The Effects of Particle Morphology (Shape and Sizes) Characteristics on its Engineering Behaviour and Sustainable Engineering Performance of Sand

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Abstract: The mechanical behaviour of granular soil is interestingly dependent on the characteristics of the particles. The wide range of particle shapes and size distribution of sand, by virtue of its sedimentological process of formation plays a significant factor in the influence on its engineering behaviour reflected in terms of its packing density, permeability, shear strength and dilatancy. In this study, research on well graded sand (SW), poorly graded uniform sand (SPu_{Kahang}), gap graded sand (SPg) from Kahang Malaysia and also (SPu_{L.Buzzard}) Leighton Buzzard sand from UK were tested in a direct shear box. The shapes were quantified using images from a digital microscope where its morphological features can lead via statistical methods to determined correlations between strength and its physical properties. The research effort focuses in obtaining its shear strength and roughness parameters and also its extreme packing (e_{min} and e_{max}). Results from published studies on related matter and also the study on permeability are presented. The findings would lead to a better way to classify the shape and size distribution for the assessment of the behaviour of sand in various engineering disciplines such a good foundation soil in geotechnical engineering, as an abrasive material in mechanical engineering, as a filter of concrete in civil engineering, as a filter in chemical engineering and occurs as oil sands in petroleum engineering.

Keywords: Morphology, sands, sustainability, shear strength, packing density permeability

1. Introduction

Sand are granular materials which comprises of quartz where it is known as the most abundant mineral on the surface of the earth because of its hardness of 7 on the Moh's scale, compared to other soil minerals such as clay minerals. This, on the other hand consist of silicate minerals and are not pure quartz. The shape and size distribution of sand particles depend on the formation history of the grains where it results from the disintegration of rocks due to water, weather and glaciers [1]. The principal sources are relatively young and unconsolidated superficial deposits which have accumulated in the geological past. These geological deposits include marine beach and lacustrine deposits, alluvial deposits, mountain slope solifluction wastes as well as those of the glacial deposits [2]. The combination of the mineralogy of the sand particles and the weathering process has resulted in different shapes and grading distribution to

Sand deposits are very important materials used in many engineering purposes where in some countries such as Sri Lanka, the Philippines and even in Malaysia are facing a scarcity of suitable sand for construction. The seriousness of this matter has led to some countries imposing fines and penalty on those persons responsible for the irresponsible and

indiscriminate quarrying of sand [3], (Fig. 1). Environmental problems may also arise from this such as coastal erosion, destruction of animal habitats, flooding and also the increase in the turbidity of water due to the disturbance of underwater sand. Due to the high demand of this material, it is important to ensure the suitability of the sand materials used in various engineering disciplines to avoid wastage.

Sand materials are used in a wide spectrum of sustainable engineering practices such as material engineering, mechanical engineering, hydraulics and most importantly in geotechnical engineering. Sand is also used in agriculture, due to its suitability as a soil with high permeability characteristics to make heavy clay soil lighter. It can also be a filtering material in ground engineering, chemical engineering and petroleum engineering. It is also popularly used for building temporary and sometimes permanent road construction [4]. The particle shape and size of sand is considered to be one of the most important factors that influence the behaviour of the soil when it is used for various purposes [5]. In this study, the morphology of sand particles is investigated to examine its significant effect on permeability and together with the critical factor of packing density on shear strength parameter. These are important parameters in civil and foundation engineering. The

study on the usage of sand in materials is also discussed. Therefore this research aims to indicate that the information on particle shape and size distribution of sand is valuable for estimating several other characteristics of granular soil.

