A Poset perspective for the evaluation of self-reported health of the elderly in Italy

Un metodo basato sul POSET per valutare lo stato di salute auto-percepita della popolazione anziana in Italia

E. Furfaro, L. Pagani and M. C. Zanarotti

Abstract Measuring health status is becoming a more and more relevant task, especially in relation to ageing societies. In this contribution, we first propose the use of a methodology based on the theory of Partially Ordered Sets that allows to build synthetic indicators out of a set of ordinal variables, respecting the ordinal nature of the variables included. Secondly, using survey data, we calculate two synthetic indicators to evaluate self-rated health status of the elderly population living in Italy.
Abstract *Misurare lo stato di salute è un compito molto importante soprattutto nelle società contemporanee caratterizzate da un aumento della popolazione in età anziana. In questo contributo, dapprima proponiamo l'uso di un approccio basato sulla teoria degli insiemi parzialmente ordinabili che permette di costruire indicatori sintetici a partire da variabili categoriali conservandone il naturale ordinamento. Successivamente, gli indicatori proposti sono stati utilizzati per valutare lo stato di salute della popolazione anziana residente in Italia.*

Key words: self-reported health, ordinal data, regression trees, POSET

1 Introduction

The adage 'health is wealth' is a timeless truth that becomes even more relevant in ageing societies: health is wealth for both individual well-being and for population prosperity, particularly in the face of the new challenges connected with population ageing. The state of health impacts all dimensions of individuals' life, and poor

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health conditions dramatically influence it as a whole. Individual health in turn impacts societies, for example by increasing the need for care and assistance. Getting older frequently means getting worse in health conditions so being able to measure and to monitor individual health is of crucial importance in ageing societies.

Measuring health implies a clear definition which may not be trivial. In 1948 the World Health Organization defined health as "a state of complete physical, mental and social well-being and not merely the absence of disease or infirmity" ([16]). It was a new and ambitious formulation because it overcame the definition of health as merely absence of disease. More recently this definition has been criticized and some new proposals have been made with the aim to define health taking into account new goals and new needs ([10]). What is clear when looking at the different definitions is that health is a complex phenomenon that involving different factors that affect health of individuals and/or communities. This has led to the deveolpment of many methods to measure health, different approaches, different purposes and levels of measurement ([12, 13]).

Health can be evaluated at individual level through one or more indicators or at aggregate level considering population indicators (life expectancy, mortality rate, incidence of some pathology, etc.). In the former case, measurements can be classified as "subjective", based on self-perceived health; or "objective", based on diagnosis by physicians and/or by other procedures like laboratory or screening examinations. In this contribution we focus on individuals' self-reported health measures, as they have been proven to be efficient tools for health status evluation ([11, 9]). Even in this case there are more possibilities. Here we consider the measurement of subjective health through a set of individual indicators. Specifically we refer to the SF-12 Questionnaire of Health Survey developed during the 90s in United States by the Medical Outcomes Trust of Boston ([15]). The SF-12 is a psychometric questionnaire based on twelve items widely used in internationla studies in the last decades. Items in the questionnaire are then summarized in two synthetic indices (or compiste indicators), one representing physical health (PCS) and the other one mental health (MCS). Please refer to Table 1 for details on the twelve items together with the labels used in this contribution, and the reduced eight-dimensional scale -as presented by [15]- useful for variables interpretation. PCS and MCS are obtained by aggregating the items into two composite indicators by means of a weighting system.

In this contribution we propose the synthesis of the two health indicators with the use of an alternative approach based on the Partially Ordered Set theory (POSET). This approach allows to build synthetic indicators out of a set of categorical variables without the need of any aggregative procedure and respecting the ordinal nature of the variables in the SF-12. After giving some basic background of the POSET theory and providing the details of the proposed indicators, we calculate them for evulating the health conditions of the ageing population living in Italy.

The rest of the contribution is organised as follows: Section 2 is devoted to briefly present the POSET approach; Section 3 summarises the main results on the ageing population who lives in Italy; while concluding remarks are contained in Section 4.

