

# Light cone QCD sum rule analysis of $B \rightarrow K\ell^+\ell^-$ decay

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## Abstract

We calculate the transition formfactors for the  $B \rightarrow K\ell^+\ell^-$  decay in the framework of the light cone QCD sum rules. The invariant dilepton mass distribution and the final lepton longitudinal polarization asymmetry are investigated. The comparison analysis of our results with traditional sum rules method predictions on the formfactors is performed.

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# 1 Introduction

Experimental observation [1] of the inclusive and exclusive radiative decays  $B \rightarrow X_s \gamma$  and  $B \rightarrow K \gamma$  stimulated the study of rare  $B$  decays on a new footing. These Flavor Changing Neutral Current (FCNC)  $b \rightarrow s$  transitions in the Standard Model (SM) do not occur at the tree level and appear only at the loop level. Therefore the study of these rare  $B$ -meson decays can provide a means of testing the detailed structure of the SM at the loop level. These decays are also very useful for extracting the values of the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements [2], as well as for establishing new physics beyond the SM [3].

Currently, the main interest on rare  $B$ -meson decays is focused on decays for which the SM predicts large branching ratios and can be potentially measurable in the near future. The rare  $B \rightarrow K \ell^+ \ell^-$  and  $B \rightarrow K^* \ell^+ \ell^-$  decays are such decays. The experimental situation for these decays is very promising [4], with  $e^+ e^-$  and hadron colliders focusing only on the observation of exclusive modes with  $\ell = e, \mu$  and  $\tau$  final states, respectively. At quark level the process  $b \rightarrow s \ell^+ \ell^-$  takes place via electromagnetic and  $Z$  penguin and  $W$  box diagrams and are described by three independent Wilson coefficients  $C_7, C_9$  and  $C_{10}$ . Investigations allow us to study different structures, described by the above mentioned Wilson coefficients. In the SM, the measurement of the forward-backward asymmetry and invariant dilepton mass distribution in  $b \rightarrow q \ell^+ \ell^-$  ( $q = s, d$ ) provide information on the short distance contributions dominated by the top quark loops and are essential in separating the short distance FCNC process from the contributing long distance effects [5] and also are very sensitive to the contributions from new physics [6]. Recently it has been emphasized by Hewett [7] that the longitudinal lepton polarization, which is another parity violating observable, is also an important asymmetry and that the lepton polarization in  $b \rightarrow s \ell^+ \ell^-$  will be measurable with the high statistics available at the B-factories currently under construction. However, in calculating the Branching ratios and other observables in hadron level, i.e. for  $B \rightarrow K \ell^+ \ell^-$  decay, we have the problem of computing the matrix element of the effective Hamiltonian,  $\mathcal{H}_{eff}$ , between the states  $B$  and  $K$ . This problem is related to the non-perturbative sector of QCD.

These matrix elements, in the framework of different approaches such as chiral theory [8], three point QCD sum rules method [9], relativistic quark model by the light-front formalism [10, 11], have been investigated. The aim of this work is the calculation of these matrix elements in light cone QCD sum rules method and to study the lepton polarization asymmetry for the exclusive  $B \rightarrow K \ell^+ \ell^-$  decays.

The effective Hamiltonian for the  $b \rightarrow s \ell^+ \ell^-$  decay, including QCD corrections [12–14] can be written as

$$\mathcal{H}_{eff} = \frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} C_i(\mu) O_i(\mu), \quad (1)$$

which is evolved from the electroweak scale down to  $\mu \sim m_b$  by the renormalization group equations. Here  $V_{ij}$  represent the relevant CKM matrix elements, and  $O_i$  are a complete set of renormalized dimension 5 and 6 operators involving light fields which govern the  $b \rightarrow s$  transitions and  $C_i(\mu)$  are the Wilson coefficients for the corresponding operators. The explicit forms of  $C_i(\mu)$  and  $O_i(\mu)$  can be found in [12–14]. For  $b \rightarrow s \ell^+ \ell^-$  decay, this

effective Hamiltonian leads to the matrix element

$$\mathcal{M} = \frac{G_F \alpha}{\sqrt{2\pi}} V_{tb} V_{ts}^* \left[ C_9^{eff} \bar{s} \gamma_\mu L b \bar{\ell} \gamma^\mu \ell + C_{10} \bar{s} \gamma_\mu L b \bar{\ell} \gamma^\mu \gamma_5 \ell - 2 \frac{C_7}{p^2} \bar{s} i \sigma_{\mu\nu} p^\nu (m_b R + m_s L) b \bar{\ell} \gamma^\mu \ell \right], \quad (2)$$

where  $p^2$  is the invariant dilepton mass, and  $L(R) = [1 - (+)\gamma_5]/2$  are the projection operators. The coefficient  $C_9^{eff}(\mu, p^2) \equiv C_9(\mu) + Y(\mu, p^2)$ , where the function  $Y$  contains the contributions from the one loop matrix element of the four-quark operators, can be found in [12–14]. Note that the function  $Y(\mu, p^2)$  contains both real and imaginary parts (the imaginary part arises when the  $c$ -quark in the loop is on the mass shell).

The  $B \rightarrow K \ell^+ \ell^-$  decay also receives large long distance contributions from the cascade process  $B \rightarrow K J/\psi(\psi') \rightarrow K \ell^+ \ell^-$ . These contributions are taken into account by introducing a Breit-Wigner form of the resonance propagator and this procedure leads to an additional contribution to  $C_9^{eff}$  of the form [15]

$$- \frac{3\pi}{\alpha^2} \sum_{V=J/\psi, \psi'} \frac{m_V \Gamma(V \rightarrow \ell^+ \ell^-)}{(q^2 - m_V^2) - i m_V \Gamma_V}.$$

As we noted earlier, in order to calculate the branching ratios for the exclusive  $B \rightarrow K \ell^+ \ell^-$  decays, the matrix elements  $\langle K | \bar{s} \gamma_\mu (1 - \gamma_5) q | B \rangle$  and  $\langle K | \bar{s} i \sigma_{\mu\nu} p^\nu (1 + \gamma_5) q | B \rangle$  must be calculated. These matrix elements can be parametrized in terms of the formfactors as follows (see also [9]):

$$\langle K(q) | \bar{s} \gamma_\mu (1 - \gamma_5) q | B(p+q) \rangle = 2q_\mu f^+(p^2) + [f^+(p^2) + f^-(p^2)] p_\mu, \quad (3)$$

$$\langle K(q) | \bar{s} i \sigma_{\mu\nu} p^\nu (1 + \gamma_5) q | B(p+q) \rangle = [P_\mu p^2 - p_\mu (Pp)] \frac{f_T(p^2)}{m_B + m_K}, \quad (4)$$

where  $p+q$  and  $q$  are the momentum of  $B$  and  $K$  and  $P_\mu = (p+2q)_\mu$ .

