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A COMPARATIVE STUDY ON DECISION-MAKING METHODOLOGY

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RÉSUMÉ

La prise de décision (DM), un processus de détermination et de sélection de décisions alternatives en fonction des informations et des préférences des décideurs (DM), apparaît largement dans notre vie personnelle et professionnelle quotidienne. Un grand nombre de méthodes DM ont été développées pour aider les DM dans leur type unique de processus de décision. Dans cette thèse, les méthodes DM associées à deux types de processus DM sont étudiées : la prise de décision sous incertitude (DMUU) et la prise de décision multicritère (MCDM).

La DMUU doit prendre la décision lorsqu'il existe de nombreuses inconnues ou incertitudes sur le type d'états de la nature (une description complète des facteurs externes) qui pourraient se produire à l'avenir pour modifier le résultat d'une décision. La DMUU comprend deux sous-catégories : la prise de décision sous incertitude stricte (DMUSU) et la prise de décision sous risque (DMUR). Cinq méthodes classiques de DM pour DMUSU sont le principe de raison insuffisante de Laplace, le Waldimin Maximin, le regret Savage Minimax, le critère d'index pessimisme-optimisme de Hurwitz et le critère de domaine de Starr. En outre, l'examen de la relation entre un jeu à deux joueurs dans la théorie des jeux et l'équilibre DMUSU et Nash Equilibrium est également considéré comme l'une des méthodes pour résoudre le DMUSU. Les méthodes DM bien connues de DMUR sont la valeur monétaire attendue, la perte d'opportunité attendue, les états de nature les plus probables et l'utilité attendue.

Le MCDM est une sous-discipline de la recherche opérationnelle, où les DM évaluent plusieurs critères conflictuels afin de trouver la solution compromise soumise à tous les critères. Un certain nombre de méthodes DM pour MCDM sont présentes de nos jours. Le processus de hiérarchie analytique (AHP), l'élimination et le choix traduisant la réalité (ELECTRE), les méthodes d'organisation du classement des préférences pour les évaluations d'enrichissement (PROMETHEE) et la technique de préférence par ordre de similitude et de solution idéale (TOPSIS) sont les plus choisies et utilisées des méthodes parmi toutes les différentes méthodes MCDM.

Ce travail de thèse se concentre sur la présentation théorique d'une étude comparative des méthodes DM et l'évaluation des performances de différentes méthodes avec un problème de décision particulier. Cette contribution peut guider les DM à rassembler les informations relatives objectives et subjectives, à structurer le problème de décision et à sélectionner la bonne méthode de DM pour prendre la décision qui convient non seulement à leurs préférences subjectives, mais aussi aux faits objectifs.

L'étude de cas utilisée ici est la sélection du plan de construction du réseau d'égouts. Il s'agit d'un problème de décision pratique représentatif et complexe qui nécessite la qualité, l'entretien du cycle de vie et les performances du réseau d'égouts sélectionné pour répondre à la planification à long terme des futurs changements climatiques et du développement urbain.

ABSTRACT

Decision making (DM), the process of determining and selecting alternative decisions based on information and the preferences of decision makers (DMs), plays a significant role in our daily personal and professional lives. Many DM methods have been developed to assist DMs in their unique type of decision process. In this thesis, DM methods associated with two types of DM processes are studied: Decision-making under uncertainty (DMUU) and Multi-criteria decision making (MCDM).

DMUU is making a decision when there are many unknowns or uncertainties about the kinds of states of nature (a complete description of the external factors) that could occur in the future to alter the outcome of a decision. DMUU has two subcategories: decision-making under strict uncertainty (DMUSU) and decision-making under risk (DMUR). Five classic DMUSU methods are Laplace's insufficient reason principle, Wald's Maximin, Savage's Minimax regret, Hurwicz's pessimism-optimism index criterion and Starr's domain criterion. Furthermore, based on a review of the relation between a two-player game in game theory and DMUSU, Nash equilibrium is considered a method for approaching DMUSU as well. The well-known DMUR DM methods are expected monetary value, expected opportunity loss, most probable states of nature and expected utility.

MCDM is a sub-discipline of operations research, where DMs evaluate multiple conflicting criteria in order to find a compromise solution subject to all the criteria. Numerous MCDM methods exist nowadays. The Analytic Hierarchy Process (AHP), the ELimination et Choix Traduisant la REalité (ELECTRE), the Preference Ranking Organization METHod for Enrichment Evaluations (PROMETHEE) and the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) are the most employed of all the various MCDM methods.

This PhD work focuses on presenting a comparative study of DM methods theoretically and evaluating the performance of different methods on a single decision problem. This

contribution can guide DMs in gathering the relative objective and subjective information, structuring the decision problem and selecting the right DM method to make the decision that suits not only their subjective preferences, but also the objective facts.

The case study used here is the selection of a sewer network construction plan. It is a representative and complex practical decision problem that requires the quality, life-cycle maintenance and performance of the selected sewer system to meet long-term planning for future climate changes and urban development.

Keywords: Decision making under strict uncertainty, Decision making under risk, Multi-criteria decision making, Sewer network planning, Laplace's insufficient reason principle, Wald's Maximin, Savage's Minimax regret, Hurwitz's pessimism-optimism index criterion, Starr's domain criterion, Nash equilibrium, AHP, TOPSIS, ELECTRE, PROMETHEE.

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LISTE DES ABRÉVIATIONS, SIGLES ET ACRONYMES

Abréviation. L'abréviation est la forme réduite d'un mot résultant du retranchement d'une partie des lettres de ce mot. Les sigles et les acronymes sont des types d'abréviations. Certains symboles alphabétiques s'apparentent aux abréviations.

Sigle. Suite d'initiales de plusieurs mots qui forment un mot unique. Un sigle se prononce alphabétiquement, c'est-à-dire avec les noms des lettres qui le composent, ou syllabiquement, comme un mot ordinaire. Dans ce dernier cas, on l'appelle acronyme.

Acronyme. Sigle prononcé comme un mot ordinaire (ACDI, UNESCO, sida), ou un mot formé de syllabes de mots différents (AFNOR, radar, algol, pergélisol).

Liste des abréviations, sigles et acronymes apparaissant dans ce guide :

DM	Decision making
DMs	Decision makers
DMUU	Decision making under uncertainty
DMUSU	Decision making under strict uncertainty
NE	Nash equilibrium
DMUR	Decision making under risk
MCDM	Multi-criteria decision making
OR	Operations research
AHP	Analytic Hierarchy Process
TOPSIS	Technique for Order Preference by Similarity to Ideal Solution
ELECTRE	Elimination Et Choix Traduisant la REalité
PROMETHEE	Preference Ranking Organization METHod for Enrichment Evaluations

CHAPTER 1 – INTRODUCTION

1.1 Introduction

Decision making (DM), the process of determining and selecting alternative decisions based on information and the preferences of decision makers (DMs), plays a significant role in our daily personal and professional lives. Every single day people make decisions. Most are relatively insignificant; for example, whether or not to add milk to one's tea. Others are more important and require a deep analysis before choosing one alternative from all the possibilities that meets the goal and has a decent probability of success. A few examples are decision making as part of budget planning in production engineering (Keefer & Kirkwood, 1978), airport location (Layard, 1972), water resource management (Liu, Gupta, Springer, & Wagener, 2008) and career choices (Gianakos, 1999).

In general, the DM process contains three basic stages: first, structure the decision problem. This includes defining the goal or the purpose of making the decision, identifying the various available alternatives, gathering the relative data and facts about the alternatives and the decision environment. Second, select one decision-making method that suits the decision problem. Third, execute the DM method and select the right alternative to make the decision. Here, DM methods refers to techniques or algorithms that effectively gather the information, provide a good understanding of the decision problem structure and rank the alternatives to find the final solution. Many DM methods have been developed to assist DMs in their unique type of decision process.

In this thesis, DM methods associated with two types of DM processes are studied:

- Decision making under uncertainty (DMUU)
  - Decision making under strict uncertainty (DMUSU)
  - Decision making under risk (DMUR)
- Multi-criteria decision making (MCDM)

DMUU is making a decision when there are many unknowns or uncertainties about the kinds of states of nature (a complete description of the external factors) that could occur in the future to alter the outcome of a decision. In other words, the consequence of the decision is highly affected by a host of conditions beyond one's control, e.g., whether a farmer harvests his crop is highly dependent on weather conditions, or decisions about launching a new product could be influenced by market forces. Furthermore, based on the degree of uncertainty, DMUU has two subcategories: decision making under strict uncertainty (DMUSU) and decision making under risk (DMUR). "Strict uncertainty" means that the likelihood of various possible future conditions is quantitatively immeasurable. "Risk" assumes that DMs can assign a probability distribution to each state of nature based on their own experiences or historical frequencies. Five classic DMUSU methods are Laplace's insufficient reason principle (Keynes, 1921), Wald's Maximin (Wald, 1950), Savage's Minimax regret (Savage, 1972), Hurwicz's pessimism-optimism index criterion (Hurwicz, 1952) and Starr's domain criterion (Starr, 1966). They were actively developed in the early 1950s. Each method proposes different ways of handling uncertainty. As the probability distribution of states of nature can be assigned in DMUR, the well-known DM methods of DMUR are the expected monetary value, the expected opportunity loss, the most probable states of nature and the expected utility (Taghavifard, Damghani, & Moghaddam, 2009).

MCDM is a sub-discipline of operations research, where DMs evaluate multiple conflicting criteria in order to find the compromise solution subject to all the criteria. For example, when purchasing a car, price, comfort, power and fuel economy are the main criteria to consider. The criteria can be quantitative and objective, such as price, or qualitative and subjective, such as comfort. Most of the time, there is no perfect option available to suit all the criteria; for example, it is unlikely that the cheapest car is the most comfortable one. Hence, MCDM methods mainly focus on helping DMs synthesize the information to find a trade-off among the conflicting criteria. A number of MCDM methods currently exist and more are being developed (Wallenius, et al., 2008) (Ishizaka & Nemery, 2013). The Analytic Hierarchy Process (AHP) (Saaty, 1980), the ELimination Et Choix Traduisant la REalité (ELECTRE) (Benayoun, Roy, & Sussman, 1966), the

Preference Ranking Organization METHod for Enrichment Evaluations (PROMETHEE) (Brans & Vincke, 1985) and the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) (Yoon & Hwang, 1995) are the most-employed MCDM methods (Kabir, Sadiq, & Tesfamariam, 2014).

1.2 Objectives and Methodologies

Defining the correct type of decision-making process is essential and is a starting point for making a good decision. Based on the information available to DMs, they first need to think about how many external factors should be incorporated into their decision-making. If there is only one external factor, Decision Making Under Uncertainty is the right choice. Moreover, based on the DMs' knowledge of this external factor, it will be clear if it is a DMUSU or DMUR problem. If there are several different external factors, i.e., different criteria or perspectives, that DMs would like to consider in evaluating each alternative, then MCDM will be the right type of decision-making process. See Figure 1-1.

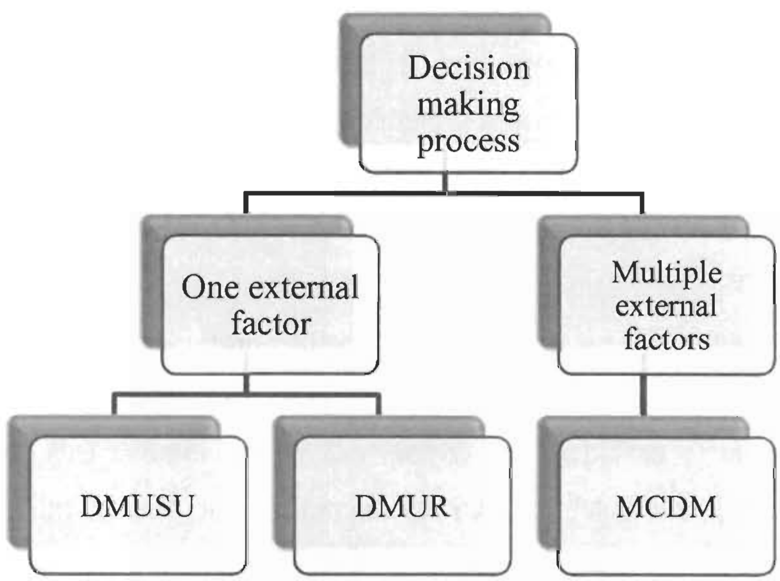

Figure 1-1: Decision-making process

Facing various DM methods corresponding to different types of DM problems, DMs are confronted with the difficult task of selecting one appropriate method, as each method has its own restrictions, particularities, preconditions and perspectives and can lead to different results when applied to an identical problem (Ishizaka & Nemery, 2013). Hence, it is worthwhile and important to present a study that can help DMs select the right decision-making method when dealing with different types of decision processes in order to find the right solution to the problem. In this way, DMs can be guided in gathering the relative objective and subjective information to structure the decision process and select the right DM method to make the decision that suits not only their subjective preferences, but also the objective facts.

To achieve this objective, the comparative study on different DM methods in this thesis is carried out via the following methodologies:

1. A full overview of the different types of decision-making processes (DMUSU, DMUR and MCDM) considered in this research is presented to clarify and distinguish them.
2. Research on the methodologies for approaching DMUSU:
    a) A full literature review and theoretical comparison of five classic methods for solving a DMUSU problem is provided in order to clearly understand each method's character, advantages and disadvantages;
    b) The relation between DMUSU and a two-player game is discussed and Nash equilibrium from game theory methodology is proposed as another option for solving DMUSU problems;
    c) All the methodologies for approaching DMUSU (five classic ones and Nash equilibrium) are applied to one particular sewer network selection problem in order to compare them during practical implementation.
3. Research on DMUR methodologies:
    a) Four well-known DMUR methodologies are explored and compared in theory. The examples of sushi restaurant planning and buying a lottery ticket are used to clearly demonstrate how to implement each method and how they differ;
    b) Expected value of perfect information is discussed in theory and a practical example of farmer's payoff is explored to explain whether DMs would be

willing to pay to get the perfect information to help them make decisions in a DMUR process.

4. Research on the methodologies for approaching MCDM:
   a) The four most commonly used MCDM methods (AHP, TOPSIS, ELECTRE and PROMETHEE) are reviewed in theory to discover each method's own limitations and particularities;
   b) AHP, TOPSIS, ELECTRE and PROMETHEE are applied to the same decision problem to evaluate and analyze the suitability of results in order to highlight the differences.
   c) During implementation, the Delphi method is used to collect all the stakeholders' opinions.
5. To summarize the above, an overall conclusion is provided to present a clear picture to DMs about how to define the types of decision processes (DMUSU, DMUR or MCDM) based on the available information. Furthermore, once the type of decision process is defined, the research can guide them in selecting a single appropriate methodology for their unique decision problem.

All the results of this research have been published or submitted via four papers listed below.

Paper 1: Literature Review in Decision Making with Uncertainty. The aim of this paper is to perform a complete literature review of all DMUU methods in order to fully understand them from a theoretical perspective, point out their advantages/disadvantages and state their particularities. Furthermore, based on a literature review of the relationship between a two-player game in game theory and DMUSU, this work proposes a link between the basic concepts in game theory and decision making and Nash equilibrium (Nash, 1950) (Nash, 1951) is considered one of the methods for approaching DMUSU. (Published in 12^{e} édition du Congrès international de Génie industriel, May 2017).

Paper 2: Decision Making Under Strict Uncertainty: Case Study in Sewer Network Planning. The goal of this research is to implement DMUSU methods and Nash equilibrium in a real-life project: selecting a suitable sewer network construction plan and comparing each method in a practical way based on the different results from each method. (Published in *International Journal of Electrical, Computer, Energetic, Electronic and Communication Engineering*, *11*(7), 2017).

Paper 3: Selecting Sewer Network Plans Using the Analytic Hierarchy Process. This work is the first step in the research on the direction of MCDM. In this paper, a single popular MCDM method is explained and implemented to discover its advantages and limitations. (Published in the 47th International Conference on Computers & Industrial Engineering, October 2017).

Paper 4: Comparison of multi-criteria group decision-making methods for urban sewer network plan selection. The paper is aimed at providing an intuitive explanation and interpretation of the most-employed MCDM methods (AHP, ELECTRE, PROMETHEE, TOPSIS). It examines four MCDM methods through a comparative study of their implementation in an urban sewer network group decision problem (forthcoming).

1.3 Organization of the Thesis

The thesis is organised as follows: Chapter 1 is the introduction, which provides a general background on DM processes to introduce the motivations, objectives and methodologies of this research. Chapter 2 contributes a literature review of the DM methods in DMUSU, DMUR, game theory and their relation. Classic DMUSU methods and their axiomatic comparison are described in detail and illustrated with examples. In game theory, the basic concepts of constituting a game and game types are introduced, followed by the description of the prisoner's dilemma, matching pennies and the pirate game. Then Nash equilibrium, a solution concept in game theory, is illustrated with examples. Using three basic elements of decision-making problems and the basic concepts of a game, a decision-making problem can be converted to a two-player game where player 1 is the decision maker and player 2 is nature. A detailed comparison of DMUR methodologies is also provided. Chapter 3 compares five classic DMUSU methods in a more practical way than axiomatic comparison. It applies each DM method to a practical sewer network planning example; results from different methods are discussed and analyzed. Moreover, NE in game theory is applied, as it is another candidate for DMUSU based on the link between DMUSU and a two-player game. Chapter 4 and Chapter 5 start the work on the topic of MCDM, where Chapter 4 proposes three theoretical categories of MCDM methods and four popular MCDM methods from each category – AHP, ELECTRE, PROMETHEE and TOPSIS – are presented. Meanwhile, Chapter 5 presents a comparative study of these methods in a practical way by applying them to a real sewer network planning case study and analyzing the suitability of results in order to highlight the differences and lead to meaningful conclusions. Chapter 6 summarizes this PhD work through concluding remarks, contributions and ideas for future research.

CHAPTER 2 – LITERATURE REVIEW ON DECISION MAKING UNDER UNCERTAINTY

2.1 Introduction

In reality, only very few decisions are made with absolute certainty. It is seldom possible for a decision maker to collect all the information and data surrounding a decision problem, thus most decisions are made with a certain risk. Based on the decision maker's knowledge of the information and data, decision making under uncertainty problems are divided into two categories: decision making under strict uncertainty (DMUSU) and decision making under risk (DMUR) (French, 1988).

These categories are limited to a decision maker facing an inert environment. However, there are situations where the environment can actively work against the decision maker. These situations belong to the realm of game theory. Game theory is considered the theory of interdependent decision making, where the outcome is related to the decisions of two or more players and no single player has full control over the outcome.

While the literature has studied different solution concepts for game theory, such as the Nash equilibrium, it is surprising that the link between decision making and game theory remains relatively uncharted. This chapter provides a literature review of these two domains and proposes a structure to better link them.

The rest of the chapter is as follows. Section 2.2 covers the decision-making literature, from formalizing a decision-making problem to describing the existing criteria. Section 2.3 covers game theory literature. Section 2.4 links decision making problems with game theory. Section 2.5 presents the conclusion and potential future work.

2.2 Decision Making Under Uncertainty

2.2.1 Decision Table

Before launching the DM process, DMs need to specify the relevant actions, states and outcomes (Peterson, 2009). In short, states (also called states of nature) refer to a complete description of the external factors that may affect the decision maker's preference for a certain action. Actions in a DM problem are considered alternative decisions, one of which is the solution to the initial problem. Outcomes are the consequences of all the possible actions under a given set of states of nature, which ultimately help decision makers to figure out which action to choose. The consequence of any decision is determined not just by the decision itself but also by a number of states of nature.

Let's assume that $d_1, d_2, \ldots, d_m$ denote the actions or decision alternatives available to the decision maker, the possible states of nature are denoted by $s_1, s_2, \ldots, s_n$, and a_{ij} represents the outcome that is the consequence of selecting decision d_i when s_j is the state, it can be a numerical value, e.g., payoff. Thus, the process can be summarized as in Table 2-1.

Table 2-1: Decision table

Consequences		**States of Nature**			
		s_1	s_2	$\cdots$	s_n
d_1		a_{11}	a_{12}	$\cdots$	a_{1n}
d_2	**Actions**	a_{21}	a_{22}	$\cdots$	a_{2n}
.		.	.	$\cdots$	.
.		.	.	$\cdots$	.
.		.	.	$\cdots$	.
d_m		a_{m1}	a_{m2}	$\cdots$	a_{mn}

The decision table clearly presents every possible combination of alternatives and states of nature. The outcomes form a $m \times n$ dimensional matrix $A = (a_{ij})_{m \times n}$ that is called the decision matrix; it helps the decision maker to visualize the decision problem and facilitates the decision-making process.

Let us consider a classic example from Savage (1972). Person A wants to make an omelette and has just broken five good eggs into a bowl. Person B would like to break the sixth egg and finish the omelette. Person B can either add the sixth egg into the bowl or not add it. With the condition of the sixth egg (good or rotten), they can have a six-egg omelette or a five-egg omelette, or no omelette. Clearly, in this example, the states of nature are the condition of the sixth egg, the alternative acts are adding the sixth egg into the bowl or not adding it, the outcomes are what kind of omelette they can have. Table 2-2 is the decision table for this example.

Table 2-2: Decision table for Savage omelette decision problem

	States of Nature	
	Good	**Rotten**
Add into bowl	Six-egg omelette	No omelette
Not add into bowl	Five-egg omelette	Five-egg omelette

2.2.2 Category

Most problems in DM fall into a specific category according to DMs' knowledge of the state of nature (French, 1988): DMUSU and DMUR.

DMUSU means that the decision maker has no information about states of nature. He is not unaware of the true states, but he cannot quantify his uncertainty in any way. He can only prepare an exhaustive list of possible states of the world. Let us take the example of the roll of dice where one must use skewed dice. The probability distribution over these skewed dice is unknown. In this example, the outcome is much more difficult to predict. The decision maker has no knowledge about the states of nature and/or cannot quantify their distribution.

DMUR is a situation where a decision maker does not know the true state of nature for certain, but can assign a probability distribution $(p(s_1), p(s_2), \dots, p(s_n))$ to each state of nature, where each state s_j describes a possible state of the world and $s_1, s_2, \dots, s_n$ is an exhaustive list of the possibilities. Think here of an unbiased dice. The exact result is unknown, but the probability distribution over the possible outcome is known. As such, the outcome remains unpredictable but the decision is based on known probabilities. The problems of decision making under risk first appeared in the analysis of gambling.

2.2.3 DMUSU Methods

Consider the following type of DMUSU problem. Let $d_1, d_2, \dots, d_m$ denote the decision alternatives available to the decision maker. The possible states of nature are denoted by $s_1, s_2, \dots, s_n$. Every specific combination of a decision d_i and a state of nature s_j has a particular payoff value $a_{ij} \in \mathbb{R}$ with $\mathbb{R}$ denoting the real numbers. The outcomes form a $(m \times n)$ dimensional payoff matrix $A = \left(a_{ij}\right)$.

In the early 1950s, there was an active discussion about methods for decision making under uncertainty. Five classic decision methods have been proposed to solve the problem of decision making under strict uncertainty, which are Laplace's insufficient reason criterion, Wald's maximin criterion, Hurwicz's pessimism-optimism index criterion, Savage's minimax regret criterion and Starr's Domain criterion. A brief introduction of each method follows.

2.2.3.1 Laplace's principle of insufficient reason

In a situation where the probabilities of the different possible states of nature are unknown, Laplace's criterion assumes that they are all equal. Thus if the decision maker chooses the i^{th} row, his expectation is given by the average $(a_{i1} + \cdots + a_{in})/n$, and he should choose the row for which this average is maximized. The alternative chosen by using the Laplace method is

$$\mathrm{d}^* = \max_i \left\{\frac{1}{n}\sum_{j=1}^{n} a_{ij}\right\} \text{ where } i = 1, \dots, m. \tag{2.1}$$

Laplace (1825) argued that "knowing nothing at all about the true state of nature" is equivalent to "all states having equal probability". This criterion is also known as the principle of indifference (Keynes, 1921). With this assumption, the decision maker can compute the average payoff for each row (the sum of the possible consequences of each alternative is divided by the number of states of nature) and then select the alternative that has the highest row average.

When DMs assume that all states of nature are equally likely, the problem shifts from uncertainty to risk. The advantage of this approach is that it transforms a difficult problem into a relatively simple one through the use of probability theory. However, with this assumption, a major drawback of this criterion is that the state space must be constructed in order to be amenable to a uniform probability distribution (Sniedovich, 2007).

2.2.3.2 Wald's Maximin

The idea behind this method is to obtain the most robust possible outcome (Wald, 1950). In short, if the player chooses the i^{th} row, then his payoff will certainly be at least $\min_j a_{ij}$. The safest possible course of action is therefore to choose a row for which $\min_j a_{ij}$ is maximized. Thus, the alternative selected (d^*) in Wald's Maximin criterion is

$$\mathrm{d}^* = \max_i \min_j a_{ij}, \text{ where } i = 1, \dots, m \text{ and } j = 1, \dots, n. \tag{2.2}$$

Wald's maximin is the rule of choosing the "best of the worst". It evaluates each decision by its associated minimum possible return. Then the decision that yields the maximum value of minimum returns (maximin) is selected.

