Liquidation, bailout, and bail-in: Insolvency resolution mechanisms and bank lending*

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Abstract

We present a dynamic, continuous-time model in which risk averse inside equityholders set a bank's lending, payout, and financing policies, and the exposure of bank assets to crashes. We examine whether bailouts encourage excessive lending and risk-taking compared to liquidation or bail-ins with debt-to-equity conversion or debt write-downs. The effects of the prevailing insolvency resolution mechanism (IRM) on the probability of insolvency, loss in default, and the bank's value suggest no single IRM is a panacea. We show how a bailout fund financed through a tax on bank dividends resolves bailouts without public money and without distorting insiders' incentives.

Keywords: liquidation, bailout, bail-in, asset sale, agency
JEL: G33, H81, G34, G32

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1 Introduction

The 2008-2010 financial crisis and the COVID-19 pandemic have highlighted the importance of an orderly insolvency resolution mechanism (hereafter IRM). Liquidation, bailout and bail-in are currently the three main IRMs. Under liquidation a bankruptcy court administers the wind-down of the firm and distributes the liquidation proceeds to its creditors. Under a bailout, the government prevents a failing firm from collapsing by injecting public money in exchange for full or partial ownership. Examples of industries that received bailouts include the financial services industry, airlines, railways, car and plane manufacturers.

In response to the criticisms on the bank bailouts during the financial crisis, regulators on both sides of the Atlantic devised new regulatory frameworks that attempt to minimize the use of public money to recapitalize failing banks. In this context the bail-in tool is probably the most important regulatory innovation.[1] Bail-ins do not rely on external funding but rescue a failing firm through an internal recapitalization. Bail-in is a statutory power in the hands of resolution authorities that permits them to write down part of the bank's liabilities or to convert the bank's liabilities into equity in order to preserve the bank as a going concern. Bail-ins came first under the spotlight in 2013 during Cyprus's banking crisis when the Bank of Cyprus converted an estimated 47.5% of uninsured deposits into full voting shares.

Existing papers on IRMs typically focus on one single type of IRM (usually liquidation or bailout). We lack papers that compare how firms behave and fare differently under the three IRMs. Important questions remain therefore unanswered. For example, conventional wisdom might predict that the guarantees and safety net provided by bailouts encourage excessive lending and risk-taking compared to liquidations and bail-ins. Is this indeed the case? How do different IRMs affect payout, leverage, the loss in default, insolvency rates and the net value created by a bank? A comparative

[1] Another regulatory innovation is the Orderly Liquidation Authority (OLA), which authorizes the FDIC (instead of a bankruptcy court) to administer swift wind-downs of systemically important financial institutions. An OLA may involve a bail-in allowing the recapitalized bank to return to private hands under new management.

analysis of this type is missing. Prior studies have also ignored the role of IRMs for payout despite the fact that regulatory restrictions on bank payout during the financial and COVID-19 crises highlight the importance of payout policy for stress testing.

This paper develops a unifying, dynamic model for IRMs that addresses the above questions from a microprudential perspective, i.e. we study the effects of insolvency regulation on an individual bank. Our paper focuses on insolvency risk. Liquidity risk does not arise in our model (see Hugonnier and Morellec (2017) for a model with liquidity requirements). Our model can be applied to banks, mortgage servicers, large levered funds, investment banks, insurers, financial market infrastructures (e.g. clearing houses and stock exchanges), and other entities that are candidates for the IRMs described in this paper.

Our paper explores how the three IRMs affect the payout rate, as well as the quantity and quality of loans when these three decision variables are set by risk averse inside equityholders. Insiders can invest in risky assets (loans) of which the return follows a jump diffusion process. The diffusion component reflects continuous shocks to loan returns, whereas the jumps correspond to rare, negative shocks (hereafter referred to as "crashes"). Crashes arrive according to an exogenous Poisson process, but the fraction of the assets that is destroyed by the crash (i.e. the crash risk exposure, a proxy for "loan quality") is a decision variable under insiders' control (e.g. through collateral requirements). Assets with a higher exposure to crash risk carry a higher expected return.

We show that insiders follow an optimal, constant asset to net worth ratio through dynamic rebalancing. In good times, banks issue additional loans financed by debt and retained earnings. In downturns or following a crash, banks with low exposure remain solvent and rebalance by selling assets and using the proceeds to pay down debt. However, banks with assets that are (too) highly exposed to crashes become insolvent. Whether insiders choose to put the bank at risk of insolvency depends on the reward for taking on crash risk. A high reward not only encourages banks to lend a lot but also to issue loans that become impaired in crashes (e.g. subprime mortgages). We show that incentives to put the bank at risk of insolvency are strongest (weakest) under the bailout (liquidation) regime because the cost of insolvency to insiders is relatively low (high). Banks are therefore most (least) likely to default under the bailout (liquidation) regime, in line with conventional wisdom.

Our findings regarding loan quality turn some of the conventional wisdom on its head, however. Insiders' limited liability in the liquidation regime creates incentives to take on as much crash risk as possible by issuing low quality loans that give the bank a higher return in good times. However, when these loans default in a crash, low recovery rates push the bank deep into insolvency. Although loan volume (and therefore leverage) remains relatively low under the liquidation regime due to the high risk-adjusted cost of borrowing, credit spreads are highest and the loss in default is most severe under the liquidation regime because insiders do not care whether the bank is a little or a lot insolvent in liquidation. Moreover, payout to inside and outside equityholders is high because insiders want to milk the bank before a crash arrives and the "music stops".

With bailouts and bail-ins, on the other hand, banks remain a going concern, and managers retain "skin in the game" even in retirement.[2] This mitigates insiders' incentives to take risks that crystallize in crashes and impair the bank's assets. This leads to the lowest loan issuance, lowest leverage, and lowest crash risk exposure under the bail-in regime. With bailouts, the implicit government guarantee keeps the bank's cost of borrowing and credit spread artificially low, causing lending activity and leverage to be highest. The payout rate to equityholders is lowest under the bailout regime because

[2]Following the bailout of the Royal Bank of Scotland, its CEO Sir Fred Goodwin retired in 2008 at the age of 50 with a pension entitlement of £693,000 per year. Had the government not stopped RBS from going bankrupt, Sir Fred would have received a yearly pension of £28,000 from the pension-protection fund, starting at age 65.

insiders are happy to reinvest profits to stimulate long term growth. This provides an argument in favor of bailouts that hitherto has not been made. Our model allows us to disentangle how each IRM affects the likelihood of default and loss in default. We show that a shift from one IRM (liquidation) to another (bailout) can increase the default probability, while reducing the loss in default. We believe these to be new findings with important policy implications.

The net value created by an individual bank is by far highest under the bailout regime. Importantly, by distributing a fraction of all bank dividends into a bailout fund through a dividend tax, it is possible to cover expected bailout costs without public money provided that banking (net of recapitalization costs) is a positive NPV activity. We therefore believe that "pre-funded" bailouts could be a viable way of rescuing insolvent banks.[3] Furthermore, a bailout fund financed by a tax on dividends does not alter equityholders' incentives (unlike deposit guarantee schemes). We are not aware of another paper that demonstrates how bailouts can be funded from taxes on dividends.

The previous discussion and our paper more generally focus mainly on bail-ins with debt-to-equity conversion. Debt write-down is an alternative loss absorption mechanism in bail-ins. In recent years the use of principal write-down (PWD) bonds has been on the rise. In a debt write-down the debt principal is reduced and bondholders do not receive any equity in return. We briefly consider debt write-downs bail-ins in section 6 and compare them with bail-ins with debt-to-equity conversion. We find that debt write-downs generate higher leverage, higher crash risk exposure, and lower insider claim values than equity-conversion bail-ins. Lenders' recovery rate and the cost of borrowing are less sensitive to changes in crash risk exposure and leverage for debt write-downs than for equity-conversion bail-ins. With debt write-downs unsecured

[3] It is not optimal for insiders in our model to issue new equity against assets in place. We refer to Bolton, Chen and Wang (2013) for a dynamic framework where firms have a motive for equity issues. Their model predicts cuts in investment and payouts in bad times and equity issues in good times.

creditors take the hit in a crash, which distorts inside equityholders' ex post incentives.

Our dynamic, continuous-time, open-horizon model captures optimal balance sheet rebalancing and recapitalizations for banks facing continuous diffusion risk and rare jump (crash) risk. To boost returns, insiders load up on ex ante risks that materialize ex post for states in which the bank will be insolvent and managers ousted. Our model delivers tractable analytical results as well as quantitative comparative statics, allowing clear comparisons across IRMs. We are unable to achieve all of this with a static two-date model.

Our paper belongs to a growing strand of dynamic, continuous-time models of banks' optimal investment and financing policies. Sundaresan and Wang (2016) adapt the framework of Leland (1994) to analyze banks' financing decisions and the effects of deposit insurance and regulatory closure on bank liability structure. Hugonnier and Morellec (2017) develop a dynamic model of banking to study the effect of liquidity and leverage requirements on the likelihood and the magnitude of bank losses in default. Gornall and Strebulaev (2018) develop a model of the joint capital structure decisions of banks and their borrowers. The interaction between banks and borrowers explains the high leverage of banks and the low leverage of firms. Vissers (2020) analyses optimal bank capital structure and the effects of deposit insurance and capital requirements in the presence of tail risk. Unlike these previous papers, we focus on endogenizing the bank's investments, payout and the riskiness of its loans. We study how different IRMs affect bank value and insiders' incentives. A complementary working paper by Berger et al. (2018) examines a regulator's optimal intervention strategy and how IRMs affect the capitalization decisions and the bank's net market value, keeping investment, payout and the riskiness of assets exogenously given. Our analysis takes the regulatory IRM as given, and examines how it affects managerial risk taking, bank lending and payout. We show that outcomes may be significantly different depending on whether or not managers control asset risk.

Earlier dynamic banking models include Merton (1977, 1978), Fries, Mella-Barral, and Perraudin (1997), Bhattacharya, Plank, Strobl, and Zechner (2002), and Décamps, Rochet and Roger (2004). These papers treat the banks' asset and liability structure as exogenous. Unlike our model, all the above papers assume risk neutral agents. We show that risk aversion dramatically reduces leverage and risk taking. For coefficients of relative risk aversion above 1 (with 1 corresponding to log utility), insiders avoid insolvency altogether and adopt safe debt. In our model high leverage results from low managerial risk aversion, high tax benefits from debt, or cheap financing because of government guarantees.

We adopt the standard assumption that losses are exogenously allocated across stakeholders. Complementary papers endogenize the sharing rule and negotiation between claimants for specific IRMs. Early examples of such continuous-time models include Anderson and Sundaresan (1996), Mella-Barral and Perraudin (1997), Mella-Barral (1999) and François and Morellec (2004). More recently, Antill and Grenadier (2019) model a firm's optimal capital structure decision in a dynamic bargaining framework in which it may later choose to enter either Chapter 11 reorganization or Chapter 7 liquidation. Colliard and Gromb (2018) model within a static framework a distressed bank's shareholders and creditors negotiating a restructuring given asymmetric information about asset quality and externalities onto the government.

Our paper is also related to research on contingent convertibles (CoCos). CoCos, like bail-in debt, are another example of contingent capital. However, as pointed out by Chen et al. (2017), CoCos provide "going concern" contingent capital (meaning that they are debt contracts designed to convert into equity well before a bank would otherwise default), whereas bail-in debt is "gone concern" contingent capital (i.e. bail-in debt converts when the firm is no longer viable). Furthermore, CoCos are financial instruments in which the trigger event and the conversion rate are identified in advance in the debt contract. For recent dynamic models of CoCos we refer to Sundaresan and

Wang (2015) and Chen et al. (2017). The latter paper also studies the effect of tail risk on equityholders incentives.

There is a large literature on bank leverage, bank capital requirements, deposit insurance and their role for bank risk taking going back to seminal papers by Gorton and Pennacchi (1990), and Dewatripont and Tirole (1994) (see Santos (2001) and VanHoose (2007), and Allen, Carletti and Leonello (2011) for a review). More recently, De Nicolo, Gamba, and Lucchetta (2014) study the effect of microprudential bank regulations on bank lending and value metrics of efficiency and welfare. Bianchi (2016) shows that the anticipation of bailouts leads to an increase in risk-taking. Moral hazard effects are limited if bailouts are systemic and broad-based (rather than idiosyncratic and targeted). Keister (2016) considers the pros and cons of bailouts with limited commitment. Caliendo et al. (2018) provide a static model of a self-financed bailout program financed out of taxes on households (not on bank dividends). Cordella et al. (2018) present a model of bank risk taking and government guarantees. In Davila and Walther (2020) large banks anticipate that their actions affect the government's bailout response and therefore take on more leverage than small banks. Thanassoulis and Tanaka (2018) study how the structure of bank executive compensation affects risk taking when banks might be too big to fail. Della Seta et al. (2020) challenge the view that short-term debt curbs moral hazard. Finally, recent papers that model the optimal design of bank resolution or restructuring include Acharya and Yorulmazer (2008), Lucchetta, Parigi, and Rochet (2018) and Walther and White (2020)).

