Magnetic Moments of Decuplet Baryons in Light Cone QCD

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November 13, 2018

Abstract

We calculate the magnetic moments of decuplet baryons containing strange quarks within the framework of light cone QCD sum rules taking into account the $SU(3)$ flavor symmetry breaking effects. It is obtained that magnetic moments of the neutral Σ^{*0} and Ξ^{*0} baryons are mainly determined by the $SU(3)$ breaking terms. A comparison of our results on the magnetic moments of the decuplet baryons with the predictions of other approaches is presented.

1 Introduction

For the determination of the fundamental parameters of hadrons from experiments, some information about physics at large distances is required. The large distance physics can not be calculated directly from fundamental QCD Lagrangian because at large distance perturbation theory can not be applied. For this reason a reliable non-perturbative approach is needed. Among non-perturbative approaches, QCD sum rules [1] occupied a special place in studying the properties of ground state hadrons. This method is applied to various problems in hadron physics and extended in many works (see for example Refs. [2, 3, 4] and references therein). The magnetic moments of hadrons are one of their characteristic parameters in low energy physics. Calculation of the nucleon magnetic moments in the framework of QCD sum rules method using external fields technique, first suggested in [5], was carried out in [6, 7]. They were later refined and extended to the entire baryon octet in [8, 9].

In [10, 11], magnetic moments of the decuplet baryons are calculated within the framework of QCD sum rules using external field method. Note that in [10], from the decuplet baryons, only the magnetic moments of Δ^{++} and Ω^- were calculated. At present, the magnetic moments of Δ^{++} [12], Δ^0 [13] and Ω^- [14] are known from experiments. The experimental information provides new incentives for theoretical scrutiny of these physical quantities.

Recently, we have calculated the magnetic moments of the Δ baryons [15] within the framework of an alternative approach to the traditional sum rules, i.e. the light cone QCD sum rules (LCQSR). In this work, the magnetic moments of other members of the decuplet which contain at least one s-quark, namely the $\Sigma^{*\pm,0}$, $\Xi^{*0,-}$ and Ω^-, are calculated within the same approach. The novel feature of the present work is that we take into account the $SU(3)$ flavor symmetry breaking effects.

A few words about the LCQSR method are in order. The LCQSR is based on the operator product expansion on the light cone, which is an expansion over the twists of the operators rather than dimensions as in the traditional QCD sum rules. The main contribution comes from the lower twist operator. The matrix elements of the nonlocal operators between the vacuum and hadronic state defines the hadronic wave functions. (More about this method and its applications can be found in [16, 17] and references therein). Note that magnetic moments of the nucleon using LCQSR approach was studied in [18].

The paper is organized as follows. In Sect. II, the light cone QCD sum rules for the magnetic moments of the decuplet baryons are derived. In Sect. III, we carry out numerical calculations. Comparison of the predictions of this approach on the magnetic moments of the decuplet baryons with the results of other methods, and the experimental results is also presented in this section.

2 Sum Rules for the Magnetic Moments of Decuplet Baryons

A sum rule for the magnetic moment can be constructed by equating two different representations of the corresponding correlator, written in terms of hadrons and quark-gluons. We begin our calculations by considering the following correlator:

$$\Pi_{\mu\nu} = i \int dx e^{ipx} \langle 0 | \mathcal{T} \eta^B_\mu(x) \bar{\eta}^B_\nu(0) | 0 \rangle_F \,, \tag{1}$$

where $\mathcal{T}$ is the time ordering operator, F means electromagnetic field and the η^B_μ's are the interpolating currents of the corresponding baryon, B, carrying the same quantum numbers. This correlator can be calculated on one side phenomenologically, in terms of the hadron parameters, and on the other side by the operator product expansion (OPE) in the deep Eucledian region, $p^2 \to -\infty$, using QCD degrees of freedom. By equating both expressions, we construct the corresponding sum rules.

Saturating the correlator, Eq. (1), by ground state baryons we get:

$$\Pi_{\mu\nu}(p_1^2, p_2^2) = \frac{\langle 0 | \eta^B_\mu | B_1(p_1) \rangle}{p_1^2 - M_1^2} \langle B_1(p_1) | B_2(p_2) \rangle_F \frac{\langle B_2(p_2) | \eta^B_\nu | 0 \rangle}{p_2^2 - M_2^2}, \tag{2}$$

where $p_2 = p_1 + q$, q is the photon momentum and M_i is the mass of the baryon B_i.

The matrix elements of the interpolating currents between the ground state and the state containing a single baryon, B, with momentum p and having spin s is defined as:

$$\langle 0 | \eta_\mu | B(p, s) \rangle = \lambda_B u_\mu(p, s) \,, \tag{3}$$

where λ_B is the residue, and u_μ is the Rarita-Schwinger spin-vector (For a discussion of the properties of the Rarita-Schwinger spin-vector see e.g.

[19]). In order to write down the phenomenological part of the sum rules from Eq. (2) it follows that one also needs an expression for the matrix element $\langle B(p_1)|B(p_2)\rangle_F$, i.e. the electromagnetic vertex of spin 3/2 baryons. In the general case, this vertex can be written as:

$$\langle B(p_1)|B(p_2)\rangle_F = \epsilon_\rho \bar{u}_\mu(p_1)\mathcal{O}^{\mu\rho\nu}(p_1,p_2)u_\nu(p_2)\,, \tag{4}$$

where ϵ_ρ is the polarization vector of the photon and the Lorentz tensor $\mathcal{O}^{\mu\rho\nu}$ is given by:

$$\begin{aligned}\mathcal{O}^{\mu\rho\nu}(p_1,p_2) &= -g^{\mu\nu}\left[\gamma_\rho(f_1+f_2)+\frac{(p_1+p_2)_\rho}{2M_B}f_2+q_\rho f_3\right]- \\ &- \frac{q_\mu q_\nu}{(2M_B)^2}\left[\gamma_\rho(G_1+G_2)+\frac{(p_1+p_2)_\rho}{2M_B}G_2+q_\rho G_3\right]\end{aligned} \tag{5}$$

where the form factors f_i and G_i are functions of $q^2=(p_1-p_2)^2$. In our problem, the values of the formfactors only at one point, $q^2=0$, are needed.

