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On the problem of application of diffracted transition radiation for indication of relativistic electron beam parameters

S. Blazhevich,[a] M. Bronnikova,[a] K. Lyushina,[a] A. Noskov[a,b,1] and R. Zagorodnyuk[a]

[a] *Belgorod State University,*
Pobedy Str., 85, Belgorod 308015, Russia

[b] *Belgorod State Technological University named after V.G. Shukhov,*
Kostyukova Str., 46, Belgorod 308012, Russia

E-mail: noskovbupk@email.com

ABSTRACT: The diffracted transition radiation (DTR) produced by a beam of relativistic electrons traversing a thin single-crystal plate in the Laue scattering geometry is considered. We have obtained the expression describing the angular density of the DTR for the case when the path length of the electron in the target is far less than the extinction length of X-rays in the crystal. It is shown that in this case the considered DTR process has the explicit kinematic character. The numerical calculations of the yield of DTR photons in the direction of Bragg scattering performed for various values of the registration solid angle show a significant influence of the electron beam divergence on the photon yield. We have arrived at a conclusion that the measured photon output of DTR radiation emitted in a given solid angle can be used for indication of the electron beam divergence. The model calculations of the electron beam divergence parameters on the base of "measured" yield of DTR photons traversing through a slit collimator are carried out. The results of the calculation show that the proposed in this work formula can be successfully used as a base for the development of methods for measuring the divergence of beams of relativistic ultrahigh-energy electrons based on DTR angular distribution.

KEYWORDS: Beam-line instrumentation (beam position and profile monitors; beam-intensity monitors; bunch length monitors); Detector modelling and simulations I (interaction of radiation with matter, interaction of photons with matter, interaction of hadrons with matter, etc); Radiation calculations

[1] Corresponding author.

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Contents

1 Introduction

While carrying out fundamental and applied experimental research with electron beams of different energies, scientists face the problem of insufficient information about the parameters of the beams. The transverse dimension and divergence are the main parameters of a beam. The main problem for physicists dealing with beams of relativistic electrons in the energy range 100–1000 MeV is to provide measurements of the transverse dimensions of the beam, since the angular divergence of the beam on modern electron accelerators is about 0.001 mrad which is an insignificant value for the electron beam having the transverse dimensions order of 10 microns or greater than that.

Nowadays, two new linear electron-positron colliders are being designed [1, 2]. In these installations, electrons and positrons will be accelerated to an energy of 250 GeV, with transverse beam dimensions assumed to be very small ($\sim$ 5–100 nm) and the main problem will be the measurement of the angular divergence of the beam. The solution of this problem will allow a more accurate interpretation of the experimental data in fundamental and applied researches.

Recently the possibilities of using parametric X-ray radiation (PXR) for the diagnostics of the transverse dimensions of beams of relativistic electrons have been experimentally investigated in [3, 4].

The influence of the divergence of the electron beam on the PXR in the crystal was experimentally investigated in Tomsk and Tokyo [5] for electron energies of 600 MeV and 800 MeV, respectively. The researchers showed that the orientation dependence of the PXR generated by relativistic electrons in the crystal is sensitive to the divergence of the beam and proposed using PXR as a simple means for determining the angular divergence of beams of charged particles of high energy. In [6], the authors suggested using parametric X-ray radiation generated in thin crystals to obtain operational information on the position and dimensions of the electron beam.

PXR in the direction of Bragg scattering is accompanied by diffracted transition radiation (DTR) [7–10] generated at the front boundary of the crystalline target.

In [11] we developed a dynamic theory of coherent X-ray radiation excited by a divergent beam of relativistic electrons traversing a single-crystal plate in the Laue scattering geometry for a general case of asymmetric reflection of the electron field relative to the target surface.

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The expressions obtained in [11] show that DTR photons emitted by electron beams with energies exceeding several hundred MeV have a smaller angular spread than those of PXR, and as a result, the angular density of the DTR becomes more sensitive to the divergence of the beam. With a further increase in the energy of the relativistic electron, the angular distribution of the DTR photons generated by the electron gets narrower, while the width of the angular distribution of the PXR photons reaches saturation and stops changing. In this case, the maximum of the PXR angular density is far beyond the angular region of DTR, in which the intensity of PXR becomes many times lower than that of DTR. Thus, DTR is the only suitable means for indicating the parameters of the ultrarelativistic electron beams with a relativistic factor $\gamma > 2000$.

