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RARE DECAY OF THE TOP $t \to cgg$ IN THE STANDARD MODEL

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Abstract

We calculate the one-loop flavor changing neutral current top quark decay $t \to cgg$ in the Standard Model. We demonstrate that the rate for $t \to cgg$ exceeds the rate for a single gluon emission $t \to cg$ by about two orders of magnitude, while the rate for $t \to cq\bar{q}$, $q = u$ is slightly smaller than for $t \to cg$.

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I. INTRODUCTION

Flavor Changing Neutral Currents (FCNC) in general, and of the top quark in particular, play an important role as a testing ground for the Standard Model (SM) and for New Physics (NP). Because of its large mass, the top quark can decay into all other quarks, accompanied by gauge or Higgs bosons, as well as into new particles predicted by NP models. The interest in FCNC for top quark physics stems from the facts that:

1. The scale of NP is closer to the top quark mass more than to any other quark.

2. In the SM, the FCNC two-body processes $t \to cg, \gamma, Z, H$ are absent at tree- level and are highly suppressed by the GIM mechanism at one loop. Their branching ratios predicted in the SM are of the order of 10^{-11} to 10^{-14} [1, 2], far away from present and even future reaches of either the Large Hadron Collider (LHC) [3, 4] or the International Linear Collider (ILC) [5]. There are many models of NP in which the branching ratios for the above 2-body FCNC decays are much larger than those obtained in the SM (see e.g. [6, 7] and references therein).

In addition to the two-body rare decays of top quark, some of its rare three-body decays e.g., $t \to cWW, cZZ, bWZ$ have been considered in the literature within the SM [8, 9, 10] and for NP [10, 11]. These three body decays are suppressed with respect to two-body decays in the SM but some of them get comparably large within models of NP, such as two-Higgs-Doublet [11], especially after including finite-width effects [10].

In this paper we will analyze another three-body rare decay, namely $t \to cgg$ within the SM framework and compare it to both $t \to cg$ and $t \to cq\bar{q}$, $q = u$. The main motivation for such a calculation comes from the phenomenon in which higher order dominates over a lower order rate, as observed in the c, b and in other systems.

For the case of charm decays, the short distance contribution to $c \to u\gamma$, exhibits a huge enhancement over the lowest order penguin diagrams [12]. One can argue then that even higher order short distance are not that important. Unfortunately for radiative D decays, even this enhancement is overshadowed by much larger long distance terms.

For b-quark decays, a study of the next-to-leading logarithmic (NLL) QCD corrections [13], yielded $\mathrm{Br}^{\mathrm{NLL}}(b \to sg) \approx 5.0 \times 10^{-3}$, whereas the leading-logarithmic (LL) result $\mathrm{Br}^{\mathrm{LL}}(b \to sg) \approx 2.2 \times 10^{-3}$, in spite of an α_s/π suppression. This large correction is dynamical in nature, since it is due to a large ratio of Wilson coefficients evaluated at m_b, $C_2/C_8 \approx 7$ (for earlier references see [14, 15, 16, 17]), Higher order dominance may also become substantial in the decays of a sequential fourth generation quark b', if it exists [15, 18, 19].

More recently [20], the one-loop, three-body, rare top quark decay $t \to u_1\bar{u}_2u_2$, where $u_i = u, c$ was calculated in the SM and found to dominate over the one-loop, two-body, rare $t \to u_1 g$ decay, by about one order of magnitude, although the latter is of lower order in α_s. Later on, we will comment about their result.

The purpose of this article is to evaluate and discuss the higher order dominance issue in rare top decays among $t \to cg, t \to cq\bar{q}$, and $t \to cgg$. The present calculations are within the SM and while, as discussed above, $t \to cg$ and $t \to cq\bar{q},\ q = u$ were calculated before, this is the first calculation of $t \to cgg$ in the SM.

The remainder of the paper is organized as follows: In Section II we present the calculation of $t \to cgg$, in Section III the decay $t \to cq\bar{q},\ q = u$ is evaluated, and in Section IV we conclude. The Appendix includes the one-loop functions which appear in Section II.

II. CALCULATION OF $t \to cgg$

The one-loop $t - c - g^*(k)$ and, in general, the $q_1 - q_2 - g^*(k)$ vertex function can be expressed, using Lorentz and gauge invariance, as [21]

$$\Gamma_\mu = F_1(k^2)\left(k^2\gamma_\mu - k_\mu \not{k}\right) P_L - iF_2(k^2) m_t\, \sigma_{\mu\nu}\, k^\nu\, P_R, \tag{2.1}$$

where $P_{L,R} \equiv (1 \mp \gamma_5)/2$ and $m_c = 0$ are assumed. The functions F_1 and F_2 are called *charge-radius* (or *monopolar*) and *dipole moment* (or *dipolar*) form factor, respectively. Note that this is not the most general vertex function. There are two more form factors, namely F_{1R} (right-handed monopolar) and F_{2L} (left-handed dipolar), which are both proportional to m_c/m_t so that for the sake of simplicity we omit them here (see [21] for the details). In our numerical analyses, however, all contributions are retained.

While $F_1(k^2)$ contribution to $q_1 \to q_2 g$, $q_1 = t$, $q_2 = c$ (i.e. for a real gluon) vanishes, both $F_1(k^2)$ and $F_2(k^2)$ give non-zero contribution to $q_1 \to q_2 q\bar{q}$ and to $q_1 \to q_2 gg$. Of course, the vertex functions $\Gamma_{\mu\nu}$ for the three-body processes are more complicated than just Γ_μ above, but if $F_2 < F_1$, there is a chance that the three-body modes will be of the same order, or even larger than $t \to cg$.

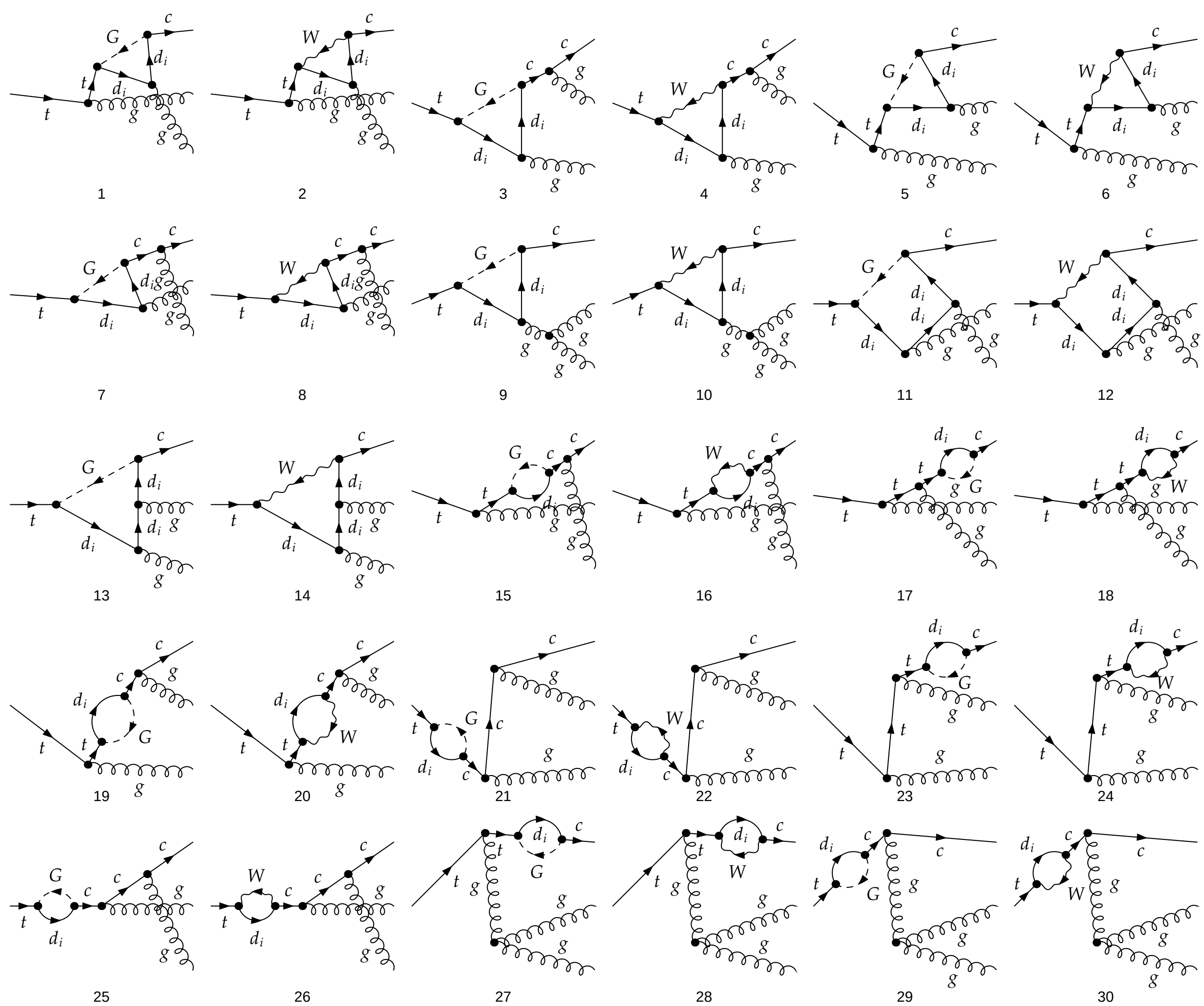

FIG. 1: The one-loop contributions to $t \to cgg$ in the SM in the 't Hooft - Feynman gauge. The ghost contributions are depicted in Fig. 2. The first 1-10 diagrams are the vertex diagrams, the diagrams 11-14 are the one-particle irreducible (1PI or box) diagrams, and the rest 15-30 are the $t - c$ self energy diagrams. The crossed diagrams are also shown explicitly.

In the SM, the decay $t \to cgg$ occurs at one-loop level and the Feynman diagrams contributing to the decay are depicted in Fig. 1 in the t' Hooft - Feynman gauge ($\xi = 1$).[1] The G field in the diagrams is the unphysical part of the Higgs field. The polarization sum

[1] Note that there will be no cross term for diagrams with triple gluon vertex since it is already counted in the vertex factor.

of the gluons is naively

$$\sum_{\lambda} \epsilon^*_\mu(k,\lambda)\epsilon_\nu(k,\lambda) = -g_{\mu\nu}. \tag{2.2}$$

However, it is well known [22, 23] that in cases where there are two or more external gluons (either for tree, like $gq \to gq$, or for loop diagrams as in the present case), the above sum leads to violation of gauge invariance. The problem is alleviated either by choosing a transverse polarization sum, or by introducing ghost fields to get rid of the unphysical gluon polarizations while keeping the simple polarization sum above. The ghost contributions are shown in Fig. 2.

