_q^2/m_W^2)$, $q = u, d, c$, the analytic expressions for all Wilson coefficients in the SM in the leading order (LO) and in the next to leading logarithmic approximation (NLO) can be found in [28]–[38]. The most recent developments in the SM calculations of $b \rightarrow s\ell^+\ell^-$ transitions and the results for the NNLL virtual corrections to the matrix elements of the operators O_1 and O_2 for this inclusive process in the kinematical region $q^2 > 4m_c^2$ have been discussed in Ref. [23, 40] and the references therein. In the low-dilepton-mass region $q^2 < 6 \text{ GeV}^2$ and also in the high-dilepton-mass region with $q^2 > 4m_c^2$, theoretical predictions for the invariant mass spectrum are dominated by the perturbative contributions, and a theoretical precision of order 10% is in principle possible.

The explicit forms of the C_i^{new} can be obtained from the corresponding expression of the Wilson coefficients in the SM by substituting $m_t \rightarrow m_{t'}$.

$C_9^{eff}(\hat{s}) = C_9 + Y(\hat{s})$, where $Y(\hat{s}) = Y_{\text{pert}}(\hat{s}) + Y_{\text{LD}}(\hat{s})$ contains both the perturbative part $Y_{\text{pert}}(\hat{s})$ and long-distance part $Y_{\text{LD}}(\hat{s})$. $Y(\hat{s})_{\text{pert}}$ is given by [29]

$$\begin{aligned} Y_{\text{pert}}(\hat{s}) &= g(\hat{m}_c, \hat{s}) C_0 \\ &\quad - \frac{1}{2} g(1, \hat{s}) (4\bar{C}_3 + 4\bar{C}_4 + 3\bar{C}_5 + \bar{C}_6) - \frac{1}{2} g(0, \hat{s}) (\bar{C}_3 + 3\bar{C}_4) \\ &\quad + \frac{2}{9} (3\bar{C}_3 + \bar{C}_4 + 3\bar{C}_5 + \bar{C}_6), \end{aligned} \quad (8)$$

$$\text{with } C_0 \equiv \bar{C}_1 + 3\bar{C}_2 + 3\bar{C}_3 + \bar{C}_4 + 3\bar{C}_5 + \bar{C}_6, \quad (9)$$

and the function $g(x, y)$ defined in [29]. Here $\bar{C}_1 - \bar{C}_6$ are the Wilson coefficients in the leading logarithmic approximation. The relevant Wilson coefficients were collected in Ref. [30]. $Y(\hat{s})_{\text{LD}}$ involves $b \rightarrow sV(\bar{c}c)$ resonances [32, 31, 33], where $V(\bar{c}c)$ are the vector charmonium states. We follow Refs. [32, 31] and set

$$Y_{\text{LD}}(\hat{s}) = -\frac{3\pi}{\alpha_{em}^2} C_0 \sum_{V=\psi(1s), \dots} \kappa_V \frac{\hat{m}_V \mathcal{B}(V \rightarrow l^+l^-) \hat{\Gamma}_{\text{tot}}^V}{\hat{s} - \hat{m}_V^2 + i\hat{m}_V \hat{\Gamma}_{\text{tot}}^V}, \quad (10)$$

where $\hat{\Gamma}_{\text{tot}}^V \equiv \Gamma_{\text{tot}}^V/m_B$ and $\kappa_V = 2.3$. The relevant properties of vector charmonium states are summarized in Table 1.

Using the expression of matrix element in equation (5) and neglecting the s-quark mass (m_s) [41]–[42], we obtain the expression for the differential decay rate as [43]:

$$\Gamma_0 = \frac{d\Gamma}{d\hat{s}} = \frac{G_F m_b^5 \alpha_{em}^2}{192\pi^3 4\pi^2} |V_{tb} V_{ts}^*|^2 (1 - \hat{s})^2 \sqrt{1 - \frac{4\hat{m}_\ell^2}{\hat{s}}} \Delta \quad (11)$$

Table 1: Masses, total decay widths and branching fractions of dilepton decays of vector charmonium states [39].

V	Mass[GeV]	Γ_{tot}^V [MeV]	$\mathcal{B}(V \rightarrow \ell^+\ell^-)$
$J/\Psi(1S)$	3.097	0.093	5.9×10^{-2} for $l = e, \mu$
$\Psi(2S)$	3.686	0.327	7.4×10^{-3} for $l = e, \mu$ 3.0×10^{-3} for $l = \tau$
$\Psi(3770)$	3.772	25.2	9.8×10^{-6} for $l = e$
$\Psi(4040)$	4.040	80	1.1×10^{-5} for $l = e$
$\Psi(4160)$	4.153	103	8.1×10^{-6} for $l = e$
$\Psi(4415)$	4.421	62	9.4×10^{-6} for $l = e$

with

$$\begin{aligned} \Delta = & 4 \frac{(2 + \hat{s})}{\hat{s}} \left(1 + \frac{2\hat{m}_\ell^2}{\hat{s}} \right) |C_7^{\text{tot}}|^2 + (1 + 2\hat{s}) \left(1 + \frac{2\hat{m}_\ell^2}{\hat{s}} \right) |C_9^{\text{tot}}|^2 \\ & + (1 - 8\hat{m}_\ell^2 + 2\hat{s} + \frac{2\hat{m}_\ell^2}{\hat{s}}) |C_{10}^{\text{tot}}|^2 + 12 \left(1 + \frac{2\hat{m}_\ell^2}{\hat{s}} \right) \text{Re}(C_9^{\text{tot}*} C_7^{\text{tot}}). \end{aligned} \quad (12)$$

3 Polarization Asymmetries

In order to compute the polarization asymmetries, one has to choose a reference frame to define the spin directions. A reference frame can be chosen in the center of mass (CM) frame of the leptons. In such a reference frame, if we suppose that ℓ^- moves in positive z direction and the fact that momentum is conserved, the s and b quarks move in the same direction. In this reference frame, the spin direction of leptons, the 4-vector $s_{\ell^-}^\mu$, after the Lorentz boost from its rest frame can be obtain as [44]:

$$s_{\ell^-}^\mu = \left\{ \frac{|p^-|}{m_\ell} s_z^-, s_x^-, s_y^-, \frac{E}{m_\ell} s_z^- \right\}, \quad s_{\ell^+}^\mu = \left\{ \frac{|p^+|}{m_\ell} s_z^+, s_x^+, s_y^+, \frac{E}{m_\ell} s_z^+ \right\}, \quad (13)$$

where \mathbf{s}^\pm and p^\pm are the unit vectors and three-momenta of leptons in the ℓ^\pm rest frames, respectively. The double-lepton polarization asymmetries \mathcal{P}_{ij} are defined as [45]

$$\mathcal{P}_{ij} = \frac{\left[\frac{d\Gamma(\mathbf{s}^+=\hat{\mathbf{i}}, \mathbf{s}^-=\hat{\mathbf{j}})}{d\hat{s}} - \frac{d\Gamma(\mathbf{s}^+=\hat{\mathbf{i}}, \mathbf{s}^-=\hat{-\mathbf{j}})}{d\hat{s}} \right] - \left[\frac{d\Gamma(\mathbf{s}^+=-\hat{\mathbf{i}}, \mathbf{s}^-=\hat{\mathbf{j}})}{d\hat{s}} - \frac{d\Gamma(\mathbf{s}^+=-\hat{\mathbf{i}}, \mathbf{s}^-=\hat{-\mathbf{j}})}{d\hat{s}} \right]}{\left[\frac{d\Gamma(\mathbf{s}^+=\hat{\mathbf{i}}, \mathbf{s}^-=\hat{\mathbf{j}})}{d\hat{s}} + \frac{d\Gamma(\mathbf{s}^+=\hat{\mathbf{i}}, \mathbf{s}^-=\hat{-\mathbf{j}})}{d\hat{s}} \right] + \left[\frac{d\Gamma(\mathbf{s}^+=-\hat{\mathbf{i}}, \mathbf{s}^-=\hat{\mathbf{j}})}{d\hat{s}} + \frac{d\Gamma(\mathbf{s}^+=-\hat{\mathbf{i}}, \mathbf{s}^-=\hat{-\mathbf{j}})}{d\hat{s}} \right]}, \quad (14)$$

where $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ are unit vectors.