## 2 Sum rules for the $B \rightarrow K$ transition formfactors

According to the QCD sum rules ideology, in order to calculate these formfactors, we start by considering the representation of a suitable correlator function in hadron and quark–gluon languages. For this purpose, we consider the following matrix elements of the  $T$ -product of two currents between the vacuum state and the  $K$ -meson:

$$\Pi_\mu^{(1)}(p, q) = i \int d^4x e^{ipx} \langle K(q) | T \{ \bar{s}(x) \gamma_\mu (1 - \gamma_5) b(x) \bar{b}(0) i \gamma_5 q(0) \} | 0 \rangle, \quad (5)$$

$$\Pi_\mu^{(2)}(p, q) = i \int d^4x e^{ipx} \langle K(q) | T \{ \bar{s}(x) i \sigma_{\mu\nu} p^\nu (1 + \gamma_5) b(x) \bar{b}(0) i \gamma_5 q(0) \} | 0 \rangle, \quad (6)$$

where  $q$  is  $K$ -meson momentum and  $p$  is the transfer momentum.

The hadronic (physical) part of eqs.(5) and (6) is obtained by inserting a complete set of states including the  $B$ -meson ground state, and higher states with  $B$ -meson quantum number:

$$\begin{aligned}
\Pi_\mu^{(1)}(p, q) &= \\
&= \langle K(q) | \bar{s}(x) \gamma_\mu (1 - \gamma_5) b(x) | B(p+q) \rangle \frac{1}{m_B^2 - (p+q)^2} \langle B(p+q) | \bar{b}(0) i \gamma_5 q(0) | 0 \rangle + \\
&\quad + \sum_h \langle K(q) | \bar{s}(x) \gamma_\mu (1 - \gamma_5) b(x) | h(p+q) \rangle \frac{1}{m_B^2 - (p+q)^2} \langle h(p+q) | \bar{b}(0) i \gamma_5 q(0) | 0 \rangle \\
&= F_1(p^2, (p+q)^2) q_\mu + F_2(p^2, (p+q)^2) p_\mu , \tag{7}
\end{aligned}$$

$$\begin{aligned}
\Pi_\mu^{(2)}(p, q) &= \\
&= \langle K(q) | \bar{s}(x) i \sigma_{\mu\nu} p^\nu (1 + \gamma_5) b(x) | B(p+q) \rangle \frac{1}{m_B^2 - (p+q)^2} \langle B(p+q) | \bar{b}(0) i \gamma_5 q(0) | 0 \rangle + \\
&\quad + \sum_h \langle K(q) | \bar{s}(x) i \sigma_{\mu\nu} p^\nu (1 + \gamma_5) b(x) | h(p+q) \rangle \frac{1}{m_B^2 - (p+q)^2} \langle h(p+q) | \bar{b}(0) i \gamma_5 q(0) | 0 \rangle \\
&= F_3((p+q)^2, p^2) [P_\mu p^2 - p_\mu (Pp)] , \tag{8}
\end{aligned}$$

where  $P_\mu = (p+2q)_\mu$ . Then, for the invariant amplitudes  $F_i$ , one can write a general dispersion relation in the  $B$  meson momentum squared,  $(p+q)^2$ , as:

$$F_i(p^2, (p+q)^2) = \int_{m_b^2}^{\infty} \frac{\rho_i(p^2, s) ds}{s - (p+q)^2} , \tag{9}$$

where the spectral densities are given by

$$\rho_1(p^2, s) = \delta(s - m_B^2) 2f^+(p^2) \frac{m_B^2 f_B}{m_b} + \rho_1^h(p^2, s) , \tag{10}$$

$$\rho_2(p^2, s) = \delta(s - m_B^2) [f^+(p^2) + f^-(p^2)] \frac{m_B^2 f_B}{m_b} + \rho_2^h(p^2, s) , \tag{11}$$

$$\rho_3(p^2, s) = \delta(s - m_B^2) \frac{f_T(p^2)}{m_B + m_K} \frac{m_B^2 f_B}{m_b} + \rho_3^h(p^2, s) . \tag{12}$$

The first terms in eqs.(10), (11) and (12) represent the ground state  $B$ -meson contribution and follow from eqs.(5) and (6) by inserting the matrix elements given in eqs.(3), (4) and by replacing

$$\langle B | \bar{b} i \gamma_5 q | 0 \rangle = \frac{f_B m_B^2}{m_b} , \tag{13}$$

in which we neglect the mass of the light quarks. In eqs.(10)-(12),  $\rho_i^h(p^2, s)$  represent the spectral density of the higher resonances and of the continuum of states. In accordance with the QCD sum rules method we invoke the quark-hadron duality prescription and replace the spectral density  $\rho_i^h$  by

$$\rho_i^h(p^2, s) = \frac{1}{\pi} \text{Im} F_i^{QCD}(p^2, s) \Theta(s - s_0) , \tag{14}$$

where  $\text{Im}F_i^{QCD}(p^2, s)$  is obtained from the imaginary part of the correlator functions calculated in QCD starting from some threshold  $s_0$ . In order to suppress the higher states and continuum contributions, we follow the standard procedure in the QCD sum rules method and apply Borel transformation  $\hat{B}$  on the variable  $(p+q)^2$  to the dispersion integral to get,

$$\begin{aligned} F_i(p^2, M^2) &= \hat{B}F_i(p^2, (p+q)^2) \\ &= \int_{m_b^2}^{\infty} \rho_i(p^2, s) e^{-s/M^2} ds . \end{aligned} \quad (15)$$

Using eqs.(10)-(12) and (14) we get

$$F_1(p^2, M^2) = 2f^+(p^2) \frac{m_B^2 f_B}{m_b} e^{-m_B^2/M^2} + \frac{1}{\pi} \int_{s_0}^{\infty} \text{Im}F_1^{QCD}(p^2, s) e^{-s/M^2} ds , \quad (16)$$

$$\begin{aligned} F_2(p^2, M^2) &= [f^+(p^2) + f^-(p^2)] \frac{m_B^2 f_B}{m_b} e^{-m_B^2/M^2} + \\ &+ \frac{1}{\pi} \int_{s_0}^{\infty} \text{Im}F_2^{QCD}(p^2, s) e^{-s/M^2} ds , \end{aligned} \quad (17)$$

$$F_3(p^2, M^2) = \frac{f_T(p^2)}{M_B + m_K} \frac{m_B^2 f_B}{m_b} e^{-m_B^2/M^2} + \frac{1}{\pi} \int_{s_0}^{\infty} \text{Im}F_3^{QCD}(p^2, s) e^{-s/M^2} ds . \quad (18)$$

The main problem is then the calculation of the correlator functions (5) and (6) in QCD. After applying the Borel transformation, the result can be written in the following form:

$$F_i(p^2, M^2) = \frac{1}{\pi} \int_{m_b^2}^{\infty} \text{Im}F_i^{QCD}(p^2, s) e^{-s/M^2} ds . \quad (19)$$

Equating the  $F_i$  in eq.(19) to the corresponding  $F_i$  in eqs.(16)-(18), we arrive at the sum rules for the formfactors, which describe the  $B \rightarrow K$  transition:

$$f^+(p^2) = \frac{m_b}{2\pi f_B m_B^2} \int_{m_b^2}^{s_0} \text{Im}F_1^{QCD}(p^2, s) e^{-(s-m_B^2)/M^2} ds , \quad (20)$$

$$f^+(p^2) + f^-(p^2) = \frac{m_b}{\pi f_B m_B^2} \int_{m_b^2}^{s_0} \text{Im}F_2^{QCD}(p^2, s) e^{-(s-m_B^2)/M^2} ds , \quad (21)$$

$$f_T(p^2) = \frac{m_b}{\pi f_B m_B^2} (m_B + m_K) \int_{m_b^2}^{s_0} \text{Im}F_3^{QCD}(p^2, s) e^{-(s-m_B^2)/M^2} ds . \quad (22)$$