In calculations, summation over spins of the Rarita-Schwinger spin vector is performed,

$$\sum_s u_\sigma(p,s)\bar{u}_\tau(p,s) = -\frac{(\not p+M_B)}{2M_B}\left\{g_{\sigma\tau}-\frac{1}{3}\gamma_\sigma\gamma_\tau-\frac{2p_\sigma p_\tau}{3M_B^2}+\frac{p_\sigma\gamma_\tau-p_\tau\gamma_\sigma}{3M_B}\right\} \tag{6}$$

Using Eqs. (2-6), one can see that the correlator contains many structures, not all of them independent. To remove the dependencies, an ordering of the gamma matrices should be chosen. For this purpose the ordering $\gamma_\mu \not p_1 \not\epsilon \not p_2 \gamma_\nu$ is chosen. With this ordering, the correlation function becomes:

$$\begin{aligned}\Pi_{\mu\nu} &= \lambda_B^2\frac{1}{(p_1^2-M_B^2)(p_2^2-M_B^2)}\left[g_{\mu\nu}\ \not p_1 \not\epsilon \not p_2 \frac{g_M}{3}+\right. \\ &+ \left.\textit{other structures with } \gamma_\mu \textit{ at the beginning and } \gamma_\nu \textit{ at the end}\right]\end{aligned} \tag{7}$$

where g_M is the magnetic form factor, $g_M/3 = f_1+f_2$. The value of g_M at $q^2=0$ gives the magnetic moment of the baryon in units of its natural magneton, $e\hbar/2m_Bc$. Hence, among the many structures in the correlator, for determination of the magnetic moments, only the structure $g_{\mu\nu}\ \not p_1 \not\epsilon \not p_2$ is needed. The appearance of the factor 3 can be understood from the fact that in the nonrelativistic limit, the maximum energy of the baryon in the presence of a uniform magnetic field with magnitude H is $3(f_1+f_2)H \equiv g_M H$ [20]. Another advantage of choosing the $g_{\mu\nu}\ \not{p}_1 \not{e} \not{p}_2$ structure is that spin 1/2 baryons do not contribute to this structure. Indeed, their overlap is given by:

$$\langle 0|\eta_\mu|J=1/2\rangle = (Ap_\mu + B\gamma_\mu)u(p) \tag{8}$$

where $(\not{p}-m)u(p)=0$ and $(Am+4B)=0$ [20, 21], and we can not construct the structure $g_{\mu\nu}\ \not{p}_1 \not{e} \not{p}_2$.

For calculating the correlator (1) from the QCD side, first of all, suitable interpolating currents should be chosen. For the baryons under study, they can be chosen as (see for example [11]):

$$\begin{aligned}
\eta_\mu^{\Sigma^{*+}} &= \frac{1}{\sqrt{3}}\epsilon^{abc}[2(u^{aT}C\gamma_\mu s^b)u^c + (u^{aT}C\gamma_\mu u^b)s^c]\,, \\
\eta_\mu^{\Sigma^{*0}} &= \sqrt{\frac{2}{3}}\epsilon^{abc}[(u^{aT}C\gamma_\mu d^b)s^c + (d^{aT}C\mu_\alpha s^b)u^c + (s^{aT}C\gamma_\mu u^b)d^c]\,, \\
\eta_\mu^{\Sigma^{*-}} &= \frac{1}{\sqrt{3}}\epsilon^{abc}[2(d^{aT}C\gamma_\mu s^b)d^c + (d^{aT}C\gamma_\mu d^b)s^c]\,, \\
\eta_\mu^{\Xi^{*0}} &= \frac{1}{\sqrt{3}}\epsilon^{abc}[2(s^{aT}C\gamma_\mu u^b)s^c + (s^{aT}C\gamma_\mu s^b)u^c]\,, \\
\eta_\mu^{\Xi^{*-}} &= \frac{1}{\sqrt{3}}\epsilon^{abc}[2(s^{aT}C\gamma_\mu d^b)s^c + (s^{aT}C\gamma_\mu s^b)d^c]\,, \\
\eta_\mu^{\Omega^-} &= \epsilon^{abc}(s^{aT}C\gamma_\mu s^b)s^c
\end{aligned} \tag{9}$$

where C is the charge conjugation operator, $a,\ b,\ c$ are color indices. It should be noted that these baryon currents are not unique, one can choose an infinite number of currents with the same quantum numbers [22, 23].

After some calculations, for the theoretical parts of the correlator, we get:

$$\begin{aligned}
\Pi_{\mu\nu}^{\Sigma^{*+}} = \ & \Pi'^{\Sigma^{*+}}_{\mu\nu} - \frac{1}{6}\epsilon^{abc}\epsilon^{def}\int d^4x e^{ipx}\langle\gamma(q)|\bar{u}^d A_i u^a \\
& \Big\{2A_i\gamma_\nu S'^{be}_s\gamma_\mu S^{cf}_u + 2A_i\gamma_\nu S'^{cf}_u\gamma_\mu S^{be}_s + \\
& +2S^{be}_s\gamma_\nu A'_i\gamma_\mu S^{cf}_u + 2A_i\mathrm{Tr}(\gamma_\nu S'^{cf}_u\gamma_\mu S^{be}_s) + \\
& +S^{be}_s\mathrm{Tr}(\gamma_\nu A'_i\gamma_\mu S^{cf}_u) + \\
& +2S^{cf}_u\gamma_\nu S'^{be}_s\gamma_\mu A_i + 2S^{cf}_u\gamma_\nu A'_i\gamma_\mu S^{be}_s + \\
& +2S^{be}_s\gamma_\nu S'^{cf}_u\gamma_\mu A_i + 2S^{cf}_u\mathrm{Tr}(\gamma_\nu A'_i\gamma_\mu S^{be}_s) +
\end{aligned}$$

$$
\begin{aligned}
& +S_s^{be}\mathrm{Tr}(\gamma_\nu S_u'^{\,cf}\gamma_\mu A_i)\Big\} + \bar{s}^e A_i s^b \\
& \Big\{2S_u^{ad}\gamma_\nu A_i'\gamma_\mu S_u^{cf} + 2S_u^{ad}\gamma_\nu S_u'^{\,cf}\gamma_\mu A_i + \\
& +2A_i\gamma_\nu S_u'^{\,ad}\gamma_\mu S_u^{cf} + 2S_u^{ad}\mathrm{Tr}(\gamma_\nu S_u'^{\,cf}\gamma_\mu A_i) + \\
& +A_i\mathrm{Tr}(\gamma_\nu S_u'^{\,ad}\gamma_\mu S_u^{ad})\Big\}\,|0\rangle \qquad (10)
\end{aligned}
$$

