International Journal of Integrated Engineering, Vol. 5 No. 2 (2013) p. 37-42

Nonlinear Simulation of Beam Elements Subjected to High Mass Low Velocity Impact Loading using the Smoothed Particle Hydrodynamics (SPH) Method

Shahrul Niza Mokhatar[1,*], Yoshimi Sonoda[2] and Zainorizuan Mohd Jaini[1]

[1] Jamilus Research Center, Universiti Tun Hussein Onn Malaysia, 86400 Johor, MALAYSIA.
[2] Department of Civil Engineering, Kyushu University, Motooka 744, Nishi-ku, Fukuoka, JAPAN.

Abstract: In this paper, a fully Lagrangian method namely as Smoothed Particle Hydrodynamics (SPH) has been utilized in order to simulate the response of reinforced concrete beam subjected to low velocity impact loading. Models based on this method are proposed to capture an important failure mechanism on compression and tension of the beam elements under dynamic (impact) loading condition. Pressure dependant criterion known as Drucker-Prager (DP) with a new Plane-Cap (PC) yield surface were employed to concrete, and (Von-Mises) criterion was applied for steel reinforcement. The constitutive equation of PC model is employed on compression, while, orthotropic constitutive equation due to the damage effect is considered on tension. Meanwhile, Dynamic Increase Factor (DIF) was defined separately for the effect of strain rate (SR) on the concrete and steel reinforcement. Shear cracking, bending cracking and compressive behavior of the beam were evaluated by using displacement-time histories and overall failure mode. Then, the applicability and efficiency of the proposed models were validated with experimental tests through numerical simulations with different velocity (height of drop-mass).

Keywords: Impact response analysis, Smoothed Particle Hydrodynamics (SPH), Drucker-Prager, Plane-Cap constitutive equation, orthotropic constitutive equation

1.0 Introduction

Reinforced concrete (RC) is common construction materials that extensively used as structural members. Furthermore, RC beams are also one of the most common structural members that used in building constructions. When impact load is applied to concrete structures, for example on RC beam, it will be deflected, at any instant; various forms of damages are occurred during the loading process. The most common concrete beams suffers from impact loads are the different types of global or localized damage, including flexural cracking, shear cracking, crushing of concrete beneath the projectile and spalling at the bottom of a concrete element. Understanding the structural behaviour especially beam element under impact loads is essential to protect this critical members from collapse and fail. Moreover, in order to ascertain a reliable impact-resistant design procedure of beam elements, a series of practical tests are required. The behaviour of RC beams under low velocity impact loads has been done experimentally by Chen et al. [1], Kishi et al. [2] and Sangi et al. [3]. However, estimating the response of RC structures to impact loading through full-scale tests is expensive in terms of providing the necessary test material, test equipment, and time to perform.

With regard to this, Banthe et al. [4], Saatci et al. [5] and Kantar et al. [6] have developed the Finite Element (FE) analysis in order to predict the low velocity impact response of RC elements. However, some of the FE techniques have certain inherent advantages and disadvantages, which strictly depend to a large extent on its particular application. Some problems in dealing with the use of mesh are process of generating/regenerating a quality mesh and difficulties to assess the reliability calculation of shear, flexural or crushing failure of RC members. As a result, the nonexistence of the mesh elements in Smoothed Particle Hydrodynamics (SPH) method and the independently calculation of relations among the particles results/beget become paramount way in the bending deformation and cracking analysis of RC structures. During recent years, the application of SPH has been widely used for the high velocity impact computation to the problems of solids mechanics. Swaddiwudhipong et al. [7] and Johnson [8] have successfully applied this kind of mesh-free method to study the perforation/penetration of target materials such as steel and plain concrete with high velocity of 170 m/s and higher. On the other hand, numerical simulations of tensile response of RC structures using mesh-free technique have also been extensively presented by Fukazawa et al. [9]. However, investigations concerned with compressive behaviour subjected to the low velocity impact loads using SPH method have not been done so much yet. Therefore, this mesh-free method is utilized in this study to simulate the compression and tension phenomena of RC beam under high mass low velocity impact loadings.

*Corresponding author: shahruln@uthm.edu.my

penerbit.uthm.edu.my/ojs/index.php/ijie

The constitutive model, an availability of material law, the improvement of stable and accuracy of non-linear numerical procedure to model RC behaviour is still debated. Moreover, the compressive failure model of RC beams under impact phenomena have not as yet been identified entirely. Therefore, an attempt has been made in the current work by developing a simple and reliable damage model, with regards to assessing the response of RC elements (beam) due to impact forces in term of shear cracking, bending failure as well as crushing in the compressive region. The numerical capabilities were extensively verified against existing experimental results conducted by Fujikake et al. [10].