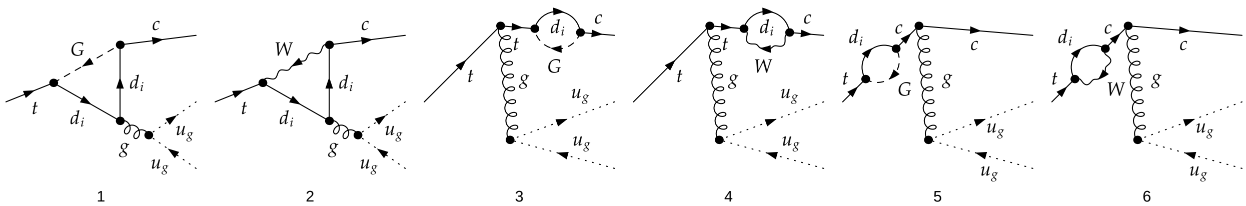

FIG. 2: The one-loop ghost contributions to $t \to cgg$ in the SM in the 't Hooft - Feynman gauge. There is a ghost contribution for each diagram of Fig. 1 with triple gluon vertex.

Fig. 2 requires some explanation. The ghosts couple only to a gluon, i.e. there is a g–ghost–ghost coupling but there is no coupling to quarks. Therefore, there is a ghost anti-ghost diagram for each diagram with a triple gluon coupling. Furthermore, since the ghost is not its anti-particle there are six diagrams with ghosts as in Fig. 2 and there is no statistical factor after phase-space integration, unlike the case of two final gluons where a statistical factor of 1/2 is inserted following phase space integration. Note also that there are no interference terms between ghost and ghostless amplitudes. [2]

In our calculation we make the GIM mechanism manifest by dropping terms independent of internal quark mass.[3] This will substantially simplify the calculation. The divergent parts originate only from the unphysical Higgs loops. As the cancellation of ultraviolet divergences should happen at the amplitude squared level for $t \to cgg$ in the 't Hooft - Feynman gauge, the analytical output is so long that the ultraviolet finiteness has to be checked numerically. We carried out the calculation in D-dimension using dimensional regularization [24] and

[2] The ghost fields, although they are bosons, obey Fermi-Dirac statistics and will have an overall -1 factor for each closed loop, similar to a fermion loop.

[3] This simplification applies to the divergent parts as well since the SM without the unitarity of the CKM matrix is non-renormalizable.

keep the decay width formulas as functions of $\epsilon = 4 - D$. Then we check the stability of the decay width by varying ϵ. Of course, this method is limited by the achievable numerical precision of the program used. We have done our calculation with the softwares FeynArts and FormCalc [25, 26] and done some partial cross-checks with FeynCalc [27]. With FormCalc the numerical check of ultraviolet finiteness is controlled by a parameter and thus easier to check. We have tested and confirmed the finiteness of our results for $t \to cgg$ (as well as for $t \to cq\bar{q},\ q = u$ and $t \to cg$).

In addition to ultraviolet divergences, we must deal with infrared and collinear divergences which exist in the calculation of $t \to cgg$. There are three possible cases producing such singularities: a) the configuration of having one of the gluons travel parallel to the charm quark, b) the configuration of having the gluons traveling parallel to each other, or c) the fact that one of the gluons in the final state could be soft. While cases a) and b) are related collinear divergence, case c) is related to the infrared singularity. To cure case a) we take a non-zero mass for the charm quark in our numerical calculation even though we present some of our analytical results in the $m_c = 0$ limit, just for simplicity.

There are two ways to approach cases b) and c). One can either do an extensive study by including QCD corrections consistently of the same order in the perturbation theory, this case up to the order of α_s^2 requires including the interference terms from two-loop $t \to cg$ (one-loop QCD corrections) with one-loop $t \to cgg$; or one can exclude the part of the phase space containing singular points by simply imposing cuts on the kinematics of the process. The former method can be achieved by dimensionally regularizing the phase space integrals with $D = 4 - \epsilon_{IR}$ and making the divergence manifest. Then the cancellation of infrared singularities is guaranteed by the Kinoshita-Lee-Nauenberg (KLN) theorem [28, 29, 30]. Basically one has to carry out a study for top quark decays similar to the $b \to sg$ evaluation done by Greub and Liniger[13]. We chose to proceed by cutting the "dangerous" integration limits as discussed below. Our calculation is therefore more in the spirit of [16].

When we carry out integration over phase space by using the momentum delta functions and azimuthal symmetry, we are left with only two non-trivial integration over variables, say, the energy of one of the gluons ($E_3 \equiv k_3^0$) and the energy of the c-quark ($E_c \equiv k_2^0$), where the energies are in rest frame of the decaying top quark. Then in (E_c, E_3) space, the region of integration becomes a triangular shape. The singularities discussed above lie at the boundaries of the region, infrared singularities at the vertices and collinear singularities along the boundary lines. Thus we have to impose cuts on both E_c and E_3. We will discuss this point further later in the section. Note that since we constrain our phase space, our result for $\mathrm{Br}(t \to cgg)$ should be seen as approximate and more conservative.

Let us now present some analytical expressions. For the sake of simplicity of the presentation, the masses m_c, m_d and m_s are assumed to be zero, though we included their contributions in our numerical study. From Figs. 1 and 2, the amplitude for the decay can be written in a compact form as

$$\begin{aligned}
A_{\rm self} &= \frac{\alpha\alpha_s V_{tb} V_{cb}^*}{4m_t^2 m_W^2 s_{23}\sin^2\theta_W t\, t_{12} R_1 R_4}\left[-F_2 m_t t_{12} R_2 R_4 - 2F_1(SP_4 - SP_5) t R_6 R_4 \right.\\
&\quad \left. + s_{23} SP_1 R_1 (2F_9 R_5 - F_{12} m_t R_3 R_4) + F_3 t_{12} R_7 + 2F_4 R_8\right],\\
A_{\rm vert} &= \frac{\alpha\alpha_s V_{tb} V_{cb}^*}{2m_W^2 s_{23}\sin^2\theta_W t t_{12} R_1 R_4}\left[2R_1(F_{13} m_t t\, t_{12} R(9) - F_9 s_{23} SP_1 R_{11}) - 2F_4 R_{13}\right.\\
&\quad \left. - F_3 t_{12} R_{14} + R_4(m_t s_{23} t_{12}(2F_{15} R_{10} + F_2 R_{12}) + 2F_1 t R_{15}) + F_{12} m_t R_{16}\right],\\
A_{\rm 1PI} &= \frac{\alpha\alpha_s V_{tb} V_{cb}^*}{2m_W^2 \sin^2\theta_W}\left[2F_9 R_{17} + F_3 R_{19} + m_t\left(-2F_{15} R_{18} + 2F_1 3 R_{20} + F_2 R_{21}\right.\right.\\
&\quad \left.\left. - 2F_{12} R_{22}\right) + 2F_4 R_{23} - 2F_1 R_{24}\right],\\
A_{\rm ghost} &= \frac{\alpha\alpha_s V_{tb} V_{cb}^*}{4m_t^2 m_W^2 \sin^2\theta_W t_{12}} R_{25}\left[F_9 R_{27} - 2F_{12} m_t R_{26}\right],
\end{aligned} \tag{2.3}$$

where

$$\begin{aligned}
F_1 &= \bar{u}(k_2,0) P_R \not\epsilon^*(k_3) u(k_1,m_t), \quad F_2 = \bar{u}(k_2,0) P_R \not\epsilon^*(k_3)\not\epsilon^*(k_4) u(k_1,m_t),\\
F_3 &= \bar{u}(k_2,0) P_R \not\epsilon^*(k_3)\not\epsilon^*(k_4)\not k_3 u(k_1,m_t), \quad F_4 = \bar{u}(k_2,0) P_R \not\epsilon^*(k_4) u(k_1,m_t),\\
F_5 &= \bar{u}(k_2,0) P_L \not\epsilon^*(k_3) u(k_1,m_t), \quad F_6 = \bar{u}(k_2,0) P_L \not\epsilon^*(k_3)\not\epsilon^*(k_4) u(k_1,m_t),\\
F_7 &= \bar{u}(k_2,0) P_L \not\epsilon^*(k_3)\not\epsilon^*(k_4)\not k_3 u(k_1,m_t), \quad F_8 = \bar{u}(k_2,0) P_L \not\epsilon^*(k_4) u(k_1,m_t),\\
F_9 &= \bar{u}(k_2,0) P_R \not k_3 u(k_1,m_t), \qquad F_{10} = \bar{u}(k_2,0) P_L u(k_1,m_t),\\
F_{11} &= \bar{u}(k_2,0) P_L \not k_3 u(k_1,m_t), \qquad F_{12} = \bar{u}(k_2,0) P_R u(k_1,m_t),\\
F_{13} &= \bar{u}(k_2,0) P_R \not\epsilon^*(k_3)\not k_3 u(k_1,m_t), \quad F_{14} = \bar{u}(k_2,0) P_L \not\epsilon^*(k_3)\not k_3 u(k_1,m_t),\\
F_{15} &= \bar{u}(k_2,0) P_R \not\epsilon^*(k_4)\not k_3 u(k_1,m_t), \quad F_{16} = \bar{u}(k_2,0) P_L \not\epsilon^*(k_4)\not k_3 u(k_1,m_t),
\end{aligned} \tag{2.4}$$

The Lorentz invariant t and s are the usual Mandelstam variables, while $t_{12} = (k_1 - k_2)^2$ and $s_{23} = (k_2 + k_3)^2$ are the generalized Mandelstam variables. The scalar products are defined as

$$\begin{aligned}
SP_1 &= \epsilon^*(k_3)\cdot\epsilon^*(k_4), \qquad SP_2 = \epsilon^*(k_3)\cdot k_1,\\
SP_3 &= \epsilon^*(k_3)\cdot k_2, \qquad SP_4 = \epsilon^*(k_4)\cdot k_1, \quad SP_5 = \epsilon^*(k_4)\cdot k_2.
\end{aligned} \tag{2.5}$$