$$
\begin{aligned}
\Pi_{\mu\nu}^{\Omega^-} = \quad & \Pi_{\mu\nu}'^{\Omega^-} + \frac{1}{2}\epsilon^{abc}\epsilon^{def}\int d^4x e^{ipx}\langle\gamma(q)|\bar{s}^f A_i s^a \\
& \Big\{2S_s^{cd}\gamma_\nu S_s'^{be}\gamma_\mu A_i + 2S_s^{cd}\gamma_\nu A_i'\gamma_\mu S_s^{be} + \\
& +2A_i\gamma_\nu S_s'^{cd}\gamma_\mu S_s^{be} + S_s^{cd}\mathrm{Tr}(\gamma_\nu S_s'^{\,be}\gamma_\mu A_i) + \\
& +S_s^{cd}\mathrm{Tr}(\gamma_\nu A_i'\gamma_\mu S_s^{be}) + A_i\mathrm{Tr}(\gamma_\nu S_s'^{\,cd}\gamma_\mu S_s^{be})\Big\}\,|0\rangle \qquad (11)
\end{aligned}
$$

where $A_i = 1$, γ_α, $\sigma_{\alpha\beta}/\sqrt{2}$, $i\gamma_\alpha\gamma_5$, γ_5, a sum over A_i implied, $S' \equiv CS^TC$, $A_i' = CA_i^TC$, with T denoting the transpose of the matrix, and S_q is the full light quark propagator with both perturbative and non-perturbative contributions. We calculate the theoretical part of the sum rules in linear order in the strange quark mass, m_s. The calculations show that, the terms quadratic in the strange quark mass give smaller contributions than the terms linear in m_s (about 8%). For the propagator of quarks, we will use the following expression:

$$
\begin{aligned}
S_q &= \langle 0|\mathcal{T}\bar{q}(x)q(0)|0\rangle \\
&= \frac{i\not{x}}{2\pi^2x^4} - \frac{m_q}{4\pi^2x^2} - \frac{\langle\bar{q}q\rangle}{12}\left(1 - \frac{im_q}{4}\not{x}\right) - \frac{x^2}{192}m_0^2\langle\bar{q}q\rangle\left(1 - \frac{im_q}{6}\not{x}\right) - \\
&- ig_s\int_0^1 dv\left[\frac{\not{x}}{16\pi^2x^2}G_{\mu\nu}(vx)\sigma_{\mu\nu} - vx_\mu G_{\mu\nu}(vx)\gamma_\nu\frac{i}{4\pi^2x^2} - \right. \\
&- \left.\frac{im_q}{32\pi^2}G_{\mu\nu}\sigma_{\mu\nu}\left(\ln\frac{-x^2\Lambda^2}{4} + 2\gamma_E\right)\right] \qquad (12)
\end{aligned}
$$

where Λ is an energy cutoff separating perturbative and non-perturbative regimes.

In Eqs. (10)-(11), the first terms, $\Pi_{\mu\nu}'^{B}$, describe diagrams in which the photon interact with the quarks perturbatively. Their explicit expressions can be obtained from the remaining terms by substituting all occurances of

$$
\bar{q}^a(x)A_iq^bA_{i\alpha\beta} \to 2\left(\int d^4yF_{\mu\nu}y_\nu S_q^{pert}(x-y)\gamma_\mu S_q^{pert}(y)\right)_{\alpha\beta}^{ba} \qquad (13)
$$

where the Fock-Schwinger gauge, $x_\mu A_\mu(x) = 0$ is used, and S_q^{pert} is the perturbative part of the quark propagator, i.e. the first two terms in Eq. (12). Here, $F_{\mu\nu}$ is the electromagnetic field strength tensor.

For customary, here we presented theoretical results only for the correlators of Σ^{*+} and Ω^- (see Eqs. (10) and (11)). The corresponding expressions for the theoretical parts of the correlators for the Σ^{*-}, Σ^{*0}, Ξ^{*0} and Ξ^{*-} baryons can be obtained from Eq. (10) as follows: For Σ^{*-}, substitute d quarks instead of u quarks; for Ξ^{*0} exchange u and s quarks; and for Ξ^{*-}, substitute s quarks instead of u quarks, and d quarks instead of s quarks. The theoretical part of the correlator for the Σ^{*0} baryon is half the sum of the theoretical parts of the correlators for the Σ^{*+} and Σ^{*-} baryons in exact $SU(2)$ flavor symmetry limit.

For calculating the QCD part of the sum rules, one needs to know the matrix elements $\langle\gamma(q)|\bar{q}A_i q|0\rangle$. Upto twist-4, matrix elements contributing to the selected $g_{\mu\nu}\ \not{p}_1 \not{\epsilon}\ \not{p}_2$ structure are expressed in terms of the photon wave functions as [24, 25, 26]:

$$\begin{aligned}
\langle\gamma(q)|\bar{q}\gamma_\alpha\gamma_5 q|0\rangle &= \frac{f}{4} e_q \epsilon_{\alpha\beta\rho\sigma}\epsilon^\beta q^\rho x^\sigma \int_0^1 du e^{iuqx}\psi(u) \\
\langle\gamma(q)|\bar{q}\sigma_{\alpha\beta} q|0\rangle &= i e_q \langle\bar{q}q\rangle \int_0^1 du e^{iuqx} \\
&\times \{(\epsilon_\alpha q_\beta - \epsilon_\beta q_\alpha)[\chi\phi(u) + x^2[g_1(u) - g_2(u)]] \\
&+ [qx(\epsilon_\alpha x_\beta - \epsilon_\beta x_\alpha) + \epsilon x(x_\alpha q_\beta - x_\beta q_\alpha)]\, g_2(u)\} \qquad (14)
\end{aligned}$$

where χ is the magnetic susceptibility of the quark condensate and e_q is the quark charge. The functions $\phi(u)$ and $\psi(u)$ are the leading twist-2 photon wave functions, while $g_1(u)$ and $g_2(u)$ are the twist-4 functions.

