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An Evolved Control Design of Complex Systems with Multi-time Delays and Multi-Interconnections

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Abstract: To guarantee the asymptotic stability of multi-time delays complex system with multi-interconnections, an evolved control design is proposed in this paper. Based on this criterion and the decentralized control scheme, a set of fuzzy controllers is then synthesized via the technique of parallel distributed compensation (PDC) to stabilize a complex system with multi-interconnections. This representation of PDC is constructed by sector nonlinearity which converts the nonlinear model to multiple rule base of the linear model and a new sufficient condition to guarantee the asymptotic stability via Lyapunov function is implemented in terms of linear matrix inequalities (LMI). Finally, a numerical example with simulations is given to demonstrate the results.

Keywords: Complex system, multi-time delays, parallel distributed compensation

1. Introduction

The mathematical models of many physical and engineering systems are frequently of high dimension, or possessing interactive dynamic phenomena. The information processing and requirements to experiment with these models for control purposes are usually excessive. It is therefore natural to seek techniques that can reduce the computational effort. The methodologies of complex systems provide such techniques through the manipulation of system structure in some way. Thus, there has been considerable interest in the research area of modeling, analysis, optimization and control of complex systems [1]. Recently, many approaches have been used to investigate the stability

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and stabilization of complex systems, as proposed in the literature [2-5].

Moreover, time delay is commonly encountered in various engineering systems; for example, systems with computer control have time delays, as it takes time for the computer to execute numerical operations. Besides, remote working, radar, electric networks, transport process, metal rolling systems, etc. all have time delays. The output in these systems responds only to an input after some time interval. The introduction of time-delay factor is often a source of instability and generally complicates the analysis. Hence, the problem of stability analysis of delay systems has been a main concern of several previously published research efforts [6-8].

We have witnessed rapidly growing interest in fuzzy control in the past few years, and there have been many successful applications. In spite of the success, it has become evident that many basic issues remain to be further addressed. Stability analysis and systematic design are certainly among the most important issues for fuzzy control systems. During the last decade, there have been significant research efforts on these issues (see [9-16] and the references therein). However, a literature search indicates that the stabilization problem of delay complex system with multi-interconnections remains unresolved.

Hence, a stability criterion in terms of Lyapunov's direct method is derived in this study to guarantee the asymptotic stability of multi-time delay fuzzy complex system with multi-interconnections. According to this criterion and the decentralized control scheme, a set of fuzzy controllers is synthesized to stabilize a multi-time delay fuzzy complex system with multi-interconnections. Each subsystem is represented by a T-S fuzzy model with multi-time delays. In this type of fuzzy model, each fuzzy implication is expressed by a linear system model, which allows us to use linear feedback control as in the case of feedback stabilization. The control design is carried out based on the fuzzy model via the concept of PDC scheme. The idea is that a linear feedback control is designed for each local linear model. The resulting overall fuzzy controller, which is nonlinear in general, is a fuzzy blending of each individual linear controller [10, 13].

This paper is organized as follows. First, the T-S fuzzy model with multi-time delays is briefly introduced and the concept of PDC scheme is utilized to design fuzzy controllers. Then, based on Lyapunov's approach, a stability criterion is derived to guarantee the asymptotic stability of multi-time delay complex system with multi-interconnections. Finally, a numerical example with simulations is given to illustrate the results, and the conclusions are drawn.

2. System Description

Consider a Multi-time delay fuzzy complex system ***F*** composed of *J* interconnected subsystems F_j, $j = 1, 2, \cdots, J$. The *j*th isolated subsystem (without interconnection) of ***F*** is represented by a T-S fuzzy model with multi-time delays. The main feature of T-S fuzzy models is to express each rule by a linear state equation, and the *i*th rule of this fuzzy model is of the following form [10, 16]:

Rule *i*: IF $x_{1j}(t)$ is M_{i1j} and $\cdots$ and $x_{gj}(t)$ is M_{igj}

$$\text{THEN } \dot{x}_j(t) = A_{ij}x_j(t) + \sum_{k=1}^{N_j} A_{ikj}x_j(t-\tau_{kj}) + B_{ij}u_j(t) \tag{2.1}$$

where $x_j^T(t) = [x_{1j}(t), x_{2j}(t), \cdots, x_{gj}(t)]$, $u_j^T(t) = [u_{1j}(t), u_{2j}(t), \cdots, u_{mj}(t)]$

$i = 1, 2, \cdots, r_j$ and r_j is the number of IF-THEN rules of the *j*th subsystem; A_{ij}, A_{ikj} and B_{ij} are constant matrices with appropriate dimensions, $x_j(t)$ is the state vector, $u_j(t)$ is the input vector, τ_{kj} denotes the time delay, M_{ipj} ($p = 1, 2, \cdots, g$) are the fuzzy sets, and $x_{1j}(t) \sim x_{gj}(t)$ are the premise variables. The final state of this fuzzy dynamic system is inferred as follows:

$$\dot{x}_j(t) = \frac{\sum_{i=1}^{r_j} w_{ij}(t)[A_{ij}x_j(t) + \sum_{k=1}^{N_j} A_{ikj}x_j(t-\tau_{kj}) + B_{ij}u_j(t)]}{\sum_{i=1}^{r_j} w_{ij}(t)} = \sum_{i=1}^{r_j} h_{ii}(t)[A_{ii}x_i(t) + \sum_{k=1}^{N_j} A_{iki}x_i(t-\tau_{ki}) + B_{ii}u_i(t)] \tag{2.2}$$

with

$$w_{ij}(t) = \prod_{p=1}^{g} M_{ipj}(x_{pj}(t)), \quad h_{ij}(t) = \frac{w_{ij}(t)}{\sum_{i=1}^{r_j} w_{ij}(t)} \tag{2.3}$$

Based on the above analysis, the *j*th subsystem F_j with interconnections can be described as follows:

$$F_i: \begin{cases} \dot{x}_j(t) = \sum_{i=1}^{r_j} h_{ij}(t)[A_{ij}x_j(t) + \sum_{k=1}^{N_j} A_{ikj}x_j(t-\tau_{kj}) + B_{ij}u_j(t)] + \phi_j(t) & (2.4a) \\ \phi_i(t) = \sum_{\substack{n=1 \\ n\neq j}}^{J} C_{ni}x_n(t), & (2.4b) \end{cases}$$

where C_{nj} is the interconnection matrix between the *n*th and *j*th subsystems.