Table 1: The SF-12 measurement model ([15]), labels used in this contribution

Summary measures	Item	Label	Scales
Physical Health	Perceived health	X_1	General Health
	Limited activities	X_2	Physical Functioning
	Difficulties in climbing several fights of stairs	X_3	Physical Functioning
	Accomplished less due to phyisical condition	X_4	Role-Physical
	Limited work due to physical condition	X_5	Role-Physical
	Pain interferes with everyday activities	X_6	Bodily Pain
Mental Health	Accomplished less because of emotional status	X_7	Role Emotional
	Less concentrated because of emotional status	X_8	Role Emotional
	Felt calm	X_9	Mental health
	Felt full of energy	X_{10}	Vitality
	Felt sad	X_{11}	Mental Health
	Emotional status compromised social life	X_{12}	Social Functioning

2 The POSET approach for building composite indicators

The theory of Partially Ordered Sets is a well established mathematical theory that has recently been leveraged to calculate synthetic measures out of a set of ordinal variables. Its use in the calculation of synthetic measures is motivated by the fact that, differently from aggregative procedures, it preserves the ordinal nature of the variables ([5]). It has been succesfully used for producing synthetic measures of wealth, life satisfaction, gender gap and for the evaluation of frailty in the elderly population ([3, 4, 2, 14]). In this section we give some basic definitions useful for the purpose of understanding our work (for more details see, among others, [6]).

A *Partially Ordered Set* (POSET) is a finite set X with a partial order relation, i.e. a binary relation "$\leq$" satisfying the properties of *(i)* reflexivity, *(ii)* antisymmetry and *(iii)* transitivity (for more details see for instance [6]). Two elements a and b of the set X are comparable if $a \leq b$ or $b \leq a$, otherwise we say they are *incomparable*. The elements of X can hence be ordered based on the partial order relation, generating *linear extensions* (see [8] for formalisation). When two elements are incomparable, they generate more linear extensions as there is more than one way to order them.

For the purpose of our study we consider S ordinal variables, each with k_s possible responses (with $s = 1,...,S$). The elements of the set X are combinations of the values of the S ordinal variables and they are called profiles. Two profiles $\mathbf{p}_a = \{p_a^1, p_a^2, ..., p_a^S\}$ and $\mathbf{p}_b = \{p_b^1, p_b^2, ..., p_b^S\}$ are comparable if and only if $p_a^s \leq p_b^s \; \forall \, s, \; s = 1,...,S$, or viceversa. In other words, $\mathbf{p}_a$ and $\mathbf{p}_b$ are comparable if and only if the values observed on $\mathbf{p}_a$ are higher or equal than those observed on $\mathbf{p}_b$ for all variables, or viceversa. Note that in our case, the S variables are those given in Table 1 that we will call elementary variables. The profiles can then be ordered, as mentioned above, generating linear extensions. A possibility for evaluating the profiles is to set a threshold profile τ so that profiles can be classified above τ or below τ, hence creating two groups. On the different linear extensions, profiles may always be classified in the same group or they may be classified differently on the different linear extensions, representing fuzzy states. Drawing on the threshold definition and on the computation of linear extensions, different synthetics measures have been proposed for the evaluation of profiles.

In this contribution, we evaluate the *Height* of a profile that is a combination of two other measures, namely *wealth* and *severity* ([3]). The severity function provides a measure of the depth of a profile into a group. In order to calculate severity, the first step is to compute, for every linear extension, the severity function, i.e. the distance between a profile and the first element lower than τ. The distance is computed on the rank of the two objects. Let $\Omega(P)$ be the set of all linear extensions on a POSET P, the severity function is given by:

$$svr_l(\mathbf{p}) = \begin{cases} r_l(\mathbf{q}_l) - r_l(\mathbf{p}), & \text{if } \mathbf{p} \leq \tau \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

where $\mathbf{q}_l$ is the first element lower than τ, $r_l(\mathbf{q}_)$ and $r_l(\mathbf{p})$ are the ranks of profile $\mathbf{p}$ and of profile $\mathbf{q}_l$ respectively on a linear extension $l \in \Omega(P)$. The severity value of a profile is then obtained by aggregating the results observed on all linear extensions:

$$svr_l(\mathbf{p}) = \frac{1}{|\Omega(P))|} \sum_{l \in \Omega} svr_l(\mathbf{p}) \tag{2}$$

With reference to health status, severity allows for an evaluation of the intensity of poor health. High values of severity indicate that not only the profile is often classified as a poor health profile, but, when classified as poor health, it positions very far from the threshold indicating the intensity of poor health. Similarly, the wealth function provides a measure of how good is the condition of those classified as good health profiles, indicating on average how far deep into the group of good health profiles a profile is positioned.

The *Height*, that is the profiles evaluation measure we use in this contribution, is then given by the following:

$$H_\tau(\mathbf{p}) = wea(\mathbf{p}) - svr(\mathbf{p}) = \frac{1}{|\Omega(P))|} \sum_{l \in \Omega} wea_l(\mathbf{p}) - \frac{1}{|\Omega(P))|} \sum_{l \in \Omega} svr_l(\mathbf{p}) \tag{3}$$

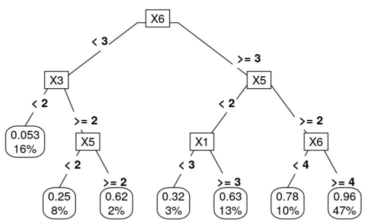
High values of H_τ correspond to profiles in good health and low values correspond those in poor health. Note that high values correspond to profiles that when classified as in good health are very far from the threshold (high values of wealth) and, if classified as poor health, are not too severe.

3 Application: evaluating health conditions of elderly population living in Italy

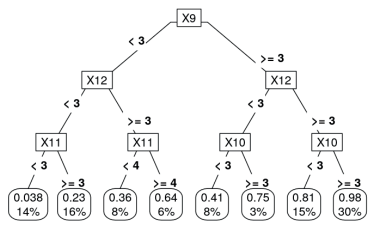
We use data from the 2013 Multipurpose Survey on Health Conditions carried out by the Italian National Institute of Statistics and we focus on people aged ≥ 65 years. The sample includes 49.811 households, for a total of 119.000 individuals, of which 27.003 are above 65 years old. Following the methodolgoy described in 2 and using the variables in the SF-12 (see Table 1), we built two indicators based on the Equation 3, one for physical health (PCS) and one for mental health (MCS). They were calculated using elementary variables $X_1 - X_6$ and $X_7 - X_{12}$ respectively and setting the threshold on the basis of external information. All the computations were carried out in the R environment, using the R package PARSEC for the computation of posetic measures ([7]).

In order to better understand which elementary variables characterise low and high values of PCS and MCS, we implemented a regression tree for each of the synthetic indicators. PCS and MCS are output variables, the elementary indicators are the regressors. Figure 1 syntheses our results, showing the percentage size of the groups obtained, along with the average value of PCS (left panel) and MCS (right panel), and the values of the elementary indicators at each splitting node. PCS and MCS were normalised to simplify interpretation. An interesting finding regard the role of X_{12} in discriminating between poor mental health profiles and good mental health profiles: in fact X_{12} represents a compromised social life that highlights the importance of social ties for healthy (mental) ageing ([1]).

Secondly, we use quantile regressions to further study the sub-groups in poorer health. Thanks to it, we investigate the role of structural and economic variables on different quantiles. In particular, we are interested in studying gender differences, territorial differences, which is a long-standing issue in Italy, and the role of social relationships which have attracted increasing interest in the context of active and health ageing ([1, 17]). We control for age, citizenship, education and type of income.



(a) Physical health indicator



(b) Mental health indicator

Fig. 1: Groups as identified by the regression tree, for PCS (left panel) and MCS (right panel). Variables labels are given in Table 1

Figure 2 and 3 show main results, with the estimated values for the coefficients (confidence intervals in the shaded area) at each quantile regression. More results are available upon request. For both PCS and MCS, the variables considered exhibit smaller coefficients on the very extreme quantiles, while the coefficients seem larger around the 20-30th percentile for physical health (Figure 2), and around the 30-40th percentile for mental health (Figure 3). This suggest that very poor health below those quantiles may be related to some other factors, such as for example having chronic diseases, and it may be independent of structural and contextual characteristics. Women generally report lower levels of mental and physical health, with significant coefficients in all percentiles, intensiying for the mentioned middle-low percentiles. Similarly, North-South territorial differences widen in such percentiles suggesting that the geographical context may play a role in fuzzy states. Being widow is also related to lower levels of both physical and mental health, with coefficients being higher in correspondance of the middle-low quantiles. Regarding living arrangements, we compared those living in couple and those who live with other people (either family members or not) with those living alone. Results highlight no differences in any quantiles with the regard to the former and poorer health