Using Eqs. (12) and (14), after some algebra, and performing Fourier transformation, the result for the structure $g_{\mu\nu}\ \not{p}_1\ \not{\epsilon}\ \not{p}_2$ can be obtained. As stated earlier, in order to construct the sum rules, we must equate the phenomenological and theoretical expressions for the correlator. Performing the Borel transformation on the variables p^2 and $(p+q)^2$ in order to suppress the contributions of the higher resonances and the continuum, the following sum rules for the magnetic moment of the baryons are obtained:

$$\begin{aligned}
g_M^{\Sigma^{*+}} &= \frac{e^{\frac{M_{\Sigma^*}^2}{M^2}}}{\lambda_{\Sigma^*}^2} \left\{ \frac{f\psi(u_0)}{12\pi^2} \left[\frac{\langle g^2G^2\rangle}{48} - M^4 f_1(\frac{s_0}{M^2}) \right] (e_s + 2e_u) + \right. \\
&+ \frac{8}{3}\langle\bar{u}u\rangle(g_1(u_0) - g_2(u_0)) \left[\langle\bar{s}s\rangle(e_s + e_u) + \langle\bar{u}u\rangle e_u\right] +
\end{aligned}$$

$$
\begin{aligned}
&+ \quad \frac{\chi\phi(u_0)\langle\bar{u}u\rangle}{6}\left[m_0^2 - 4M^2 f_0(\frac{s_0}{M^2})\right](\langle\bar{s}s\rangle(e_s+e_u)+\langle\bar{u}u\rangle e_u)+ \\
&+ \quad \frac{2}{3}\langle\bar{u}u\rangle(e_s\langle\bar{u}u\rangle+2e_u\langle\bar{s}s\rangle)+\frac{\langle g^2G^2\rangle M^2}{768\pi^4}f_0(\frac{s_0}{M^2})(e_s+2e_u)+ \\
&+ \quad \frac{3M^6}{64\pi^2}f_2(\frac{s_0}{M^2})(e_s+2e_u)+\frac{m_sM^2}{4\pi^2}f_0(\frac{s_0}{M^2})(e_u\langle\bar{s}s\rangle-e_s\langle\bar{u}u\rangle)- \\
&- \quad \frac{m_s\langle\bar{u}u\rangle}{8\pi^2}\left(\gamma_E-\ln\frac{\Lambda^2}{M^2}\right)\left[m_0^2e_s+\frac{e_u}{9}\langle g^2G^2\rangle\phi(u_0)\chi\right]+ \\
&+ \quad \frac{m_s\langle\bar{u}u\rangle M^2}{\pi^2}f_0(\frac{s_0}{M^2})\left[e_s\gamma_E-2e_u(g_1(u_0)-g_2(u_0))-e_u\right]+ \\
&+ \quad \frac{e_um_s\langle\bar{u}u\rangle}{4\pi^2}\left(m_0^2-\frac{2}{9}(g_1(u_0)-g_2(u_0))\frac{\langle g^2G^2\rangle}{M^2}+\right. \\
&+ \quad \left.\frac{8}{3}\pi^2 f\psi(u_0)+\chi\phi(u_0)M^4f_1(\frac{s_0}{M^2})\right)\Bigg\}\ , \qquad (15)
\end{aligned}
$$

$$
\begin{aligned}
g_M^{\Xi^{*0}} \quad &= \quad \frac{e^{\frac{M^2_{\Xi^*}}{M^2}}}{\lambda^2_{\Xi^*}}\left\{\frac{f\psi(u_0)}{12\pi^2}\left[\frac{\langle g^2G^2\rangle}{48}-M^4f_1(\frac{s_0}{M^2})\right](e_u+2e_s)+\right. \\
&+ \quad \frac{8}{3}\langle\bar{s}s\rangle(g_1(u_0)-g_2(u_0))\left[\langle\bar{u}u\rangle(e_u+e_s)+\langle\bar{s}s\rangle e_s\right]+ \\
&+ \quad \frac{\chi\phi(u_0)\langle\bar{s}s\rangle}{6}\left[m_0^2-4M^2f_0(\frac{s_0}{M^2})\right](\langle\bar{u}u\rangle(e_u+e_s)+\langle\bar{s}s\rangle e_s)+ \\
&+ \quad \frac{2}{3}\langle\bar{s}s\rangle(e_u\langle\bar{s}s\rangle+2e_s\langle\bar{u}u\rangle)+\frac{\langle g^2G^2\rangle M^2}{768\pi^4}f_0(\frac{s_0}{M^2})(e_u+2e_s)+ \\
&+ \quad \frac{3M^6}{64\pi^2}f_2(\frac{s_0}{M^2})(e_u+2e_s)+ \\
&- \quad \frac{m_s}{\pi^2}(g_1(u_0)-g_2(u_0))(\langle\bar{s}s\rangle e_s+\langle\bar{u}u\rangle e_u)\left(2M^2f_0(\frac{s_0}{M^2})+\frac{\langle g^2G^2\rangle}{18M^2}\right)- \\
&- \quad \frac{m_s\chi\phi(u_0)}{72\pi^2}\langle g^2G^2\rangle(\langle\bar{s}s\rangle e_s+\langle\bar{u}u\rangle e_u)\left(\gamma_E-\ln\frac{\Lambda^2}{M^2}\right)+ \\
&- \quad \frac{m_0^2m_se_s}{8\pi^2}(\langle\bar{s}s\rangle+\langle\bar{u}u\rangle)\left(\gamma_E-\ln\frac{\Lambda^2}{M^2}\right)+ \\
&+ \quad m_s\left(\frac{2}{3}f\psi(u_0)+\frac{m_0^2}{4\pi^2}\right)(\langle\bar{u}u\rangle e_s+\langle\bar{s}s\rangle e_u) \\
&+ \quad \frac{m_s}{4\pi^2}M^2f_0(\frac{s_0}{M^2})\left[4\gamma_Ee_s(\langle\bar{s}s\rangle+\langle\bar{u}u\rangle)-(5\langle\bar{u}u\rangle e_s+3\langle\bar{s}s\rangle e_u)\right]
\end{aligned}
$$

$$
\begin{aligned}
&+ \quad \frac{m_s \chi \phi(u_0)}{4\pi^2} M^4 f_1(\frac{s_0}{M^2})(\langle \bar{s}s \rangle e_s + \langle \bar{u}u \rangle e_u) \Bigg\} \ , \qquad (16) \\
g_M^{\Omega^-} \quad &= \quad \frac{e_s}{\lambda_\Omega^2} e^{\frac{M_\Omega^2}{M^2}} \left\{ \frac{f\psi(u_0)}{4\pi^2} \left[\frac{\langle g^2 G^2 \rangle}{48} - M^4 f_1(\frac{s_0}{M^2}) \right] + \right. \\
&+ \quad 8\langle \bar{s}s \rangle^2 [g_1(u_0) - g_2(u_0)] + \\
&+ \quad \frac{\chi\phi(u_0)\langle \bar{s}s \rangle^2}{2} \left[m_0^2 - 4M^2 f_0(\frac{s_0}{M^2}) \right] \\
&+ \quad 2\langle \bar{s}s \rangle^2 + \frac{\langle g^2 G^2 \rangle M^2}{256\pi^4} f_0(\frac{s_0}{M^2}) + \frac{9M^6}{64\pi^4} f_2(\frac{s_0}{M^2}) + 2f\psi(u_0) m_s \langle \bar{s}s \rangle - \\
&- \quad \frac{m_s \langle \bar{s}s \rangle}{6\pi^2} \langle g^2 G^2 \rangle \left[\frac{g_1(u_0) - g_2(u_0)}{M^2} + \chi\phi(u_0) \left(\gamma_E - \ln \frac{\Lambda^2}{M^2} \right) \right] - \\
&- \quad \frac{6}{\pi^2} m_s \langle \bar{s}s \rangle (g_1(u_0) - g_2(u_0)) M^2 f_0(\frac{s_0}{M^2}) + \\
&+ \quad \frac{3m_0^2}{8\pi^2} m_s \langle \bar{s}s \rangle \left(2 - \gamma_E + \ln \frac{\Lambda^2}{M^2} \right) - \\
&- \quad \left. \frac{3(1-\gamma_E)}{\pi^2} m_s \langle \bar{s}s \rangle M^2 f_0(\frac{s_0}{M^2}) + \frac{3\chi\phi(u_0)}{4\pi^2} m_s \langle \bar{s}s \rangle M^4 f_1(\frac{s_0}{M^2}) \right\} . \qquad (17)
\end{aligned}
$$