One can calculate  $\text{Im}F_i^{QCD}(p^2, s)$ , in the deep Euclidean region, where both  $p^2$  and  $(p+q)^2$  are negative and large. The leading contribution to the operator product expansion comes from the contraction of the  $b$ -quark operators expressed in eqs.(5) and (6) to the free  $b$ -quark propagator

$$\langle 0|T\{b(x)\bar{b}(0)\}|0\rangle = -i \int \frac{d^4k}{(2\pi)^4} e^{-ikx} \frac{\not{k} + m_b}{m_b^2 - k^2} .$$

Then we have

$$\begin{aligned} \Pi_\mu^{(1)}(p, q) &= i \int \frac{d^4x d^4k}{(2\pi)^4(m_b^2 - k^2)} e^{i(p-k)x} \times \\ &\times \langle K(q) | [-m_b \bar{s} \gamma_\mu (1 - \gamma_5) q + \bar{s} \gamma_\mu \not{k} (1 + \gamma_5) q] | 0 \rangle, \end{aligned} \quad (23)$$

$$\begin{aligned} \Pi_\mu^{(2)}(p, q) &= - \int \frac{d^4x d^4k}{(2\pi)^4(m_b^2 - k^2)} e^{i(p-k)x} p^\nu \times \\ &\times \langle K(q) | [-\bar{s} \sigma_{\mu\nu} \not{k} (1 - \gamma_5) q + m_b \bar{s} \sigma_{\mu\nu} (1 + \gamma_5) q] | 0 \rangle. \end{aligned} \quad (24)$$

Note that, as can be seen from eqs.(23) and (24), the problem is reduced to the calculation of the matrix elements of the gauge-invariant, nonlocal operators sandwiched in between the vacuum and the  $K$  meson states. These matrix elements define the  $K$  meson light cone wave functions. Following [16–17], we define the  $K$ -meson wave functions as:

$$\begin{aligned} \langle K(q) | \bar{s}(x) \gamma_\mu (1 - \gamma_5) q(0) | 0 \rangle &= i q_\mu f_K \int_0^1 du e^{iuqx} [\varphi_K(u) + x^2 g_1(u)] - \\ &- f_K \left( x_\mu - \frac{x^2 q_\mu}{qx} \right) \int_0^1 du e^{iuqx} g_2(u). \end{aligned} \quad (25)$$

In eq.(25)  $\varphi_K(u)$  is the leading twist two,  $g_1(u)$  and  $g_2(u)$  are the twist four  $K$  meson wave functions, respectively. The second matrix element of eq.(23), can be split into two matrix elements using the identity  $\gamma_\mu \gamma_\nu = g_{\mu\nu} - i \sigma_{\mu\nu}$ , and the result can be evaluated using the twist three wave functions defined as [16–19]:

$$\langle K(q) | \bar{s}(x) i \gamma_5 q(0) | 0 \rangle = \frac{f_K m_K^2}{m_s + m_q} \int_0^1 du e^{iuqx} \varphi_p(u), \quad (26)$$

$$\langle K(q) | \bar{s}(x) \sigma_{\mu\nu} (1 + \gamma_5) q(0) | 0 \rangle = i (q_\mu x_\nu - q_\nu x_\mu) \frac{f_K m_K^2}{6(m_s + m_q)} \int_0^1 du e^{iuqx} \varphi_\sigma(u). \quad (27)$$

The matrix elements in eq.(24) can be easily calculated using the identities,

$$\begin{aligned} \sigma_{\mu\nu} &= -\frac{i}{2} \epsilon_{\mu\nu\rho\beta} \sigma^{\rho\beta} \gamma_5 \\ \sigma_{\mu\nu} \gamma_\rho &= i(\gamma_\mu g_{\rho\nu} - \gamma_\nu g_{\rho\mu}) + \epsilon_{\mu\nu\rho\beta} \gamma^\beta \gamma_5, \end{aligned}$$

to express it in terms of the wave functions in eq.(25).

Substituting the matrix elements (25–27) into the eqs.(23) and (24) and integrating over the variables  $x$  and  $k$  we get the following expressions for  $F_i(p^2, (p+q)^2)$ :

$$\begin{aligned} F_1(p^2, (p+q)^2) &= m_b f_K \int_0^1 \frac{du}{\Delta} \left[ \varphi_K(u) - \frac{8m_b^2 [g_1(u) + G_2(u)]}{\Delta^2} + \frac{2u g_2(u)}{\Delta} \right] + \\ &+ f_K \mu_K \int_0^1 \frac{du}{\Delta} \left[ u \varphi_\rho(u) + \frac{1}{6} \varphi_\sigma(u) \left( 1 + \frac{2m_b^2 - 2upq - 2q^2 u^2}{\Delta} \right) \right], \end{aligned} \quad (28)$$

$$F_2(p^2, (p+q)^2) = m_b f_K \int_0^1 \frac{du}{\Delta} \left\{ \left[ \frac{2g_2(u)}{\Delta} + \frac{\mu_K}{m_b} \left( \varphi_p(u) - \frac{(2pq + 2q^2u)}{6\Delta} \varphi_\sigma(u) \right) \right] \right\}, \quad (29)$$

$$F_3(p^2, (p+q)^2) = -f_K \int_0^1 \frac{du}{\Delta} \left\{ \frac{1}{2} \left[ \varphi_K(u) - \frac{4[g_1(u) + G_2(u)]}{\Delta} \left( 1 + \frac{2m_b^2}{\Delta} \right) \right] + m_b \mu_K \frac{\varphi_\sigma(u)}{6\Delta} \right\}, \quad (30)$$

where

$$q^2 = m_K^2, \quad \Delta = m_b^2 - (p+qu)^2, \quad \mu_K = \frac{m_K^2}{m_s + m_q}, \quad G_2(u) = -\int_0^u g_2(v)dv.$$

There are also contributions to the above considered wave functions from multi-particle meson wave functions. Here we consider only the operator  $\bar{q}Gq$ , which gives the main contribution and corresponds to the quark–antiquark–gluon components in the kaon (for more detail see [20]). In this approximation, the  $b$ -quark propagator is defined as (see [18]):

$$\begin{aligned} \langle 0 | T \{ b(x) \bar{b}(0) \} | 0 \rangle &= iS_b^0(x) - ig_s \int \frac{d^4k}{(2\pi)^4} e^{-ikx} \times \\ &\times \int_0^1 du \left[ \frac{1}{2} \frac{\not{k} + m_b}{(m_b^2 - k^2)^2} G^{\mu\nu}(ux) \sigma_{\mu\nu} + \frac{1}{m_b^2 - k^2} ux_\mu G^{\mu\nu}(ux) \gamma_\nu \right]. \end{aligned} \quad (31)$$