To apply the DTR for the electron beam indication it is important to define which of its characteristics can be used. The use of some integral characteristics of DTR would significantly simplify the indication process. In the present paper we demonstrate the possibility of using the integral yield of DTR photons in a given solid angle. The diffracted transition radiation of relativistic electrons in a single-crystal target is studied in the Laue scattering geometry. We consider the case of a very thin target, when multiple scattering of electrons by atoms of the target is negligible. It is important as it provides the conditions for measurement with very small distortions in the measured parameters. We derived the expression describing the angular density of the DTR in the case when the path length of the electron in the target is substantially less than the extinction length. It is shown that in this case the expression has an obvious kinematic character.

The DTR at such small target thicknesses has never been considered before. Traditionally DTR was considered only for the case when the path length of the electron is much larger than the extinction length when the DTR waves undergo dynamical diffraction in a single crystal [7–11]. In those cases, the formulas used in [7–11] are naturally dynamical.

In this paper, we study the possibility of using the integral yield of the collimated DTR emitted by the beam of ultrarelativistic electrons traversing a very thin monocrystalline target for beam divergence analysis.

We assume that the contribution of PXR in the coherent radiation is negligible in these conditions and we do not consider it.

2 Geometry of the radiation process

We consider the beam of relativistic electrons traversing the crystal plate (figure 1). The interaction of each electron in the beam with the target is treated as independent. Therefore, the spectral-angular density of radiation, generated by the electron beam can be obtained by averaging the expression for the spectral-angular density of the radiation generated by a separate electron in the beam through all its possible trajectories in the target. In figure 1, $\mathbf{V}$ is the vector of electron velocity (in units of the velocity of light in free space), $\mathbf{e}_1$ is the unit vector of the electron beam axis, the unit vector $\mathbf{n}$ defines the wave vector direction of incident pseudo photon of the electron coulomb field, $\mathbf{n_g}$ is the unit vector in the direction of the photon diffraction. the unit vector $\mathbf{e}_2$ defined by the Bragg relations $\mathbf{e}_1\theta_0 = 0$ and $\mathbf{e}_1\mathbf{e}_2 = \cos 2\theta_B$. The angles ψ, θ_0 and θ define the position of the electron velocity $\mathbf{V}$and incident pseudo photon relative to the beam axis $\mathbf{e}_1$ and the direction of diffracted photon relative to the beam axis $\mathbf{e}_2$ correspondently.

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The angular variables are decomposed into components parallel and perpendicular to the plane of the figure 1: $\theta = \theta_{\parallel} + \theta_{\perp}$, $\theta_0 = \theta_{0\parallel} + \theta_{0\perp}$, $\psi = \psi_{\parallel} + \psi_{\perp}$.

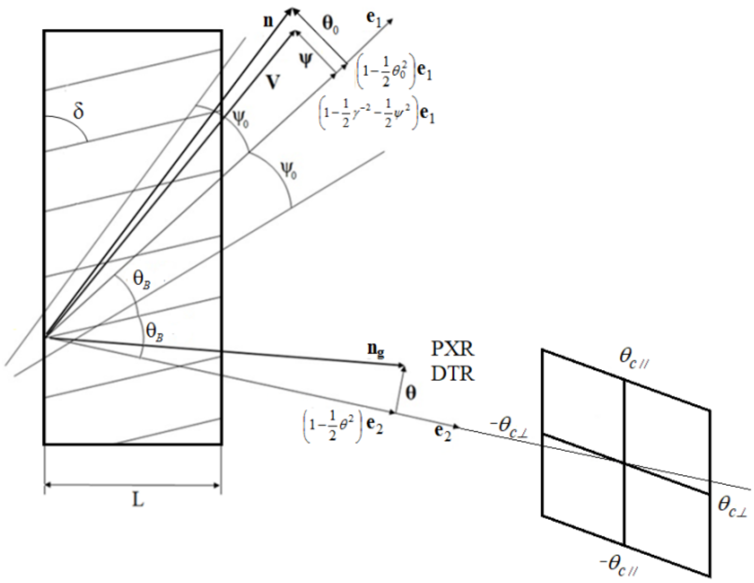

Figure 1. Geometry of the radiation process. Solid angle of DTR photon registration is $\Delta\Omega = 4\theta_{c\parallel} \cdot \theta_{c\perp}$, where $\theta_{c\parallel}$ and $\theta_{c\perp}$ are the limits of the collimator relative to the collimator center situated on the axis $\mathbf{e}_2$ (see the limits of integration in formulas (4.4)).

3 Angular density of DTR in a thin monocrystalline plate

We use the formula for spectral-angular density of DTR, obtained in our work [11] to derive the expression for DTR angular density from a thin target:

$$\frac{dN^{(s)}_{\mathrm{DTR}}}{d\Omega} = \frac{e^2 \omega_B \chi_0'^2 \chi_{\mathbf{g}}'^2 C^{(s)2}}{4\pi \sin^2 \theta_B} \times$$
$$\times \frac{\Omega^{(s)2}}{\left(\gamma^{-2} + (\theta_\perp - \psi_\perp)^2 + (\theta_\parallel + \psi_\parallel)^2 - \chi_0'\right)^2 \left(\gamma^{-2} + (\theta_\perp - \psi_\perp)^2 + (\theta_\parallel + \psi_\parallel)^2\right)^2} \varepsilon \frac{L}{\sin(\delta - \theta_B)}. \tag{3.1}$$

where $\Omega^{(1)} = \theta_\perp - \psi_\perp$, $\Omega^{(2)} = \theta_\parallel - \psi_\parallel$, $C^{(1)} = 1$, $C^{(2)} = \cos 2\theta_B$, θ_B is the angle of Bragg scattering, ω_Bi s Bragg frequency, $\chi_g = \chi_g' + i\chi_g''$ is the coefficient of the Fourier expansion of the crystal dielectric susceptibility in terms of the reciprocal-lattice vectors $\mathbf{g}$, $\chi_0 = \chi_0' + i\chi_0''$ is the average dielectric susceptibility, $\varepsilon = \frac{\sin(\delta+\theta_B)}{\sin(\delta-\theta_B)}$. We use the Heaviside-Lorentz system of units.

4 Influence of the beam divergence on the yield of DTR

Since the DTR of electrons at ultrahigh energies has a narrow angular distribution, it is advisable to consider the possibility of using the dependence of the yield of DTR photons per electron within

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the given solid angle in the direction of Bragg scattering on the initial divergence of the electron beam. To do this, we average the expressions for the number of emitted photons over the possible rectilinear trajectories of the electron in the beam. As an example, we average the DTR with Gaussian angular distribution

$$f(\psi) = \frac{1}{\pi\psi_0^2} e^{-\frac{\psi^2}{\psi_0^2}}, \tag{4.1}$$

where the parameter ψ_0 will be called the divergence of the beam of radiating electrons (see figure 1). The angle ψ_0 defines the cone that delimits the part of the electron beam beyond which the electron density decreases more than a factor e in comparison with the density on the beam axis.

The base expression for number $\langle N_{\mathrm{DTR}}\rangle$ of the DTR photons emitted into the solid angle $4(\theta_{c_\parallel} \cdot \theta_{c\perp})$ is

$$\langle N_{\mathrm{DTR}}\rangle = \frac{1}{\pi\psi_0^2} \int_{-\theta_{c_\parallel}}^{\theta_{c_\parallel}} \int_{-\theta_{c\perp}}^{\theta_{c\perp}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \sum_{s=1}^{2} \frac{dN_{\mathrm{DTR}}^{(s)}}{d\Omega} (\psi_\perp, \psi_\parallel, \theta_\perp, \theta_\parallel) \cdot e^{-\frac{\psi_\perp{}^2 + \psi_\parallel{}^2}{\psi_0^2}} d\psi_\perp d\psi_\parallel d\theta_\perp d\theta_\parallel \tag{4.2}$$

This expression contains the summing of DTR angular density over two photon polarization projections, the integrating over all photon emission angles and the averaging over the angular distribution (4.1) of electrons in the beam.

The curves in figure 2, plotted by the formula (4.2) (solid curves) using the angular density in the form (3.1) demonstrate the dependence on the divergence of the beam of electrons incident on the crystal of the reduced to one incident electron average number $\langle N_{\mathrm{DTR}}\rangle$ of DTR photons in Bragg direction (the $\mathbf{e}_2$ axis) emitted to the rectangular collimator with the collimation angles $2\theta_{c\perp}$ and $2\theta_{c_\parallel}$ (see in figure 1). The curves are plotted for various values of the registration solid angle under condition $\theta_{c\perp} = \theta_{c\parallel}$. The calculations were performed for a C(111) diamond target with a thickness of $5 \cdot 10^{-5}$ cm and for the Bragg energy of $\omega_B = 10.9$ keV.