As is stated earlier, the sum rules for $\Sigma^{*\pm}$, and Ξ^{*-} can be obtained from Eq. (15) and Eq. (16), respectively as follows: To obtain the sum rules for Σ^{*-} and Σ^{*0} from Eq. (15), replace e_u by e_d and $(e_u + e_d)/2$ respectively. To obtain the sum rules for Ξ^{*-}, replace e_u by e_d in Eq. (16).

In Eqs. (15)-(17), the functions

$$
f_n(x) = 1 - e^{-x} \sum_{k=0}^{n} \frac{x^k}{k!} \qquad (18)
$$

are used to subtract the contributions of the continuum and s_0 is the continuum threshold,

$$
\begin{aligned}
u_0 \quad &= \quad \frac{M_2^2}{M_1^2 + M_2^2} \\
\frac{1}{M^2} \quad &= \quad \frac{1}{M_1^2} + \frac{1}{M_2^2}
\end{aligned}
$$

As we are working with just a single baryon, the Borel parameters M_1^2 and M_2^2 should be taken to be equal, i.e. $M_1^2 = M_2^2$, from which it follows that $u_0 = 1/2$.

3 Numerical Analysis

From the sum rules, one sees that, besides several constants, one needs expressions for the photon wave functions in order to calculate the numerical value of the magnetic moment of the decuplet baryons. It was shown in [24, 25] that they do not deviate much from the asymptotic form, hence, we shall use the following photon wave functions [25, 26]:

$$\begin{aligned}
\phi(u) &= 6u\bar{u} \\
\psi(u) &= 1 \\
g_1(u) &= -\frac{1}{8}\bar{u}(3-u) \\
g_2(u) &= -\frac{1}{4}\bar{u}^2
\end{aligned}$$

where $\bar{u} = 1 - u$. The values of the other constants that are used in the calculation are: $f = 0.028\,GeV^2$, $\chi = -4.4\,GeV^{-2}$ [27] (in [28], χ is estimated to be $\chi = -3.3\,GeV^{-2}$), $\langle g^2G^2\rangle = 0.474\,GeV^4$, $\langle\bar{u}u\rangle = \langle\bar{s}s\rangle/0.8 = -(0.243)^3\,GeV^3$, $m_0^2 = (0.8 \pm 0.2)\,GeV^2$ [29], $\lambda_{\Sigma^*} = 0.043\,GeV^3$, $\lambda_{\Xi^*} = 0.053\,GeV^3$, $\lambda_\Omega = 0.068\,GeV^3$ [30]. For the energy cut-off, Λ, we will take $\Lambda = 0.5\,GeV$.

Having fixed the input parameters, our next task is to find a region of Borel parameter, M^2, where dependence of the magnetic moments on M^2 and the continuum threshold s_0 is rather weak and at the same time higher states and continuum contributions remain under control. We demand that these contributions are less then 35%. Under this requirement, the working region for the Borel parameter, M^2, is found to be $1.1\,GeV^2 \leq M^2 \leq 1.4\,GeV^2$ for Σ^* baryons and $1.1\,GeV^2 \leq M^2 \leq 1.7\,GeV^2$ for Ξ^* and Ω^- baryons. In the case of Ξ^* and Ω^- baryons, the working region of the Borel parameter is wider due to the relatively large masses of these baryons.

In Figs. 1-6, we present the dependence of the magnetic moment of each baryon on the Borel parameter, M^2 for three values of the continuum threshold and for the cases $m_s = 0$ and $m_s = 0.15\,GeV$. The magnetic moments depend weekly on the value of the continuum threshold, they change at most 6% by a variation of s_0 and are also very weakly dependent on M^2. From these figures we can deduce the following conclusions. When we take into account mass of strange quarks, the results for the magnetic moments of charged decuplet baryons change about 25%, but for the neutral decuplet baryons, the situation changes drastically, i.e. the results increase by more than a factor of four. This fact can be explained in the following way. In exact $SU(3)$ limit, magnetic moments of Σ^{*0} and Ξ^{*0} are proportional to $(e_u + e_d + e_s)$ and $(e_u + 2e_s)$, respectively. For example, the Ξ^{*0} case is evident from Eq. (16) if in this equation we put $m_s \to 0$ and $\langle \bar{u}u \rangle = \langle \bar{s}s \rangle$. In other words, magnetic moments of Σ^{*0} and Ξ^{*0} are exactly zero in $SU(3)$ symmetry limit. (In Figs. 2 and 4, they are slightly different from zero . This is due to the fact that in the calculations we take $\langle \bar{s}s \rangle \neq \langle \bar{q}q \rangle$ $(q = u, d)$). Hence, the main contribution to the magnetic moments of Σ^{*0} and Ξ^{*0} come from $SU(3)$ breaking terms (the mass of s-quark, s-quark condensate, etc.). For this reason, for the magnetic moments of the neutral decuplet baryons $SU(3)$ breaking effects play an essential role. Note that, all the graphs are plotted for $\chi = -4.4\,GeV^2$ and $m_0^2 = 0.8\,GeV^2$. Our final results on the magnetic moments of the decuplet baryons at $m_s = 0.15\,GeV$ is presented in Table 1. For completeness, in this table, we also depicted our previous predictions on the magnetic moments of Δ baryons and also the predictions of other methods. The quoted errors in Table 1, are due to the uncertainties in m_0^2, s_0, variation of the Borel parameter M^2 and the neglected m_s^2 terms. One final remark is that our predictions on the magnetic moment of Ξ^{*0} differ from the QCD sum rule results not just in magnitude, but also, more essentially, by sign.