With our choice of reference frame Eq. (13), the decay happens in two dimensions, i.e., yz plane. In this frame, just the components of spin can be in \hat{x} direction. Therefore, any terms including the spin along \hat{x} direction are the result of either dot product of two spins or triple-product correlation with one spin along \hat{x} direction (i.e., \mathcal{P}_{xx} , \mathcal{P}_{xy} and \mathcal{P}_{xz}). This holds even in the presence of any extension of SM. Among these quantities, \mathcal{P}_{xy} and \mathcal{P}_{xz} are interesting as they probe the imaginary parts of the products of Wilson coefficients [44].

The \mathcal{P} 's take the form

$$\begin{aligned} \mathcal{P}_{xx} = & \frac{1}{\Delta} \left\{ 24 \text{Re}(C_7^{\text{tot}} C_9^{\text{tot}*}) \frac{\hat{m}_\ell^2}{\hat{s}} + 4 |C_7^{\text{tot}}|^2 \frac{(-1 + \hat{s})\hat{s} + 2(2 + \hat{s})\hat{m}_\ell^2}{\hat{s}^2} \right. \\ & \left. + (|C_9^{\text{tot}}|^2 - |C_{10}^{\text{tot}}|^2) \frac{(1 - \hat{s})\hat{s} + 2(1 + 2\hat{s})\hat{m}_\ell^2}{\hat{s}} \right\}, \end{aligned} \quad (15)$$

$$\mathcal{P}_{yx} = \frac{-2}{\Delta} \text{Im}(C_9^{\text{tot}} C_{10}^{\text{tot}*}) (1 - \hat{s}) \sqrt{1 - \frac{4\hat{m}_\ell^2}{\hat{s}}}, \quad (16)$$

$$\mathcal{P}_{xy} = \mathcal{P}_{yx}, \quad (17)$$

$$\mathcal{P}_{zx} = \frac{-3\pi}{2\sqrt{\hat{s}}\Delta} \hat{m}_\ell \left\{ 2 \text{Im}(C_7^{\text{tot}} C_{10}^{\text{tot}*}) + \text{Im}(C_9^{\text{tot}} C_{10}^{\text{tot}*}) \right\}, \quad (18)$$

$$\begin{aligned} \mathcal{P}_{yy} = & \frac{1}{\Delta} \left\{ 24 \text{Re}(C_7^{\text{tot}} C_9^{\text{tot}*}) \frac{\hat{m}_\ell^2}{\hat{s}} - 4(|C_9^{\text{tot}}|^2 + |C_{10}^{\text{tot}}|^2) \frac{(1 - \hat{s})\hat{m}_\ell^2}{\hat{s}} \right. \\ & \left. + (|C_9^{\text{tot}}|^2 - |C_{10}^{\text{tot}}|^2) \left((-1 + \hat{s}) + \frac{6\hat{m}_\ell^2}{\hat{s}} \right) \right. \\ & \left. + 4 |C_7^{\text{tot}}|^2 \frac{((1 - \hat{s})\hat{s} + 2(2 + \hat{s})\hat{m}_\ell^2)}{\hat{s}^2} \right\}, \end{aligned} \quad (19)$$

$$\mathcal{P}_{zy} = \frac{3\pi}{2\sqrt{\hat{s}}\Delta} \left\{ 2 \text{Re}(C_7^{\text{tot}} C_{10}^{\text{tot}*}) - |C_{10}^{\text{tot}}|^2 + \text{Re}(C_9^{\text{tot}} C_{10}^{\text{tot}*}) \hat{s} \right\} \hat{m}_\ell \sqrt{1 - \frac{4\hat{m}_\ell^2}{\hat{s}}}, \quad (20)$$

$$\mathcal{P}_{xz} = -\mathcal{P}_{zx}, \quad (21)$$

$$\mathcal{P}_{yz} = \frac{3\pi}{2\sqrt{\hat{s}}\Delta} \left\{ 2 \text{Re}(C_7^{\text{tot}} C_{10}^{\text{tot}*}) + |C_{10}^{\text{tot}}|^2 + \text{Re}(C_9^{\text{tot}} C_{10}^{\text{tot}*}) \hat{s} \right\} \hat{m}_\ell \sqrt{1 - \frac{4\hat{m}_\ell^2}{\hat{s}}}, \quad (22)$$

$$\begin{aligned} \mathcal{P}_{zz} = & \frac{1}{2\Delta} \left\{ 12 \text{Re}(C_7^{\text{tot}} C_9^{\text{tot}*}) \left(1 - \frac{2\hat{m}_\ell^2}{\hat{s}} \right) + \frac{4 |C_7^{\text{tot}}|^2 (2 + \hat{s}) \left(1 - \frac{2\hat{m}_\ell^2}{\hat{s}} \right)}{\hat{s}} \right. \\ & \left. + (|C_9^{\text{tot}}|^2 + |C_{10}^{\text{tot}}|^2) \left(1 + 2\hat{s} - \frac{6(1 + \hat{s})\hat{m}_\ell^2}{\hat{s}} \right) \right. \\ & \left. + \frac{2(|C_9^{\text{tot}}|^2 - |C_{10}^{\text{tot}}|^2)(2 + \hat{s})\hat{m}_\ell^2}{\hat{s}} \right\}. \end{aligned} \quad (23)$$

Except \mathcal{P}_{zz} which is two times smaller than the one obtained in Ref. [44], the other coefficients \mathcal{P}_{ij} 's can be obtained from results in Ref. [44] by the replacement of $C_i^{\text{tot}} \rightarrow C_i^{\text{eff}}$ where $i = 7, 9, 10$.

4 Single and Double Lepton Polarization Forward-Backward Asymmetries

Equipped with the definition of the spin directions in the CM frame of leptons, we can evaluate the forward-backward asymmetries corresponding to various polarization components of the ℓ^- and/or ℓ^+ spin by writing [44]:

$$A_{FB}(\mathbf{s}^+, \mathbf{s}^-, \hat{s}) = A_{FB}(\hat{s}) + \left[\mathcal{A}_x^- s_x^- + \mathcal{A}_y^- s_y^- + \mathcal{A}_z^- s_z^- + \mathcal{A}_x^+ s_x^+ + \mathcal{A}_y^+ s_y^+ + \mathcal{A}_z^+ s_z^+ \right]$$

$$\begin{aligned}
& + \mathcal{A}_{xx} s_x^+ s_x^- + \mathcal{A}_{xy} s_x^+ s_y^- + \mathcal{A}_{xz} s_x^+ s_z^- \\
& + \mathcal{A}_{yx} s_y^+ s_x^- + \mathcal{A}_{yy} s_y^+ s_y^- + \mathcal{A}_{yz} s_y^+ s_z^- \\
& + \mathcal{A}_{zx} s_z^+ s_x^- + \mathcal{A}_{zy} s_z^+ s_y^- + \mathcal{A}_{zz} s_z^+ s_z^- \Big]. \quad (24)
\end{aligned}$$