The functions $R_1, ..., R_{27}$ are defined in terms of Passarino-Veltman functions [31] and are given explicitly in the appendix.[4] The decay width can be written as

$$
\begin{aligned}
d\Gamma(t \to cgg) &= \frac{1}{2m_t} \sum_{\text{spins}} |\mathcal{M}|^2 d\Phi_3(k_1; k_2, k_3, k_4) \\
d\Phi_3(k_1; k_2, k_3, k_4) &= \frac{d^3k_2}{(2\pi)^3 2k_2^0} \frac{d^3k_3}{(2\pi)^3 2k_3^0} \frac{d^3k_4}{(2\pi)^3 2k_4^0} (2\pi)^4 \delta^{(4)}(k_1 - k_2 - k_3 - k_4), \qquad (2.6)
\end{aligned}
$$

where $|\mathcal{M}|^2$ is straightforward to calculate from the amplitude given in Eq. (2.3). The phase space $d\Phi_3$ can be expressed in terms of energies of the third and fourth particles, chosen to be the gluon pair in the rest frame of top quark. In this frame, one can take the production plane as the $x - z$ plane and choose the c quark momentum along the z-axis. After rearranging the volume elements and carrying out angular and momenta integrals, we implement the phase space cuts discussed previously

$$
d\Phi_3(k_1; k_2, k_3, k_4) = \frac{1}{32\pi^3} \int_{(k_3^0)^{\min}}^{(k_3^0)^{\max}} dk_3^0 \int_{(k_2^0)^{\min}}^{(k_3^0)^{\max}} dk_2^0, \qquad (2.7)
$$

where the limits are

$$
\begin{aligned}
(k_2^0)^{\min} &= \text{Max}\left[Cm_t, \frac{\sigma - |\mathbf{k}_3|}{2} \right], \\
(k_2^0)^{\max} &= \frac{\sigma + |\mathbf{k}_3|}{2}(1 - 2C), \\
(k_3^0)^{\min} &= Cm_t, \\
(k_3^0)^{\max} &= \frac{m_t}{2}(1 - 2C), \qquad (2.8)
\end{aligned}
$$

with $\sigma = m_t - k_3^0$ (recall that for simplicity we have assumed $m_c = 0$). Here C is our cutoff parameter, which we initially take as $C = 0.001$ and then study the effect of increasing its value. To calculate the branching ratio, we assume that $t \to bW$ is the dominant decay mode of the top quark and use $\Gamma(t \to bW) = 1.55$ GeV. For the numerical analysis, we have used the parameters [32] given in Table I.

[4] The result is expressed in terms of Passarino-Veltman functions [31] but the reduction to the scalar function A_0, B_0, C_0, and D_0 has not been carried out. This is indeed one advantage of using FormCalc which does not require such reduction to save substantial CPU time.

TABLE I: The parameters used in the numerical calculation.

$\alpha_s(m_t)$	$\alpha(m_t)$	$\sin\theta_W(m_t)$	$m_c(m_t)$	$m_b(m_t)$	$m_t(m_t)$
0.106829	0.007544	0.22	0.63 GeV	2.85 GeV	174.3 GeV

Our result is

$$\mathrm{Br}(t \to cgg) \equiv \frac{\Gamma(t \to cgg)}{\Gamma(t \to bW)} = 1.02 \times 10^{-9}, \tag{2.9}$$

while for the two-body decay $t \to cg$ we get

$$\mathrm{Br}(t \to cg) = 5.73 \times 10^{-12}. \tag{2.10}$$

for the same parameter set. At this point, a comment is needed. For completeness, and to check our procedure, we recalculated the two-body decay $t \to cg$ and our numerical value is around one order of magnitude smaller than that of the Ref. [1] (see Fig. 2 of [1]). The main source of such disprepancy lies in the value for bottom quark mass. The pole mass $m_b = 5$ GeV is used in Ref. [1], while we have used the running bottom quark mass at m_t scale, $m_b = 2.85$ GeV. Since the branching ratio is proportional to m_b^4 (due to GIM suppression), our results differ by one order of magnitude ($(2.85/5)^4 \sim 0.1$). The original SM calculation of $t \to cg$ in Ref. [1] was later updated [33], where the running bottom quark mass was used and $\mathrm{Br}(t \to cg) = 4.6 \times 10^{-12}$ was obtained.

There is however one cautionary remark about the $t \to cgg$ decay. The branching ratios were calculated as function of the cutoff (see below). One should calculate the rate for $t \to cgg$ by doing a complete higher order calculation. QCD corrections should be at most of the order of 10%, which is the order of magnitude of QCD corrections in $t \to bW$. One can view C as a detector cutoff. The cutoff dependence of the branching ratio of $t \to cgg$ is given in Table II. As seen from Table II, the branching ratio is sensitive, but not significantly so, to the C parameter.

In general, contributions from ghost diagrams are quite suppressed with respect to the rest of the diagrams and the 1PI diagrams slightly dominate the self energy and vertex type diagrams depicted in Figs. 1.

TABLE II: The cutoff dependence of the branching ratio of $t \to cgg$ for various C values.

C (in m_t units)	0.001	0.003	0.005	0.01
$\mathrm{Br}(t \to cgg)$	1.02×10^{-9}	9.04×10^{-10}	8.76×10^{-10}	7.78×10^{-10}

Within the range $(0.001-0.01)$ for the cutoff C, the rate for the three-body decay $t \to cgg$ is more than two orders of magnitude larger than the rate for the two-body decay $t \to cg$. This is higher order dominance par excellence. However, the branching ratio for the $t \to cgg$ channel in the SM, is still too small to be observable in any conceivable experiment. Thus any experimental sighting of $t \to cgg$ will indicate the appearance of NP. The three-body decay $t \to cgg$ has another important difference with respect to $t \to cg$: A promising ratio for $t \to cgg$ might lead to a sizable cross section for single top production via $gg \to t\bar{c}$ at a future hadron collider, especially since most of the interesting events will be fed by partonic sub-processes originating from gluon-gluon collisions. We are currently investigating the decay $t \to cgg$ and the effect of its crossed partonic sub-process $gg \to t\bar{c}$ on single top production at the LHC, within the minimal supersymmetric SM [34].

III. THE DECAY $t \to cq\bar{q},\ q = u$

Using the same procedure and the same parameters as for $t \to cgg$, we have also calculated the branching ratio of $t \to cq\bar{q}, q = u$. Unlike $t \to cgg$ case, this decay arises dominantly from the $tcg^\star$ vertex, where $g^\star$ then decays to $\to q\bar{q}$ pair. There are of course diagrams mediated by electroweak gauge bosons, γ and Z or the neutral Higgs boson, but their contributions in the SM are negligible. The dominant diagrams are given in Fig. 3, while the box diagrams displayed in Fig. 4, were found to be at least 3 orders of magnitude smaller than the diagrams in Fig. 3.

Compared with $t \to cgg$, it is a much simpler calculation, mainly since there are no external gluons and thus no ghosts. The only non-trivial issue is to demonstrate the ultraviolet finiteness of the decay. This can be checked either analytically at the amplitude level, or numerically for the branching ratio. We followed the second method.

We find that $t \to cq\bar{q},\ q = u$ remains slightly smaller than $t \to cg$. Thus we disagree

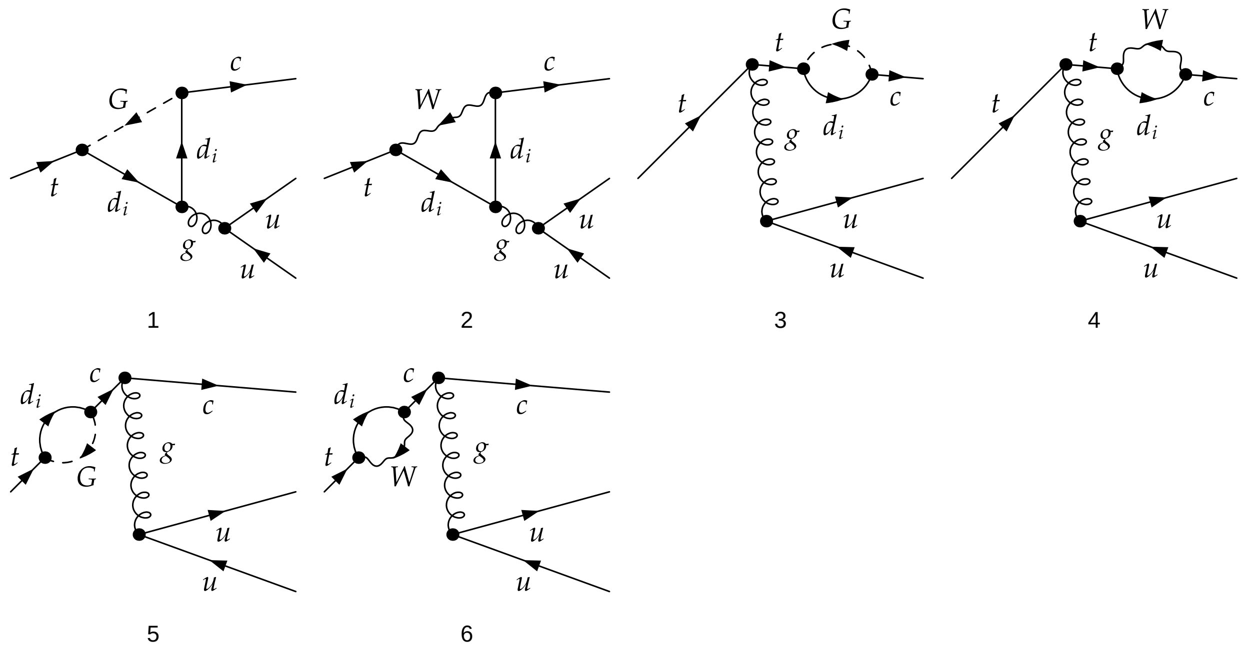

FIG. 3: The dominant one-loop contributions to $t \to cq\bar{q}$ in the SM in the 't Hooft - Feynman gauge.