Appendix A

In this appendix, derivation of the rules for Fourier and Borel transformation which we have used in our calculations will be presented.

In coordinate representation, the structures that contribute to the structure $g_{\mu\nu}\not{p}_1\not{e}\not{p}_2$ are $x_\mu x_\nu \not{x}\not{e}\not{q}$ and $g_{\mu\nu}\not{x}\not{e}\not{q}$. Let us start with the following expressions:

$$\int d^4x e^{iPx} \frac{x_\mu x_\nu x_\alpha}{(-x^2)^n} \tag{19}$$

and

$$\int d^4x e^{iPx} \frac{x_\alpha}{(-x^2)^n} \tag{20}$$

for arbitrary n (there are also terms proportional to $\ln(-x^2)$, these terms will be discussed later). Note that we are interested only in the part of the Fourier transforms that are proportional to $g_{\mu\nu}$. In Eqs. (19) and (20), $P^2 = (p+uq)^2 = p_1^2\bar{u} + p_2^2 u$ where $\bar{u} = 1-u$. The derivation will be demonstrated for Eq. (19), as generalization is quite trivial. One can replace every occurance of x_β by $-i\frac{\partial}{\partial P_\beta}$.

$$\begin{aligned}\int d^4x e^{iPx} \frac{x_\mu x_\nu x_\alpha}{(-x^2)^n} &= \left(-i\frac{\partial}{\partial P_\alpha}\right)\left(-i\frac{\partial}{\partial P_\mu}\right)\left(-i\frac{\partial}{\partial P_\nu}\right)\frac{(-i)}{\Gamma(n)} \times \\ &\times \int d^4x \int_0^\infty dt e^{-iPx} t^{n-1} e^{-tx^2}\end{aligned} \tag{21}$$

where we have switched to the Euclidean space in the integral and used the identity

$$\frac{1}{y^n} = \frac{1}{\Gamma(n)}\int_0^\infty t^{n-1}e^{-ty} \tag{22}$$

In Eq. (21) one should be careful in taking the derivatives as the derivatives are with respect to the Minkowskian four vector P but the integrand is expressed in terms of the Euclidean vector P. The four dimensional integral is now a trivial Gaussian integration. After performing the integration over Euclidean space time, and taking the derivatives, the coefficient of $g_{\mu\nu}P_\alpha$ is found to be

$$\int d^4x e^{iPx} \frac{x_\mu x_\nu x_\alpha}{(-x^2)^n} \rightarrow \frac{\pi^2}{4\Gamma(n)}\int_0^\infty dt t^{n-5} e^{-\frac{P^2}{4t}} \tag{23}$$

Using the Borel transformation of the exponential

$$B_{p_1^2} B_{p_2^2} e^{-\frac{P^2}{4t}} = \delta\left(\frac{1}{M_1^2} - \frac{\bar{u}}{4t}\right) \delta\left(\frac{1}{M_2^2} - \frac{u}{4t}\right) \tag{24}$$

and carrying out the t integration, one obtains

$$\int d^4 x e^{iPx} \frac{x_\mu x_\nu x_\alpha}{(-x^2)^n} \to \frac{\pi^2}{\Gamma(n)} \left(\frac{M^2}{4}\right)^{n-3} M^2 \delta(u - u_0) \tag{25}$$

where

$$\begin{aligned} M^2 &= \frac{M_1^2 M_2^2}{M_1^2 + M_2^2} \\ u_0 &= \frac{M_1^2}{M_1^2 + M_2^2} \end{aligned}$$

Similarly

$$\begin{aligned} \int d^4 x e^{iPx} \frac{x_\alpha}{(-x^2)^n} &\to -\frac{2\pi^2}{\Gamma(n)} \left(\frac{M^2}{4}\right)^{n-2} M^2 \delta(u - u_0) && (26) \\ \int d^4 x e^{iPx} \frac{\ln(-x^2) x_\alpha}{(-x^2)^n} &\to -\frac{2\pi^2}{\Gamma(n)} \left(\frac{M^2}{4}\right)^{n-2} M^2 \times \\ &\times \left\{\ln\left(\frac{M^2}{4}\right) - \frac{d}{dn} \ln\Gamma(n)\right\} \delta(u - u_0) && (27) \\ \int d^4 x e^{iPx} \frac{\ln(-x^2) x_\mu x_\nu x_\alpha}{(-x^2)^n} &\to \frac{\pi^2}{\Gamma(n)} \left(\frac{M^2}{4}\right)^{n-3} M^2 \times \\ &\times \left\{\ln\left(\frac{M^2}{4}\right) - \frac{d}{dn} \ln\Gamma(n)\right\} \delta(u - u_0) && (28) \end{aligned}$$

The corresponding transformation rules for terms containing $\ln(-x^2)$ have been obtained by making use of the identity

$$\ln(-x^2) = -\frac{\partial}{\partial\epsilon} \frac{1}{(-x^2)^\epsilon}\bigg|_{\epsilon=0} \tag{29}$$

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Figure Captions

Fig. 1. The dependence of the magnetic moment of Σ^{*+} on the Borel parameter, M^2, (in units of nuclear magneton) for three different values of the continuum threshold, s_0, and for the cases $m_s = 0$ and $m_s = 0.15\,GeV$.

Fig. 2. The same as Fig. 1, but for Σ^{*0}.

Fig. 3. The same as Fig. 1, but for Σ^{*-}.

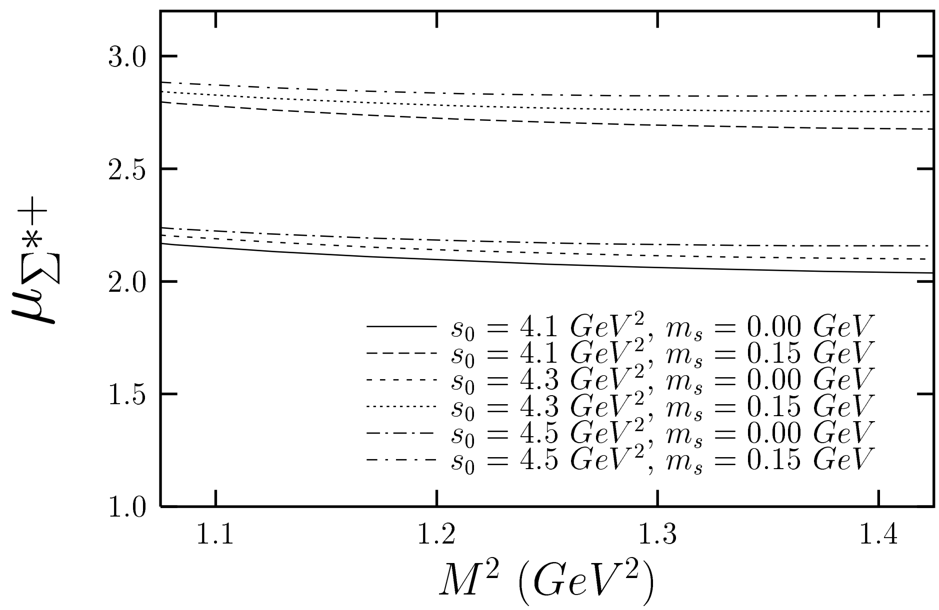
Fig. 4. The same as Fig. 1, but for Ξ^{*0}.