The different polarized forward-backward asymmetries are then calculated as follows:

$$\mathcal{A}_x^+ = 0, \quad (25)$$

$$\mathcal{A}_y^+ = \frac{2}{\Delta} \text{Re}(C_9^{tot} C_{10}^{tot*}) \frac{(1 - \hat{s}) \hat{m}_\ell}{\sqrt{\hat{s}}} \sqrt{1 - \frac{4 \hat{m}_\ell^2}{\hat{s}}}, \quad (26)$$

$$\begin{aligned}
\mathcal{A}_z^+ &= \frac{1}{\Delta} \left\{ 6 \text{Re}(C_7^{tot} C_9^{tot*}) - \frac{6 |C_7^{tot}|^2}{\hat{s}} - 3 (|C_9^{tot}|^2 - |C_{10}^{tot}|^2) \hat{m}_\ell^2 \right. \\
& - 12 \text{Re}(C_7^{tot} C_{10}^{tot*}) \frac{\hat{m}_\ell^2}{\hat{s}} - 6 \text{Re}(C_9^{tot} C_{10}^{tot*}) \frac{\hat{m}_\ell^2}{\hat{s}} \\
& \left. - \frac{3}{2} (|C_9^{tot}|^2 + |C_{10}^{tot}|^2) \hat{s} \left(1 - \frac{2 \hat{m}_\ell^2}{\hat{s}}\right) \right\}, \quad (27)
\end{aligned}$$

$$\mathcal{A}_x^- = 0, \quad (28)$$

$$\mathcal{A}_y^- = \mathcal{A}_y^+, \quad (29)$$

$$\begin{aligned}
\mathcal{A}_z^- &= \frac{1}{\Delta} \left\{ -6 \text{Re}(C_7^{tot} C_9^{tot*}) - \frac{6 |C_7^{tot}|^2}{\hat{s}} - 3 (|C_9^{tot}|^2 - |C_{10}^{tot}|^2) \hat{m}_\ell^2 \right. \\
& + 12 \text{Re}(C_7^{tot} C_{10}^{tot*}) \frac{\hat{m}_\ell^2}{\hat{s}} + 6 \text{Re}(C_9^{tot} C_{10}^{tot*}) \frac{\hat{m}_\ell^2}{\hat{s}} \\
& \left. - \frac{3}{2} (|C_9^{tot}|^2 + |C_{10}^{tot}|^2) \hat{s} \left(1 - \frac{2 \hat{m}_\ell^2}{\hat{s}}\right) \right\}, \quad (30)
\end{aligned}$$

$$\mathcal{A}_{xx} = 0, \quad (31)$$

$$\mathcal{A}_{xy} = \frac{-6}{\Delta} (2 \text{Im}(C_7^{tot} C_{10}^{tot*}) + \text{Im}(C_9^{tot} C_{10}^{tot*})) \frac{\hat{m}_\ell^2}{\hat{s}}, \quad (32)$$

$$\mathcal{A}_{xz} = \frac{2}{\Delta} \text{Im}(C_9^{tot} C_{10}^{tot*}) \frac{(1 - \hat{s}) \hat{m}_\ell}{\sqrt{\hat{s}}} \sqrt{1 - \frac{4 \hat{m}_\ell^2}{\hat{s}}}, \quad (33)$$

$$\mathcal{A}_{yx} = -\mathcal{A}_{xy}, \quad (34)$$

$$\mathcal{A}_{yy} = 0, \quad (35)$$

$$\mathcal{A}_{yz} = \left(2 |C_9^{tot}|^2 - \frac{8 |C_7^{tot}|^2}{\hat{s}}\right) \frac{(1 - \hat{s}) \hat{m}_\ell}{\Delta \sqrt{\hat{s}}}, \quad (36)$$

$$\mathcal{A}_{zx} = \mathcal{A}_{xz}, \quad (37)$$

$$\mathcal{A}_{zy} = \mathcal{A}_{yz}, \quad (38)$$

$$\mathcal{A}_{zz} = \frac{-3}{\Delta} (2 \text{Re}(C_7^{tot} C_{10}^{tot*}) + \text{Re}(C_9^{tot} C_{10}^{tot*}) \hat{s}) \sqrt{1 - \frac{4 \hat{m}_\ell^2}{\hat{s}}}. \quad (39)$$

Here, \mathcal{A}_{zz} coincides with $-\mathcal{A}_{FB}$ in the SM and consequential extension of the SM (SM4) [44, 20]. A significant difference between \mathcal{A}_{zz} and (\mathcal{A}_{FB}) appears when the new type of interactions are taken into account in the effective Hamiltonian, i.e, the tensor type and scalar type interactions differ between \mathcal{A}_{zz} and (\mathcal{A}_{FB}) (see ref. [46]).

r_{sb}	0.01	0.02
$m_{t'}$ (GeV)	529	385

Table 2: The experimental limit of $m_{t'}$ for $\phi_{sb} = \pi/3$ [21]

r_{sb}	0.01	0.02
$m_{t'}$ (GeV)	373	289

Table 3: The experimental limit of $m_{t'}$ for $\phi_{sb} = \pi/2$ [21]

Note that, \mathcal{A}_{ij} coefficients calculated in Ref. [44] can again be obtained by the replacement $C_i^{tot} \rightarrow C_i^{eff}$ where $i = 7, 9, 10$.

5 Numerical analysis

We try to analyze the dependency of the various asymmetries on the fourth generation quark mass ($m_{t'}$) and the product of quark mixing matrix elements ($V_{t'b}^* V_{t's} = r_{sb} e^{i\phi_{sb}}$). We will use the next-to-leading order logarithmic approximation for the Wilson coefficients C_i^{eff} and C_i^{new} [30, 37] at the scale $\mu = m_b = 4.8 \text{ GeV}$. It is worth to mention that, beside the short distance contribution, C_9^{eff} has also long distance contributions resulting from the real $\bar{c}c$ resonant states of the J/ψ family. In the present study, we do take the long distance effects into account by using the approaches of Refs. [32, 31]

The input parameters we used in this analysis are as follows:

$|V_{tb} V_{ts}^*| = 0.04166$, $m_t = 175 \text{ GeV}$, $m_W = 80.41 \text{ GeV}$ and $\Gamma_B = 4.22 \times 10^{-13} \text{ GeV}$. In order to perform quantitative analysis of the physical observables, numerical values for the new parameters ($m_{t'}$, r_{sb} , ϕ_{sb}) are necessary. Using the experimental values of $B \rightarrow X_s \gamma$ and $B \rightarrow X_s \ell^+ \ell^-$, the bound on $r_{sb} \sim \{0.01 - 0.03\}$ has been obtained [8, 21] for $\phi_{sb} \sim \{0 - 2\pi\}$ and $m_{t'} \sim \{200, 600\}$ (GeV)(see table 2). Also considering Δm_{B_s} , ϕ_{sb} receives a strong restriction ($\phi_{sb} \sim \pi/2$) [11].

In order to do simplify analysis of the observables, we must eliminate some of the variables. From explicit expressions of the various physical observables, we see that they depend on four variables $m_{t'}$, r_{sb} , ϕ_{sb} and \hat{s} . Therefore, it may experimentally be difficult to study these dependencies at the same time. For this reason, we will do two types of analysis: first, we choose fixed values for $m_{t'} = 400 \text{ GeV}$, $r_{sb} \sim \{0.01, 0.02\}$ and $\phi_{sb} \sim \{\pi/3, \pi/2\}$ and look at the \hat{s} dependency of the FB asymmetries. Note that, zero point position of the FB asymmetries in terms of the \hat{s} is less sensitive to the hadronic uncertainties in exclusive decay channels. Second, we eliminate the \hat{s} dependence by performing integration over \hat{s} in the allowed region, i.e., we consider the averaged values of the various asymmetries. The

average gained over \hat{s} is defined as:

$$\langle \mathcal{P}(\mathcal{A}) \rangle = \frac{\int_{4\hat{m}_\ell^2}^{(1-\sqrt{\hat{r}_K})^2} \mathcal{P}(\mathcal{A}) \frac{d\mathcal{B}}{d\hat{s}} d\hat{s}}{\int_{4\hat{m}_\ell^2}^{(1-\sqrt{\hat{r}_K})^2} \frac{d\mathcal{B}}{d\hat{s}} d\hat{s}}.$$