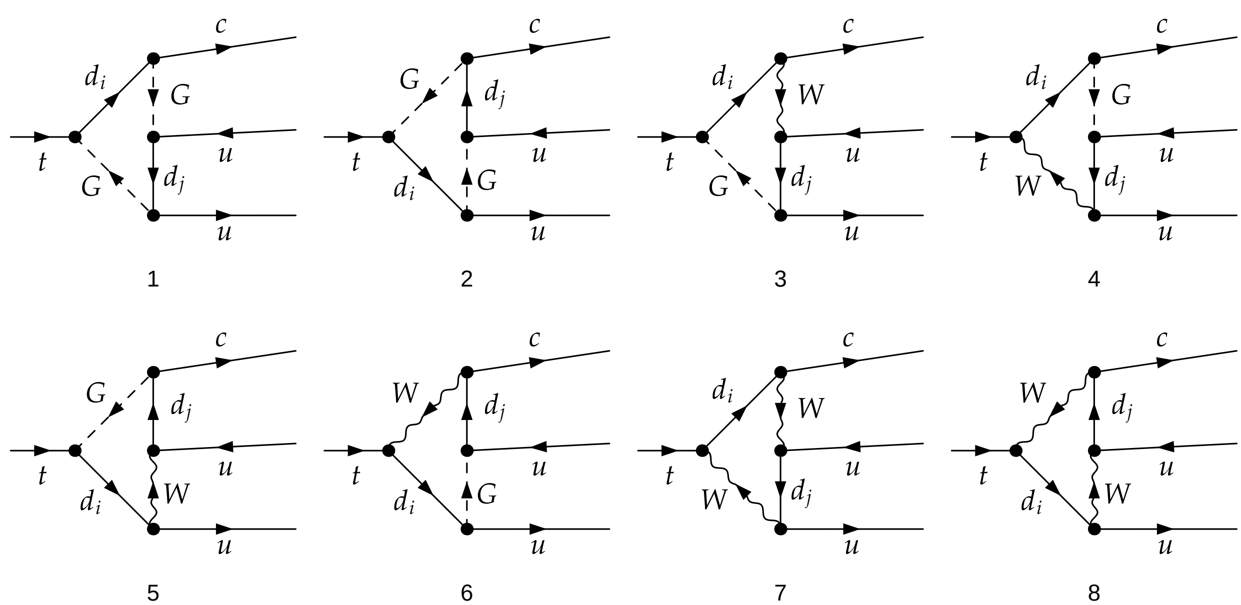

FIG. 4: The sub-dominant one-loop contributions to $t \to cq\bar{q}$ in the SM in the 't Hooft - Feynman gauge.

with the recent study by Cordero-Cid et al. [20] where $t \to cq\bar{q},\ q = u$ is found to be almost an order of magnitude larger than $t \to cg$. The reason for the discrepancy might be their use of different parameters, which unfortunately are not spelled out in their paper. Since we mainly concentrate on the $t \to cgg$ decay mode in this study, we prefer to skip the details of this calculation and present only our result here. For the parameter set chosen before, we found

$$\mathrm{Br}(t \to cq\bar{q},\ q = u) = 3.96 \times 10^{-12}. \tag{3.1}$$

As seen, unlike $\mathrm{Br}(t \to cgg)$, this ratio stays slightly smaller but still comparable to $\mathrm{Br}(t \to cg)$. The decay $t \to cq\bar{q}$ has been analyzed and compared with $t \to cg$ in both the SM and version II of the Two-Higgs-Doublet model by Deshpande, Margolis and Trottier [21].[5] They show that for $m_t = 175$ GeV, $\mathrm{BR}(t \to cq\bar{q})$ in the SM is slightly (~ 1.2 times) bigger than $\mathrm{Br}(t \to cg)$ when there is a sum over $q = u, c, d, s, b$. Our results agree with theirs. We get $\sum_{q=1}^{5} \mathrm{Br}(t \to cq\bar{q})$ as 1.56×10^{-11}, which becomes bigger than $\mathrm{Br}(t \to cg)$.

IV. CONCLUSIONS

In this study, we have discussed higher order dominance in the rare top decays $t \to cgg$ within the SM framework, a phenomenon known more than a decade ago in b-physics. For completeness, we also calculated $t \to cq\bar{q},\ q = u$. Using running quark masses, we have found $\mathrm{Br}(t \to cgg) = 1.02 \times 10^{-9}$ with a cutoff $C = 0.001$ for a top mass $m_t = 174.3$ GeV. We considered the sensitivity of the ratio with respect to the cutoff parameter C and found that even for $C = 0.01$ it is still more than two orders of magnitude larger than the two-body decay $\mathrm{Br}(t \to cg)$ which we calculated as 5.73×10^{-12} with the same set of parameters. By comparison, we found $\mathrm{Br}(t \to cq\bar{q},\ q = u)$ to be smaller, 3.96×10^{-12} and comparable to $\mathrm{Br}(t \to cg)$. However, when we sum over $q\bar{q}$ pairs for all five quarks, it becomes slightly larger than $\mathrm{Br}(t \to cg)$.

If higher order dominance is still valid for a viable NP model in the sense that $t \to cg$ is much smaller than $t \to cgg$ yet larger than its value in the SM, we may have a glimpse of NP at work either in the decay $t \to cgg$ or in production at the LHC through the partonic sub-process $gg \to t\bar{c}$.

[5] Some of this decay mode ($q = d, b$) has also been considered by Eilam et. al. in the SM [35].

Acknowledgments

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APPENDIX: THE ONE-LOOP R-FUNCTIONS

The explicit form of the functions defined in Eq. (2.3) are given below. As in the paper, expressions are given for $m_c = 0$, while we use a non-zero c-quark mass in our numerical calculations. We define $T_d = (T_a T_b)_{ij}$, $T_e = (T_b T_a)_{ij}$ as products of color matrix elements. Here a and b are gluon indexes running from 1 to 8 with color indexes $i, j = 1, 2, 3$

$$
\begin{aligned}
R_1 &= m_t^2 - s_{23}, \\
R_2 &= 2m_t^2 m_W^2 s_{23}(-tT_d + m_t^2 T_e - s_{23}T_e) + ((m_b^2 - m_W^2)(m_b^2 + 2m_W^2)(m_t^2 - s_{23})s_{23}T_e \\
&\quad -m_t^2(m_b^4 - 2m_W^4 + m_b^2(m_W^2 - 2s_{23}))tT_d)B_0^{(1)} - (m_b^4 + 2(m_t^2 - m_W^2)m_W^2 \\
&\quad +m_b^2(m_W^2 - m_t^2))(m_t^2 - s_{23})s_{23}T_e B_0^{(2)} + 2m_b^2 m_t^2 s_{23}(s_{23} - m_t^2)T_e B_0^{(3)} \\
&\quad +m_t^2(m_b^4 + m_b^2(m_W^2 - s_{23}) + 2m_W^2(s_{23} - m_W^2))tT_d B_0^{(5)}, \\
R_3 &= (m_b^2 - m_W^2)(m_b^2 + 2m_W^2)(tT_d - (t + 2t_{12})T_e)(B_0^{(1)} - B_0^{(2)}) + m_t^2(m_b^2(tT_d(B_0^{(2)} \\
&\quad -2B_0^{(1)}) + T_e(2tB_0^{(1)} - tB_0^{(2)} - 2t_{12}B_0^{(2)} + 4t_{12}B_0^{(3)})) \\
&\quad -2m_W^2(tT_d - (t + 2t_{12})T_e)(B_0^{(2)} - 1)), \\
R_4 &= m_t^2 - t, \\
R_5 &= 2m_t^2 m_W^2((m_t^2 - t)t(T_d - T_e) - (m_t^2 - 2t)t_{12}T_e) + (m_b^4 + m_b^2(m_W^2 - 2m_t^2) \\
&\quad -2m_W^4)t((m_t^2 - t)(T_d - T_e) + t_{12}T_e)B_0^{(1)} - (m_b^4 + 2(m_t^2 - m_W^2)m_W^2 \\
&\quad +m_b^2(m_W^2 - m_t^2))(m_t^2 - t)(tT_d - (t + t_{12})T_e)B_0^{(2)} + m_t^2(-m_b^4 - 2m_W^2(t - m_W^2) \\
&\quad -m_b^2(-2m_t^2 + m_W^2 + t))t_{12}T_e B_0^{(3)}, \\
R_6 &= 2m_W^2(m_t^2(m_t^2 - 2s_{23})t_{12}T_d + m_t^2(m_t^2 - s_{23})s_{23}(T_d - T_e)) + (m_b^4 + m_b^2(m_W^2 - 2m_t^2) \\
&\quad -2m_W^4)s_{23}((m_t^2 - s_{23})(T_d - T_e) - t_{12}T_d)B_0^{(1)} - (m_b^4 + 2(m_t^2 - m_W^2)m_W^2 \\
&\quad +m_b^2(m_W^2 - m_t^2))(m_t^2 - s_{23})(t_{12}T_d + s_{23}(T_d - T_e))B_0^{(2)} + m_t^2(m_b^4 \\
&\quad +2m_W^2(s_{23} - m_W^2) + m_b^2(-2m_t^2 + m_W^2 + s_{23}))t_{12}T_d B_0^{(5)}, \\
R_7 &= 2m_t^2 m_W^2(m_t^4(tT_d + s_{23}T_e) + 2s_{23}t(tT_d + s_{23}T_e) - m_t^2(T_e s_{23}^2 + 2t(T_d + T_e)s_{23}
\end{aligned}
$$