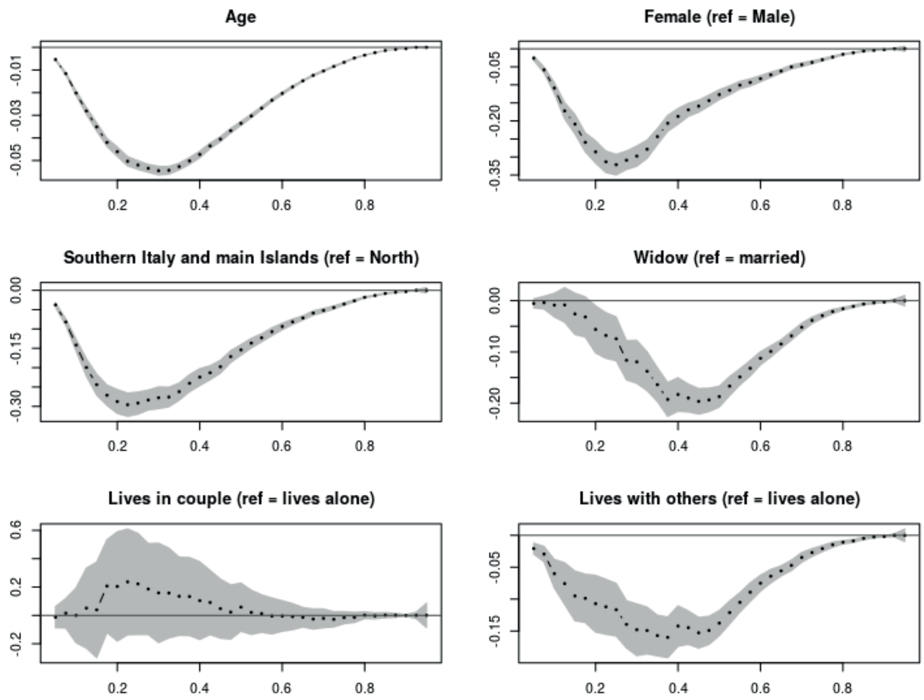


Fig. 2: Physical health indicator: estimated quantile regression coefficients at each quantile. Gray area represents the 95% confidence interval.

status for the latter. This may seem counterintuitive, but it can be interpreted as evidence of the need of people in poor health to stay with others, highlighting the need for social support.

4 Final remarks

This contribution addresses the important task of synthetically measuring health conditions. We propose the use of a methodology based on the POSET theory that allows to create synthetic indicators out of a set of ordinal variables. We build a posetic version of PCS and MCS out of the variables included in the SF-12 questionnaire, widely considered a valuable starting point for the analysis of health conditions. After defining the indicators and identifying which elementary ordinal variables discriminate between good and poor health profiles, we calculated the syntehtic indicators to provide a synthetic measure of the health status of the elderly population living in Italy.

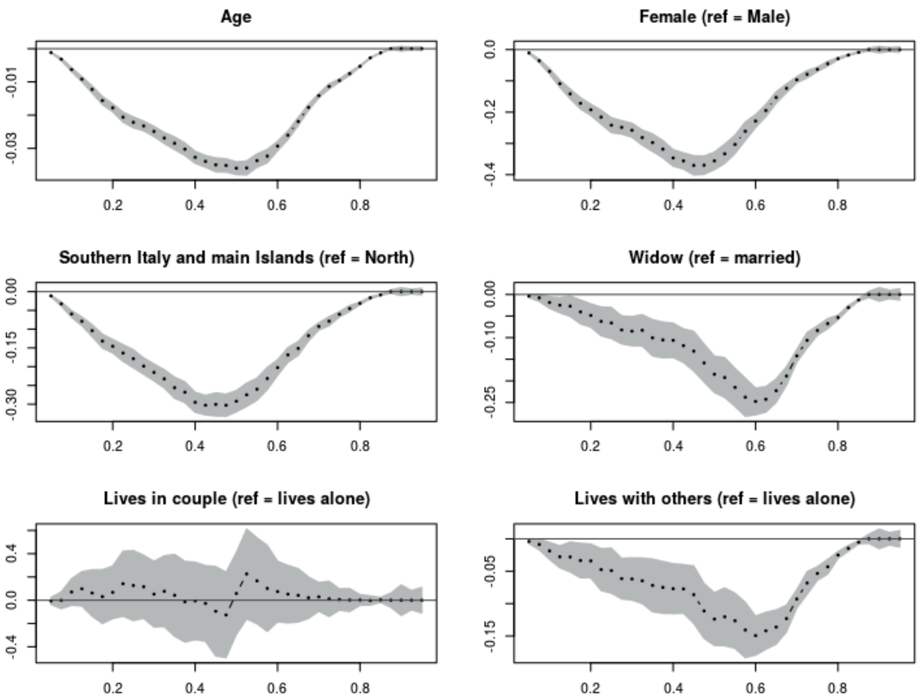


Fig. 3: Mental health indicator: estimated quantile regression coefficients at each quantile. Gray area represents the 95% confidence interval.

By means of quantile regressions, we found that the middle-low quantiles seem to be largely affected by gender and territorial differences, suggesting that it is in fuzzy situations that structural and contextual variables matter. We also looked at the social dimension, highligthing lower levels of health for widowed people and finding that people with lower health tend to be in larger households.

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