It is clear that when we take into account the second term in the right hand side of eq.(31), new matrix elements appear. Substituting eq.(31) into eqs.(5) and (6), and using the identity

$$\begin{aligned} \gamma_\mu \gamma_\nu \sigma_{\rho\lambda} &= (\sigma_{\mu\lambda} g_{\nu\rho} - \sigma_{\mu\rho} g_{\nu\lambda}) + i(g_{\mu\lambda} g_{\nu\rho} - g_{\mu\rho} g_{\nu\lambda}) - \\ &- \epsilon_{\mu\nu\rho\lambda} \gamma_5 - i\epsilon_{\nu\rho\lambda\alpha} g^{\alpha\beta} \sigma_{\mu\beta} \gamma_5, \end{aligned}$$

one can express the resulting new matrix elements in terms of the three particle wave functions [16–20]:

$$\begin{aligned} \langle K(q) | \bar{s}(x) g_s G_{\mu\nu}(ux) \sigma_{\alpha\beta} \gamma_5 q(0) | 0 \rangle &= if_{3K} [(q_\mu q_\alpha g_{\nu\beta} - q_\nu q_\alpha g_{\mu\beta}) - (q_\mu q_\beta g_{\nu\alpha} - q_\nu q_\beta g_{\mu\alpha})] \times \\ &\times \int \mathcal{D}\alpha_i \varphi_{3K}(\alpha_i) e^{iqx}, \end{aligned} \quad (32)$$

$$\begin{aligned} \langle K(q) | \bar{s}(x) \gamma_\mu \gamma_5 g_s G_{\alpha\beta}(ux) q(0) | 0 \rangle &= f_K \left[ q_\beta \left( g_{\alpha\mu} - \frac{x_\alpha q_\mu}{qx} \right) - q_\alpha \left( g_{\beta\mu} - \frac{x_\beta q_\mu}{qx} \right) \right] \times \\ &\times \int \mathcal{D}\alpha_i \varphi_\perp(\alpha_i) e^{iqx} + \\ &+ f_K \frac{q_\mu}{qx} (q_\alpha x_\beta - q_\beta x_\alpha) \int \mathcal{D}\alpha_i \varphi_\parallel(\alpha_i) e^{iqx}, \end{aligned} \quad (33)$$

$$\begin{aligned}
\langle K(q)|\bar{s}(x)\gamma_\mu g_s \tilde{G}_{\alpha\beta}(ux)q(0)|0\rangle &= if_K \left[ q_\beta \left( g_{\alpha\mu} - \frac{x_\alpha q_\mu}{qx} \right) - q_\alpha \left( g_{\beta\mu} - \frac{x_\beta q_\mu}{qx} \right) \right] \times \\
&\times \int \mathcal{D}\alpha_i \tilde{\varphi}_\perp(\alpha_i) e^{iqx\omega} + \\
&+ if_K \frac{q_\mu}{qx} (q_\alpha x_\beta - q_\beta x_\alpha) \int \mathcal{D}\alpha_i \tilde{\varphi}_\parallel(\alpha_i) e^{iqx\omega}, \quad (34)
\end{aligned}$$

where  $\tilde{G}_{\alpha\beta} = \frac{1}{2}\epsilon_{\alpha\beta\rho\lambda}G^{\rho\lambda}$ ,  $\mathcal{D}\alpha_i = d\alpha_1 d\alpha_2 d\alpha_3 \delta(1 - \alpha_1 - \alpha_2 - \alpha_3)$ , and  $\omega = \alpha_1 + u\alpha_3$ . Here  $\varphi_{3K}(\alpha_i)$  is a twist three wave function and the remaining functions  $\varphi_\perp$ ,  $\varphi_\parallel$ ,  $\tilde{\varphi}_\perp$  and  $\tilde{\varphi}_\parallel$  are all twist four wave functions.

When we substitute eqs.(32)–(34) into eqs.(5) and (6), perform integration over  $x$  and  $k$ , add (28), (29), (30), apply the Borel transformation over  $(p+q)^2$  and equate the obtained results to (16), (17) and (18), we get the following sum rules for the  $B \rightarrow K$  transition formfactors:

$$\begin{aligned}
f^+(p^2) &= \frac{m_b f_K}{2f_B m_B^2} e^{\frac{m_B^2}{M^2}} \left\{ \int_\delta^1 \exp\left(-\frac{m_b^2 - p^2(1-u) + q^2 u(1-u)}{uM^2}\right) \frac{du}{u} \times \right. \\
&\times \left[ m_b \left( \varphi_K(u) - \frac{8m_b^2 [g_1(u) + G_2(u)]}{2u^2 M^4} + \frac{2g_2(u)}{M^2} \right) + \right. \\
&+ \left. \mu_K \left( u\varphi_\rho(u) + \frac{\varphi_\sigma(u)}{6} \left( 2 + \frac{m_b^2 + p^2 - q^2 u^2}{uM^2} \right) \right) \right] + \\
&+ f_{3K} \int_0^1 du \int \mathcal{D}\alpha_i \theta(\omega - \delta) \exp\left(-\frac{m_b^2 - p^2(1-\omega) + q^2 \omega(1-\omega)}{\omega M^2}\right) \times \\
&\times \left[ \frac{(2u-1)\varphi_{3K}}{f_K} \frac{3q^2}{\omega M^2} + \frac{2u\varphi_{3K}}{f_K} \left( -\frac{1}{\omega^2} + \frac{m_b^2 - p^2 - q^2 \omega^2}{\omega^3 M^2} \right) + \right. \\
&+ \left. \frac{m_b}{f_{3K}} \left( \frac{2\varphi_\perp - \varphi_\parallel + 2\tilde{\varphi}_\perp - \tilde{\varphi}_\parallel}{\omega^2 M^2} + \frac{2q^2 \alpha_3 (\Psi_\perp + \Psi_\parallel + \tilde{\Psi}_\perp + \tilde{\Psi}_\parallel)}{\omega^2 M^4} \right) \right] \left. \right\}, \quad (35)
\end{aligned}$$

$$\begin{aligned}
f^+(p^2) + f^-(p^2) &= \frac{m_b f_K}{f_B m_B^2} e^{\frac{m_B^2}{M^2}} \left\{ \int_\delta^1 \exp\left(-\frac{m_b^2 - p^2(1-u) + q^2 u(1-u)}{uM^2}\right) \frac{du}{u} \times \right. \\
&\times \left[ 2m_b \frac{g_2(u)}{uM^2} + \mu_K \left( \varphi_\rho(u) + \frac{\varphi_\sigma(u)}{6u} - \frac{\varphi_\sigma(u)}{6u^2 M^2} (m_b^2 - p^2 + q^2 u^2) \right) \right] + \\
&+ \int_0^1 du \int \mathcal{D}\alpha_i \theta(\omega - \delta) \exp\left(-\frac{m_b^2 - p^2(1-\omega) + q^2 \omega(1-\omega)}{\omega M^2}\right) \frac{q^2}{\omega^2 M^2} \times \\
&\times \left[ \frac{f_{3K}(2u-3)\varphi_{3K}}{f_K} + 2m_b \alpha_3 \frac{\Psi_\perp + \Psi_\parallel + \tilde{\Psi}_\perp + \tilde{\Psi}_\parallel}{\omega M^2} \right] \left. \right\}, \quad (36)
\end{aligned}$$