Fig. 5. The same as Fig. 1, but for Ξ^{*-}.

Fig. 6. The same as Fig. 1, but for Ω^-.

Table 1: Comparisons of decuplet baryon magnetic moments from various calculations: this work (LCQSR),QCDSR [11] lattice QCD (Latt) [31], chiral perturbation theory (χPT) [32], light-cone relativistic quark model (RQM) [33], non-relativistic quark model (NQM) [34], chiral quark-soliton model (χQSM) [35], chiral bag-model(χB) [36]. For the magnetic moments of Δ baryons in LCQSR, we have used the result of [15]. All results are in units of nuclear magnetons.

Baryon	Δ^{++}	Δ^{+}	Δ^{0}	Δ^{-}	Σ^{*+}	Σ^{*0}	Σ^{*-}	Ξ^{*0}	Ξ^{*-}	Ω^{-}
Exp.	4.5±1.0		~ 0							-2.024±0.056
LCQCD	4.4±0.8	2.2±0.4	0.00	-2.2±0.4	2.7±0.6	0.20±0.05	-2.28±0.5	0.40±0.08	-2.0±0.4	-1.65±0.35
QCDSR	4.39±1.00	2.19±0.50	0.00	-2.19±0.50	2.13±0.82	0.32±0.15	-1.66±0.73	-0.69±0.29	-1.51±0.52	-1.49±0.45
Latt.	4.91±0.61	2.46±0.31	0.00	-2.46±0.31	2.55±0.26	0.27±0.05	-2.02±0.18	0.46±0.07	-1.68±0.12	-1.40±0.10
χPT	4.0±0.4	2.1±0.2	-0.17±0.04	-2.25±0.25	2.0±0.2	-0.07±0.02	-2.2±0.2	0.1±0.04	-2.0±0.2	input
RQM	4.76	2.38	0.00	-2.38	1.82	-0.27	-2.36	-0.60	-2.41	-2.48
NQM	5.56	2.73	-0.09	-2.92	3.09	0.27	-2.56	0.63	-2.2	-1.84
χQSM	4.73	2.19	-0.35	-2.90	2.52	-0.08	-2.69	0.19	-2.48	-2.27
χB	3.59	0.75	-2.09	-1.93	2.35	-0.79	-3.87	0.58	-2.81	-1.75



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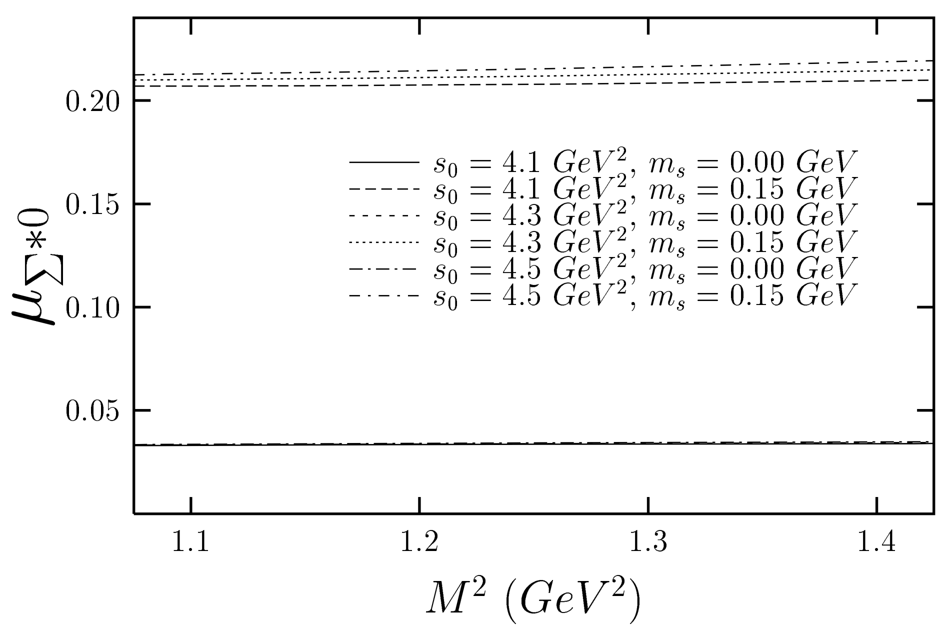

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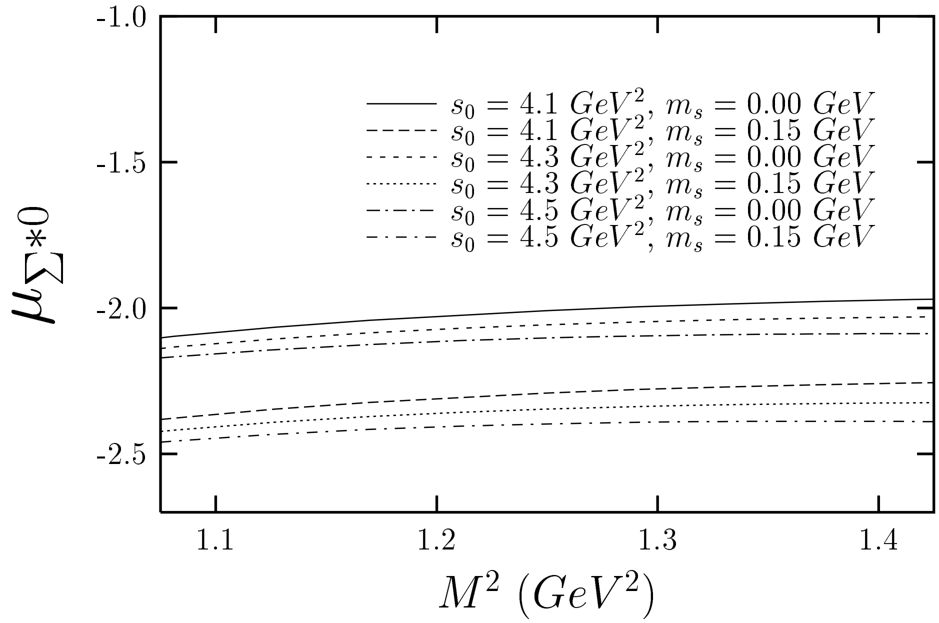