We analysis the uncertainties among the SM parameters namely, product of quarks mixing angles $V_{tb}V_{ts}^*$ and quarks mass (m_b and m_c). With present bound on $0.03966 < |V_{tb}V_{ts}^*| < 0.04166$ [39] we find that the uncertainties of lepton polarization and FB asymmetries are negligible (less than 0.1%). There are rather weak dependency on the m_c/m_b which we show those by relevant figures. We present figures for only those observables that has significant dependence on the new parameters. The extra signs(+, \times , \square , \otimes) in figures show the experimental limit on $m_{\ell'}$, considering the 1σ level deviation from the measured branching ratio of $B \rightarrow X_s \ell^- \ell^+$ (see Table 1,2). *Note that, \mathcal{A}_z^- , $\mathcal{A}_{xz} = \mathcal{A}_{zx}$, $\mathcal{A}_{xy} = -\mathcal{A}_{yx}$, $\mathcal{A}_{yz} = \mathcal{A}_{zy}$ and \mathcal{P}_{zz} , \mathcal{P}_{xz} for μ channel and $\mathcal{A}_y^+ = \mathcal{A}_y^-$, $\mathcal{A}_{xz} = \mathcal{A}_{zx}$, $\mathcal{A}_{yz} = -\mathcal{A}_{zy}$, $\mathcal{P}_{yx} = \mathcal{P}_{yx}$ and \mathcal{P}_{zz} for τ channel do not deviate from the SM3 values over than 10%. Hence, we do not present their predictions in the figures.* A typical deviation of order -5% to -10% for studying the ratio of physical observables such as asymmetries, as we can see from figures and above mentioned discussion, can not be covered by the uncertainties among the SM parameters [22] i.e., the uncertainties of quark quark masses, quarks mixing angles and higher order calculations of the Wilson coefficients. Here, large parts of the uncertainties partially cancel out.

From these figures, we deduce the following results:

5.1 Differential Polarized FB Asymmetries

Figs 1–3 depict the \hat{s} dependency of the single or double lepton polarization FB asymmetries for fixed value of the 4th generation quark mass ($m_{\ell'} = 400$ GeV) and different values of the r_{sb} and ϕ_{sb} .

- $\mathcal{A}_z^-(\hat{s})$ for τ channel strongly depend on the SM4 parameters. The discrepancy of $\mathcal{A}_z^-(\hat{s})$ with respect to the m_c/m_b is almost negligible (see figs. 1).
- Magnitude of $\mathcal{A}_{zz}(\hat{s})$ is suppressed by the 4th generation for μ channel (see figs. 2). The zero point position of $\mathcal{A}_{zz}(\hat{s})$ for μ channel stays the same as the SM3 case. This point is especially important for the exclusive decays where the hadronic uncertainty almost vanishes at this point. The deviation is considerable for low and high \hat{s} values where both are in the non-resonance region (see fig. 2).
- $\mathcal{A}_{xy}(\hat{s}) = -\mathcal{A}_{yx}(\hat{s})$ strongly depends on SM4 parameters. The deviation can be a high as a factor of seven in the low \hat{s} region. At high \hat{s} region the sensitivity is less than the low \hat{s} region (see fig. 3). Moreover, there is rather weak dependency on the value of m_c/m_b .

5.2 Averaged Double Lepton Polarization Asymmetries

- Taking into account the 4th generation, the value of $\langle \mathcal{P}_{xx} \rangle$, $\langle \mathcal{P}_{yz} \rangle$ and $\langle \mathcal{P}_{yy} \rangle$ show strong dependency on the new parameters for both μ and τ channels. While $\langle \mathcal{P}_{xx} \rangle$, $\langle \mathcal{P}_{yz} \rangle$ and $\langle \mathcal{P}_{yy} \rangle$ are increasing for $\phi_{sb} = \pi/2$, they increase/decrease in different regions of parameter space for $\phi_{sb} = \pi/3$ (see figs. 4–8). The dependency on the value of m_c/m_b is ignorable.
- Due to inclusion of 4th generation, the value of $\langle \mathcal{P}_{xz} \rangle$ gets sizable deviation from the SM3 value (which is almost zero) (see fig. 9). Compared to the SM3 prediction, the τ channel obtains the maximum value about -0.2 (-0.3) when $\phi_{sb} \sim \pi/2(\pi/3)$ (see fig. 9). Also, there is weak dependency on the value of m_c/m_b .
- The non-zero values of $\langle \mathcal{P}_{yx} \rangle$ in the SM3 has their origin in the higher order QCD corrections to the C_9^{eff} . Since this function is proportional to imaginary part of the C_9^{eff} , its value is negligible. But, it exceeds the SM3 value sizeably with the SM4 contribution. This is because of the new weak phase and new contribution to the Wilson coefficients coming from the 4th generation. Furthermore, the maximum value of $\langle \mathcal{P}_{yx} \rangle$ for μ channel is almost independent from the values of r_{sb} and depend on ϕ_{sb} (see fig. 10). Note that its value also is sensitive to the value of m_c/m_b .

Finally, the quantitative estimate about the accessibility of the various physical observables in experiments are in order. To observe an asymmetry A at the $n\sigma$ level, the required number of $B\bar{B}$ pairs is given as:

$$N = \frac{n^2}{\mathcal{B}_{s_1 s_2} \langle \mathcal{A} \rangle^2},$$

where $s_i (i = 1, 2)$ is the efficiency of the lepton and \mathcal{B} is the branching ratio.

Typical values of the efficiencies of the τ -leptons range from 50% to 90% for their various decay modes [47]. It should be noted, here, that the error in τ -lepton polarization is estimated to be about (10 ÷ 15)% [48]. So, the error in measurement of the τ -lepton asymmetries is approximately (20 ÷ 30)%, and the error in obtaining the number of events is about 50%. Equipped with the expression of N , it can be understood that in order to detect the asymmetries in the μ and τ channels at 3σ level with the asymmetry of $\mathcal{A} = 10\%$ and efficiency of $\tau \sim 0.5$, the minimum number of required events are $N \sim 10^8$ and $N \sim 10^9$ for μ and τ leptons, respectively.

On the other hand, the number of $B\bar{B}$ pairs, that are produced at LHC, are about $\sim 10^{12}$. As a result of comparison of these numbers and N , we conclude that a typical asymmetry of ($\mathcal{A} = 10\%$) is detectable at LHC.

6 Conclusion

To sum up, we present the various asymmetries in inclusive $b \rightarrow s\ell^+\ell^-$ transition using the SM with the 4th generation of quarks. The results are:

- The zero point position of the polarized single or double lepton polarization FB asymmetry coincide with each other in the SM3 and SM4. Furthermore, there are sizable discrepancies, specially, in the non–resonance region between the result of the SM3 and SM4.
- Some of the double–lepton polarization and polarized double or single lepton polarization Forward–Backward asymmetries which are already accessible at LHC depict the strong dependency on the 4th generation quark mass and product of quark mixing.
- While the magnitude of asymmetries, which is proportional to the real part of the product of Wilson coefficients, is generally suppressed by the 4th generation parameters. The situation for the asymmetries proportional to the imaginary part of the product of Wilson coefficients is enhanced
- We examine the uncertainties among the SM parameters and we show that the discrepancy of order 10% in principle can not be covered by the uncertainties among the input parameters.

Thus, the study of such strong dependent asymmetries can serve as good test for the predictions of the SM3 and indirect search for the 4th generation up type quarks t' .

7 Acknowledgment

The authors would like to thank T. M. Aliev and A. Ozpineci for their useful discussions. One of the authors (M.B.) would like to thank TUBITAK, Turkish Scientific and Research Council, for their financial support provided under the project 103T666. Also, V. Bashiry would like to thank Ministry of Education of North Cyprus (Turkish Republic), for their partial financial support through the project.

