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Comparison of Equivalent Stress Methods with Critical Plane Approaches for Multiaxial High Cycle Fatigue Assessment

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Abstract

Several equivalent stress methods and more advanced critical plane criteria are compared in terms of their performance in fatigue life estimations under uniaxial and biaxial loadings in which the effect of phase is investigated. For this purpose a MATLAB code is written which transforms the multiaxial cyclic stress state into a uniaxial cyclic stress to use with the equivalent stress based methods. For critical plane approaches the program searches a damage parameter on all material planes. The maximum damage parameter is then compared with a material allowable obtained from uniaxial fatigue tests for life estimation. In addition, various methods for calibration of the material coefficient k and prominent stress history enclosure methods to calculate the shear stress amplitude are also studied for determining their effect on the performance of critical plane approaches.

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Keywords: multiaxial fatigue; high cycle fatigue; equivalent stress; critical plane; non-proportional loading; shear stress amplitude

1. Introduction

Many critical engineering parts designed with safe life methodology such as rotor blades, pressure vessels, railroad wheels, crankshafts and bolted joints experience cyclic loading that leads to biaxial or triaxial stress states. Fatigue

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failure under such stress states is called multiaxial fatigue. Multiaxial fatigue is a complex problem due to multiaxial stress state, non-proportional loading and directional characteristics of materials and the fatigue process. Therefore, appropriate damage models should be used for accurate life estimations.

Research concerning multiaxial fatigue and appropriate life estimation methods have been carried out for several decades and significant improvements were made. At first, extensions of static failure criteria were developed which reduce multiaxial cyclic stress state to an equivalent cyclic stress history. Once equivalent stress history is obtained, fatigue life could be estimated from S-N curves. Although equivalent stress methods provide estimations that are in good agreement with test data for proportional loading, experimental studies show that they fail to account the directivity of non-proportional loading and effects of shear and tensile stresses on fatigue life. In order to cope with non-proportional loading and also considering physical mechanisms of fatigue crack initiation, critical plane methods were developed. According to critical plane theory, shear stresses/strains are the main cause of crack initiation as they induce movement of dislocations along slip lines while normal stresses/strains are responsible for crack opening since they reduce the friction between crack surfaces. Therefore, most of the proposals contain linear or nonlinear combination of mean and/or alternating values of shear and normal stresses/strains with weighting material constants.

Equivalent stress methods are frequently used instead of critical plane approaches in the industry for fatigue assessment due to their simplicity and speed. However, as mentioned above, their accuracy for non-proportional loading histories is questionable. Authors Pedersen (2016) and Papuga et al. (2012) compared the performances of equivalent stress methods and critical plane approaches. However, very few of the comparisons in literature include the most basic equivalent stress methods such as Maximum Principal Stress or Von Mises criterion. Therefore, in this paper, those basic equivalent stress methods and several critical plane approaches are investigated. Their performance evaluations are carried out by proportional and non-proportional loading histories with different phases which are collected from Papuga (2005).

2. Multiaxial Loading

Load histories can be classified as proportional or non-proportional loading. Any loading that causes a change in principal stress directions and/or principal stress ratio in time is called non-proportional loading whereas principal directions and stress ratio remains constant for proportional loading. For instance, a loading history with two loading channels both having the same frequency and phase without mean stresses can be called proportional.

Fig 1 shows three of the loading histories and related stress paths that are common. First loading is proportional in which the stress path (σ vs. τ) is a horizontal line that intersects the origin. Second loading illustrates the 90° out of phase loading in which the stress path takes the form of a circle and which is considered as the most damaging type of loading history as stated in Socie et al. (2000). The last loading shown in Fig 1 exemplifies a case of phase and frequency ratio difference combination.

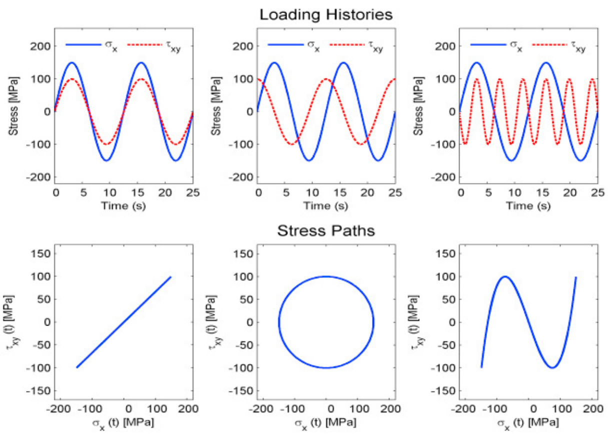

Fig 1. Load histories and stress paths

3. Equivalent Stress Theories

3.1. Absolute Maximum Principal Stress Criterion

Absolute Maximum Principal Stress criterion is an attempt to correlate multiaxial test data by means of the static yield criteria, maximum normal stress theory. An equivalent uniaxial stress history is produced from principal stresses and the sign of the equivalent stress at a time is the sign of the absolute maximum principal stress. Criterion can be expressed as

$$\sigma_{eq} = sign * \sigma_{AMP} \tag{1}$$

where σ_1, σ_2 and σ_3 are principal stresses ($\sigma_1 > \sigma_2 > \sigma_3$) that are eigenvalues of the stress tensor. Mean and alternating values of the equivalent stress can be obtained from

$$\sigma_{eq\_mean} = \frac{\sigma_{eq\_\max} + \sigma_{eq\_\min}}{2} ; \sigma_{eq\_alternating} = \frac{abs(\sigma_{eq\_\max} - \sigma_{eq\_\min})}{2} \tag{2}$$

3.2. Signed Von Mises Criterion

Signed Von Mises is based on the well-known static failure criteria, octahedral shear stress theory. Multiaxial stress state is transformed into a uniaxial stress history and a signing procedure is applied in order to correctly reflect the real load spectrum. Bishop (2000) states that signing should be applied according to principal stresses whereas Papuga defines the method signed by the sign of the first stress invariant (I_1) Papuga et al. (2012). In this study, methodology of signing procedure with principal stress signs is adapted. Signed von Mises formulated in terms of principal stresses as follows,

$$\sigma_{eq} = (sign) * \frac{1}{\sqrt{2}} \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2} \tag{3}$$

Signed von Mises is calculated for the loading history and once the uniaxial stress history is obtained, mean and alternating values may be obtained just as in Absolute Maximum Principal Criterion.

4. Critical Plane Theories

4.1. Background information

Critical plane theories involve calculation of mean and alternating values of shear and normal stresses on every material plane to find the maximum value of a proposed damage parameter. This task could be achieved by successful coordinate transformations of the stress state and the plane, where the damage parameter is maximized, is called the critical plane. For any plane Δ defined with normal vector $\mathbf{n}_{x'}$, the unit vectors on material plane, $\mathbf{n}_{x'}$, $\mathbf{n}_{y'}$, and $\mathbf{n}_{z'}$ may be expressed in terms of spherical angles θ and φ (Fig 2a) as

$$n_{x'} = \begin{bmatrix} \sin(\varphi)\cos(\theta) \\ \sin(\varphi)\sin(\theta) \\ \cos(\varphi) \end{bmatrix} ; n_{y'} = \begin{bmatrix} -\sin(\theta) \\ \cos(\theta) \\ 0 \end{bmatrix} ; n_{z'} = \begin{bmatrix} -\cos(\varphi)\cos(\theta) \\ -\cos(\varphi)\sin(\theta) \\ \sin(\varphi) \end{bmatrix} \tag{4}$$

Stress vector **t** acting on the plane may be found from Cauchy's theorem as **t**=**σn**$_{x'}$ and can be decomposed into normal and shear stresses

$$t = \tau + \sigma_n n_{x'} = \tau_{x'y'} n_{y'} + \tau_{x'z'} n_{z'} + \sigma_n n_{x'} \tag{5}$$

where each component can be written as

$$\tau_{x'y'} = t n_{y'} ; \tau_{x'z'} = t n_{z'} ; \sigma_n = t n_{x'} \tag{6}$$

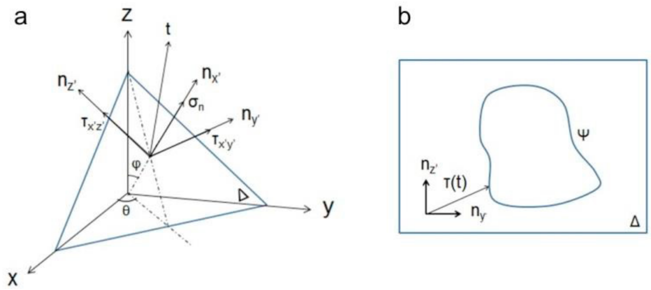

Fig 2. a) Material plane, stress vector and its components b) Resultant shear stress path

For proportional loading, calculation of mean and alternating values of normal and resultant shear stress (τ) is an easy task since both shear stresses vary proportionally in magnitude without any change in their direction. An important thing to mention is that direction of the normal stress vector is always fixed for any material plane Δ, thus evaluation of the normal stress vector is straightforward for any loading history. However, the same is not true for resultant shear stress vector. For non-proportional loading as both the direction and magnitude changes, sophisticated methods are required which are discussed in the following chapter.

4.2. Calculation of shear stress amplitude

There are several methods for determining the alternating and mean values of the resultant shear stress. In this study two of those methods are investigated. Minimum Circumscribe Circle (MCC) method was first proposed by Dan Vang and later by Papadopoulos. The idea of the MCC is basically environing the shear stress path Ψ with a circle, thus the radius of the circle gives the alternating value of the shear stress and mean value is the magnitude of the vector joining the center of the MCC and the origin. Determining the MCC is actually a min-max optimization problem as stated in Araújo et al (2011) and Bernasconi, et al. (2005). Mean shear stress vector (τ_m) may be obtained by minimizing an arbitrary shear vector τ* which maximizes the norm of the difference (τ-τ*) as follows,

$$\tau_m = \min_{\tau^*}(\max_t \|\tau(t) - \tau^*\|) \tag{7}$$

After center of the MCC is obtained, radius which corresponds to the alternating value of shear stress (τ_a) is the maximum value of the norm of the difference (τ-τ*) which can be formulated as

$$\tau_a = \max_t \|\tau(t) - \tau_m\| \tag{8}$$

Minimum circumscribed circle gives a unique solution and currently it is the most popular method as mentioned in Araújo et al. (2011). However, the method has some drawbacks. One drawback is that MCC requires complicated optimization algorithms. Another drawback is MCC method may not distinguish between proportional and non-proportional loading i.e. method bounds some proportional and non-proportional stress histories with the same MCC as stated in Castro et al. (2014). Fig 3a shows the definition of MCC. Fig 3b illustrates stress histories where MCC method fails. For MCC, two stress paths (Ψ_1, Ψ_2) are shown in Fig 3b. Ψ_1 is a non-proportional stress history while Ψ_2 is a proportional stress history. As can be seen from the figure same alternating shear stress is calculated for both histories which do not reflect the reality since experimental studies show non-proportional histories are more damaging than proportional ones as mentioned in Dantas et al. (2011) and Lönnqvist et al. (2007).

Maximum Rectangular Hull (MRH), which is first introduced by Araujo (2011), does not have drawbacks stated above. Main idea of the MRH is to enclose the shear stress path Ψ with a rectangular hull (RH) and finding the maximum by 2D rotation on material plane Δ. Half sides of the rectangular hull for an orientation of α may be obtained from (Fig 3c):

$$a_k(\alpha) = \frac{1}{2}(\max_t(\tau_k(\alpha,t)) - \min_t(\tau_k(\alpha,t)); k = 1.2 \tag{9}$$

In equation 9, τ_1 and τ_2 corresponds to $\tau_{x'y'}$ and $\tau_{x'z'}$ respectively. For each rectangular hull alternating value of the shear stress is defined as:

$$\tau_a(\alpha) = \sqrt{a_1^{\,2}(\alpha) + a_2^{\,2}(\alpha)} \tag{10}$$

Maximum Rectangular Hull is defined as the hull where orientation α maximizes the alternating shear stress. Once the MRH is obtained, like in MCC distance from origin to the center of MRH gives the mean value of resultant shear stress. In this study, maximum rectangular hull search is carried out with 1^o increments from 0^o up to 90^o.

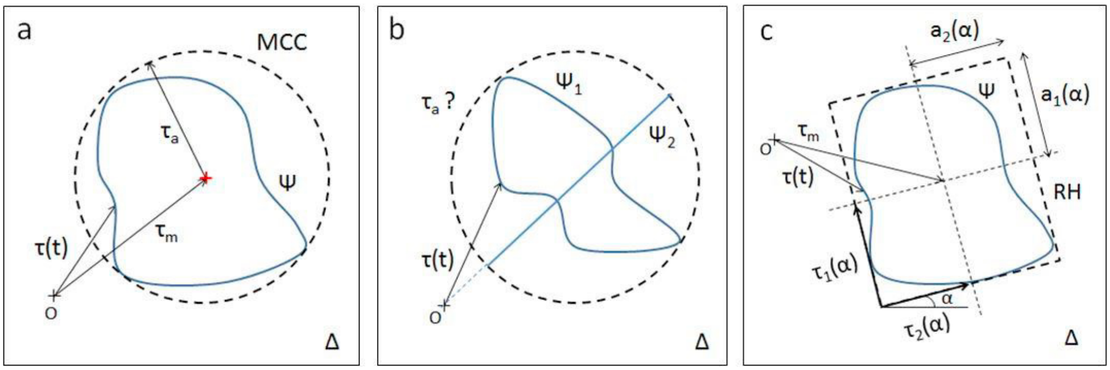

Fig 3. Left hand side is the definition and right hand side is the drawback of methods: a) Minimum Circumscribed Circle (MCC) and b) Rectangular Hull (RH)

4.3. Findley Criterion

Findley proposed a damage parameter that is a linear combination of shear stress amplitude and maximum normal stress acting on material planes and it was one of the first critical plane theories. Parabolic forms were also studied by Findley; however, linear formulation was found to be sufficient for the experimental data investigated. Findley defines the critical plane as the material plane where damage parameter is maximized. Findley damage parameter is as follows,

$$\max_{\theta,\varphi}(\tau_a + k\sigma_{n,\max}) = f \tag{11}$$

where k and f are material constants and may be determined from at least two uniaxial tests. For fully reversed tension-compression (or bending) and fully reversed torsion, k and f can be formulated as

$$k = \frac{2 - r_{-1}}{2\sqrt{r_{-1} - 1}};\ f = \frac{1}{2}\sigma_{-1}\frac{1}{\sqrt{r_{-1} - 1}} \tag{12}$$

σ_{-1} and $\tau_{-1}$are fully reversed bending and fully reversed torsion endurance limits respectively and r_{-1} is the ratio (σ_{-1}/τ_{-1}). For fully reversed bending and repeated bending, k can be formulated as

$$k = \frac{0.5(1 - r_0^{\,2})}{\sqrt{r_0(5r_0 - 2 - 2r_0^{\,2})}} \tag{13}$$

where r_0 is the ratio (σ_0/σ_{-1}) in which σ_0 is repeated bending endurance limit. . If experimental data is not available for repeated bending, σ_0 can be obtained by Smith, Watson and Topper (SWT) formulation mentioned in Papuga (2005). Therefore, critical plane analysis can be carried out with only one uniaxial test data for this second k calibration method.

4.4. Matake Criterion

Matake proposed a damage parameter similar to Findley criterion. The only difference in the proposal is the critical plane definition. Matake defines the critical plane as the material plane that maximizes shear stress amplitude. Matake damage parameter is as follows,

$$\max_{\theta,\varphi}(\tau_a) + k\sigma_{n,\max} = f \tag{14}$$

Same notation is used for material parameters as in Findley criterion. However, k and f takes different values due to distinction of critical plane definitions of Findley and Matake criterion. Material parameters k and f, for fully reversed bending and torsion can be obtained as

$$k = \frac{2 - r_{-1}}{r_{-1}};\ f = \tau_{-1} \tag{15}$$

For fully reversed bending and repeated bending, k and f can be formulated as

$$k = \frac{r_0 - 1}{1 - 2r_0};\ f = \frac{1}{2}\sigma_{-1}\frac{r_0}{2r_0 - 1} \tag{16}$$

where again, like in Findley method, r_0 is the ratio (σ_0/σ_{-1}).

5. Results and Discussion

For evaluation of equivalent stress and critical plane methods, experimental data obtained from references are used and fatigue index error (FIE) is introduced. FIE shows the deviation of equivalent stress or damage parameter from experimental endurance limits. For equivalent stress criterion FIE is as follows:

$$FIE(\%) = \frac{\sigma_{a,eq} - \sigma_{-1}}{\sigma_{-1}} * 100 \tag{17}$$

where $\sigma_{a,eq}$ is the alternating value of equivalent stress after mean stress correction and f_{-1} is the fully reversed axial fatigue endurance limit. For critical plane methods FIE can be expressed as

$$FIE(\%) = \frac{DP - f}{f} * 100 \tag{18}$$

where DP is the damage parameter calculated according to related critical plane criterion and f is the material parameter obtained from experimental endurance limits. A negative value of FIE means that the criterion predicts no failure; although it actually occurred in the experiment. Therefore, such estimation is evaluated as non-conservative while opposite is true for positive values of FIE.

For this study, special attention is given to phase effect and 57 test data, which are resulted from constant amplitude proportional and non-proportional bending-torsion loading, conducted on un-notched smooth specimens reported by Papuga (2005) are gathered.

A statistical analysis is carried out to identify the trends of equivalent stress and critical plane methods. Table 1 shows the mean, alternating and standard deviation values of FIE calculated from the experimental data set for each method. For critical plane methods, calculations are executed with a combination of different shear stress amplitude methods and material parameter k calibrations. As seen from Table 1, critical plane methods, Findley and Matake with shear stress amplitude calculation method MRH and k calibration with fully reversed bending and torsion endurance limits gave the best overall results with positive mean values and low standard deviations. On the other hand, equivalent stress methods overestimate the fatigue life which is evident from negative mean values. Absolute Maximum Principal is the worst method in this study with a mean value of -15.84 and standard deviation of 12.48. Although for both Findley and Matake, changing the shear stress amplitude method or k calibration worsens the results, Matake method is much more sensitive to such changes. As can be seen from Table 1, for Matake, using different calibration decreases mean value to -2.09 and increases standard deviation to 8.50 meanwhile using MCC decreases mean to a value of -1.31 and increases the standard deviation up to 9.39.

Table 1. Fatigue Index Errors Calculated from Experimental Data

HCF Method	Absolute Maximum Principal	Signed Von Mises	Findley			Matake		
τ_a Calculation Method	-	-	MRH	MRH	MCC	MRH	MRH	MCC
k Calibration Method	-	-	σ_{-1}, τ_{-1}	σ_{-1}, σ_0	σ_{-1}, τ_{-1}	σ_{-1}, τ_{-1}	σ_{-1}, σ_0	σ_{-1}, τ_{-1}
Mean	-15.84	-3.15	3.15	0.65	2.01	3.48	-2.09	-1.31
Range	43.94	45.01	22.98	23.49	27.45	24.77	40.71	48.77
Standard Deviation	12.48	10.56	5.09	5.28	5.97	5.53	8.50	9.39

6. Conclusion

Comparison of several equivalent stress and critical plane methods is made in the current study. For this purpose experimental data which only focus on phase effect is obtained from literature. Absolute Maximum Principal and Signed von Mises criterion are selected as equivalent stress methods since they are used commonly in industry due to their simplicity and speed. For critical plane criteria Findley and Matake methods are found to be appropriate for comparison as they both have the same formulation but with different definitions of critical plane. Furthermore, effect of various shear stress amplitude calculation methods is investigated by implementing the most popular and accepted method MCC and one of the newly proposed method MRH. Since finding torsional test data is a difficult task for most of the engineering materials, in addition to classical calibration type of material parameter k, another calibration which only requires uniaxial bending or tension-compression endurance limits for different R ratios is investigated.

- Both Absolute Maximum Principal and Signed Von Mises methods tend to give results either highly non-conservative or highly scattered for the experimental data set studied. This is mainly due to the instantaneous signing procedure which cannot take into account all the effects of real loading scenario.
- Critical plane methods, Findley and Matake correlated fatigue data quite well with positive mean values and small scatter in fatigue index errors.
- Results obtained with different shear stress amplitude calculation methods or calibration of k worsen the results obtained by Findley method; however, FIE (%) mean values are still positive and a small increase in standard deviation is observed. Therefore, any combination can be used for Findley but the best results are obtained with MRH and calibration of k with fully reversed bending and torsion endurance limits.
- Matake method is excessively sensitive to shear stress amplitude calculation method and calibration of material coefficient k as results are shifted to non-conservative side with almost doubled standard deviation of FIE (%). This behavior is due to the definition of critical plane which is defined as the material plane with highest shear stress amplitude.

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