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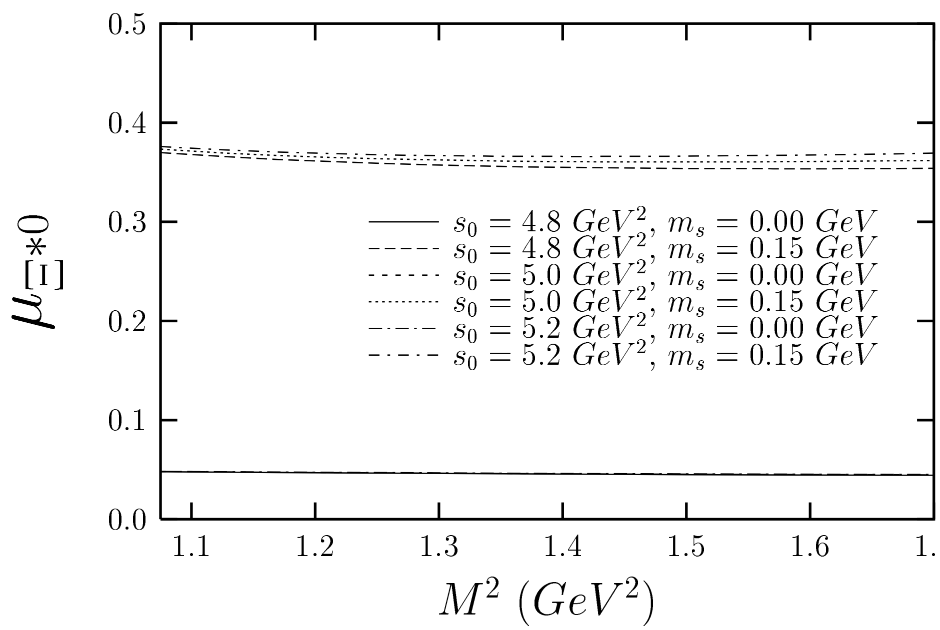

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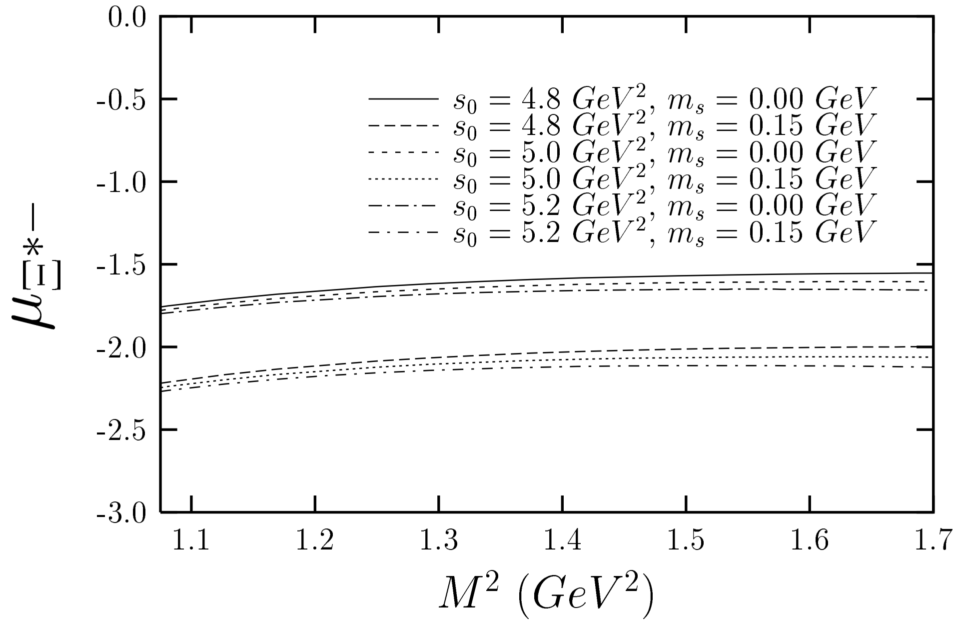

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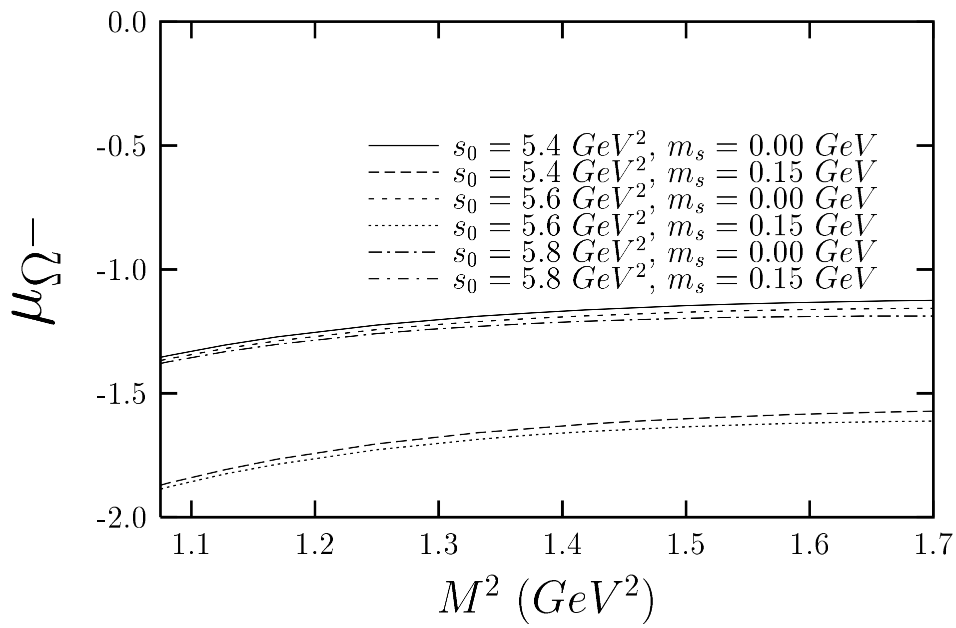

Figure 6: