# Inverse Problem: Comparison between Linear Back-Projection Algorithm and Filtered Back-Projection Algorithm in Soft-Field Tomography

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Abstract: An image reconstruction algorithm in process tomography is very important in ensure that the reconstructed image is satisfied. The back-projection algorithm is the most popular algorithm applied in process tomography including a linear back-projection (LBP) algorithm and a filtered back-projection (FBP) algorithm. The objectives of this paper are: (1) to compare the result from LBP and FBP algorithm in soft-field tomography, and (2) to investigate the effect of FBP algorithm to a reconstructed image in soft-field tomography. Also, a comparison between hard-field tomography (ultrasonic tomography) and soft-field tomography (electrical resistance tomography, ERT) had been discussed in this paper. The ERT had been conducted based on a simulation using COMSOL Multiphysics live link with MATLAB software. As a result, a nature behavior of soft-field gave a main factor of inaccurate position of reconstructed image using FBP algorithm. When it multiplied with the concentration profile from LBP result, a "hill" surface sensitivity distribution and a color scale shifted in FBP gave inaccurate result. Therefore, it is believed that it gave a reason of why the FBP algorithm is not the concern on reconstructing image in soft-field tomography.

Keywords: Non-iterative algorithm, filtered back-projection, linear back-projection, hard-field, soft-field

# 1. Introduction

The image reconstruction of process tomography, can be divided into two parts: forward problem and inverse problem [1]. The forward problem is solved first in order to know the theoretical value of each of the sensors output based on the signal projection, while the algorithm is solved later in the inverse problem to obtain a tomogram. After the sensitivity map of the system solved, the tomogram is obtained by using either non-iterative or iterative algorithm in the inverse problem part [2].

A non-iterative algorithm is a direct method of image reconstruction and back-projection algorithm is categorized as a non-iterative algorithm [3]. It is a famous technique in tomography application consisting of a Linear Back-Projection (LBP) algorithm and a filtered back-projection (FBP) algorithm. LBP is applied in many tomography processes, such as in ultrasonic tomography [4]–[7], optical tomography [8]–[11], electrical resistance tomography [12]–[15] and electrical capacitance tomography [16]–[20]. The advantages of the LBP algorithm is its low computational complexity, and it can generate an image at high speed [21]. In reconstructing the image using LBP, each sensitivity matrix is multiplied with it corresponding sensor reading [22]. The back projected data values are smeared back across the unknown density function (image) and overlap each other to increase the projection data density [5]. Hence, the main disadvantage of LBP algorithm is that it produces a blurred image, also known as the smearing effect [23].

The FBP algorithm is used to sharpen the reconstructed image obtained from the LBP algorithm [24]. It is applied mostly in hard-field tomography

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compared to soft-field tomography such as in ultrasonic tomography [25]–[27], X-ray tomography [28]–[30] and optical tomography [8], [31], [32], but was only discussed in electrical impedance tomography in 1992 [33].

Therefore, this paper compares the reconstructed images obtained from LBP and FBP algorithms in softfield tomography, and investigates a factor of FBP algorithm is not widely applied in the soft-field tomography compared in the hard-field tomography. The comparison is made based on non-invasive electrical resistance tomography and ultrasonic tomography system.

### 2. Methodology

The FBP algorithm in this paper was based on M. H. Fazalul Rahiman [25] that is applied in ultrasonic tomography application. This filter matrix has the same dimension as the sensitivity matrix so that it produces a weighting for the individual pixel. Hence, it can produce a uniform concentration profile for equal sensor output. The filter matrix, F is expressed by Equation 1 [25].

$$F = \frac{P_m}{W} \tag{1}$$

where  $P_m$  is the maximum pixel magnitude in total matrix, W. Therefore, the FBP algorithm can be solved by multiplying the filter matrix by the LBP result [25]:

$$G_{FBP}(x, y) = F(x, y) \times G_{LBP}(x, y)$$
(2)

where  $G_{FBP}(x, y)$  is the FBP concentration profile, and  $G_{LBP}(x, y)$  is the LBP concentration profile and it can be expressed by Equation 3 [22].

$$G_{LBP}(x, y) = \sum_{Tx=1}^{N} \sum_{Rx=1}^{N} \overline{M}_{Tx, Rx}(x, y) \times S_{Tx, Rx}$$
(3)

 $\overline{M}_{Tx,Rx}(x,y)$  is the normalized sensitivity matrice for the view of Tx,Rx, is the sensor loss amplitude of receiver Rx th for projection Tx th and N is the number of sensors or electrodes applied.

A soft-field tomography in this paper is based on current research on non-invasive electrical resistance tomography. Sixteen electrodes are applied on the periphery of pipe wall non-invasively and it had been simulated using COMSOL Multiphysics live link with MATLAB software. Air was used as gas phantom so that it gives same material applied in Ref. [25].

#### 3. Results and Discussion

A hard-field tomography in this paper is referred from Ref. [25]. A concentration profile of liquid-gas bubble column had been determined by using ultrasonic tomography. The reconstructed image from LBP had been improved using FBP algorithm. The results in Ref. [25] proved that the reconstructed image using FBP algorithm can remove the smearing effect of the image with its current position. Nevertheless, if comparison is made between those sensitivity distributions for hard-field tomography and soft-field tomography (see Table 1); the distribution for soft-field tomography using FBP produces an inverse curve surface, while a flat surface of sensitivity distribution occurs in hard-field tomography. This is due to the nature of soft-field behavior that produces a curve line of each pair projections of the sensitivity map [34-35]. Thus, it makes the sensitivity distribution becomes a curve surface instead of a flat surface. Observation of the same pixel gives a higher pixel value resulting in color scale ratio changes. This give the color scale in FBP shifted. Consequently, it not produces a weighting for the sensitivity distribution.

As an example, tomograms with different positions for electrical resistance tomography system had been reconstructed as shown in Fig. 1. Comparisons between reconstructed images from LBP and FBP algorithm are clearly seen in Fig. 1. The position of the phantom from LBP algorithm can be identified. But, the size and shape of the phantom cannot be exactly as the reference image. When the phantom is placed near to the pipe wall, the shape and size of the reconstructed image affected due to the smearing effect and hence an ideal circle shape likes a reference image cannot be obtained.

However, the tomogram obtained for FBP is not accurate. The color scale ratio that changes in FBP influences the reconstructed image. It is believed that when every pixel from the curved filter matrix is multiplied with the LBP result, the color scale in FBP shifted.

#### 4. Conclusion

In this study, effect of reconstructed image using FBP algorithm for soft-field tomography had been discussed. It is believed that the nature of soft-field behavior gave a main factor of inaccurate reconstructed image obtained from the FBP algorithm. A "hill" surface produced by a filter matrix affects the final result of reconstructed image when it multiplied with LBP result. The reconstructed image is not accurate due to the color scale shifted when the FBP algorithm is applied to the electrical resistance tomography. Consequently, it is believed that it gave a reason of the FBP algorithm is not the main concern among researchers that conduct a research in soft-field tomography. However, more research work should be undertaken to investigate this problem.

## Acknowledgement

The authors would like to thank the Ministry of Higher Education and University Malaysia Pahang for funding the study. Very special thanks go to the Universiti Teknologi Malaysia and PROTOM research group for their generous support and cooperation



Table 1 Comparison sensitivity distribution between hard-field and soft-field tomography

Fig. 1 Example of comparison between reconstructed image using LBP versus FBP

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