2020

Analyzing Concrete Beam Design: Verifying Predictions in T&EE Classrooms

Andrew J. Hughes
California State University, San Bernardino, andrew.hughes@csusb.edu

Chris Merrill
Illinois State University, cpmerri@ilstu.edu

Recommended Citation
Hughes, A. J. & Merrill, C (2020). Analyzing Concrete Beam Design: Verifying Predictions in T&EE Classrooms. Technology and Engineering Teacher, 79(5), 14-19.

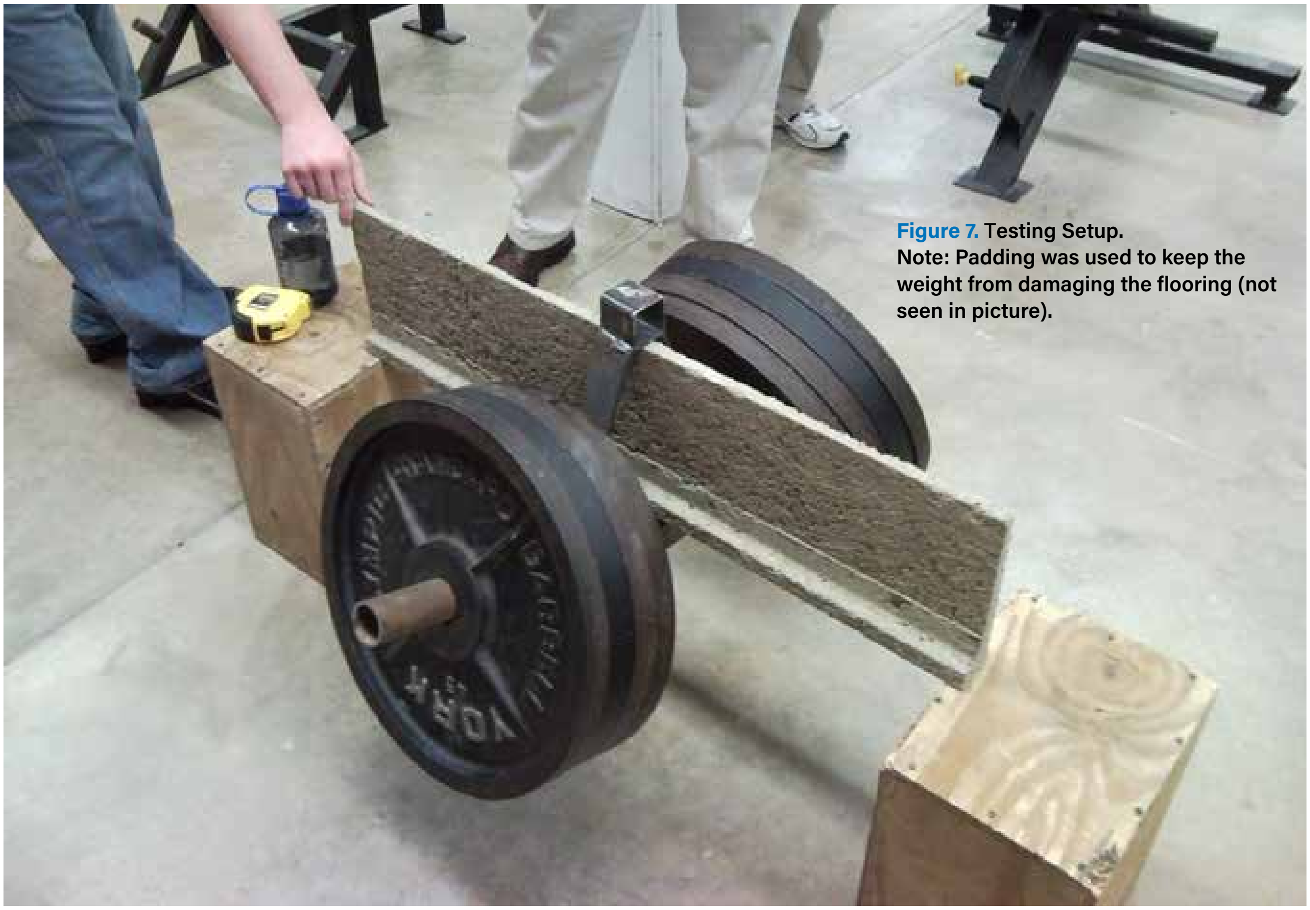


Figure 7. Testing Setup.
Note: Padding was used to keep the weight from damaging the flooring (not seen in picture).

analyzing concrete beam design:

verifying predictions in T&EE classrooms

Students design the beam, cut and assemble wooden forms, mix and pour concrete, and then test the beam after a seven-day cure.

Part one of this article appeared in the November 2019 issue of Technology and Engineering Teacher.

by
Andrew J. Hughes and Chris Merrill, DTE

Design is often accepted as a fundamental aspect of engineering (Dym, et al., 2005). The design process is frequently portrayed as a set of steps. However, the design process is more complex than just a set of steps in a relatively fixed process. The complex nature of design, design thinking, questioning, and decision making is exactly what technology and engineering classrooms are well suited to address. When addressing the question—"Why is technology and engineering education (T&EE) so important?"—the authors believe T&EE's importance relates to our discipline's ability to solve complex problems by balancing theory and practice in engaging hands-on learning scenarios like designing, fabricating, and testing a concrete beam.

In the previous article, students were exposed to beams with relatively uniform single polygon rectangular cross-sectional areas in the moment-of-inertia lab. In the case of an I-beam, both the flanges and web have individual moment-of-inertia quantities. These individual inertia quantities combine to determine the I-beam's moment of inertia about a specified axis (commonly the neutral axis, I_{NA}). *Note: For all beams in this article, the centroidal axis and neutral axis are the same.*

The focus of this second article of two is another moment-of-inertia lab and a final concrete-beam design challenge. The lab further develops students' understanding of additional engineering principles involved in beam design. This article starts by providing background information on the moment of an area with respect to an axis. Then it transitions into the lab activity for the purpose of students determining how the moment-of-inertia quantity is affected by the distribution of area relative to the centroidal axis. *Note: The previous article uses the same lab setup presented.* The final concepts and skills learned in this article will lead your students into concrete beam and form design as well as fabrication and testing. This article introduces the straightforward mathematics used to precisely predict the amount of weight the concrete beam will hold during testing.

Moment of an Area

In mechanics, the moment of inertia represents the second type of moment for an area. *Note: This was covered in the previous article.* The first type of moment for an area is represented by the location of the centroid. The centroid of an object is the geometric center of an area in a two-dimensional space. In objects with homogeneous density, the centroid is located at the same position as the center of gravity. To enhance the students' ability to design a beam, they will need to develop a deeper understanding that the moment-of-inertia quantity is affected by the distribution of the area and mass relative to an axis (usually the centroidal axis). The three coordinate centroidal axes pass through the centroid (Figure 1).

The parallel-axis theorem states that the moment of inertia of an area may be determined using an axis other than the centroidal axis (usually the neutral axis or reference axis) only if the axis is parallel to a centroidal axis. The neutral axis or neutral surface represents a planar area of a beam that does not experience any change in length under a transverse (normal) force causing bending. The neutral surface of a beam is located where the cross-sectional area above the neutral surface is equal to the cross-sectional area below the neutral surface. For a beam with a symmetrical cross-sectional area, the neutral surface is located in the middle of the height (Figure 2). With a simply supported beam, the beam material above the neutral axis experiences compression when a transverse force is applied. The beam material below the neutral axis experiences tension. Again, the material at the neutral axis does not experience either compression or tension when a transverse force is applied (Figure 3). The bottom surface of the beam, the surface contacting the beam supports, is often referred to as the reference axis. The parallel axis theory is applied when calculating the moment of inertia about the neutral axis (I_{NA}) and beam design optimization.

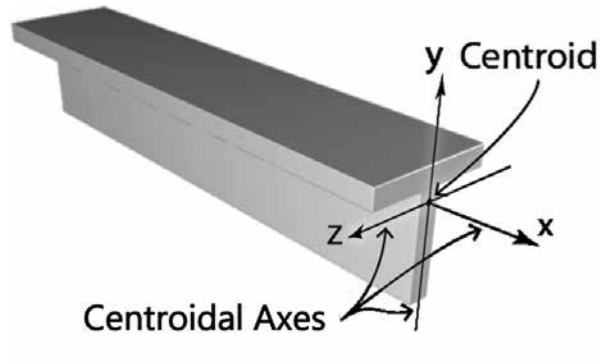


Figure 1. Centroid and Centroidal Axes.