References

- [1] P. Q. Hung, Marc Sher, arXiv:hep-ph/0711.4353.
- [2] Y. Kikukawa, M. Kohda and J. Yasuda, arXiv:0901.1962 [hep-ph].
- [3] E. Arik, S. Atağ, Z. Z. Aydin, A. Çelikel, Z. Çiçek, A. K. Çiftçi, A. Mailov, S. Sultansoy, and G. Ünel, *Phys. Rev.* **D58**, (1998) 117701.
- [4] P. Q. Hung, *Phys. Rev. Lett.* **80** (1998) 3000.
- [5] W. S. Hou, arXiv:0803.1234 [hep-ph].
- [6] T. Aaltonen *et al.* [CDF Collaboration], *Phys. Rev.* **D** (2007) 071006.
- [7] W.M. Yao, *et al.*, (Particle Data Group), *J. Phys. G: Nucl. Part. Phys.* **33**, (2006) 1.
- [8] A. Arhrib and W. S. Hou, *Eur. Phys. J.* **C27** (2003) 555.
- [9] V. Bashiry and K. Azizi, *JHEP* **0707**, (2007) 064 [arXiv:hep-ph/0702044].
- [10] T. M. Aliev, A. Özpineci and M. Savcı, *Eur. Phys. J.* **C29** (2003) 265.
- [11] W. S. Hou, H-n. Li, S. Mishima and M. Nagashima, arXiv:hep-ph/0611107.
- [12] W. S. Hou, M. Nagashima and A. Soddu, *Phys. Rev. Lett.* **95** (2005) 141601.
- [13] W. j. Huo, C. D. Lu and Z. j. Xiao, arXiv:hep-ph/0302177.
- [14] W. J. Huo, arXiv:hep-ph/0203226.
- [15] Y. Dincer, *Phys. Lett.* **B505** (2001) 89.
- [16] T. M. Aliev, A. Özpineci and M. Savcı, *Nucl. Phys.* **B585** (2000) 275.
- [17] C. S. Huang, W. J. Huo and Y. L. Wu, *Mod. Phys. Lett.* **A14** (1999) 2453.
- [18] S. Abachi *et al.* [D0 Collaboration], *Phys. Rev. Lett.* **78** (1997) 3818.
- [19] D. London, *Phys. Lett.* **B234** (1990) 354.
- [20] V. Bashiry and F. Falahati, arXiv:hep-ph/0707.3242.
- [21] F. Zolfagharpour and V. Bashiry, arXiv:hep-ph/0707.4337.
- [22] K. S. M. Lee and F. J. Tackmann, arXiv:0812.0001 [hep-ph].
- [23] Z. Ligeti and F. J. Tackmann, *Phys. Lett.* **B653**, 404 (2007); Tobias Hurth, arXiv:hep-ph/0703226; C. Greub, V. Pilipp, and C. Schüpbach, *JHEP* **0812**, (2008) 040.
- [24] A. Ghinculov, T. Hurth, G. Isidori and Y. P. Yao, *Nucl. Phys. B* **685** (2004) 351 [hep-ph/0312128].

- [25] For a recent review, see: T. Hurth, *Int. J. Mod. Phys. A* **22** (2007) 1781 [hep-ph/0703226]; and references therein.
- [26] C. W. Bauer and C. N. Burrell, *Phys. Lett. B* **469**, 248 (1999) [hep-ph/9907517];
- [27] M. Neubert, *JHEP* **0007**, 022 (2000) [hep-ph/0006068].
- [28] G. Buchalla, A. J. Buras and M. E. Lautenbacher, *Rev. Mod. Phys.* **68** (1996) 1125.
- [29] A. J. Buras and M. Munz, *Phys. Rev. D* **52**, 186 (1995) [arXiv:hep-ph/9501281].
- [30] A. J. Buras, M. Misiak, M. Munz and S. Pokorski, *Nucl. Phys. B* **424**, 374 (1994) [arXiv:hep-ph/9311345].
- [31] C. S. Lim, T. Morozumi and A. I. Sanda, *Phys. Lett. B* **218**, 343 (1989).
- [32] A. Ali, T. Mannel and T. Morozumi, *Phys. Lett. B* **273**, 505 (1991).
- [33] F. Kruger and L. M. Sehgal, *Phys. Lett. B* **380**, 199 (1996) [arXiv:hep-ph/9603237].
- [34] B. Grinstein, M. J. Savage and M. B. Wise, *Nucl. Phys.* **B319** (1989) 271.
- [35] G. Cella, G. Ricciardi and A. Viceré, *Phys. Lett.* **B258** (1991) 212.
- [36] C. Bobeth, M. Misiak, and J. Urban, *Nucl. Phys.* **B574** (2000) 291.
- [37] H. H. Asatrian, H. M. Asatrian, C. Greub, and M. Walker, *Phys. Lett.* **B507** (2001) 162.
- [38] M. Misiak, *Nucl. Phys.* **B393** (1993) 23, M. Misiak, *Nucl. Phys.* **B439** (1995) 461(E).
- [39] C. Amsler et al., [Particle Data Group], *Phys. Lett.* B667, 1 (2008).
- [40] T. Huber, T. Hurth, E. Lunghi, arXiv:0807.1940.
- [41] A. Ali, G. Hiller, L. T. Handako, and T. Morozumi, *Phys. Rev.* **D55** (1997) 4105.
- [42] A. F. Falk, M. E. Luke, and M. J. Savage, *Phys. Rev.* **D49** (1994) 3367.
- [43] K. S. Babu, K. R. S. Balaji, I. Schienbein, *Phys. Rev.* **D68** (2003) 014021.
- [44] W. Bensalem, D. London, N. Sinha and R. Sinha, *Phys. Rev.* **D67** (2003) 034007.
- [45] S. Fukae, C.S. Kim, T. Yoshikawa, *Phys. Rev.* **D61** (2000) 074015.
- [46] T. M. Aliev, V. Bashiry and M. Savci, *JHEP* **0405**, (2004) 037 [arXiv:hep-ph/0403282].
- [47] G. Abbiendi *et. al*, OPAL Collaboration, *Phys. Lett.* **B492**, (2000) 23.
- [48] A. Rouge, Workshop on τ lepton physics, Orsay, France (1990).

Figure captions

Fig. (1) The dependence of the $\mathcal{A}_z^-(\hat{s})$ for the $b \rightarrow s\tau^+\tau^-$ decay on \hat{s} for the fourth generation quark mass $m_{t'} = 400$ GeV for two different values of $\phi_{sb} = 60^\circ, 90^\circ$ and $r_{sb} = 0.01, 0.02$.

Fig. (2) The dependence of the $\mathcal{A}_{zz}(\hat{s})$ for the $b \rightarrow s\mu^+\mu^-$ decay on \hat{s} for the fourth generation quark mass $m_{t'} = 400$ GeV for two different values of $\phi_{sb} = 60^\circ, 90^\circ$ and $r_{sb} = 0.01, 0.02$.

Fig. (3) The dependence of the $\mathcal{A}_{xy}(\hat{s}) = -\mathcal{A}_{yx}(\hat{s})$ for the $b \rightarrow s\tau^+\tau^-$ decay on \hat{s} for the fourth generation quark mass $m_{t'} = 400$ GeV for two different values of $\phi_{sb} = 60^\circ, 90^\circ$ and $r_{sb} = 0.01, 0.02$.

Fig. (4) The dependence of the $\langle \mathcal{P}_{xx} \rangle$ on the fourth generation quark mass $m_{t'}$ for two different values of $\phi_{sb} = 60^\circ, 90^\circ$ and $r_{sb} = 0.01, 0.02$ for μ lepton.

Fig. (5) The same as in Fig. (4), but for the τ lepton.

Fig. (6) The same as in Fig. (5), but for the $\langle \mathcal{P}_{yz} \rangle$.

Fig. (7) The dependence of the $\langle \mathcal{P}_{yy} \rangle$ on the fourth generation quark mass $m_{t'}$ for two different values of $\phi_{sb} = 60^\circ, 90^\circ$ and $r_{sb} = 0.01, 0.02$ for μ lepton.

Fig. (8) The same as in Fig. (7), but for the τ lepton.

Fig. (9) The same as in Fig. (8), but for the $\langle \mathcal{P}_{xz} \rangle$.

Fig. (10) The same as in Fig. (7), but for the $\langle \mathcal{P}_{yx} \rangle$.