$$
\begin{aligned}
& +t^2T_d)) - (m_b^4 + m_b^2(m_W^2 - 2m_t^2) - 2m_W^4)s_{23}t(-tT_d - s_{23}T_e + m_t^2(T_d + T_e))B_0^{(1)} \\
& -(m_b^4 + 2(m_t^2 - m_W^2)m_W^2 + m_b^2(m_W^2 - m_t^2))(m_t^2 - s_{23})(m_t^2 - t)(tT_d + s_{23}T_e)B_0^{(2)} \\
& +m_t^2(m_t^2 - s_{23})s_{23}(m_b^4 + 2m_W^2(t - m_W^2) + m_b^2(-2m_t^2 + m_W^2 + t))T_eB_0^{(3)} + m_t^2(m_b^4 \\
& +2m_W^2(s_{23} - m_W^2) + m_b^2(-2m_t^2 + m_W^2 + s_{23}))(m_t^2 - t)tT_dB_0^{(5)}, \\
R_8 \;=\; & 2m_t^2m_W^2(s_{23}(s_{23} - m_t^2)(SP_2 - SP_3)t(t - m_t^2)(T_d - T_e) - t_{12}(m_t^4(SP_3tT_d + s_{23}SP_2T_e) \\
& +2s_{23}t(SP_3tT_d + s_{23}SP_2T_e) - m_t^2(SP_3t(2s_{23} + t)T_d + s_{23}SP_2(s_{23} + 2t)T_e))) \\
& +(m_b^4 + m_b^2(m_W^2 - 2m_t^2) - 2m_W^4)s_{23}t((m_t^2 - s_{23})(SP_2 - SP_3)(m_t^2 - t)(T_d - T_e) \\
& +t_{12}(m_t^2SP_3T_d - SP_3tT_d + m_t^2SP_2T_e - s_{23}SP_2T_e))B_0^{(1)} - (m_b^4 + 2(m_t^2 - m_W^2)m_W^2 \\
& +m_b^2(m_W^2 - m_t^2))(m_t^2 - s_{23})(m_t^2 - t)(s_{23}(SP_2 - SP_3)t(T_d - T_e) - t_{12}(SP_3tT_d \\
& +s_{23}SP_2T_e))B_0^{(2)} - m_t^2(m_t^2 - s_{23})s_{23}SP_2(m_b^4 + 2m_W^2(t - m_W^2) \\
& +m_b^2(-2m_t^2 + m_W^2 + t))t_{12}T_eB_0^{(3)} - m_t^2(m_b^4 + 2m_W^2(s_{23} - m_W^2) \\
& +m_b^2(-2m_t^2 + m_W^2 + s_{23}))SP_3(m_t^2 - t)tt_{12}T_dB_0^{(5)}, \\
R_9 \;=\; & s_{23}SP_5T_e(2m_W^2(C_1^{(1)} + C_{12}^{(1)}) - m_b^2(C_0^{(1)} + C_1^{(1)} - C_{12}^{(1)})) + SP_4(m_t^2 - t) \\
& \times T_d(2m_W^2(C_{12}^{(4)} + C_{22}^{(4)}) + m_b^2(C_0^{(4)} + C_{12}^{(4)} + 2C_2^{(4)} + C_{22}^{(4)})), \\
R_{10} \;=\; & SP_3tT_d(m_b^2(C_0^{(3)} + C_1^{(3)} - C_{12}^{(3)}) - 2m_W^2(C_1^{(3)} + C_{12}^{(3)})) - (m_t^2 - s_{23}) \\
& \times SP_2T_e(2m_W^2(C_{12}^{(2)} + C_{22}^{(2)}) + m_b^2(C_0^{(2)} + C_{12}^{(2)} + 2C_2^{(2)} + C_{22}^{(2)})), \\
R_{11} \;=\; & m_W^2(m_t^2 - t)t(T_d - T_e) - (m_b^2 + 2m_W^2)(m_t^2 - t)tB_0^{(4)}(T_d - T_e) \\
& +(m_t^2 - t)t(((m_b^2 - m_t^2 - m_W^2)(m_b^2 + 2m_W^2) + 2m_W^2t_{12})C_0^{(5)} + 2(m_b^2 + 2m_W^2)C_0^{(5)} \\
& +2m_W^2(t_{12} - m_t^2)C_1^{(5)} - (m_b^2m_t^2 + 4m_W^2m_t^2 - 2m_W^2t_{12})C_2^{(5)})(T_d - T_e) \\
& -m_W^2(m_t^2 - 2t)t_{12}T_e + (m_b^2 + 2m_W^2)(m_t^2 - 2t)t_{12}T_eB_0^{(3)} + tt_{12}T_e(-m_b^2m_t^2C_0^{(1)} \\
& +2(m_b^2 + 2m_W^2)C_0^{(1)} + 2m_W^2(m_t^2 - s_{23} - t_{12})C_1^{(1)}) + (m_t^2 - t)t_{12}T_e(m_b^2m_t^2C_0^{(2)} \\
& -2(m_b^2 + 2m_W^2)C_0^{(2)} + (m_b^2m_t^2 - 2m_W^2t)C_2^{(2)}), \\
R_{12} \;=\; & m_W^2(-tT_d + m_t^2T_e - s_{23}T_e) - (m_b^2 + 2m_W^2)(m_t^2 - s_{23})T_eB_0^{(3)} + (m_b^2 + 2m_W^2) \\
& \times tT_dB_0^{(5)} + m_b^2(m_t^2 - s_{23})tT_eC_0^{(1)} + tT_d(m_b^2s_{23}C_0^{(3)} - 2(m_b^2 + 2m_W^2)C_0^{(3)} \\
& -2m_W^2s_{23}C_1^{(3)}) - (m_t^2 - s_{23})T_e(m_b^2((m_t^2 - t)(C_0^{(2)} + C_2^{(2)}) - 2C_0^{(2)}) \\
& -4m_W^2C_0^{(2)}) - (m_t^2 - s_{23})tT_d(m_b^2(C_0^{(4)} + C_2^{(4)}) - 2m_W^2C_2^{(4)}), \\
R_{13} \;=\; & m_W^2s_{23}(s_{23} - m_t^2)(SP_2 - SP_3)t(t - m_t^2)(T_d - T_e) - (m_b^2 + 2m_W^2)(m_t^2 - s_{23}) \\
& \times s_{23}(SP_2 - SP_3)(m_t^2 - t)tB_0^{(4)}(T_d - T_e) + (m_t^2 - s_{23})s_{23}(SP_2 - SP_3)(m_t^2 - t)t
\end{aligned}
$$

$$
\begin{aligned}
&\times(((m_b^2-m_t^2-m_W^2)(m_b^2+2m_W^2)+2m_W^2t_{12})C_0^{(5)}+2(m_b^2+2m_W^2)C_0^{(5)}\\
&+2m_W^2(t_{12}-m_t^2)C_1^{(5)}-(m_b^2m_t^2+4m_W^2m_t^2-2m_W^2t_{12})C_2^{(5)})(T_d-T_e)\\
&-m_W^2t_{12}(m_t^4(SP_3tT_d+s_{23}SP_2T_e)+2s_{23}t(SP_3tT_d+s_{23}SP_2T_e)-m_t^2(SP_3t(2s_{23}+t)T_d\\
&+s_{23}SP_2(s_{23}+2t)T_e))+(m_b^2+2m_W^2)(m_t^2-s_{23})s_{23}SP_2(m_t^2-2t)t_{12}T_eB_0^{(3)}\\
&+(m_b^2+2m_W^2)(m_t^2-2s_{23})SP_3(m_t^2-t)tt_{12}T_dB_0^{(5)}-(m_t^2-s_{23})s_{23}SP_2tt_{12}T_e\\
&\times(m_b^2m_t^2C_0^{(1)}-2(m_b^2+2m_W^2)C_0^{(1)}+2m_W^2(-m_t^2+s_{23}+t_{12})C_1^{(1)})\\
&+s_{23}SP_3(m_t^2-t)tt_{12}T_d(2(m_b^2+2m_W^2)C_0^{(3)}+m_b^2m_t^2C_1^{(3)}-(m_b^2+2m_W^2)s_{23}C_{12}^{(3)})\\
&+(m_t^2-s_{23})SP_3(m_t^2-t)tt_{12}T_d(m_b^2m_t^2C_0^{(4)}-2(m_b^2+2m_W^2)C_0^{(4)}+(m_b^2m_t^2\\
&+2m_W^2(-m_t^2+t+t_{12}))C_2^{(4)})-(m_t^2-s_{23})s_{23}SP_2(m_t^2-t)t_{12}T_e\\
&\times(2(m_b^2+2m_W^2)C_0^{(2)}+(m_b^2+2m_W^2)(m_t^2-t)C_{12}^{(2)}+m_b^2m_t^2C_2^{(2)}\\
&+m_t^2(m_b^2+2m_W^2)C_{22}^{(2)}),\\
R_{14} \;=\;& m_W^2(m_t^4(tT_d+s_{23}T_e)+2s_{23}t(tT_d+s_{23}T_e)-m_t^2(T_es_{23}^2\\
&+2t(T_d+T_e)s_{23}+t^2T_d))-(m_b^2+2m_W^2)(m_t^2-s_{23})s_{23}(m_t^2-2t)T_eB_0^{(3)}\\
&-(m_b^2+2m_W^2)(m_t^2-2s_{23})(m_t^2-t)tT_dB_0^{(5)}+(m_t^2-s_{23})s_{23}tT_e(m_b^2m_t^2C_0^{(1)}\\
&-2(m_b^2+2m_W^2)C_0^{(1)}+2m_W^2(-m_t^2+s_{23}+t_{12})C_1^{(1)})+s_{23}(m_t^2-t)tT_d(m_b^2m_t^2C_0^{(3)}\\
&-2(m_b^2+2m_W^2)C_0^{(3)}-2m_W^2s_{23}C_1^{(3)})-(m_t^2-s_{23})s_{23}(m_t^2-t)T_e(m_b^2m_t^2C_0^{(2)}\\
&-2(m_b^2+2m_W^2)C_0^{(2)}+(m_b^2m_t^2-2m_W^2t)C_2^{(2)})-(m_t^2-s_{23})(m_t^2-t)tT_d\\
&\times(m_b^2m_t^2C_0^{(4)}-2(m_b^2+2m_W^2)C_0^{(4)}+(m_b^2m_t^2+2m_W^2(-m_t^2+t+t_{12}))C_2^{(4)}),\\
R_{15} \;=\;& m_W^2(SP_4-SP_5)((m_t^2-2s_{23})t_{12}T_d+(m_t^2-s_{23})s_{23}(T_d-T_e))-(m_b^2+2m_W^2)\\
&\times(m_t^2-s_{23})s_{23}(SP_4-SP_5)(T_d-T_e)B_0^{(4)}-(m_b^2+2m_W^2)(m_t^2-2s_{23})(SP_4-SP_5)\\
&\times t_{12}T_dB_0^{(5)}+s_{23}(SP_4-SP_5)t_{12}T_d(m_b^2m_t^2C_0^{(3)}-2(m_b^2+2m_W^2)C_0^{(3)}\\
&-2m_W^2s_{23}C_1^{(3)})-(m_t^2-s_{23})s_{23}SP_5t_{12}T_e(m_b^2C_{12}^{(1)}+2m_W^2(C_1^{(1)}+C_{12}^{(1)}))\\
&+(m_t^2-s_{23})t_{12}T_d(-m_b^2m_t^2(SP_4-SP_5)C_0^{(4)}+(m_b^2+2m_W^2)(2(SP_4-SP_5)C_0^{(4)}\\
&-s_{23}SP_4C_{12}^{(4)})-(m_b^2m_t^2(SP_4-SP_5)+2m_W^2(m_t^2(SP_5-SP_4)-SP_5(t+t_{12})\\
&+SP_4(s_{23}+t+t_{12})))C_2^{(4)})+(m_t^2-s_{23})s_{23}(SP_4-SP_5)(T_d-T_e)\\
&\times(((m_b^2-m_t^2-m_W^2)(m_b^2+2m_W^2)+2m_W^2t_{12})C_0^{(5)}+2(m_b^2+2m_W^2)C_0^{(5)}\\
&+2m_W^2(t_{12}-m_t^2)C_1^{(5)}-(m_b^2m_t^2+4m_W^2m_t^2-2m_W^2t_{12})C_2^{(5)}),\\
R_{16} \;=\;& m_W^2(m_t^2-s_{23})s_{23}SP_1(m_t^2-t)(tT_d-(t+2t_{12})T_e)+2(m_b^2+2m_W^2)
\end{aligned}
$$