Robbed Sand-less

Exclusive

Months Sand of and rieft for core is great a single singl



Fig. 1 Illegal quarrying in (a) Malaysia and (b)
Philippines [3]

2. Sand in Sustainable Engineering

As discussed in the previous section, sand is widely use in a sustainable manner in materials engineering. It is a major constituent in many engineering materials such as concrete blocks, composite materials, manufacture of glass, ceramics and many others. San can be a sustainable engineering material, when it is used in an effectively way as a raw material. Fig. 2 shows a sustainable composite material consisting of sands in a resin. Another way of using sand effectively which utilises the shape and size of sand particle are sandpaper (Fig. 3). Sandpapers are sustainable material due to its low cost and availability of the constituent material. The shapes and sizes sand particles are important to

determine the grading scales of sandpaper. The abrasive grading scales for sand paper follows the classification of the Federation of European Producers Association (FEPA) where grading is according to the average size of the sand particles. Digital microscope was used in this study obtain microscopic images of the sand used in sandpaper with different grades shown in Fig. 4. It is seen that, for the same grade of sandpaper, the size of the particles is almost similar. The shape of the sand particles can be seen as fairly angular with rough edges. The reason for this is to maximize its usage as an abrasive material for smoothening purposes.



Fig. 2 Compostie consist of sand material and resins



Fig. 3 Different grades of sandpaper

Sand is also sustainably used in mechanical engineering as a casting material. Sand casting is a metal casting process characterized by using principally sand as the mould material. It is used to make metal parts such as bronze, brass, aluminium and others. The process is relatively cheap and sufficiently refractory and the moulding material is formed with a suitable bonding agent such as clay to develop strength. In most operations, the sand can be sustainably used many times, requiring the addition of only small amounts of sand each time. The sands used however must be according to the casting classification to ensure quality of castings, reduce cost and to improve productivity of extremely important significance.

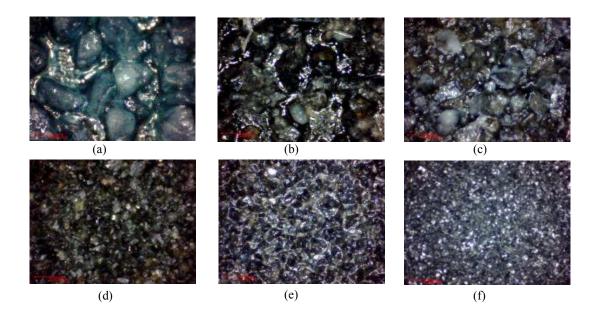


Fig. 4 Microscopic images (x50) of sandpaper with different grading, (a) grade 80, (b) grade 120, (c) grade 240, (d) grade 320, (e) grade 400, (f) grade 1000.

Other than that, hydraulic engineering in sandy soils is important in the use of agriculture, flow in catchment areas and as good drainage soil material due to it's a high permeability characteristic. The shape and sizes of the sand particles can affect the permeability coefficient of the soil with the use of empirical equations, thus helping to determine the suitability of the soil so that it can be used in a sustainable and in an effective way. Permeability is the measure of the ease with which water moves through aquifer material, certain relationships exist between permeability and the statistical parameters that describe the grain size and shape distribution of the porous mediums [6]. General estimates base on readily information such as effective size of soil particle is sufficient using the Hazen and USBR formula shown below where C is a constant and D₁₀ is diameter of the 10 % passing particle-size and D₂₀ is the diameter of the 10 % passing particle-size respectively [7].

Hazen
$$k = C_H D_{10}^2$$
 (1)
USBR $k = 36 \times D_{20}^{2.3}$ (2)

In Fig. 5, the U.S Bureau of Reclamation has worked out a means of finding the range of the permeability coefficient value in the individual of soil nature [7]. It can be seen that for sandy soil, poorly graded sand (SP) show the highest permeability coefficient than the rest. In the work of Goktepe and Sezer [5], the effect of particle shape and size

distribution on the permeability of the soil is investigated using a constant head-permeability test. The materials used for testing were separated into poorly graded uniform coarse, medium and fine sand and also well graded sand. The equation suggested by Chapuis [8] as shown is used to determine the coefficient of permeability (k).

$$k = 1.5 \times d_{10}^2 \times \frac{e^2}{1+e} \times \frac{1+e_{max}}{e_{max}^2}$$
 (3)