2.0 Nonlinear analysis procedure

This part explains a reliable non-linear numerical method and a simple constitutive model of concrete. Four basic schemes to present the localized failure of RC beam subjected to low velocity impact loads are: (i) the accurate strength of steel reinforcement and concrete in tension and compression are represented separately by a parameter, namely DIF (ii) linear pressure-sensitive yield surface Drucker-Prager (DP) with a new volume dependent Plane-Cap (PC) hardening function are utilized (iii) strain softening in tension is implemented during post-peak regime by adopting φ to degrade the material's stiffness (iv) two kinds of constitutive equations is developed to simulate the crushing, shear cracking as well as bending cracking. All of these features incorporated under SPH method.

2.1 Smoothed Particle Hydrodynamic

SPH technique is utilized in this study to examine its ability in the analysis of low velocity impact loads. Generally, the procedure of SPH method uses a kernel interpolation to approximate the field variables at any particle in a support domain, h. For example to calculate particle A, the integral arbitrary function of this particle is shown below;

$$f(x^A) = \int_{\Omega} f(x^B) W(x^A - x^B, h) dx^B \tag{1}$$

where $f(x^A)$ and $f(x^B)$ are function of estimating particle A, and function that represent the neighboring particle B, respectively. The particle A is weighted by the kernel function $W(x^A\text{-}x^B,h)$. Then, the function of integral form in equation (1) is approximated by summation of all neighbouring particle B located in the support domain h, the mass of the particle B (m^B) as well as its density of particle B (ρ^B). The form of function can be written as;

$$f(x^A) = \sum_{B=1}^{N} \frac{m^B}{\rho^B} f(x^B) W(x^A - x^B, h) dx^B \tag{2}$$

Finally, the function of particle A in equation (2) is approximated by using the first-order partial differentiation of kernel function, where the derivative is only calculate for the kernel function such as;

$$\nabla . f(x^A) = \sum_{B=1}^{N} \frac{m^B}{\rho^B} f(x^B) . \nabla W(x^A - x^B, h) dx^B \tag{3}$$

In this study, in order to get the stable condition of the calculation, B-spline function is employed for the kernel function as shown in equation (4).

$$W(q,h) = \frac{1}{h} \times \begin{cases} \frac{2}{3} - q^2 + \frac{1}{2} q^3 & 0 \le q < 1 \\ \frac{1}{6}(2-R)^3 & 1 \le q < 1 \\ 0 & q \ge 2 \end{cases} \tag{4}$$

where q is the distance between two particles $(x^A\text{-}x^B)$.

The calculations scheme is described as follows:

i. The interactive particles in the influence area are defined prior updating the time increment. Where, only a fixed number of particles are within the support domain used in the particle approximations.

ii. The derivative of kernel function in equation (4) is calculated and an evaluated particle is approximated as in equation (2).

iii. The SPH equation for the acceleration of particle A is computed under the force thread in the calculations scheme as;

$$\frac{dv_i^A}{dt} = \sum_{B=1}^{N} m^B \left(\frac{\sigma_{ij}^A}{(\rho^A)^2} + \frac{\sigma_{ij}^B}{(\rho^B)^2} + \Pi^{AB} \delta_{ij} \right) \frac{\partial W}{\partial x_i^A} \tag{5}$$

In equation (5), the artificial viscosity of Monaghan Π^{AB} is used in order to simulate shock and to prevent the unnecessary penetration for particle during impact. This viscosity formulation and details are also given in the textbook by Liu et al. [11].

iv. Compute the strain rate tensor and rotation rate tensor as given in [11].

$$\varepsilon_{ij}^A = \frac{1}{2} \sum_{B=1}^{N} \frac{m^B}{\rho^B} \left(v_i^{BA} \frac{\partial W}{\partial x_j^A} + v_j^{BA} \frac{\partial W}{\partial x_i^A} \right) \tag{6}$$

and

$$R_{ij}^A = \frac{1}{2} \sum_{B=1}^{N} \frac{m^B}{\rho^B} \left(v_i^{BA} \frac{\partial W}{\partial x_j^A} - v_j^{BA} \frac{\partial W}{\partial x_i^A} \right) \tag{7}$$

where $v_i^{BA} = v_i^B - v_i^A$ and $v_j^{BA} = v_j^B - v_j^A$ are the velocity vector at particle A and B.

v. The plasticity theory for yield criterion, the flow rule and hardening rule of both materials concrete and steel is calculated under the stress thread. The explanation of this calculation is described in the following sub-section.

vi. The strain and stress calculation is updated due to the time increment and by applying the constitutive equation, respectively.

3.0 Concrete Model

The strength of concrete is predicted by using pressure dependent Drucker-Prager (DP) yield criterion. Meanwhile, a (PC) yield surface is utilized to control the volumetric expansion in order to simulate the crushing phenomena (in compression area). Refer to Fig. 1, equation (8) and (9).

$$f_{DP} = \sqrt{\frac{3}{2}\sigma'_{ij}\,\sigma'_{ij}} + \alpha\frac{1}{3}\sigma_{mm} - k \tag{8}$$

where σ'_{ij} is the deviatoric stress tensor, k and α are the positive material constant that corresponds to the Fig. 1. DP surface is open in that a pure hydrostatic pressure with no limitation in a compression region. It can be also employed without affecting yield. However, in the highly compressible conditions, the closed (capped) surface is necessary to model the appropriate behaviour of concrete material. Besides, the cap surface can be utilized to simulate the volumetric change under large hydrostatic compression. The PC model and its hardening rule are shown in Fig. 1 and Fig. 2, respectively. The equation of this model is written as below,

$$f_{PC} = I_1 - I_1^h\left(\varepsilon_v^p\right) = 0 \tag{9}$$

where I_1^h is a hardening parameter of the PC related to the inelastic volumetric plastic strain ε_v^p.

Based on the Fig. 1, k and α can be defined as DP parameters, and h is hardening path. In Fig. 2, the hardening mechanism is expressed by the relationships between inelastic volumetric strain ε_v^p and the first stress invariant I_1 as below;

$$I_1^h(\varepsilon_v^p) = \left[\frac{h'}{\left(1+e^{\varepsilon_v^p}\right)}\right] \tag{10a}$$

where,

$$h' = 2.5\sigma_c \tag{10b}$$

$$\varepsilon_v^p = \varepsilon_{11}^p + \varepsilon_{22}^p + \varepsilon_{33}^p = \varepsilon_{ij}^p\delta_{ij} \tag{10c}$$

In this model, two kinds of constitutive models are developed to simulate the crushing and bending/flexural cracking. For crushing phenomena, the new PC model is used as below;

$$d\sigma_{ij} = \left[D_{ijkl}^e - \frac{[3\lambda+2\mu]\delta_{ij}[3\lambda+2\mu]\delta_{kl}}{[9\lambda+6\mu] + \dfrac{3h'e^{\varepsilon_v^p}}{\left(1+e^{\varepsilon_v^p}\right)^2}}\right]\delta_{mk}\delta_{nl}\,d\varepsilon_{kl} \tag{11}$$

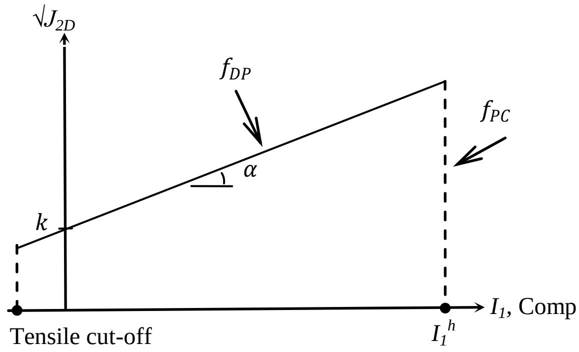


Fig. 1: PC surface on an I_1 versus $\sqrt{J_{2D}}$ plot

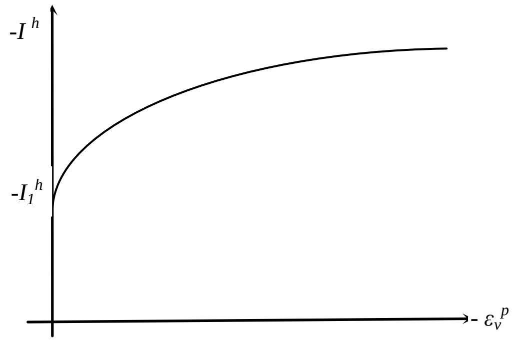


Fig 2: Nonlinear hardening rule for PC yield surface

Meanwhile, the mechanism of cracking on a tension side is simulated by the softening technique, where, the damage variable is multiplied to the undamaged elastic stiffness tensor during plasticity. In which, the variables were employed during the linear-softening phase to give the real flexural cracking behavior of the concrete beam after impact phenomena. Thus, orthotropic constitutive equation is applied in tension as in equation (12). Three principle strains ε_x, ε_y, ε_z (positive value) components are used for the damage factor, $d_i = \varepsilon_i$; $i = x, y, z$, where, these principal strains corresponds with 3 directions x,y,z are calculated based on the local coordinate and defined φ for the damage variable, the formulation is shown in equation (13).

$$d\sigma ij = \left[\varphi_{x,y,z}\, D_{ijkl}^e\right] d\varepsilon kl \tag{12}$$

where,

$$\varphi_x = \sqrt{(1-d_x)} \tag{13a}$$

$$\varphi_y = \sqrt{(1-d_y)} \quad (13b)$$

$$\varphi_z = \sqrt{(1-d_z)} \quad (13c)$$

3.1 Strain rate

In order to define the accurate strength of material during the short period, Dynamic Increase Factor (DIF) has been employed for the effect of strain rate on the compression and tensile strength of concrete. The constitutive equation of the DIF in compression and tension given by Bischoff et al. [12] and Zhou et al. [13] are expressed in equation (14) and (15), respectively.

$$CDIF = \left(\frac{\dot{\varepsilon}cd}{\dot{\varepsilon}cs}\right)^{1.026\eta} \quad \text{for } \dot{\varepsilon}cd \le 30s^{-1} \quad (14a)$$

$$CDIF = \gamma(\dot{\varepsilon}cd)^{1/3} \quad \text{for } \dot{\varepsilon}cd > 30s^{-1} \quad (14b)$$

In this calculation, the value of $\dot{\varepsilon}_{cs}$ is 30.0e^{-6} s^{-1}. Meanwhile, logγ = 6.156η – 0.49, and η is equal to [5.0 + 3.0(f_{cu}/4.0)]$^{-1}$. For tensile strength, the empirical formula for DIF, defined as:

$$TDIF = \left(\frac{\dot{\varepsilon}td}{\dot{\varepsilon}ts}\right)^{1.016\theta} \quad \text{for } \dot{\varepsilon}td \le 30s^{-1} \quad (15a)$$

$$TDIF = \beta\left(\frac{\dot{\varepsilon}td}{\dot{\varepsilon}ts}\right)^{\frac{1}{3}} \quad \text{for } \dot{\varepsilon}td > 30s^{-1} \quad (15b)$$

The value of $\dot{\varepsilon}_{ts}$ on the tension side is 3.0 x 10^{-6} s^{-1} and logβ = 7.11θ – 2.33, where θ is derived from 1/(10 + 6f_{cu}/f'). In which the value of 10 N/mm^2 is taken for f'. Besides, the DIF equation for steel reinforcement (rebar) in this study can be written as;

$$SDIF = \left(\frac{\dot{\varepsilon}}{10^{-4}}\right)^{\alpha fy}, \quad (16a)$$

where

$$\therefore \alpha fy = 0.074 - 0.040\left(\frac{fy}{4.22x10^{7}}\right) \quad (16b)$$

where f_y is the static yield strength of steel reinforcement.

4.0 Experimental Work

Tests were carried out by Fujikake et al. [10] investigating high mass low-velocity impact behavior of RC beam and resulting in dynamic response of the total structures. The size of the beam is 250 mm and 150 mm in depth and width, while 1700 mm in length (1400 mm span). The details of reinforcement arrangement and test setup used in the tests can be referred in the literature. The drop mass is acted vertically from a certain height about 1.2 m (correspond to 4.9 m/s velocity), and its weight is 400 kg. As a result, the failure mode of the RC beam can be obtained in Table 1.

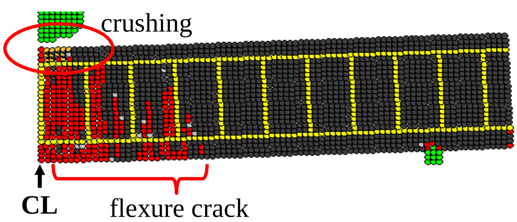
Table 1: Reinforcement size and its failure characteristics

Beam specimens	Comp. Num-size (mm)	Tension Num-size (mm)	Vel. (m/s)	Failure mode
S1616	2-D16	2-D16	4.9	Vertical flexure crack, crushing
S1322	2-D13	2-D22		Vertical flexure crack, total crushing

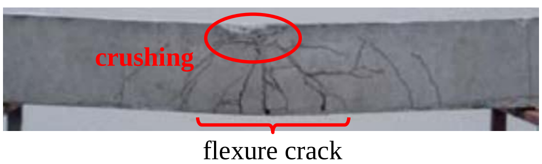
*Comp. = compression, Vel.=Velocity, Num.=Number

5.0 Analysis Results

In experimental tests, shear design strength of specimens is larger than that of bending strength in order to generate bending failure, Fujikake et al. [10]. Thus, the flexural behavior is investigated critically in the bottom side of a specimen S1616 as shown in Fig. 3(b). In this study, the numerical result in Fig. 3(a) shows the comparable results with Fig. 3(b). Flexural bending cracks for specimen S1616 was simulated by using the proposed constitutive equation in the tension region. The model utilizes the decreasing of elastic material's stiffness as explained in the previous section. Conversely, in the compression side, some local failure in the impact region also can be calculated similarly with an experimental overall response. The degree of compaction in this region is controlled by the hardening rule of the PC model.



(a) Numerical results (quarter model)



(b) Experiment results (full view)

Fig. 3 Comparisons between simulation and experiment

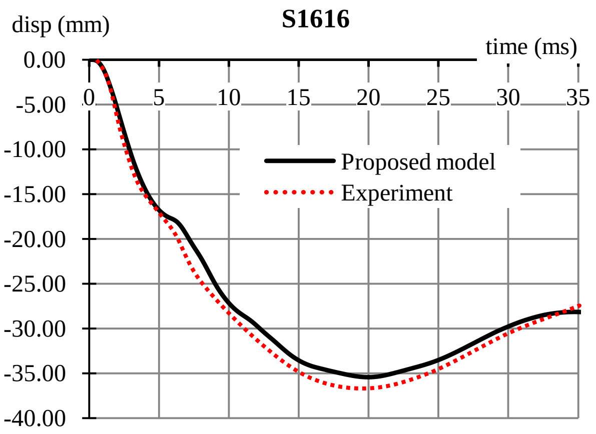
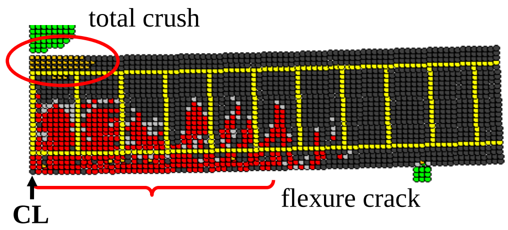


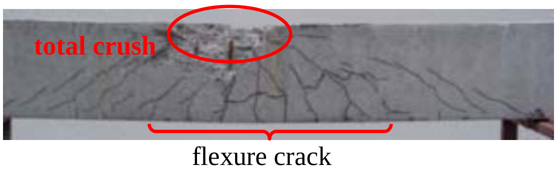
Fig. 4 Mid-span displacement-time histories

The investigation of accuracy of numerical results was extended to the displacement-time relationships as shown in Fig. 4. Based on the graph, it is indicated that the maximum mid-span displacement and its curve are accurately analyzed with the experimental tests.

The proposed numerical analyses were further validated with the lower amount of longitudinal compressive reinforcement in order to calculate the increasing of damage failure in the impact region. Fujikake et al. [10] has been discovered that the degree of local failure in the impact region is related to the increasing amount of tensile reinforcement. As shown in Fig. 5(b), the specimen S1322 resulting the failure with critically crushing at the impact point as compared to the specimen S1616. This phenomenon can be analyzed by using the proposed constitutive equation of PC model [refer Fig. 5(a)], where the calculation of volumetric plastic change in the hydrostatic compression is capable to simulate this mechanism.



(a) Numerical results (quarter model)



(b) Experiment results (full view)

Fig. 5 Comparisons between simulation and experiment

The mid-span displacement of a specimen S1322 obtained from the analyses, and experimental tests are again compared. In general, the shape of the displacement curves from the analysis are in reasonable agreement with the tests, however, there is some difference in the peak displacement value about 2 mm. This comparison can be attained in Fig. 6. It shows that the strong dependence on hydrostatic pressure can decrease the displacement values under a certain impact phenomena.