$$
\begin{aligned}
&\times(m_t^2-s_{23})s_{23}SP_1(m_t^2-t)t_{12}T_eB_0^{(3)}-(m_b^2+2m_W^2)(m_t^2-s_{23})\\
&\times s_{23}SP_1(m_t^2-t)t(T_d-T_e)B_0^{(4)}-2(m_t^2-s_{23})s_{23}tt_{12}T_e(m_b^2(m_t^2SP_1C_0^{(1)}\\
&-SP_1tC_0^{(1)}+2SP_2SP_5(C_0^{(1)}+C_1^{(1)}-C_{12}^{(1)}))-4m_W^2SP_2SP_5(C_1^{(1)}\\
&+C_{12}^{(1)}))-4s_{23}SP_3(SP_4-SP_5)(m_t^2-t)tt_{12}T_d(m_b^2(C_0^{(3)}+C_1^{(3)}-C_{12}^{(3)})\\
&-2m_W^2(C_1^{(3)}+C_{12}^{(3)}))+2(m_t^2-s_{23})s_{23}SP_1(m_t^2-t)t_{12}T_e(m_b^2((m_t^2-t)(C_0^{(2)}\\
&+C_2^{(2)})-2C_0^{(2)})-4m_W^2C_0^{(2)})+4(m_t^2-s_{23})SP_3SP_4(m_t^2-t)tt_{12}T_d(2m_W^2(C_{12}^{(4)}\\
&+C_{22}^{(4)})+m_b^2(C_0^{(4)}+C_{12}^{(4)}+2C_2^{(4)}+C_{22}^{(4)}))+(m_t^2-s_{23})s_{23}(m_t^2-t)t(T_d-T_e)\\
&\times(m_b^4SP_1C_0^{(5)}+2m_W^2(-m_W^2SP_1C_0^{(5)}-2s_{23}SP_1C_0^{(5)}-4SP_2SP_5C_0^{(5)}+2SP_1C_0^{(5)}\\
&-2s_{23}SP_1C_1^{(5)}-4SP_2SP_5C_1^{(5)}+m_t^2SP_1C_{12}^{(5)}-2s_{23}SP_1C_{12}^{(5)}-4SP_2SP_5C_{12}^{(5)}\\
&-SP_1t_{12}C_{12}^{(5)}-2m_t^2SP_1C_2^{(5)}-2s_{23}SP_1C_2^{(5)}-8SP_2SP_5C_2^{(5)}+2SP_1tC_2^{(5)}+SP_1t_{12}C_2^{(5)}\\
&-(4SP_2SP_5+SP_1(m_t^2-2t-t_{12}))C_{22}^{(5)}+4SP_3SP_4(C_0^{(5)}+C_1^{(5)}+C_{12}^{(5)}+2C_2^{(5)}\\
&+C_{22}^{(5)}))+m_b^2(4(SP_3SP_4-SP_2SP_5)(-C_1^{(5)}+C_{12}^{(5)}+C_{22}^{(5)})-m_t^2SP_1(C_0^{(5)}+C_1^{(5)}\\
&-C_{12}^{(5)}+C_2^{(5)}+C_{22}^{(5)})+SP_1(m_W^2C_0^{(5)}+2C_0^{(5)}+(2s_{23}+t_{12})(C_1^{(5)}-C_{12}^{(5)})\\
&+(2t+t_{12})C_{22}^{(5)}))),\\
R_{17}\;=\;&2m_W^2SP_1T_d(C_0^{(6)}-C_1^{(6)})+m_b^2SP_1T_e(C_2^{(7)}-C_0^{(7)})+2T_d(m_b^2(SP_1(D_0^{(1)}+D_2^{(1)}\\
&+m_W^2(D_2^{(1)}-D_0^{(1)}))+SP_3(SP_5(D_{112}^{(1)}+D_{12}^{(1)}+D_{122}^{(1)})+SP_4(D_{122}^{(1)}+D_{123}^{(1)}-D_{13}^{(1)}\\
&+D_{22}^{(1)}+D_{222}^{(1)}+D_{223}^{(1)}))+SP_2(SP_5(D_{123}^{(1)}+D_{13}^{(1)})+SP_4(D_{223}^{(1)}+D_{23}^{(1)}+D_{233}^{(1)})))\\
&+m_W^2(m_W^2SP_1(D_0^{(1)}-D_2^{(1)})+2(SP_3(SP_5(D_{112}^{(1)}+D_{12}^{(1)}+D_{122}^{(1)})+SP_4(2D_{12}^{(1)}\\
&+D_{122}^{(1)}+D_{123}^{(1)}+D_{13}^{(1)}+2D_2^{(1)}+3D_{22}^{(1)}+D_{222}^{(1)}+D_{223}^{(1)}+2D_{23}^{(1)}))+SP_2(SP_5(D_{123}^{(1)}\\
&-D_{13}^{(1)})+SP_4(D_{223}^{(1)}+D_{23}^{(1)}+D_{233}^{(1)})))+SP_1(-4D_0^{(1)}+2D_2^{(1)}-s_{23}(D_{12}^{(1)}+2D_{22}^{(1)})\\
&+(-2m_t^2+t+t_{12})D_{23}^{(1)}+m_t^2D_3^{(1)})))+T_e(m_b^4SP_1(D_0^{(2)}-D_3^{(2)})-m_b^2(m_t^2SP_1\\
&\times(D_0^{(2)}-D_{13}^{(2)}+D_2^{(2)}-D_{23}^{(2)})+SP_1(-4D_0^{(2)}+2D_3^{(2)}+(s_{23}+t_{12})D_{13}^{(2)}+m_W^2(D_0^{(2)}\\
&-D_3^{(2)})-t(D_{23}^{(2)}+2D_{33}^{(2)}))+2(SP_3(SP_4(D_{12}^{(2)}+D_{123}^{(2)})+SP_5(D_{113}^{(2)}+D_{13}^{(2)}\\
&+D_{133}^{(2)}))+SP_2(SP_4(D_{223}^{(2)}+D_{23}^{(2)}+D_{233}^{(2)})+SP_5(-D_{12}^{(2)}+D_{123}^{(2)}+D_{133}^{(2)}+D_{233}^{(2)}+D_{33}^{(2)}\\
&+D_{333}^{(2)}))))-2m_W^2(m_t^2SP_1(D_0^{(2)}+D_1^{(2)}+D_2^{(2)}+D_3^{(2)})-SP_1(t_{12}(D_0^{(2)}+D_1^{(2)}\\
&+D_2^{(2)}+D_3^{(2)})-2(D_0^{(2)}+D_3^{(2)}))+2(SP_3(SP_4(D_{123}^{(2)}-D_{12}^{(2)})+SP_5(D_{113}^{(2)}+D_{13}^{(2)}\\
&+D_{133}^{(2)}))+SP_2(SP_4(D_{223}^{(2)}+D_{23}^{(2)}+D_{233}^{(2)})+SP_5(D_{12}^{(2)}+D_{123}^{(2)}+2D_{13}^{(2)}\\
&+D_{133}^{(2)}+2D_{23}^{(2)}+D_{233}^{(2)}+2D_3^{(2)}+3D_{33}^{(2)}+D_{333}^{(2)}))))),
\end{aligned}
$$