In the next section, the concept of PDC scheme is utilized to design fuzzy controllers and a stability criterion is proposed to guarantee the asymptotic stability of multi-time delay fuzzy complex system with multi-interconnections.

3. Parallel Distributed Compensation

On the basis of the decentralized control scheme, a set of fuzzy controllers is synthesized via the technique of parallel distributed compensation (PDC) to stabilize the Multi-time delay fuzzy complex system with multi-interconnections. The concept of PDC scheme is that each control rule is distributively designed for the corresponding rule of a T-S fuzzy model. The fuzzy controller shares the same fuzzy sets with the fuzzy model in the premise parts [11]. Since each rule of the fuzzy model is described by a linear state equation, linear control theory can be used to design the consequent parts of a fuzzy controller. The resulting overall fuzzy controller, nonlinear in general, is achieved by fuzzy blending of each individual linear controller.

Hence, the *j*th fuzzy controller can be described as follows:

Rule *i*: IF $x_{1j}(t)$ is M_{i1j} and $\cdots$ and $x_{gj}(t)$ is M_{igj} THEN $u_j(t) = -K_{ij}x_j(t)$, (3.1)

where $i = 1, 2, \cdots, r_j$. The final output of this fuzzy controller is

$$u_j(t) = -\frac{\sum_{i=1}^{r_j} w_{ij}(t)K_{ij}x_j(t)}{\sum_{i=1}^{r_j} w_{ij}(t)} = -\sum_{i=1}^{r_j} h_{ij}(t)K_{ij}x_j(t) \qquad (3.2)$$

Substituting Eq. (3.2) into Eq. (2.4), we have the *j*th closed-loop system with multi-interconnections:

$$\dot{x}_j(t) = \sum_{i=1}^{r_j}\sum_{f=1}^{r_j} h_{ij}(t)h_{fj}(t)[(A_{ij} - B_{ij}K_{fj})x_j(t) + \sum_{k=1}^{N_j} A_{ikj}x_j(t-\tau_{kj})] + \phi_j(t). \qquad (3.3)$$

The purpose of this paper is two-fold: to stabilize the closed-loop nonlinear system and to attenuate the influence of the external disturbance on the state variable. According to [10], the disturbance attenuation problem, which is characterized by means of the so-called L_2 gain of a nonlinear system, is defined as follows: Given a real number $\gamma > 0$, it is said that the exogenous input is locally attenuated by γ if there exists a neighborhood *U* of $\boldsymbol{x} = 0$ such that for every positive integer *N* and for which the state trajectory of the closed-loop nonlinear system starting from $\boldsymbol{x}(0) = 0$ remains in *U*

$$\sum_{j=1}^{J}\int_0^{t_f} x_j^T(t)Q_j x_j(t)dt \le \eta_j^2 \sum_{j=1}^{J}\int_0^{t_f} \phi_j^T(t)\phi_j(t)dt$$

where $\boldsymbol{Q}$ is a positive definite weighting matrix. The physical meaning is finding an L_2 gain less than or equal to a prescribed number γ (strictly less than 1).

If the initial condition is also considered, the inequality (3.1) can be modified as

$$\sum_{j=1}^{J}\int_0^{t_f} x^T(t)Q_j x_j(t)dt \le \sum_{j=1}^{J} x^T(0)P_j x_j(0) + \eta^2 \sum_{j=1}^{J}\int_0^{t_f} \phi_j^T(t)\phi_j(t)dt$$

where $\boldsymbol{P}$ are some positive definite matrices.

Prior to examination of asymptotic stability of the multi-time delay fuzzy complex system with multi-interconnections, a useful concept is given below.

Lemma 1 [17]**:** For any real matrices *X* and *Y* with appropriate dimensions, we have

$$X^TY + Y^TX \le \kappa X^TX + \kappa^{-1}Y^TY$$

where κ is a positive constant.

Theorem 1: The Multi-time delay fuzzy complex system $\boldsymbol{F}$ is asymptotically stable, if there exist positive constants α_j and β , $j = 1, 2, \cdots, J$ and the feedback gains K_{ij}'s are chosen to satisfy

(I) $\bar{\lambda}_j \equiv \max_k \lambda_M(\bar{Q}_{kj}) < 0 \quad \text{for } k = 1, 2, \cdots, N_j;\ j = 1, 2, \cdots, J$ (3.4a)

$\lambda_{ij} \equiv \lambda_m(Q_{ij}) > 0 \quad \text{for } i = 1, 2, \cdots, r_j;\ j = 1, 2, \cdots, J$ (3.4b)

$\lambda_{ifj} \equiv \lambda_m(Q_{ifj}) > 0 \quad \text{for } i < f \le r_j;\ j = 1, 2, \cdots, J$ (3.4c)

or

(II)
$$\Lambda_j \equiv \begin{bmatrix} -\bar{\lambda}_j & 0 & 0 & \cdots & 0 \\ 0 & \lambda_{1j} & 1/2\lambda_{12j} & \cdots & 1/2\lambda_{1r_j j} \\ 0 & 1/2\lambda_{12j} & \lambda_{2j} & \cdots & 1/2\lambda_{2r_j j} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 1/2\lambda_{1r_j j} & 1/2\lambda_{2r_j j} & \cdots & \lambda_{r_j j} \end{bmatrix} > 0 \quad \text{for } j = 1, 2, \cdots, J \qquad (3.5)$$

where $\bar{Q}_{kj} \equiv \alpha_j^{-1} I - R_{kj}$, (3.6)

$$Q_{ij} \equiv -\left\{(A_{ij} - B_{ij}K_{ij})^T P_j + P_j(A_{ij} - B_{ij}K_{ij}) + R_j + \alpha_j P_j \bar{A}_{ij} P_j + \sum_{n=1}^{J} \beta_n^{-1}\left[\left(\frac{J-1}{J}\right)I + \beta_n P_j C_{nj} C_{nj}^T P_j\right]\right\} \qquad (3.7)$$

$$Q_{ifj} = -\frac{1}{2}\left\{G_{ifj}^T P_j + P_j G_{ifj} + R_j + \alpha_j P_j \hat{A}_{ifj} P_j + \sum_{n=1}^{J} \beta_n^{-1}\left[\left(\frac{J-1}{J}\right)I + \beta_n P_j C_{nj} C_{nj}^T P_j\right]\right\} \qquad (3.8)$$

with $R_j = \sum_{k=1}^{N_j} R_{kj}$, $\bar{A}_{ij} = \sum_{k=1}^{N_j} A_{ikj} A_{ikj}^T$, $\hat{A}_{ifj} = \frac{\bar{A}_{ij} + A_{fj}}{2}$

$G_{ifj} = \frac{(A_{ij} - B_{ij}K_{fj}) + (A_{fj} - B_{fj}K_{ij})}{2}$, $P_j = P_j^T > 0$, $R_{kj} = R_{kj}^T > 0$,

and $\lambda_M(\bar{Q}_{kj})$ denotes the maximum eigenvalue of $\bar{Q}_{kj}$. Moreover, $\lambda_m(Q_{ij})$ and $\lambda_m(Q_{ifj})$ denote the minimum eigenvalues of Q_{ij} and Q_{ifj}, respectively.

The Appendix show the proof of the above Theorem and Evolved Bat Algorithm (EBA) is proposed based on the bat echolocation fuzzy complex system in the natural world. Unlike other swarm intelligence algorithms, the strong point of EBA is that it only has one parameter, which is called the medium, needs to be determined before employing the algorithms to solve problems. Choosing different medium determines different searching step size in the evolutionary process. In this study, we choose the air to be the medium because it is the original existence medium in the natural environment where bats live. The operation of EBA can be summarized in the following steps:

Initialization: the artificial agents are spread into the solution space by randomly assigning coordinates to them.

Movement: the artificial agents are moved. A random number is generated and then it is checked whether it is larger than the fixed pulse emission rate. If the result is positive, the artificial agent is moved using the random walk process $x_i^t = x_i^{t-1} + D$, where x_i^t indicates the coordinate of the i-th artificial agent at the t-th iteration, x_i^{t-1} represents the coordinate of the i-th artificial agent at the last iteration, and D is the moving distance that the artificial agent goes in this iteration $D = \gamma \cdot \Delta T$ where γ is a constant corresponding to the medium chosen in the experiment, and $\Delta T \in [-1, 1]$ is a random number. $\gamma = 0.17$ is used in our experiment because the chosen medium $x_i^{t_R} = \beta\left(x_{\text{best}} - x_i^t\right)$, $\beta \in [0, 1]$ where β is a random number; x_{best} indicates the coordinate of the near best solution found so far throughout all artificial agents; and $x_i^{t_R}$ represents the new coordinates of the artificial agent after the operation of the random walk process.

4. Example

Consider a multi-time delay fuzzy complex system composed of three interconnected subsystems which are described as follows.

Subsystem 1:

Rule 1: If $x_{11}(t)$ is M_{111} Then $\dot{x}_1(t) = A_{11}x_1(t) + \sum_{k=1}^{3} A_{1k1}x_1(t-\tau_{k1}) + B_{11}u_1(t)$,

Rule 2: If $x_{11}(t)$ is M_{211} Then $\dot{x}_1(t) = A_{21}x_1(t) + \sum_{k=1}^{3} A_{2k1}x_1(t-\tau_{k1}) + B_{21}u_1(t)$

with $x_1^{-}(t) = [x_{11}(t)\ \ x_{21}(t)]$, $\tau_{11} = 0.3$ (sec), $\tau_{21} = 0.5$ (sec), $\tau_{31} = 0.7$ (sec),

$$A_{11} = \begin{bmatrix} -9 & 1 \\ [illegible] & [illegible] \end{bmatrix}, A_{21} = \begin{bmatrix} -35 & -4 \\ [illegible] & [illegible] \end{bmatrix}, A_{111} = \begin{bmatrix} 1.2 & -0.3 \\ [illegible] & [illegible] \end{bmatrix}, A_{121} = \begin{bmatrix} 0.9 & 0.3 \\ [illegible] & [illegible] \end{bmatrix}, A_{131} = \begin{bmatrix} 2.2 & 0.5 \\ [illegible] & [illegible] \end{bmatrix},$$

$$A_{211} = \begin{bmatrix} 1 & -0.1 \\ [illegible] & [illegible] \end{bmatrix}, A_{221} = \begin{bmatrix} 0.7 & 0.1 \\ [illegible] & [illegible] \end{bmatrix}, A_{231} = \begin{bmatrix} 1.7 & 0.3 \\ [illegible] & [illegible] \end{bmatrix}, B_{11} = \begin{bmatrix} 0.55 \\ [illegible] \end{bmatrix}, B_{21} = \begin{bmatrix} 0.3 \\ [illegible] \end{bmatrix} \quad (4.1)$$

Subsystem 2:

Rule 1: If $x_{12}(t)$ is M_{112} Then $\dot{x}_2(t) = A_{12}x_2(t) + \sum_{k=1}^{3} A_{1k2}x_2(t-\tau_{k2}) + B_{12}u_2(t)$,

Rule 2: If $x_{12}(t)$ is M_{212} Then $\dot{x}_2(t) = A_{22}x_2(t) + \sum_{k=1}^{3} A_{2k2}x_2(t-\tau_{k2}) + B_{22}u_2(t)$

with $x_2^{-}(t) = [x_{12}(t)\ \ x_{22}(t)]$, $\tau_{12} = 0.4$ (sec), $\tau_{22} = 0.6$ (sec), $\tau_{32} = 0.8$ (sec),

$$A_{12} = \begin{bmatrix} -10 & 1 \\ [illegible] & [illegible] \end{bmatrix}, A_{22} = \begin{bmatrix} -34 & -4 \\ [illegible] & [illegible] \end{bmatrix}, A_{112} = \begin{bmatrix} 0.8 & 0.2 \\ [illegible] & [illegible] \end{bmatrix}, A_{122} = \begin{bmatrix} 1 & 0.3 \\ [illegible] & [illegible] \end{bmatrix}, A_{132} = \begin{bmatrix} 0.7 & -0.1 \\ [illegible] & [illegible] \end{bmatrix},$$

$$A_{212} = \begin{bmatrix} 0.7 & 0.1 \\ [illegible] & [illegible] \end{bmatrix}, A_{222} = \begin{bmatrix} 0.9 & 0.2 \\ [illegible] & [illegible] \end{bmatrix}, A_{232} = \begin{bmatrix} 0.6 & 0 \\ [illegible] & [illegible] \end{bmatrix}, B_{12} = \begin{bmatrix} 0.5 \\ [illegible] \end{bmatrix}, B_{22} = \begin{bmatrix} 0.36 \\ [illegible] \end{bmatrix} \quad (4.2)$$

Subsystem 3:

Rule 1: If $x_{13}(t)$ is M_{113} Then $\dot{x}_3(t) = A_{13}x_3(t) + \sum_{k=1}^{3} A_{1k3}x_3(t-\tau_{k3}) + B_{13}u_3(t)$,

Rule 2: If $x_{13}(t)$ is M_{213} Then $\dot{x}_3(t) = A_{23}x_3(t) + \sum_{k=1}^{3} A_{2k3}x_3(t-\tau_{k3}) + B_{23}u_3(t)$

with $x_3^{-}(t) = [x_{13}(t)\ \ x_{23}(t)]$, $\tau_{13} = 0.5$ (sec), $\tau_{23} = 0.8$ (sec), $\tau_{33} = 1.1$ (sec),

$$A_{13} = \begin{bmatrix} -12.5 & 1 \\ [illegible] & [illegible] \end{bmatrix}, A_{23} = \begin{bmatrix} -33 & -4 \\ [illegible] & [illegible] \end{bmatrix}, A_{113} = \begin{bmatrix} 3.5 & 0.2 \\ [illegible] & [illegible] \end{bmatrix}, A_{123} = \begin{bmatrix} 1.1 & 0.1 \\ [illegible] & [illegible] \end{bmatrix}, A_{133} = \begin{bmatrix} 1.1 & -0.2 \\ [illegible] & [illegible] \end{bmatrix}$$

$$A_{213} = \begin{bmatrix} 2.8 & 0 \\ -0.6 & 0.3 \end{bmatrix}, A_{223} = \begin{bmatrix} 1 & 0 \\ -0.2 & 0.6 \end{bmatrix}, A_{233} = \begin{bmatrix} 0.8 & 0 \\ 0.1 & 0.4 \end{bmatrix}, B_{13} = \begin{bmatrix} [illegible] \\ -2.2 \end{bmatrix}, B_{23} = \begin{bmatrix} [illegible] \\ 1.4 \end{bmatrix} \quad (4.3)$$

Moreover, the interconnection matrices among three subsystems are given in the following:

$$C_{21} = \begin{bmatrix} 0.3 & 0.1 \\ -1.2 & 1 \end{bmatrix}, \quad C_{31} = \begin{bmatrix} -1 & 0.9 \\ 0.1 & 0.5 \end{bmatrix}, \quad C_{12} = \begin{bmatrix} 5.1 & -1.4 \\ 1.6 & 3 \end{bmatrix},$$

$$C_{32} = \begin{bmatrix} 2 & 0.2 \\ 2.3 & -3.8 \end{bmatrix}, \quad C_{13} = \begin{bmatrix} 1.5 & 0.2 \\ 0.3 & 0.5 \end{bmatrix}, \quad C_{23} = \begin{bmatrix} 0.1 & 0.2 \\ 1.1 & 0.3 \end{bmatrix}. \quad (4.4)$$

Therefore, the multi-time delay fuzzy complex system with multi-interconnections can be summarized as

$$F:\begin{cases}\dot{x}_1(t)=\sum_{i=1}^{2}h_{i1}(t)[A_{i1}x_1(t)+B_{i1}u_1(t)+\sum_{k=1}^{3}A_{ik1}x_1(t-\tau_{k1})]+\phi_1(t) & (4.5a)\\ \dot{x}_2(t)=\sum_{i=1}^{2}h_{i2}(t)[A_{i2}x_2(t)+B_{i2}u_2(t)+\sum_{k=1}^{3}A_{ik2}x_2(t-\tau_{k2})]+\phi_2(t) & (4.5b)\\ \dot{x}_3(t)=\sum_{i=1}^{2}h_{i3}(t)[A_{i3}x_3(t)+B_{i3}u_3(t)+\sum_{k=1}^{3}A_{ik3}x_3(t-\tau_{k3})]+\phi_3(t) & (4.5c)\\ \phi_j(t)=\sum_{\substack{n=1\\n\neq j}}^{3}C_{nj}x_n(t). & (4.5d)\end{cases}$$

In order to stabilize the multi-time delay fuzzy complex system with multi-interconnections, three fuzzy controllers which are synthesized via the concept of PDC scheme are described as follows.

Fuzzy controller of subsystem 1:

Rule 1: If $x_{11}(t)$ is M_{111} Then $u_1(t)=-K_{11}x_1(t)$, (4.6a)

Rule 2: If $x_{11}(t)$ is M_{211} Then $u_1(t)=-K_{21}x_1(t)$. (4.6b)

Choosing the closed-loop eigenvalues $(-24.7,\,-8)$ for $A_{11}-B_{11}K_{11}$ and the closed-loop eigenvalues $(-17,\,-28)$ for $A_{21}-B_{21}K_{21}$, we have $K_{11}=\begin{bmatrix}-0.9708 & -12.4924\end{bmatrix}$ and $K_{21}=\begin{bmatrix}-27.153 & -14.4128\end{bmatrix}$.

Fuzzy controller of subsystem 2:

Rule 1: If $x_{12}(t)$ is M_{112} Then $u_2(t)=-K_{12}x_2(t)$, (4.7a)

Rule 2: If $x_{12}(t)$ is M_{212} Then $u_2(t)=-K_{22}x_2(t)$. (4.7b)

Choosing the closed-loop eigenvalues $(-22,\,-8.3)$ for $A_{12}-B_{12}K_{12}$ and the closed-loop eigenvalues $(-12,\,-29)$ for $A_{22}-B_{22}K_{22}$, we have $K_{12}=\begin{bmatrix}1.6209 & -11.2448\end{bmatrix}$ and $K_{22}=\begin{bmatrix}-26.5499 & -16.442\end{bmatrix}$.

Fuzzy controller of subsystem 3:

Rule 1: If $x_{13}(t)$ is M_{113} Then $u_3(t)=-K_{13}x_3(t)$, (4.8a)

Rule 2: If $x_{13}(t)$ is M_{213} Then $u_3(t)=-K_{23}x_3(t)$. (4.8b)

Choosing the closed-loop eigenvalues $(-21,\,-10)$ for $A_{13}-B_{13}K_{13}$ and the closed-loop eigenvalues $(-20,\,-22)$ for $A_{23}-B_{23}K_{23}$, we have $K_{13}=\begin{bmatrix}0.3457 & -9.2396\end{bmatrix}$ and $K_{23}=\begin{bmatrix}-23.571 & -11.3776\end{bmatrix}$.

For the purpose of fulfilling the stability conditions of Theorem 1, selecting the proper common positive definite matrix P and the control force K becomes the key problem to be dealt with. In this paper, we use EBA to discover the proper solutions. In this case, the obtained solutions can be classified into two categories: feasible and infeasible. It means that designing the fitness function in a binary operation form is a simpler way to answer to the need of this application. In this paper, the fitness function is designed based on the stability criterion derived from the LMI conditions via the Lyapunov function approach. The AND logical operation is employed in the fitness function for examining the solutions to produce the binary classification results on the discovered solutions. The fitness function is formulated as follows:

$$P_1=\begin{bmatrix}0.88 & 0.22\\0.22 & 0.65\end{bmatrix},\; P_2=\begin{bmatrix}0.8 & 0.22\\0.22 & 0.5\end{bmatrix},\; P_3=\begin{bmatrix}1 & 0.22\\0.22 & 0.4\end{bmatrix} \quad (4.9)$$

with $\alpha_1=4$, $\alpha_2=5$, $\alpha_3=2.3$, $\beta=1.5$ and $R_{kj}=\begin{bmatrix}1.1 & 0\\0 & 1.1\end{bmatrix}$ for $k,\, j=1,\,2,\,3$. Then, we have the matrices $\overline{Q}_{kj}$'s from Eq. (3.6):

$$\bar{Q}_{11}=\begin{bmatrix}-0.85 & 0\\ 0 & -0.85\end{bmatrix},\quad \bar{Q}_{21}=\begin{bmatrix}-0.85 & 0\\ 0 & -0.85\end{bmatrix},\quad \bar{Q}_{31}=\begin{bmatrix}-0.85 & 0\\ 0 & -0.85\end{bmatrix},\quad \bar{Q}_{12}=\begin{bmatrix}-0.9 & 0\\ 0 & -0.9\end{bmatrix}$$

$$\bar{Q}_{22}=\begin{bmatrix}-0.9 & 0\\ 0 & -0.9\end{bmatrix},\quad \bar{Q}_{32}=\begin{bmatrix}-0.9 & 0\\ 0 & -0.9\end{bmatrix},\quad \bar{Q}_{13}=\begin{bmatrix}-0.6652 & 0\\ 0 & -0.6652\end{bmatrix},\quad \bar{Q}_{23}=\begin{bmatrix}-0.6652 & 0\\ 0 & -0.6652\end{bmatrix},$$

$$\bar{Q}_{33}=\begin{bmatrix}-0.6652 & 0\\ 0 & -0.6652\end{bmatrix}. \tag{4.10}$$

Substituting Eqs. ($4.1-4.4, 4.9$) and the feedback gains K_{ij} 's in Eqs. ($4.6-4.8$) into Eqs. ($3.7-3.8$) yields

$$Q_{11}=\begin{bmatrix}1.5149 & -3.2767\\ -3.2767 & 18.2477\end{bmatrix},\quad Q_{21}=\begin{bmatrix}21.6541 & -14.5346\\ -14.5346 & 15.7639\end{bmatrix},\quad Q_{121}=\begin{bmatrix}16.625 & 22.9224\\ 22.9224 & 27.1598\end{bmatrix},$$

$$Q_{12}=\begin{bmatrix}3.7232 & -3.9969\\ -3.9969 & 6.7598\end{bmatrix},\quad Q_{22}=\begin{bmatrix}15.2709 & -9.5061\\ -9.5061 & 7.4943\end{bmatrix},\quad Q_{122}=\begin{bmatrix}10.1033 & 8.894\\ 8.894 & 7.785\end{bmatrix},$$

$$Q_{13}=\begin{bmatrix}2.9129 & -2.8386\\ -2.8386 & 5.1018\end{bmatrix},\quad Q_{23}=\begin{bmatrix}18.5592 & -7.7059\\ -7.7059 & 6.3643\end{bmatrix},\quad Q_{123}=\begin{bmatrix}18.6927 & 18.7692\\ 18.7692 & 15.826\end{bmatrix}. \tag{4.11}$$

From Eq. (3.5), we have

$$\Lambda_1=\begin{bmatrix}0.85 & 0 & 0\\ 0 & 0.8961 & -1.6274\\ 0 & -1.6274 & 3.879\end{bmatrix},\Lambda_2=\begin{bmatrix}0.9 & 0 & 0\\ 0 & 0.966 & -0.0251\\ 0 & -0.0251 & 1.1121\end{bmatrix},\Lambda_3=\begin{bmatrix}0.6652 & 0 & 0\\ 0 & 0.9651 & -1.5645\\ 0 & -1.5645 & 2.6353\end{bmatrix} \tag{4.12}$$

and the eigenvalues of them are given below:

$$\lambda(\Lambda_1)=0.85,\ 0.18,\ 4.595>0, \tag{4.13}$$

$$\lambda(\Lambda_2)=0.9,\ 0.9618, 1.1162>0, \tag{4.14}$$

$$\lambda(\Lambda_3)=0.6652,\ 0.0268,\ 3.5736>0. \tag{4.15}$$

Although the inequality (3.4c) is not satisfied, the matrices Λ_j ($j=1, 2, 3$) are all positive definite. Therefore, based on condition (II) of Theorem 1, the fuzzy controllers (4.6) $-$ (4.8) can asymptotically stabilize the multi-time delay fuzzy complex system with multi-interconnections. Simulation results of each subsystem are illustrated in Figs. 4.1–4.3 with random initial conditions.

5. Conclusions

A stability criterion is derived in this paper from Lyapunov's direct method for multi-time delay fuzzy complex system with multi-interconnections with the guarantee of asymptotic stability. In principle, both the conditions of this criterion can be used to test the asymptotic stability of multi-time delay fuzzy complex system with multi-interconnections. It is therefore reasonable to check the asymptotic stability with either one of the conditions and then, if it fails, to resort the other. On the basis of this stability criterion and the decentralized control scheme, a set of fuzzy controllers is synthesized via the concept of PDC scheme to stabilize a multi-time delay fuzzy complex system with multi-interconnections. Finally, a numerical example with simulations is provided to illustrate the results.

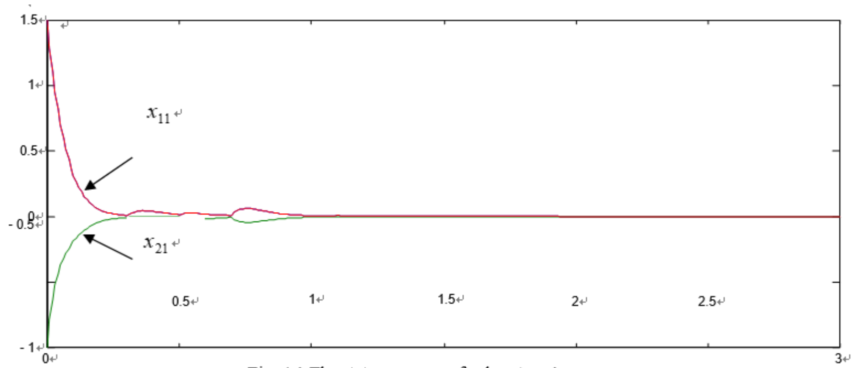

Fig. 4.1 The state response of subsystem 1.

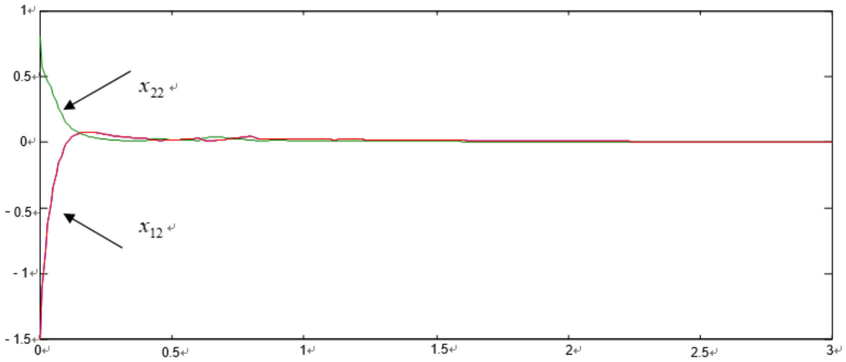

Fig. 4.2. The state response of subsystem 2.

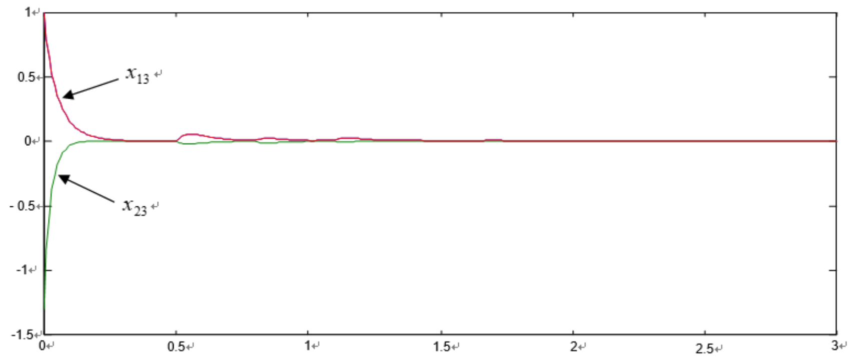

Fig. 4.3. The state response of subsystem 3.

Appendix: Proof of Theorem 1

(I): Let the Lyapunov function for the Multi-time delay fuzzy complex system with multi-interconnections be defined as

$$V=\sum_{j=1}^{J}v_j(t)=\sum_{j=1}^{J}\left\{x_j^T(t)P_jx_j(t)+\sum_{k=1}^{N_j}\int_0^{\tau_{kj}}x_j^T(t-\tau)R_{kj}x_j(t-\tau)d\tau\right\} \qquad \text{(A1)}$$

where $P_j=P_j^T>0$ and the weighting matrix $R_{kj}=R_{kj}^T>0$. We then evaluate the time derivative of V on the trajectories of Eq. (3.3) to get

$$\dot V=\sum_{j=1}^{J}\dot v_j(t)=\sum_{j=1}^{J}[\dot x_j^T(t)P_jx_j(t)+x_j^T(t)P_j\dot x_j(t)+\sum_{k=1}^{N_j}(x_j^T(t)R_{kj}x_j(t)-x_j^T(t-\tau_{kj})R_{kj}x_j(t-\tau_{kj}))]$$

$$=\sum_{j=1}^{J}\sum_{i=f=1}^{r_j}h_{ij}^2(t)x_j^T(t)[(A_{ij}-B_{ij}K_{ij})^TP_j+P_j(A_{ij}-B_{ij}K_{ij})+R_j]x_j(t)$$

$$+\sum_{j=1}^{J}\sum_{i\neq f}^{r_j}h_{ij}(t)h_{fj}(t)x_j^T(t)[(A_{ij}-B_{ij}K_{fj})^TP_j+P_j(A_{ij}-B_{ij}K_{fj})+R_j]x_j(t)$$

$$+\sum_{j=1}^{J}[\phi_j^T(t)P_jx_j(t)+x_j^T(t)P_j\phi_j(t)]-\sum_{j=1}^{J}\sum_{k=1}^{N_j}[x_j^T(t-\tau_{kj})R_{kj}x_j(t-\tau_{kj})]$$

$$+\sum_{j=1}^{J}\sum_{i=1}^{r_j}\sum_{f=1}^{r_j}\sum_{k=1}^{N_j}h_{ij}(t)h_{fj}(t)[x_j^T(t-\tau_{kj})A_{ikj}^TP_jx_j(t)+x_j^T(t)P_jA_{ikj}x_j(t-\tau_{kj})]$$

$$\le\sum_{j=1}^{J}\sum_{i=1}^{r_j}h_{ij}^2(t)x_j^T(t)[(A_{ij}-B_{ij}K_{ij})^TP_j+P_j(A_{ij}-B_{ij}K_{ij})+R_j]x_j(t)$$

$$+\sum_{j=1}^{J}\sum_{i\neq f}^{r_j}h_{ij}(t)h_{fj}(t)x_j^T(t)[(A_{ij}-B_{ij}K_{fj})^TP_j+P_j(A_{ij}-B_{ij}K_{fj})+R_j]x_j(t)$$

$$+\sum_{j=1}^{J}[\phi_j^T(t)P_jx_j(t)+x_j^T(t)P_j\phi_j(t)]-\sum_{j=1}^{J}\sum_{k=1}^{N_j}[x_j^T(t-\tau_{kj})R_{kj}x_j(t-\tau_{kj})]$$

$$+\sum_{j=1}^{J}\sum_{i=1}^{r_j}\sum_{f=1}^{r_j}\sum_{k=1}^{N_j}h_{ij}(t)h_{fj}(t)[\alpha_jx_j^T(t)P_jA_{ikj}A_{ikj}^TP_jx_j(t)+\alpha_j^{-1}x_j^T(t-\tau_{kj})x_j(t-\tau_{kj})]$$

(by Lemma 1) (A2)

$$=\sum_{j=1}^{J}\sum_{i=1}^{r_j}h_{ij}^2(t)x_j^T(t)[(A_{ij}-B_{ij}K_{ij})^TP_j+P_j(A_{ij}-B_{ij}K_{ij})+R_j+\alpha_jP_j\bar A_{ij}P_j]x_j(t)$$

$$+\sum_{j=1}^{J}\sum_{i\neq f}^{r_j}h_{ij}(t)h_{fj}(t)x_j^T(t)[(A_{ij}-B_{ij}K_{fj})^TP_j+P_j(A_{ij}-B_{ij}K_{fj})+R_j+\alpha_jP_j\bar A_{ij}P_j]x_j(t)$$

$$+\sum_{j=1}^{J}[\phi_j^T(t)P_jx_j(t)+x_j^T(t)P_j\phi_j(t)]-\sum_{j=1}^{J}\sum_{k=1}^{N_j}\left[x_j^T(t-\tau_{kj})R_{kj}x_j(t-\tau_{kj})\right]$$

$$+\sum_{j=1}^{J}\sum_{k=1}^{N_j}\alpha_j^{-1}x_j^T(t-\tau_{kj})x_j(t-\tau_{kj})=D_1+D_2+D_3+D_4 \qquad \text{(A3)}$$

where

$$D_1\equiv\sum_{j=1}^{J}\sum_{i=1}^{r_j}h_{ij}^2(t)x_j^T(t)[(A_{ij}-B_{ij}K_{ij})^TP_j+P_j(A_{ij}-B_{ij}K_{ij})+R_j+\alpha_jP_j\bar A_{ij}P_j]x_j(t), \qquad \text{(A4)}$$

$$D_2\equiv\sum_{j=1}^{J}\sum_{i\neq f}^{r_j}h_{ij}(t)h_{fj}(t)x_j^T(t)[(A_{ij}-B_{ij}K_{fj})^TP_j+P_j(A_{ij}-B_{ij}K_{fj})+R_j+\alpha_jP_j\bar A_{ij}P_j]x_j(t)$$

$$=\sum_{j=1}^{J}\sum_{i<f}^{r_j}h_{ij}(t)h_{fj}(t)x_j^T(t)[2(G_{ifj}^TP_j+P_jG_{ifj}+R_j+\alpha_jP_j\hat A_{ifj}P_j)]x_j(t)$$