Note that Wald's maximin model of uncertainty is extremely conservative. It does not provide a faithful representation of how we operate in reality. It may lead to exceedingly costly solutions resulting from over-protection against uncertainty.

2.2.3.3 Savage's Minimax regret criterion

Let us define $r_{ij} = \max_{k=1,\dots,m} a_{kj} - a_{ij}$ for all i, j, and a regret matrix $R = (r_{ij})$ that measures the difference between the payoff that could have been obtained if the true state of nature had been known and the payoff that is actually obtained. Now apply the Wald minimax criterion to regret matrix R. That is, choose a row for which $\max_j r_{ij}$ is minimized. Thus, the decision in terms of Savage Minimax regret is:

$$\mathrm{d}^* = \min_i \{\max_j \{r_{ij}\}\}, \text{where } i = 1, \dots, m \text{ and } j = 1, \dots, n. \tag{2.3}$$

Savage (Savage, 1951) argued that by using the values payoff a_{ij} to guide choice, the decision maker is actually comparing the value of the consequence of an action under one state of nature with the values of all other consequences, whatever states of nature they occur under. Nevertheless, the actual state of nature is beyond the control of the decision maker. The consequence of an action should only be compared with the consequences of other actions under the same state of nature. A particular consequence a_{ij} may be poor in the context of the complete decision table, but it may be the best consequence that can result from any action if s_j is the true state. Thus, Savage defined the regret of a consequence $r_{ij} = \max_{k=1,\dots,m} a_{kj} - a_{ij}$.

The regret matrix only reflects the difference between each payoff and the best possible payoff in a column; hence, the disadvantage of Savage's minimax regret criterion is that it does not consider the row differences.

2.2.3.4 Hurwicz's pessimism-optimism index criterion

Hurwicz's criterion (Hurwicz, 1951) (Hurwicz, 1952) is defined as follows. Select a constant $0 \leq \alpha \leq 1,$ which is a coefficient of the player's optimism. For each row $i,$ let a_i denote the smallest component and A_i the largest, then Hurwicz's measurement H_i is defined as:

$$H_i = \alpha A_i + (1 - \alpha) a_i \text{ where } i = 1, \cdots, m. \quad (2.4)$$

And the decision is obtained where:

$$\mathrm{d}^* = \max_i \{ H_i \} \quad (2.5)$$

In Hurwicz's criterion, the decision maker considers both the best and the worst possible results, weighted according to the decision maker's attitude (optimistic or pessimistic) towards the decision. The weighting is made using a constant, named the coefficient of the optimist ($0 \leq \alpha \leq 1$). When $\alpha = 1$, then the decision maker is completely optimistic and Hurwicz's criterion is reduced to the minimax method; when $\alpha = 0$, the decision maker is pessimistic and Hurwicz's criterion becomes Wald's maximin.

The formula of Hurwicz's measurement H_i shows that this criterion only considers the highest and the lowest payoff for each alternative. It does not take other non-extreme payoffs into account. Therefore, two decisions with the same minimal and maximal profits always obtain an identical Hurwicz's measurement, even if one of them contains many small payoffs and the other one has many high payoffs (Gaspars-Wieloch, 2014).

2.2.3.5 Starr's Domain

Starr introduced the Domain method for DMUSU in 1963 (Starr, 1963). While its philosophical foundation and its usefulness are well known (Schneller & Sphicas, 1983), it remains relatively unpopular compared to the previous methods.

Define the set D (the domain) of all possible probability distributions associated with the states of nature $s_j,\ j = 1, \dots, n$, as $D = \{p = (p_j) \in R_+^n \mid \sum p_j = 1\}$. This set is called the fundamental probability simplex (FPS). For any given distribution p, we may define the expected monetary value of the i^{th} decision:

$$E^p(d_i) = \sum_{j=1}^{n} p_j a_{ij} \qquad (2.6)$$

Then

$$D_i = \{p \in D | E^p(d_i) \geq E^p(d_k)\ \forall\ k \neq i\} \qquad (2.7)$$

is the set of all probability distributions p for which the i^{th} decision is chosen according to the Bayesian expected value criterion. Let $V(D_i)$ denote the volume of the set D_i. In Starr's criterion, the r^{th} decision is the one to choose if $V(D_r) \geq V(D_i)\ \ \forall i\ \neq r$. In other words, Starr's criterion selects the decision that is most likely to have a higher expected payoff value than all the others.

When the number of states of nature $n \leq 3$, the volume can be computed by graphical method. For $n > 3$, alternatively, one can use the Monte-Carlo sampling algorithm to approximate the volume. Cohen and Hickey (1979) present an algorithm that can find exact convex polyhedral volumes. Starr (1966) also proposes using simulation with random sampling of points in the FPS. Although there are algorithms that can rapidly approximate large-dimension volume, it remains difficult for decision makers to clearly understand this approach. As such, the main drawback for DMs is the ease of appropriation.

2.2.4 Axiomatic Comparison for DMUSU Methods

Consider a decision-making problem in Table 2-3. Laplace's insufficient reason chooses d_1, Wald's Maximin chooses d_2, Savage's Minimax chooses d_4, Hurwicz's criterion chooses d_2 if $\alpha < \frac{1}{4}$ and d_3 if $\alpha > \frac{1}{4}$ and Starr's Domain chooses d_1.

Table 2-3: Milnor's example (Milnor, 1954)

	Decision table			
	s_1	s_2	s_3	s_4
d_1	2	2	0	1
d_2	1	1	1	1
d_3	0	4	0	0
d_4	1	3	0	0

These five classic DMUSU methods are quite different in their definition and furthermore can provide different results for the same decision problem. The differences among them have been revealed by Milnor's axioms (Milnor, 1954). He presents 10 axioms, which are considered requirements for an ideal and reasonable decision-making method. He proves the compatibility of Laplace, Wald, Hurwicz and Savage with these 10 axioms. The axiomatic characterization of Starr's domain criterion with Milnor's 10 axioms has been discussed in Schneller and Sphicas (1983).

Milnor's 10 axioms are defined below:

- AXIOM 1. Ordering. The criterion should impose a complete order $\geq$ on the rows.
- AXIOM 2. Symmetry. The order is independent of the labelling of the rows and columns.
- AXIOM 3. Strong Domination. If for every j, $a_{i_1 j} > a_{i_2 j}$ then $d_{i_1} \geq d_{i_2}$
- AXIOM 4. Continuity. If the matrices $(a_{ij})^k$ converge componentwise to (a_{ij}) and if for every k, $d_{i_1}^k > d_{i_2}^k$ then $d_{i_1} \geq d_{i_2}$

- AXIOM 5. Matrix Linearity. The ordering relation is unchanged if the matrix (a_{ij}) is transformed to (b_{ij}) by the linear transformation $b_{ij} = wa_{ij} + u,\ w > 0$.
- AXIOM 6. Row Adjunction. The order of "old" strategies of (a_{ij}) is not changed by adjoining a new strategy (row) to (a_{ij}).
- AXIOM 7. Column Additivity. The order is not changed if a constant value is added to every entry in a column of (a_{ij}).
- AXIOM 8. Column Duplication. The order is unchanged if a new state of nature column, identical to an old column, is adjoined to (a_{ij}).
- AXIOM 9. Convexity. If there are three strategies, d_{i_1}, d_{i_2} and d_{i_3}, such that d_{i_1} and d_{i_2} are equivalent under the order of the criterion, and d_{i_3} obeys the property that $d_{i_3} = (d_{i_1} + d_{i_2})/2$ for each j, then d_{i_3} is equivalent to d_{i_1} and d_{i_2}.
- AXIOM 10. Dominated Row Adjunction. The order of the "old" strategies is not changed by adjoining a new dominated strategy (row), providing that no component of this new row is greater than the corresponding components of all old rows.

Milnor's summary of the relation between the ten axioms and five classic criteria is in Table 2-4. The √ symbol indicates that the corresponding axiom and criteria are compatible. Each criterion is characterized by the axioms marked √√. It is shown that none of the five classic criteria have all ten axioms. Wald's criterion fails Axiom 7, Hurwicz's fails Axiom 7 and Axiom 9, Savage's fails Axiom 6, Laplace's fails Axiom 8, Starr's domain fails Axiom 6, Axiom 7 and Axiom 8. The axiomatic approach theoretically points out each classic criterion's drawbacks.

Table 2-4: Axioms

Axioms	Laplace	Wald	Hurwicz	Savage	Starr
1. Ordering	√√	√√	√√	√√	√√
2. Symmetry	√√	√√	√√	√√	√√
3. Strong Domination	√√	√√	√√	√√	√√
4. Continuity	√	√√	√√	√√	√√
5. Linearity	√	√	√√	√	√√
6. Row adjunction	√√	√√	√√		
7. Column additivity	√√			√√	
8. Column duplication		√√	√√	√√	
9. Convexity	√	√√		√√	√√
10. Dominated row adjunction	√	√	√	√√	√√

Definitions of all classic DMUSU methods and their axiomatic characterization have been introduced. Laplace's insufficient reason transfers a DMUSU problem into an easy DMUR problem; however, an obvious drawback to this criterion is that it is very sensitive to how states are individuated. Wald's Maximin and Hurwicz's criterion focus only on extreme payoffs to the exclusion of others, while Savage's Minimax considers all payoffs, but does not have the ability to factor the raw differences. Starr's Domain runs into complexity of computation when there are more than three states.

2.2.5 DMUR Methods

When the decision maker has some knowledge about the states of nature, s/he can assign subjective probability estimates for the occurrence of each state. In such cases, the problem is classified as decision making with risk (Rowe, 1988). These probabilities may be subjective or they may reflect historical frequencies. Here, the same notations are used as in the previous section for decision alternatives $d_1, d_2, \dots, d_m$, states of nature $s_1, s_2, \dots, s_n$, and $m \times n$ dimensional decision matrix $A = (a_{ij})$ where a_{ij} is the outcome of decision d_i associated with state of nature s_j. Furthermore, let us use

$(p(s_1), p(s_2), \dots, p(s_n))$ to describe the probability distribution of the states of nature. Decision rules for approaching DMUR have been discussed in the literature (Taghavifard, Damghani, & Moghaddam, 2009).

2.2.5.1 The Expected Monetary Value rule

We consider decision matrix $A = (a_{ij})$ the monetary payoff matrix. The Expected Monetary Value (EMV) is computed by multiplying each monetary value (payoff) by the probability for the relevant state of nature and summing the results. This value is computed for each alternative, and the one with the highest value is selected as the final decision, i.e.

$$\text{EMV}_i = \sum_{j=1}^{n} p(s_j) a_{ij}\text{, where } i = 1, \dots, m. \tag{2.8}$$

Thus, the decision chosen according to the expected monetary value principle is

$$\text{d}^* = \max_i \{\text{EMV}_i\}. \tag{2.9}$$

The principle of EMV remains the most useful of all the decision rules for DMUR. Here is an example of a DMUR problem solved by this method. Consider the following DMUR problem: a sushi restaurant needs to decide how much sushi (quantified by small amount, medium amount or large amount) it needs to make every day. Its profit depends on demand that can be low, moderate, or high. The probability of the demand is $0.3, 0.5, 0.2$. Table 2-5 shows the profit value per day (in \$) for the possible situations.

Table 2-5: Sushi Restaurant Payoff Matrix

	Low (p = 0.3)	**Moderate (p = 0.5)**	**High (p = 0.2)**
Small	5000	5000	5000
Medium	4200	5200	5200
Large	3400	4400	5400

$$\text{EMV (small)} = 0.3 * 5000 + 0.5 * 5000 + 0.2 * 5000 = 5000;$$

$$\text{EMV (medium)} = 0.3 * 4200 + 0.5 * 5200 + 0.2 * 5200 = 4900;$$
$$\text{EMV (large)} = 0.3 * 3400 + 0.5 * 4400 + 0.2 * 5400 = 4300.$$

Therefore, according to the EMV rule, the small amount of sushi should be chosen.

2.2.5.2 The Expected Opportunity Loss Rule

The principle of Expected Opportunity Loss (EOL) is nearly identical to the EMV approach, except that instead of payoff matrix $A = (a_{ij})$, the opportunity loss (or regrets) matrix $R = (r_{ij})$ where $r_{ij} = \max_{k=1,\dots,m} a_{kj} - a_{ij}$ for all i, j is used. The expected opportunity loss is computed for each alternative and the alternative with the smallest expected loss is selected as the final choice, i.e.

$$\mathrm{EOL_i} = \sum_{j=1}^{n} p(s_j) r_{ij} \text{ where } i = 1, \dots, m. \tag{2.10}$$

Thus, the decision using the expected opportunity loss principle is

$$\mathrm{d}^* = \min_i \{\mathrm{EOL_i}\}. \tag{2.11}$$

The regret matrix for Table 2-5 is shown in Table 2-6:

Table 2-6: Sushi Restaurant Regret Matrix

	Low (p = 0.3)	**Moderate (p = 0.5)**	**High (p = 0.2)**
Small	0	200	400
Medium	800	0	200
Large	1600	800	0

The EOL for each row is:

$$\text{EOL (small)} = 0.3 * 0 + 0.5 * 200 + 0.2 * 400 = 180;$$
$$\text{EOL (medium)} = 0.3 * 800 + 0.5 * 0 + 0.2 * 200 = 280;$$
$$\text{EOL (large)} = 0.3 * 1600 + 0.5 * 800 + 0.2 * 0 = 880.$$

The smallest EOL is 180. Hence, making the small amount of sushi is the decision to be taken.

The EOL approach resulted in the same alternative as the EMV approach. The two methods always result in the same choice, because maximizing the payoffs is equivalent to minimizing the opportunity loss.

2.2.5.3 The Most Probable States of Nature Rule

In this decision rule, only the state of nature with the highest probability is taken into account, and in that column, the alternative with the biggest payoff is the final decision, i.e.

$$d^* = \max_{i=1,\cdots,m} \{a_{ik}\} \tag{2.12}$$

where k is the state of nature index, which has the highest probability: $p(s_k) = \max_{j=1,\cdots,n} p(s_j)$.

According to this decision rule, for the example in Table 2-5, the state of moderate demand has the highest probability. In that column, the best profit is located in the second row, thus the alternative selected is to produce the medium amount of sushi.

Since the most probable states of nature rule takes only one uncertain state of nature into account it may lead to bad decisions.

2.2.5.4 The Expected Utility Rule

Consider the following DMUR problem: there are two types of lottery, wherein Lottery A guarantees you receive one million dollars and Lottery B entitles you to a fifty per cent chance of winning either three million dollars or nothing. See Table 2-7.

Table 2-7: Buying Lottery tickets

	50%	50%
Lottery A	1 million dollars	1 million dollars
Lottery B	3 million dollars	0

The expected monetary values for the two lotteries are:

$$\text{EMV(Lottery A)} = 50\% \cdot 1 + 50\% \cdot 1 = 1 \text{ million dollars;}$$
$$\text{EMV(Lottery B)} = 50\% \cdot 3 + 50\% \cdot 0 = 1.5 \text{ million dollars.}$$

$\text{EMV(Lottery A)} < \text{EMV(Lottery B)}$, thus, the EMV principle dictates buying a ticket for lottery B. However, many of us would prefer lottery A, where we are sure to have one million dollars.

When dealing with a risky decision problem (e.g., the decision can only be made once or the amounts of money involved in the problem are big), the expected monetary value criterion cannot encompass the full range of reasoning behind a decision as a human would. Thus, the decision dictated by EMV may be different from what the decision maker himself would choose. In this case, it is helpful to introduce the concept of utility.

Utility is an abstract concept that cannot be directly observed. Utility represents the subjective attitude of the individual to risk, it implies how valuable the outcome is from the decision maker's point of view (Peterson, 2009). We use $u(a_{ij})$ to present the utility value of outcome a_{ij}. The principle of expected utility (EU) is obtained from the principle of EMV by replacing the monetary value a_{ij} by its utility $u(a_{ij})$, i.e.:

$$\mathrm{EU}_i = \sum_{j=1}^{n} p(s_j)u(a_{ij}) \text{ where } i = 1, \dots, m. \tag{2.13}$$

Thus, the chosen decision according to the expected utility principle is

$$\mathrm{d}^* = \max_i\{\mathrm{EU}_i\}. \tag{2.14}$$

Back to the example in Table 2-7, suppose that the lottery ticket buyer himself expressed the utilities of the outcomes with the following:

$$\mathrm{u}(1 \text{ million dollars}) = 0.7;$$
$$\mathrm{u}(3 \text{ million dollars}) = 1;$$
$$\mathrm{u}(0 \text{ million dollars}) = 0.$$

Therefore, the expected utility values for the two lotteries are:

$$\mathrm{EU}(\text{Lottery A}) = 50\% * 0.7 + 50\% * 0.7 = 0.7;$$
$$\mathrm{EU}(\text{Lottery B}) = 50\% * 1 + 50\% * 0 = 0.5.$$

$\mathrm{EU}(\text{Lottery A}) > \mathrm{EU}(\text{Lottery B})$, therefore, the EU principle dictates that buying a ticket for lottery A is the better option.

In summary, the computation of the four decision rules for DMUR is similar. The difference is that each decision rule maximizes or minimizes different objects, i.e. the expected monetary value, the expected opportunity loss, the expected utility. The decision maker needs to choose which object they want to consider based on the property of each individual DMUR problem.

2.2.6 Expected Value of Perfect Information

In DMUR, the probabilities of the states of nature represent the decision maker's degree of uncertainty and personal judgment on the occurrence of each state, but which state will actually occur when a decision alternative is applied is still unknown. Knowledge of when each state will actually happen, known as perfect information for decision making, can help the decision maker to choose the most profitable alternative every time. In decision theory, the expected value of perfect information (EVPI) is the amount that the decision maker would be willing to pay in order to get the perfect information (Hubbard, 2007).

For a DMUR problem, when there is no knowledge of the perfect information, the decision maker will choose the decision with the largest EMV; hence, the expected value without perfect information (EV) is:

$$\mathrm{EV} = \max_{i}\{\mathrm{EMV}_i\}, \text{where } \mathrm{EMV}_i = \sum_{j=1}^{n} p(s_j)a_{ij}\,. \qquad (2.15)$$

If the decision maker had perfect information, s/he would choose the decision with the best payoff for each specific state. Thus, the expected value with perfect information (EV|PI) is defined by multiplying the best outcome in each column by its probability and summing the results:

$$\mathrm{EV|PI} = \sum_j p_j(\max_j a_{ij}). \qquad (2.16)$$

The difference between EV|PI and EV is called the expected value of perfect information (EVPI):$\mathrm{EVPI} = \mathrm{EV|PI} - \mathrm{EV}$.

Hence, EVPI indicates how much more value the decision maker can get by knowing perfect information. If the decision maker is offered perfect information for a price higher than EVPI, it is better for him to refuse it (Riggs, Rentz, Kahl, & West, 1986).

Let us present one example from Quirk (Quirk, 1976) and compute the expected value of perfect information. Suppose a farmer can harvest his entire crop today at a cost of \$10,000 or half today, half tomorrow at a cost of \$2,500 per day. The harvested crop is worth \$50,000. The payoff decision matrix for this problem is shown in Table 2-8.

Table 2-8: Farmer's payoff

States of Nature / Decisions	Heavy rain tomorrow p = 55%	No heavy rain tomorrow p = 45%
Decision A: Harvest all today	\$40,000	\$40,000
Decision B: Harvest over two days	\$22,500	\$45,000

Let's assume the probability of heavy rain tomorrow is 55%, hence 45% for no heavy rain tomorrow.

$$\mathrm{EMV_A} = 0.55 * (\$40000) + 0.45 * (\$40{,}000) = \$40{,}000;$$
$$\mathrm{EMV_B} = 0.55 * (\$22500) + 0.45 * (\$45{,}000) = \$32{,}625;$$
$$\mathrm{EV} = \max_i(\mathrm{EMV_A}, \mathrm{EMV_B}) = \$40{,}000;$$
$$\mathrm{EV|PI} = 0.55 * \$40{,}000 + 0.45 * \$45{,}000 = \$42{,}250.$$

Hence, the expected value of perfect information is: $\mathrm{EVPI} = \mathrm{EV|PI} - \mathrm{EV} = \$2{,}250$.

The conclusion is that if someone provides the accurate weather forecast for tomorrow at a price of less than \$2,250, the farmer will want to purchase this information.

2.3 Game Theory

Game theory is a mathematical study of a strategy-choosing situation (i.e. game), where each player's strategy choice interacts with the other's. Thus, game theory is considered the theory of interdependent decision making, where the outcome is related to the decisions of two or more players and no single player has full control over the outcome.

Considering decision making problems as a game has been explored in the literature (Luce & Raiffa, 1957) (Kelly, 2003) (Aliprantis & Chakrabarti, 2000).

Game theory has been widely used in economics (Friedman, 1998), psychology (Camerer, 2003) and political science (Morrow, 1994) as well as logistics (Reyes, 2005), computer science (Shoham, 2008), biology (Durlauf & Blume, 2010) and so on. This subject originated from zero-sum games, in which the gains of one player are exactly equal to the losses of the others. John von Neumann first established game theory as a unique field in his 1928 paper (von Neumann, 1928). Later, his 1944 book *Theory of Games and Economic Behavior* (von Neumann & Morgenstern, 1944) came to be considered the ground-breaking text that created the interdisciplinary research field of game theory (Mirowski, 1992).

2.3.1 Basic Concepts

The basic concepts are the features that constitute a game. Here we briefly give their definitions.

- **Players:** participants who choose a strategy in a game.
- **Strategies per player:** each player makes his/her choice from a set of possible actions, known as pure strategies. The set of pure strategies available to each player is called a strategy set.
- **Payoffs:** the outcome received by a player after his/her strategy choice or strategy combination.

2.3.2 Game Types

2.3.2.1 Cooperative/Non-cooperative game

A cooperative game is where the players can form and respect mutually binding agreements. For example, the legal system requires each player to respect his or her agreements. Games that are not cooperative are known as non-cooperative games, i.e., players cannot keep their agreements and act independently.

2.3.2.2 Zero/non-zero sum game

In a zero-sum game, you win exactly as much as your opponent(s) loses. The total benefit to all players in the game, for every combination of strategies, always adds up to zero. Typical examples are casino games and classic board games like Go and chess. Non-zero-sum games are where a gain by one player does not necessarily correspond to a loss by another; the total benefit to all players is not zero.

2.3.2.3 Simultaneous/Sequential game

In simultaneous games, all players choose their strategy at the same time, or if they do not choose at the same time, the players who choose later do not know the choices of the players who chose earlier (making them effectively simultaneous). A typical example of a simultaneous game is Rock-Paper-Scissors. In sequential games (or dynamic games), players who choose later have some knowledge of earlier actions. It does not need to be perfect information about every previous action; it might be very little information. Chess is a sequential game.

2.3.2.4 Perfect information and imperfect information

Perfect-information games are a subset of sequential games. A perfect-information game is where all the players have full information about the actions previously chosen by the other players. Chess is a perfect-information game. Simultaneous games obviously cannot be games of perfect information. Games that are not perfect-information games are known as imperfect-information games.

2.3.2.5 Pure and mixed strategy

A pure strategy provides a complete definition of how a player will play a game. In particular, it determines the move a player will make in any situation s/he could face. A player's strategy set is the set of pure strategies available to that player. A mixed strategy means to play a pure strategy with probability between zero and one. This allows a player to randomly select a pure strategy. Since probabilities are continuous, there are infinite mixed strategies available to a player.

2.3.3 Classic Games

2.3.3.1 Prisoner's dilemma

The Prisoner's dilemma is one of the games studied in game theory, which was presented by Poundstone (Poundstone, 1992), as follows.

> "Two members of a criminal gang are arrested and imprisoned. Each prisoner is in solitary confinement with no means of communicating with the other. The prosecutors lack sufficient evidence to convict the pair on the principal charge. They hope to get both sentenced to a year in prison on a lesser charge. Simultaneously, the prosecutors offer each prisoner a bargain. Each prisoner is given the opportunity either to betray the other by testifying that the other committed the crime, or to cooperate with the other by remaining silent. The offer is:
> If A and B each betray the other, each of them serves 2 years in prison.
> If A betrays B but B remains silent, A will be set free and B will serve 3 years in prison (and vice versa).
> If A and B both remain silent, both of them will only serve 1 year in prison (on the lesser charge)."

Both prisoners have two options – "cooperate" or "defect." In this game, each prisoner gains when both cooperate; however, if only one of them cooperates, the one who defects will gain more. If both defect, both lose. See Table 2-9.

Table 2-9: The prisoner's dilemma

B / A	cooperate	defect
Cooperate	Each serves 1 year	Prisoner A: 3 years Prisoner B: goes free
Defect	Prisoner A: goes free Prisoner B: 3 years	Each serves 2 years

Based on the game type definitions, the prisoner's dilemma is a non-cooperative, simultaneous and non-zero-sum game.

