

Leveraging latent persistency in United States patent and trademark applications to gain insight into the evolution of an innovation-driven economy

Iraj Daizadeh¹

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ABSTRACT

Objective. An understanding of when one or more external factors may influence the evolution of innovation tracking indices (such as US patent and trademark applications (PTA)) is an important aspect of examining economic progress/regress. Using exploratory statistics, the analysis uses a novel tool to leverage the long-range dependency (LRD) intrinsic to PTA to resolve when such factor(s) may have caused significant disruptions in the evolution of the indices, and thus give insight into substantive economic growth dynamics.

Design/Methodology/Approach. This paper explores the use of the Chronological Hurst Exponent (CHE) to explore the LRD using overlapping time windows to quantify long-memory dynamics in the monthly PTA time series spanning 1977 to 2016.

Results/Discussion. The CHE is found to increase in a clear S-curve pattern, achieving persistence ($H \sim 1$) from non-persistence ($H \sim 0.5$). For patents, the inflection occurred over a span of 10 years (1980-1990), while it was much sharper (3 years) for trademarks (1977-1980).

Conclusions/Originality/Value. This analysis suggests (in part) that the rapid augmentation in R&D expenditure and the introduction of the various patent-directed policy acts (e.g., Bayh-Dole, Stevenson-Wydler) are the key factors behind persistency, latent in PTA. The post-1990's exogenous factors

seem to be simply maintaining the high degree and consistency of the persistency metric. These findings suggest investigators should consider latent persistency when using these data and the CHE may be an important tool to investigate the impact of substantive exogenous variables on growth dynamics.

Keywords: Innovation, Hurst, trademarks, patents, persistency, economy

¹ Takeda Pharmaceuticals, United States. Email: iraj.daizadeh@takeda.com, ORCID: 0000-0003-3648-023X.

INTRODUCTION

Intellectual property (IP)-based metrics (initially patents and more recently trademarks) have been used as proxy measures of global politico-socio-economic innovative behavior [on the micro (firm)-, meso (research institution, cluster)-, and/or macro (country/regional)-level] for decades (see, e.g., Dziallas and Blind, 2019). These and related indices focus on the time-dependent ebbs and flows of absolute counts of observations and/or derivatives thereof. Importantly, it is understood that these data integrate one or more extrinsic forces and/or factors that are either directly or indirectly influenced the chronological evolution of these data in some way (see, e.g., Daizadeh, 2007, 2009, 2021). Indeed, a vast majority of this research either test or generate a hypothesis of how such extrinsic impulses – such as the promulgation of a specific policy framework, gross effects of research and development expenditures, sector-specific technology dynamics (e.g., disruptive versus incremental) – may affect the time course of these IP-metrics (see, e.g., Grimaldi and Cricelli, 2020; Flikkema, et al., 2015; Daizadeh, 2007, 2021). Unfortunately, to the author’s knowledge, there are limited-to-no inquiries exploring the intrinsic make-up of these time-series data.

Even in the simplest of cases, the interpretative potential of time series data, such as those of patents and trademarks, may be challenging, however, due to their intrinsic behavior (s) (see, e.g., Cheng et al., 2015). Importantly and simultaneously, some of these properties are also of interest due to the potential to elucidate interesting dynamics. For example, memory characteristics across length scales (short, medium, and long) suggest the existence of various economic cycles (see, e.g., Alvarez-Ramirez et al., 2020). Concordantly, these same memory characteristics can obfuscate true signal detection, due to – for example – mistakenly considering long-memory and non-stationary complex dynamics (see, e.g., Saha et al., 2020). Other properties such as linearity, normality, and seasonality of the time series may further exacerbate misinterpretations. Statistical time series practitioners exert great effort in developing methods that would increase the certitude of the interpretation by accommodating one or more of these properties. Resolving the suite of issues is the first step in establishing robust statistical approaches and increased confidence in model interpretability.

The Hurst exponent is a statistical time-series tool that has been used to better understand memory effects in time series data and has been applied to various fields including earth sciences (see, e.g., Sliho, et al., 2020), economics (see, e.g., Wu and Chen, 2020), and others. While there are other ways, the Hurst exponent (a measurement of memory) is classically defined as $H \sim \ln(R / S)^t / \ln(t)$, where R and S are the rescaled range and standard deviation, respectively, and t is a time window. An $H=0.5$, an $H<0.5$, and an $H>0.5$ indicate a random walk (non-persistent), an anti-persistent, and a persistent (trend reinforcing) time series behavior, respectively (see, e.g., Mandelbrot and Wallis, 1968, 1969).

To the author’s knowledge, there have not been any investigations of the Hurst exponent with regards to either patents or trademarks. In terms of patents, however, some elements of memory effects have been explored in the context of economic cycles by several authors (see, e.g., Korotayev, 2011; Haustein and Neuwirth, 1982, Alvarez-Ramirez, 2020, Epicoco, 2020, Daizadeh, 2021a). Korotayev et al (2011) demonstrated a Kondratieff (K)-wave pattern when investigating the dynamics of the annual number of global patents per million population from 1900 to 2008. As summarized by (Alvarez-Ramirez, 2020), Korotayev showed that patents presented with “a steady increase during the upswing phase of Kondratieff’s cycle, and a pronounced decrease during the downswing phase.” Haustein and Neuwirth (1982) found that “industrial production on patents with a lag of 9 years.” More recently, Epicoco (2020) fitted the information and communications technologies cycle with that of the economy using patent and productivity data and proposed “the current productivity slowdown may be a

signal that the economic system needs to change its leading technologies.”

Carbone, Castelli, and Stanley (2004) proposed a time-dependent Hurst exponent based on a detrending moving average (DMA). Here, the Hurst exponent is a log-log slope of the DMA standard deviation against window (see equations 1 and 2 therein). The authors conclude from an analysis of artificial and observed time series of financial data that the time-variability is much “richer” than anticipated from a mono-fractal approach. To accommodate non-stationary effects, the work was subsequently extended with the aid of detrended fluctuation analysis over non-overlapping window lengths by Alvarez-Ramirez and colleagues (2020). Effectively, these approaches to time-dependent Hurst exponent calculations are model parameterized.

In this work, and as described in the Methodology section below, a non-model parameterized chronological Hurst exponent (CHE) is proposed that when applied to a given time series may identify significant changes in the persistency of memory. The method is straightforward to implement since it simply uses a standard estimate of the Hurst exponent calculated from the initial time point to month a , where a is a monthly increment. The output of the CHE calculation is described as the time series plot of each of the Hurst exponents, allowing a qualitative view of the Hurst exponent over a given time period (see Methods). This approach allows for an arbitrary method to calculate the Hurst exponent while taking into regard the nuances of the time series. Here, the method is applied to US patent and trademark applications (PTA) from 1977-2016. The date range selected is chronologically broad (a period of over 40 years) and used in prior work, and thus presented here. Interested readers may extend the data range accordingly, as all data and R Programs used for this manuscript are available (Appendix; Daizadeh, 2021b).

As described in the Results section below, from the application of this novel tool to PTA, it is found that the CHE evolves in a highly descriptive and idiosyncratic S-like pattern: from non-persistent ($H \sim 0.5$) to saturation (trend reinforcing persistent ($H \sim 1$) level via a quickly evolving inflection period (see Figures 1-3). For patents, the inflection occurred over a span of 10 years (1980-1990), while it was much sharper (3 years) for trademarks (1977-1980). As will be further discussed below, these findings suggest: investigators should consider latent persistency when using these data; exogenous factors after the identified inflection points for these indices have only incrementally strengthened intrinsic memory; and, the CHE may be an important tool to investigate the impact of substantive exogenous variables on growth dynamics. Qualitative correlation of the timing of the inflection points for patents and trademarks suggests the importance of research and development expenditure.

METHODOLOGY

The data were comprised of the monthly number of US patent applications (Patents) and the monthly number of US trademarks filings (Trademarks) (together, PTA) from 1977 to 2016, and were obtained from the United States Patent and Trademark Office (USPTO) as described below:

- Patents:
 - Website: <http://patft.uspto.gov/netahtml/PTO/search-adv.htm>
 - Search pattern: Application Filing Date: "APD/MM/\$/YYYY"
- Trademarks:
 - Website: <http://tmsearch.uspto.gov/bin/gate.exe?f=tess&state=4804:57thz4.1.1>
 - Search pattern: Filing Date: "(YYYYMM\$)[FD]"

Note: MM/YYYY is the 2/4 digital representation for month/year. The two searches resulted in 472 datapoints – representing monthly observations over the period of study (approximately 40 years) – for each variable and imported into R for processing (Appendix; Daizadeh, 2021b).

The methodology followed standard implementation, and default parameters were used throughout. The general algorithm for the analysis is as follows:

- Load time series (R package ‘tseries’ (Trapletti and Hornik, 2019)), identify and replace outliers with an average of prior and posterior-month values (R package ‘tsoutliers’ (López-de-Lacalle, 2019). Note: 3 outliers were determined for Trademarks (September 1982; November 1989; and June 1999) and 4 for Patents (September 1982, June 1995, October 2007, and March 2013).
- Calculate descriptive statistics [including standard deviation, kurtosis, and skew (R package ‘moments’ (Komsta and Novomestky, 2015))] and auto/serial correlation (base R package).
- Calculate intrinsic variables: normality (R package ‘nortest’ (Gross and Ligges, 2015)), stationarity (R package ‘forecast’ (Hyndman, et al., 2020; Hyndman and Khandakar, 2008; R package ‘aTSA’ (Qiu, 2015)), seasonality (R package ‘seastests’ (Ollech, 2019)), and non-linearity (R package ‘nonlinearTseries’ (Garcia, 2020))
- Calculate chronological Hurst exponent: Determine Hurst exponent based on Hyndman implementation (R package ‘tsfeatures’ (Hyndman et al, 2020)) using the following algorithm:
 - for (i in start:end) { hurst-IP[i] <- hurst (time[1:(1+i*1)]) }, where IP is either Trademarks or Patents; start = September, 1977; end = December 2016; I = monthly increments
 - Note: The Hyndman approach – one of several methods to calculate (estimate) the Hurst Exponent (Shang, 2020) – is defined as 0.5 plus the maximum likelihood estimation of the fractional differencing order (see Hyndman et al, 2020); thus, it has properties that differ than Hurst’s original definition (e.g., no singularities at certain scales). In principle, any approach should produce qualitatively the same result as that outline above, albeit additional work is required to confirm the approaches sensitively.

RESULTS AND DISCUSSION

General Statistics

Generally, the time series were similar in structure with a general cobra-like structure (see upper graphs of Figures 1, 2, and 3) and similarities in the shape of the distributions (e.g., approximately symmetric (skew) and platykurtic) (see Table 1). The time series showed clear long-memory tendency as presented in the auto and serial correlation functions with lag much great than 2. Lastly, both time series were non-normal, non-stationary [with a single difference ((t-1) – t) bringing them into stationarity – that is, integration of order 1 typical of econometric data], seasonal, and non-linear (see Table 2).

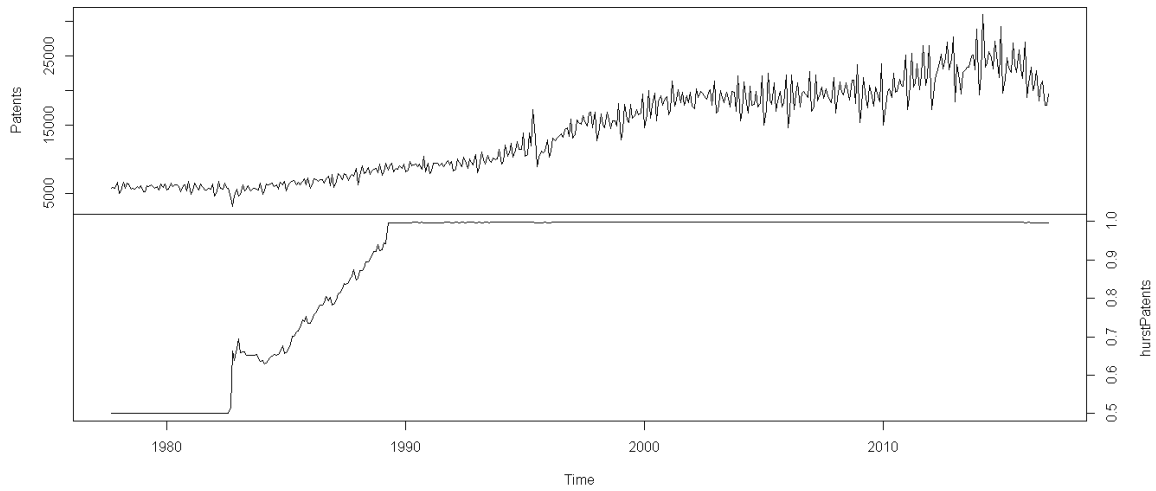


Figure 1. The monthly number of Patents (top graph) from 1977-2016 with its corresponding chronological Hurst values (bottom graph).