T

, (A5)

$$D_3 = \sum_{j=1}[\phi_j(t)P_j x_j(t) + x_j(t)P_j\phi_j(t)] = \sum_{j=1}\sum_{n\neq j}[x_n(t)C_{nj}P_j x_j(t) + x_j(t)P_j C_{nj} x_n(t)]$$

$$\sum_{j=1}^{J}\sum_{n\neq j}^{J}[\beta\, x_n(t)x_n(t) \quad \beta \quad x_j(t)P_jC_{nj}C_{nj}P_jx_j(t)] \text{ (by Lemma 1)}$$

$$=\sum_{j=}^{J}\sum_{n=}^{J}[\beta\,(\frac{J-1}{J})x^T_j(t)x_j(t)+\beta^{-1}x^T_j(t)P_jC_{nj}C^T_{nj}P_jx_j(t)]$$

$$=\sum_{j=1}^{J}\sum_{n=1}^{J}\sum_{i=1}^{r_j}\sum_{f=1}^{r_j}h_{ij}(t)h_{fj}(t)x_j^T(t)[\beta\,(\frac{J-1}{J})I+\beta^{-1}P_jC_{nj}C^T_{nj}P_j]x_j(t)$$

$$=\sum_{j=1}^{J}\sum_{n=1}^{J}\sum_{i=f=1}^{r_j}h^2_{ij}(t)x_j^T(t)[\beta\,(\frac{J-1}{J})I+\beta^{-1}P_jC_{nj}C^T_{nj}P_j]x_j(t)$$

$$+\sum_{j=1}^{J}\sum_{n=1}^{J}\sum_{i\neq f}^{r_j}h_{ij}(t)h_{fj}(t)x_j^T(t)[\beta\,(\frac{J-1}{J})I+\beta^{-1}P_jC_{nj}C^T_{nj}P_j]x_j(t),\qquad \text{(A6)}$$

$$4\sum_{j=1}^{J}\sum_{k=1}^{N_j} \quad j \quad kj \quad j \quad kj \quad j \quad kj \qquad \sum_{j=1}^{J}\sum_{k=1}^{N_j} \quad M \quad kj\,\| \quad j \quad kj\,\|^2 .\qquad \text{(A7)}$$

Substituting Eqs. (A4-A7) into Eq. (A3) yields

$$\dot V\le\sum_{j=1}^{J}\sum_{i=1}^{r_j}h^2_{ij}(t)x^T_j(t)\{(A_{ij}-B_{ij}K_{ij})^TP_j+P_j(A_{ij}-B_{ij}K_{ij})+R_j+\alpha_jP_j\bar A_{ij}P_j$$

$$+\sum_{n=1}^{J}[\beta\,(\frac{J-1}{J})I+\beta^{-1}P_jC_{nj}C^T_{nj}P_j]\}x_j(t)$$

$$+\sum_{j=1}\sum_{i<f}h_{ij}(t)h_{fj}(t)x^T_j(t)\{2(G^T_{ifj}P_j+P_jG_{ifj}+R_j+\alpha_jP_j\hat A_{ifj}P_j)$$

$$+2\sum_{n=1}^{J}[\beta\,(\frac{J-1}{J})I+\beta^{-1}P_jC_{nj}C^T_{nj}P_j]\}x_j(t)+\sum_{j=1}^{J}\sum_{k=1}^{N_j}\quad\|\quad\|^2$$

$$=-\sum_{j=1}^{J}\left\{\sum_{i=1}^{r_j}h^2_{ij}(t)x^T_j(t)Q_{ij}x_j(t)+\sum_{i<f}^{r_j}h_{ij}(t)h_{fj}(t)x^T_j(t)Q_{ifj}x_j(t)-\sum_{k=1}^{N_j}\lambda_m(\bar Q_{kj})\|x_j(t-\tau_{kj})\|^2\right\}$$

$$\le-\sum_{j=1}^{J}\left\{[\sum_{i=1}^{r_j}h^2_{ij}(t)\lambda_m(Q_{ij})+\sum_{i<f}^{r_j}h_{ij}(t)h_{fj}(t)\lambda_m(Q_{ifj})]\|x_j(t)\|^2-\bar\lambda_j\sum_{k=1}^{N_j}\|x_j(t-\tau_{kj})\|^2\right\}\qquad\text{(A8)}$$

Based on Eq. (3.4), we have $\dot V<0$ and the proof of condition (I) is thereby completed.

(II): According to Eq. (A8), we get

$$\dot V\le-\sum_{j=1}^{J}\left\{[\sum_{i=1}^{r_j}h^2_{ij}(t)\lambda_m(Q_{ij})+\sum_{i<f}^{r_j}h_{ij}(t)h_{fj}(t)\lambda_m(Q_{ifj})]\|x_j(t)\|^2-\bar\lambda_j\sum_{k=1}^{N_j}\|x_j(t-\tau_{kj})\|^2\right\}$$

$$=-\sum_{j=1}^{J}\left\{\begin{bmatrix}\sqrt{\sum_{k=1}^{N_j}\|x_j(t-\tau_{kj})\|^2} & h_{1j}(t)\|x_j(t)\| & h_{2j}(t)\|x_j(t)\| & \cdots & h_{r_jj}(t)\|x_j(t)\|\end{bmatrix}\right.$$

$$\left.\begin{bmatrix}-\bar\lambda_j & 0 & 0 & \cdots & 0\\ 0 & \lambda_{1j} & 1/2\lambda_{12j} & \cdots & 1/2\lambda_{1r_jj}\\ 0 & 1/2\lambda_{12j} & \lambda_{2j} & \cdots & 1/2\lambda_{2r_jj}\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ 0 & 1/2\lambda_{1r_jj} & 1/2\lambda_{2r_jj} & \cdots & \lambda_{r_jj}\end{bmatrix}\begin{bmatrix}\sqrt{\sum_{k=1}^{N_j}\|x_j(t-\tau_{kj})\|^2}\\ h_{1j}(t)\|x_j(t)\|\\ h_{2j}(t)\|x_j(t)\|\\ \vdots\\ h_{r_jj}(t)\|x_j(t)\|\end{bmatrix}\right\}=-\sum_{j=1}^{J}H^T_j\Lambda_jH_j,\qquad\text{(A9)}$$

$$T \quad \Big[\ \sum^{N_j}\|\quad\|^2 \qquad \|\quad\|\ \Big]$$

in which $H_j \equiv \left[\sqrt{\sum_{k=1} x_j(t-\tau_{kj})} \quad h_{1j}(t)\|x_j(t)\| \quad h_{2j}(t)\|x_j(t)\| \quad \cdots \quad h_{r_j j}(t)\, x_j(t) \right]$. The Lyapunov derivative is negative if the matrices Λ_j ($j = 1,\ 2,\ \cdots,\ J$) are positive definite, which completes the proof of condition (II).

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