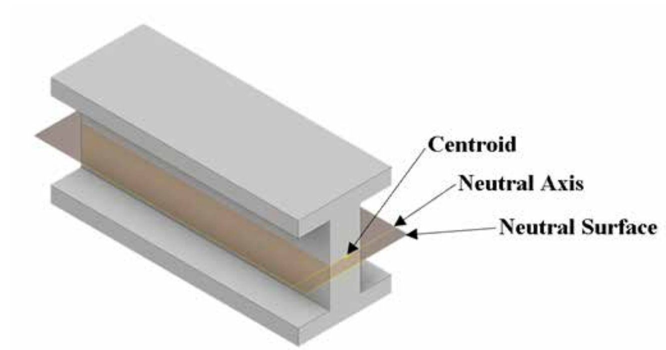


Figure 2. Neutral Axis and Surface.

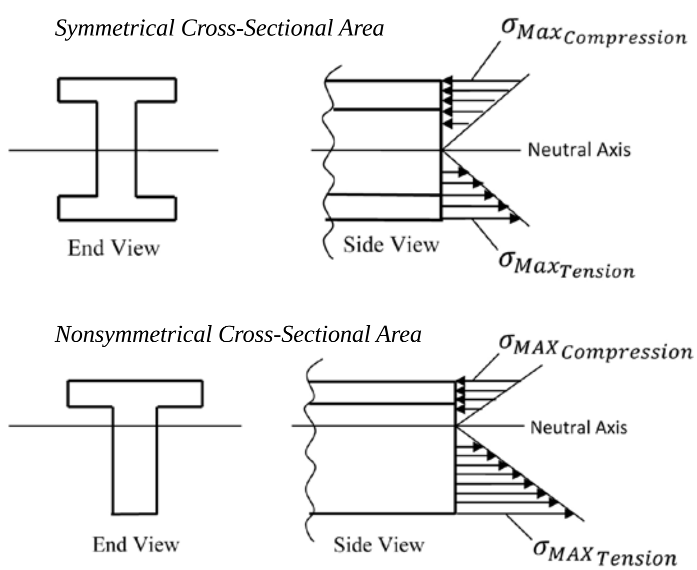


Figure 3. Hooke's Law for Distribution of Bending Stress.
Note: As a graphical representation, you can see that the tension and compression forces are not equal on the nonsymmetrical cross-sectional area. The cross section is experiencing a higher stress in tension based on the design.

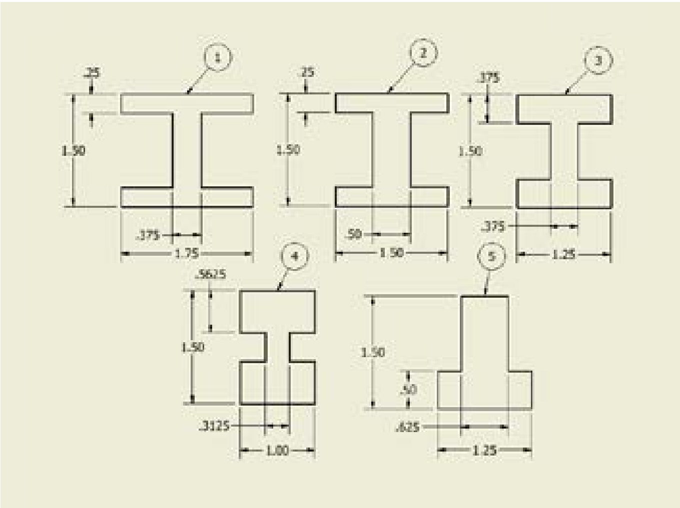
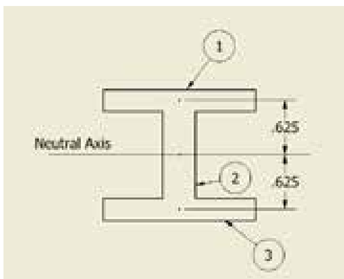


Figure 4. Beam cross sections for lab.
Note: The beams were cut out using a dado blade.

Beam 1:



$$I_1 = \frac{wh^3}{12} = \frac{(1.75\,in)(.25\,in)^3}{12} = \frac{.0273in^4}{12} = .002279in^4$$

$$I_2 = \frac{wh^3}{12} = \frac{(.375\,in)(1\,in)^3}{12} = \frac{.375in^4}{12} = .03125in^4$$

$$I_3 = \frac{wh^3}{12} = \frac{(1.75\,in)(.25\,in)^3}{12} = \frac{.0273in^4}{12} = .002279in^4$$

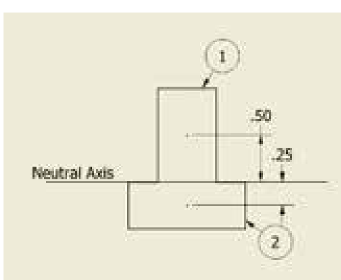
$$A_1d^2 = wh(d^2) = (1.75\,in)(.25\,in)(.625\,in)^2 = .1709\,in^4$$

$$A_2d^2 = wh(d^2) = (.375\,in)(1\,in)(0\,in)^2 = 0\,in^4$$

$$A_3d^2 = wh(d^2) = (1.75\,in)(.25\,in)(.625\,in)^2 = .1709\,in^4$$

$$I_{NA} = \Sigma I + \Sigma Ad^2 = .002279in^4 + .03125in^4 + .002279in^4 + .1709in^4 + .1709in^4 = .3776in^4$$

Beam 5:



$$I_1 = \frac{wh^3}{12} = \frac{(.625\,in)(1\,in)^3}{12} = \frac{.625in^4}{12} = .0521in^4$$

$$I_2 = \frac{wh^3}{12} = \frac{(1.25\,in)(.5\,in)^3}{12} = \frac{.156in^4}{12} = .013in^4$$

$$A_1d^2 = wh(d^2) = (.625\,in)(1\,in)(.5\,in)^2 = .15625\,in^4$$

$$A_2d^2 = wh(d^2) = (1.25\,in)(.5\,in)(.25\,in)^2 = .039\,in^4$$

$$I_{NA} = \Sigma I + \Sigma Ad^2 = .0521in^4 + .013in^4 + .15625\,in^4 + .039\,in^4 = .2604in^4$$

Figure 5. Calculating Moment of Inertia about the neutral axis.

Moment-of-Inertia (about specific axis) Lab

The purpose of this lab is for students to determine how the moment-of-inertia quantity is affected by the distribution of area relative to the centroidal axis. In this activity, students will be comparing the deflection amounts of beams with relatively uniform mass and cross-sectional areas comprised of multiple polygons. The outcome of this lab is students' understanding that changes in deflection are based on the distribution of area relative to the centroidal axis. The students are given beams that are 40 inches long cut from 2 by 4s to the specified cross-sectional dimensions for testing (Figure 4). The students should notice that each beam is the same height and has a similar cross-sectional area. The differences and similarities between each beam's dimensions should be discussed. Ask the students which beam they believe will deflect least under the same transverse load and why. Students will set up each beam on two desks 36 inches apart (needs to be consistent), clamp one end to the desk, measure the unloaded distance from the center bottom of the beam to the floor, then load a weight at the middle of the beam (usually 45 pounds), and again measure the distance from the floor to the middle of the bottom of the beam. Students then calculate the *deflection percentage* in the same way as the moment-of inertia lab from the first article. By the end of the lab, students are going to determine that the beam with the highest moment of inertia has less deflection than a beam with a lower moment of inertia using the same load.

After measuring the deflections of Beams 1, 2, and 3, students will notice that Beam 1 deflects the least and Beam 3 deflects the most. The students will also notice that there is little difference in the deflection amount of each beam. At this point students will not know that these beams have similar I_{NA}. Based on the students' experience with the lab from the previous article, when questioned about these results they may start to think that, due to the height and area being similar, the difference in deflection is the result of changing overall width. This is partially true, but width, height, and area are only three of the four variables in determining a beam's moment of inertia about the centroidal axis. Discussing the relationship between Beams 1, 2, and 3 deflections and dimensions can help students make reasonable connections with their moment-of-inertia values about I_{NA}. This is where the students learn (see) that the cross-sectional area's relationship to the neutral axis is important.

Provide the students with the equations for calculating the moment of inertia about the neutral axis for Beams 1 and 5 (Figure 5). The students will see that the moment-of-inertia quantities are calculated separately for each polygon of the beam's cross section. The distribution of the area is also calculated for each polygon. Moment-of-inertia and distribution-of-area quantities are then summed, resulting in the beam's moment-of-inertia quantity about the centroidal axis/neutral axis. The students will notice that the web portion of Beam 1 has a higher inertia value compared to the flanges. However, due to the symmetrical distribution of the cross-sectional area, the centroid of the web is located on the beam's neutral axis. This results in the web's Ad^2 value being zero and not adding to the beam's moment of inertia about the neutral axis. This should indicate to students that Beam 1 could be redesigned so that the centroid of the web is not located on the neutral axis and/or less material is used in the web and more is used in the flanges to increase the beam's moment of inertia. The Ad^2 value is also zero for Beams 2, 3, and 4 (Figure 4).

Students should notice that the centroids for each polygon in the cross-sectional area of Beam 5 are not located on the neutral axis. This allows for the Ad^2 value for each polygon to be something other than zero and add to the beam's moment of inertia about the neutral axis. However, the students should also notice that Beam 5 has a lower moment of inertia about the neutral axis than Beam 1. This implies that the cross-sectional area of Beam 5 is not optimized. Ask the students to design the cross-sectional area of a beam with the highest possible moment-of-inertia quantity given the constraints: maximum height 1.5 inches, maximum width 1.75 inches, .25 inch minimum dimension, and the cross sectional area consists of at least 2 polygons. The students should try to reach an I_{NA} value higher than Beam 1. *Note:* Students will want to use the moment of inertia about the centroidal axis equations presented in Figure 6 of the previous article. The optimized answer, having the lowest cross-sectional area but highest moment of inertia about the neutral axis, will be two trapezoids with one on top of the other like an hourglass. After students have designed a few cross-sectional areas of beams, they will have a comprehension of controlling inertia about the neutral axis. It is possible to have students cut their beam design out of 2 by 4s and test the deflection.

What is Concrete?

Remember 1, 2, and 3. A standard mixture of concrete contains a ratio of 1 cement, 2 small aggregate (sand), and 3 large aggregate (stone). After a seven-day cure, a standard concrete mixture has the strength to resist 350 psi (lbs/in^2) in tension and 3500 psi in compression. However, these strength numbers may vary slightly. The two strength numbers are based on a standard mixture of concrete. If the ratio of 1 cement, 2 sand, and 3 stone varies even slightly, then these numbers can vary plus or minus about 100 psi in tension and 500 psi in compression. More cement and less sand are usually the adjusted materials to increase strength of concrete. The strength of concrete is also reduced by adding too much water. A dry but "workable" mixture is the desired consistency, representing an ideal ratio of water to large and small aggregate. For a 40-pound bag of standard concrete mixture, less than half of a gallon of water should be added (about 1.5 quarts). Also, the two strength numbers were deter-

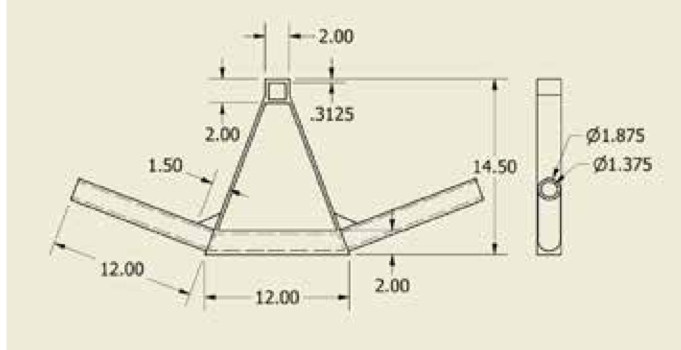


Figure 6. Welded metal frame used for testing concrete beams.
Note: This device will only allow for about 700 pounds.

mined for concrete in which slightly less than 50% of the total surface area was exposed to air during the cure. If the concrete beam form allows more or less than 50% of the total surface area to be exposed to air, this will also affect the strength of concrete. If the concrete beam form absorbs water from the concrete mixture, this can also affect concrete strength. Adding oil to the wooden form helps to reduce water absorption and helps the form to release the beam. What does all this information mean? Curing of concrete is a chemical process in which heat is created. Anything that will change this curing process can in the end change the strength of the concrete.

Concrete Beam Design Challenge

The challenge is to design the strongest concrete beam possible based on the constraints. In this challenge, students design the beam, cut and assemble wooden forms, mix and pour concrete, and then test the beam after a seven-day cure. The beam will be simply supported with traverse concentrated loads. The beam must be designed and built with the following constraints:

- Beam may not weigh more than 20 pounds.
- Beam must span a minimum distance of 36 inches.
- Beam must hold at least 350 pounds or a minimum moment of 6300 lb. in.
- Only material allowed is standard concrete mixture and water supplied in class.
- Beam must allow for the use of the testing devices (Figures 6 [this page] and 7 [pg. 14]).

The maximum beam weight of 20 pounds provides students with the maximum cross-sectional area possible (Figure 8). The density of a standard mixture of concrete is $.08\frac{lb}{in^3}$. Dividing 20 lbs. by $.08\frac{lb}{in^3}$ gives the student the maximum volume of the beam, which is to 250 in^3. Now the students need to consider the length of the beam. The span distance is 36 inches, so the beam length must be greater. Based on experience, the minimum recommended length is 36.5625 inches. Dividing 250 in^3 by 36.5625 inches provides the recommended maximum cross sectional area of 6.8376 in^2. Now the students will design their beam's cross-sectional area with the greatest I_{NA}.

$$weight = density\ x\ volume$$

$$20\ lbs = .08\frac{lb}{in^3}\ x\ volume$$

$$volume = \frac{20\ lbs}{.08\frac{lb}{in^3}} = 250in^3$$

$$volume = height\ x\ width\ x\ length$$

$$height\ x\ width = \frac{volume}{length}$$

$$cross\ sectional\ area = \frac{250in^3}{36.5625in} = 6.8376in^2$$

Figure 8. Calculating cross-sectional area.

Once the students have their I_{NA} value, they will use the bending stress formula (or flexure formula) to determine the maximum bending moment that causes failure. The bending stress formula is based on Hooke's Law for distribution of bending stress, elastic section modulus, radius of curvature, and distance from the bottom of the beam to the neutral axis (c) (Figure 9). In this case, students will be calculating from the moment (M). The σ_{MAX} value is 350 $\frac{lb}{in^2}$ because the beam will fail in tension before it fails in compression (in most cases). When solving the *bending stress formula*, students should notice that the stress max (σ_{MAX}) is multiplied by their I_{NA}, so having a high I_{NA} is important. However, the students should also notice that they are then dividing that value by the distance from the bottom of their beam to the neutral axis (c), so reducing c is also important. The students can optimize how much their beam holds by finding a balance between a higher I_{NA} and lower c values. The students will find that these two variables are basically negatively correlated.

$$\sigma_{MAX} = \frac{Mc}{I_{NA}}$$

$$350\frac{lb}{in^2} = \frac{Mc}{I_{NA}}$$

Figure 9. Bending Stress Formula.

Once students have the maximum moment (M) that will result in beam failure, they will need to calculate the load that will cause failure (Figure 10). The concrete beam should fail at or very near the calculated weight. If the calculations are completed correctly and the form results in a beam with the same size and shape as what was calculated, the difference between weight calculated and the weight causing failure should be within 1%. Additionally, concrete can be considered brittle. While loading the beam, pumping, side loading, or uneven loading of the beam can result in premature failure. Basically, students need to be careful while loading the beam.

$$M = \frac{Load\ x\ span\ distance}{2}$$

$$Load = \frac{2M}{span\ distance}$$

Figure 10. Load Causing Failure.

Beam Form Design

The concrete beam forms can be quite basic or challenging to make. If the beam has a specific angle like 47.5 degrees, the form will also need to have that same angle. This will require the students to learn how to set up the machinery precisely to cut specific angles (Figure 11). To make the forms, students were asked to use 2 x 4s and lauan plywood, often left over after the deconstruction of school play and musical sets. The forms were built so that concrete could be sandwiched inside. The bottom (as seen in Figure 11) would have one side, usually a thin top piece made from a 2 x 4, and end caps. The form would be laid on its side and concrete would be filled in and roughly formed into the shape of the currently unattached side. Then the side would be added and screwed into place, sandwiching the concrete. The following day, the form would be stood upright, each side and top would be carefully removed, and the beam would spend six more days curing before testing (Figure 12).

Figure 11. Measuring, cutting, and assembly.

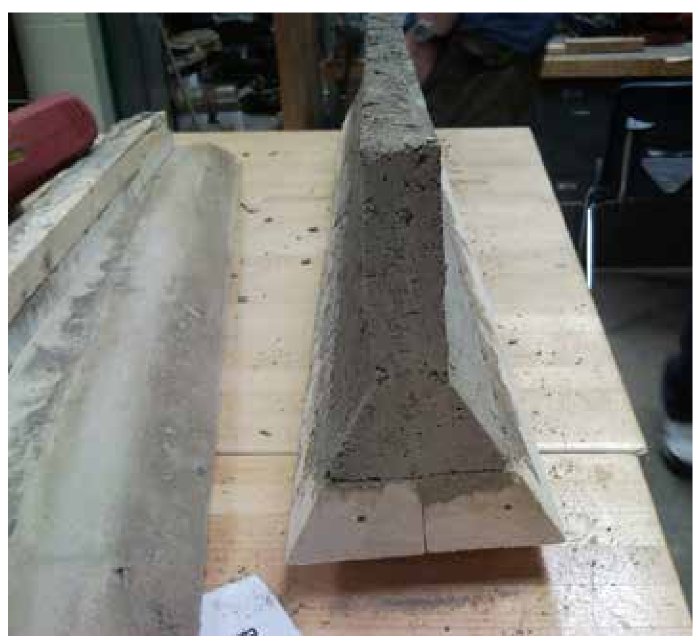


Figure 12. Form sides, top, and ends removed.

Conclusion

At this point, students should be able to design a concrete beam based on the moment of inertia about the neutral axis and optimize the beam design based on the straightforward mathematics involved. The students should also be able to design and fabricate a concrete form to produce their designed beam. The only thing left to do is test the beam and compare the calculated weight the beam should hold versus the weight held during testing. This is an activity that combines the application of mathematics and science in terms of engineering principles as well as producing and testing a design using more than just paper, cardboard, glue, and scissors.

The authors let their students take their broken beams home after testing under strict stipulations to not get in trouble. The parents would see this mass of concrete that their child was proudly displaying and boasting about in a similar way as we all used to with our birdhouses, folding stools, bookshelves, candleholders, benches, gun racks/cabinets, and many other woodshop projects.

References

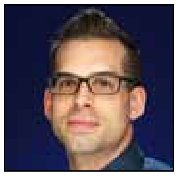
Cheng, F. H. (1997). *Statics and strength of materials* (2nd ed). Westerville, OH: Glencoe/McGraw-Hill.

Dym, C. L., Agogino, A. M., Eris, O., Frey, D. D., & Leifer, L. J. (2005). Engineering design thinking, teaching, and learning. *Journal of Engineering Education, 94*(1), 103-120.

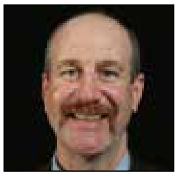
International Technology Education Association (ITEA/ITEEA). (2000/2002/2007). *Standards for technological literacy: Content for the study of technology.* Reston, VA: Author.

Morrow, H. W., Kokernak, R. P., & Morrow, H. W. (2011). *Statics and strength of materials* (7th ed.). Pearson Prentice Hall.

NGSS Lead States. (2013). *Next generation science standards: For states, by states.* Washington, DC: The National Academies Press.



Andrew J. Hughes, Ed. D., *is an assistant professor in the Designated Subjects, Career and Technical Education, and Adult Education Program, Department of Educational Leadership and Technology at the California State University of San Bernardino. He can be reached at andrew.hughes@csusb.edu.*



Chris Merrill, Ph. D., *is a professor in the Department of Technology at Illinois State University, Normal, IL. He can be reached at cpmerri@ilstu.edu.*

This is a refereed article.

Technology and Engineering Teacher Statement of Ownership