$$
\begin{aligned}
R_{18} &= m_b^2(SP_3(T_e(D_{12}^{(2)}-D_1^{(2)})+T_d(-D_1^{(1)}+D_{12}^{(1)}+D_{13}^{(1)}+D_{22}^{(1)}+D_{23}^{(1)}))\\
&+SP_2(T_e(D_{22}^{(2)}+D_{23}^{(2)}-D_3^{(2)})+T_d(D_{23}^{(1)}+D_{33}^{(1)})))+2m_W^2(SP_3(T_d(D_0^{(1)}+D_1^{(1)}\\
&-D_{12}^{(1)}-D_{13}^{(1)}-D_{22}^{(1)}-D_{23}^{(1)}+D_3^{(1)})+T_e(D_0^{(2)}+D_1^{(2)}-D_{12}^{(2)}+D_2^{(2)}+D_3^{(2)}))\\
&-SP_2(T_e(D_2^{(2)}+D_{22}^{(2)}+D_{23}^{(2)})+T_d(D_{23}^{(1)}+D_3^{(1)}+D_{33}^{(1)}))),\\
R_{19} &= m_b^4(T_d(D_2^{(1)}-D_0^{(1)})+T_e(D_3^{(2)}-D_0^{(2)}))+m_b^2(T_d(C_0^{(6)}-C_1^{(6)}-6D_0^{(1)}+3m_W^2(D_0^{(1)}\\
&-D_2^{(1)})-s_{23}(D_{12}^{(1)}+2D_{22}^{(1)})+(t+t_{12})D_{23}^{(1)}+m_t^2(D_0^{(1)}-2D_{23}^{(1)}+D_3^{(1)}))\\
&+T_e(C_0^{(7)}-C_2^{(7)}-6D_0^{(2)}+(s_{23}+t_{12})D_{13}^{(2)}+m_t^2(D_0^{(2)}-D_{13}^{(2)}\\
&+D_2^{(2)}-D_{23}^{(2)})+3m_W^2(D_0^{(2)}-D_3^{(2)})-t(D_{23}^{(2)}+2D_{33}^{(2)})))+2m_W^2(T_d\\
&\times(-C_0^{(6)}+C_1^{(6)}+6D_0^{(1)}+m_W^2(D_2^{(1)}-D_0^{(1)})+s_{23}(D_{12}^{(1)}+2D_{22}^{(1)})\\
&-tD_{23}^{(1)}+m_t^2(D_0^{(1)}+D_1^{(1)}+D_2^{(1)}+2D_{23}^{(1)})-t_{12}(D_0^{(1)}+D_1^{(1)}+D_2^{(1)}+D_{23}^{(1)}\\
&+D_3^{(1)}))+T_e(-C_0^{(7)}+C_2^{(7)}+6D_0^{(2)}-s_{23}D_{13}^{(2)}+m_W^2(D_3^{(2)}-D_0^{(2)})-t_{12}(D_0^{(2)}+D_1^{(2)}\\
&+D_{13}^{(2)}+D_2^{(2)}+D_3^{(2)})+m_t^2(D_0^{(2)}+D_1^{(2)}+D_{13}^{(2)}+D_{23}^{(2)}+D_3^{(2)})+t(D_{23}^{(2)}+2D_{33}^{(2)}))),\\
R_{20} &= 2m_W^2(SP_5(T_d(D_0^{(1)}+D_1^{(1)}-D_{13}^{(1)}+D_2^{(1)}+D_3^{(1)})+T_e(D_0^{(2)}+D_1^{(2)}-D_{12}^{(2)}-D_{13}^{(2)}\\
&+D_2^{(2)}-D_{23}^{(2)}-D_{33}^{(2)}))-SP_4(T_e(D_2^{(2)}+D_{22}^{(2)}+D_{23}^{(2)})+T_d(D_{23}^{(1)}+D_3^{(1)}+D_{33}^{(1)})))\\
&+m_b^2(SP_4(T_e(D_{22}^{(2)}+D_{23}^{(2)})+T_d(-D_2^{(1)}+D_{23}^{(1)}+D_{33}^{(1)}))+SP_5(T_d(D_{13}^{(1)}-D_1^{(1)})\\
&+T_e(-D_1^{(2)}+D_{12}^{(2)}+D_{13}^{(2)}+D_{23}^{(2)}+D_{33}^{(2)}))),\\
R_{21} &= T_d(2m_W^2(C_0^{(6)}+C_2^{(6)})-m_b^2C_2^{(6)})+T_e((m_b^2-2m_W^2)(C_1^{(7)}+C_2^{(7)})-m_b^2C_0^{(7)})\\
&+T_d(m_b^4D_3^{(1)}+2m_W^2(-2D_0^{(1)}-s_{23}(D_0^{(1)}+D_1^{(1)}-D_{13}^{(1)}-2D_{23}^{(1)})+m_t^2D_3^{(1)}\\
&+m_W^2(D_0^{(1)}+D_3^{(1)})-(-2m_t^2+t+t_{12})D_{33}^{(1)})+m_b^2(2D_0^{(1)}+s_{23}(D_1^{(1)}-D_{13}^{(1)}+D_2^{(1)}\\
&-2D_{23}^{(1)})-m_W^2(2D_0^{(1)}+3D_3^{(1)})+(-2m_t^2+t+t_{12})D_{33}^{(1)}))+T_e(m_b^4(D_0^{(2)}-D_2^{(2)}\\
&-D_3^{(2)})+m_b^2(4D_0^{(2)}+(s_{23}+t_{12})(D_1^{(2)}-D_{12}^{(2)}-D_{13}^{(2)})+m_t^2(-D_0^{(2)}-D_1^{(2)}+D_{12}^{(2)}\\
&+D_{13}^{(2)}-D_2^{(2)}+D_{22}^{(2)}+D_{23}^{(2)})+m_W^2(3(D_2^{(2)}+D_3^{(2)})-D_0^{(2)})+t(D_{22}^{(2)}+3D_{23}^{(2)}-D_3^{(2)}\\
&+2D_{33}^{(2)}))-2m_W^2(4D_0^{(2)}-t_{12}(D_{12}^{(2)}+D_{13}^{(2)})+m_t^2(D_{12}^{(2)}+D_{13}^{(2)}+D_{22}^{(2)}+D_{23}^{(2)})\\
&+m_W^2(D_2^{(2)}+D_3^{(2)})+s_{23}(D_0^{(2)}+D_1^{(2)}-D_{12}^{(2)}-D_{13}^{(2)}+D_2^{(2)}+D_3^{(2)})+t(D_2^{(2)}+D_{22}^{(2)}\\
&+3D_{23}^{(2)}+D_3^{(2)}+2D_{33}^{(2)}))),\\
R_{22} &= 2m_W^4SP_1T_d(D_0^{(1)}+D_3^{(1)})+m_b^2(-SP_1(2T_d(D_3^{(1)}-D_0^{(1)})+T_e(C_0^{(7)}-C_1^{(7)}-C_2^{(7)}\\
&+2(-2D_0^{(2)}+D_2^{(2)}+D_3^{(2)})-(s_{23}+t_{12})(D_1^{(2)}-D_{12}^{(2)}-D_{13}^{(2)})+m_t^2(D_0^{(2)}+D_1^{(2)}\\
&-D_{12}^{(2)}-D_{13}^{(2)}+D_2^{(2)}-D_{22}^{(2)}-D_{23}^{(2)})+m_b^2(-D_0^{(2)}+D_2^{(2)}+D_3^{(2)})-t(D_{22}^{(2)}
\end{aligned}
$$

$$
\begin{aligned}
&+3D_{23}^{(2)} - D_{3}^{(2)} + 2D_{33}^{(2)}))) - 2(SP_3(SP_5(T_e(-D_{11}^{(2)} + D_{112}^{(2)} + D_{113}^{(2)} + D_{123}^{(2)} + D_{133}^{(2)}) \\
&-T_d(D_{11}^{(1)} - D_{113}^{(1)} + 2D_{12}^{(1)} - D_{123}^{(1)} + D_{22}^{(1)} + D_{23}^{(1)})) + SP_4(T_e(D_{122}^{(2)} + D_{123}^{(2)}) \\
&+T_d(-D_{1}^{(1)} + D_{123}^{(1)} + D_{13}^{(1)} + D_{133}^{(1)} - D_{2}^{(1)} + D_{223}^{(1)} + 2D_{23}^{(1)} + D_{233}^{(1)} + D_{33}^{(1)}))) \\
&+SP_2(SP_4(T_e(D_{22}^{(2)} + D_{222}^{(2)} + 2D_{223}^{(2)} + D_{23}^{(2)} + D_{233}^{(2)}) + T_d(D_{233}^{(1)} + D_{33}^{(1)} + D_{333}^{(1)})) \\
&+SP_5(T_e(-D_{1}^{(2)} + D_{122}^{(2)} + 2D_{123}^{(2)} + D_{133}^{(2)} + D_{223}^{(2)} + D_{23}^{(2)} + 2D_{233}^{(2)} + D_{33}^{(2)} + D_{333}^{(2)}) \\
&-T_d(D_{13}^{(1)} - D_{133}^{(1)} + D_{23}^{(1)} + D_{33}^{(1)}))))) + m_W^2(SP_1(2T_d(C_0^{(6)} + C_2^{(6)} - 2D_0^{(1)} - 2D_3^{(1)} \\
&+s_{23}(D_{13}^{(1)} + D_{2}^{(1)} + 2D_{23}^{(1)}) - m_b^2(D_0^{(1)} + D_3^{(1)}) + (2m_t^2 - t - t_{12})(D_3^{(1)} + D_{33}^{(1)})) \\
&-T_e(4(D_0^{(2)} + D_2^{(2)} + D_3^{(2)}) + m_b^2(D_0^{(2)} - D_2^{(2)} - D_3^{(2)}) + 2s_{23}(D_0^{(2)} + D_1^{(2)} + D_2^{(2)} \\
&+D_3^{(2)}))) - 4(SP_3(SP_5(T_e(D_1^{(2)} + D_{11}^{(2)} + D_{112}^{(2)} + D_{113}^{(2)} + 2D_{12}^{(2)} + D_{123}^{(2)} + 2D_{13}^{(2)} \\
&+D_{133}^{(2)}) + T_d(D_1^{(1)} + D_{11}^{(1)} + D_{113}^{(1)} + 2D_{12}^{(1)} + D_{123}^{(1)} + 2D_{13}^{(1)} + D_2^{(1)} + D_{22}^{(1)} + D_{23}^{(1)})) \\
&+SP_4(T_e(D_{122}^{(2)} + D_{123}^{(2)}) + T_d(D_0^{(1)} + D_1^{(1)} + D_{123}^{(1)} + D_{13}^{(1)} + D_{133}^{(1)} + D_2^{(1)} + D_{223}^{(1)} \\
&+2D_{23}^{(1)} + D_{233}^{(1)} + 2D_3^{(1)} + D_{33}^{(1)}))) + SP_2(SP_4(T_e(D_{22}^{(2)} + D_{222}^{(2)} + 2D_{223}^{(2)} + D_{23}^{(2)} + D_{233}^{(2)}) \\
&+T_d(D_{233}^{(1)} + D_{33}^{(1)} + D_{333}^{(1)})) + SP_5(T_d(D_{13}^{(1)} + D_{133}^{(1)} + D_{23}^{(1)} + D_3^{(1)} + D_{33}^{(1)}) + T_e(D_0^{(2)} \\
&+D_1^{(2)} + 2D_{12}^{(2)} + D_{122}^{(2)} + 2D_{123}^{(2)} + 2D_{13}^{(2)} + D_{133}^{(2)} + 3D_2^{(2)} + 2D_{22}^{(2)} + D_{223}^{(2)} \\
&+5D_{23}^{(2)} + 2D_{233}^{(2)} + 3(D_3^{(2)} + D_{33}^{(2)}) + D_{333}^{(2)}))))),
\end{aligned}
$$

$$
\begin{aligned}
R_{23} \;=\; &2m_W^4(SP_2(T_dD_0^{(1)} - T_e(D_0^{(2)} + D_2^{(2)} + D_3^{(2)})) - SP_3(T_d(D_0^{(1)} + D_1^{(1)} + D_2^{(1)}) \\
&-T_eD_0^{(2)})) + m_b^2(-SP_2T_eC_0^{(7)} - SP_3(T_d(C_0^{(6)} - m_b^2D_0^{(1)} - 4D_0^{(1)} - 2D_1^{(1)} - 2D_2^{(1)} \\
&-s_{23}(D_{12}^{(1)} + D_{22}^{(1)}) + (t + t_{12})(D_{13}^{(1)} + D_{23}^{(1)}) + m_t^2(D_0^{(1)} + D_1^{(1)} - D_{13}^{(1)} + D_2^{(1)} - D_{23}^{(1)} \\
&+D_3^{(1)})) - T_e(C_0^{(7)} + C_1^{(7)} + C_2^{(7)} + 2D_1^{(2)} + (m_b^2 - m_t^2)D_1^{(2)} - t(D_{12}^{(2)} + D_{13}^{(2)}))) \\
&-SP_2(-T_e(m_b^2D_0^{(2)} + 2(2D_0^{(2)} + D_2^{(2)} + D_3^{(2)}) - (s_{23} + t_{12})D_{12}^{(2)} + m_t^2(-D_0^{(2)} \\
&+D_{12}^{(2)} + D_{13}^{(2)})) + T_d(C_2^{(6)} - 2D_3^{(1)} + s_{23}(D_{13}^{(1)} + D_{23}^{(1)}) - m_b^2D_3^{(1)} + m_t^2(D_3^{(1)} + D_{33}^{(1)})) \\
&+T_e((s_{23} + t_{12})D_{13}^{(2)} + m_t^2(D_2^{(2)} - D_{22}^{(2)} - D_{23}^{(2)} + D_3^{(2)}) - t(D_{23}^{(2)} + D_{33}^{(2)})))) \\
&+m_W^2(SP_3(T_d(2C_2^{(6)} + m_b^2(D_0^{(1)} + 2(D_1^{(1)} + D_2^{(1)})) + 2(2(D_1^{(1)} + D_2^{(1)}) - s_{23}(D_{12}^{(1)} \\
&+D_2^{(1)} + 2D_{22}^{(1)}) + t(D_{12}^{(1)} + D_{13}^{(1)} + D_2^{(1)} + D_{23}^{(1)}) + t_{12}(D_0^{(1)} + 2D_1^{(1)} + D_{11}^{(1)} + 2D_{12}^{(1)} \\
&+D_{13}^{(1)} + 2D_2^{(1)} + D_{23}^{(1)} + D_3^{(1)}) - m_t^2(D_0^{(1)} + 2D_1^{(1)} + D_{11}^{(1)} + 3D_{12}^{(1)} + 3D_{13}^{(1)} + 3D_2^{(1)} \\
&+D_{22}^{(1)} + 3D_{23}^{(1)} + 2D_3^{(1)}))) + T_e(2C_0^{(7)} - m_b^2(2D_0^{(2)} + D_1^{(2)}) + 2(-2D_0^{(2)} + 2D_1^{(2)} \\
&+s_{23}D_{13}^{(2)} + (t + t_{12})(D_{12}^{(2)} + D_{13}^{(2)}) + tD_3^{(2)} + m_t^2(D_0^{(2)} + D_1^{(2)} - D_{12}^{(2)} - D_{13}^{(2)} + 2D_2^{(2)} \\
&+D_3^{(2)})))) + SP_2(-2T_e(C_0^{(7)} + C_1^{(7)} + C_2^{(7)}) + T_d(2C_0^{(6)} - m_b^2(2D_0^{(1)} + D_3^{(1)})
\end{aligned}
$$