$$\begin{aligned}
f_T(p^2) &= \frac{m_b(m_B + m_K)f_K}{f_B m_B^2} e^{\frac{m_B^2}{M^2}} \left\{ \int_\delta^1 du \exp\left(-\frac{m_b^2 - p^2(1-u) + q^2 u(1-u)}{uM^2}\right) \times \right. \\
&\times \left[ -\mu_K \frac{m_b \varphi_\sigma(u)}{6u^2 M^2} - \frac{1}{2} \frac{\varphi_K(u)}{u} + 2 \left( \frac{m_b^2}{uM^2} + 1 \right) \frac{[g_1(u) + G_2(u)]}{u^2 M^2} \right] + \\
&+ \int_0^1 du \int \mathcal{D}\alpha_i \theta(\omega - \delta) \exp\left(-\frac{m_b^2 - p^2(1-\omega) + q^2 \omega(1-\omega)}{\omega M^2}\right) \times \\
&\times \left. \left[ \frac{u(\varphi_\parallel - 2\tilde{\varphi}_\perp)}{\omega^2 M^2} + \frac{\varphi_\parallel + \tilde{\varphi}_\parallel - 2\varphi_\perp - 2\tilde{\varphi}_\perp}{\omega^2 M^2} \right] \right\}, \tag{37}
\end{aligned}$$

where  $\delta = \frac{m_b^2 - p^2}{s_0 - p^2}$ . The functions  $\Psi_\perp(\tilde{\Psi}_\perp)$ ,  $\Psi_\parallel(\tilde{\Psi}_\parallel)$  in eqs.(35)–(37) are defined in the following way

$$\begin{aligned}
\Psi_\perp(\tilde{\Psi}_\perp) &= - \int_0^u \varphi_\perp(v) (\tilde{\varphi}_\perp(v)) dv, \\
\Psi_\parallel(\tilde{\Psi}_\parallel) &= - \int_0^u \varphi_\parallel(v) (\tilde{\varphi}_\parallel(v)) dv.
\end{aligned}$$

Note that the formfactor  $f^+(p^2)$  is investigated in [18] and [19] (In [18], it is necessary to make the simple replacement  $\pi \rightarrow K$ ), in light cone sum rules. Our results coincide with theirs if we let  $q^2 = 0$ .

At the end of this section we calculate the differential decay rate with the longitudinal polarization of the final leptons and we obtain

$$\begin{aligned}
\frac{d\Gamma}{dp^2} &= \frac{G^2 \alpha^2}{2^{12} \pi^5} \frac{|V_{tb} V_{ts}^*|^2 v \sqrt{\lambda}}{m_B} \left\{ m_B^2 (2m_\ell^2 + m_B^2 s) (|A|^2 + |C|^2) \frac{\lambda}{3s} + \right. \\
&+ 8m_B^2 m_\ell^2 [r |C|^2 + s |D|^2 + \text{Re}(C^* D)(1 - r - s)] + \\
&\left. - \frac{2}{3} \text{Re}(A^* C) m_B^4 \xi v \lambda \right\}, \tag{38}
\end{aligned}$$

where  $\lambda = 1 + r^2 + s^2 - 2r - 2s - 2sr$ ,  $r = m_K^2/m_B^2$ ,  $s = p^2/m_B^2$ ,  $\xi$  is the longitudinal polarization of the final lepton,  $m_\ell$  and  $v = \sqrt{1 - \frac{4m_\ell^2}{p^2}}$  are its mass and velocity, respectively. In eq.(38)  $A$ ,  $C$  and  $D$  are defined as follows:

$$\begin{aligned}
A &= 2C_9^{eff} f^+ - C_7 \frac{4m_b f_T}{m_B + m_K}, \\
C &= 2C_{10} f^+, \\
D &= C_{10} (f^+ + f^-).
\end{aligned} \tag{39}$$

For the dileptonic decays of the  $B$  mesons, the longitudinal polarization asymmetry  $P_L$  of the final state  $\ell$ , is defined by

$$P_L(p^2) = \frac{\frac{d\Gamma}{dp^2}(\xi = -1) - \frac{d\Gamma}{dp^2}(\xi = 1)}{\frac{d\Gamma}{dp^2}(\xi = -1) + \frac{d\Gamma}{dp^2}(\xi = 1)}. \tag{40}$$

where  $\xi = -1(+1)$  corresponds to the left (right) handed lepton. If in eq.(40), we let  $m_\ell = 0$ , our results coincide with the results in [21] and if  $m_\ell \neq 0$ , they coincide with the ones in [11].

### 3 Numerical analysis

The main input parameters in the sum rules (35)–(37) are the kaon wave functions on the light cone. For kaon wave functions we use the results of ref. [16, 17, 19]:

$$\begin{aligned}
\varphi_K &= 6u(1-u) \left\{ 1 + 0.52 \left[ 5(2u-1)^2 - 1 \right] + 0.34 \left[ 21(2u-1)^4 - 14(2u-1)^2 + 1 \right] \right\} , \\
\varphi_p &\simeq 1 , \\
\varphi_\sigma &\simeq 6u(1-u) , \\
g_1(u) &\simeq \frac{5}{2} \delta^2 u^2 (1-u)^2 , \\
g_2(u) &\simeq \frac{10}{3} \delta^2 u(1-u)(2u-1) , \\
\varphi_{3K}(\alpha_i) &\simeq 360 \alpha_1 \alpha_2 \alpha_3^2 , \\
\varphi_\perp(\alpha_i) &\simeq 10 \delta^2 (\alpha_1 - \alpha_2) \alpha_3^2 , \\
\varphi_\parallel(\alpha_i) &\simeq 120 \delta^2 \epsilon (\alpha_1 - \alpha_2) \alpha_1 \alpha_2 \alpha_3 , \\
\tilde{\varphi}_\perp(\alpha_i) &\simeq 10 \delta^2 \alpha_3^2 (1 - \alpha_3) , \\
\tilde{\varphi}_\parallel(\alpha_i) &\simeq -40 \delta^2 \alpha_1 \alpha_2 \alpha_3 ,
\end{aligned}$$

here  $\delta^2(\mu_b) \simeq 0.17 \text{ GeV}^2$  at  $\mu_b \simeq \sqrt{m_B^2 - m_b^2} = 2.4 \text{ GeV}^2$ , which follows from the QCD sum rules analysis (for more detail see [18]),  $\epsilon(\mu_b) \simeq 0.36$ . In order to estimate  $\mu_K$ , we use the PCAC relation for the pseudo Goldstone bosons

$$\frac{\mu_K}{\mu_\pi} = \frac{(\langle qq \rangle + \langle \bar{s}s \rangle) f_K^2}{2 \langle qq \rangle f_\pi^2} \simeq 0.62 \quad (q = u \text{ or } d) .$$

Here we use  $f_\pi \simeq 133 \text{ MeV}$ ,  $f_K \simeq 160 \text{ MeV}$  and we assume  $\langle \bar{q}q \rangle \div \langle \bar{s}s \rangle = 1 \div 0.8$ , which follows from QCD sum rules for strange hadrons [22]. As a result we have

$$\mu_K \simeq 1 \text{ GeV} \text{ when } \mu_\pi = \frac{m_\pi^2}{m_u + m_d} \simeq 1.6 \text{ GeV} .$$

For the values of the other parameters, we choose:  $f_B \simeq 0.14 \text{ GeV}$ , which is obtained from 2–point QCD sum rules analysis [18, 23],  $m_b \simeq 4.7 \text{ GeV}$  and  $s_0 \simeq 35 \text{ GeV}^2$ , and  $|V_{tb} V_{ts}^*| \simeq 0.045$ .

Before giving numerical results on the formfactors, we must first determine the region for the Borel mass parameter  $M^2$ , for which the sum rules yields reliable results. The lower limit of this region is determined by the requirement that, the terms proportional to  $M^{-2n} (n > 1)$  remain subdominant. The upper limit of  $M^2$  is determined by requiring the higher resonance and continuum contributions to be less than  $\sim 30\%$  of the total result. Our numerical results show that both requirements are satisfied in the region  $8 \text{ GeV}^2 \leq M^2 \leq 16 \text{ GeV}^2$  and for the numerical analysis we use  $M^2 = 10 \text{ GeV}^2$ . When  $p^2$  approaches

the region  $m_b^2 - O(1 \text{ GeV}^2)$  a breakdown of the stability is expected, similar to the  $B \rightarrow \pi$  case (see [18, 19]).

In fig.1, we present the  $p^2$  dependence of the formfactors  $f^+(p^2)$ ,  $f^-(p^2)$  and  $f_T(p^2)$ . At zero momentum transfer, the QCD prediction for the formfactors are

$$\begin{aligned} f^+(p^2 = 0) &= 0.29 , \\ f^-(p^2 = 0) &= -0.21 , \\ f_T(p^2 = 0) &= -0.31 . \end{aligned}$$

In fig.2a(b) we present the  $p^2$  dependence of the differential Branching ratios for the  $B \rightarrow K\mu^+\mu^-$  and  $(B \rightarrow K\tau^+\tau^-)$  decay with and without long distance effects. In both cases summation over the final lepton polarization is performed. Note that  $p^2$  dependence of the differential branching ratio  $B \rightarrow K\tau^+\tau^-$  is analysed in [11] and [24] using the light front formalism and heavy meson chiral theory, respectively. In [11] both short and long distance contributions are considered while in [24] only the short distance contributions are taken into account and in both works the obtained spectrum is fully symmetric while in the present work the spectrum we obtain seems to be slightly asymmetric as a result of a highly asymmetric resonance-type behaviour due to the nonperturbative contributions. Performing the integration over  $p^2$  in eq.(38) and using the values of the life times  $\tau_{B_d} = (1.56 \pm 0.06) \times 10^{-12}$  s,  $\tau_{B_u} = (1.62 \pm 0.06) \times 10^{-12}$  s [25], for the branching ratios, including only the short distance contributions, we get:

$$\begin{aligned} B(B_d \rightarrow K^0\mu^+\mu^-) &= (3.1 \pm 0.9) \times 10^{-7} , \\ B(B_d \rightarrow K^0\tau^+\tau^-) &= (1.7 \pm 0.4) \times 10^{-7} , \\ B(B_u \rightarrow K^+\mu^+\mu^-) &= (3.2 \pm 0.8) \times 10^{-7} , \\ B(B_u \rightarrow K^+\tau^+\tau^-) &= (1.77 \pm 0.40) \times 10^{-7} , \end{aligned}$$

where theoretical and experimental errors have been added quadratically.

In fig.3 we display the lepton longitudinal polarization asymmetry  $P_L$  as a function of  $p^2$  for the  $B \rightarrow K\mu^+\mu^-$  and  $B \rightarrow K\tau^+\tau^-$  decays, at  $m_t = 176 \text{ GeV}$ , with and without the long distance contributions. From this figure one can see that  $P_L$  vanishes at the threshold due to the kinematical factor  $v$  and the value of  $P_L$  for the  $B \rightarrow K\mu^+\mu^-$  decay varies in the region  $(0 \div -0.7)$  and  $(0 \div -0.1)$  for the  $B \rightarrow K\tau^+\tau^-$  decay, when long distance effects are excluded.

## 4 Conclusion

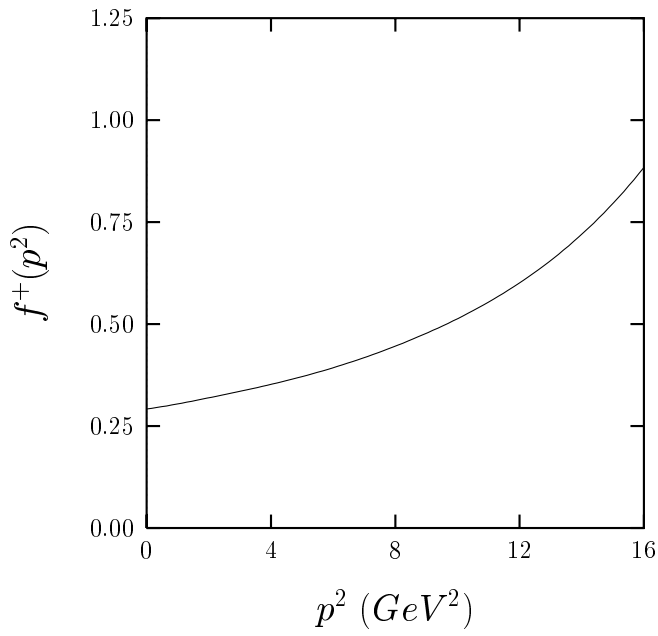
In this work, we calculate the transition formfactors for the exclusive  $B \rightarrow K\ell^+\ell^-$  ( $\ell = \mu, \tau$ ) decay in the framework of the light cone QCD sum rules, and investigate the longitudinal polarization asymmetries of the muon and tau in this decay. From a comparison of our results with the traditional QCD sum rule predictions (see [9]), we observe that the behaviour of the formfactors are similar and the value of  $f^+$  in both approaches coincides, while  $f_T$  differs two times than that of [9] at  $p^2 = 0$ . It is important to note that for a more refined analysis, it is necessary to take into account the SU(3) breaking effects: the differences

between  $f_K, \mu_K$  and  $f_\pi, \mu_\pi$  and the differences of pion and kaon wave functions. These SU(3) breaking terms can lead to differences between  $B \rightarrow \pi$  and  $B \rightarrow K$  formfactors. But we expect that these effects can change the results about 15–20% and this lies at the accuracy level of the sum rules method.

