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Evaluation of Rayleigh Damping Coefficients for Laminated Rubber Bearing Components Using Finite Element and Experimental Modal Analysis

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Abstract: Laminated rubber bearing is a significant device found in structures such as bridges and buildings. It is used to isolate the foundation and the superstructure from seismic loads. Since it is made up from a combination of rubbers and steel plates in alternate layers which make the laminated rubber bearing a complex material, the measurements of damping of the laminated rubber bearing is difficult in practice. Damping is a dissipation of energy or energy losses in the vibration of the structure when the structures are excited with external dynamic loading. An accurate value of damping is very important as damping plays a crucial role in fixing the borderline between stability and instability in many structural systems. Therefore, it is essential to determine the accurate value of damping in structural analysis. Finite element and experimental modal analysis is one of the methods that can be used to determine dynamic properties including damping in any structures. Hence, the main objectives of this research are to determine the natural frequencies, mode shapes, Rayleigh's damping coefficients, α and β and to evaluate the performance of the laminated rubber bearing components i.e. steel and rubber plates using finite element and experimental modal analysis. Based on the finding, the finite element modal analysis with the added Rayleigh's damping coefficients α and β shows a good agreement with the experimental modal analysis in term of natural frequencies and mode shapes. The value of natural frequencies reduces after the Rayleigh's damping coefficients are added into the finite element modal analysis and the minimum and maximum displacement are found to be unaffected by the Rayleigh's damping coefficients for rubber plate. It can be concluded that modal analysis method can be used to estimate the accurate values of damping ratio and to determine the Rayleigh's damping coefficients α and β as well.

Keywords: Damping, Rayleigh's damping, laminated rubber bearing, finite element modal analysis, experimental modal analysis.

1. Introduction

Laminated rubber bearing is a device that isolates a structure from seismic loads. It is made up of a combination of rubber layer and steel plate that are laminated together alternately. Rubber layer is an almost incompressible material that has high horizontal flexibility while steel plate is a solid material that has high vertical stiffness [1,2]. Therefore, the laminated rubber bearing provides a very high vertical stiffness, while still maintaining its high flexibility in the horizontal direction which is required to lengthen the time period of a structure during seismic event. The combination of rubber and steel plates in alternate layers will make the laminated rubber bearing a complex material, thus the measurement of damping for the laminated rubber bearing is difficult in practice.

Damping is one of the most important aspects to make sure the system remains stable and undamaged. Talbot and Woodhouse [3] stated that damping is important in structural engineering since it controls the amplitude of resonant vibration response. When a structure is excited with a force vibration near its natural frequency, the small force will exert a high stress on the structure which can lead to the failure of the structure. However, with sufficient amount of damping, the stress can be reduced [4, 5]. Damping is a dissipation of energy or energy losses in a vibration system [6].

The understanding of damping mechanism in vibration analysis is still an issue although a lot of literature related to this mechanism is available. The determination of damping, C in the equation of motion that contains mass, M and stiffness, K is still less comprehensive as compared to the determination of mass

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and stiffness. This is because predicting vibration parameters with respect to damping is difficult in practice.

The importance of laminated rubber bearing in seismic isolated structure has led many researchers in trying to understand the behavior and to improve the design of laminated rubber bearing. The key point in the design of an effective base isolation system is the understanding of the isolator characteristics. Thus, the main objectives of this research are to determine the natural frequencies, mode shapes, Rayleigh's damping coefficients, α and β and to evaluate the performance of the laminated rubber bearing components i.e. steel and rubber plates using finite element and experimental modal analysis.

2. Rayleigh Damping

Rayleigh damping is a damping that is proportional to mass and stiffness. It is also known as proportional damping [7,8] and is a type of viscous damping. The concept of Rayleigh damping was introduced by Lord Rayleigh to solve the damping problem in a vibrating system numerically and the concept is commonly used in finite element method [9]. The formulation of Rayleigh damping is given by [10]:

$$C = \alpha M + \beta K \quad (1)$$

where M and K are mass and stiffness respectively while α is mass coefficient and β is stiffness coefficient. With this formulation, the damping ratio is the same for axial, bending and torsional response. Alpha, α and beta, β are calculated from the following equation [10]:

$$\zeta = \frac{\alpha}{2\omega} + \frac{\beta\omega}{2} \quad (2)$$

where ζ is damping ratio and ω is natural frequency. This damping ratio, ζ and natural frequency, ω value can be determined from the modal analysis method. In relation to Equation (2) and Equation (1), it is important to note that if β = 0, as the natural frequencies increase, the damping ratio will decrease and if α = 0, as the natural frequencies increase, the damping ratio will also increase. This means that mass proportional term gives inverse damping ratio proportional to the response frequency while stiffness proportional term gives linear damping ratio proportional to the response frequency [11]. Fig. 1 shows the relationship of mass proportional term and stiffness proportional term related to damping ratio and the natural frequency of the structure. Thus, it is important to select the appropriate values of α and β to determine the Rayleigh damping value in a structure for dynamic analysis.

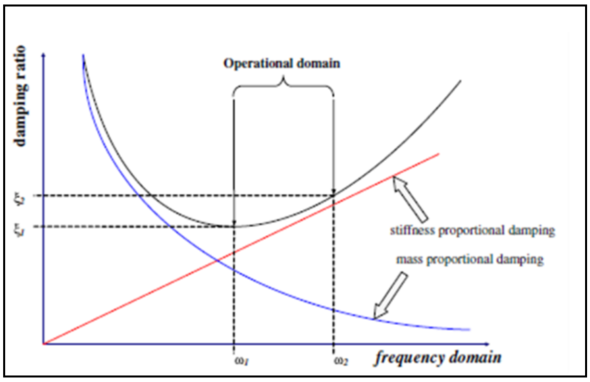


Fig. 1: Relationship of Mass and Stiffness Proportional Term with Damping Ratio and Natural Frequencies [10].

3. Modal Analysis

Modal analysis is a process of determining the dynamic characteristic of a system in term of its natural frequencies, mode shapes and damping [12]. Dynamic characteristic helps to better understand how the systems behave and how to adjust or improve the design of the system. Modal analysis consists of both theoretical and experimental technique [12,13]. In the theoretical modal analysis, the dynamic characteristics of a system are determined by using analytical or numerical process such as formulation or finite element analysis. For experimental modal analysis, the dynamic characteristics of a system are determined by using real physical system and field measurements.

As far as this research concerns, both finite element and experimental modal analysis are used to determine the dynamic properties of steel and rubber plate from laminated rubber bearing. One steel plate and one rubber plate were used. "Hard" rubber is the type of rubber used for the rubber materials. For the steel and rubber plates, the dimension is 200mmx230mm. The thickness of the rubber plate is 10mm meanwhile the thickness of the steel plate is 3mm. Table 1 shows the details of the steel and rubber plate used for this research.

Table 1: Details of steel plate and rubber plate

Materials	Young's Modulus (Pa)	Poisson's Ratio	Density (kg/m^3)
Steel Plate	2.1e11	0.3	7850
Rubber Plate	5e7	0.49	1100

3.1 Finite Element Modal Analysis

Finite Element is an effective numerical way in solving complex differential equations. In finite element modal analysis, only linear elements and linear material properties are considered. Any nonlinearity is ignored even if it is defined. External excitation and damping are also ignored in this analysis. Free vibration Equation of Motion for single degree of freedom (SDoF) system without damping can be written as [13]:

$$[\mathbf{M}]\{\ddot{x}\} + [\mathbf{K}]\{x\} = \{0\} \quad (3)$$

Free vibration solution is mathematically non-trivial solution. It should take the form as:

$$\{x\} = \{X\} \sin \omega t \quad (4)$$

By substituting Equation (4) into Equation (3), a simple algebraic matrix equation can be expressed as below:

$$([\mathbf{K}] - \omega^2 [\mathbf{M}]) \{X\} = \{0\} \quad (5)$$

In Equation (5), {X} cannot be 0, thus:

$$|[\mathbf{K}] - \omega^2 [\mathbf{M}]| = \{0\} \quad (6)$$

where [**K**] and [**M**] are the stiffness and mass matrix of the systems and ω^2 is the eigenvalue that determines the natural frequency of the system and {X} is the eigenvector that determines the mode shape of the system.

In this research, steel and rubber plate were modeled and analyzed using ANSYS workbench 14.0. Modal analysis was chosen as the type of analysis used to analyze this plate. For steel plate, it was assigned to be linear elastic material properties (Young's modulus 210 GPa; Poisson's ratio of 0.3; Density of 7850 kg/m3). Rubber is a hyperplastic material with low shear modulus and very high bulk modulus. The Poison's ratio of rubber is close to 0.5 and is considered almost incompressible. The Young's modulus of 50 MPa, Poisson's ratio of 0.49 and density of 1100 kg/m3 were assigned for the material properties of rubber plate. The materials were meshed using SOLID 185 elements and the meshing size was 2.5 mm. Fig. 2 shows the example of mashed steel plate model in finite element modal analysis.

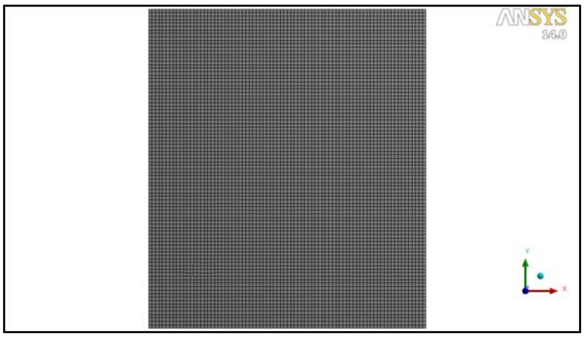


Fig. 2: Meshed Steel Plate Model

3.2 Experimental Modal Analysis

In experimental modal analysis, the steel and rubber plate were tested in a free-free boundary condition. To stimulate the free-free condition for both plates, every corner of the steel and rubber plate were hanged by nylon string and rubber bands. Then, the rubber bands were connected to springs and the springs were then connected to a fixed place. Four rubber bands, four nylon strings and four numbers of springs were used to hang the steel and rubber plates in a horizontal direction. Before hanging, a few points were marked on the steel and rubber plates to place the accelerometers and to excite the impact hammer. For this testing, 15 points in total were picked based on the nodes from the finite element model. One PCB Piezotronics Impact Hammer Model 086C03 with a medium plastic tip was used for excitation while three accelerometers were used to record the responses. The excitation was based on 'fixed hammer, roving accelerometers' which means the excitation of impact hammer was fixed at one point while the accelerometer roving at every points except at the reference accelerometer. One accelerometer was fixed at point 2 as a reference point but the opposite direction from the other point and two other accelerometers roving at other points. The force was excited by the impact hammer at point 2 which was at the same direction as the reference point as shown in Fig. 3. A total of 15 points was measured and the responses were recorded by the accelerometer. An average of three responses was recorded for every excitation to reduce the differences between the excitations. All responses were then transferred to the B&K Data Acquisition to get the Frequency Response Function (FRF). From the FRF, the dynamic properties which were the natural frequencies, the mode shapes, and the damping ratio of steel and rubber plates were determined using the PULSE Reflex software.

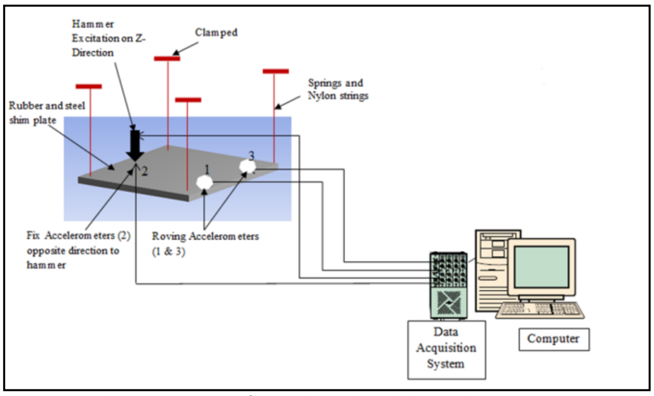


Fig. 3: Full Diagram of Steel and Rubber Plates Testing

3.3 Determination and Evaluation of Rayleigh Damping Coefficients

Natural frequencies and damping ratio from the experimental modal analysis were used in determining the Rayleigh damping coefficients α and β. The formula used in determining α and β is:

$$\alpha = 2\omega 1\omega i\ (\zeta 1\omega i - \zeta i\omega 1) / (\omega i2 - \omega 12) \quad (7)$$

and

$$\beta = 2\ (\zeta i\omega i - \zeta 1\omega 1) / (\omega i2 - \omega 12) \quad (8)$$

The values of α and β were determined for all modes and the graph of damping factor versus frequency were plotted to select the best values of α and β. Noted that the values of natural frequencies must be in radian. After determining the coefficients for steel and rubber plates, the values of α and β from both plates were inserted back into the finite element modal analysis to observe the changing of natural frequencies, mode shapes, minimum and maximum displacement with consideration of Rayleigh damping in the analysis. The results were then compared.

4. Results and Discussions

The results for steel and rubber plate are extracted from both the finite element modal analysis and the experimental modal analysis. The results of natural frequencies and damping ratio from the experimental modal analysis are used in determining the Rayleigh's damping coefficients, α and β for the steel and rubber plates as shown in Table 2, Fig. 4 and Fig. 5. The coefficients are then inserted in the finite element modal analysis and the performances of the steel and rubber plates with the addition of Rayleigh's damping coefficients are evaluated as shown in Table 3 and Table 4. First and foremost, the patterns of the mode shapes from both methods are compared. The mode shapes from the experimental modal analysis must be as close as possible to the finite element modal analysis. The value of the natural frequencies is then compared if the mode shapes from both methods are similar. Then, the relative error of the natural frequencies and minimum and maximum displacement between both methods is calculated as shown in Table 6, Table 7, Table 8 and Table 9.

Table 2: Natural Frequencies and Damping Ratio from Experimental Modal Analysis

Mode No.	Steel Plate		Rubber Plate	
	Natural Frequency (rad/sec)	Damping Ratio (%)	Natural Frequency (rad/sec)	Damping Ratio (%)
E1	1818.103	0.16	132.5124	6.78
E2	2584.588	0.18	170.023	2.65
E3	5214.039	0.11	263.9566	4.58
			322.3902	2.13

Table 3: Comparison of Results for Steel Plate

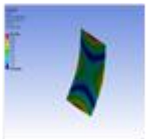
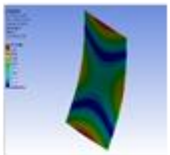
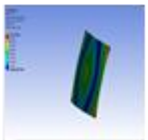
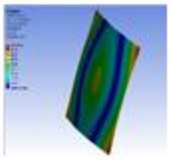
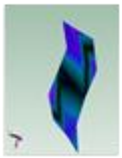
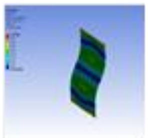
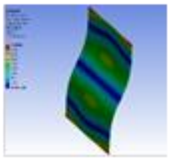
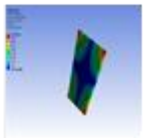
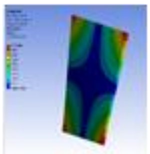
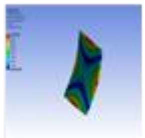
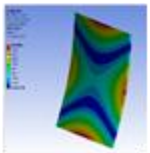
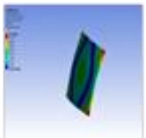
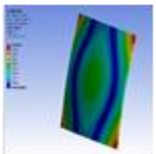
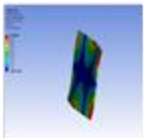
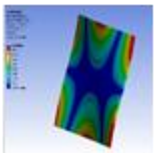
Experimental Modal Analysis		Finite Element Modal Analysis Without Damping Coefficients		Finite Element Modal Analysis With Damping Coefficients	
Mode No.	Mode Shapes & Natural Frequencies (Hz)	Mode No.	Mode Shapes & Natural Frequencies (Hz)	Mode No.	Mode Shapes & Natural Frequencies (Hz)
E1	 289.36 Hz	F7	 296.56 Hz	FD7	 292.82 Hz
E2	 411. 35 Hz	F8	 422.53 Hz	FD8	 418.27 Hz
E3	 829. 84 Hz	F9	 864.36 Hz	FD9	 852.76 Hz

Table 4: Comparison of Results for Rubber Plate

Experimental Modal Analysis		Finite Element Modal Analysis Without Damping Coefficients		Finite Element Modal Analysis With Damping Coefficients	
Mode No.	Mode Shapes & Natural Frequencies (Hz)	Mode No.	Mode Shapes & Natural Frequencies (Hz)	Mode No.	Mode Shapes & Natural Frequencies (Hz)
E1	 21.09 Hz	F7	 27.63 Hz	FD7	 23.35 Hz
E2	 27.06 Hz	F8	 39.68 Hz	FD8	 34.43 Hz
E3	 42.01 Hz	F9	 63.13 Hz	FD9	 58.87 Hz
E4	 51.31 Hz	F10	 77.52 Hz	FD10	 73.24 Hz

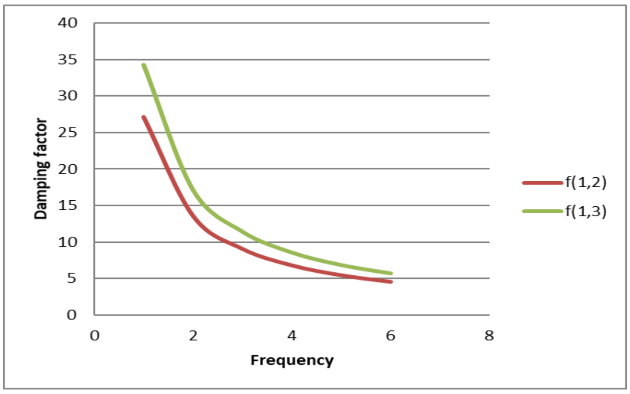


Fig. 4: Graph from First Mode of Damping Ratio for Steel Plate

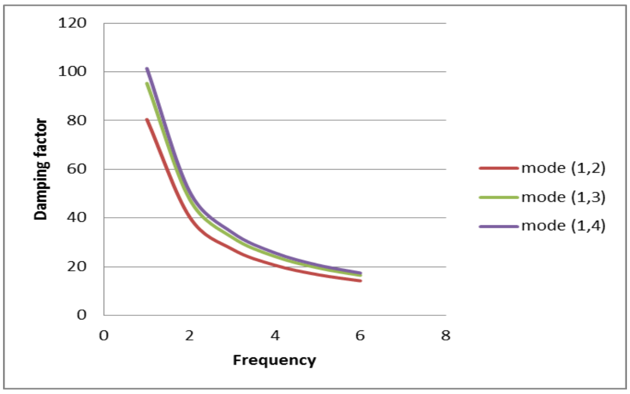


Fig. 5: Graph from First Mode of Damping Ratio for Rubber Plate

Table 6: Relative Error between Experimental and Finite Element with and without Rayleigh Damping Coefficients for Steel Plate

Relative Error of Natural Frequency (%)			
Mode No.	**Without Damping Coefficients**	**Mode No.**	**With Damping Coefficients**
F7	2.49	FD7	1.20
F8	2.72	FD8	1.68
F9	4.16	FD9	2.76

Table 7: Minimum and Maximum Displacement for Steel Plate

Minimum Displacement (mm)				Maximum Displacement (mm)			
Mode No.	Without Damping Coefficients	Mode No.	With Damping Coefficients	Mode No.	Without Damping Coefficients	Mode No.	With Damping Coefficients
F7	0.95272	FD7	0.030128	F7	2187.5	FD7	69.173
F8	0.017326	FD8	0.00054771	F8	2470.1	FD8	78.113
F9	2.7341e-6	FD9	4.3185e-7	F9	2385.1	FD9	75.424

Table 8: Relative Error between Experimental and Finite Element with and without Rayleigh Damping Coefficients for Rubber Plate

Relative Error of Natural Frequency (%)			
Mode No.	**Without Damping Coefficients**	**Mode No.**	**With Damping Coefficients**
F7	31.01	FD7	10.72
F8	46.64	FD8	27.24
F9	50.27	FD9	40.13
F10	51.08	FD10	42.74

Table 9: Minimum and Maximum Displacement for Rubber Plate

Minimum Displacement (mm)				Maximum Displacement (mm)			
Mode No.	Without Damping Coefficients	Mode No.	With Damping Coefficients	Mode No.	Without Damping Coefficients	Mode No.	With Damping Coefficients
F7	8.7044e-9	FD7	8.7044e-9	F7	122.13	FD7	122.13
F8	0.015022	FD8	0.015022	F8	103.64	FD8	103.64
F9	0.0014	FD9	0.0014	F9	126.4	FD9	126.4
F11	3.3812e-8	FD10	3.3812e-8	F11	113.46	FD10	113.46

In general, the entire mode shapes of steel and rubber plates produced by the finite element modal analysis show a good agreement and a similar pattern with the experimental modal analysis. All the vibration patterns contain both twisting and bending modes. The relative error of natural frequencies between the experimental modal analysis and the finite element modal analysis for steel plate shows a small discrepancy but for rubber plate, it shows a very large discrepancy of around 30% in average. This is highly due to the hyperplastic material of rubber that contains high nonlinearity and high damping capacity.

Damping ratio produced in the experimental modal analysis also shows a good agreement between both plates. Rubber plate has a large damping ratio as compared to steel plate. It is well known that rubber material has high damping capacity which is always used in the absorption of energy during vibration of a structure. In the finite element modal analysis, damping ratio is an input but not an output. Thus, the only way to get damping ratio is from assumption or experimental modal analysis. In this research, the first mode was used in determining the Rayleigh damping coefficients α and β. The first mode was chosen because in modal analysis, the first mode is very crucial. It determines how the first shape pattern will produce when the external frequency is excited with the same natural frequency as the first mode. It is also called as resonance.

In general, the entire mode shapes produced in the finite element modal analysis with the added Rayleigh damping coefficients for both materials show a good agreement with the experimental and the finite element modal analysis without the addition of Rayleigh damping coefficients.

Theoretically, after the Rayleigh damping coefficients was added in the finite element modal analysis, the values of natural frequencies will reduce as close as possible to the natural frequencies from the experimental modal analysis. The reduction in the value of natural frequencies can be seen in Table 3 for steel plate and Table 4 for rubber plate.

The values of minimum and maximum displacement show some reductions for the steel plate after the Rayleigh damping coefficients were added in the finite element modal analysis but for the rubber plate, the values remain the same. Hence, the Rayleigh damping coefficients does not affect the minimum and maximum displacement of the rubber plate.

5. Conclusion

Both finite element and experimental modal analyses were applied and compared to one another, thus making the results more convincing. The two sets of results were carefully compared and proved that they fit each other quite well. Although the natural frequencies have shown quite a large discrepancy for rubber plate but the mode shapes produced by both methods have shown a good agreement between each other.

The Rayleigh's damping coefficients α and β were successfully determined using the results of natural frequencies and damping ratio from experimental modal analysis. The first mode of damping ratio which is 0.16% was used for steel plate and the coefficients produced is 431.375 for α and 0.0000455 for β. For rubber plate, the first damping ratio which is 6.78% was used and the coefficients produced is 1273.443 for α and 0.029809 for β. The relative errors of natural frequencies between finite element and experimental modal analyses reduced after the coefficients were included in the finite element modal analysis. It can be concluded that the first mode of damping ratio from the experimental modal analysis can be used in determining the Rayleigh damping coefficients for the steel and rubber plate.

The performance of steel and rubber plates with Rayleigh damping coefficients using finite element modal analysis was successfully evaluated. The mode shapes of the both plates with the addition of Rayleigh damping showed a good agreement with the experimental modal analysis. The value of natural frequencies also was reduced closer to the value of natural frequencies from the experimental modal analysis. This study also shows that the Rayleigh damping coefficients do not affect the minimum and maximum displacement of the rubber plate.

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