$$
\begin{aligned}
&+2(-2D_0^{(1)}+2D_3^{(1)}+s_{23}D_{13}^{(1)}+t_{12}D_{13}^{(1)}+s_{23}D_2^{(1)}+(s_{23}+t+t_{12})D_{23}^{(1)}-m_t^2(D_{13}^{(1)}\\
&+2D_{23}^{(1)}+D_{33}^{(1)})))+T_e(m_b^2(D_0^{(2)}+2(D_2^{(2)}+D_3^{(2)}))+2(2(D_2^{(2)}+D_3^{(2)})+s_{23}(D_{12}^{(2)}\\
&+D_{13}^{(2)}+D_{23}^{(2)}+D_3^{(2)})-m_t^2(D_0^{(2)}+D_1^{(2)}+D_{12}^{(2)}+D_{13}^{(2)}+3D_2^{(2)}+2D_{22}^{(2)}+3D_{23}^{(2)}\\
&+2D_3^{(2)})+t_{12}(D_0^{(2)}+D_1^{(2)}+D_{12}^{(2)}+D_{13}^{(2)}+2D_2^{(2)}+D_{22}^{(2)}+2(D_{23}^{(2)}+D_3^{(2)}))\\
&-t(D_{23}^{(2)}+D_3^{(2)}+2D_{33}^{(2)}))))),\\
R_{24} \;=\; &-m_b^4(SP_4(T_dD_0^{(1)}+T_eD_2^{(2)})+SP_5(T_d(-D_0^{(1)}+D_1^{(1)}+D_2^{(1)})+T_eD_3^{(2)}))\\
&+2m_W^2(SP_5(T_e(C_0^{(7)}-C_1^{(7)}-C_2^{(7)}-2(3D_0^{(2)}+D_1^{(2)}+D_3^{(2)})+m_W^2(2D_0^{(2)}+D_1^{(2)})\\
&-t_{12}(D_1^{(2)}+D_{11}^{(2)})+s_{23}D_{13}^{(2)}+m_t^2(D_0^{(2)}+2D_1^{(2)}+D_{11}^{(2)}+D_{12}^{(2)}+3D_2^{(2)}+D_3^{(2)})\\
&+t(D_{12}^{(2)}+2(D_{13}^{(2)}+D_3^{(2)})))-T_d(C_1^{(6)}+4D_0^{(1)}+2D_1^{(1)}+m_W^2D_2^{(1)}+s_{23}(D_{12}^{(1)}+D_2^{(1)}\\
&+2D_{22}^{(1)})-t(D_{13}^{(1)}+D_{23}^{(1)})-t_{12}(D_0^{(1)}+D_1^{(1)}+D_2^{(1)}+D_{23}^{(1)}+D_3^{(1)})+m_t^2(D_0^{(1)}\\
&+D_1^{(1)}+D_{13}^{(1)}+D_2^{(1)}+2D_{23}^{(1)}+D_3^{(1)})))+SP_4(T_d(C_0^{(6)}+C_1^{(6)}+C_2^{(6)}-2D_2^{(1)}\\
&-2D_3^{(1)}+s_{23}(D_{12}^{(1)}+D_{13}^{(1)}+2(D_2^{(1)}+D_{22}^{(1)}+D_{23}^{(1)}))+m_W^2(D_0^{(1)}+D_2^{(1)}+D_3^{(1)})\\
&+m_t^2(D_0^{(1)}+D_1^{(1)}+D_2^{(1)}+D_{23}^{(1)}+2D_3^{(1)}+D_{33}^{(1)})-t_{12}(D_0^{(1)}+D_1^{(1)}+D_2^{(1)}\\
&+D_{23}^{(1)}+2D_3^{(1)}+D_{33}^{(1)}))-T_e(C_0^{(7)}+m_W^2D_0^{(2)}-2D_0^{(2)}+2D_2^{(2)}-s_{23}D_{12}^{(2)}+m_t^2D_2^{(2)}\\
&+tD_3^{(2)})))+m_b^2(SP_4(T_e(C_1^{(7)}-2D_2^{(2)}-(s_{23}+t_{12})D_{12}^{(2)}+m_W^2(2D_0^{(2)}\\
&+D_2^{(2)})+tD_{23}^{(2)}+m_t^2(D_{12}^{(2)}+D_2^{(2)}+2D_{22}^{(2)}+D_{23}^{(2)}))+T_d(C_0^{(6)}-4D_0^{(1)}-2D_2^{(1)}\\
&-2D_3^{(1)}-s_{23}(D_{12}^{(1)}+D_{13}^{(1)}+D_{22}^{(1)}+D_{23}^{(1)})+m_t^2(D_0^{(1)}+D_3^{(1)})-m_W^2(D_0^{(1)}+2(D_2^{(1)}\\
&+D_3^{(1)}))))+SP_5(T_e(C_2^{(7)}+2D_0^{(2)}-2D_1^{(2)}-2D_3^{(2)}+m_W^2(-4D_0^{(2)}-2D_1^{(2)}+D_3^{(2)})\\
&+t(D_{33}^{(2)}-D_{12}^{(2)})+m_t^2(D_{12}^{(2)}+D_{13}^{(2)}+2D_{23}^{(2)}+D_3^{(2)}+D_{33}^{(2)}))-T_d(2C_0^{(6)}+C_2^{(6)}-6D_0^{(1)}\\
&+2D_1^{(1)}+m_W^2(D_0^{(1)}-D_1^{(1)}-3D_2^{(1)})-2s_{23}(D_{12}^{(1)}+D_{22}^{(1)})+(t+t_{12})(D_{13}^{(1)}\\
&+D_{23}^{(1)})+m_t^2(D_0^{(1)}-2(D_{13}^{(1)}+D_{23}^{(1)})+D_3^{(1)})))),\\
R_{25} \;=\; &T_d-T_e,\\
R_{26} \;=\; &m_b^2(s_{23}(C_{12}^{(5)}-C_1^{(5)})+(m_t^2-t)C_{22}^{(5)})+2m_W^2(s_{23}(C_0^{(5)}+C_1^{(5)}+C_{12}^{(5)}+C_2^{(5)})\\
&+(m_t^2-t)(C_2^{(5)}+C_{22}^{(5)})),\\
R_{27} \;=\; &(m_b^4+m_b^2(m_W^2-2m_t^2)-2m_W^4)B_0^{(1)}-(m_b^4+2(m_t^2-m_W^2)m_W^2+m_b^2(m_W^2-m_t^2))B_0^{(2)}\\
&+2m_t^2(m_b^2+2m_W^2)B_0^{(4)}+2m_t^2(-((m_b^2-m_t^2-m_W^2)(m_b^2+2m_W^2)+2m_W^2t_{12})C_0^{(5)}\\
&-2(m_b^2+2m_W^2)C_0^{(5)}+2m_W^2(m_t^2-t_{12})C_1^{(5)}+(m_b^2m_t^2+4m_W^2m_t^2-2m_W^2t_{12})C_2^{(5)}).
\end{aligned}
$$

where

$$
\begin{aligned}
B_0^{(1)} &= B_0(m_b^2, m_W^2), \quad B_0^{(2)} = B_0(m_t^2, m_b^2, m_W^2), \quad B_0^{(3)} = B_0(t, m_b^2, m_W^2), \\
B_0^{(4)} &= B_0(t_{12}, m_b^2, m_b^2), \quad B_0^{(5)} = B_0(s_{23}, m_b^2, m_W^2), \\
C_\beta^{(1)} &= C_\beta(0, t, 0, m_b^2, m_W^2, m_b^2), \ C_\beta^{(2)} = C_\beta(0, t, m_t^2, m_b^2, m_b^2, m_W^2), \\
C_\beta^{(3)} &= C_\beta(0, s_{23}, 0, m_b^2, m_W^2, m_b^2), \ C_\beta^{(4)} = C_\beta(0, s_{23}, m_t^2, m_b^2, m_b^2, m_W^2), \\
C_\beta^{(5)} &= C_\beta(0, t_{12}, m_t^2, m_W^2, m_b^2, m_b^2), \ C_\beta^{(6)} = C_\beta(0, 0, t_{12}, m_W^2, m_b^2, m_b^2), \\
C_\beta^{(7)} &= C_\beta(t_{12}, 0, 0, m_W^2, m_b^2, m_b^2), \ \beta = 0, 1, 12, 2, 22, \\
D_\lambda^{(1)} &= D_\lambda(0, 0, 0, m_t^2, s_{23}, t_{12}, m_W^2, m_b^2, m_b^2, m_b^2), \\
D_\lambda^{(2)} &= D_\lambda(0, t_{12}, 0, t, m_t^2, 0, m_W^2, m_b^2, m_b^2, m_b^2), \\
\lambda &= 0, 1, 11, 12, 13, 2, 22, 23, 3, 33, 003, 112, 113, 122, 123, 133, 222, 223, 233, 333.
\end{aligned}
$$

The coefficient functions $C_{i,ij}$ and $D_{i,ij,ijk}$ are symmetric functions and can further be decomposed into the scalar functions A_0, B_0, C_0, and D_0. See [36] for details.

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