2.3.3.2 Matching pennies

Matching pennies is a two-player game. Each player has a penny and they are shown simultaneously. If the pennies match (either heads or tails), player A will get the penny from B (i.e., A wins one penny [+1], B loses one penny [-1]). If the pennies do not match, player B receives the penny from A (i.e., B wins one penny [+1], A loses one penny [-1]). This game is represented in Table 2-10. Obviously, this is a zero-sum game, in which one player's gain is exactly equal to the other one's loss.

Table 2-10: Matching pennies

Player B / Player A	Heads	Tails
Heads	+1, -1	-1, +1
Tails	-1, +1	+1, -1

2.3.3.3 Pirate Game

The pirate game is a simple mathematical multi-player game as follows. Five rational pirates, A, B, C, D and E have to decide how to distribute 100 gold coins. There is a strict order of seniority among the pirates: A is senior to B, who is senior to C, who is senior to D, who is senior to E. The most-senior pirate, A, will propose a coin-distribution method. Then the pirates, including A, vote on whether to accept this distribution. If the distribution is accepted, the coins are disbursed and the game ends. If not, the proposer is thrown overboard from the pirate ship and dies, and the next most-senior pirate makes a new proposal to begin the game again (Talbot Coram & Goodin, 1998) (Stewart, 1999).

Each pirate clearly knows the previous pirate's move and the total benefit of all the players is not zero; hence, this game is a perfect information and non-zero-sum game.

2.3.4 Nash Equilibrium (NE)

Nash equilibrium (NE) is a solution concept in game theory to solve a game involving two or more players. If each player has chosen a strategy and no player has anything to gain by changing strategies while the other players keep theirs unchanged, then the current set of strategy choices and the corresponding payoffs constitute a Nash equilibrium (Nash, 1950) (Nash, 1951). That means a Nash equilibrium can be seen as a rule that no one would want to break even in the absence of an effective police force. Take the example of two cars driving perpendicularly at a traffic light junction. In this situation, Nash equilibrium would mean one car respects the green light and the other respects the red light. NE can be divided into two types. Pure-strategy Nash equilibrium is the equilibrium where all players are playing pure strategies. Mixed-strategy Nash equilibrium is the equilibrium where at least one player is playing a mixed strategy. The definition of pure strategy and mixed strategy can be found in the previous section. John Nash stated that every game in which the set of actions available to each player is finite has at least one mixed-strategy equilibrium (Nash, 1950). The following are some examples to illustrate this concept.

2.3.4.1 Example I Pure NE in a Coordination game

Consider the two-player game shown in Table 2-11: each player has two actions. If both players choose action 1, each of them gains 2, and if they both choose action 2, each gets 1, if the players choose different actions from each other, they gain nothing. In this game, there are four possible pure strategy sets: action 1, action 1; action 1, action 2); action 2, action 1; and action 2, action 2. Therefore action 1, action 1 is a Nash equilibrium since no one can get a higher payoff by unilaterally changing their strategy. The same applies to the strategy set action 2, action 2, which is also a Nash equilibrium. This game has two Nash equilibria and all the players are playing pure strategies in the equilibrium; they are pure Nash equilibria.

Table 2-11: Coordination Game

Two-player game		Player 2 Action 1	Player 2 Action 2
Player 1	Action 1	2, 2	0, 0
Player 1	Action 2	0, 0	1, 1

2.3.4.2 Example II Mixed-Strategy NE in Matching Pennies

The game matching pennies was described in the previous section. Let us take a look at all the pure strategy sets in this game. Heads, Heads cannot be a Nash equilibrium, because if player B knows that player A reveals heads, he will want to switch to tails. Heads, Tails cannot be a Nash equilibrium either, because player A wants to change to tails if player B plays tails. The same is true for Tails, Heads and Tails, Tails. Therefore, there is no pure-strategy Nash equilibrium in this game.

According to John Nash, there must be a mixed-strategy Nash equilibrium in every game. In Spaniel (2011) and von Ahn (2008), an algorithm for computing mixed-strategy Nash equilibrium is given. For each individual player:

1. Assign a variable to each strategy that denotes the probability that a player will choose that strategy.
2. The total sum of the probabilities for each strategy available to a player is 1.

3. Based on the randomization of the player's choice, the expected payoff for a player should be the same.
4. This creates a group of equations from which the probabilities of choosing each strategy can be computed.

Now, let us apply the above algorithm in order to find the mixed-strategy NE for the game matching pennies.

For player A,

- Assign p to be the probability that player A plays Heads; $1 - p$ is the probability that he plays Tails;
- If player B chooses Heads, the expected payoff for player A is $(+1) * p + (-1) * (1 - p) = 2p - 1$;
- If player B chooses Tails, the expected payoff for player A is $(-1) * p + (+1) * (1 - p) = 1 - 2p$;
- The above two expected payoffs are equal; we get $= \frac{1}{2}$.

The same is true for player B: if we assign q as the probability that player B plays Heads,

$1 - q$ is the probability that he plays Tails, then we arrive at $= \frac{1}{2}$.

Note, a robust response strategy is one that achieves maximal expected performance against a particular set of opponent strategies. Thus, according to the concept of NE, each strategy in a NE must be the best response to the rest of the strategies in that player's strategy set. Therefore, we can evaluate a strategy based on the comparison between this strategy and the strategy in the NE. In the literature, the two existing methods for performing this comparison are Exploitability and Distance to Nash (Davis, Burch, & Bowling, 2014) (Lupien St-Pierre, Hoock, Liu, Teytaud, & Teytaud, 2016).

2.4 The Relation between Decision Making and Game Theory

The relation between decision-making problems and game theory has been discussed directly or indirectly in the literature. Milnor (1954) considers DMUSU problems to be a game against nature. A decision matrix $A = (a_{ij})$ is given, in which the decision maker as player 1 must choose a row. A column will be chosen by player 2, "Nature", a fictitious player having no known objective and no known strategy. Luce and Raiffa (1957) propose that decision-making problems can be considered a two-person non-zero-sum, non-cooperative game: player 1 and player 2 can be referred to as the decision maker and neutral nature separately. Thus, some solution concepts for two-player games can be applied indirectly to decision-making problems. Aliprantis and Chakrabarti (2000) mention that game theory is considered the theory of mutual interdependent decision making, which means that a player's outcome depends not only on his/her actions but also on the decisions the other player makes. Kelly (2003) divides games into three categories: games of skill, games of chance and games of strategy. Games of skill, like decision making under certainty, are one-player games where the player fully controls all the outcomes. Games of chance are games played by an individual player against neutral nature and further categorized as either involving risk or involving uncertainty; thus, games of chance belong to decision making under risk or strict uncertainty in decision theory. Games of strategy are defined as games between two or more players, not including nature, each of whom has partial control over the outcomes.

Now it is time to introduce the connection between game theory and decision making. As explained in the previous sections, the basic concepts for a decision-making problem are: (1) alternative decisions, (2) states of nature, (3) consequences of each decision for each state of nature. These correspond, respectively, to the basic concepts of a two-player strategic game: (1) strategies (alternatives) for player 1, (2) strategies (alternatives) for player 2 and (3) payoffs for each player from possible strategy combinations. See Figure 2-1.

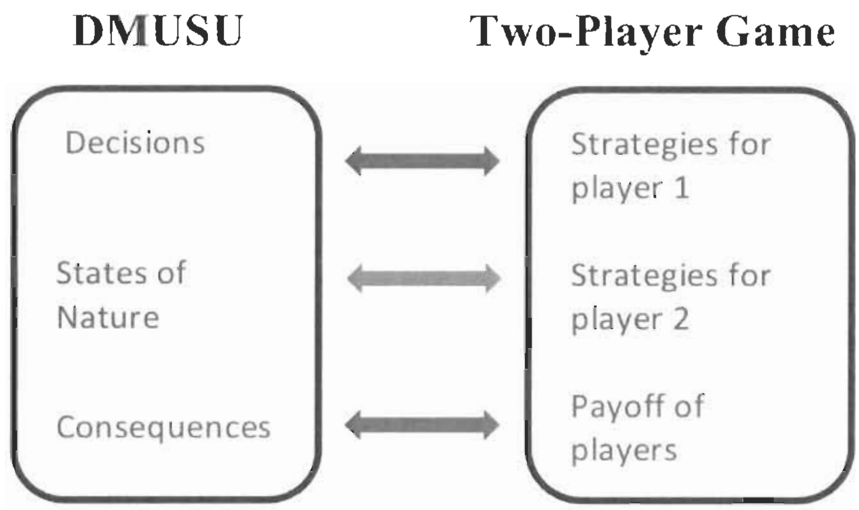

Figure 2-1： Relationship between Decision Making and Game Theory

From this perspective, DM can be converted to a two-player game where player 1 is the decision maker and player 2 is nature. Furthermore, it is a non-cooperative, non-zero-sum game since one of the players in this game is neutral nature.

2.5 Conclusion

This chapter is divided into three parts: decision-making problems, game theory and their relation. Decision-making problems are categorized as decision making under strict uncertainty and decision making under risk. Classic decision rules for decision-making problems are introduced and compared with examples. In game theory, the basic concepts of constituting a game and game types are introduced, followed by a description of the prisoner's dilemma, matching pennies and the pirate game. Then Nash equilibrium, a solution concept in game theory, is illustrated with examples. With three basic elements of decision-making problems and the basic concepts of a game, decision-making problems can be converted to a two-player game where player 1 is the decision maker and player 2 is nature.

CHAPTER 3 – COMPARATIVE STUDY OF DMUSU METHODS: A CASE STUDY IN SEWER NETWORK PLANNING

3.1 Introduction

After the review and introduction in chapters 1 and 2, this chapter focuses on the comparison of five classic methods for DMUSU and NE in a more practical way than axiomatic comparison.

Different methods may arrive at different decisions for the same DM problem. Hence, a good understanding of what the decision-making process involves and how to choose effective decision rules can be helpful in order to make better decisions and have a higher probability of success.

At this point, practical DMs need to think about which method to use. They could choose their preferred method based on the axiomatic characterization; however, axiomatic comparisons are very theoretical and mathematical for practical DMs. In order to find an easy way to help them to choose one suitable DM method for a single DMUSU problem, our work is carried out in the following steps:

- Apply all the DM methods to one DMUSU problem and analyze their results;
- Based on the connection between DM and game theory, consider a DMUSU problem a two-player game and apply NE to find the decision;
- According to the concept of NE, the choice made by NE is the best response;
- Compare the decision indicated by classic DM methods with the decision indicated by NE.

The practical decision problem of selecting a sewer network plan is used here to illustrate how each decision method is implemented in a real-life project. The city's civil engineer proposed four sewer network construction alternatives in order to direct more rainfall water in one particular area to the river. The city needs to make a decision to choose one alternative and construct it in this area. Because the city has no information about weather conditions, this DM problem is structured into DMUSU. With the existing data and analysis, a decision matrix is generated to which five classic DM methods and NE are applied.

The remaining parts of this chapter are organized as follows: Section 3.2 briefly recalls the definition of five classic DMUSU methods and Nash equilibrium; Section 3.3 gives a full description of the case study: sewer network planning; Section 3.4 shows how to structure this real project into a DMUSU problem; Section 3.5 applies each DMUSU method and NE to the problem and selects the final plan; Section 3.6 discusses and analyzes results from the various methods.

3.2 Five classic methods for DMUSU and Nash equilibrium

Five classic methods for solving DMUSU problems and Nash equilibrium are the following:

1. Laplace's Principle of Insufficient Reason: It assumes that the probabilities of the different possible states of nature are all equal. The selected decision is the one that has the maximum of the average.
2. Wald's Maximin: It evaluates each decision by the minimum possible return associated with the decision. Then, the decision that yields the maximum value of the minimum returns (maximin) is selected.
3. Savage's Minimax Regret: It defines a regret matrix that measures the difference between the payoff that could have been obtained if the true state of nature had been known and the payoff that is actually obtained. Then the minimax criterion is applied to the regret matrix.
4. The Hurwicz's Pessimism-Optimism Index Criterion: It selects a coefficient of the player's optimism. Then, it computes Hurwicz's measurement for each decision and selects the one for which Hurwicz's measurement is maximized.

5. Starr's Domain: It selects the decision that is most likely to have a higher expected payoff value than all the others.
6. Nash equilibrium: If each player has chosen a strategy and no player has anything to gain by changing strategies while the other players keep theirs unchanged, then the current strategy set choices and the corresponding payoffs constitute a NE.

3.3 Problem Statement: Sewer Network Planning

A pumping station is located next to the river and northwest of Highway 40. This pumping station receives combined sewer water (rainfall and sanitary flow) from one particular area. See Figure 3-1.

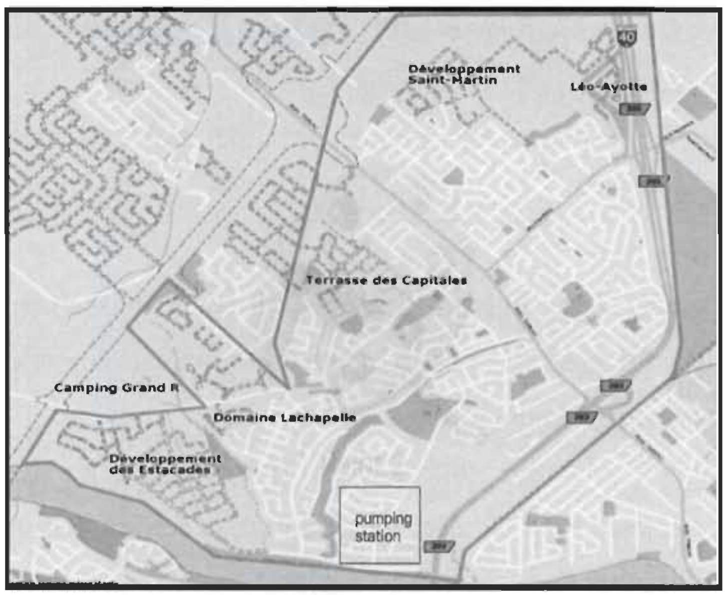

Figure 3-1 : Pumping station and its area

The local city would like to reduce the rainfall flow channelled to the pumping station in order to improve its sanitary flow capacity. To meet this goal, the city wants to gather the rainfall water for the area and direct it to the river. Thus, there will be less rainfall water taking space in the pumping station and more space for the sanitary flow. The city's civil engineering department has proposed four construction plans for building this new rainfall pipe:

1. Plan 1 is to build a new rainfall water pipe along Barkoff Street from Boulevard des Ormeaux going directly to the river. With this plan, rainfall water flows from this segment will be directed to the river. See black solid line in Figure 3-2;
2. Plan 2 is to extend the existing rainfall water pipe along rue Vachon to the river, such that rainfall water for this segment is directed to the river. See grey solid line in Figure 3-2;
3. Plan 3 includes the construction of Plan 1. Furthermore, it will extend the rainfall pipe to the northeast to du Parc Road. Plan 3 is the black solid line and black dashed line in Figure 3-2;
4. Plan 4 includes the construction of Plan 2. In addition, it will extend the rainfall pipe to the northeast along Morin Road and Highway 40. Plan 4 is the grey solid line and grey dashed line in Figure 3-2.

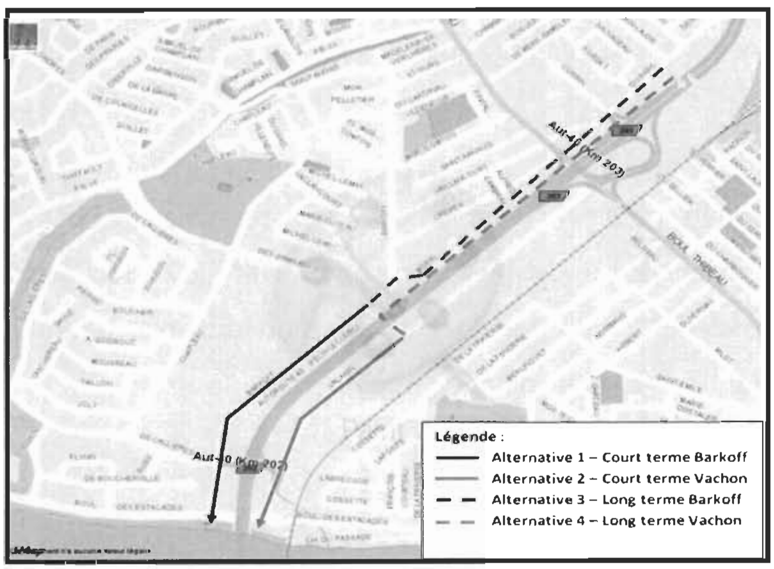

Figure 3-2 : Construction Plans

The total cost for each construction plan is listed in Table 3-1.

Table 3-1: Total Cost of Each Plan

Plan	Total cost (CAD)
P1	1,884,753
P2	437,606
P3	4,127,967
P4	2,680,820

In order to evaluate how much rainfall water is relieved from the pumping station in each plan, civil engineers modelled the current sewer network of the area and the possible alternatives (Plan 1 to 4) using Sanitary and Combined Sewer Modelling Software (SewerGEMS), a fully-dynamic, multi-platform (GIS, CAD and Stand-Alone) modelling solution.

The process is as follows. In SewerGEMS, start by setting up the baseline rain: 9 mm of rain in a three-hour period. Second, execute the model of the current sewer network and each alternative respectively with this rainfall. Third, gather the value of the rainfall flow channelled to the pumping station per second for each model. Last, compare the different values.

The results are shown in the following figures, where the higher line indicates the rainfall flow channelled to the pumping station with the current sewer network, the lower line indicates the same value but for each individual plan, and the grey area is the reduced rainfall flow from the pumping station.

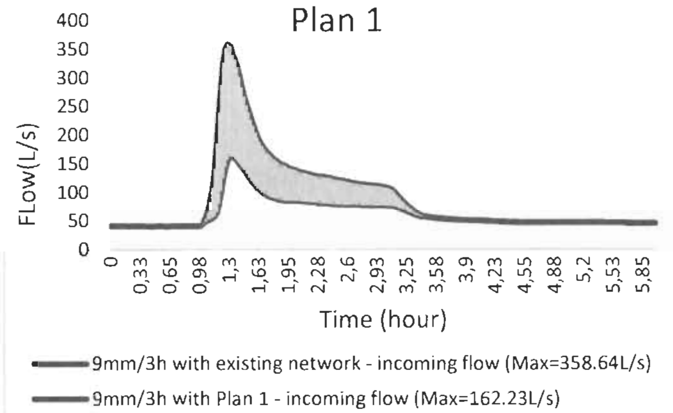

Figure 3-3： Plan 1 vs. current sewer network with 9mm/3hrs rainfall

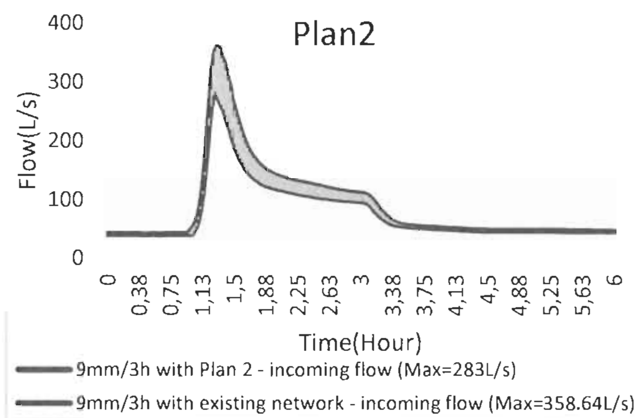

Figure 3-4： Plan 2 vs. current sewer network with 9mm/3hrs rainfall

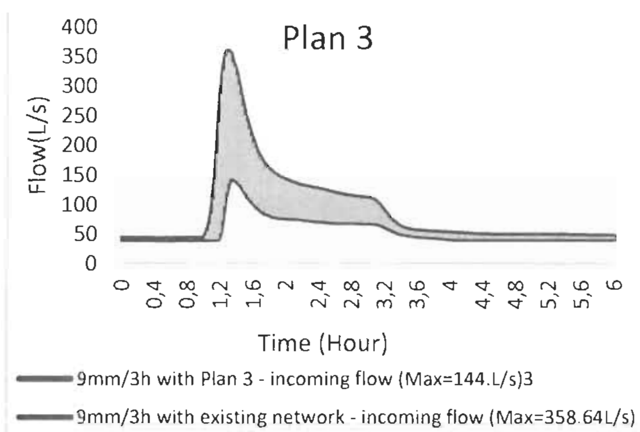

Figure 3-5： Plan 3 vs. current sewer network with 9mm/3hrs rainfall

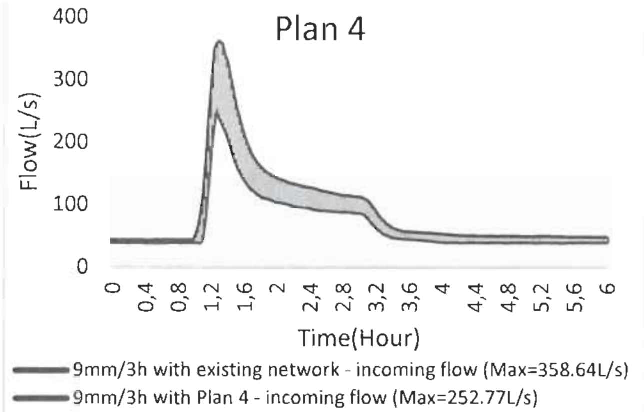

Figure 3-6： Plan 4 vs. current sewer network with 9mm/3hrs rainfall

These figures directly show the reduction of rainfall flows for each plan at the pumping station (the order of the reduced rainfall flow is Plan 3 > Plan 1 > Plan 4 > Plan 2), which also means how much capacity is improved for containing sanitary flow.

In reality, it is not always practical or beneficial to choose the plan with the biggest reduction because of the cost per volume saved. Moreover, the first unit of volume saved is clearly of importance, yet the millionth might not be as important. Thus, a weighted sum of the volume saved is more representative of the city's needs. In addition, from a pragmatic point of view, the functional level of the pumping station should be considered.

3.4 Converting the Case Study to a DMUSU Problem

In order to select one of the four plans, the city is actually facing a DMUSU problem, where weather conditions can be considered states of nature. The decision maker (the city) has no information about their true states, and the probabilities of the states of nature is quantitatively immeasurable.

To form the DMUSU problem, three basic concepts (states of nature, decision alternatives and outcomes) should be specified. As mentioned before, the rainfall is the states of nature, which cannot be quantified by the decision maker, but a list can be provided. Based on their preference, states of nature considered in this process are s_1= 7.2mm over a period of 3 hours; s_2 = 8.1mm over a period of 3 hours; s_3 = 9mm over a period of 3 hours; s_4 = 9.9mm over a period of 3 hours.

Clearly, the decision alternatives are the four construction plans: d_1=Plan 1; d_2=Plan 2; d_3=Plan 3; d_4=Plan 4.

Outcomes are the consequences of each plan under each rainfall scenario, which is the value encompassing the cost, the amount of reduced rainfall water and the functional level of the pumping station. To do this, four steps are used to compute the outcomes of this DMUSU problem:

Step 1. Set up the rainfall condition s_1, s_2, s_3, s_4 in SewerGEMS. Then, execute each decision (d_1 to d_4) respectively with each state of nature. Next, gather the maximum incoming rainfall flow channeled to the pumping station (liters per second) for each decision under each rainfall condition. See Table 3-2.

Table 3-2: Maximum Incoming Rainfall Flow in Pumping Station

	s_1	s_2	s_3	s_4
d_1	107.2	133.11	162.23	195.01
d_2	176.36	226.25	283	342.41
d_3	92.12	116.13	144.3	175.29
d_4	152.03	198.44	252.77	307.13

Step 2. Set the incoming rainfall flow of the current sewer network under rainfall scenario 9mm/3hrs: 358.64L/s as the base value. Compute the reduced incoming rainfall flow for each plan under each rainfall scenario using the difference between the base value and the value in Table 3-2. Results are presented in Table 3-3.

Table 3-3: Reduced Incoming Rainfall Flow in Pumping Station

	s_1	s_2	s_3	s_4
d_1	251.44	225.53	196.41	163.63
d_2	182.28	132.39	75.64	16.23
d_3	266.52	242.51	214.34	183.35
d_4	206.61	160.2	105.87	51.51

Step 3. Because the first unit of volume saved is clearly of importance, yet the millionth might not be as important, a weighted sum method is used to modify the data in Table 3-3 to obtain more representative data that fits the city's needs. Weighted factors are set up in Table 3-4.

Table 3-4: Weighted Factors

Reduced rainfall flow	Qty (L/s)	Weight
Need	80.000	1.000
Possible future use	120.000	0.500
Not necessary		0.100

Thus, from Table 3-3, the first 80 L/s are worth their exact weight. Values between 80L/s and 120L/s, while nice to save, are not relevant to the current situation. Thus, half weight is given, i.e., 80+ (value-80) * 0.5. There should never be any need for volumes beyond 120L/s, thus, they become 80 +40*0.5 + (value-120) * 0.1. Table 3-5 presents the weighted results:

Table 3-5: Weighted Reduced Incoming Rainfall Flow in Pumping Station

	s_1	s_2	s_3	s_4
d_1	113.144	110.553	107.641	104.363
d_2	106.228	101.239	75.64	16.23
d_3	114.652	112.251	109.434	206.335
d_4	108.661	104.02	92.93	51.51

Step 4. Generate Table 3-6 by dividing the total cost of each plan by the weighted reduced incoming flow values in Table 3-5. The values in Table 3-6 are the cost per weighted litre per second for each alternative plan under each state of nature, which is the desired outcome of the DMUSU.

Table 3-6: DMUSU's Decision Matrix for Sewer Network Planning

\$/(L/s)	s_1	s_2	s_3	s_4
d_1	16658.00	17048.41	17509.62	18059.59
d_2	4119.50	4322.50	5785.38	26962.79
d_3	36004.32	36774.44	37721.07	38820.40
d_4	24671.41	25772.17	28846.19	52044.66

3.5 Plan Selection Using Five DMUSU and NE Criteria

In this section, five DMUSU and NE criteria are applied to the decision matrix formalized in Table 3-6 in order to make decision on which plan to choose.

1. Laplace's Principle of Insufficient Reason

As a reminder, according to Laplace's criterion, when the probabilities of conditions are not known, the probabilities of states of nature are accepted as equal. Thus, the expectation of each decision is computed through the average $(a_{i1} + a_{i2} + a_{i3} + a_{i4})/4$. The decision chosen is the smallest average. Hence, Plan 2 should be chosen for the city based on Laplace's Principle. See Table 3-7.

Table 3-7: Selected Plan (**) according to Laplace

\$/(L/s)	s1	s2	s3	s4	Laplace average
d_1	16658.00	17048.41	17509.62	18059.59	17318.91
d_2	4119.50	4322.50	5785.38	26962.79	10297.54**
d_3	36004.32	36774.44	37721.07	38820.40	37330.06
d_4	24671.41	25772.17	28846.19	52044.66	32833.61

2. Wald's Maximin

Wald's criterion is an approach best summarized as a pessimistic decision maker. Instead of maximin, minimax is applied since the idea is to minimize the cost. Hence, Plan 1 is the selected plan for the city based on Wald's maximin. See Table 3-8.

Table 3-8: Selected Plan (**) according to Wald's Maximin

$/(L/s)	s1	s2	s3	s4	Maximum cost for each row
d_1	16658.00	17048.41	17509.62	18059.59	18059.59**
d_2	4119.50	4322.50	5785.38	26962.79	26962.79
d_3	36004.32	36774.44	37721.07	38820.40	38820.40
d_4	24671.41	25772.17	28846.19	52044.66	52044.66

3. Savage's Minimax Regret

Savage's regret criterion minimizes the probable regrets for the decision maker. For the cost matrix, regret is calculated by $r_{ij} = a_{ij} - \min_{k=1,\dots,m} a_{kj}$ for all i, j. The regret matrix of this problem is presented in Table 3-9. The selected plan is Plan 2 according to this rule.

Table 3-9: Selected Plan (**) according to Savage's Minimax Regret

$/(L/s)	s_1	s_2	s_3	s_4	Maximum regret for each row
d_1	12538.50	12725.91	11724.24	0	12725.91
d_2	0	0	0	8903.20	8903.20**
d_3	31884.82	32451.93	31935.69	20760.81	32451.93
d_4	20551.92	21449.66	23060.81	33985.15	33985.15

4. Hurwicz's Pessimism-Optimism Index Criterion

With Hurwicz's rule, the decision maker's attitude is between pessimistic and optimistic and measured by one optimistic coefficient $0 < \alpha < 1$. For the cost matrix, in each row,

a_i denotes the smallest component and A_i the largest, then Hurwicz's measurement H_i is defined as: $H_i = \alpha a_i + (1-\alpha)A_i$ where $i = 1, \cdots, m$.

The selected plan is $\min_i H_i$. Hence, Plan 1 is the one to be chosen if $\alpha \leq 0.4152$ and Plan 2 is the one to be chosen if $\alpha > 0.4152$. See Table 3-10.

Table 3-10: Selected Plan (**) according to Hurwicz's Criterion

$/(L/s)	s_1	s_2	s_3	s_4	Hurwicz's measurement H_i
d_1	16658.00	17048.41	17509.62	18059.59	$18059.59 - 1401.59\alpha$** if $\alpha \leq 0.4152$
d_2	4119.50	4322.50	5785.38	26962.79	$26962.79 - 22843.29\alpha$** if $\alpha > 0.4152$
d_3	36004.32	36774.44	37721.07	38820.40	$38820.4 - 2816.08\alpha$
d_4	24671.41	25772.17	28846.19	52044.66	$52044.67 - 27373.25\alpha$

5. Starr's Domain

Starr's domain criterion computes the volume of the set D_i for each decision and chooses the decision with the highest volume; in this way, it actually selects the decision that is most likely to have a higher expected payoff value than all the others. In this example, Starr's criterion is applied to a modified matrix, which is the cost matrix times minus one. The dimension of the decision matrix is 4×4; the simulation with random sampling of points in the FPS is implemented to approximate the volume. The selected plan according to this criterion is Plan 2. See Table 3-11.

Table 3-11: Selected Plan (**) according to Starr's Domain

$/(L/s)	s_1	s_2	s_3	s_4	Domain
d_1	-16658.00	-17048.41	-17509.62	-18059.59	0.0368
d_2	-4119.50	-4322.50	-5785.38	-26962.79	0.4632**
d_3	-36004.32	-36774.44	-37721.07	-38820.40	0.0000
d_4	-24671.41	-25772.17	-28846.19	-52044.66	0.0000

6. Nash equilibrium

Consider the city to be player 1 and nature to be player 2 and the DMUSU problem becomes a two-player game. The representation of the game is a matrix, which shows players, strategies and payoffs, while in this example only the cost matrix is given. Hence, when applying NE in this example, consider a new matrix which is the cost matrix times minus one. This new matrix indicates how much player 1 loses using each strategy. NE chooses Plan 1 with 100% probability. See Table 3-12.

Table 3-12: Selected plan (**) according to NE

$/(L/s)	s_1	s_2	s_3	s_4	NE
d_1	-16658.00	-17048.41	-17509.62	-18059.59	100%**
d_2	-4119.50	-4322.50	-5785.38	-26962.79	0
d_3	-36004.32	-36774.44	-37721.07	-38820.40	0
d_4	-24671.41	-25772.17	-28846.19	-52044.66	0

3.6 Analysis and Conclusion

This section summarizes all the results according to the different decision rules and NE.

Table 3-13: Summary

Criterion	The selected plan
Laplace's principle of insufficient reason	P2
Wald's criterion	P1
Savage's Minimax regret criterion	P2
Hurwicz's criterion	P1, if $\alpha \leq 0.4152$ P2, if $\alpha > 0.4152$
Starr's Domain criterion	P2
Nash equilibrium	P1

Table 3-13 shows that P2 is an selected choice according to the criteria of Laplace, Savage, Hurwicz if $\alpha > 0.4152$ and Starr, while the criteria of Wald, NE and Hurwicz if $\alpha \leq 0.4152$ find the selected choice to be P1. It is worth noting that P2 is selected most often, but most civil engineers intuitively rooted for P3 from a purely city planning perspective.

On the other hand, the fact that NE points toward P1 is a compelling argument for this alternative. As a reminder, NE is a strategy where regardless of the choice of one's opponent, there is no incentive to change one's strategy. In other words, regardless of the state of nature, NE says that P1 is the best choice. This is a strong recommendation. The main drawback of NE is that it can recommend a mixed strategy (several alternatives with different probabilities). Such a recommendation is hardly helpful to decision makers. However, in this specific case, the fact that NE is 100% behind Plan 1 (i.e. a pure strategy) is reassuring for the decision maker.

From the theoretical definition and practical implementation of each method, the following conclusions will aid DMs in their DMUSU decision process. First, DMs need to list and organize all the information they have in order to define the decision goal and decision alternatives. Furthermore, they need to think about what kind of external factors are considered states of nature, plus their degree of knowledge thereof. Thus, they can clearly determine whether it is a DMUSU or DMUR problem. Second, DMs need to clarify their preferences and decide which method to choose. For DMs who are very conservative and don't want the chance of a loss, Wald's maximin is the right decision method; for DMs who prefer to quantify their attitude, Hurwicz introduces the coefficient of decision maker's optimism; for DMs who want to evaluate how much they would regret choosing an alternative and want to minimize that regret, Savage should be considered; for DMs who think the likelihood of each state of nature is equal, Laplace is the simplest criterion to implement; for DMs who are more convinced by the method with strong quantitative proof, Starr's Domain should be selected; lastly, a choice made by NE is supposed to be robust according to its definition, so it can be used as a reference or a recommendation to support other methods.

CHAPTER 4 – MULTI-CRITERIA DECISION-MAKING METHODS

4.1 Introduction

Multi-criteria decision making (MCDM) is the most well-known branch of operations research (OR), which deals with decision problems in the presence of a number of decision criteria (Belton & Stewart, 2002) (Keeney & Raiffa, 1976). It is a procedure that structures and solves decision problems by combining the performance of each decision alternative under multiple conflicting, qualitative and/or quantitative decision criteria and outcomes into a compromise choice. In MCDM, DMs' behaviour is more active; they understand and decide which dimensions or perspectives (criteria) they want to consider for evaluating decision alternatives. Conversely, in DMUU, DMs believe that a series of external factors (states of nature) significantly impact the outcomes of decisions; they are more passive and more focused on future uncertainties.

The relevant MCDM methods aim to help DMs solve MCDM problems; they are widely applied in different types of real-life problems, where groups of decision alternatives are considered against conflicting criteria (Triantaphyllou & Mann, 1995). A good number of MCDM methods have been developed to provide techniques for DMs during the decision process. They incorporate all the objective and subjective information in order to find a compromise selected solution. According to the literature, the available methods can be grouped into three categories (Ishizaka & Nemery, 2013) (Belton & Stewart, 2002):

- Full aggregation methods: each criterion is assigned a weight, which indicates the importance of the criterion, then a numerical score for each alternative is calculated and the one with the highest score prevails [e.g., the analytic hierarchy process (AHP) (Saaty, 1980)].
- Outranking methods: each pair of alternatives is compared for each criterion to rank the alternatives [e.g., the ELimination Et Choix Traduisant la REalité (ELECTRE) (Benayoun, Roy, & Sussman, 1966), the Preference Ranking Organization Method for Enrichment Evaluations (PROMETHEE) (Brans & Vincke, 1985).

- Goal, aspiration or reference level methods: these methods identify how far each alternative is from the ideal goal or aspiration [e.g., the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) (Yoon & Hwang, 1995)].

The purpose of this chapter is to review four MCDM methods in reality: AHP, TOPSIS, ELECTRE and PROMETHEE. Sections 4.2 to 4.5 respectively describe each of the above MCDM methods with an intuitive explanation and interpretation. They also discuss each method's advantages and limitations. Section 4.6 is the conclusion for this chapter.

4.2 AHP

The Analytic hierarchy process (AHP), developed by Thomas L. Saaty in "A scaling method for priorities in hierarchical structures" (Saaty, 1977) (Saaty, 1980), is one of the most extensively used MCDM methods. It helps DMs understand the problem and choose one decision to suit their goal. Its strength lies in its simplicity and ease to understand. In general, AHP first deconstructs the original decision problem into a hierarchical structure containing the decision goal, the alternatives and the criteria; then it uses pairwise comparison techniques to obtain the priorities of all the elements in the decision problem; finally, it synthesizes all the judgments and summarizes a set of overall priorities in order to make the final decision. This method is widely used around the world in a broad range of applications (Vaidyaa & Kumar, 2006), such as selection (Lai, Wong, & Cheung, 2002), evaluation (Akarte, Surendra, & Ravi, 2001), cost/benefit analysis (Wedley, Choo, & Schoner, 2001), allocations (Saaty, Vargas, & Dellmann, 2003), forecasting (Rossetti & Selandari, 2001), etc.

AHP is completed in four steps to obtain the ranking of all the decision alternatives. This method first structures the decision problem into a hierarchy of all the elements of the problem, which are: the overall goal of the problem, a group of decision alternatives for achieving the goal and a group of criteria that connects the alternatives to the goal; second, it calculates priorities among the elements of this hierarchy by making a series of judgments based on pairwise comparisons of the elements; third, the judgments in step two are checked for consistency; fourth and finally, it synthesizes these judgments to obtain the ranking of all the alternatives with regard to the goal and makes the final decision. The following subsections give a brief introduction to each step.

Step 1. Structure the Problem into a Hierarchy. In AHP, DMs first specify the overall goal of the problem, the list of criteria they want to consider and the available decision alternatives. They then structure the complex decision problem into a hierarchy where the top level is the overall goal, the second level is the criteria and the lowest level represents the alternatives; see Figure 4-1. In a more complex hierarchy, criteria can be further divided into sub-criteria, sub-sub-criteria and so on; hence, more additional levels can be added. Nevertheless, the hierarchy must be at least three levels (Saaty & Vargas, 2001).

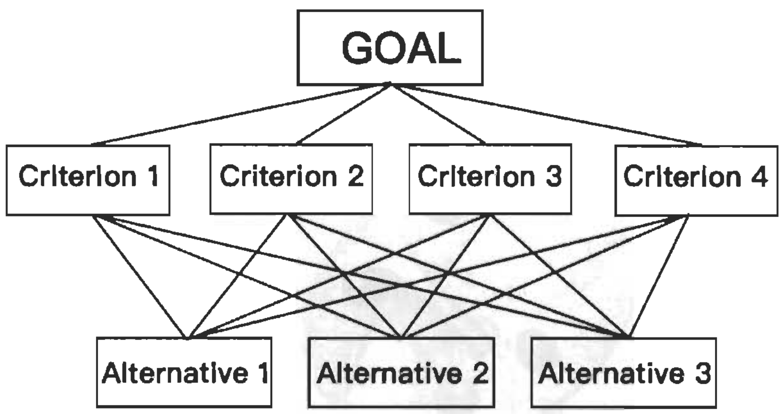

Figure 4-1: AHP hierarchy structure

Step 2. Perform the Priority Calculation. A priority is represented by an absolute number between zero and one that indicates the importance of each alternative with regard to one specific criterion and the importance of each criterion with regard to the top goal in the decision problem. The technique used in the priority calculation is called pairwise comparison. This technique generally consists in comparing all the alternatives in pairs to judge which alternative is preferable. It is often used in psychology (Yokoyama, 1921) (Thurstone, 1927). It is believed that pairwise comparison is a more efficient and accurate way to evaluate the preference between two alternatives than simultaneously comparing all the alternatives (Ishizaka & Labib, 2011). The fundamental scale of pairwise comparison used in AHP is a 1-9 fundamental scale (Saaty & Vargas, 2001), see Table 4-1.

Table 4-1: The Fundamental Scale for Pairwise Comparison in AHP

Degree of Importance	**Definition**
1	Equal importance
3	Moderate importance
5	Strong importance
7	Very strong importance
9	Extreme importance
Degrees of 2, 4, 6 and 8 can be used to express intermediate values. Degrees of 1.1, 1.2, 1.3, etc. can be used for alternatives that are very close in importance.	

The priority calculation in AHP involves the following tasks:

1. Starting from the second level of the hierarchical structure, comparing the nodes at each level two by two with respect to their contribution to the nodes above them and collecting the results into a positive square $n \times n$ matrix $S = (s_{ij})$, where n is the number of alternatives when computing the alternative priority and the number of criteria when computing the criteria priority. The diagonal elements of the matrix are 1 and s_{ji} is the reciprocal of s_{ij}, i.e. $s_{ji} = \frac{1}{s_{ij}}$.
2. Computing the priority vector of each pairwise comparison matrix. Saaty (Saaty, 2003) explains that a priority vector must remain invariant under multiplication by a positive constant and it should be unchanging under the hierarchical structure for its own judgment matrix so that one does not keep getting new priority vectors from that matrix. In the same paper, Saaty also proves that the principal right eigenvector (also known as right Perron vector) is a necessary representation of the priority vector derived from a positive reciprocal pairwise matrix S when S is a small perturbation of a consistent matrix. Teknomo (2006) introduces a way to compute this eigenvector by hand and Seshadri (2009) provides a function to compute this eigenvector through the Matlab software.

Step 3. Check the Consistency of the Pairwise Comparison Matrix. The pairwise comparison matrix may be inconsistent because in making a pairwise comparison judgment, a human is more likely to be cardinally inconsistent because s/he cannot give precise estimations. Furthermore, several successive pairwise comparisons may contradict each other; for example, A is preferred to B twice and B to C four times, but A is preferred to C only six times when compared pairwise; another example could be a situation where A is preferred to B and B to C but C is preferred to A. Be aware that AHP doesn't insist on 100% consistency because people are not robots unable to change their minds with new evidence and unable to look within for judgments that represent their thoughts, feelings and preferences. AHP allows inconsistency; however, the consistency level of the pairwise comparison matrix needs to meet a certain level. This is because the principal eigenvector can represent the priority vector when the matrix is a small perturbation of a consistent matrix (Saaty, 2003).

The consistency check consists in:

1. Computing the consistency index (CI) by: $= \frac{\lambda_{max}-n}{n-1}$, where λ_{max} is the largest eigenvalue of the matrix and n is the number of independent rows in the matrix. If the matrix is perfectly consistent then $CI = 0$.
2. The more pairwise comparison judgments, the greater the chance that the consistency error is increasing. Thus, Saaty (1980) proposes using consistency ratio (CR):$CR = \frac{CI}{RI}$, where RI is the average CI values from a random simulation of pairwise comparison matrices. Table 4-2 shows RI values derived from simulations (Alonso & Lamata, 2006). In AHP, if CR is smaller than or equal to 0.1, the inconsistency is acceptable; if CR is greater than 0.1, the subjective pairwise comparison judgment must be revised.

Table 4-2: RI values derived from simulations

n	500	100,000	500,000
3	0.58	0.525	0.525
4	0.90	0.880	0.880
5	1.12	1.109	1.109
6	1.24	1.248	1.248
7	1.32	1.342	1.342

Step 4. Synthesize the Final Priorities. After the previous steps, the priorities of the criteria with respect to the goal and the priorities of the alternatives with respect to the criteria are known; the next step is to calculate the priorities of the alternatives with respect to the goal that represent the alternatives' relative ability to achieve the decision goal. The calculation is a straightforward matter of multiplying and adding: (1) for each criterion C_j, multiply the priority of C_j with respect to the goal by the priority vector of all the alternatives with respect to C_j; (2) for each alternative A_i, add all the i^{th} elements from the results of (1), the sum is the priority of A_i with respect to the global goal; (3) the alternative with the highest priority with respect to the goal is considered the final decision choice.

The AHP method is a well-structured technique to help DMs understand and analyze complex decision problems. It selects the best decision from a number of alternatives evaluated with several criteria. In this process, DMs use simple pairwise comparison judgments to develop overall priorities for ranking the alternatives. It has received the most academic attention and been frequently used around the world in a large variety of applications due to its simplicity, ease to understand and the quality assurance provided by the consistency check. The disadvantages of AHP are that the potential compensation between good scores on some criteria and bad scores on others cause the loss of information (Machairs, Witte, & Ampe, 2008) and the complexity and time of computation depends on the number of criteria and alternatives (Chou, Chang, & Shen, 2008).

4.3 TOPSIS

The Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS), from the group of goal, aspiration or reference level methods, was first presented by Hwang and Yoon in 1981 (Hwang & Yoon, 1981). The basic principle of this method is that the best alternative is the one that is the shortest distance to the ideal solution and the furthest distance from the anti-ideal solution (Ishizaka & Nemery, 2013) (Kabir, Sadiq, & Tesfamariam, 2014). The ideal solution maximizes the benefit criteria and minimizes the cost criteria, whereas the anti-ideal solution maximizes the cost criteria and minimizes the benefit criteria (Kabir, Sadiq, & Tesfamariam, 2014) (Kabir & Sumi, 2012). It is applied across many fields such as supply chain management and logistics (Chen, Lin, & Huang, 2006), (Dalalah, Hayajneh, & Batieha, 2011); design, engineering and manufacturing systems (Lin, Wang, Chen, & Chang, 2008); business and marketing management (Peng, Wang, Kou, & Shi, 2011); energy management (Kaya & Kahraman, 2011), etc.

The TOPSIS process is built with five computation steps (Ishizaka & Nemery, 2013). It first generates the decision matrix that contains the performances of the alternatives for the different criteria. Then the decision matrix is normalized and weighted. The distances to the ideal and anti-ideal solution are calculated. Finally, the relative closeness is computed by the ratio of these distances. The details of each step are:

Step 1. The decision matrix is generated as $A = (a_{ij})_{m \times n}$ which contains m alternatives, denoted as $d_1, d_2, \dots, d_m$, and n criteria, denoted as $c_1, c_2, \dots, c_n$, with the performance of each alternative on a criterion given as a_{ij}.

Step 2. The decision matrix needs to be normalized in order to be able to compare the measure on different units (e.g., dollars, days and km). Distributive normalization is one of the normalization methods; it calculates the normalized matrix $R = (r_{ij})_{m\times n}$ using the following equation:

$$r_{ij} = \frac{a_{ij}}{\sqrt{\sum_{k=1}^{m} a_{kj}^2}}, i = 1,2,\cdots,m, j = 1,2,\cdots,n \tag{4.1}$$

Step 3. The weights are taken into account. The weighted normalized matrix is $\mathrm{T} = (\mathrm{t_{ij}})_{\mathrm{m\times n}}$ by

$$\mathrm{t_{ij}} = \mathrm{r_{ij}} \cdot \omega_{\mathrm{j}}, \mathrm{i} = 1,2,\cdots,\mathrm{m}, \mathrm{j} = 1,2,\cdots,\mathrm{n} \tag{4.2}$$

where $\omega_1, \omega_2, \cdots, \omega_{\mathrm{n}}$ is a set of weights associated with the criteria and $\sum_{\mathrm{j}=1}^{\mathrm{n}} \omega_{\mathrm{j}} = 1$.

Step 4. The ideal solution S^+ and the anti-ideal solution S^- are defined as follows:

$$S^+ = \{t_j^+ | j = 1,2,\cdots,n\} = \left\{\left(\min_i t_{ij} \middle| j \in J^-\right), \left(\max_i t_{ij} \middle| j \in J^+\right)\right\}, \tag{4.3}$$

$$S^- = \{t_j^- | j = 1,2,\cdots,n\} = \left\{\left(\max_i t_{ij} \middle| j \in J^-\right), \left(\min_i t_{ij} \middle| j \in J^+\right)\right\}, \tag{4.4}$$

where J^+ and J^- are related to the benefit and cost criteria respectively.

Step 5. Finally, the n-dimensional Euclidean distance from the alternative i to the ideal solution S^+ and the anti-ideal solution S^-, denoted as D_i^+ and D_j^- in the following equations is calculated:

$$D_i^+ = \sqrt{\sum_{j=1}^{n} (t_{ij} - t_j^+)^2} \tag{4.5}$$

$$D_i^- = \sqrt{\sum_{j=1}^{n} (t_{ij} - t_j^-)^2} \tag{4.6}$$

Step 6. The relative closeness of each alternative to the ideal solution is obtained by

$$C_i = \frac{D_i^-}{(D_i^+ + D_i^-)} \tag{4.7}$$

if $C_i = 1$, alternative i is the ideal solution, if $C_i = 0$, alternative i is the anti-ideal solution. Then, rank the alternatives based on the values of C_i; the maximum value refers to the best solution to the problem.

The advantage of this method is that it requires minimal input from DMs and its output is easy to understand; the drawback is that vector normalization is needed to solve multi-dimensional problems (Kabir, Sadiq, & Tesfamariam, 2014).

4.4 ELECTRE

One of the famous outranking methods is ELimination Et Choix Traduisant la REalité (ELECTRE). The ELECTRE is a family of MCDM methods containing ELECTRE I, ELECTRE II, ELECTRE III, ELECTRE IV, ELECTRE IS and ELECTRE TRI. The two main procedures in ELECTRE methods are: a multiple criteria aggregation procedure that builds one or several outranking relation(s) in order to compare each pair of alternatives in a comprehensive way; an exploitation procedure that can provide results based on how the problem is being addressed: choosing, ranking or sorting (Figueira, Mousseau, & Roy, 2005). ELECTRE I was first presented by B. Roy in 1968 (Roy, 1968), which triggered the development of other ELECTRE methods in order to deal with different types of decision problems: ELECTRE I is made for selection problems; ELECTRE TRI for assignment problems; ELECTRE II, III and IV for ranking problems. ELECTRE III is the most popular of the ELECTRE methods and a well-established partial ranking method, as it considers imprecise data and uncertainties (Kabir, Sadiq, & Tesfamariam, 2014) (Salminen, Hokkanen, & Lahdelma, 1998) and has many successful real-world applications such as environmental and energy management (Figueira, Mousseau, & Roy, 2005) (Karagiannidis & Papadopoulos, 2008), strategic planning (Kangas & Pykäläinen, 2001), water and wastewater management (Carriço, Covas, Almeida, Leitão, & Alegre, 2012).

4.4.1 ELECTRE III Procedure in Theory

ELECTRE III constructs and exploits outranking relations between alternatives based on the weights of the criteria, the indifference, the preference and the veto thresholds provided by DMs. An outranking relation, where a outranks b (denoted by aSb), indicates that there are sufficient reasons to prove that a is at least as good as b and there are no important arguments disproving this (Roy, 1974). An outranking degree $S(a, b)$ between a and b measures the power of the statement "a outranks b". It is a grade between 0 and 1, where the closer $S(a, b)$ is to 1, the more a outranks b. This outranking degree $S(a, b)$ is computed with two perspectives: the concordance and the discordance of the statement that a outranks b. The concordance and discordance are evaluated separately while incorporating the decision maker's preference on various (often conflicting) criteria. DMs need to provide the indifference and preference thresholds for calculating the concordance index and the veto threshold for the discordance index (Ishizaka & Nemery, 2013) (Tzeng & Huang, 2011).

All the criteria have to be maximized without loss of generality. Let's define $A = (a, b, c, \dots, n)$ to be a set of alternatives and n criteria, denoted as $(g_1, g_2, \dots, g_n)$ for a MCDM problem; $g_j(a)$ represents the performance or the outcome of the alternative $a \in A$ for the criterion g_j; thus, the multi-criteria evaluation of alternative a is represented by the vector $g(a) = (g_1(a), g_2(a), \dots, g_n(a))$. Let $q(g)$ and $p(g)$ be the indifference and preference thresholds, respectively. For one pair of alternatives if $g(a) \geq g(b)$, then

$$\boxed{\begin{array}{l} g(a) \succ g(b) + p(g(b)) \Leftrightarrow aPb \\ g(a) + q(g(b)) \prec g(a) \prec g(b) + p(g(b)) \Leftrightarrow aQb \\ g(b) \prec g(a) \prec g(b) + q(g(b)) \Leftrightarrow aIb \end{array}}$$

where P represents a strong preference, Q represents a weak preference, I represents indifference.

With all the denotations introduced so far, the ELECTRE III procedure is presented below (Ishizaka & Nemery, 2013), (Roy & Bouyssou, 1993).

Step 1. The partial concordance index $C_j(a,b)$ measures the statement "a outranks b" or "a is at least as good as b" on the specific criterion g_j and is calculated by

$$C_j(a,b) = \begin{cases} 0 \ \ if \ g_j(b) - g_j(a) > p_j \\ 1 \ \ if \ \ g_j(b) - g_j(a) \le q_j \\ \frac{p_j-(g_j(b)-g_j(a))}{p_j-q_j} \ \ otherwise \end{cases} \tag{4.8}$$

where p_j, q_j ($p_j > q_j$) denote respectively the preference and indifference thresholds for criterion g_j. The higher $C_j(a,b)$, the more a outranks b on criterion g_j. It is a value between 0 and 1. When $C_j(a,b) = 0$, this means that the performance of alternative b on g_j is higher than the performance of a augmented with preference threshold p_j and there is a strict preference for b over a, i.e., a does not outrank b; when it equals 1, the performance of b on g_j is less than the performance of a augmented with indifference threshold q_j and a and b are indifferent, i.e., a is at least as good as b; when it is between 0 and 1, the performance of b on g_j is between the performance of a augmented with indifference threshold q_j and the performance of a augmented with preference threshold p_j and b is slightly preferred to a.

Step 2. The global concordance index $C(a,b)$ combines all the partial concordance indices on the different criteria together with their corresponding criteria weights. Hence, it is the weighted sum of all the partial concordance indices and measures the concordance of the statement “a is at least as good as b” with all the criteria:

$$C(a,b) = \frac{\sum_{j=1}^{n} w_j C_j(a,b)}{\sum_{j=1}^{n} w_j} \tag{4.9}$$

Step 3. The partial discordance index $d_j(a,b)$ measures the discordance with the statement “a is at least as good as b” for criterion g_j and is computed as follows:

$$d_j(a,b) = \begin{cases} 1 \ if \ g_j(b) - g_j(a) > v_j \\ 0 \ if \ g_j(b) - g_j(a) \leq p_j \\ \frac{g_j(a) - g_j(b) - p_j}{v_j - p_j} \ otherwise \end{cases} \tag{4.10}$$

where v_j (satisfying $v_j > p_j$) is the veto threshold for criterion g_j. The higher the discordance index, the more discordant this statement. Its value is between 0 and 1. When $d_j(a,b) = 1$, it means that $g_j(b)$ is higher than $g_j(a) + v_j$, the difference between b and a exceeds the veto threshold and the statement “a is at least as good as b” is completely discordant. When $d_j(a,b) = 0$, the statement “a is at least as good as b” is correct and there is no discordance. When $d_j(a,b)$ is between 0 and 1, the performance of b is between $g_j(a) + p_j$ and $g_j(a) + v_j$; therefore, b is slightly preferred to a.

Step 4. The outranking degree $S(a,b)$ is ready to be computed. It summarizes the concordance and discordance index into one measurement of the statement “a outranks b” as below:

$$S(a,b) \begin{cases} C(a,b) & if \ C(a,b) \geq d_j(a,b) \\ C(a,b) \cdot \prod \left[\frac{1 - d_j(a,b)}{1 - C(a,b)}\right] & if C(a,b) < d_j(a,b) \end{cases} \tag{4.11}$$

Step 5. To obtain the ranking order of the alternatives, descending distillation and ascending distillation must first be determined, then the final ranking is obtained by combining both orders.

Descending distillation

- Determine the maximum value of the credibility index: $\lambda_{max} = \max S(a,b)$;
- Calculate $\lambda = \lambda_{max} - (0.3 - 0.15\lambda_{max})$. where -0.15 and 0.3 are the preset up values of distillation coefficients, α and β;
- For each alternative a, determine its λ-strength, i.e. the value of alternative b with $S(a,b) > \lambda$;
- For each alternative a, determine its λ-weakness, i.e. the value of alternative b with $\left(1 - (0.3 - 0.15\lambda)\right) * S(a,b) > S(b,a)$;
- For each alternative, determine its qualification, i.e. the difference between λ-strength and λ-weakness;
- The set of alternatives with the largest qualification is called the first distillate (D_1);
- If D_1 has more than one alternative, repeat the process on the set D_1 until all alternatives have been classified. If there is a single alternative, then this is the most preferred one. Then continue with the original set of alternatives minus the set D_1, repeating until all alternatives have been classified;

Ascending distillation

- This is computed in the same way as descending distillation but the lowest qualification is used to form the first distillate.

ELECTRE III has many advantages for decision-making problems. Compared to ELECTRE II, the ELECTRE III implements a structured procedure to extract the relationship between decision alternatives. Its main advantage is that ELECTRE III is an interactive method, which means DMs directly participate in the decision process. Another advantage is that ELECTRE III avoids compensation between criteria and any normalization process, which distorts the original data; the drawback is that it requires various technical parameters such that it is not always easy to fully understand them (Ishizaka & Nemery, 2013).

4.5 PROMETHEE

The PROMETHEE, another family of outranking methods, ranks alternatives by computing a positive outranking flow and a negative outranking flow for each alternative. Seven different methods in the PROMETHEE group have been developed and used by decision makers. PROMETHEE I (partial ranking) and PROMETHEE II (complete ranking) were first published in 1982 by Brans (Brans J. , 1982), then in 1985, Brans and Mareschal developed PROMETHEE III (ranking based on intervals) and PROMETHEE IV (continuous case) (Brans & Vincke, 1985). They subsequently suggested PROMETHEE GAIA, which provides geometrical representation in support of the PROMETHEE methodology in 1988 (Mareschal & Brans, 1988). In 1992 and 1995, the same authors proposed another two versions: PROMETHEE V (including segmentation constraints) (Brans & Mareschal, 1992) and PROMETHEE VI (representation of the human brain) (Brans & Mareschal, 1995). In this section, PROMETHEE I and PROMETHEE II are fully described below.

4.5.1 PROMETHEE I & II Procedure in Theory

4.5.1.1 Essential concepts of the PROMETHEE method

According to the literature (Ishizaka & Nemery, 2013) (Brans J. , 1982), PROMETHEE methods follow three main steps: (1) computing the preference degrees for every ordered pair of alternatives on each criterion, (2) computing the unicriterion flows, (3) computing the global flows. The global flows give DMs a ranking order of the alternatives and a graphical representation of the decision problem. The three steps are explained in greater detail below.

Step 1. Unicriterion preference degrees. The unicriterion preference degree is a grade (between 0 and 1) that shows that an alternative is preferred over another on a certain criterion from the decision maker's own point of view. A preference degree of 1 denotes a strong preference for one of the alternatives for this criterion. If there is no preference at all, then the preference degree is 0. On the other hand, if there is some preference but not a strong preference, then the preference degree lies somewhere between 0 and 1.

DMs evaluate each alternative on every specific criterion with numerical values or scaled values (e.g., good, average, poor, etc.), then PROMETHEE uses pairwise comparisons to identify the differences between evaluations of each alternative on one specific criterion and preference function to explore the relation between the difference and the preference. There are a few different types of preference functions; of them, the linear function is the most common. The linear preference function requires two parameters: an indifference threshold q and a preference threshold p. If the difference between the evaluations of a criterion is smaller than the indifference threshold, then the decision maker sees no difference between these two alternatives (i.e. the preference degree is 0). If the difference is higher than the preference threshold, then the preference is strong (i.e. the preference degree is 1). The preference function gives the value of the preference degree for differences that fall between the indifference and preference threshold. See Figure 4-2.

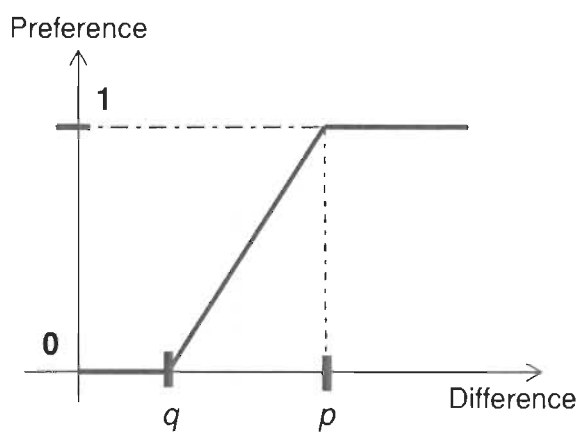

Figure 4-2: Linear Preference Function

Step 2. Unicriterion positive, negative and net flows. With the unicriterion pairwise preference degree, it is hard to determine the ranking of all the alternatives, especially when there are many. Therefore, it is necessary to summarize all the unicriterion pairwise preference degrees into unicriterion positive, negative and net flows, which present that an alternative is preferred over all other alternatives.

A unicriterion positive flow of an alternative is a score between 0 and 1, which shows that an alternative is preferred (based on the decision maker's preference) over all other alternatives on that particular criterion. The higher the positive flow, the better the action compared to the others. It is an average combination of all the preferences of an alternative compared to the others (excluding the preference degree compared with itself). Hence, it is the normalized sum of all the row elements and always lies between 0 and 1.

A unicriterion negative flow expresses that the other actions are preferred to this one. The negative flow is thus computed by taking an average combination of all the preference degrees of the actions compared to that particular action (excluding the preference degree compared with itself). It corresponds to the average of the entire column except for the diagonal element. This score thus always lies between 0 and 1. Note that the unicriterion negative flow needs to be minimized; the lower the negative flow, the more preferred the alternative.

Unicriterion net flow considers both the positive and the negative flows. The net flow of an alternative is calculated by the positive flow minus the negative flow. It represents the balance between an alternative's global strength and its global weakness; hence it should be maximized. It always lies between –1 and 1 according to the method of computation.

Step 3. Global flows. In the previous steps, only one criterion is considered. In order to include all the criteria, DMs need to specify a weight for each criterion so that a weighted sum of all the unicriterion positive, negative and net flows can be calculated into global positive flows, global negative flows and global net flows respectively.

A global positive score indicates that an alternative is globally preferred to all the other alternatives when considering several criteria. Since the weights are normalized, the global positive score always lies between 0 and 1.

Similarly, a global negative score indicates that other alternatives are preferred over a given alternative. The negative score always lies between 0 and 1 and must be minimized.

The global net flow of an alternative, obtained by subtracting the negative flows from the positive flows, includes both perspectives (preferred over other alternatives and other alternatives preferred).

4.5.1.2 The PROMETHEE I Ranking

The PROMETHEE I ranking depends on the global positive and negative flows. It follows four different rules to analyze the flows of two alternatives and conclude their ranking order:

- An alternative has a better rank than the other one if its global positive flow score is higher and its global negative flow score is lower simultaneously than the scores of the other alternative.
- An alternative has a worse rank than the other one if both the global positive and negative flow are worse.
- Two alternatives are considered to be incomparable if one alternative has a higher global positive score but a lower global negative score (or vice versa).
- Two alternatives are considered indifferent if they have identical global positive and negative flows.

4.5.1.3 The PROMETHEE II Ranking

The PROMETHEE II ranking is based on the global net flows only and leads to a complete ranking of the actions (i.e., the incomparable status does not exist). Hence, the alternatives can be ordered from best to worst.

4.5.1.4 Summary

The decision process of PROMETHEE I and II is the following: first, DMs define which criteria they want to consider in their decision making; second, all the alternatives are evaluated according to those criteria. Third, by specifying the preference function and associated parameters, the pairwise criterion preference degrees can be computed; fourth, unicriterion flows are calculated from the pairwise criterion preference degrees; last, the unicriterion flows are summarized into global flows. Then the ranking order is obtained based on whether PROMETHEE I or PROMETHEE II is chosen.

The PROMETHEE method allows direct operation on the variables included in the decision matrix without requiring any normalization and is applicable even when there is insufficient information. However, its main drawback is that it is time consuming and difficult for DMs to have a clear view of the problem, especially when there are many criteria involved (Kabir, Sadiq, & Tesfamariam, 2014) (Brans & De Smet, 2005).

4.6 Conclusion

This chapter explains AHP, TOPSIS, ELECTRE III and PROMETHEE I&II in theory. It gives a clear description of their mathematical algorithms. Furthermore, each method's advantages and limitations are underlined in order to provide a high-level overview of what kind of decision environment each method is suited for. In general, computation is difficult for AHP when there are quite a number of criteria and alternatives; TOPSIS involves fewer inputs, but it requires vector normalization for multi-dimension criteria. ELECTRE III uses original data without any normalization requirements, but it has various technical parameters such that it is not always easy to fully understand; PROMETHEE I&II are applicable even when there is insufficient information, but can be time consuming as well when many criteria are involved. In the next chapter, these MCDM methods will be implemented in order to perform a deep comparative analysis.

CHAPTER 5 – A COMPARATIVE STUDY OF MCDM METHODS

5.1 Introduction

Due to the number of MCDM methods available, DMs are confronted with the difficult task of selecting the appropriate MCDM method, as each method has its own limitations, particularities, hypotheses, premises and perspectives and can lead to different results when applied to an identical problem (Ishizaka & Nemery, 2013). Hence, it is worth evaluating the performance of different methods using a single decision problem. The aim of this chapter is to present a comparative study of four MCDM methods (AHP, TOPSIS, ELECTRE, PROMETHEE) by applying them to one real-world sewer network planning case study and analyzing the suitability of results in order to highlight the differences and reach meaningful conclusions. The purpose of this chapter is to help DMs fully understand each MCDM method's particularities, strengths and weaknesses in a practical way and choose the suitable MCDM method for their unique decision problem.

A sewer network system is the infrastructure that transports sewage, rainwater or stormwater. The main part of this system encompasses components such as manholes, pumping stations and large pipes in a combined sewer (sewage and rainwater) or sanitary sewer (sewage only) system. Sewer water infrastructure asset management has major impacts on protecting public health and sustaining our environments (Cardoso, Silva, Coelho, Almeida, & Covas, 2012) (Ugarelli, Venkatesh, Brattebø, Di Federico, & Saegrov, 2010) (Grigg, 2012). Deciding on the right sewer network plan is challenging, especially when considering the following requirements (Zheng, Egger, & Lienert, 2016): first, the selected sewer system plan's quality, life-cycle maintenance and performance need to meet the sustainability requirements for society, the economy, and the environment (Ashley, Blackwood, Butler, & Jowitt, 2008); second, the decision should involve all the stakeholders' preferences (Reed, 2008); third, the decision making must incorporate uncertainty, i.e., information is imperfect or unknown (Gregory, et al., 2012); fourth, long-term planning for future climate changes, urban development in the context of population increase or decrease, numerous environmental pollutants, etc., must be a factor.

Multi-criteria decision making (MCDM) is able to meet all the above challenges (Keeney & Raiffa, 1976) (Belton & Stewart, 2002) for a sewer network plan decision problem. It is a procedure that structures and solves decision problems by combining the performance of each decision alternative for multiple conflicting, qualitative and/or quantitative decision criteria and outcomes into a compromise choice. The relevant MCDM methods have been developed to help DMs solve MCDM problems. They are widely applied in different types of real-life problems where groups of decision alternatives are considered against conflicting criteria (Triantaphyllou & Mann, 1995). The application of MCDM methods in water and wastewater infrastructure management has steadily increased in the literature since 1990, where the analytic hierarchy process (AHP) (Saaty, The Analytic Hierarchy Process, 1980), the elimination et choix traduisant la realité (ELECTRE) (Benayoun, Roy, & Sussman, 1966), the preference ranking organization methods for enrichment evaluations (PROMETHEE) (Brans & Vincke, 1985) and the technique for order preference by similarity to Ideal Solution (TOPSIS) (Yoon & Hwang, 1995) are the most employed of all the various MCDM methods (Kabir, Sadiq, & Tesfamariam, 2014).

The remaining parts of this chapter are organized as follows: section 5.2 gives a brief description of AHP, TOPSIS, ELECTRE III and PROMETHEE II; section 5.3 provides the details of constructing the sewer network decision problem (introduced in Section 2.3) into a MCDM problem and using four MCDM methods for this case study to compare and analyze their results.

5.2 MCDM Methods

The following methods have been selected for the purposes of this chapter, as they are widely used MCDM methods in decision problems for water and wastewater infrastructure management: AHP, TOPSIS, ELECTRE and PROMETHEE.

- AHP

AHP contains four steps as shown in Figure 5-1. In its first step, it structures the original decision problem into a hierarchical structure. The overall goal of the problem is at the top level of the hierarchy; the next level contains the criteria representing the different dimensions from which the alternatives can be considered; while the bottom level is filled with decision alternatives, which are the different choices available to the decision maker. The second step is to calculate the priority of each criterion with respect to the goal and the priority of each alternative with respect to one specific criterion. The technique of pairwise comparison with a 1 – 9 fundamental scale (Saaty & Vargas, 2001) is used to obtain pairwise comparison matrix $S = (s_{ij})$, which is a positive reciprocal matrix, i.e. $s_{ji} = \frac{1}{s_{ij}}$. Saaty proves that the principal right eigenvector of S sufficiently represents the priority vector when S is a small perturbation of a consistent matrix (Saaty, 2003). Hence, the third step is to perform a consistency check of pairwise comparison matrices. This requires computing the consistency index (CI) by: $CI = \frac{\lambda_{max} - n}{n-1}$, where λ_{max} is the largest eigenvalue of the matrix and n is the number of independent rows in the matrix. Then the random index RI (see Table 3-2), which is the average CI values from a random simulation of pairwise comparison matrices (Alonso & Lamata, 2006), is introduced. If $\frac{CI}{RI} \leq 0.1$, the inconsistency is acceptable; if $\frac{CI}{RI} > 0.1$, the subjective pairwise comparison judgment needs to be revised. The last step is to summarize a set of overall priorities in order to make the final decision. The alternative with the highest priority with respect to the goal is considered the final decision choice.

AHP has received the most academic attention and been frequently used around the world in a large variety of applications due to its simplicity, ease to understand and the quality assurance provided by the consistency check. AHP is used in 28.3% of publications regarding water and wastewater (Kabir, Sadiq, & Tesfamariam, 2014) (Huang, Keisler, & Linkov, 2011). The disadvantages of AHP are: the potential compensation between good scores on some criteria and bad scores on others causes the loss of information (Machairs, Witte, & Ampe, 2008); and the complexity and time of computation depends on the number of criteria and alternatives (Chou, Chang, & Shen, 2008).

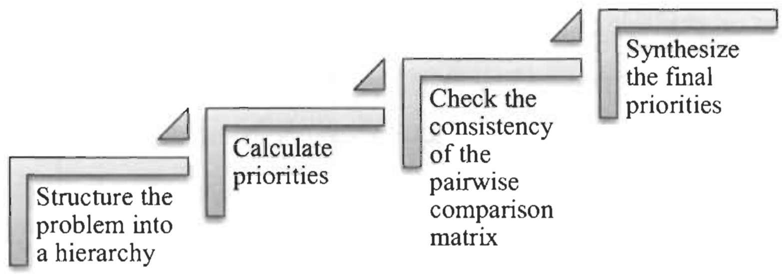

Figure 5-1: AHP

- TOPSIS

The TOPSIS process as shown in Figure 5-2 first generates the decision matrix $A = (a_{ij})_{m \times n}$. Then, it calculates the normalized matrix $R = (r_{ij})_{m \times n}$ and the weighted normalized matrix $T = (t_{ij})_{m \times n}$. The ideal solution S^+ and the anti-ideal solution S^- are defined based on the weighted normalized matrix. Subsequently, it computes the n-dimensional Euclidean distance from the alternative i to the ideal solution $S^+$and the anti-ideal solution S^- in order to obtain each alternative's relative closeness to the ideal solution. The rank of the alternatives is based on the relative closeness value.

The application of this method in water and wastewater management can be found in Afshar, Marino, & Saadatpour (2011) for ranking projects in the Karun river basin; Coutinho-Rodrigues, Simão, & Antunes (2011) for selecting the water supply system investment option for an urban development/expansion project; and in Srdjevic, Mederios, & Faria (2004) for ranking water management scenarios.

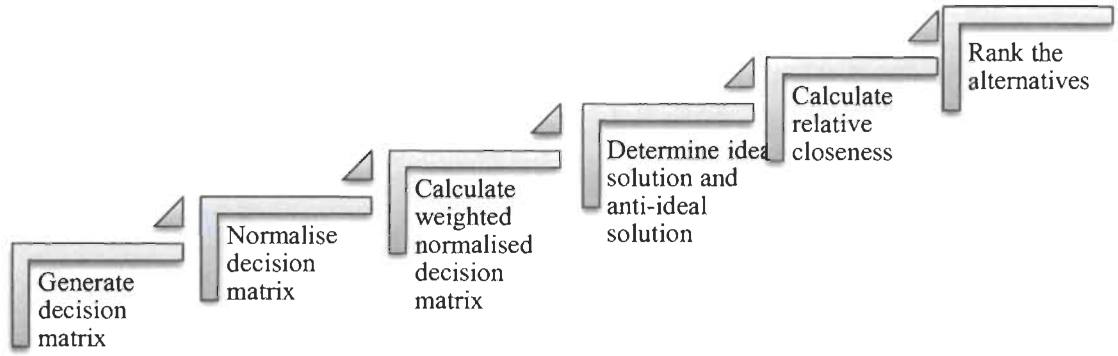

Figure 5-2: TOPSIS Process

- ELECTRE III

To use ELECTRE III as shown in Figure 5-3, DMs need to define criteria indifference (q), preference (p) and veto (v) thresholds where $(v \geq q \geq p)$ and the weight (w_j) for each criterion j. The main ELECTRE III steps are shown in Figure 5-3. The concordance index, denoted as $C(a, b)$, is evaluated by an overall comparison of the performances of each pair of a and b alternatives for all criteria. It varies from 0 to 1; a value of 0 means that alternative a is worse than alternative b for all criteria. The concordance index is computed by a weighted comparison of the performances for each criterion $c_j(a, b)$ individually; the discordance index for one criterion j, denoted as $D_j(a, b)$, describes the situation where alternative a is better than b generally, but for criterion j, alternative a is worse than b. The estimation of credibility scores is based on the concordance and discordance indices in one of the following two scenarios: first, the degree of outranking is equal to the concordance index if there is no criterion that is discordant or where no veto threshold is used; second, the degree of outranking is equal to the concordance with a reduction as the level of discordance increases above a threshold value. The distillation procedure comprises two parts: Descending Distillation, where the alternatives are ordered from the best rankings to the worst, and Ascending Distillation, which is to order the alternatives from the worst rankings to the best. The final complete ranking result comes from the combination of Descending Distillation and Ascending Distillation.

ELECTRE methods have been applied in approximately 15.1% of publications regarding water and wastewater: Carriço, et al. (2012) used ELECTRE TRI and ELECTRE III to prioritize rehabilitation interventions on the sanitary sewer system in Lisbon; Trojan and Morais (2012) applied ELECTRE II to prioritize alternatives for maintenance of water distribution networks; ELECTRE I is implemented in Morais, & Almeida (2006) for the decision on a city water supply system.

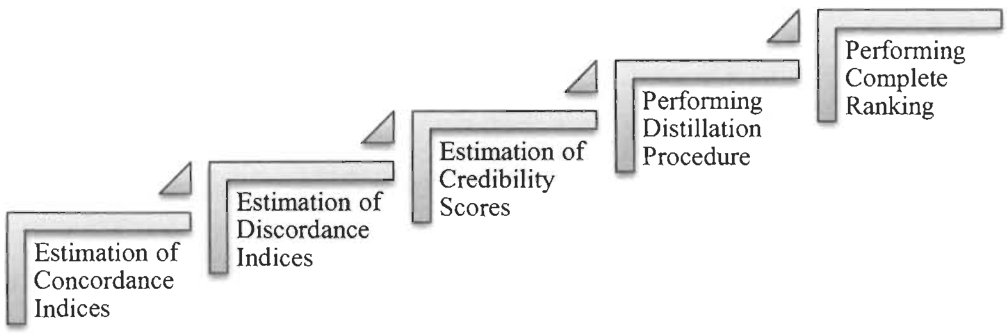

Figure 5-3: ELECTRE III Process

- PROMETHEE

The PROMETHEE I or II process as shown in Figure 5-4 first looks into each pair of alternatives for one criterion and computes the unicriterion pairwise preference degree, which is a score (between 0 and 1) showing that the decision maker prefers one alternative over the other one for the considered criterion. Then, it summarizes all the unicriterion pairwise preference degrees into unicriterion positive, negative and net flows, which demonstrate that an alternative is preferred over all other alternatives. In the previous steps, only one criterion is considered at a time. Now, all the criteria are taken into account at the same time in order to compute the global flow. To do so, DMs first need to define the relative importance or weight of each criterion w_j, where $\sum_{j=1}^{n} w_j = 1$. Then, DMs calculate the weighted sum of all the unicriterion positive, negative and net flows into global positive, negative and net flows. The PROMETHEE I ranking is dependent on the global positive flows and the global negative flows. The PROMETHEE II ranking is dependent on global net flows only. In this chapter, PROMETHEE II is used, since alternatives can be ranked from the best to the worst, resulting in a complete ranking of the alternatives.

PROMETHEE has been applied in 13.2% of publications regarding water and wastewater: Morais, & de Almeida (2007) used PROMETHEE V to rank alternative strategies for municipal water distribution systems to reduce leakage; PROMETHEE II was applied in Khelifi, et al. (2006) to select groundwater remediation technologies; implemented PROMETHEE and GAIA for the selection of a wastewater treatment plant.

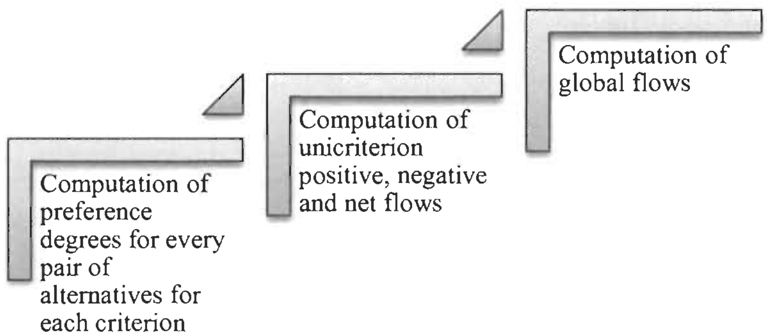

Figure 5-4: PROMETHEE Process

5.3 MCDM Problem Case Study

This case study was provided by the civil engineering team from the city of Trois-Rivières (introduced in Section 3.3). The decision problem is to select one construction plan to reduce the rainfall flow channeled to the pumping station so that it can accommodate a greater sanitary flow. In order to define this project as a MCDM problem, eight professionals participated in structuring and analyzing the decision alternatives and criteria: one project manager, two civil engineers, two sanitary engineers, two road operators and one environment/weather expert.

5.3.1 Structuring the MCDM Problem

To meet the goal, a rainfall water pipe needs to be designed to guide rainfall water to the local river instead of the pumping station. Civil engineers and sanitary engineers propose four designs; they are referred to as alternatives 1, 2, 3 and 4. Alternatives 1 and 2 are the short-term plans, while alternatives 3 and 4 are their respective long-term extensions. Briefly, Alternative 1 is to build a new rainfall water pipe along Barkoff street from Boulevard des Ormeaux flowing directly to the river (see solid black line in Figure 3-2); Alternative 2 is to extend the existing rainfall water pipe along Vachon street to the river (see grey solid line in Figure 3-2); Alternative 3 includes the construction of Alternative 1, but will further extend the rainfall pipe to the northeast to du Parc road (see solid and dashed black lines in Figure 3-2); Alternative 4 includes the construction of Alternative 2, while extending the rainfall pipe to the northeast along Morin road and Highway 40 (see solid and dashed grey line in Figure 3-2).

In order to identify evaluative criteria, the group of experts held a meeting to brainstorm the values and objectives of the problem in order to come up with a list of criteria, and descriptions of why each of them has been chosen as a criterion. In addition, they identified whether they are quantitative or qualitative (criteria source) and whether they are to be minimizing or maximizing (aim). In this way, five criteria were identified on which to base their decision, see Table 5-1.

Table 5-1: Criteria for Case Study

		Source	**Status**	**Aim**
C1	Dynamic performance	Quantitative	Positive	Maximize
C2	Cost of construction	Quantitative	Negative	Minimize
C3	Cost of maintenance	Qualitative	Negative	Minimize
C4	Environmental impact	Qualitative	Negative	Minimize
C5	Potential future profit	Qualitative	Positive	Maximize

Dynamic performance is a positive quantitative variable, and it represents by how much rainfall flow volume can be reduced in the pumping station. This criterion is evaluated based on the amount of rainfall water relieved from the pumping station under 9mm/3h rainfall conditions (refer to Figure 3-3 to 3-6).

The cost of construction is a negative quantitative variable defining how much it costs to implement a plan. It covers the cost of the duration of work, manpower, materials, and machines, etc. Note that the cost of construction for each alternative is listed in Table 3-1.

The cost of maintenance is a negative qualitative variable defining the cost of possible maintenance. For example, regular inspections or repairing damage due to human fault or extreme weather issues. It is not limited to a monetary valuation, as it also includes societal and environmental considerations.

Environmental impact is a negative qualitative variable that includes the disruption to current inhabitants and existing industries, for example, noise, traffic, air or water pollution, water supply disruptions, etc.

Potential future profit is a positive qualitative variable indicating the possible benefit a plan could provide after its implementation. For example, more population, or capacity during extreme weather (heavy rain), etc. It is not limited to a monetary valuation as it also includes societal and environmental considerations.

Before going through any MCDM method, the overall opinions of the expert team are as follows: of the four construction plans, Plan 3 is most expensive in terms of cost of construction. However, this plan has the best potential future profit and leads to the maximum pumping station capacity. Plan 2 has the lowest construction costs but it would become more expensive if expansion is required. The costs of Plan 1 and Plan 4 fall in the middle but their maintenance costs and environmental impact are not low.

5.3.2 Implementation of the MCDM Methods

The entire AHP and TOPSIS processes are implemented manually since neither method is based on complex algorithms. ELECTRE and PROMETHEE can be implemented by performing all the computation steps in a spreadsheet, but it is not easy work. A number of user-friendly software packages are available that successfully apply the ELECTRE and PROMETHEE methods. In this paper, the *Chemdecide* decision framework (Hodgett, 2016) for the ELECTRE III method and the *Smart-picker* decision software (Brussels, 2011) for PROMETHEE II are used.

During the implementation process, in order to take into account all of the eight professionals' opinions, the Delphi technique is applied. The Delphi method, originally developed by Dalkey in 1969 (Dalkey, 1969), is a structured communication technique to extract and refine group judgments. The Delphi method uses three essential elements: anonymous response, iteration and controlled feedback, and statistical group responses. Each member of the group answers the questionnaire in two or more rounds. After each round, each participant revises his/her previous answers based on the anonymized summary of the previous round until a stable result is achieved, i.e., the results from the last two rounds are the same. This technique is built to minimize the biasing effects of irrelevant communications, dominant individuals and group pressure towards conformity.

The next section contains a detailed description of implementing each MCDM method. This leads to a comparative analysis of MCDM methods

5.3.2.1 AHP

As there are five criteria, AHP requires 10 pairwise comparisons to calculate criteria weights. Furthermore, with four alternatives, six pairwise comparisons for each of the five criteria are needed. Each professional provides her/his pairwise comparison results, then the Delphi method is used to collect all the results to form the final six pairwise comparison matrices. Although this required a significant number of inputs, the consistency is checked and the resulting pairwise comparisons are consistent.

Figure 5-5 shows the criteria weight resulting from using pairwise comparison. Dynamic performance has the highest weight, followed by potential future profit and cost of construction. Environmental impact and cost of maintenance have the lowest weights. All the professionals are comfortable with the weight distribution among the criteria. Figure 5-6 displays the alternatives' performance for each criterion. P3 and P1 are the top two in terms of dynamic performance, followed by P4, which is less than half of P3, and P2 is the lowest of all. Regarding the cost of construction, cost of maintenance and environmental impact criteria, the alternatives have relatively similar normalized score behaviour, where the least expensive project (P2) clearly outperforms the other alternatives, while P3, the most expensive project, has the lowest score, and P1 and P4 are in the middle. For potential future profit, P3 has the highest score—almost three times more than the runner up, P1. P4 is in third position, which is less than half of P1 and two times higher than the last one, P2.

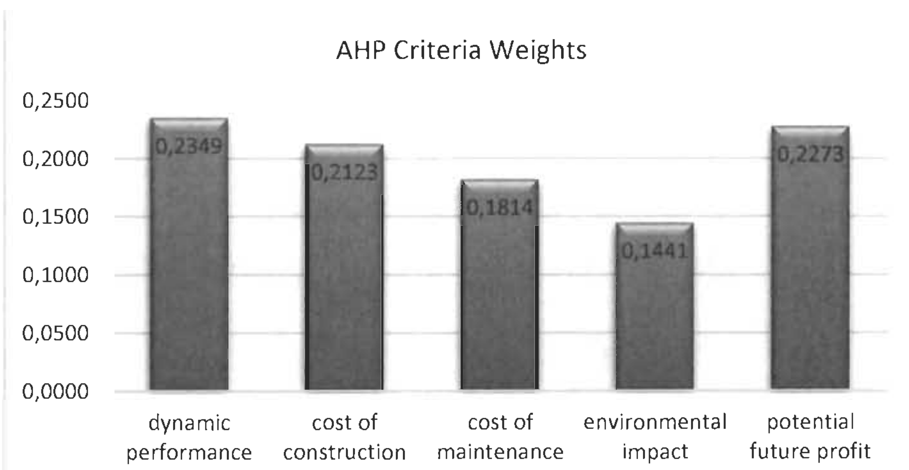

Figure 5-5: AHP: Criteria weights using pairwise comparison

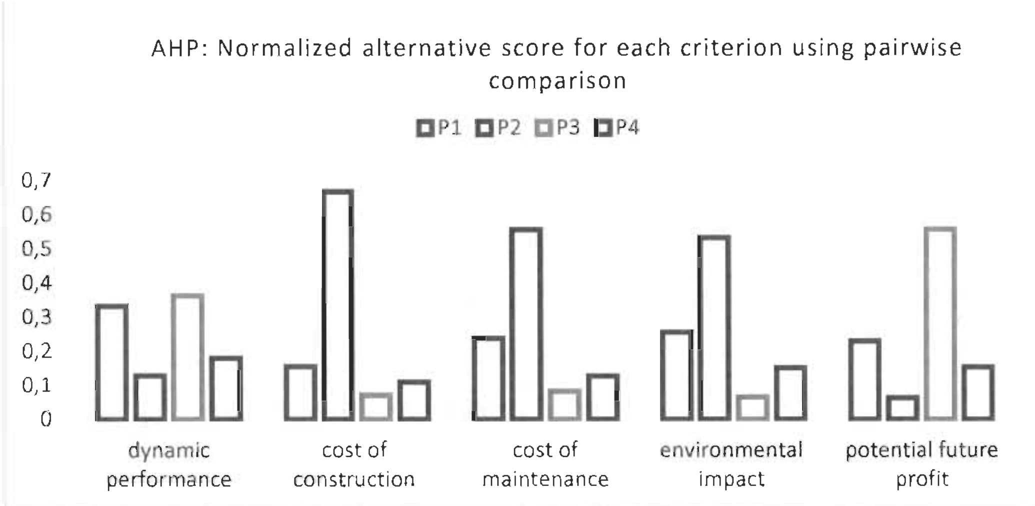

Figure 5-6: AHP: Normalized alternative score for each criterion using pairwise comparison

The results from Figures 5-5 and 5-6 summarize the final score and derive the rank of the alternatives, shown in Figure 5-7, where P2 is the selected alternative according to the AHP methodology, followed by P3 and P1. P4 receives the lowest score.

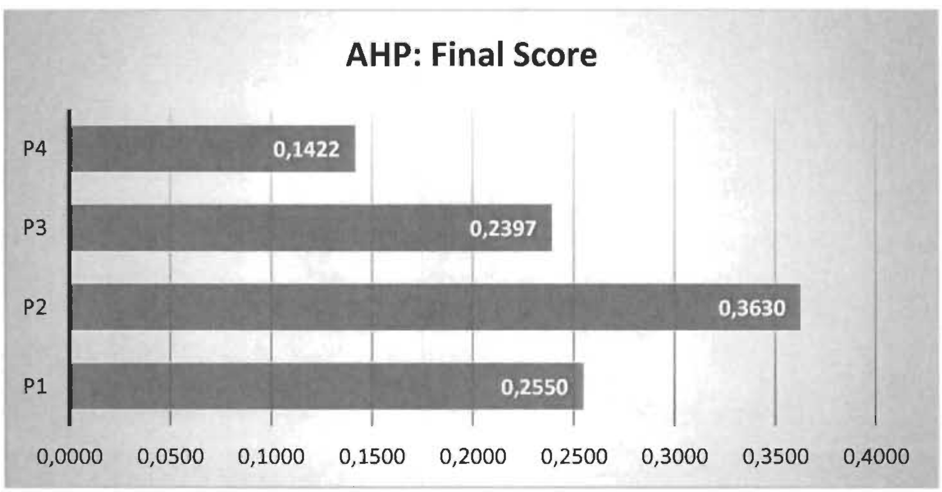

Figure 5-7: AHP: Results for sewer network planning case study

5.3.2.2 TOPSIS

When implementing the TOPSIS process, each professional can assign criteria weighting based on his/her own knowledge. Professionals choose a value between 0% and 100%;

the higher the percentage, the greater the criterion's weighting. For simplicity, the total sum of the assigned weighting of the five criteria must equal 100%. With three rounds of the Delphi technique, each professional finalized his/her assignment, and the final criteria weighting is calculated by taking the average from all professionals; the result is shown in Figure 5-8. The weighting is almost equally distributed among dynamic performance, cost of construction, cost of maintenance and potential future profit, while environmental impact received a lower weighting.

After deciding the criteria weighting, the TOPSIS process also requires all professionals to provide their opinions on the alternatives' performance for each criterion in order to form the decision matrix. Furthermore, due to the normalization in TOPSIS, the alternatives' performance for different criteria must be expressed in the same measurement unit. Hence, in order to formalize their opinion, all professionals are asked to rate the alternative between 1 and 10 for each criterion, where 1 denotes extremely poor performance and 10 denotes excellent performance. For example, Alternative P1 is rated by each expert (columns in Table 5-2) for each criterion (rows in Table 5-2), and P1's final rating for one criterion is the average of all the professionals' scores. The final column "Average" in Table 5-3 is the final score for P1 for different criteria. Note that the scores in Table 5-2 are from each expert and are also derived through the Delphi technique.

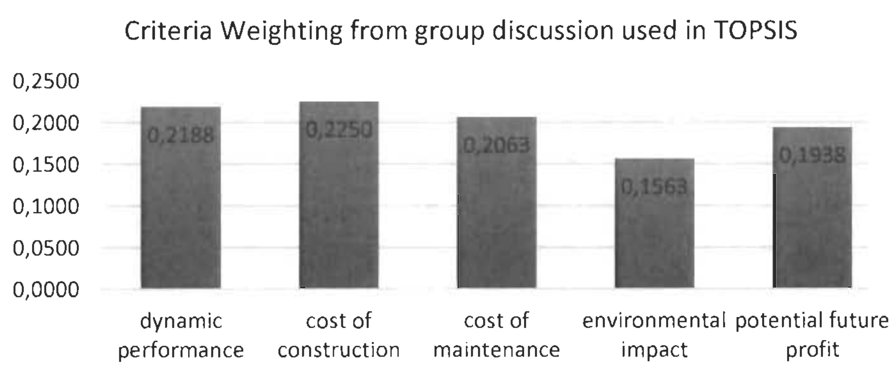

Figure 5-8: TOPSIS: Criteria weighting from the group discussion

Table 5-2: Professionals' ratings for P1 in TOPSIS

P1	Project manager	Civil engineer 1	Civil engineer 2	Road operator 1	Road operator 2	Weather and environment expert	Sanitary engineer 1	Sanitary engineer 2	**Average**
Dynamic performance	8	8	8	7	7	7	7	7	7.375
Cost of construction	6	6	6	7	6	6	6	6	6.125
Cost of maintenance	7	7	6	6	6	6	6	6	6.25
Environmental impact	7	7	7	7	7	6	7	7	6.875
Potential future profit	7	7	8	8	8	7	7	7	7.375

This process is repeated for all the other alternatives, and the decision matrix is formed by the average rate of each alternative for each criterion; see Table 5-3. Figure 5-9 illustrates the decision matrix for Table 5-3 for a better overview. P1 received above 6 for all the criteria. P2 has a very good rate (over 8) in terms of cost of construction, which is reasonable since its construction cost is significantly lower than the others. P3 has very good rates for the dynamic performance and potential future profit criteria (both are over 8), while it does not have any advantages for cost of construction and environmental impact. P4 receives relatively similar rates for all criteria and the average is 4.5.

Table 5-3: TOPSIS decision matrix

Alternatives / Criteria	**P1**	**P2**	**P3**	**P4**
Dynamic performance	7.375	4.875	8.375	5
Cost of construction	6.125	8.5	3	4.5
Cost of maintenance	6.25	7.75	5.125	5.125
Environmental impact	6.875	7.375	3.25	3.875
Potential future profit	7.375	2.875	8.375	5.125

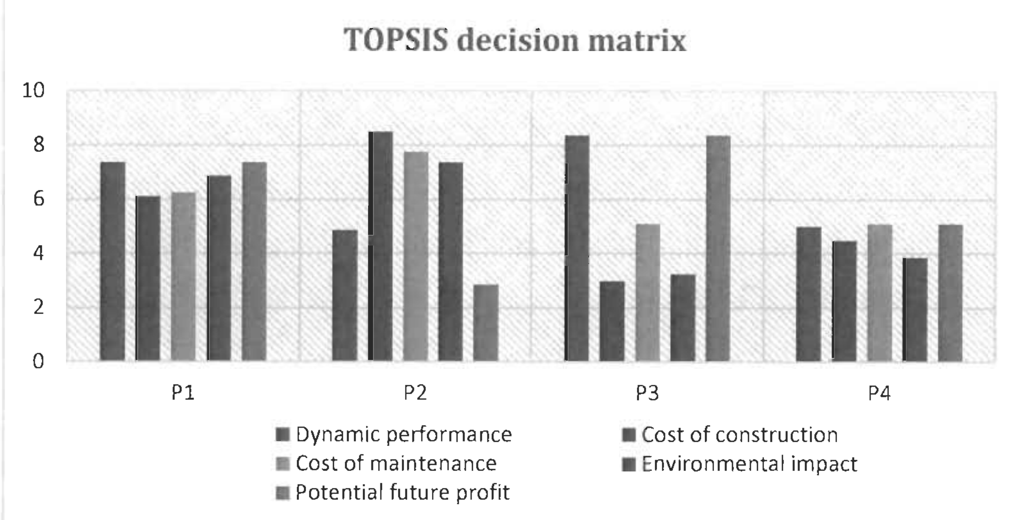

Figure 5-9: TOPSIS decision matrix

After the decision matrix is built, the next steps in TOPSIS are: deriving the standardized matrix; next, considering the weights of the criteria to get the weighted standardized matrix; followed by finding the ideal solution S^+ and anti-ideal solution S^- in order to calculate the Euclidean distance from each alternative to the ideal solution S^+ and the anti-ideal solution S^-, i.e. D_i^+ and D_j^-; finally, obtaining the relative closeness. The selected choice is the one with the highest relative closeness value. Table 5-4 shows the result from TOPSIS, where P1 receives the highest relative closeness value, i.e., it is the alternative that is the farthest from the anti-ideal solution and nearest to the ideal solution.

Table 5-4: TOPSIS results for sewer network planning case study

TOPSIS Results	**P1**	**P2**	**P3**	**P4**
Rank	1st	2nd	3rd	4th
Relative closeness	0.6663	0.5538	0.4462	0.2672

5.3.2.3 ELECTRE III

The *Chemdecide* decision framework is introduced and developed in Hodgett (2016), where Hodgett explained the workflow for ELECTRE III and illustrated how to use the software by applying it to an equipment selection decision problem. The *Chemdecide* framework contains four different tools, one related to structuring the decision-making problem and the other three associated with the analysis provided by three different MCDM methodologies; one of the methodologies is ELECTRE III. The problem-structuring tool requires the user to designate a goal, a set of alternatives and a defined set of criteria (including whether the criterion is qualitative or quantitative and minimizing or maximizing). The analysis tool requires the decision maker to input the criteria weights and the alternatives' performances.

It is time consuming and unrealistic to ask each expert to use the software. Since all experts have attended several group meetings to structure the decision problem and to decide the criteria weights for AHP and TOPSIS, the project manager is aware of each professional's perspective; he represents the group as the user to provide the inputs to the software. His inputs are concluded and gathered to include the perspectives of all the professionals. The complete description of this software framework can be found in Hodgett (2016). The following is a brief list of the steps in using this software to implement the sewer network planning case study.

Step 1. Choose the decision setup tool to enter the goal of the sewer network planning, all the available alternatives, plus five criteria and indicate whether each criterion is qualitative or quantitative and minimizing or maximizing.

Step 2. Choose the ELECTRE III analysis tool. Open the structured problem from Step 1. Then make selections using the slider bars to indicate which criterion is more important, i.e. higher weighting. Here, the project manager decided to use the weighting (in Figure 5-8) derived from the group discussion during the TOPSIS process to define the criteria weights. See Figure 5-10. The weights are not exactly the same because they are entered using a slider bar.

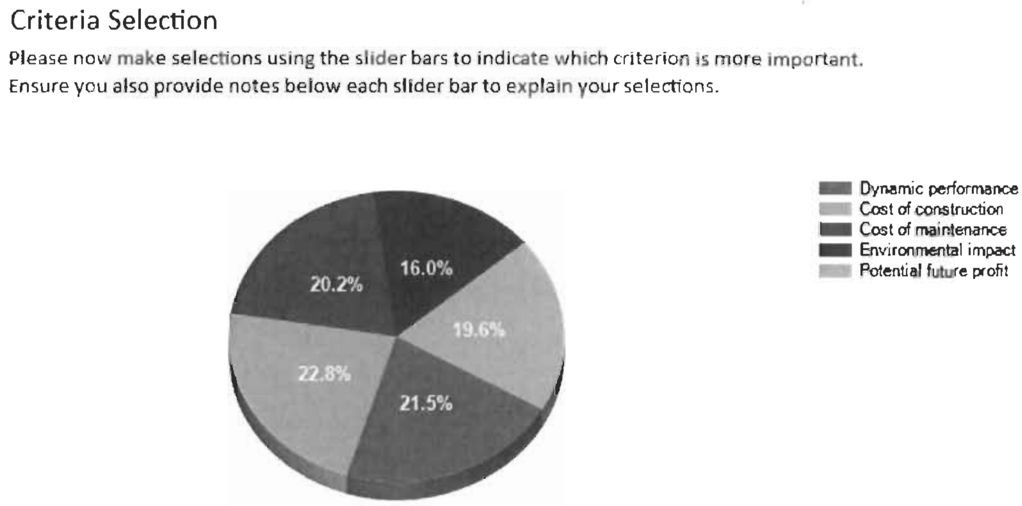

Figure 5-10: Chemdecide software framework ELECTRE III: criteria weights

Step 3. For each quantitative criterion, enter its true quantitative data source (numerical value and unit) as well as the indifference, preference and veto thresholds. Two alternatives are considered indifferent if their difference is smaller than or equal to the indifference threshold; Alternative A is preferred to Alternative B if their difference is larger than the indifference threshold and smaller than or equal to the preference threshold; Alternative A is vetoed in favour of Alternative B if their difference is larger than the preference threshold and smaller than or equal to the veto threshold. In this case, the user does not know the meaning thresholds; the tool has already provided the explanation to make sure the user entered reasonable inputs. For each qualitative criterion, the user indicates his/her preference for each alternative using the slider bar. The slider bar assigns an evaluation of extremely poor, very poor, average, good, very good, and excellent. Figures 5-11 and 5-12 provide some insight into the above description.

The user has entered all the information in the above steps. The software generates a report showing the results as in Table 5-5. It shows that ELECTRE III assigns both P1 and P2 first rank: the descending order proposes P1 as the best alternative, while the ascending order proposes P2 as the best alternative.

Table 5-5: ELECTRE III: results of sewer network planning case study

	Descending Order	Ascending Order	Final Order
1st	P1	P2	P1 P2
2nd	P2	P1	P3
3rd	P3 P4	P3	P4
4th		P4	

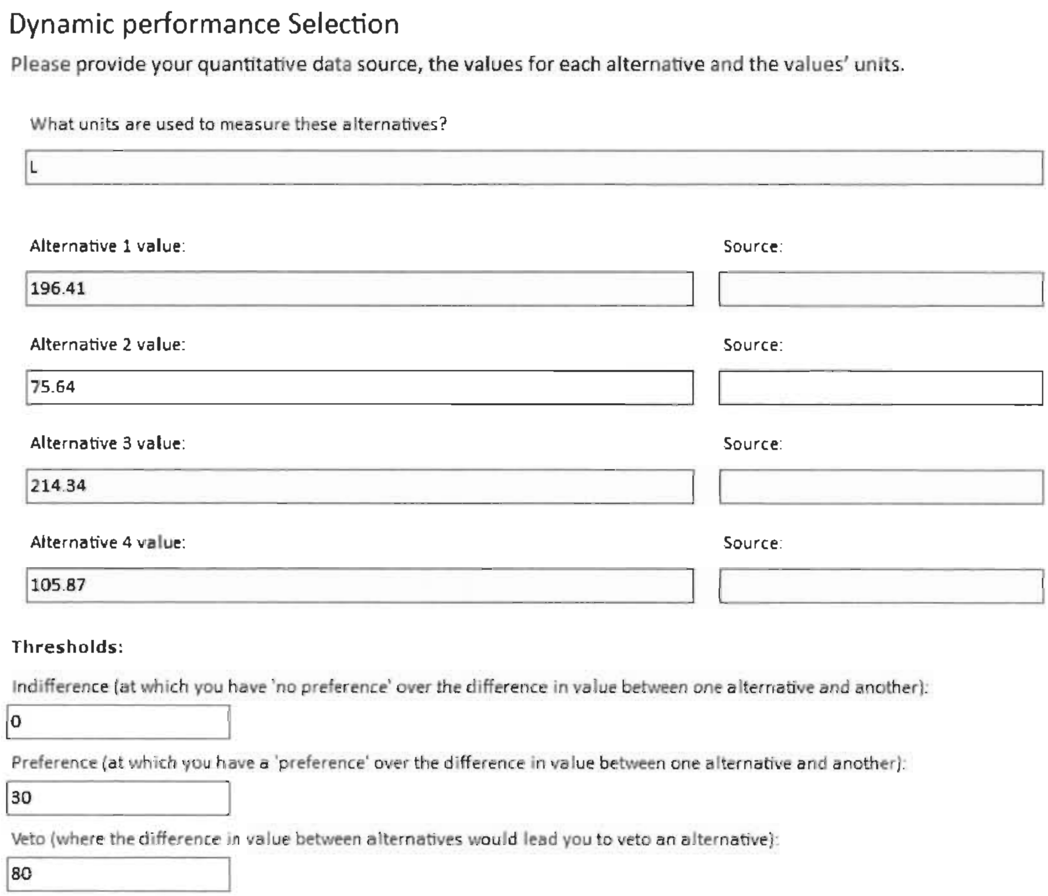

Figure 5-11: Chemdecide ELECTRE III quantitative criterion

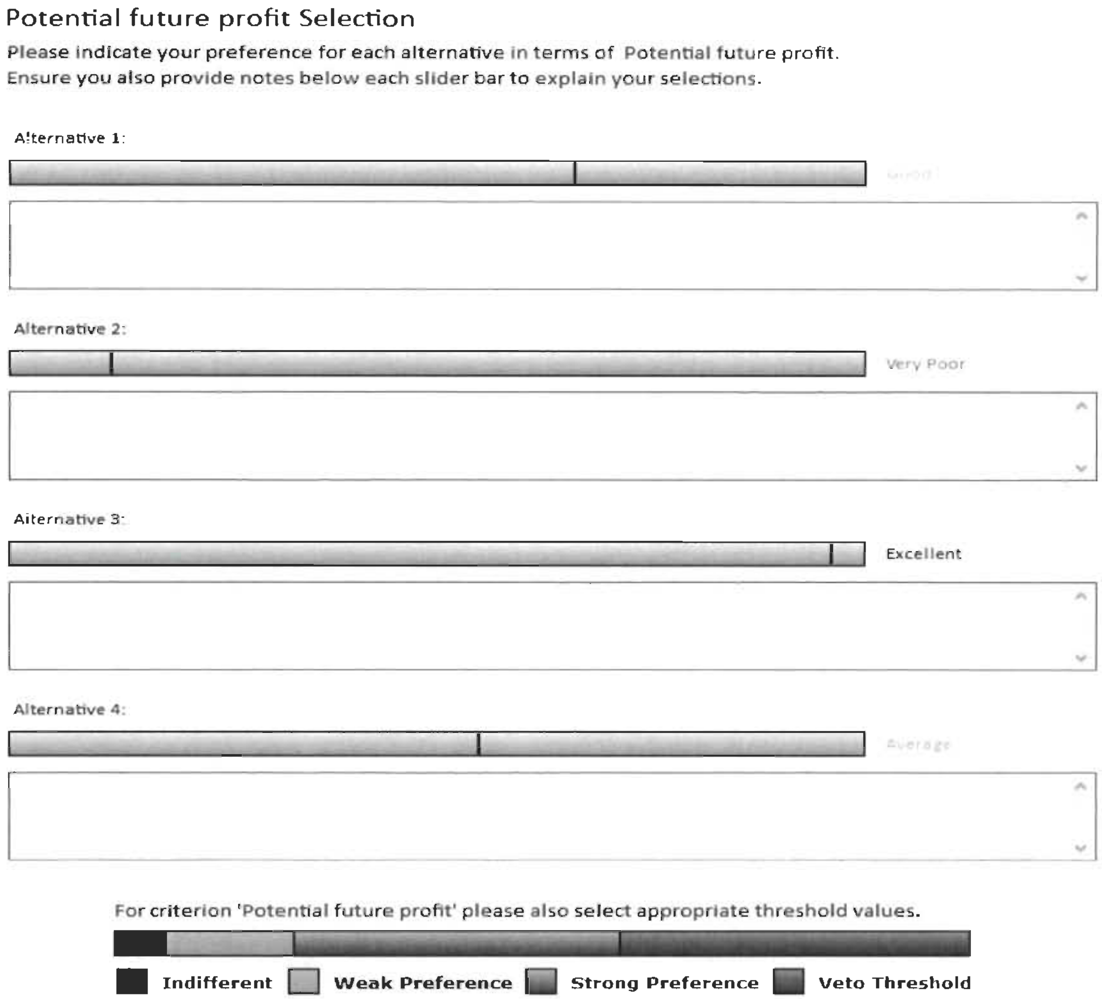

Figure 5-12: Chemdecide ELECTRE III qualitative criterion

5.3.2.4 PROMETHEE II

Although all the PROMETHEE II computations can be performed manually, for simplicity's sake, and because DMs can have a different experience using a manual decision-making process, a software tool is chosen to aid professionals in implementing this MCDM method. The current available software for PROMETHEE are *Decision Lab*, *D-Sight*, *Smart Picker Pro* and *Visual Promethee* (Ishizaka & Nemery, 2013). From these, *Smart Picker Pro* (Brussels, 2011), developed by a team from the engineering department at the Free University of Brussels, is chosen. Its user-friendly interface allows DMs to model the decision problem step by step and enter their preferences, e.g., the criteria weighting and other preference parameters. It reflects the user preferences entered into the software. Also, unlike other software, it is available as a free trial version (www.smart-picker.com) with time-unlimited use. However, its trial version is limited to a maximum of five alternatives and four criteria, but this is sufficient to comprehend its application. *Smart Picker Pro* does not require much understanding of the PROMETHEE II method itself, which makes it very easy to use. The algorithm behind this tool is PROMETHEE I (partial ranking) and PROMETHEE II (complete ranking). As previously mentioned, PROMETHEE II is the method used from the PROMETHEE family in this case study. Full instructions for this software can be found in Ishizaka and Nemery (2013) or the HELP menu in the tool.

As was the case for ELECTRE III, the project manager represents the whole project group in using the software. The essential operating steps for the tool in solving the sewer network planning decision problem are listed below.

Step 1. Enter the performance of alternatives for different criteria. See Figure 5-13. The performance of alternatives for qualitative criteria (dynamic performance and cost of construction) are based on the true experiment value, while the performances for quantitative criteria are evaluated on a scale of Very Good, Good, Average, Bad or Very Bad; the corresponding scores for this scale are 4, 3, 2, 1, 0 respectively. Ultimately, both quantitative and qualitative criteria are quantified. It is worth mentioning that in the PROMETHEE method, there is no need to restrict all the performances measured to the same unit.

All Data | Parameters | Weights

Performances of the actions

	Name	C1	C2	C3	C4	C5
		dynamic performance	cost of construction	cost of maintanence	environmental impact	potential future profit
A1	p1	196.41	1884753.0	good	average	average
A2	p2	75.64	437606.0	good	good	bad
A3	p3	214.34	4127967.2	bad	bad	very good
A4	p4	105.87	2680820.57	bad	bad	average

Figure 5-13: Smart picker pro PROMETHEE II: performance of alternatives

Step 2. Set up the preference parameters, such as: maximize or minimize, to indicate whether it is a positive or negative criterion; preference function: linear function is selected for all criteria; indifference and preference threshold; see Figure 5-14 for the setup of one criterion.

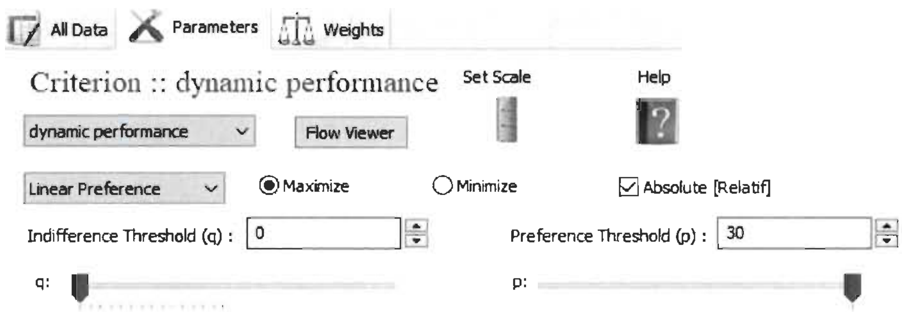

Figure 5-14: Smart Picker Pro PROMETHEE II: preference parameter setup for dynamic performance

Step 3. Set the criterion weight values. In this case, the project manager decided to use the weights derived from the group discussion during the TOPSIS process to define the criteria weights. In *Smart Picker Pro*, users set the weights using a slider bar. See Figure 5-15. Note that the weights are not exactly the same values as shown in TOPSIS, because the slider bar cannot provide the exact value and causes bias.

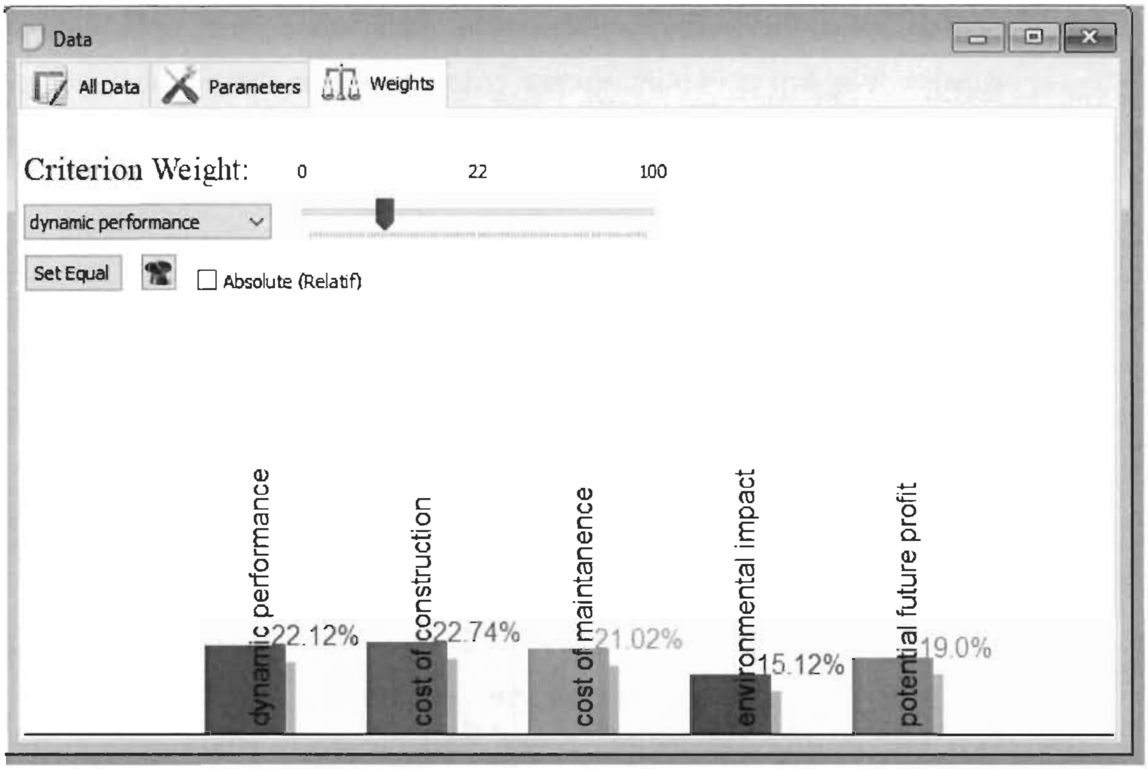

Figure 5-15: Smart Picker Pro PROMETHEE II: criteria weights

With the above steps, all the decision problem inputs are ready for *Smart Picker Pro* to analyze and show the final ranking result. The result is shown in Figure 5-16. P1, ranked in first position, has the highest net flow, which is much higher than the runner up, P2; this ensures its first position over all other alternatives. P3 and P4 received negative net flows far behind the first two.

Results | Processed Data

Actions	Net Flows	Position
A1 - p1	0.30621	1.0
A2 - p2	0.07286	2.0
A3 - p3	-0.07731	3.0
A4 - p4	-0.30175	4.0

Figure 5-16: PROMETHEE II: final results for sewer network planning case study

5.3.3 Results Summary and Post-Analysis Interview

Figure 5-7, Table 5-4, Table 5-5 and Figure 5-16 show the results of AHP, TOPSIS, ELECTRE III and PROMETHEE II respectively. All of them recommend alternatives P1 and P2 over P3 and P4. Table 5-6 groups all the results together. It shows that AHP chose P2 over P1 as the best option; TOPSIS and PROMETHEE II prefer P1 over P2; ELECTRE II could not provide a conclusive decision between P1 and P2, where both are given first ranking.

Table 5-6: Comparison of results from four MCDM methods

	1st	**2nd**	**3rd**	**4th**
AHP	P2	P1	P3	P4
TOPSIS	P1	P2	P3	P4
ELECTRE III	P1 P2		P3	P4
PROMETHEE II	P1	P2	P3	P4

The whole project team is interviewed to review their experiences and discuss the results. On reflection, for AHP, they agreed that pairwise comparison is indeed an efficient and accurate way to evaluate the preference between two alternatives rather than simultaneously evaluating all alternatives. However, numerous pairwise comparisons are required. Even though there is a consistency check to guarantee the subjective judgments from pairwise comparison, professionals still feel somewhat less confident with their inputs during the long pairwise comparison process. They stated that AHP is a good option for a decision involving only a few criteria and alternatives. During the process of TOPSIS, experts also needed to have team meetings to decide criteria weighting and use a 1-10 scale to score the performance of each alternative for different criteria. They felt more comfortable and confident in evaluating their preference since it is less complex than pairwise comparison in terms of the number of inputs and measurement scale. This is also why the project manager used the criteria weights from TOPSIS for the other two MCDM methods instead of the weights from the pairwise comparison. They also wanted to mention that TOPSIS requires all performances for different criteria to be in the same measurement unit, even the quantitative criteria, which means their true experimental values cannot be input into the decision matrix, but are instead transferred to a 1-10 scale. This also causes bias for the final score. The two software tools for ELECTRE III and PROMETHEE II are easy to operate and understand, which is the opposite of their complex underlying algorithms. The project manager found that the whole experience with software tools for the decision-making process was positive in terms of organization. It helped him to have a clear structure of the decision problem and give all necessary and correct inputs. Moreover, he had a clear view of the relations between the input values and the outcomes so he is aware of which factors had more impact during the process. Therefore, using software tools definitely reduced the disadvantages of these two methods. The result from PROMETHEE II is clearly indicated via each alternative's net flow value, while ELECTRE III does not give a specific score to each alternative. Besides, ELECTRE III could not make a definite decision between P1 and P2, which made it more clear from the decision maker's point of view.

5.3.4 Comparative Analysis and Discussion

In order to fully understand the decision reached by different MCDM methods, a deep comparative analysis is carried out on two factors: criteria weights obtained during the different MCDM processes and alternatives' scores for each criterion assigned by different methods.

5.3.4.1 Comparison of criteria weights

In Figure 5-17, each criterion's weight derived from AHP, TOPSIS, ELECTRE III and PROMETHEE II are displayed together for a clear picture for comparison.

In general, the weight allocations for different criteria are consistent in TOPSIS, ELECTRE III and PROMETHEE II. Inconsistency occurs during AHP, which places considerable attention on the maximizing criteria (dynamic performance, potential future profit) compared to the other three minimizing criteria. As mentioned before, the user input the criteria weights derived from TOPSIS for ELECTRE III and PROMETHEE II.

In Figure 5-17, there are still slight differences among them that could be caused by manual operation errors.

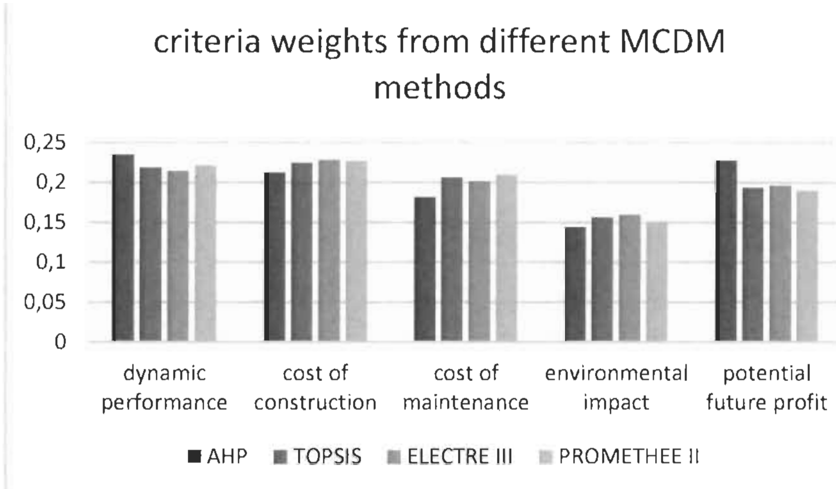

Figure 5-17: Comparison of criteria weights

5.3.4.2 Comparison of alternative scores

Figure 5-18 provides an overview of the differences for each alternative evaluated via different MCDM processes. Note that all scores have been normalized in order to make the comparison more persuasive.

For the two quantitative criteria (dynamic performance and cost of construction), the alternative scores in AHP, ELECTRE III and PROMETHEE II are consistent because the true experimental numerical values are used as input. However, in the TOPSIS process, since the decision matrix needs to be measured in the same unit, the inability to use true values for quantitative criteria causes inaccuracy.

For the other three qualitative criteria (cost of maintenance, environmental impact and potential future profit), alternative scores show a number of inconsistencies in the four MCDM methods. One explanation is that it is difficult to stay consistent when making subjective judgments on alternatives for qualitative criteria in different processes. The difficulty can be the result of decision-maker fatigue after prolonged attention and mental effort. Vohs, et al. (2005) argue that making decisions from different alternatives for various criteria requires energy, tires out decision makers and thereby impairs self-regulation. Vohs, et al. (2005) refer to this situation as decision fatigue and conclude that "self-regulation was poorer among those who had made choices than among those who had not". Another explanation for the inconsistency is that decision makers might feel that the impact of scores for qualitative criteria are minor. However, to have a sound, reliable decision result from a structured decision analysis requires decision makers to express their preferences more carefully.

Nevertheless, it is worth mentioning that AHP has the most inconsistencies for qualitative criteria, with the majority of scores showing higher or lower criteria weights than the other three MCDM methods. This happened even though all of the decision makers' pairwise comparisons are theoretically consistent, i.e. the consistency ratio is less than 0.1. Therefore, either the decision makers placed emphasis on their preferences on purpose or there are inaccuracies in the 1-9 fundamental scale proposed by Saaty and Vargas (2001). In fact, Salo, & Hamalainen (1997) point out that there is an uneven dispersion of values in Saaty's AHP selection scale. They conclude that the difference in selecting between the scale of 1 and 2 is 15 times greater than the difference in selecting between the scale of 8 and 9. This indicates that Saaty's AHP selection scale is responsible for the overemphasized criteria weights and alternative scores in the case study.

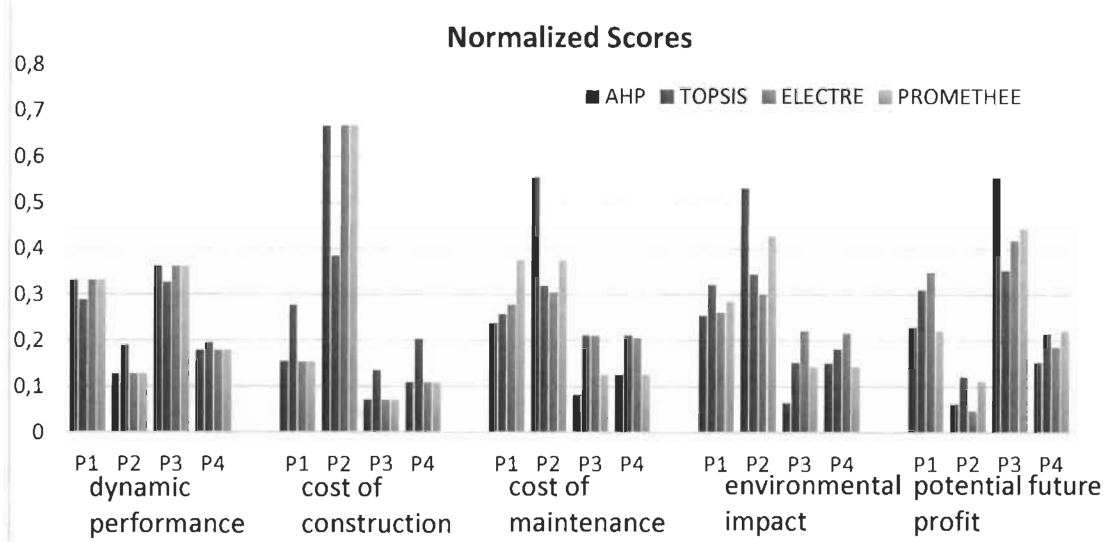

Figure 5-18: Comparison of alternative scores from the four MCDM methods

5.4 Conclusion

Making a decision on a sewer network construction project is important for urban development, public health and environmental sustainability. It has been suggested that a group of decision makers should apply an effective and efficient MCDM method for the sewer network decision problem. However, different methods have their own limitations, hypotheses, premises and perspectives, which leads to different decision results when applied to an identical problem. This chapter provides a comparative study on four different MCDM methods (AHP, TOPSIS, ELECTRE III and PROMETHEE II) from their distinctive theoretical algorithms and from their implementation on one sewer network planning group decision problem. AHP and TOPSIS were implemented via spreadsheets, while ELECTRE III and PROMETHEE II were applied via available software tools due to their complex algorithms. A number of conclusions can be drawn:

– Five criteria require 10 pairwise comparisons to determine the criteria weights in AHP, which is more time consuming. The other three methods only need 10 inputs. By increasing the number of criteria and alternatives, AHP is not a practical method to implement.