$m_{\tau} = 400 \text{ GeV}$

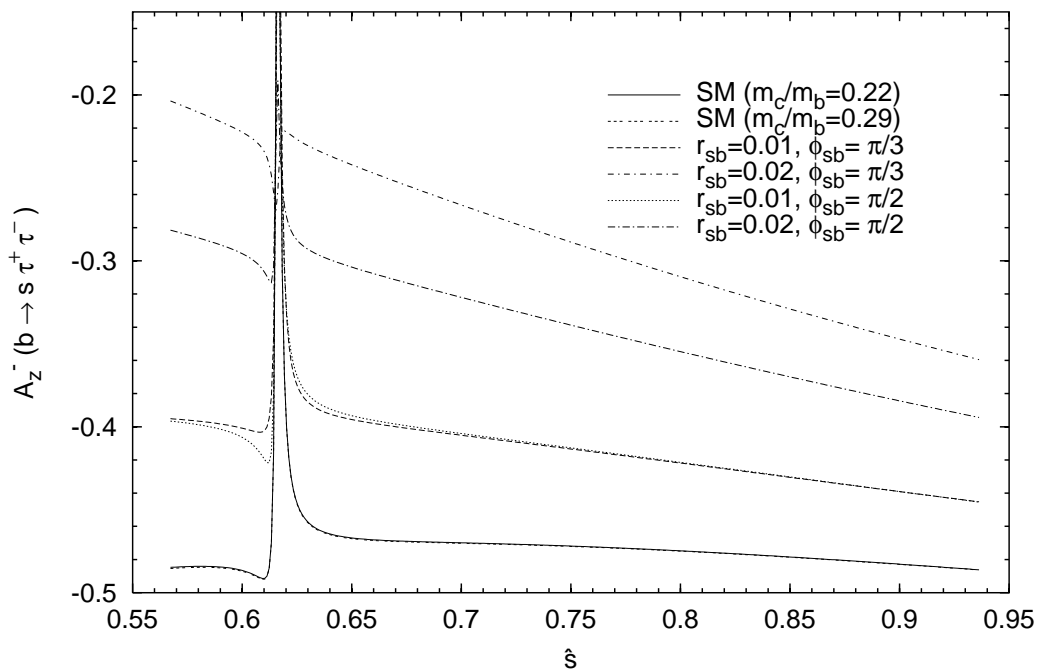


Figure 1:

$m_{\tau} = 400 \text{ GeV}$

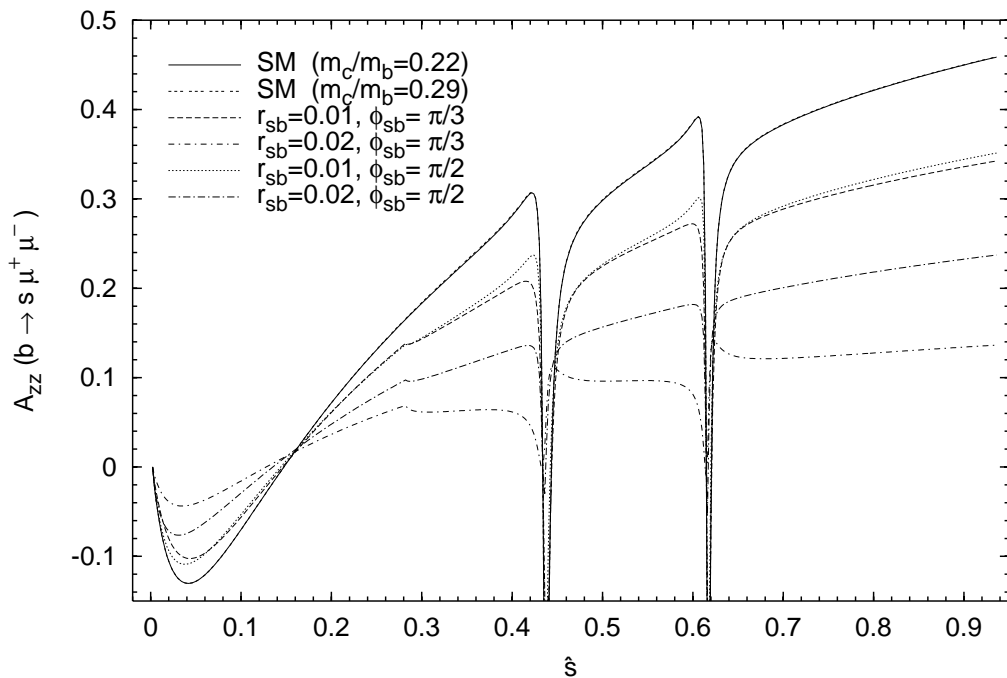


Figure 2:

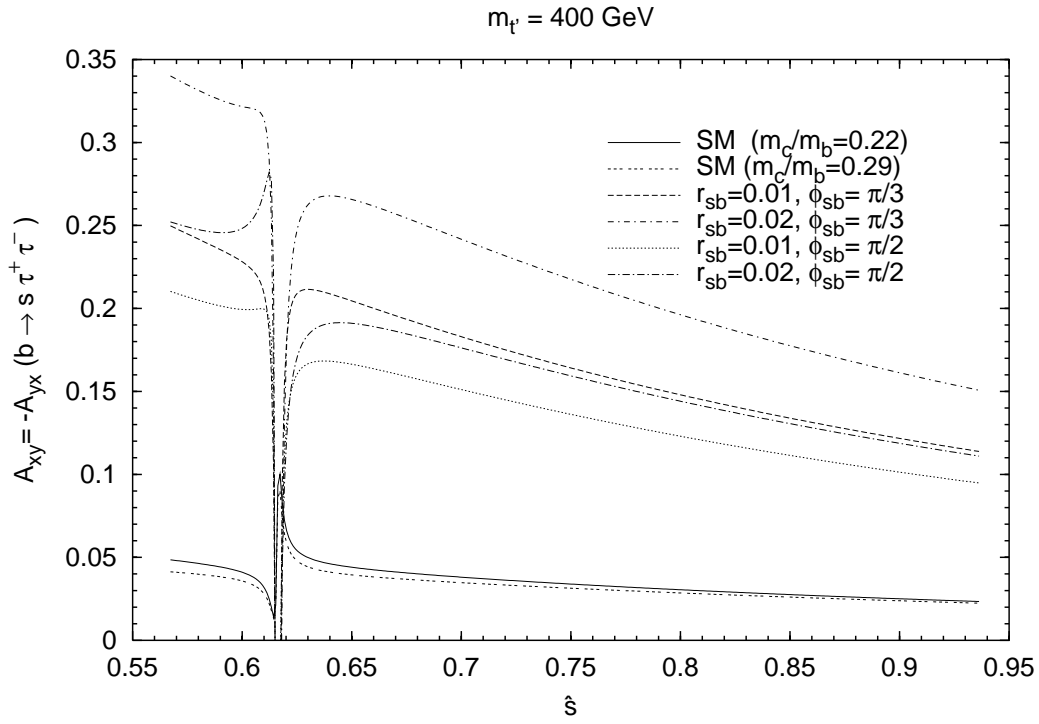


Figure 3:

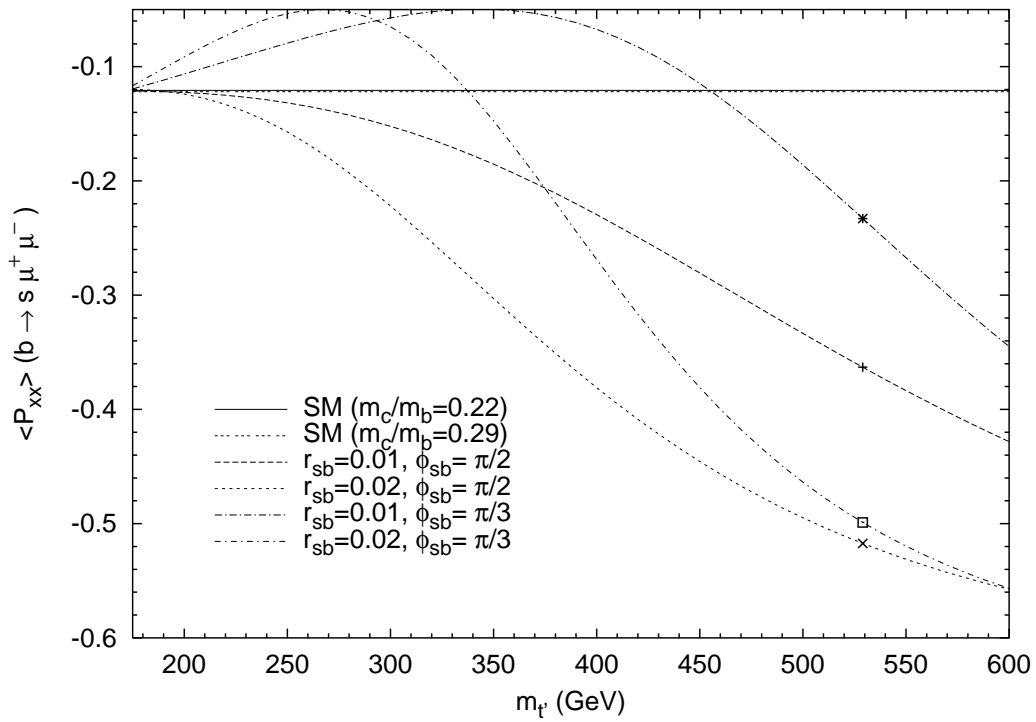


Figure 4:

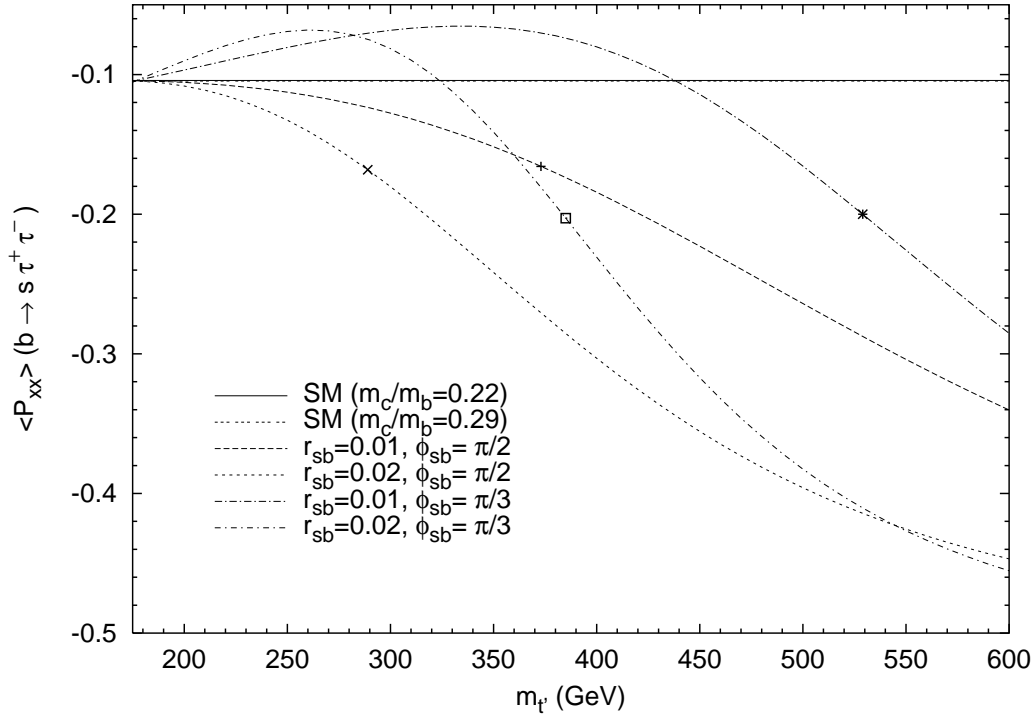


Figure 5:

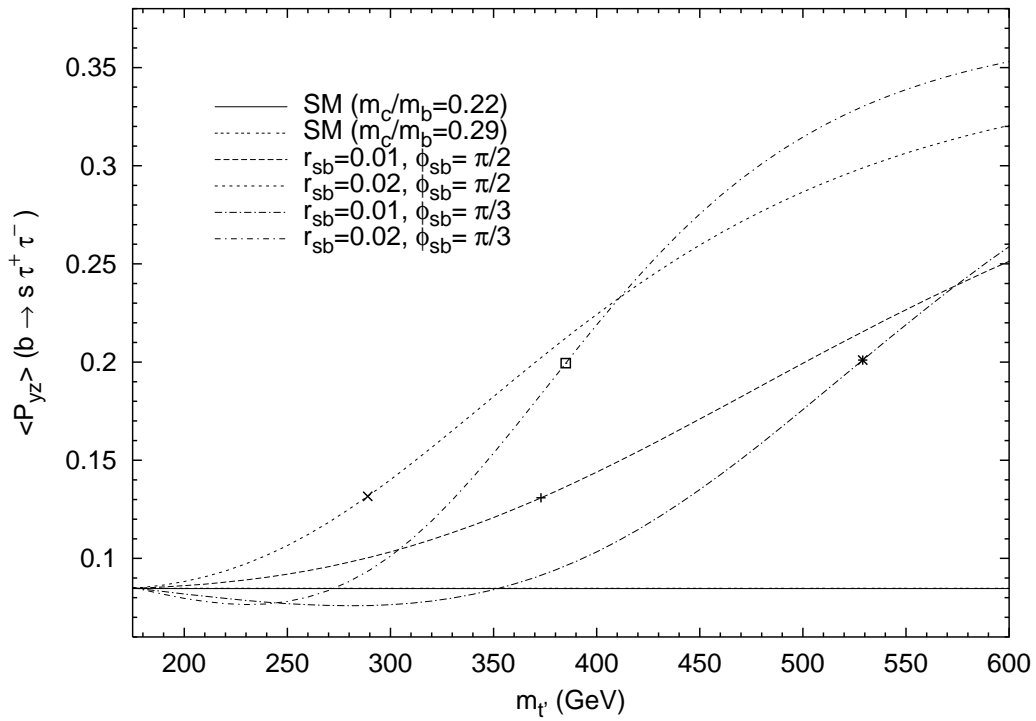


Figure 6:

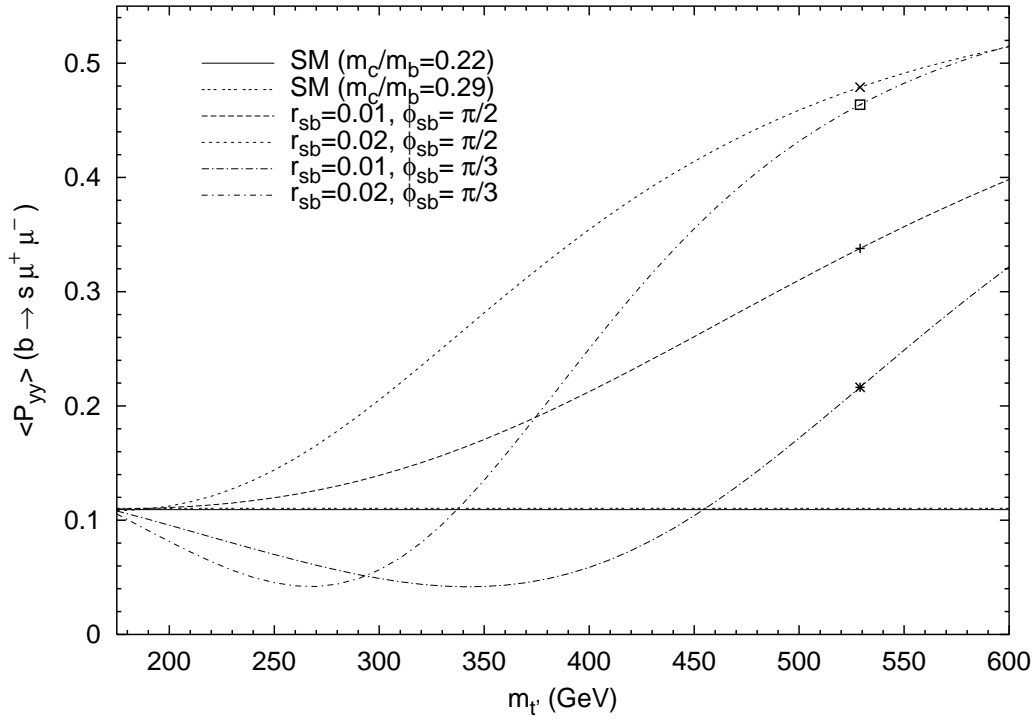


Figure 7:

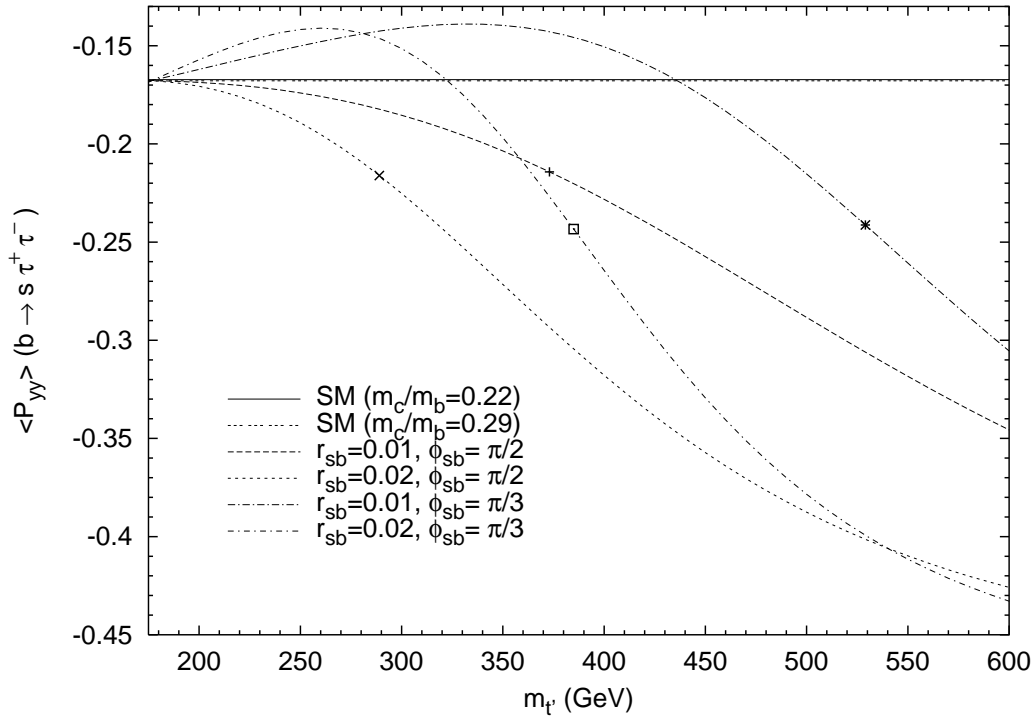


Figure 8:

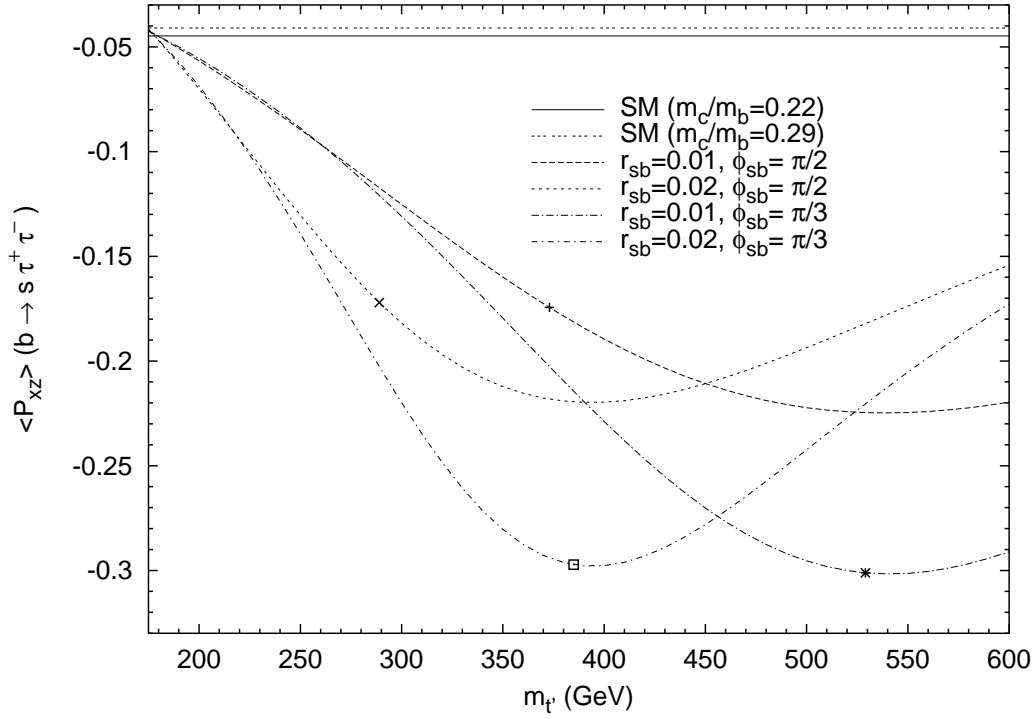


Figure 9:

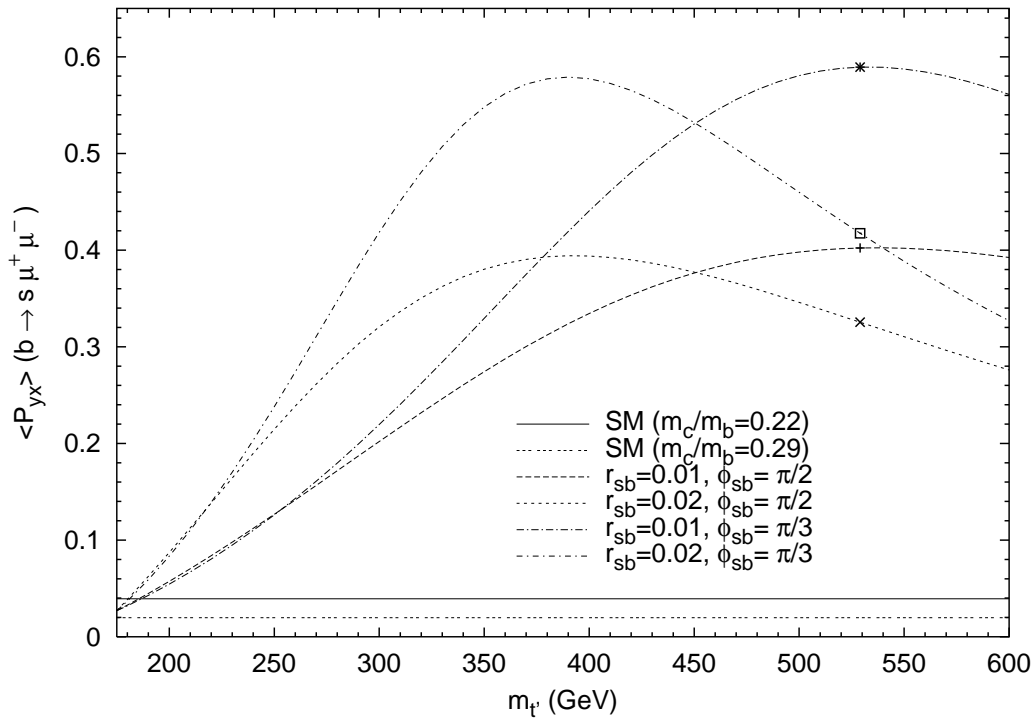


Figure 10: