SOFTWARE FOR TESTING STATISTICAL HYPOTHESES

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Introduction

The theory of statistical methods of data processing is aimed at solving real problems, so it constantly creates new statements of mathematical problems of statistical data analysis, develops and justifies new methods. The relevance of testing statistical hypotheses is that the trend of data forecasting is developing. Due to the active development of information technologies, there is a growing need for software for automating statistical data processing.

Therefore, the goal of the work was to write software that would allow users to accept or reject the hypothesis.

Algorithm description

To solve the presented problem, an application was developed in the software environment for mathematical and engineering calculations - Mathcad.

Statistical tests based on hypothesis testing are used to study statistical properties.

A statistical hypothesis is a statement about the properties of the General population [1].

The null (main) hypothesis is called *H0*, and the competing (alternative) hypothesis is called that contradicts the null hypothesis and is designated *H1*.

When testing a hypothesis, two types of errors are possible: first, the hypothesis can be rejected, although it is actually true, such an error is called a first-kind error; second, the hypothesis can be accepted, although it is actually incorrect, such an error is called a second-kind error.

Let's get a sample $X (x_1, x_2,..., x_N)$ that can be generated by one of the mutually exclusive events *A0* and *A1*. Let's assume that the event *A0* corresponds to the hypothesis *H0*, and the event *A1 - H1*. It is necessary to make a decision *Z* (estimate), which of the hypotheses corresponds to the obtained sample. To do this, you need to set a certain rule (criterion) for making a decision about the hypotheses being tested. It should be noted that any experiment, strictly speaking, does not confirm or reject the hypothesis, but only contradicts or does not contradict it. Here we consider the situation of choosing two alternative hypotheses with known probability distributions of the analyzed samples. Such hypotheses are called simple hypotheses. The decision criterion must be formed in advance, before the selection is received.

Testing

To test statistical hypotheses, an experiment was conducted, the essence of which was to make a decision to identify the transmitted symbol "0" or "1" at the output of a unipolar binary signal.

In the program, the noise level (events A0) is set by the parameter σ=1 - the initial variance of the sample, and the signal (events A1) - by the parameter d. It follows that in order to provide the necessary signal-to-noise ratio, this parameter d must be equated to λ.

Under noisy conditions, the TCS system must make a decision about the transmitted symbol (which of the A0 or A1 events occurred at the transmitting end of the line) in each time slot. This is the procedure for evaluating the simple null and alternative hypotheses H0, H1 corresponding to events A0 and A1. It is clear that under these conditions it is not possible to make measurements without errors (BER). The General level of BER consists of errors of the 1st(α) and 2nd (β) kind, the level of which strongly depends on the threshold level (critical area) of "TresH". In the program, this threshold is regulated by the l tuner. The number of signal level measurements during the time slot T is set by the parameter N - sample size.

The characteristics of the digital signal are shown in the table 1.

Table 1. The characteristics of the digital signal

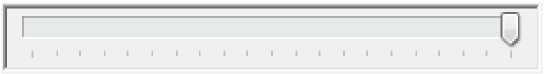
Speed B, Mbit / s	10.111
Signal-to-noise ratio, λ	1.275
Error coefficient, BER	16.8

Given that the noise variance determines its power, it is necessary to establish a test criterion for the simple two-event hypothesis considered here and determine the boundaries of the decision areas for the null and alternative hypotheses in a single measurement (a single sample size N=1). The results are illustrated with graphs.

In this experiment, we will study two sequences corresponding to "1" and "0", which have a spread of values according to the specified variance.

Using a special built-in function *trunc*, one of the two sequences was selected, relative to which the probability will be calculated to accept the correct value.

Next, you need to select the boundaries for determining decision areas, as well as select the optimal value of the threshold measuring device. In order to correctly determine the boundaries, you must select the 3σ rule, then the range boundaries will be (0-3σ; 1.275+3σ). The choice of step the threshold level will be carried out with the help of the tuner. For example, 10 discrete values were selected, as shown in figure 1.

$L :=$ 

$$TresH := \frac{L}{7.843} = 1.275$$

Fig. 1. Tuner selection threshold

Thus, if the average value of the active sample exceeds the threshold, then the H1 hypothesis corresponding to the logical "1"will be considered correct. If the value is less than the threshold, the "0" hypothesis will be considered correct.

Taking into account the settings set, the graph of the test criterion for the hypothesis of two events will take the form shown in figure 2.it should also be taken into account that for a single measurement, there can be no question of the standard deviation (RMS), since one measurement is not enough to calculate the RMS. Therefore, for a single measurement, the COE is selected equal to one.

The results of testing the algorithm are shown in figure 2.

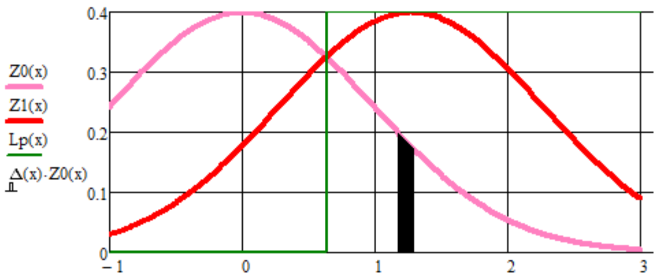


Fig. 2. Graph with the results of determining the hypothesis

Figure 2 shows two curves that correspond to the normal distribution law and define logical " 0 "and logical"1". It also follows from this graph that for a single measurement with a COE equal to one, the optimal threshold value will be a value equal to 0.6 (i.e., at the intersection of the graphs). When you select this value, you will notice that the criterion determines that the H1 hypothesis corresponding to the logical "1"is correct.

The calculation of errors for a single measurement and the optimal threshold level is shown in figure 3.

$$\alpha := \int_{TresH}^{10\sigma} Z0(x)\,dx = 0.262$$

$$\beta := \int_{-(10\sigma)}^{TresH} Z1(x)\,dx = 0.262$$

$$BER := \alpha + \beta = 0.524$$

Fig. 3. Calculation of the coefficients α, β, and BER

In statistical radar problems, when selecting signals from noise, an error of the first kind (significance level) is called the probability of a false alarm, and an error of the second kind is called the probability of missing a signal.

Now, using the l coefficient tuner, we will conduct a simulation experiment to study the dependence of the ber coefficient on the variation of the threshold level near the optimal level and make a table with the results obtained.

Table 2. The results of the simulation experiment

№	L	BER
1	0	0.5
2	0.128	0.356
3	0.256	0.231
4	0.384	0.135
5	0.512	0.072
6	0.64	0.04
7	0.768	0.039
8	0.896	0.077
9	1.024	0.166
10	1.152	0.313

After obtaining the data using the experiment, it is necessary to plot the dependence of the BER coefficient on the variation of the threshold level near the optimal level.

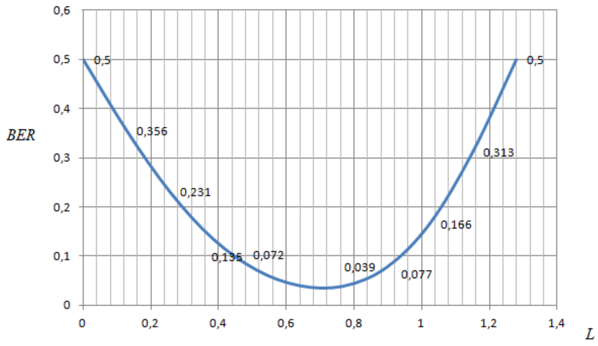


Fig. 4. Graph of ber dependence on the threshold level L

From figure 4, we can see that the minimum bit error probability falls on the threshold value of 0.64. as the sample size increases, the width of the ber minimum will become wider.

This is due to the fact that as the sample size increases, the average value becomes more accurate, which reduces the variance that determines the width of distributions.

Conclusion

As a result of the experiment, we can conclude that the algorithm for testing statistical hypotheses is workable.

During the experiment, the hypothesis of accepting the symbol "1" was found to be correct.

Examples for which this software was developed are tasks for transmitting binary code characters, pseudo-random sequences, etc.

References

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