- The criteria weights and scores of the four methods are inconsistent, with AHP showing the greatest variation (Figure 3-14 and Figure 3-15). This is most likely because of inaccuracies in AHP's 1-9 fundamental scale, decision fatigue and decision makers' perception that qualitative criteria with low weights have minor impact on the decision results.
- There are visible differences in the results of the four methods (Table 4-6). It needs to address out that ELECTRE III was unable to provide a conclusive result, identifying both P1 and P2 as the best alternatives. PROMETHEE II and TOPSIS prefer P1, while AHP selects P2 as the best option. In general, P2 receives extremely high scores on three criteria and extremely low scores on the other two criteria, while P1 has a more or less average evaluation on different criteria. When considering this, decision makers all prefer P1 over P2.
- TOPSIS requires all the performances for different criteria to be expressed in the same measurement unit. This makes decision makers feel TOPSIS is limited when the true numerical experimental values cannot be used as input directly.
- PROMETHEE is the favoured method for decision makers in terms of the decisive result identifying P1 as the best option and decision makers' satisfaction with the implementation process.

The comparison of the different MCDM methods directly helped the whole project team to make an informed decision. By going through this process, all the experts became more knowledgeable about their decision and the uncertainty associated with each sewer network plan. The results clearly show that there is a risk in following the results of just one MCDM method; therefore, if time permits, it is advisable to approach a sewer network group decision problem using different decision-making methods. However, if time is a limitation then the results indicate that PROMETHEE II is the method that most effectively provided an accurate representation of the decision makers' preferences. The conclusion of this comparative study should also encourage industry professionals to cooperate with academic researchers in order to examine the compatibility of a wider range of MCDM methods with sewer water infrastructure management. More case studies are required to test and validate the theories, since the recommendations presented in this paper are based on only one sewer network decision problem.

CHAPTER 6 – CONCLUSION AND FUTURE RESEARCH

This work first discussed in detail the definitions, differences and perspectives of three different types of decision-making processes (DMUSU, DMUR and MCDM), in order to guide DMs in structuring their decision problems into the right type, which is essential for making a good decision. Once DMs formulate their decision problems into the right type, it is time for them to think about which DM methods associated with this type of decision-making process to implement. Hence, this work provides a study of the comparative research on various DM methods within each type of decision process from detailed theoretical algorithm to practical implementation. Note that this work does not compare the methodologies from different types of DM processes, simply because this work has focused on the discussion of differences among types of DM processes from the beginning. The outline of this research work can be seen in Figure 6-1. How the results of this research help DMs in their decision problems is summarized in the following subsections.

6.1 Decide the Type of Decision Process

The two main types of decision process considered here are DMUU and MCDM. Three basic elements for DMUU are states of nature, alternatives and outcomes. Based on DMs' knowledge of states of nature, DMUU contains two sub types: DMUSU, where DMs need to make a decision without any information about the probabilities of the various states of nature, and DMUR, where DMs can subjectively assign the probabilities of the states of nature. MCDM is a sub-discipline of operations research, where DMs evaluate multiple conflicting criteria in order to find a compromise solution subject to all the criteria. MCDM mainly focuses on helping DMs synthesize information to find a trade-off among the conflicting criteria.

In order to decide which type of decision process, this study advises DMs to consider first, what kind of external criteria they want to involve to evaluate the options; second, how much they know about those criteria; third, how actively they want to be involved in the whole process, i.e., inputting their own opinions during the process. One example is a farmer's decision problem of whether or not to harvest tomorrow. If only weather matters for the farmer, then, he needs to consider how much he knows about the weather tomorrow. If he does not know or is not willing to research weather conditions, then he would structure his decision problem according to DMUSU. However, if he can subjectively estimate the weather conditions (the percentage of likelihood of rain), he could consider DMUR. If there are other perspectives or criteria that the farmer needs to consider (e.g., cost, profit, etc.), then he can structure the decision into MCDM to list the cost of harvesting tomorrow, and the cost of not harvesting, as well as the profits for both scenarios.

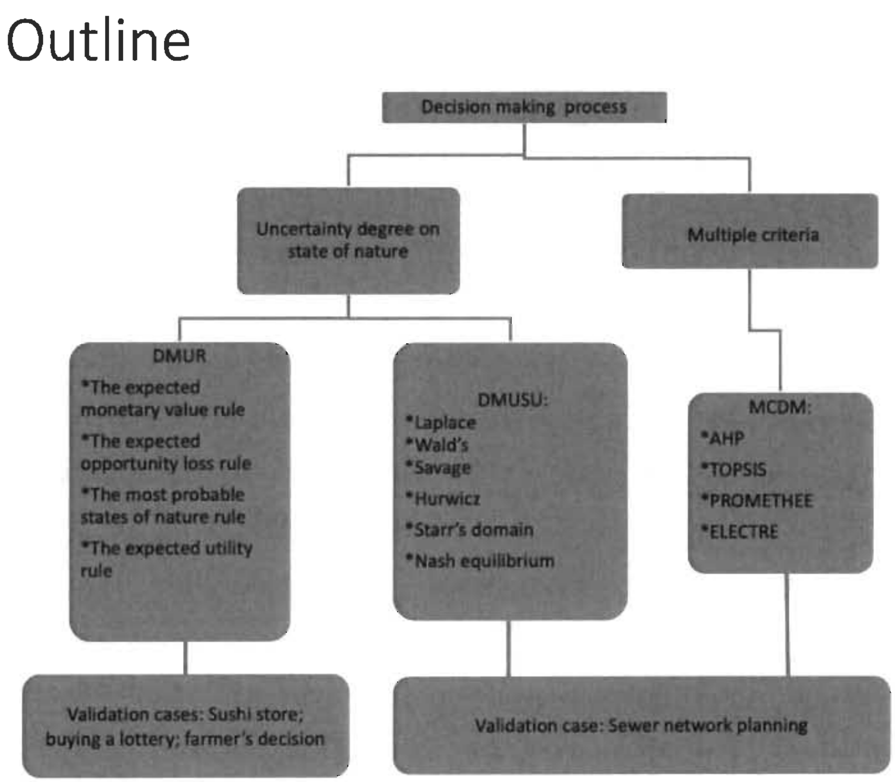

Figure 6-1: Outline of the research work

6.2 Decide Which Methodology to Use

Once the type of decision process is selected, it is time to choose which methodology under this type to employ.

For DMUSU, Laplace's principle of insufficient reason, Wald's Maximin, Savage's Minimax regret, Hurwicz's method and Starr's Domain are introduced and compared. Furthermore, a DMUSU problem is considered a two-player game, and NE is considered a method as well. The theoretical comparison of each method is summarized as follows: Laplace's principle of insufficient reason transforms a difficult problem into a simple one by assuming that all states of nature are equally alike. The need to construct the state space to be amenable to a uniform probability distribution is a major drawback of this method. Wald's Maximin is extremely conservative and does not provide a faithful representation of how people operate in reality. It could lead to exceedingly costly results from over-protection against uncertainty. Savage's Minimax regret method suggests the consequences of one action should be compared with the consequences of other actions under the same state of nature. Accordingly, it only reflects the difference between each payoff and the best possible payoff in a column. Hurwicz' method takes into account both the best and the worst possible results, weighted according to the decision maker's attitude (optimistic or pessimistic) towards the decision. This method only considers the highest and the lowest payoff for each alternative. It does not take other non-extreme payoffs into account. Therefore, two decisions with the same minimal and maximal profits always obtain an identical Hurwicz's measurement, even if one of them results in many small payoffs and the other one has many high payoffs. Starr's Domain has the disadvantage of complexity of computation when there are more than three states. Since a DMUSU problem can be considered a two-player non-cooperative and non-zero-sum game, NE becomes one of the solution options for solving a DMUSU problem. Pure-strategy NE is where all players are playing pure strategies, and mixed-strategy NE is where at least one player is playing a mixed strategy. All that said, if the DM's attitude is more conservative, Wald's and Savage's methods are correct. Wald's method uses the payoff matrix. If DMs would like to have a picture of their level of regret after making such a choice, they can use Savage's Minimax. If DMs would like to use a numerical value to represent their attitude, they can choose Hurwicz's method. Starr's Domain method is suitable where there are few states of nature. Laplace's method is quite intuitive and simple to use. NE is an algorithm from game theory.

For DMUR, the principle of the EMV rule is nearly identical to the EOL rule, except that one is using a payoff matrix, the other is using an opportunity-loss matrix. The most probable state of nature rule takes only one uncertain state of nature into account; it may lead to bad decisions. The expected utility rule is a better choice when dealing with a risky decision problem (e.g., the decision can only be made once or significant amounts of money are involved in the problem), as the expected monetary value criterion cannot encompass the full range of reasoning behind a decision as a human would. Thus, the decision chosen by EMV can be different from the one the decision maker himself would choose. In short, the computation of four decision rules for DMUR is similar. The difference is that each decision rule maximizes or minimizes different objects, i.e., the expected monetary value, the expected opportunity loss, the expected utility. The decision maker needs to choose which object s/he wants to consider based on the property of each individual DMUR problem.

For MCDM, AHP requires many inputs for pairwise comparisons, which is a time-consuming process. Therefore, this method should be chosen only for a small number of criteria and alternatives. Furthermore, the potential compensation between good scores on some criteria and bad scores on others causes the loss of information. The advantage of TOPSIS is that it requires only a few inputs from the decision maker and its output is easy to understand. The drawback is that vector normalization is needed for solving multi-dimensional problems. The main advantage of ELECTRE is that it avoids compensation between criteria and any normalization process, which distorts the original data. The drawback is that it requires various technical parameters such that it is not always easy to fully understand it. The PROMETHEE method allows direct operation on the variables included in the decision matrix without requiring any normalization and is applicable even when there is insufficient information. However, its main drawback is that it is time consuming and difficult for decision makers to have a clear view of the problem, especially when there are many criteria involved.

6.3 Further Comments on the Case Study: Sewer Network Selection

Making a decision on a sewer network construction project is important for urban development, public health and environmental sustainability. In this work, the same sewer network plan selection problem is structured into two different types of DM processes: DMUSU and MCDM. It is worth mentioning that if the probability of the different rainfall weather conditions can be assigned by the DMs, this practical problem can also be structured as DMUR. This shows that the same specific decision-making problem can be structured into different types of decision processes based on available information and on DMs' subjective preferences.

The practical comparison within each type of decision process is carried out using the same project; this can effectively show each method's limitations, hypotheses and differences.

Three basic elements for DMUSU are states of nature, alternatives and outcomes, where DMs need to make decisions without any information about the probabilities of the various states of nature. Laplace's principle of insufficient reason, Wald's criterion, Savage's Minimax regret criterion, Hurwicz's criterion and Starr's Domain criterion are introduced and compared. Furthermore, DMUSU problems are considered two-player games, and NE is used as well to find the selected decision. While different methods recommend different alternatives, the fact that the NE is 100% behind Alternative 1 is a compelling argument for choosing it. While Alternative 2 is the most-recommended alternative, it is interesting to note that Alternative 3 is not selected for any of the criteria. However, most civil engineers intuitively rooted for Alternative 3 from a purely city planning point of view. Further studies should compare this approach on more projects to evaluate if a trend is emerging. Also, from a pragmatic point of view, it is advisable to adapt the current decision process to include the comparison of these five DMUSU methods (and NE) to give a better depth to the decision. The next step is clearly to form a portfolio of decision policies and evaluate the robustness of such an approach compared to the individual criterion or the city's current decision process.

Since a sewer network plan selection problem is a complex decision problem that needs to be considered from different perspectives by different professionals, it is also restructured into a MCDM group decision problem. The Delphi technique is introduced in order to reach an opinion from a team. Of all the various MCDM methods, AHP, TOPSIS, ELECTRE III and PROMETHEE II are selected to implement, as they are the most-used MCDM methods in sewer network infrastructure asset management. The purpose is to conduct a comparative study of these methods on a single decision problem in order to address their limitations, hypotheses, premises and perspectives and help DMs to select the proper decision-making method for their decision problem. AHP requires many inputs because of pairwise comparisons, which is time-consuming. This method should be selected only when there are few criteria and alternatives. The AHP method also shows more inconsistency in the decision process than other methods. This could be the inaccuracy of the 1-9 scale. Inconsistency in TOPSIS, PROMETHEE II and ELECTRE III could be caused by decision maker fatigue in a long decision process or decision makers' perception that qualitative criteria with low weights have minimal impact on the decision result. ELECTRE III is not considered a favourable method, as it cannot provide a conclusive result for this particular decision problem. The limitation of TOPSIS is that it requires all the performances under different dimension criteria to be evaluated by the same measurement unit. By doing this, it loses information from the true value. PROMETHEE is considered the favoured method for decision makers for its conclusive decision result and the reflection of the decision makers' preferences. Furthermore, as it does not require all the performances to be expressed in the same unit, it is more in line with the true facts than others.

6.4 Future Research

The following future research related to this PhD study can be considered:

- Nash equilibrium implemented in DMUSU problem brings another perspective for solving DM problems. It is interesting to provide a mathematic proof in theory to see further, how decision-making and game theory are related with each other;

- Applying the considered DM methods in this thesis into more real life projects from different industry area can solidify the comparative conclusion;
- More focus can be given to DMUSU in order to make the system more resilient to cope with sudden changes or any type of crisis, because the effect of perturbation in these scenarios is exponential.
- Other different MCDM methods are also worth to study and implement.

6.5 Final Remarks

The results clearly show that there is a risk in following the results of one particular DMUSU method or one particular MCDM method. Therefore, if time allows, it is advisable to structure the decision problem into different types of DM problems and use different decision-making methods. However, if time is a limitation, through this research, decision makers have obtained sufficient knowledge about various DM methods to make their own choice of which method to use. The results of this PhD work should encourage industry professionals to work together with academic researchers in order to explore and compare other available DM methods for various practical decision problems to validate the theories and recommendations.

The whole PhD work can be illustrated by the diagram in Figure 6-1. The initial motivation and objective of this research is to help DMs choose the right decision-making methodology that suits the subjective preferences and the objective information, so that an selected decision can be made to balance the whole situation. This work suggests DMs first define the goal of the decision problem and check what kind of information is available to use, in order to clarify if they want to use the decision-making process with uncertainty, i.e., DMUU, or if they know a list of criteria from which the alternatives should be evaluated, i.e., MCDM. Second, they have a list of DM methodologies to choose from, depending on the type of decision-making process. Based on the comparative results of this work, DMs can confidently choose the appropriate method based on each methodology's characteristics and the decision maker's own preference.

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APPENDIX I

Matlab codes for DMUSU Methods

```
%% Laplace's insufficient reason criterion
%M is decision matrix, indexX indicates the index of the selected decision.
function indexX = laplace_insufficient_reason(M)
v=sum(M,2); %v is a column vector containing the sum of each row.
[a,indexX]=max(v);
end
%

%% Wald Maximin function
%M is decision matrix, indexX indicates the index of the selected decision.
function [v,indexX,indexY] = maximin(M)
[s,idy]=min(M,[],2);
[v,indexX]=max(s);
indexY=idy(indexX);
end
%

%% Savage function payoff
function [v,indexX,indexY]=savageMinimax(M)
tmpM=ones(size(M,1),1)*max(M)-M;
[v,indexX,indexY]=minimax(tmpM);
end
%

%hurwicz on positive flow matrix M e.g. payoff
%alpha is the degree of optimism, 1-alpha is the degree of pessimism
%for each row i, determine a P_i = alpha * best payoff + (1-alpha)*worst payoff
function row_number = hurwiczpositiveflow(M,alpha)
[nr, nc] = size(M);
h=ones(nr,1);
for i =1:1:nr
h(i)=max(M(i,:))*alpha + min(M(i,:))*(1-alpha);
end
[v,row_number] =max(h);
%
```

```
%% Starr function
% A is the decision matrix
function [v,idx,count] = starr(A)
[r,c]=size(A);

count=zeros(1,r);
total = 1000000;

for i=1:total
%Monte-Carlo
mc = sort(rand(1,c-1));
mc1=[0,mc];
mc2=[mc,1];
mcs=mc2-mc1;

score=sum(A*mcs',2);
idx=find(score==max(score));
count(idx)=count(idx)+1;
end
count
idx=find(count==max(count));
v=count(idx)/total;

end
%

%computes the mixed nash equilibrium for two players zero-sum games
function [v,p,q] = mixedNE4(A)
[r,c]=size(A);%r:row ; c:coloumn
AA = [-A', ones(c,1)];
```

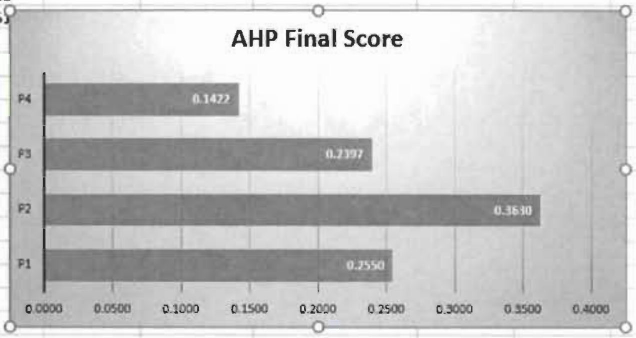
```
Aeq = [ones(1,r),0];
AA_octave = [AA;Aeq];
b = zeros(c,1);
beq = 1;
b_octave=[b;beq];
lb = [zeros(r,1);-inf];
f = [ zeros(r,1);-1];
options = optimset('Display', 'off');
s = 1;
p = linprog(f,AA,b,Aeq,beq,lb,[],[],options); % for matlab
v = p(r+1);
p = p(1:r);

if nargout > 2
   [w,q] = mixedNE4(-A');
end

end
```

APPENDIX II

Excel file for data collection during MCDM Implementation

	A	B	C	D	E	F	G	H
1		C1	C2	C3	C4	C5	total priority	:sum of each row
2	P1	0.07790929	0.03281252	0.04305546	0.036620284	0.06465168	0.255049234	
3	P2	0.02996511	0.14143328	0.10066487	0.076553183	0.01436001	0.362976456	**
4	P3	0.08510092	0.01499193	0.01479652	0.009252518	0.11558568	0.239727565	
5	P4	0.04195116	0.02306777	0.02292248	0.021639629	0.03266571	0.142246745	
6		0.23492648					1	
8	this is the matrix to compare with topsis normalised decision matrix this is not multiplied with the weight of criteria							
18	0.33163265	0.28626302	0.23729947	0.2541917	0.228923914			
19	0.12755102	0.47093393	0.55481283	0.5313772	0.061910123			
20	0.3622449	0.0587186	0.0815508	0.06422433	0.556468712			
21	0.17857143	0.18408446	0.1263369	0.15020676	0.15269725			

	Alternatives	Final Score
27	P1	0.2550
28	P2	0.3630
29	P3	0.2397
30	P4	0.1422



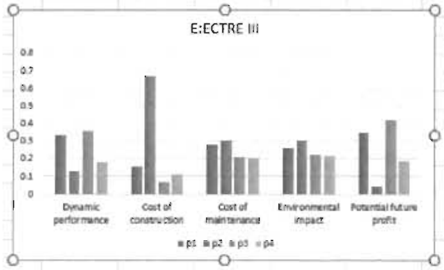
91	distance with ideal solution	P1	P2	P3	P4
92	Dynamic performance	0.000276125	0.00338253	0	0.00314523
93	Cost of construction	0.002054133	0	0.01101607	0.00582668
94	Cost of maintenance	0.000631423	0	0.00193373	0.00193373
95	Environmental impact	4.80014E-05	0	0.00326709	0.00235207
96	Potential future profit	0.000236124	0.00714274	0	0.00249406
98	S_ideal	0.056971982	0.10259275	0.12734559	0.12550608

101	distance with non ideal solution	P1	P2	P3	P4
102	Dynamic performance	0.00172578	0	0.00338253	4.3144E-06
103	Cost of construction	0.003556325	0.01101607	0	0.00081938
104	Cost of maintenance	0.000355176	0.00193373	0	0
105	Environmental impact	0.002523071	0.00326709	0	7.5002E-05
106	Potential future profit	0.004781507	0	0.00714274	0.00119538
108	S_nonideal	0.11376229	0.12734559	0.10259275	0.04576102

	p1	p2	p3	p4
s_ideal	0.056971982	0.10259275	0.12734559	0.12550608
s_nonideal	0.11376229	0.12734559	0.10259275	0.04576102
relative close	0.666311976	0.55382494	0.44617506	0.26719094
best is	0.666311976			

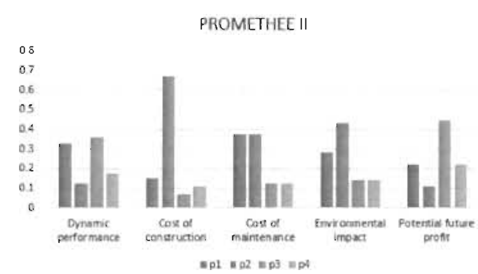
TOPSIS	p1	p2	p3	p4
Relative closeness	0.6663	0.5538	0.4462	0.2672

	A	B	C	D	E	F	G	H
1	electre III criteria							
2	dynamic per	0.2150						
3	cost of cons	0.2280						
4	cost of main	0.2020						
5	environment	0.1600						
6	potential fut	0.1960						
7								
8	dynamic perfomance							
9	p1	196.4100						
10	p2	75.6400						
11	p3	214.3400						
12	p4	105.8700						
13								
14								
15	cost of construction			cost of maintance		environment		future
16	p1	0.23		62		59		75
17	p2	1		68		68		10
18	p3	0.106		47		50		90
19	p4	0.1631		46		49		40
20								
21								
22	196.4100	0.23	62	59	75			
23	75.6400	1	68	68	10			
24	214.3400	0.106	47	50	90			
25	105.8700	0.1631	46	49	40			
26	592.2600	1.4991	223	226	215			
27	c1	c2	c3	c4	c5			
28	0.331628	0.153425389	0.2780269	0.2610619	0.3488372	p1		
29	0.1277142	0.667066907	0.3049327	0.300885	0.0465116	p2		
30	0.3619019	0.070709092	0.2107623	0.2212389	0.4186047	p3		
31	0.178756	0.108798613	0.206278	0.2168142	0.1860465	p4		



	Dynamic performance	Cost of construction	Cost of maintenance	Environmental impact	Potential future profit
p1	0.331628	0.1534254	0.2780269	0.2610619	0.3488372
p2	0.1277142	0.6670669	0.3049327	0.300885	0.0465116
p3	0.3619019	0.0707091	0.2107623	0.2212389	0.4186047
p4	0.178756	0.1087986	0.206278	0.2168142	0.1860465

	A	B	C	D	E	F
1		Dynamic performance	Cost of construction	Cost of maintenance	Environmental impact	Potential future profit
2	p1	196.41	0.23	3	2	2
3	p2	75.64	1	3	3	1
4	p3	214.34	0.106	1	1	4
5	p4	105.87	0.1631	1	1	2
6	sum	592.26	1.4991	8	7	9
7						
8		Dynamic performance	Cost of construction	Cost of maintenance	Environmental impact	Potential future profit
9	p1	0.331628	0.15342539	0.375	0.28571429	0.22222222
10	p2	0.12771418	0.66706691	0.375	0.42857143	0.11111111
11	p3	0.36190187	0.07070909	0.125	0.14285714	0.44444444
12	p4	0.17875595	0.10879861	0.125	0.14285714	0.22222222



94					
95		AHP	TOPSIS	ELECTRE	PROMETHEE
96	P1	0.33163265	0.28780488	0.331628	0.331628
97	P2	0.12755102	0.1902439	0.12771418	0.12771418
98	P3	0.3622449	0.32682927	0.36190187	0.36190187
99	P4	0.17857143	0.19512195	0.17875595	0.17875595
100		AHP	TOPSIS	ELECTRE	PROMETHEE
101	P1	0.15455333	0.27683616	0.15342539	0.15342539
102	P2	0.66617814	0.38418079	0.66706691	0.66706691
103	P3	0.07061488	0.13559322	0.07070909	0.07070909
104	P4	0.10865365	0.20338983	0.10879861	0.10879861
105		AHP	TOPSIS	ELECTRE	PROMETHEE
106	P1	0.23729947	0.25773196	0.27802691	0.375
107	P2	0.55481283	0.31958763	0.30493274	0.375
108	P3	0.0815508	0.21134021	0.21076233	0.125
109	P4	0.1263369	0.21134021	0.20627803	0.125
110		AHP	TOPSIS	ELECTRE	PROMETHEE
111	P1	0.2541917	0.32163743	0.26106195	0.28571429
112	P2	0.5313772	0.34502924	0.30088496	0.42857143
113	P3	0.06422433	0.15204678	0.22123894	0.14285714
114	P4	0.15020676	0.18128655	0.21681416	0.14285714
115		AHP	TOPSIS	ELECTRE	PROMETHEE
116	P1	0.22892391	0.31052632	0.34883721	0.22222222
117	P2	0.06191012	0.12105263	0.04651163	0.11111111
118	P3	0.55646871	0.35263158	0.41860465	0.44444444
119	P4	0.15269725	0.21578947	0.18604651	0.22222222
120					
121					
122					
123