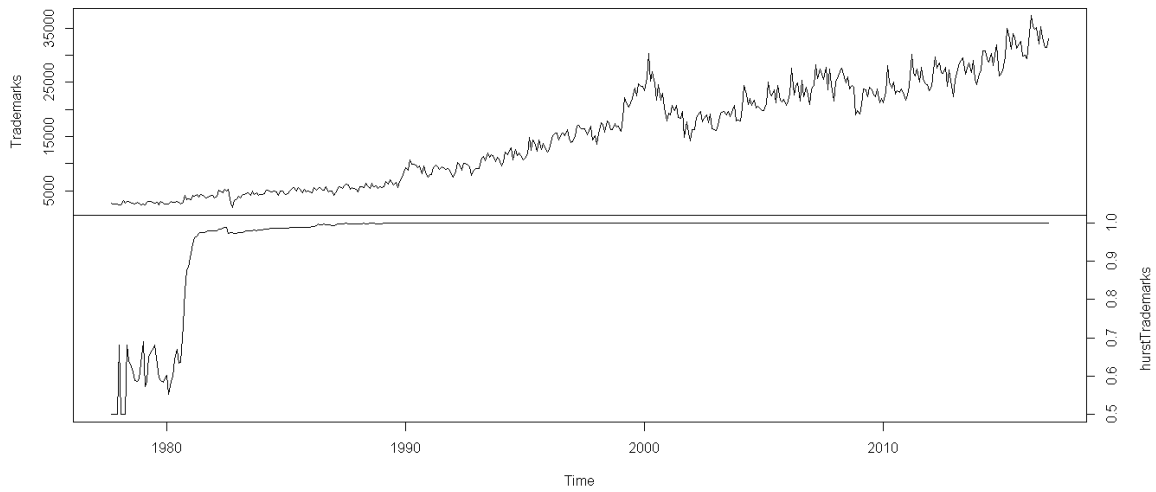


Figure 2. The monthly number of Trademarks (top graph) from 1977-2016 with its corresponding chronological Hurst values (bottom graph).

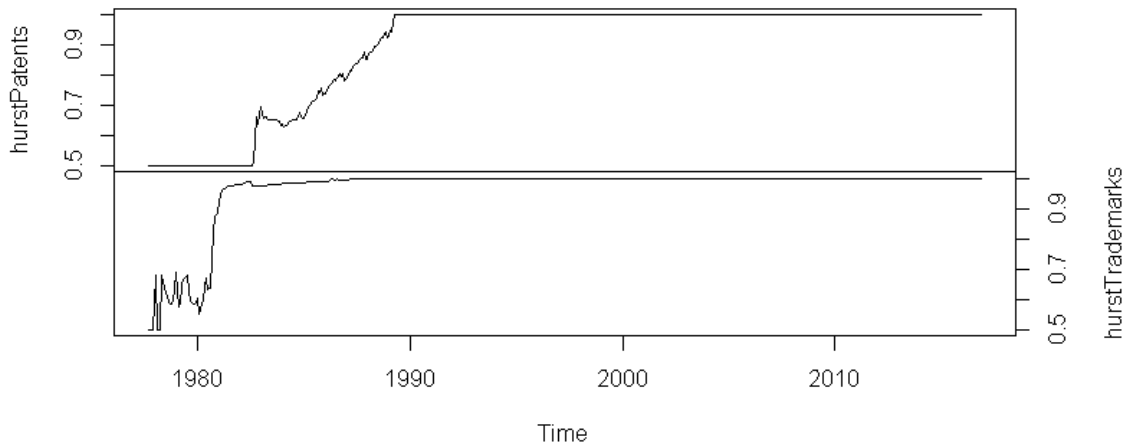


Figure 3. Comparison of chronological Hurst values between Patents and Trademarks.

Table 1. Descriptive statistics of Patents and Trademarks.

| | Patents | Trademarks |
|--------------------------|----------------|-------------------|
| Minimum | 3134 | 1895 |
| 1 st Quantile | 7598 | 5537 |
| Median | 14468 | 15456 |
| Mean | 13930 | 15276 |
| 3 rd Quantile | 19422 | 23574 |
| Maximum | 30969 | 37317 |
| Stand Deviation | 6523.347 | 9418.054 |
| Skewness | 0.1854975 | 0.2020644 |
| Kurtosis | 1.757654 | 1.763723 |

Table 2. General results of intrinsic parameters of Patents and Trademarks (see Appendix; Daizadeh, 2021b).

| | Patents | Trademarks |
|-----------|----------------|-------------------|
| Normality | Non-normal | Non-normal |

| | Patents | Trademarks |
|---------------|--|--|
| Stationarity | Non-stationary (order of integration: 1) | Non-stationary (order of integration: 1) |
| Seasonality | Seasonal | Seasonal |
| Non-linearity | Non-linear | Non-linear |

Chronological Hurst Exponent

Examining Figures 1 and 2, the time evolution of PTA shows a general trend-reinforcing pattern, edging generally up with time. There are variations of course partly due to the identified seasonal effects. The Chronological Hurst Exponent (CHE) correspondingly evolves, with non-persistence ($H \sim 0.5$) near the beginning of the time series (notice: this is where the absolute counts of the PTA are effectively static). As the time series evolves, the CHE quickly jumps to saturation ($H \sim 1$). A Hurst exponent at such a high relative value suggests that the trend will continue to persist in the original time series – that is, the exponent reflects the similarity of prior absolute values of PTA to future values; in other words: as prior timepoints values edge up, it is likely that the next value will also be up. This is consistent with the long auto and serial correlations in the original time series.

Co-positioning PTA CHE (as in Figure 3), a similarity in S-structure is presented. The scaling regimes of are trichotomized in both time series into the following periods: (1) non-persistence (albeit for Trademarks mild persistence is observed with H values oscillating between 0.5 and 0.7); (2) time-varying persistence (an inflection, where H slopes from 0.5/0.6 to near 1); and, (3) persistence ($H \sim 1$). Specifically, it is found that:

- Period 1: Weak to no persistence prior to 1980s:
 - Patents: The flat H exponent, as shown in Figure 1 (lower graph) and Figure 3 (upper graph), reflects the neutral acceleration in the time series.
 - Trademarks: The variation in H exponent (~ 0.5 to ~ 0.7) reflects a positive trajectory as depicted in Figures 2 (upper graph) and 3 in the reference time series.
- Period 2: A time-varying Hurst exponent up to and during the 1980s and 1990s
 - Patents: The value of the H exponent began to grow with a rapid linear rate of approximately 0.1-0.2 Hurst values per year, commencing with an explosive charge in the mid-1980s, and terminating with saturation before 1990.
 - Trademarks: The value of the H exponent grew at an exceptional rate at roughly 0.3 over two years, from effective baseline to near saturation by 1983.
- Period 3: Saturated persistence for trademarks beginning by 1983 and for patents in the 1990s throughout the duration of the reporting period 2016.

Focusing on the time-varying Period 2, while more work needs to be done to better our understanding of the dynamics in the 1980s and 1990s that affected these proxy measures of innovativeness, some thoughts present themselves for potentially testable causality hypotheses:

- Economic: Total spending on research and development grew from \$60B USD in 1975 to \$100B by 1985, concomitantly while the contribution from industry to the percent of Gross Domestic Product rose from 1% to nearly 1.5% during that same time (see Figure 1 in Hunt, 1999). Indeed, patent activity (number of Patent Applications) also doubled grew from 100k

(1980) to 200k (2000) (see Figure 2 in *ibid*). Could industrial R&D expenditures directly drive the persistence of Patents and Trademarks?

- Policy: There were significant competitive policy initiatives during the 1980s including the Bayh-Dole Act (Patent and Trademark Law Amendments Act (Pub. L. 96-517, December 12, 1980), Public Law 96-480, Stevenson-Wydler Technology Innovation Act (as amended in 1986 and 1990), and others (see Table 1 in Coriat and Orsi, 2002). Was the persistency initiated by one or more of these policies (including, for example, new patentability types: software and/or business plans)?
- Technology dynamics: S-shaped growth for individual technological innovations may contribute to the overall Patent evolution dynamic (see, e.g., Anderson, 1999). A linear combination of such patent growths may be affecting the persistence of the overall Patents structure. To the author's knowledge, there is limited to no information on any S-shaped distribution for Trademarks, the potential for future queries. Do technology dynamics associated with innovation cumulatively affect persistence in the innovation metrics?

CONCLUSION

The goal for the use of a novel method – termed the chronological Hurst exponent (CHE) – to investigate long-term memory dynamics in time series data is to describe the overall persistency trend and, importantly, to identify the key dates in which there is a change in persistency (anti-persistence, non-persistence, persistence). Here, the CHE was applied to two innovative-tracking, intellectual property-driven data: the monthly numbers of patent and trademark applications. CHE is a simple to use method, trivially constructed in any programming language (Appendix; Daizadeh, 2021b) and easily applied to longitudinal data. Additional work is required to better understand the approach, including using different memory quantification tools (e.g., different versions of the Hurst exponent calculation) and different (longer) datasets with different characteristics.

In summary, this paper presents the first use of the CHE as well as the initial results of its application to PTA. The importance of persistency as an intrinsic factor of time series data should not be underestimated as effects such as trend-reinforcing behavior may lead to biased results when interpreting time-series data, as the momentum of the time series may cover salient investigative concerns (recall that the Hurst exponent is a measure of self-similarity). It is a hope of this work that investigators continue to examine the innovative accomplishments of the late 20th century as well as consider persistency measures in current assessments of techno-economic progress.

Conflict of Interest Statement

The author is an employee of Takeda Pharmaceuticals; however, this work was completed independently of his employment. The views expressed in this article may not represent those of Takeda Pharmaceuticals.

Statement of Data Consent

The data generated during the development of this study has been included in the manuscript.

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APPENDIX: R CODE FOR TRANSPARENCY AND REPRODUCIBILITY

```
# Start: R code with dataset #
```

```
#Trademarks
```

```
#Go to TESS: http://tmsearch.uspto.gov/bin/gate.exe?f=tess&state=4804:57thz4.1.1
```

```
# Manually search and collect Number of Trademarks as follows:
```

```
#By Filing Date: "(198712$)[FD]" - Where 198712$ is the %Y%m$.
```

```
#Patents
```

```
#Go to PATFT: http://patft.uspto.gov/netahtml/PTO/search-adv.htm
```

```
#By Application Filing Date: "APD/12$/2018"
```

```
#The patent and trademark filings data were collected from Sept 1977 to Dec 2018.
```

```
#Confirm version of R:
```

```
> citation()
```

```
R Core Team (2019). R: A language and environment for statistical computing. R Foundation for Statistical Computing, Vienna, Austria. URL https://www.R-project.org/.
```

```
> version
```

```
_platform x86_64-w64-mingw32
```

```
arch x86_64
```

```
os mingw32
```

```
system x86_64, mingw32
```

```
status
```

```
major      3
```

```
minor      6.1
```

```
year       2019
```

```
month      07
```

```
day        05
```

```
svn rev    76782
```

```
language      R
version.string R version 3.6.1 (2019-07-05)
nickname      Action of the Toes
```

```
#Read into R:
```

```
> IP<- read.csv("../data.csv", sep=",")
```

```
#Confirm dataframe - length/variables and shrink by 24m to avoid so-called 'patent cliff':
```

```
> str(IP)
```

```
'data.frame':  496 obs. of  3 variables:
```

```
$ Date: Factor w/ 496 levels "1/1/1978","1/1/1979",...: 455 42 84 126 1 168 209 250 291 332
```

```
...
```

```
$ Number.of.Trademark.Applications: int  2669 2597 2552 2604 2386 2370 3126 2738 3028 3088 ...
```

```
$ Number.of.Patent.Applications  : int  5760 5898 5731 6630 5064 5439 6660 5799 6487 6419 ...
```

```
# Shrinking by 24 months dues to so-called "patent-cliff"
```

```
> TrademarksTotal<-IP$Number.of.Trademark.Applications[1:472]
```

```
> PatentsTotal<-IP$Number.of.Patent.Applications[1:472]
```

```
#Convert to Time-Series, decompose time-series, and perform descriptive statistics
```

```
> tsTrademarks<-ts(TrademarksTotal,start=c(1977,9),frequency=12)
```

```
> tsPatents<-ts(PatentsTotal,start=c(1977,9),frequency=12)
```

```
> tsTrademarks
```

```
      Jan Feb  Mar  Apr  May  Jun  Jul  Aug  Sep  Oct  Nov  Dec
1977                2669 2597 2552 2604
1978 2386 2370 3126 2738 3028 3088 2708 2638 2465 2793 2636 2362
1979 2518 2350 2920 2968 2953 2794 2741 2829 2438 2956 2676 2596
1980 2469 2607 3035 2893 2797 3094 2883 2590 2928 4081 3412 3559
1981 3329 4113 3906 4297 3871 4358 4077 3815 3688 3879 4162 4140
1982 3594 4009 5128 4868 4576 5244 4942 5264 15843 1895 3126 3529
1983 3915 3597 4224 4297 4389 4543 4192 4893 4265 4634 4189 4296
1984 4281 4472 5167 5068 4762 4837 4914 4803 4064 4896 4922 4474
1985 4296 4408 5087 5417 5537 4914 5537 5215 4627 5218 4747 4904
1986 4785 4677 5569 5153 5397 5630 5152 5124 5715 4787 4950 4931
1987 4188 4826 5528 5780 5372 5860 6269 5990 5309 5624 5454 5286
1988 4758 5756 5730 5618 6358 5706 5473 6341 5776 5899 5384 5744
1989 5546 5761 6615 6230 7071 6496 5995 6567 5613 6687 11400 8450
1990 9209 8797 10687 9936 9836 9707 9319 9634 8167 9567 8417 7546
1991 7906 7986 9192 9734 9441 8962 9371 9260 8884 9073 8805 7972
1992 7486 8612 10248 9940 8776 10078 10019 9701 9348 7853 8599 9107
1993 9143 9143 11059 11352 10594 11897 11160 11628 11448 10374 11248 10923
1994 9599 10388 12173 11664 12292 12902 10728 12536 11443 11932 11339 10634
1995 10961 11857 14860 12391 14323 13644 12325 14434 12631 13726 12926 12072
1996 12582 13872 15117 15636 15565 14433 15447 15590 15234 16299 14727 13810
```

| | | | | | | | | | | | | |
|------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 1997 | 14060 | 15229 | 16962 | 17079 | 16460 | 16465 | 16425 | 15363 | 16089 | 16901 | 14367 | 15163 |
| 1998 | 13510 | 15465 | 17624 | 17224 | 15805 | 17862 | 17515 | 16270 | 16331 | 17426 | 16737 | 16897 |
| 1999 | 16002 | 18431 | 22143 | 20957 | 20515 | 24106 | 21999 | 23891 | 22563 | 24753 | 24225 | 24272 |
| 2000 | 23412 | 25815 | 30423 | 25204 | 26986 | 24831 | 21769 | 24626 | 21644 | 22962 | 20340 | 17822 |
| 2001 | 19248 | 18923 | 20732 | 19781 | 20778 | 18551 | 18311 | 19608 | 14770 | 17727 | 15587 | 14181 |
| 2002 | 16273 | 16357 | 18555 | 18946 | 19622 | 17734 | 18359 | 18958 | 17639 | 19111 | 16487 | 16278 |
| 2003 | 16170 | 17077 | 19352 | 19461 | 19563 | 19045 | 19687 | 18614 | 19518 | 20752 | 17857 | 18083 |
| 2004 | 17848 | 20221 | 24380 | 22857 | 20851 | 21971 | 20794 | 21686 | 20329 | 20545 | 20163 | 19727 |
| 2005 | 19852 | 21156 | 25021 | 23106 | 22583 | 23621 | 21283 | 24405 | 21769 | 21419 | 21833 | 20716 |
| 2006 | 21532 | 22810 | 27578 | 22750 | 24386 | 24861 | 21580 | 25319 | 22311 | 24077 | 22658 | 20909 |
| 2007 | 23774 | 24365 | 28236 | 25640 | 27467 | 26565 | 25520 | 27860 | 23629 | 27407 | 24521 | 21614 |
| 2008 | 25011 | 25689 | 27048 | 27577 | 26586 | 24960 | 26086 | 23857 | 24425 | 24124 | 19074 | 19873 |
| 2009 | 19111 | 20849 | 23869 | 23573 | 22337 | 24084 | 23578 | 22625 | 22336 | 23795 | 21248 | 21987 |
| 2010 | 21221 | 23097 | 28105 | 25119 | 23953 | 25080 | 22796 | 23413 | 23092 | 23739 | 23151 | 21794 |
| 2011 | 22686 | 24232 | 30145 | 26896 | 26187 | 27407 | 25024 | 27708 | 25636 | 24807 | 24550 | 23480 |
| 2012 | 24295 | 27305 | 29660 | 27660 | 28521 | 26754 | 26636 | 27794 | 24270 | 27369 | 24447 | 22310 |
| 2013 | 25382 | 26630 | 28633 | 29002 | 29609 | 26566 | 27697 | 28612 | 26811 | 28991 | 25622 | 24625 |
| 2014 | 26543 | 27243 | 30853 | 30752 | 29450 | 28680 | 30393 | 28186 | 30059 | 32019 | 26212 | 26558 |
| 2015 | 27182 | 29681 | 35025 | 33786 | 31123 | 34043 | 33316 | 31327 | 32138 | 32584 | 29909 | 30094 |
| 2016 | 29461 | 32456 | 37317 | 35506 | 34861 | 35211 | 32027 | 35356 | 33293 | 31394 | 31452 | 33063 |

> tsPatents

Jan Feb Mar Apr May Jun Jul Aug Sep Oct Nov Dec

| | | | | | | | | | | | | |
|------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 1977 | | | | | | 5760 | 5898 | 5731 | 6630 | | | |
| 1978 | 5064 | 5439 | 6660 | 5799 | 6487 | 6419 | 5671 | 5831 | 5697 | 6012 | 5756 | 6210 |
| 1979 | 5303 | 5240 | 6131 | 6071 | 6247 | 6087 | 5726 | 5999 | 5541 | 6459 | 5913 | 6399 |
| 1980 | 5492 | 5619 | 6491 | 5971 | 6292 | 6325 | 6077 | 5328 | 6143 | 6246 | 5414 | 6726 |
| 1981 | 4899 | 5351 | 6548 | 5857 | 5583 | 6377 | 5907 | 5539 | 5489 | 5726 | 5602 | 6354 |
| 1982 | 4677 | 5313 | 6793 | 5724 | 5702 | 6543 | 5894 | 5834 | 10870 | 3134 | 4604 | 5692 |
| 1983 | 4698 | 4900 | 6279 | 5451 | 5657 | 6202 | 5390 | 5621 | 5740 | 5648 | 5544 | 6507 |
| 1984 | 4940 | 5557 | 6360 | 6216 | 6391 | 6522 | 6033 | 6230 | 5708 | 6631 | 6319 | 6774 |
| 1985 | 5381 | 6111 | 6664 | 6807 | 6895 | 6257 | 6713 | 6290 | 6814 | 7204 | 6247 | 7251 |
| 1986 | 5730 | 6296 | 7137 | 7026 | 6837 | 6982 | 6994 | 6486 | 7150 | 7544 | 6200 | 7814 |
| 1987 | 5918 | 6753 | 7899 | 7633 | 7027 | 7852 | 7784 | 6968 | 7510 | 7814 | 7539 | 8614 |
| 1988 | 6315 | 7682 | 8992 | 7850 | 8040 | 8737 | 7843 | 8262 | 8551 | 8721 | 8082 | 9321 |
| 1989 | 7616 | 8117 | 9466 | 8379 | 9123 | 9378 | 8133 | 8702 | 8588 | 9083 | 8706 | 9308 |
| 1990 | 8151 | 8472 | 9707 | 9061 | 9197 | 9331 | 8963 | 9269 | 8533 | 10444 | 8128 | 9421 |
| 1991 | 7879 | 8270 | 9363 | 9413 | 9406 | 9080 | 9394 | 8954 | 9327 | 9737 | 9155 | 9704 |
| 1992 | 8237 | 8485 | 10025 | 9591 | 8881 | 10282 | 9822 | 8666 | 10009 | 9586 | 9180 | 10693 |
| 1993 | 8063 | 8728 | 11064 | 9963 | 9246 | 10500 | 9793 | 9531 | 10232 | 9904 | 10034 | 11401 |
| 1994 | 9243 | 9614 | 12007 | 10453 | 10893 | 12276 | 10303 | 11300 | 12499 | 11444 | 11444 | 13760 |
| 1995 | 10445 | 10716 | 13835 | 11945 | 17222 | 28123 | 8860 | 10340 | 11124 | 10913 | 11152 | 12746 |
| 1996 | 10346 | 11112 | 13067 | 12689 | 13000 | 13285 | 13677 | 13195 | 14326 | 14524 | 13319 | 15858 |
| 1997 | 13121 | 13531 | 15641 | 15287 | 15015 | 16286 | 15456 | 14815 | 16658 | 16871 | 14529 | 16885 |
| 1998 | 12722 | 13746 | 16543 | 15062 | 14511 | 16769 | 15660 | 14426 | 15607 | 15603 | 14861 | 18092 |
| 1999 | 12865 | 14204 | 17997 | 15791 | 15337 | 17893 | 15963 | 16308 | 17160 | 16447 | 16702 | 19417 |
| 2000 | 14586 | 16451 | 20021 | 15922 | 17884 | 19641 | 15521 | 18278 | 19273 | 17834 | 18532 | 19122 |
| 2001 | 16493 | 17127 | 21285 | 17692 | 18593 | 20122 | 18202 | 19753 | 17856 | 19239 | 18210 | 19660 |
| 2002 | 17694 | 17281 | 20278 | 18963 | 19891 | 19568 | 19240 | 18754 | 19562 | 20034 | 17799 | 21302 |
| 2003 | 16740 | 16898 | 19990 | 18907 | 18242 | 19748 | 18830 | 17542 | 19825 | 19708 | 16896 | 22063 |
| 2004 | 15528 | 16931 | 21274 | 18212 | 16712 | 20593 | 17779 | 17943 | 19884 | 17521 | 18130 | 22102 |

```

2005 14968 17021 22490 18546 17987 21090 16914 18462 19787 17592 18133 22188
2006 14550 16701 22216 17208 19402 21150 17560 19571 19828 19312 18999 22766
2007 16930 17539 22271 18427 19480 20506 18797 20267 19143 25045 18195 21820
2008 16717 18825 21168 19762 19318 21035 19896 18546 21495 21413 18120 23736
2009 15375 17696 21848 18816 17438 20736 19164 17736 20407 19603 18327 23801
2010 14972 17696 19786 20199 19019 22414 19760 19867 21604 20657 20612 25143
2011 17261 18992 25367 20428 20964 23822 20058 22268 26472 20658 21926 26522
2012 17222 19167 21736 22546 23921 25237 23054 24756 27051 22929 24437 27708
2013 18316 23728 42788 19501 22634 22932 23350 23313 25042 25126 23015 28838
2014 19391 22253 30969 23387 24040 25581 24700 23086 27091 25141 21855 29294
2015 19599 21643 24767 23184 22546 26852 23783 22590 25856 23159 21726 27049
2016 18970 20933 23298 19948 20837 22912 18417 20752 21299 17774 17791 19436

```

```

> plot(decompose(tsTrademarks,type="additive"))
> plot(decompose(tsPatents,type="additive"))

```

#Identify outliers

#Javier López-de-Lacalle (2019). tsoutliers: Detection of Outliers in Time Series. R package version 0.6-8.

<https://CRAN.R-project.org/package=tsoutliers>

```
library(tsoutliers)
```

```
> TrademarksOutliers<-tso(tsTrademarks,types = c("AO","LS","TC"),maxit.iloop=10)
```

```
> PatentsOutliers<-tso(tsPatents,types = c("AO","LS","TC"),maxit.iloop=10)
```

```
> TrademarksOutliers
```

Series: tsTrademarks

Regression with ARIMA(2,1,1)(0,1,2)[12] errors

Coefficients:

```

      ar1  ar2  ma1  sma1  sma2  AO61  LS147  LS262
-1.0107 -0.5826 0.4306 -0.4939 -0.2779 12137.5320 4527.2969 3409.3950
s.e. 0.0834 0.0440 0.1067 0.0480 0.0458 669.2913 681.1637 674.9746
sigma^2 estimated as 868751: log likelihood=-3790.79
AIC=7599.58 AICc=7599.98 BIC=7636.74

```

Outliers:

```

type ind  time coefhat tstat
1 AO 61 1982:09 12138 18.135
2 LS 147 1989:11 4527 6.646
3 LS 262 1999:06 3409 5.051

```

```
> PatentsOutliers
```

Series: tsPatents

Regression with ARIMA(3,0,0)(2,1,2)[12] errors

Coefficients:

```

      ar1  ar2  ar3  sar1  sar2  sma1  sma2  AO61  AO214
      0.2731 0.2776 0.4185 0.7318 -0.3230 -1.4584 0.6303 5591.8986 15515.5416
s.e. 0.0480 0.0438 0.0468 0.1133 0.0673 0.1146 0.0879 773.2661 764.2898
      AO362  AO427
      5058.3040 17555.5353
s.e. 757.5541 799.8064
sigma^2 estimated as 917822: log likelihood=-3812.79
AIC=7649.58 AICc=7650.27 BIC=7699.15

```

Outliers:

```
type ind  time coefhat  tstat
1 AO 61 1982:09  5592 7.232
2 AO 214 1995:06 15516 20.301
3 AO 362 2007:10  5058 6.677
4 AO 427 2013:03 17556 21.950
```

```
> plot(TrademarksOutliers); X11(); plot(PatentsOutliers)
```

```
#Clean/smooth data - replace identified outliers (X) with average of prior (X(t-1)) and posterior (X(t+1))
```

```
> Trademarks<-tsTrademarks; Patents<-tsPatents
> Trademarks[61]= (Trademarks[62]+Trademarks[64]) / 2
> Trademarks[147]= (Trademarks[146]+Trademarks[148]) / 2
> Trademarks[262]= (Trademarks[261]+Trademarks[263]) / 2
> Patents[61]= (Patents[62]+Patents[64]) / 2
> Patents[214]= (Patents[213]+Patents[215]) / 2
> Patents[362]= (Patents[361]+Patents[363]) / 2
> Patents[427]= (Patents[426]+Patents[428]) / 2
```

```
> plot(decompose(Trademarks,type="additive"))
> plot(decompose(Patents,type="additive"))
```

```
> library(moments); citation("moments")
```

```
#Lukasz Komsta and Frederick Novomestky (2015). moments: Moments, cumulants, skewness, kurtosis #and related tests. R package version 0.14. https://CRAN.R-project.org/package=moments
```

```
#Use fitted output from tsoutliers
```

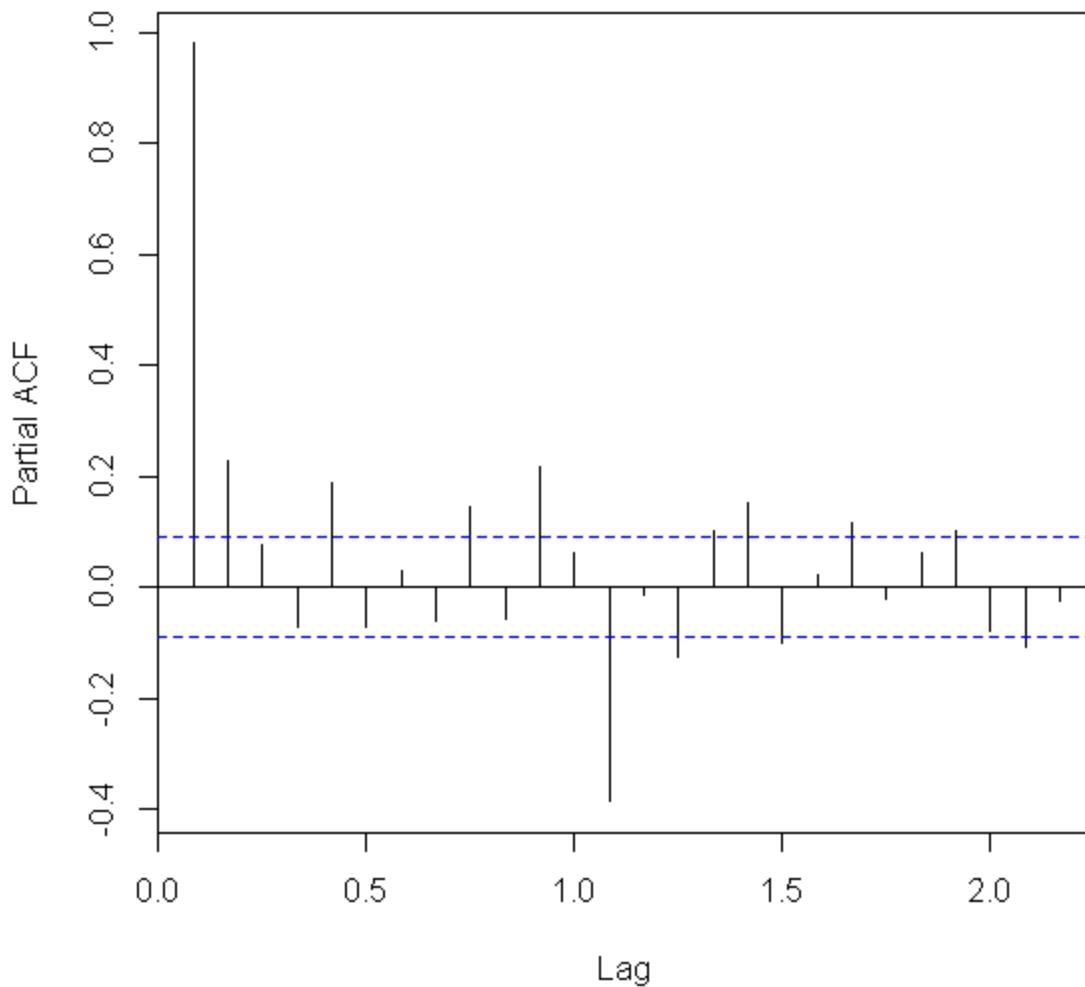
```
> summary(Trademarks); sd(Trademarks); skewness(Trademarks); kurtosis(Trademarks)
Min. 1st Qu. Median Mean 3rd Qu. Max.
1895  5537 15456 15276 23574 37317
[1] 9418.054
[1] 0.2020644
[1] 1.763723
> summary(Patents); sd(Patents); skewness(Patents); kurtosis(Patents)
Min. 1st Qu. Median Mean 3rd Qu. Max.
3134  7598 14468 13930 19422 30969
[1] 6523.347
[1] 0.1854975
[1] 1.757654
```

```
#note: skew/kurtosis comparative - no need to transform
```

```
#auto/serial correlation
```

```
acf(Trademarks);pacf(Trademarks)
```


Series Trademarks



```
acf(Patents);pacf(Patents)
```

```
#Perform normality, stationarity, seasonality, long-memory, and non-linearity tests  
> #normality test
```

```
> library(nortest);citation("nortest")
```

To cite package 'nortest' in publications use: Juergen Gross and Uwe Ligges (2015). nortest: Tests for Normality. R package version 1.0-4. <https://CRAN.R-project.org/package=nortest>

```
> ad.test(Trademarks) #null normality
```

```
Anderson-Darling normality test  
data: Trademarks  
A = 11.055, p-value < 2.2e-16
```

```
> ad.test(Patents) #null normality
```

Anderson-Darling normality test

data: Patents

A = 13.102, p-value < 2.2e-16

```
> cvm.test(Trademarks)
```

Cramer-von Mises normality test

data: Trademarks

W = 1.7112, p-value = 7.37e-10

Warning message:

In cvm.test(Trademarks) : p-value is smaller than 7.37e-10, cannot be computed more accurately

```
> cvm.test(Patents)
```

Cramer-von Mises normality test

data: Patents

W = 2.2197, p-value = 7.37e-10

Warning message: In cvm.test(Patents) : p-value is smaller than 7.37e-10, cannot be computed more accurately

```
> #stationary test
```

```
>library(forecast)
```

```
>ndiffs(Trademarks, test= "kpss"); ndiffs(Trademarks, test= "adf"); ndiffs(Trademarks, test= "pp")
```

```
[1] 1
```

```
[1] 1
```

```
[1] 1
```

```
>ndiffs(Patents, test= "kpss"); ndiffs(Patents, test= "adf"); ndiffs(Patents, test= "pp")
```

```
[1] 1
```

```
[1] 1
```

```
[1] 1
```

```
> library(aTSA)
```

Attaching package: 'aTSA'

The following object is masked from 'package:forecast': forecast

The following object is masked from 'package:graphics': identify

```
> citation("aTSA")
```

To cite package 'aTSA' in publications use: Debin Qiu (2015). aTSA: Alternative Time Series Analysis. R package version 3.1.2. <https://CRAN.R-project.org/package=aTSA>

```
> stationary.test(Trademarks, method="kpss")
```

KPSS Unit Root Test alternative: nonstationary

Type 1: no drift no trend

lag stat p.value

5 0.59 0.1

Type 2: with drift no trend

lag stat p.value

5 0.751 0.01

Type 1: with drift and trend

lag stat p.value

5 0.181 0.023

Note: p.value = 0.01 means p.value \leq 0.01: p.value = 0.10 means p.value \geq 0.10

> stationary.test(Patents, method="kpss")

KPSS Unit Root Test alternative: nonstationary

Type 1: no drift no trend

lag stat p.value

5 5.88 0.01

Type 2: with drift no trend

lag stat p.value

5 5.53 0.01

Type 1: with drift and trend

lag stat p.value

5 0.524 0.01

Note: p.value = 0.01 means p.value \leq 0.01: p.value = 0.10 means p.value \geq 0.10

#Now the first diffs

stationary.test(diff(Trademarks), method="kpss")