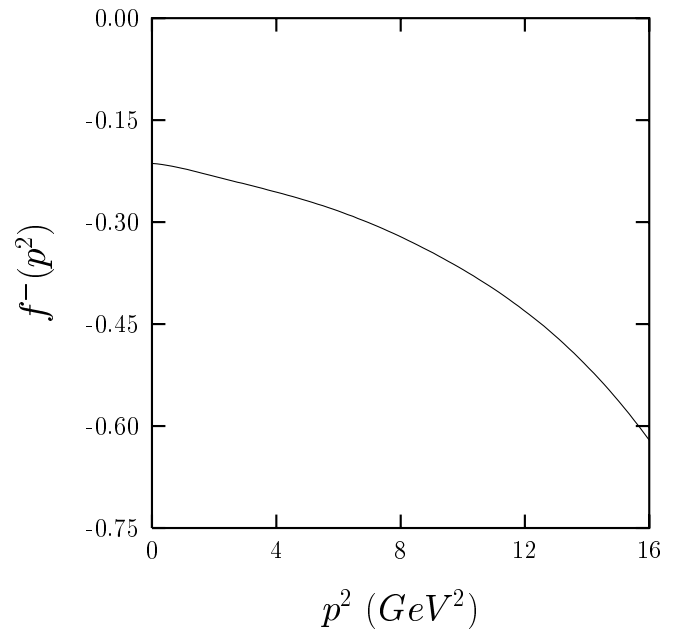
Few words about the possibility of the experimental observation of this decay are in order. Experimentally, to observe an asymmetry  $P_L$  of a decay with the branching ratio  $B$  at the  $n\sigma$  level, the required number of events is  $N = \frac{n^2}{B P_L^2}$  (see [11]). For example, to observe the  $\tau$  lepton polarization at the exclusive channel  $B \rightarrow K\tau^+\tau^-$  at the  $3\sigma$  level, one needs at least  $N \simeq 5 \times 10^9 B \bar{B}$  decays. Since in the future  $B$ -factories, it is expected that  $\sim 10^9$   $B$ -mesons would be created per year, it is possible to measure the longitudinal polarization asymmetry of the  $\tau$  lepton.

## Figure Captions

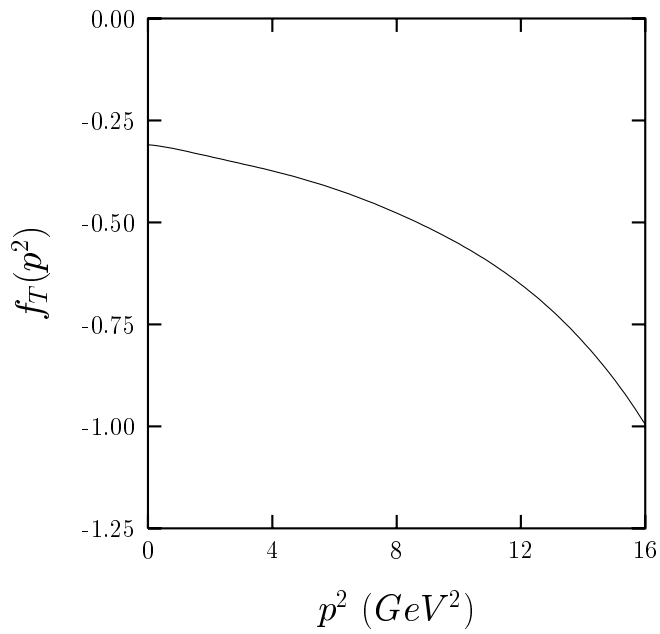
1. The  $p^2$  dependence of the formfactors  $f^+(p^2)$ ,  $f^-(p^2)$  and  $f_T(p^2)$ .
2. a) Invariant mass squared distribution of the lepton pair for the decay  $B \rightarrow K\mu^+\mu^-$ .  
b) The same as in a), but for the decay  $B \rightarrow K\tau^+\tau^-$ .  
Here and in all of the following figures the solid line corresponds to the short distance contributions only and the dashed line to the sum of both short and long distance contributions.
3. a) The longitudinal polarization asymmetry  $P_L$  for the  $B \rightarrow K\mu^+\mu^-$  decay.  
b) The same as in a), but for the  $B \rightarrow K\tau^+\tau^-$  decay.



(a)

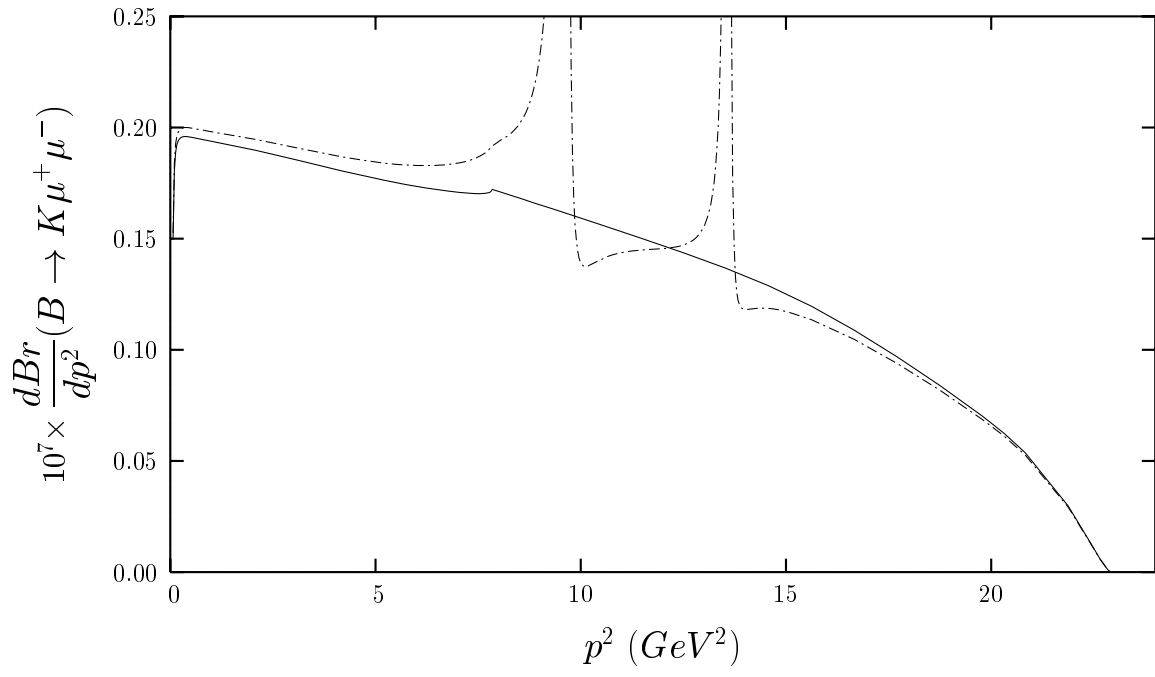


(b)