In figure 2 we can see that the total yield of DTR practically does not depend on the divergence of the beam under a sufficiently large collimation angle. The optimal collimation must supply the maximal dependence of the DTR yield on divergence what corresponds to the maximum of the derivative $|dN_{\mathrm{DTR}}/d\psi_0|$.

A more detailed consideration of DTR properties allows us to conclude that the yield of DTR photons emitted by an ultrahigh-energy electron beam from a thin single-crystal target into the given solid angle can be used for the beam divergence indication. For example, we showed that in the region of the divergence parameter $1/\gamma < \psi_0 < 10/\gamma$ the value of collimation angles $(\theta_{c\parallel} = \theta_{c\perp} = 5/\gamma)$ can be used without changing it.

To clarify the obtained values of the divergence parameters as a second step, one should change the collimation angle to a more optimal one and repeat the calculations.

For the indication of a two-parametric angular distributions, when $\psi_{0\parallel} \neq \psi_{0\perp}$ one should use the different collimation angles $\theta_{c\parallel} \neq \theta_{c\perp}$ and the two-dimensional Gaussian function for averaging the DTR angular density:

$$f(\psi) = \frac{1}{\pi\psi_{0\perp}\psi_{0\parallel}} e^{-\left(\frac{\psi_\perp{}^2}{\psi_{0\perp}^2} + \frac{\psi_\parallel{}^2}{\psi_{0\parallel}^2}\right)} \tag{4.3}$$

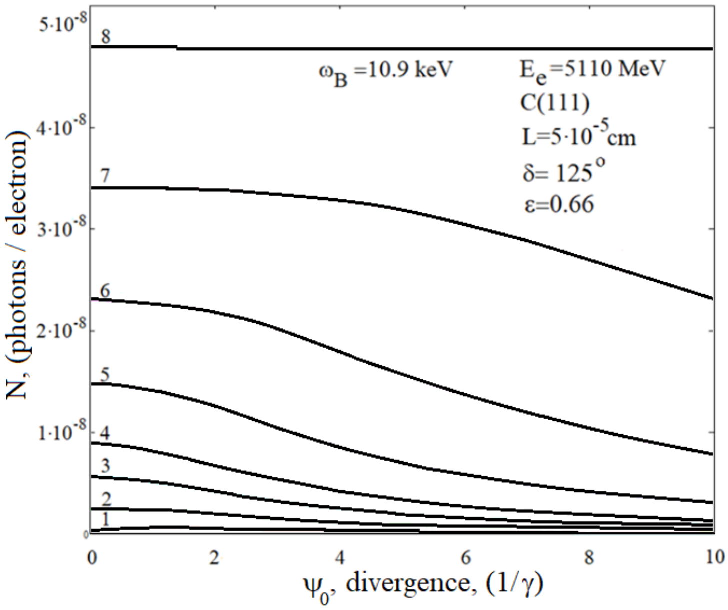

Figure 2. The dependence of DTR photon yield $N_{\rm ph}$ on the electron beam divergence ψ_0 calculated by formula (4.2) using the angular density in form (3.1). Parameter γ is electron Lorentz factor, $\gamma = 10^4$. Solid angle of registration is $\Delta\Omega = 4(\theta_{c\parallel}\cdot\theta_{c\perp})$. $\theta_{c\parallel} = \theta_{c\perp} = 1 - 0.5\gamma^{-1}, 2 - 1\gamma^{-1}, 3 - 1.5\gamma^{-1}, 4 - 2\gamma^{-1}, 5 - 3\gamma^{-1}, 6 - 5\gamma^{-1}, 7 - 10\gamma^{-1}, 8 - 100\gamma^{-1}$.

To obtain an unambiguous solution to this problem, it is necessary to carry out at least two independent measurements of the DTR photon yield, for example, using two slit collimators situated perpendicular one to another. In this case two calculations of photon yield should be done for indication of the divergence parameters $\psi_{0\perp}$ and $\psi_{0\parallel}$:

$$\left\langle N_{\rm DTR}\left(\psi_{0\perp},\psi_{0\parallel}\right)\right\rangle_{c\perp} = \frac{1}{\pi\psi_{0\perp}\psi_{0\parallel}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\theta_{c\perp}}^{\theta_{c\perp}}\int_{-\infty}^{\infty}\sum_{s=1}^{2}\frac{dN^{(s)}_{\rm DTR}}{d\Omega}\left(\psi_\perp,\psi_\parallel,\theta_\perp,\theta_\parallel\right)\cdot e^{-\left(\frac{\psi_\perp{}^2}{\psi_{0\perp}^2}+\frac{\psi_\parallel^2}{\psi_{0\parallel}^2}\right)}d\psi_\perp d\psi_\parallel d\theta_\perp d\theta_\parallel$$

$$\left\langle N_{\rm DTR}\left(\psi_{0\perp},\psi_{0\parallel}\right)\right\rangle_{c\parallel} = \frac{1}{\pi\psi_{0\perp}\psi_{0\parallel}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\theta_{c\parallel}}^{\theta_{c\parallel}}\sum_{s=1}^{2}\left(\frac{dN^{(s)}_{\rm DTR}}{d\Omega}\left(\psi_\perp,\psi_\parallel,\theta_\perp,\theta_\parallel\right)\right)\cdot e^{-\left(\frac{\psi_\perp{}^2}{\psi_{0\perp}^2}+\frac{\psi_\parallel{}^2}{\psi_{0\parallel}^2}\right)}d\psi_\perp d\psi_\parallel d\theta_\perp d\theta_\parallel \tag{4.4}$$

The objective function can be constructed in the form

$$F\left(\psi_{0\perp},\psi_{0\parallel}\right) = \left(\left\langle N_{\rm DTR}\left(\psi_{0\perp},\psi_{0\parallel}\right)\right\rangle_{c\perp} - N_{\rm DTR}^{\rm measured}{}_{c\perp}\right)^2 + \left(\left\langle N_{\rm DTR}\left(\psi_{0\perp},\psi_{0\parallel}\right)\right\rangle_{c\parallel} - N_{\rm DTR}^{\rm measured}{}_{c\parallel}\right)^2. \tag{4.5}$$

In table 1, we present the results of the calculation of the electron beam divergence parameters in the process of the objective function (4.5) minimization by Hooke-Jeeves Method. The form of the objective function (4.5) used for modeling the process of electron beam divergence parameters indication is shown in figure 3, where one can see the pronounced minimum correspondent to sought values of the divergence parameters.

To realize the resolution of beam divergence definition close to the optimal one, the angular width of the slit collimator must be chosen either of order of $\theta_{c\perp} = \theta_{c\parallel} \approx 1/\gamma$ for $\psi_{0\perp} \approx \psi_{0\parallel} \leq 1/\gamma$, or $\theta_{c\perp} \approx \psi_{0\perp}$ for $\psi_{0\perp} > 1/\gamma$ and $\theta_{c\parallel} \approx \psi_{0\parallel}$ for $\psi_{0\parallel} > 1/\gamma$.

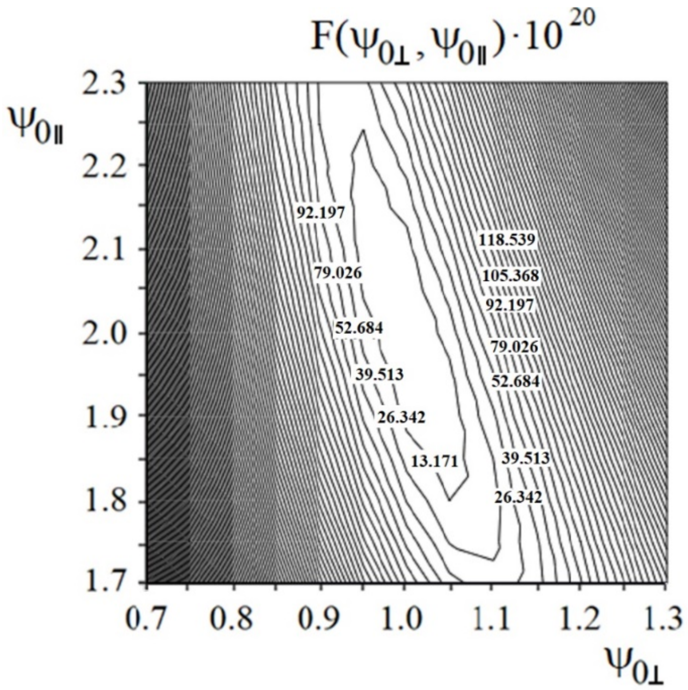

Figure 3. The objective function (4.5) used for definition of divergence parameters $\psi^*_{0\perp}$ and $\psi^*_{0\parallel}$by "measured" numbers of DTR photons $N^{\text{measured}}_{\text{DTR}}{}_{c\perp} = 6.942 \cdot 10^{-9}$ and $N^{\text{measured}}_{\text{DTR}}{}_{c\parallel} = 9.932 \cdot 10^{-9}$ (which was preliminary calculated by (3.1) for $\psi_{0\perp} = 1/\gamma$ and $\psi_{0\parallel} = 2/\gamma$). The parameters $\psi_{0\perp}$ and $\psi_{0\parallel}$ are expressed in units of $(1/\gamma)$ radians. The values of the objective function in minimum $F(1/\gamma, 2/\gamma) = 0$.

Table 1. Calculated values of the divergence parameters $\psi_{0\perp}$ and $\psi_{0\parallel}$ in process of its approximation to "experimental" values: $(\psi_{0\perp})^{(\text{exp})} = 1/\gamma$ and $\left(\psi_{0\parallel}\right)^{(\text{exp})} = 2/\gamma$ by the computer program searching the minimum of objective function with Hooke-Jeeves Minimization Method. ${\psi_{0\perp}}^{(0)} = {\psi_{0\parallel}}^{(0)} = 0.1/\gamma$ are starting values of the parameters, $\delta{\psi_{0\perp}}^{(0)} = \delta{\psi_{0\parallel}}^{(0)} = 0.2/\gamma$ are the starting increments of the parameters, $\varepsilon\varepsilon = 10^{-4} \cdot (1/\gamma)$ is prescribed accuracy of approximation $\left(\max(\delta\psi_{0\perp}, \delta\psi_{0\parallel}) \geq \varepsilon\varepsilon\right)$. Objective function $F\left(\psi_{0\perp}, \psi_{0\parallel}\right)$. "Experimental" values of electron beam divergence parameters: $\psi^{(\text{exp})}_{0\perp} = 5 \cdot 10^{-6}$ radian $= 1/\gamma$. $\psi^{(\text{exp})}_{0\parallel} = 1 \cdot 10^{-5} = 2/\gamma$. $N^{\text{measured}}_{\text{DTR}}{}_{c\perp} = 6.942 \cdot 10^{-9}$ $N^{\text{measured}}_{\text{DTR}}{}_{c\parallel} = 9.932 \cdot 10^{-9}$. $\gamma = 2 \cdot 10^5$. The target is a single crystal diamond C(111), $L_t = 5 \cdot 10^{-5}$ cm.

	Calculated values			
Iteration number (i)	$\gamma \cdot {\psi_{0\perp}}^{(i)}$	$\gamma \cdot {\psi_{0\parallel}}^{(i)}$	$\gamma \cdot \delta{\psi_{0\perp}}^{(i)}$	$\gamma \cdot \delta\psi_{0\parallel}\,(i)$
0	0.1	0.1	0.4	0.4
5	0.9	1.7	0.1	0.05
10	1.05	1.875	$6.25 \cdot 10^{-3}$	0.0125
15	1.00625	1.9 75	$3.125 \cdot 10^{-3}$	$6.25 \cdot 10^{-3}$
20	1.0015625	1.996875	$3.9063 \cdot 10^{-4}$	$1.5625 \cdot 10^{-3}$
25	1.0001953	1.9992187	$9.7656 \cdot 10^{-5}$	$1.953125 \cdot 10^{-4}$

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5 Conclusion

We study the diffracted transition radiation emitted by a beam of relativistic electrons traversing a thin single-crystal plate in the Laue scattering geometry. We have obtained the expression describing the average number of photons of DTR emitted by a divergent beam of relativistic electrons of ultrahigh energies in a thin single-crystal plate within the given solid angle in the direction of Bragg scattering. Numerical calculations demonstrating the dependence of the yield of collimated DTR photons on the beam divergence have been done. For defining the divergency parameters of the electron beam with angular distribution described by two-dimensional normal distribution with different values of dispersions, we have used the expressions for the yield of DTR photons into the slit collimators situated perpendicular one to other. We have constructed the objective function for calculation of the electron beam divergence parameters. The model calculations of the electron beam divergence parameters on the base of "measured" yield of DTR photons traversing through the slit collimators are carried out. The efficiency of algorithm for the beam divergence parameters valuation based on Hooke-Jeeves minimization method has been demonstrated. The expressions obtained can be successfully used as a base for the development of methods for measuring the divergence of beams of relativistic ultrahigh-energy electrons based on DTR angular distribution.

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