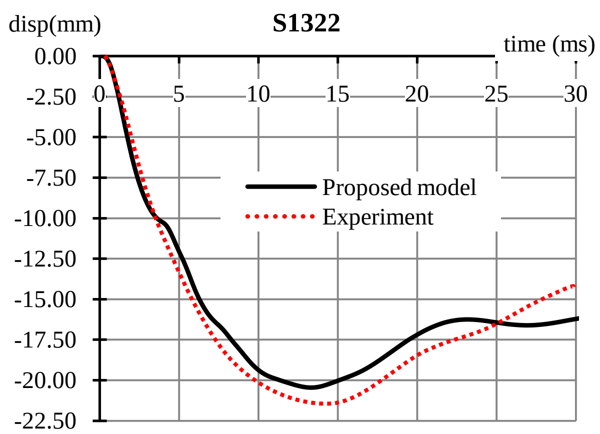


Fig. 6 Mid-span displacement-time histories

6.0 Conclusions

The results have shown that an application of the SPH method incorporated with a new DP-PC model is able to simulate crushing phenomenon under the compaction (compression) region, while the tensile-cut off followed by softening technique that utilizing orthotropic constitutive equation can give the reliable calculation of flexure/bending mechanism. Besides, the PC model defines the motion of subsequent yield surface during plastic loading by controlling the volumetric change. However, to obtain more reasonable and realistic estimation of damage behavior of RC elements, further study on SPH calculations and other numerical technique should be conducted numerously.

References

[1] Chen, Y., and May, I.M., Reinforced concrete members under drop-weight impacts. *Proc. of the Institution of Civil Engineers*, Volume 162(1), (2009), pp. 45-56.

[2] Kishi, N., Ohno, T., Mikami, H., and Ando, T. Effects of boundary conditions on impact behaviors of reinforced concrete beams subjected to falling-weight impact loads, *Proc. of Japan Society of Civil Engineers*, Volume 731/I-63, (2003), pp. 299-316.

[3] Sangi, A.J., Khan, R.A., and May, I.M. Behaviour of RC beams under multiple impact loads. *Proc. of the 1st International Conference of Protective Structures*, Manchester, United Kingdom, (2010).

[4] Bathe, K.J., Walczak, J., Welch, A., and Mistry, N. Nonlinear analysis of concrete structures. *Computers and Structures*, Volume 32(3/4), (1989), pp. 563-590.

[5] Saatci, S., and Vecchio F.J. Nonlinear finite element modeling of reinforced concrete structures under

impact loads. *ACI Structural Journal*, Volume 106, (2009), pp. 717-725.

[6] Kantar, E., Erdem, R.T., and Anil, O. Nonlinear finite element analysis of impact behavior of concrete beam. *Mathematical and Computational Applications,* Volume 16(1), (2011), pp.183-193.

[7] Swaddiwudhipong, S., Islam, M.J., and Liu, Z.S. High velocity penetration/perforation using coupled smoothed particle hydrodynamics-finite element method, *International Journal of Protective Structures*, Volume 1(4), (2010), pp. 489- 506.

[8] Johnson, G.R. Numerical algorithms and material models for high-velocity impact computations. *International Journal of Impact Engineering*, Volume 38(6), (2011), pp. 456-472.

[9] Fukazawa, J., and Sonoda, Y. An accuracy of impact failure response of reinforced concrete beam using ASPH method, *Journal of Structural Engineering Japan Society of Civil Engineers,* Volume 57A, (2011), pp. 1205-1212.

[10] Fujikake, K., Li, B., and Soeun, S. Impact response of reinforced concrete beam and its analytical evaluation. *Journal of Structural Engineering ASCE*, Volume 135, (2009), pp. 938-950.

[11] Liu, G.R., and Liu, M.B. Smoothed particle hydrodynamics: a meshfree particle method. *World Scientifiv Publishing*, Toh Tuck Link, Singapore, (2003).

[12] Bischoff, P.H., and Perry, S.H. Compressive behaviour of concrete at high strain rates. *Journal of Materials and Structures*, Volume 24, (1991), pp. 425-450.

[13] Zhou, X.Q., and Hao, H. Modelling of compressive behaviour of concrete-like materials at high strain rate. *International Journal of Solids and Structures*, Volume 45, (2008), pp. 4648-4661.