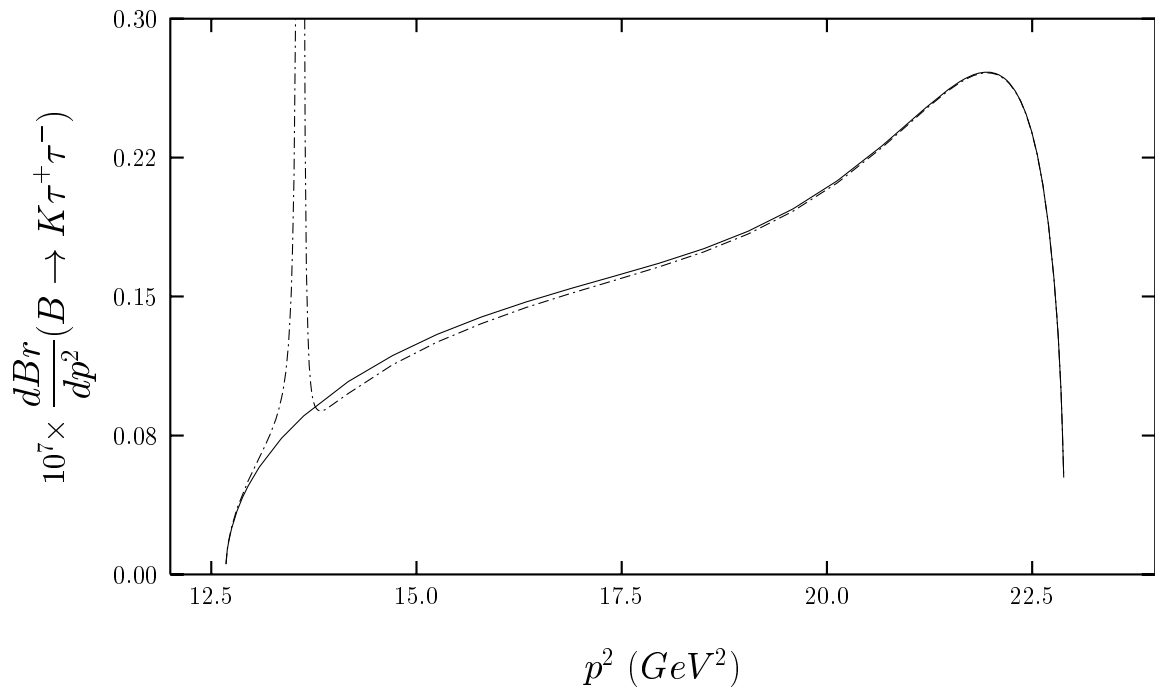


(c)

**Figure 1**

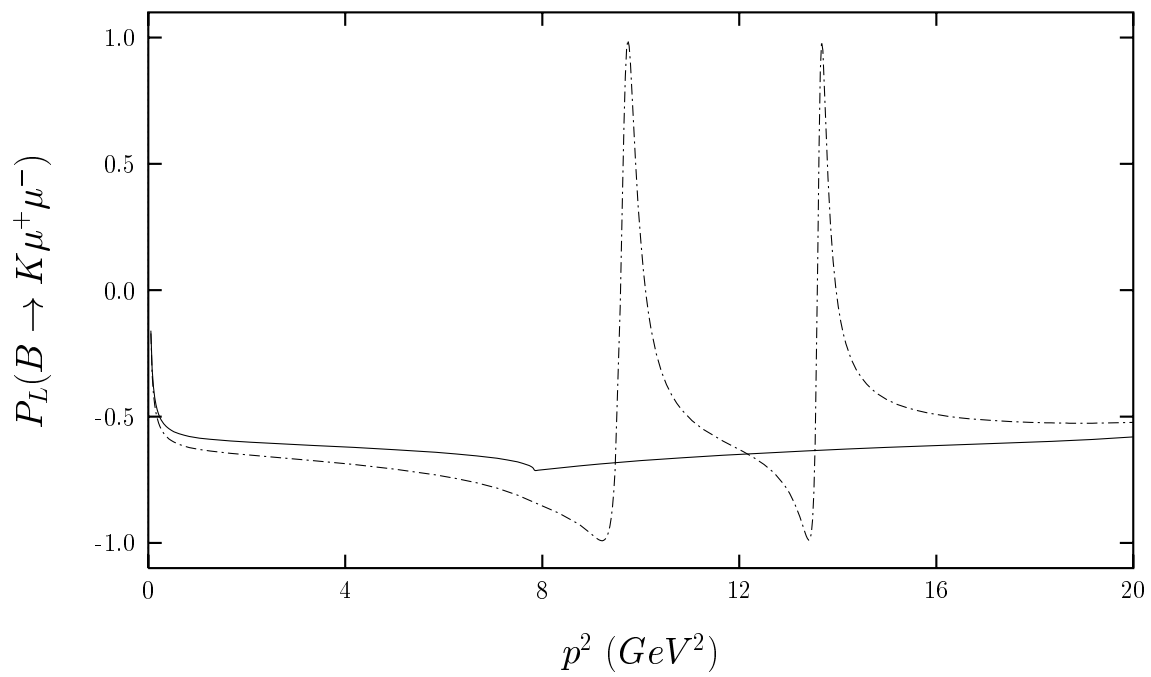


(a)

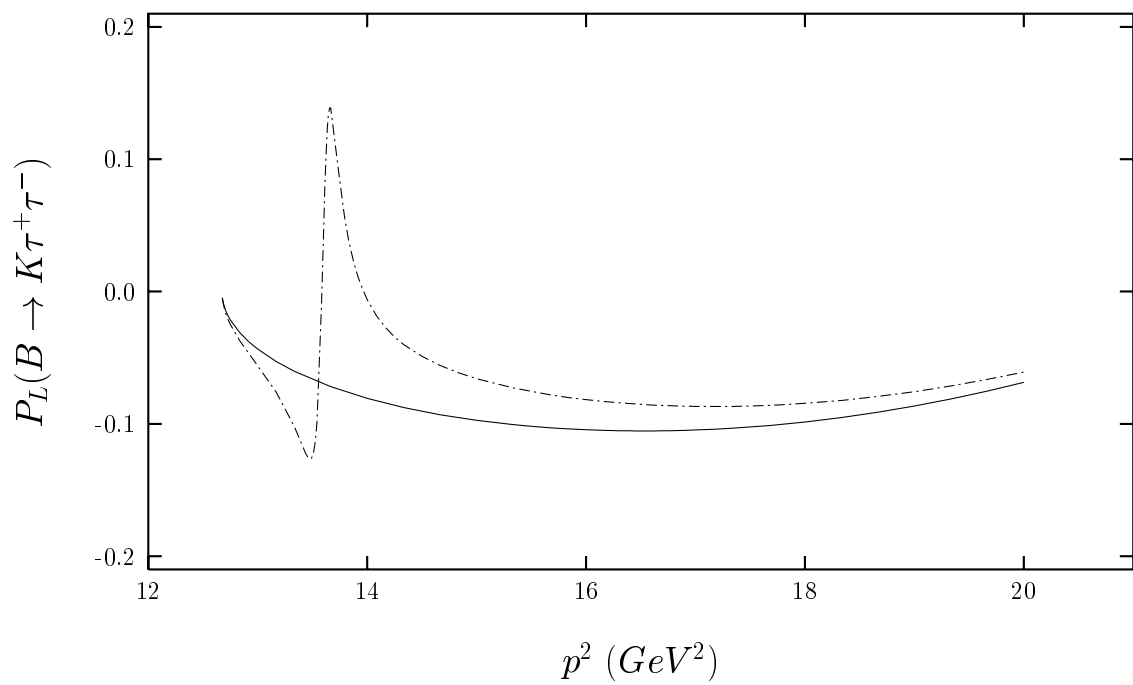


(b)

**Figure 2**



(a)



(b)

**Figure 3**



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