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Relationship Between Thermal Conductivity and Free Electrons in Metal

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Abstract

An experiment was designed and conducted to explore the relationship between thermal conductivity with free electrons in metal. In the experiment, copper, iron, aluminum, and titanium rods with close diameters were used to carry the experiment. Each rod was heat up by a heat unit at one end while cooled on the other end with a heat sink to maintain a steady state. DC current was applied to rods in the direction along as well as against the heat flow. Thermal conductivities were measured in these two situations for each rod. Results showed electrons do dominate thermal flow inside metal.

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I. Introduction

When talking about thermal conduction inside metal, most textbooks [1] state it is the free electrons and phonons (or crystal lattices) who carry heat from high temperature region to low temperature region. Most textbooks briefly mention that, due to the existence of free electron in metal; therefore, the majority of metals are good heat conductors. The primary evidence for this statement comes from the relationship between the thermal and electrical conductivities of metals, which is described by the Wiedemann-Franz law [12-14]. Furthermore, different materials, different mechanisms can dominate the conductive heat transfer, which could involve electrons, phonons, or both. Generally speaking, phonons dominate heat transfer in insulators, while electrons dominate hear transfer in metals. This experiment was designed to directly show the influence of electrons on heat transfer in a conductive metal.

Heat transfers from high temperature place to low temperature place by three ways: conduction, convection, and radiation. Inside metal, heat transfer is mainly by conduction. There are many discussions regarding the travel speed of heat in metals. When a metal bar is heated at one end, it will take some time for the bar to reach an equilibrium state. Not as fast as sound travels. This can be explained as following. Although phonons do participate the heat transfer process, they do not carry majority heat with them. So, most heat must be carried by electrons. It can be imagined that the free electrons inside metals will have a small drift velocity v_{dh} due to the temperature gradient. Combine the fact that the heat transfer speed is somewhat slower than speed of sound in metal. This also hints that electrons dominate heat transfer in metal. The current experiment is used to verify this argument. In the experiment, thermal conductivities were measured for three different metal bars while running DC current in and against the heat flow direction to investigate how much the heat transfer is affected by the direction of electric current flow.

II. Theory

In terms of solid physics and thermodynamics, solid heat conduction is composed of two parts. One part is the phonon produced by lattice vibration and the other part is the thermal motion of electron. The number of electrons in metal is huge and the speed of thermal random motion is about 100,000 m/s. On the contrary, Speed of sound in copper is about 2000-4000m/s which is comparatively slow. Therefore, electrons move much faster at the high temperature where they dominate the thermal conductivity. Electrons random speed is much higher than the directional motion (from high T to low T). The reason for the heat transfer from the hot end to the cold end is that the electrons at the high temperature region collide with the electrons at the low temperature region and pass energy to the electrons at the low temperature region. Most people accepted that in metal, due to the huge number of free electrons, heat transfer mainly by electron, while for insulator, the heat transfers basically by phonons.

In the steady-state method, a sample's temperature is controlled so that the temperature is ideally only a function of position along the sample (namely one dimension). In other words, the temperature gradient is an only a function of position. There are several different techniques belong to this method such as: the absolute technique [1], the comparative technique [1], and the radial heat flow technique,[1]. The absolute technique was used in this experiment. As absolute methods do not require calibration, the lowest uncertainty can be obtained by using this method.[2] Fourier's

Law is applied when using steady-state method to measure thermal conductivity. Fourier's Law states as the following:

$$K = QL/A\Delta T \quad (1)$$

Here, Q is the heat flow rate though the sample, A is the cross-sectional area of the sample, L is the distance between the hot and cold points at which temperatures are measured, and ΔT is the temperature difference between the hot and cold points. Fig. 1 illustrates Fourier's Law. The main challenge of absolute method is to determine the heat flow through the sample under the condition of parasitic heat loss Q_{loss}, and accurately measure the temperature difference ΔT.

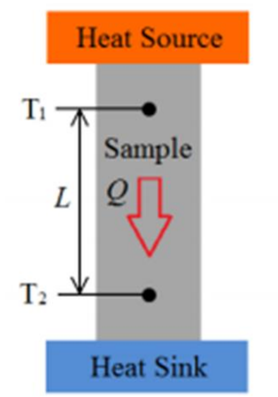

Fig. 1, Illustrate Fourier's law of thermal conductivity, the heat flow goes through the sample from heat sources to heat sink. (This figure is reproduced from another paper which will be shown in reference section) [1]

In this thesis, the effect of electrical current on the heat conduction was explored. After the sample reaches the steady state, which is the dynamic equilibrium of the heat flow, electric current was turn on in and against the direction of the heat flow.

When the direction of electric current and heat flow is the same, the moving direction of electrons is opposite to the heat flow. If the heat flux is sufficiently influenced by electric electrons movement, Q will be decrease. From Fourier's law, one can see that when Q decreases and other quantities remain unchanged, the thermal conductivity will decrease. When the current flow is reversed, electric current will help to increase thermal speed v_{dh} of electron. Then the heat conduction should be increased. The goal of this experiment is to design a system in which the electrical current can show a measurable influence to the thermal conduction.

If the current can affect the efficiency of heat transfer, the electric drift velocity v_d and the electron speed v_{dh} under temperature gradient should have same order of magnitude.

III. Experimental Set up

In order to achieve the goal that has been mentioned in theory part. An experimental setup had been designed and constructed which is improved base on the absolute method, so that current can pass through the sample while both the temperatures and the current are being detected. The main body of the setup is showed in fig. 1. Then, for testing if electric current influence the thermal conductivity of metal, wires are connected to the top and bottom of the metal rod, so that electric current can run through the metal bar.

In this experiment, four one-foot metal rods: copper, iron, aluminum, and titanium were tested. Each rod was wrapped by insulation materials (US Energy products AD5 Reflective Foam Core Insulation) to reduce the heat lost to the air. One end of each bar was attached to a heat reservoir which was made from a big chunk of copper rod to provide a stable heat source. A small hole was drilled in the heat reservoir so that a heater can be put in. The heater is a 50 W, 110 V AC unit (Fig. 2). At another end of each bar a heat sink (Fig. 3 and 4) was attached. The heat sink was built by a heat dispenser and a fan which were recycled from a used computer. The heater was plugged into a Variac so that its output power can be changed. DC current was applied to each rod by home-made copper clamps. Two temperature reading systems were used to do the temperature measurement. One was a laser temperature gun and another one was a K-type thermo couple, which has ±2.20C or ±0.75% accuracy [8].

The distance from top temperature measuring point to bottom temperature measuring point is shorter than the entire rod and is L=0.21m. The cross-sectional area is $A=6.017*10^{-4}m^2$. To get a reasonable heat flow Q_{eff} along the bar, the thermal conductivity of copper was used to calibrate the amount of heat transferred from high temperature region to low temperature region which is *K=401W/mK* [2]. Apply Fourier's Law, the total amount of heat that cross the sample is $Q_{eff} = 66.7$ W when the ΔT is 58 ^{o}C. The reason of choosing 58 ^{o}C as the ΔT to calculate the Q_{eff} is it took four hours for the temperature to stabilize. After the steady state was reached, temperature of both top and bottom of the bar were measured every ten minutes.

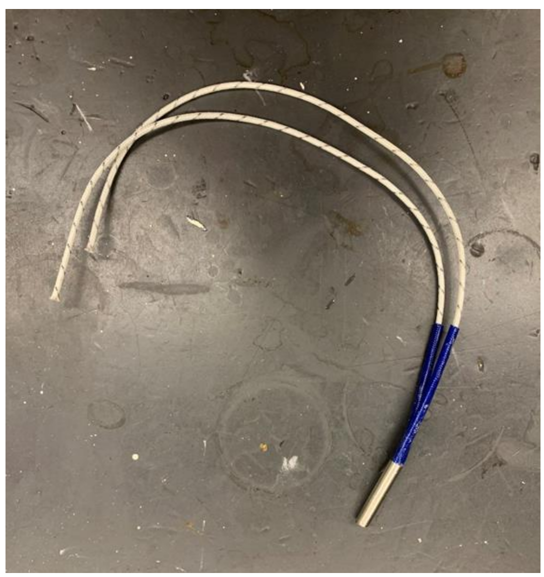

Fig. 2, Heating unit (50 W, 110VAC)

Due to the limitation of experimental conditions, even if the sample surface is covered with thermal insulation material, the use of temperature gun will be affected by the smoothness and color of the object surface, as well as by the air flow. Therefore, thermocouple directly in contact with the object can give more accurate temperature measurement. In addition, calibration showed the thermo couple gave a more accurate temperature reading by compare to ice water temperature and boiling water temperature. So, the temperature data used in this paper were collected from the thermo couple.

Fig. 3 Heat sink

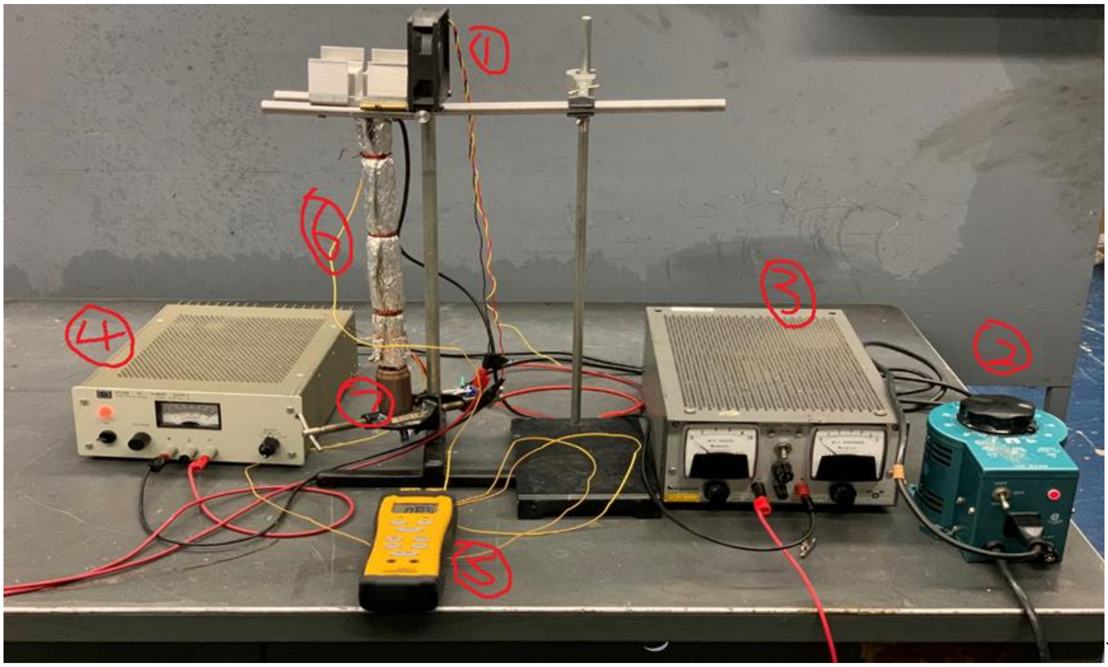

Fig. 4 Experimental set up overview showing the main components: (1. Heat sink: fan. 2.Power supply of the heat source. 3.Current supply 4. Power supply of the fan 5. K-type thermometer 6. Insulated materials 7. Heat source: resistor heaters)

IV. Data Collection

In order to determine the thermal conductivity of the sample under the influence of DC current, several steps needed to be followed. A copper rod was used first because the copper has the best electrical conductivity. The sample was heated until a steady state was established, which took about 4 hours. When this state is reached, temperatures of each end of the sample were then recorded. Next, DC current was run in the opposite direction of the heat flow, which means the electrons flow the same direction of the heat flow, and the temperatures were again collected. The temperature difference was checked every minute. These data are shown in Fig.5. Then, the DC current was stopped, and the sample was allowed to return to equilibrium. The temperature difference was recorded, and the data shown in Fig.6 were collected just after the moment we turn the current off.

Then, the DC current was run in the opposite direction and the temperatures were again collected versus time. Once again, the temperatures readings were collected both when running DC current in the direction of heat flow and then, against the heat flow. The data was shown in Table 1, and the trends can be seen in Fig.7. The experiment was repeated using the Aluminum and Iron bars. These data are shown in Table II and Table III.

Table I The data of temperatures of two ends of Copper with respect of time. ΔT has a clear trend of change

State of current	Time (min)	Top (°C)	Bot (°C)	ΔT (°C)	State of current	Time (min)	Bot (°C)	Top (°C)	ΔT (°C)
No Current	0	41	104	63	No Current	0	101	46	55
	18	40	98	58		60	91	44	47
5A B-T	24	43	96	53		76	104	45	59
	26	46	99	53		86	100	41	59
	29	41	93	52		90	92	45	47
	32	39	96	57		150	91	39	52
	40	37	96	59	5A T-B	154	96	43	53
5A T-B	41	44	90	46		157	92	43	49
	43	41	94	53		180	92	38	54
	45	42	98	56	No Current	181	93	42	51
	75	40	100	60	5A B-T	182	87	41	46
						184	92	46	46
						188	92	43	49

Table II. The data of temperatures of two ends of Aluminum with respect of time. The temperature difference is obviously change when the current flow through

State of Current	Time (min)	ΔT (°C)
No Current	1	59
	4	58
	5	58
	6	58
5A B-T	7	57
	8	56
	10	56
	11	56
	12	56
	13	57
	15	58
	17	58
	18	58
5A T-B	19	59
	20	58
	21	57
	22	58
	25	57
Turn off	28	58
	29	57
	30	58
	35	58

Table III The data of temperatures of two ends of Iron with respect of time. When the current flow through the sample from bot to top, there is an obvious downward trend of ΔT, and vice versa

State of current	Time (min)	ΔT(°C)
No Current	0	84
	3	83
5A B-T	6	84
	7	82
	8	79
	9	75
	10	77
	11	82
	12	81
	13	81
	15	80
	17	81
	21	82
	23	83
	24	84
	25	83
5A T-B	27	82
	28	82
	29	81
	30	81
	31	81
	32	80
	33	79
	34	79
	35	79
Turn off	36	80
	38	81
	39	81
	41	82
	42	83
	45	83

One can tell in Eq. (1), Q is an important quantity in calculating thermal conductivity. When heat generated from the heater, it will transfer to the bar through the heat reservoir and go to the heat sink. During this process, even though insulation materials were used, there were still many other facts could affect the heat flow rate through the metal bar. Although the power of the heater is known, the heat conduction efficiency of the heated reservoir is not known. In addition, the loss of heat radiation, heat convection and heat conduction cannot be calculated. Therefore, K can only be assumed to be a fixed value to calculate the effective value. The Q calculated by this is the actual thermal power called Q_{eff}, which means that all radiation losses and heat dissipation losses are included. To get a reasonable data of Q, an effective flow rate Q was used instead a real Q. To do so, pure copper bar was used to do the calibration. From literature, pure copper has a thermal conductivity *K=401W/mK* was treated as known number, experimental data were plug into Eq. (1) to get Q_{eff}. Then this calibrated heat flow rate was used to finish the rest experimental work. Then this calibrated heat flow rate was used to finish the rest experimental work. Since Q is from calculation, the thermal conductivity of copper rod will depend on what the Q_{eff} is. The uncertainty of the K by calculation is from the difference between the real heat flow Q and Q_{eff}. In addition, due to the limitation of experimental equipment and environment, heat loss is inevitable. Therefore, there is a certain gap between the real temperature and the temperature recorded by us. In addition, the difference of pressure between thermal coupling and copper surface will lead to different temperature.

V. Results and Discussion

After doing the experiment, the results are shown in the following figures. The data points of the experiment were put in excel to make plots. From what was shown in figure.5, the temperature difference changes with the respect of time when the 5A electric current went through. When the current flow went opposite direction of heat flow, electrons drift velocity is in the same direction as the heat flow. The temperature difference decreases, which suggests that the heat was transferring faster. This could indicate that the motion of electrons increased the rate of the heat transfer. In addition, Eq. (1) showed that temperature difference ΔT is inversely proportional to thermal conductivity K. When the temperature difference ΔT decreased, we can describe this as an increase of the effective thermal conductivity K. This would support the theory that electrons dominate the thermal conductivity.

When the current stopped passing through the sample, it was obvious from Figure. 6 to see that the temperature difference is rising back towards its former equilibrium value. This could be because, when the current flow is off, the electrons no longer have an average drift velocity in the direction of heat flow. That caused the heat transfer to slow down. One end of the sample (copper) was very high temperature, another end of the sample is still comparatively low temperature. Also, Eq. (1) showed that when the temperature difference ΔT increased, we can describe this as a return of the effective thermal conductivity K to its previous value. This also agree with the theory that electrons dominate the thermal conductivity.

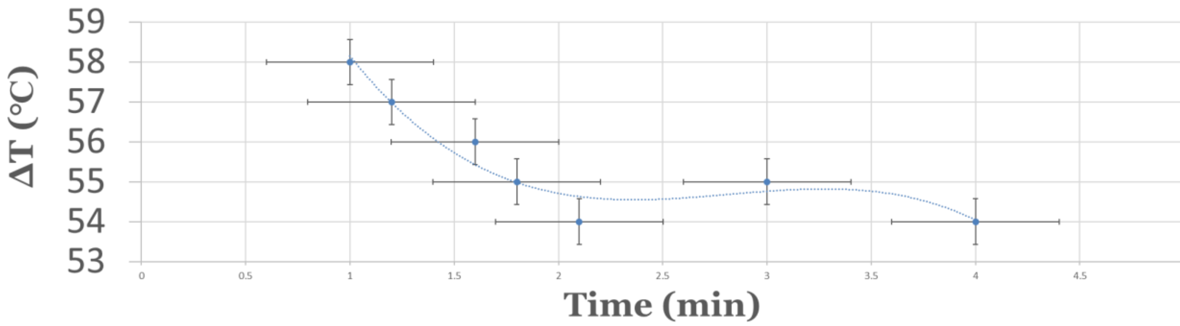

Fig. 5 Curve of ΔT vs time when electrons follow the heat flow

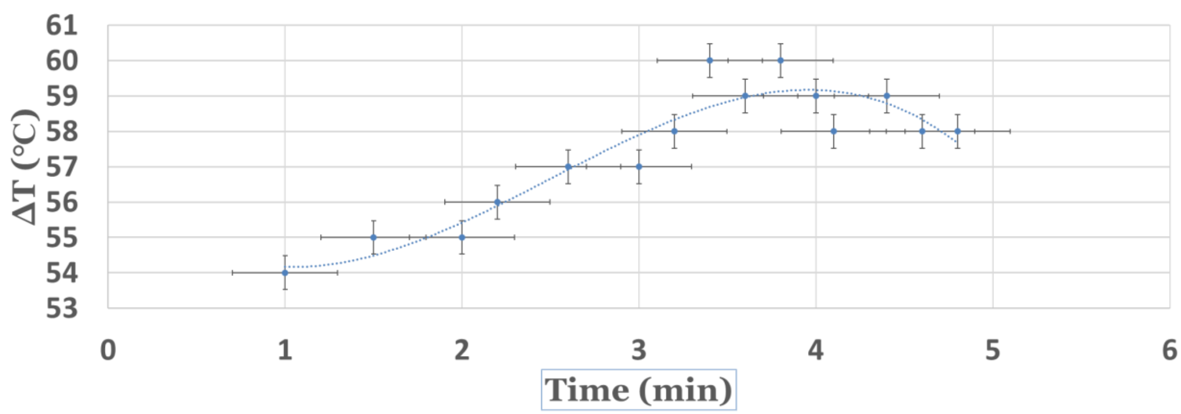

Fig. 6 Curve of ΔT vs time when turning the current off

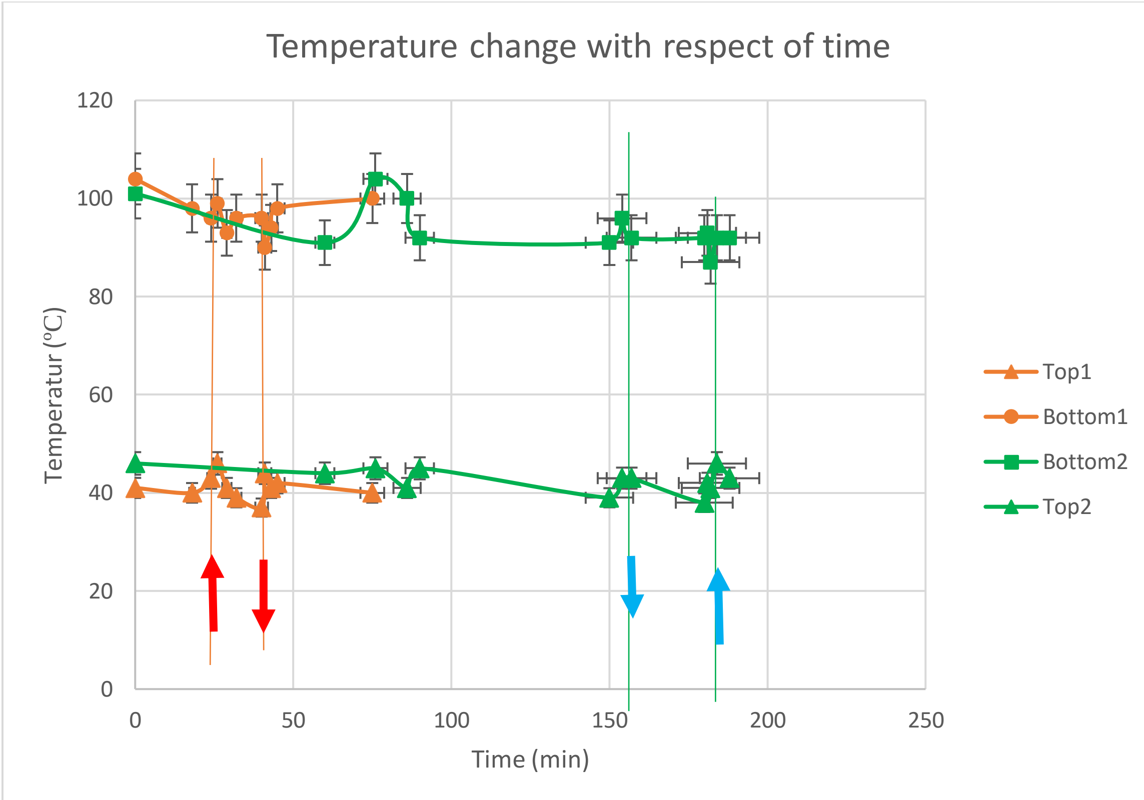

Fig.7 Temperature change with respect of time (Copper's both top and bottom temperature)

After obtaining these data, which are consistent with the hypothesis theory, the comparison of electron drift velocity and electron thermal motion velocity should also be considered

These results support the current theory that electrons dominate thermal conductivity of solid metal. It was found that the thermal motion of electrons is very fast. The velocity of thermal motion of electrons be calculated from the formula of electrons energy Eq. (2). Eq. (0) can be changed to Eq. (3) for calculating the velocity of thermal motion of electron. In this formula, k=1.380649 × 10^{-23} J/K. For temperature T, the average temperature between the top temperature and the bottom temperature of the bar is used. A random point, which the top temperature is 39 ºC (about 312.15 K), and the bottom temperature is 96 ºC (about 369.15 K). The average temperature is 340.65 K. The mass of an electron is 0.51 MeV/c^2. Then, the v_{rms} is equal to $1.246*10^5$ m/s which is very fast.

The formula of drift velocity of electron in metal under the action of electric field is shown in Eq. (4). A is the cross-sectional area of copper rod ($6.017*10^{-4}$ m^2). The electron density of copper n is equal to 8.4×10^{28} m^{-3}, and I is the current of 5 amperes. After calculating the v_d in this experiment is $0.62*10^{-6}$ m/s. The drift velocity of electrons is much slower than thermal motion velocity of electrons.

Under the action of electric field, the velocity of electron is only $0.62*10^{-6}$ m/s. However, the velocity of electrons in thermal motion is $1.246*10^5$ m/s. The ratio of these two velocities is about equal to $5*10^{-12}$. If electrons do not dominate the heat conduction of metal, it is impossible to see the change of heat conduction rate in such a small change.

Even though the electron thermal movement speed is much faster than the electron directional movement speed under the action of low electric field, the temperature difference and thermal conductivity of the sample still change when the current state (whether there is current passing through) is changed. As mentioned in our theoretical part, it is possible that the number of directional electrons is very large although the speed is very slow. This causes a large number of electrons to help the thermal movement and thus increase the thermal conductivity. Although the current data still show the same pattern when iron and aluminum are used as experimental samples, this paper only calculates the ratio of electron drift velocity under the action of electric field to the velocity under the action of thermal motion in copper samples due to limited conditions.

VI. Conclusion

From the results that has been shown in previous parts. When the current flow went opposite direction of heat flow, electrons follow the heat flow. The temperature difference decreases.

This project preliminary proved that the thermal conductivity is dominated by electrons. The assumption mentioned in the theory and the assumption mentioned in the book (Introduction to thermodynamics) are consistent.

The shortcomings of the experiment are that the reservoir heater rod is too small, it will produce a thermal gradient, the heater is not heating evenly, it will produce errors when measure temperatures. In addition, how much Q actually flow from one end to another was unknown. In this experiment, it is assumed that the thermal conductivity of copper is standard, and it is deduced from this. What needs to be done in the future. Take more data at the moment of connecting current and breaking current and enlarge them to scale time unit of seconds. Also, the resistivity of the contacts to the rod needs to be measured, and the heating effect of the current itself needs to be included in the analysis.

For getting more development in the future, the first this that need to do further is to calculate the drift velocity of electron under the action of electric field and the thermal velocity of electron of all other metals that has been tested in this experiment. Second, taking more data of just connected current and just off current, and enlarge them to scale, time unit is seconds. Thirdly, by changing the measuring temperature, the error of thermistor K can be reduced, and the error of existing thermistor is about 3%. Finally, Making the reservoir bigger to keep the temperature constant will be helpful if using absolute method. Because using this way can reduce the temperature gradient difference.

VII. Acknowledgement

In concluding this document, I would like to thank Joseph Overstreet for help me to build up the experimental setup. Much thanks to Jennifer Heath for her help and put forward valuable opinions to making my paper more scientifically. I am extremely grateful for Tianbao Xie for all the hard work and dedication he has put into the project. Thanks to Tianbao Xie, Jennifer Heath, and Joelle Murray for editing this document help me to have a better understanding of scientific writing.

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