In Fig. 6, the coefficient of permeability of the coarse-grained sands is notably higher than that of medium and fine sands. The lowest coefficients were observed for well graded sands. In the analysis of the soil with similar gradation, it was observed that the coefficient of permeability increased with increases in the void ratio. The Kozeny-Carman equation is also used in fluid dynamics to calculate the pressure drops of a fluid through a soil bed and it takes into account the shape of soil particles as shown in equation 4, where Δp is the pressure drop, L is the total height of the bed, V_o is the superficial velocity, μ is the viscosity of the fluid, E is the porosity of the bed, Φ_s is the sphericity of the particles in the packed bed, and D_p is the diameter of the related spherical particle.

$$\frac{\Delta p}{L} = \frac{150V_0 \mu (1 - \epsilon)^2}{\Phi_s^2 D_n^2 \epsilon^3} \tag{4}$$

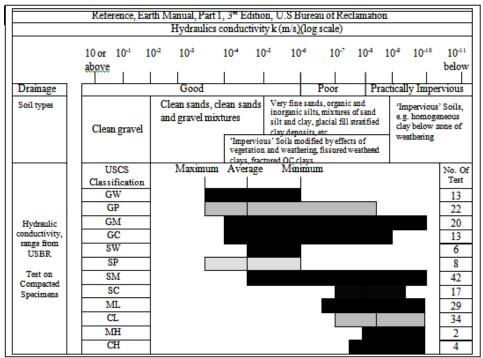
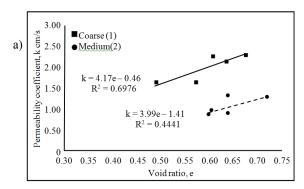


Fig. 5 The range of the permeability of coefficient on the classification of soil [8]



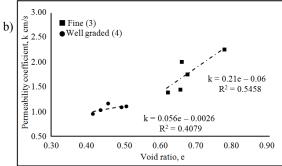


Fig. 6 Relationships of the permeability coefficient and void ratio (a) coarse and medium sand, (b) fine and well graded sand [5]

3. Effect of particle morphology of sand on its sustainable geotechnical engineering use

• Particle size distribution

Soil classification is important in determining the nature of the soil physical properties. In this study, classifications of the samples using sieve analysis are in accordance with BS 1377-1: 1990 [9]. Fig. 7 shows the particle size distribution curves of all the materials used in this study for the investigations of shape parameters, packing density and shear strength characteristics. The test samples are from river sand obtained from Kahang, Johor and comprise of three different particle size distribution classifications which are well graded sands (SW), uniformly graded sands (SPu_{Kahang}), and gap graded sands (SPg). The original test sands are classed as well graded sands. However careful separation of the sand particles using sieves enabled a new gap graded sand to be obtained. Leighton Buzzard sand from the United Kingdom was also used which is documented as being uniformly graded sand and is referred to as (SPu_{L. Buzzard}). Data of the particle size distribution and its packing density of the experimental results and also from published experimental results were compiled and used to compare in Table 1.

• Particle shape determination

According to Krumbein and sloss [10], Cho et al [11], the distinguishing features in the shapes of the particles are based on its sphericity and also its roundness. The definition of these two important scales is given below;

- Sphericity (S), $(S = \frac{r_{max-in}}{r_{min-cit}})$ determined by obtaining the radius of the largest inscribed sphere (r_{max}) relative to the radius of the smallest circumscribed sphere (r_{min}) .
- smallest circumscribed sphere (r_{min}).

 Roundness (R), ($R = \frac{\Sigma_{r_i} N}{r_{max-in}}$) is quantified as the average radius of curvature of features relative to the radius of the maximum sphere that can be inscribed in the particle.

The relationship between the sphericity and roundness can be further defined and evaluated in the form of dimensionless parameters as shown in Fig. 8 [10,11]. Comparison of the shapes is made between the uniformly graded sands (SPu_{Kahang} and SPu_{L.Buzzard}) of similar uniformity coefficient (Cu), in order to

minimise variables and allowing correlations to be made between the strength characteristics and the shapes of the particles. The digital microscope (Figure 8b) was used to obtain microscopic pictures to be taken with a magnification of x50 as illustrated in Fig. 9. Fig. 10 shows the extreme shaped particles. Fifty randomly picked sand particles of the two types of sand were studied and the statistical results are given in Fig. 11 where the cumulative frequency graph shows the distribution of the particle shape parameters of the samples. The database in Table 2 shows the effect of particle shape on soil properties of the tested sand and also sands from previously published results obtained by Cho et al. [11]. It is noteworthy that all these have similar uniformity coefficient (Cu) with the tested sample in this study.

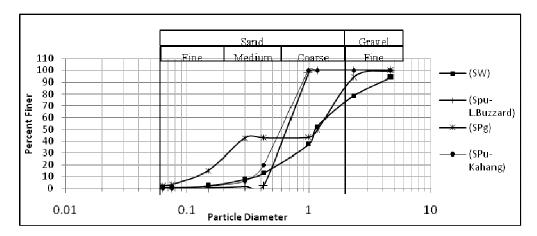
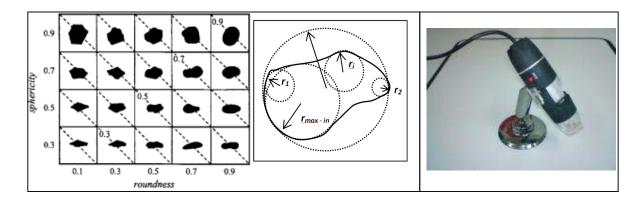


Fig. 7 Particle size distribution curves for the test sands

Table 1 Properties of the test samples (adopted from Bolton, [12])

Type of sand	Gradation			Void Ratio		References		
	D_{60}	D_{10}	C _u	e _{max}	e _{min}			
SW	1.5	0.38	3.95	0.914	0.398	-		
$SPu_{L \; Buzzard}$	0.71	0.48	1.48	0.725	0.574	-		
SPg	1.5	0.13	11.53	0.792	0.4	-		
SPu_{Kahang}	0.65	0.35	1.86	0.948	0.574	-		
Blasted river	0.29	0.12	2.42	0.79	0.47	Cornforth (1964, 1973)		
Mersey river	0.2	0.1	2.0	0.82	0.49	Rowe (1969)		
Monterey no. 20	0.3	0.15	2.0	0.78	0.57	Marachi, Chan, Seed &		
						Duncan (1969)		
Monterey no.0	0.5	0.3	1.67	0.86	0.57	Lade & Duncan (1973)		
Welland river	0.14	0.10	0.014	0.94	0.62	Barden et al. (1969)		
Mol	0.21	0.14	2.24	0.89	0.56	Ladanyi (1960)		
Berlin	0.25	0.11	1.5	0.75	0.46	De Beer (1965)		
Guinea Marine	0.41	0.16	2.56	0.90	0.52	Comforth (1973)		
Portland river	0.36	0.23	1.57	1.10	0.63	Comforth (1973)		
Karlsruhe	0.38	0.20	1.9	0.82	0.54	Hettle (1981)		
medium sand								
Sacramento river	0.22	0.15	1.47	1.03	0.61	Lee & Seed (1967)		



(a) (b) Fig. 8 (a) Particle shape definition and (b) digital microscope used for analysing (Cho et al, [11]; as presented in Krumbein and Sloss, [10])

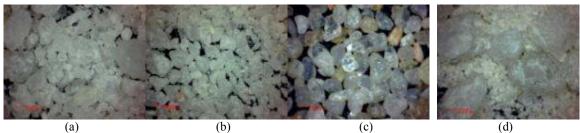
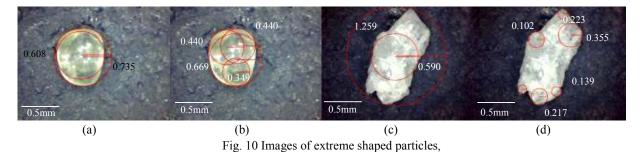
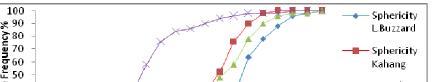


Fig. 9 Magnified pictures (x50) of the sand particles (a) SW sand (b) SPu_{Kahang} sand (c) SPu_{L.Buzzard} sand (d) GPg sand.



(a) & (b) $SPu_{L.Buzzard}$ sand particle with Sphericity = 0.827 and Roundness = 0.78 (c) & (d) SPu_{Kahang} sand particle with Sphericity = 0.469 and Roundness = 0.325



Roundness L.Buzzard Roundness Kahang 0 0.2 0.4 $\begin{array}{ccc} 0.5 & 0.6 & 0.7 \\ \text{Paramater range} \end{array}$ 0.91 1.1

Fig. 11 Cumulative frequency of the shape parameters for the tested soils

Table 2 Data of the average particle shape parameters and its effect on soil properties

		(aData c	ompiled	d from C	ho et al.	[11]		
		<u>Particle</u>		<u>Effective</u>		<u>Packing</u>		<u>Friction</u>
Soil type	C_{u}	sha	<u>ape</u>		<u>oe, S</u>	e_{max}	e_{min}	<u>angle</u>
		R	S	R	S			$\Phi_{ m cr}$
SPu _(L.Buzzard)	1.48	0.68	0.73	0.54	0.58	0.725	0.574	≈27
SPu _(Kahang)	1.86	0.41	0.64	0.24	0.52	0.948	0.574	≈34.8
Nevada sanda	1.8	0.60	0.85	-	-	0.850	0.570	31
Ticino sanda	1.5	0.40	0.80	-	-	0.990	0.574	37
Margaret river	1.9	0.70	0.70	-	-	0.870	-	33
sand ^a								
ASTM 20/30	1.4	0.80	0.90	-	-	0.690	-	32
sand ^a								
ASTM graded	1.7	0.80	0.90	-	-	0.820	0.500	30
sand ^a								
Blasting sand ^a	1.9	0.30	0.55	-	-	1.025	0.698	34
Glass bead ^a	1.4	1.00	1.00	-	-	0.720	0.542	21
Ottawa #20/30	1.2	0.90	0.90	-	-	0.742	0.502	27
sand ^a								
Ottawa F-110	1.7	0.70	0.70	-	-	0.848	0.535	31

Analysis of the Results from the Tested Samples

sanda

In geotechnical engineering, sand is considered to be a good soil foundation and therefore construction on this soil can be economically and environmentally sustainable. However, the influence of particle shapes and sizes on the engineering behaviour of sand has not been systematically assessed yet. Fig. 12, shows the relationship between the specific particle diameter size(D₆₀, D₁₀) and uniformity coefficient (Cu) on its packing density of the sand samples (experimental results from this study are labelled 'A' in the graphs, the rests of the data follows the data compiled by Bolton [12]). It seems that there is no apparently definitive relation between the extreme void ratios and the specific particle diameters of (D_{10}) . However, the minimum void ratio (emin) tends to have a slight reduction with the increase of the specific diameter (D₆₀) and also on the uniformity coefficient (Cu), but show no definitive relation on the maximum void ratio (e_{max}). Although the correlations are fairly low, e_{min} can still be regarded to be the most sensitive to grain size distribution as agreed by Simony and Houlsby [13] with gap graded sands showing the lowest values. This is due to the big difference in the distribution of particle sizes in the sample where the smaller sand particles occupy the voids present between the larger particles making it to be denser.

Fig. 13 shows the relationship of the shape parameters of the mean sphericity and roundness of the sand particles against its maximum and minimum void ratios (experimental results from this study are labelled 'A' in the graphs, the rests of the data follows the data compiled by Cho, et. al. [11]). The e_{max} and e_{min} decrease as the sphericity and roundness of the particle increases, showing that irregularity obstructs the particle mobility and their ability to attain densely packed conditions. The increase in e_{max} will also eventually increase the relative density of the sample as observed by Goktepe and Sezer [5].

Direct shear tests were done on natural dry samples to determine its shear strength characteristics. Samples with different relative densities were subjected to a normal stress (σ). Particle crushing during shearing was avoided in this study by using low level normal stress [12]. Fig. 14(a) shows test on the (SW) sample that high relative density shows a peak shear stress (τ_{peak}) on the sample. Fig. 14(b) shows a negative vertical displacement, which means that the samples are in an expansion mode where it increases in volume as shear displacement occur. This is called dilation, noticing that low relative density doesn't show any dilation but it is however in compression. Bolton[12], Simoni and Houlsby [13] and Hamidi et al, [14] computed the angle of dilation (ψ) by relating the horizontal displacement (h) and vertical displacement (v) to calculate the rate of dilation (dv/dh) with the following equation:

$$\tan \psi = \frac{dv}{dh} \tag{5}$$

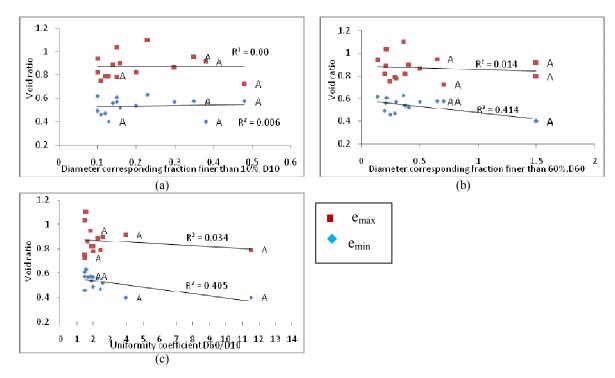


Fig. 12 Influence of the specific particle size characteristics of a soil mass (a) D₁₀, (b) D₆₀, and (c) uniformity coefficient (Cu) and its minimum and maximum void ratio.

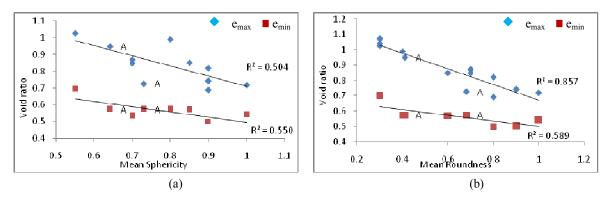


Fig. 13 Relationship of the average (a) sphericity and (b) roundness of the sand particles against its maximum and minimum void ratios.

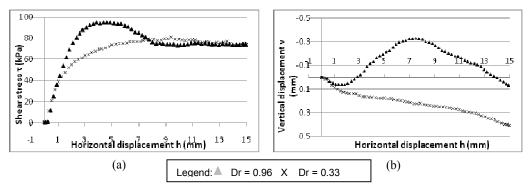


Fig. 14 Typical direct shear test result of 100kPa normal stress of the, (a) shear stress (τ), (b) vertical displacement (mm), versus horizontal displacement of well graded sand (SW)

Fig. 15 shows the variation of the peak friction angle (\emptyset_{peak}) with the relative density for all the samples with 100 kPa normal stress (σ). It is seen that the peak friction angle is the lowest for SPu(L.Buzzard) sand followed by SPu(Kahang) which shows lower Øpeak than the SW and SPg sands. The same goes for the critical friction angle (\emptyset_{cr}). The determination \emptyset_{cr} in this study uses the \emptyset_{peak} values of different densities plotted against maximum dilatancy angle (ψ_{max}) as shown in Fig. 16. The best fit line is then drawn, giving the \emptyset_{cr} values as the shearing resistance of a sample which would exhibit zero dilatancy. This indicates that the relative density, grading characteristics and the shape of the particles have a significant effect on the shear strength of the soil. Fig. 17 shows the correlation between the uniformity coefficient (Cu) and the critical angle of friction (\emptyset_{cr}) only for Kahang sands with different normal stresses. It can be seen that as Cu increases, \emptyset_{cr} would also increase until it reaches a peak, and a further increase in Cu has caused a reduction in the critical friction angle. The shapes of the sand particles also play a significant role in the shear strength of the soil and it does not entirely dependent on the mineral-to-mineral friction ([11],[15]). Fig. 18 shows the relationship of the sphericity and roundness to the critical friction angle (Ø_{cr}). The shear strength values of SPu_(L.Buzzard) and SPu(Kahang) sands (Labelled 'A' in the graph) and the compiled data from Cho et al. [11] are summarize in the graph. The graph shows that the roundness of the particles plays a more significant effect on the critical friction angle (\emptyset_{cr}) than the sphericity of the sample.

The increase in shear strength with density is primarily due to the increased tendency of the sample to dilate and the work done in overcoming frictional forces [16]. Fig. 19(a) shows the variation of

maximum dilation angle (ψ) with relative densities of Kahang sands with 50 kPa normal stresses. SW sand has the highest dilation angles compared to the other sand samples with similar relative density (Dr). SPu_{Kahang} sand tend to show higher dilation angles than SPg sand which has the highest uniformity coefficient (Cu). Fig. 19(b) shows the SPu_(l.Buzzard) has a higher mean sphericity and roundness values compared to SPu(Kahang) and it shows lower dilatancy values. This can be explained as angular particles and the grading of the particles tend to be more interlocking and it obstructs the mobility of the particles, making it to expand in volume when shear displacement is induced. Smooth and rounder particles have the ease to move around each other, which explains its low dilation angles.

4. Conclusions

The shape and size distribution characteristics of sand particles play an important role in the behaviour of sand and its sustainable usage in many engineering disciplines. The sustainable approach in avoiding the permanent lost of sand is by replacing it with the manufacture of artificial sands. Artificial sands are obtained by crushing raw materials into suitable particle sizes and also by deep heat treatment of palletised waste mixture. Therefore in today's environment, it is important that the materials used for these purposes must be economically and environmentally sustainable.

It can be concluded that:

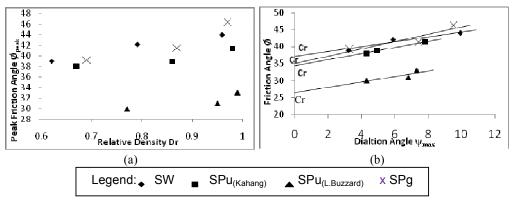


Fig. 15 Peak friction angle versus relative density of the samples with 100kPa normal stress

Fig. 16 The determination of critical friction angle with 100kPa normal stress

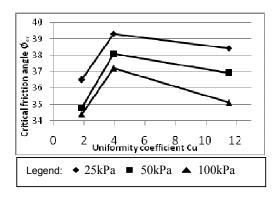


Fig. 17 The correlation between (Cu) and (\mathcal{O}_{cr}) of Kahang sand

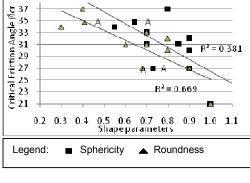
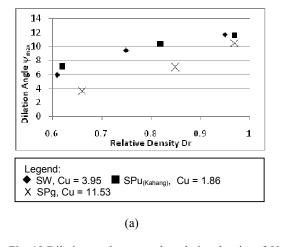


Fig. 18 The relationship of the sphericity and roundness to (\emptyset_{cr})



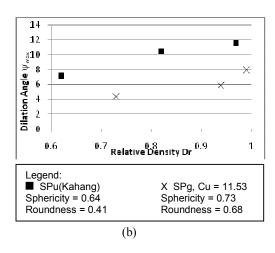


Fig. 19 Dilation angle versus the relative density of 50kPa normal stresses of sand with (a) different uniformity coefficient (Cu) (b) and shape parameters.