KPSS Unit Root Test alternative: nonstationary

Type 1: no drift no trend

lag stat p.value

5 0.642 0.1

Type 2: with drift no trend

lag stat p.value

5 0.0294 0.1

Type 1: with drift and trend

lag stat p.value

5 0.015 0.1

Note: p.value = 0.01 means p.value \leq 0.01: p.value = 0.10 means p.value \geq 0.10

```
stationary.test(diff(Patents), method="kpss")
```

```
KPSS Unit Root Test
```

```
alternative: nonstationary
```

```
Type 1: no drift no trend
```

```
lag stat p.value
```

```
5 0.799 0.1
```

```
-----
```

```
Type 2: with drift no trend
```

```
lag stat p.value
```

```
5 0.0858 0.1
```

```
-----
```

```
Type 1: with drift and trend
```

```
lag stat p.value
```

```
5 0.0716 0.1
```

```
-----
```

```
Note: p.value = 0.01 means p.value <= 0.01 : p.value = 0.10 means p.value >= 0.10
```

```
#Long memory
```

```
> library(LongMemoryTS); citation("LongMemoryTS")
```

```
To cite package 'LongMemoryTS' in publications use: Christian Leschinski (2019).
```

```
LongMemoryTS: Long Memory Time Series. R package version 0.1.0.
```

```
https://CRAN.R-project.org/package=LongMemoryTS
```

```
#Qu Test
```

```
> Qu.test(diff(Trademarks),m)
```

```
$W.stat [1] 1.432811
```

```
$CriticalValues
```

```
eps=.02 eps=.05
```

```
alpha=.1 1.118 1.022
```

```
alpha=.05 1.252 1.155
```

```
alpha=.025 1.374 1.277
```

```
alpha=.01 1.517 1.426
```

```
> Qu.test(diff(Patents),m)
```

```
$W.stat [1] 2.334971
```

```
$CriticalValues
```

```
eps=.02 eps=.05
```

```
alpha=.1 1.118 1.022
```

```
alpha=.05 1.252 1.155
```

```
alpha=.025 1.374 1.277
```

```
alpha=.01 1.517 1.426
```

```
> #multivariate local Whittle Score
```

```
> MLWS(diff(Trademarks), m=m)
```

```
$B
```

```
      [,1]
[1,] 1
```

```
$d [1] -0.3782234
```

```
$W.stat [1] 1.502709
```

```
$CriticalValues
```

```
alpha=.1 alpha=.05 alpha=.025 alpha=.01
1.118 1.252 1.374 1.517
```

```
> MLWS(diff(Patents), m=m)
```

```
$B
```

```
      [,1]
[1,] 1
```

```
$d [1] -0.3440901
```

```
$W.stat [1] 2.43958
```

```
$CriticalValues
```

```
alpha=.1 alpha=.05 alpha=.025 alpha=.01
1.118 1.252 1.374 1.517
```

```
> #seasonality tests
```

```
> library(seastests); citation("seastests")
```

To cite package 'seastests' in publications use: Daniel Ollech (2019). seastests: Seasonality Tests. R package version 0.14.2. <https://CRAN.R-project.org/package=seastests>

```
> summary(wo(Trademarks))
```

```
Test used: WO
Test statistic: 1
P-value: 0 0 0
```

The WO - test identifies seasonality

```
> summary(wo(Patents))
```

```
Test used: WO
Test statistic: 1
P-value: 0 0 0
```

The WO - test identifies seasonality

```
> isSeasonal(Trademarks,"qs"); isSeasonal(Trademarks,"fried");
isSeasonal(Trademarks,"welch");
```

```
[1] TRUE
[1] TRUE
[1] TRUE
```

```
> isSeasonal(Patents,"qs"); isSeasonal(Patents,"fried"); isSeasonal(Patents,"welch");
```

```
[1] TRUE
[1] TRUE
[1] TRUE
```

#Nonlinearity Tests

```
> library(nonlinearTseries); citation("nonlinearTseries")
```

Attaching package: 'nonlinearTseries' The following object is masked from 'package:aTSA':
estimate

The following object is masked from 'package:grDevices': contourLines

To cite package 'nonlinearTseries' in publications use: Constantino A. Garcia (2020).
nonlinearTseries: Nonlinear Time Series Analysis. R package version 0.2.10.
<https://CRAN.R-project.org/package=nonlinearTseries>

```
> nonlinearityTest(Trademarks)
```

```
** Teraesvirta's neural network test **
Null hypothesis: Linearity in "mean"
X-squared = 13.65589 df = 2 p-value = 0.001083081
```

```
** White neural network test **
Null hypothesis: Linearity in "mean"
X-squared = 14.43927 df = 2 p-value = 0.00073207
```

```
** Keenan's one-degree test for nonlinearity **
Null hypothesis: The time series follows some AR process
F-stat = 0.04386686 p-value = 0.8342031
```

```
** McLeod-Li test **
Null hypothesis: The time series follows some ARIMA process
Maximum p-value = 0
```

```
** Tsay's Test for nonlinearity **
Null hypothesis: The time series follows some AR process
F-stat = 3.041166 p-value = 8.100652e-10
```

```
** Likelihood ratio test for threshold nonlinearity **
Null hypothesis: The time series follows some AR process
Alternative hypothesis: The time series follows some TAR process
X-squared = 51.55873 p-value = 0.03278644
```

```
> nonlinearityTest(Patents)
```

```
** Teraesvirta's neural network test **
```

Null hypothesis: Linearity in "mean"
X-squared = 110.8611 df = 2 p-value = 0

** White neural network test **
Null hypothesis: Linearity in "mean"
X-squared = 108.337 df = 2 p-value = 0

** Keenan's one-degree test for nonlinearity **
Null hypothesis: The time series follows some AR process
F-stat = 3.379455 p-value = 0.06672325

** McLeod-Li test **
Null hypothesis: The time series follows some ARIMA process
Maximum p-value = 0

** Tsay's Test for nonlinearity **
Null hypothesis: The time series follows some AR process
F-stat = 6.296795 p-value = 1.162674e-15

** Likelihood ratio test for threshold nonlinearity **
Null hypothesis: The time series follows some AR process
Alternative hypothesis: The time series follows some TAR process
X-squared = 78.45917 p-value = 2.60603e-05

```
> library(tsfeatures); citation("tsfeatures")
```

To cite package 'tsfeatures' in publications use: Rob Hyndman, Yanfei Kang, Pablo Montero-Manso, Thiyanga Talagala, Earo Wang, Yangzhuoran Yang and Mitchell O'Hara-Wild (2020). tsfeatures: Time Series Feature Extraction. R package version 1.0.2. <https://CRAN.R-project.org/package=tsfeatures>

```
hurstTrademarks=0;hurstPatents=0
```

```
endT<-length(Trademarks); endP<-length(Patents)
```

```
for (i in 1:endT) { hurstTrademarks[i] <- hurst (Trademarks[1:(1+i*1)]) }
```

```
for (i in 1:endP) { hurstPatents[i] <- hurst (Patents[1:(1+i*1)]) }
```

```
hurstTrademarks<-ts(hurstTrademarks,start=c(1977,9),end=c(2016,12), frequency=12)
```

```
hurstPatents<-ts(hurstPatents,start=c(1977,9),end=c(2016,12), frequency=12)  
plot(hurstTrademarks); plot(hurstPatents)
```

```
> library(tseries);citation("tseries")
```

'tseries' version: 0.10-47 'tseries' is a package for time series analysis and computational finance. See 'library(help="tseries")' for details.

Attaching package: 'tseries' The following objects are masked from 'package:aTSA': adf.test, kpss.test, pp.test

To cite in publications use:

Adrian Trapletti and Kurt Hornik (2019). tseries: Time Series Analysis and Computational Finance. R package version 0.10-47.

```
plot(ts.intersect(Patents, hurstPatents),main="", yax.flip=TRUE)
```

```
plot(ts.intersect(Trademarks, hurstTrademarks),main="", yax.flip=TRUE)
```

```
plot(ts.intersect(hurstPatents, hurstTrademarks),main="", yax.flip=TRUE)
```

```
##### End #####
```