- The usage of sand as an abrasive material relies significantly on the average size distribution of the sand particles so that it can be graded accordingly.
- The effect on the permeability in sands tends to show that the permeability coefficient increases with void ratio for the samples that were tested by Geoktepe and Sezer [5]. The size distribution shows that the coefficient of permeability of the uniformly graded coarse-grained sands was higher than that of medium and fine sands. The lowest coefficients were observed for well graded sands.
- The effect of uniformity coefficient (Cu) of the sand samples and from published studies on the packing density show that the (e_{min}) tend to give a better relationship between the void ratio and the uniformity coefficient than (e_{max}). The increase in sphericity and roundness of the particles also decreases the minimum (e_{min}) and maximum (e_{max}) void ratio.
- Direct shear box testing shows that gradation has a significant effect on the (\$\mathcal{O}_{peak}\$) and (\$\mathcal{O}_{cr}\$). The increases in Cu also tend to increase the (\$\mathcal{O}_{cr}\$)

values, but it reaches a peak and then further decreases as where the sand would be classified as gap graded. The gradation and shape characteristics of the coarse sands also have an influence in the occurrence of dilatancy. Higher sphericity and roundness values tend to have lower dilation angles as compared to the other soils.

As a result, the particle gradation and shapes of the coarse grained sands really needs to be properly classified and looked further into as it has a significant effect on its engineering performance. It is also to ensure that its usage will be economical and environmentally sustainable in the various aspect of engineering.

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References

- [1] Blyth, F.G.H. and de Freitas, M.H. *A geology for engineers*. Pitman Press, Great Britain (1974).
- [2] Archer, A.A., Sand and gravel as aggregate. Miner. Dossier Miner. Resour. Consult. Comm. No. 4. H.M.S.O. (1972)
- [3] Ann., Prize offered for arrest in illegal quarrying. *The Brunei Times*. (2012, January 28), Pg. A11.
- [4] Boateng, J,A, and Erol,T,. Sustainable use of oil sands for geotechnical construction and road building. *Journal of ASTM International (JAI)*, Volume 9, Issue 2 (2012).
- [5] Goktepe, A.B, & Sezer, A. Effect of particle shape on density and pearmeability of sands: *Proceedings of the Institution of Civil Engineers, Geotechnical Engineering* 163, December 2010, Issue GE6, (2010). pg 307-320.
- [6] Masch, F.D. and Denny, K.J., Grain size distribution and its effect on the permeability of unconsolidated sands. *Water Resources Research*, Vol. 2, No. 4, (1966). PP. 665-677.
- [7] Ho.W.R.K, A critical assessment of empirical formulae for determining hydraulic conductivity from soils. University of East London, Masters thesis (2005).
- [8] Chapuis R.P. Predicting the Saturated Hydraulic Conductivity of Sand and Gravel Using Effective Diameter and Void Ratio. Can Geotech J 41: (2004). Pg. 787-795.
- [9] British Standard Institution (BSI). BS 1377: British Standard Method of Test for Soils for Civil Engineering Purposes.
- [10] Krumbein, W. C. And Sloss L. L. Stratigraphy and Sedimentation. San Francisco: (1963). W. H. Freeman and Company.
- [11] Cho, G.C, Dodds, J. & Santamarina, J.C. Particle shape effects on packing density, stiffness and strength: Natural and crushed sands. *Journal of Geotechnical and Geoenvironmental Engineering*, ASCE, Vol. 132, No 5. (2006).
- [12] Bolton, M.D. "The strength and dilatancy of sands" *Géotechnique*., 36(1), (1986). 65-78.
- [13] Simoni, A. & Houlsby, G.T. The direct shear strength and dilatancy on sand-gravel mixtures. *Geotechnique*. (2006)
- [14] Hamidi, A. Alizadeh, M. & Soleimani, S.M. Effect of particle crushing on shear strength and dilation characteristics of sand-gravel mixtures: *International Journal of Civil Engineering*. Vol. 7, No. 1, (March 2009).
- [15] Chan, L. C. Y.., & Page, N. W. Particle fractal and load effects on internal friction in powders *Powder Technol. and Geological Engineering*. Vol 24, (1997).523-549.
- [16] Wijeyesekera, D.C., Hachouf, K. and Warnakulasuriya, S. The significance of dilatancy of the backfill on the soil structure interaction pf polypropylene reinforcement, Proceedings of the 2nd international Conference

on ground improvement techniques, (1988). pg 537-544, Singapore.