2 Model setup

Consider a financial firm (hereafter referred to as "bank") such as a bank, large levered fund, or investment bank outside the regulated banking industry that is a candidate for the IRMs studied in this paper. The bank invests in risky assets (e.g. loans) that generate an after-tax rate of return given by the following jump-diffusion process:

$$\frac{dP_t}{P_t} = [(\mu' + \kappa'\lambda f)dt + \sigma' dB_t](1-\tau) - f dy_t \equiv (\mu + \kappa\lambda f)dt + \sigma dB_t - f dy_t \quad (1)$$

where P_t is the value of a unit of loan, B_t is a Brownian motion and y_t is a pure Poisson jump process with intensity λ and $E[dy_t] = \lambda dt$. The parameter τ is the corporate tax rate (with $0 \leq \tau < 1$).[4] The other parameters satisfy the conditions $\mu, \lambda, \sigma > 0$ and $0 \leq f \leq 1 < \kappa$. Hence, most of the time the after-tax return follows a continuous diffusion process with drift $\mu + \lambda\kappa f$ and volatility σ, but occasionally the loan portfolio is subject to a large negative shock (i.e. crash). f depends on the quality of the loans issued. The bank optimally sets f, which can be controlled through collateral requirements (highly secured loans have a low f, whereas loans with poor collateral generate a high f). κ is an exogenously given parameter that determines the risk premium associated with crash risk. This premium also captures any tax deductible provisions for loan losses. The expected return is given by: $E\left[\frac{dP_t}{P_t}\right] = (\mu + \lambda f(\kappa - 1))\, dt > \mu dt$. We assume $\kappa > 1$, which means that banks are compensated with a higher expected return for issuing loans with a higher exposure to crash risk (i.e. higher f).[5] κ reflects internal determinants such as firm size and bank capital, and external determinants such as industry competition. Empirical evidence (e.g. Berger et al. (1987)) suggests that bank profitability increases in size but may be subject to decreasing returns to scale. Size has shown to be closely related to capital adequacy since large banks tend to raise less expensive capital and hence appear more profitable (Short (1979)). Profitability is also positively related to industry concentration (Bourke (1989)).[6]

[4] Our model does not rely on the presence of taxes. Corporate taxes help us generate more realistic numbers for the comparative statics regarding leverage.

[5] We remain agnostic as to whether crash risk is systematic (e.g. economic downturns) or idiosyncratic (e.g. fraud or operational risk) in nature. In the latter case the risk premium could represent payouts from relationship banking relative to transaction or capital market lending (see Boot and Thakor (2000)).

[6] Internet appendix 3.4 shows how we can explicitly model the dependency of κ on the above determinants by adopting the following expression: $\kappa \equiv \kappa_0\, (A_t/N_t)^\theta$ where κ_0 represents a measure of the Herfindahl index and $\theta < 1$ reflects decreasing returns to scale.

At each instant in time t, given the amount of equity capital N_t in place, risk averse inside equityholders (which includes managers) decide the asset to equity capital ratio l_t that determines the bank's total amount of assets $A_t \equiv l_t N_t$. Once the investment policy (and therefore A_t) is set, total net debt $D_t \equiv (l_t - 1)N_t$ follows immediately as a residual from the balance sheet equation $A_t = N_t + D_t$.[7] As pointed out by Sundaresan and Wang (2016) banks share many common features with non-financial corporations, but are different in that they finance their assets not only with market debt, but also with deposits that are protected or insured against bank failure. In what follows the senior, secured debt (deposits) and junior market debt are denoted by D_{1t} and D_{2t}, respectively, with $D_t \equiv D_{1t} + D_{2t}$. We assume that senior debt is fully secured by the bank's assets in a crash, i.e. $D_{1t} \leq (1-f)A_t$. This upper limit in deposits ensures that the bank is not subject to a bank run when a crash occurs. We assume that the bank can continuously rebalance the outstanding amount of debt by accessing wholesale funding markets (e.g. through issuance of short-term commercial paper or through the interbank lending market). This reflects the short term nature of most bank financing, setting banks apart from non-financial firms that are mostly funded by long term debt. Interest on debt ρ_t is a tax deductible expense, which equals the after-tax risk-free rate ρ if the debt is safe. A higher, risk-adjusted rate of interest (to be derived) is paid on risky debt. The weighted average after-tax cost of debt under IRM_j is denoted by ρ_{jt}. Finally, there is a liquid secondary market for the bank's assets (loans) (in contrast to the market for assets of bricks and mortar firms). We assume therefore that the bank can rebalance in a frictionless fashion its asset portfolio. Our model could be extended to allow for illiquidity that arises in crashes by introducing proportional transaction costs on asset sales (see internet appendix 3.3 and footnote 19).

Assume that outsiders and insiders own, respectively, a fraction α and $1-\alpha$ of the

[7] We assume there are no regulatory constraints on insiders' choice for l_t. We show in internet appendix 3.1 that capital requirements reduce the optimal investment and risk exposure in a fairly trivial fashion without affecting the ranking of optimal policies across IRMs.

firm's equity.[8] Hence, if q_t denotes the payout yield to insiders, then the combined payout yield to outsiders and insiders equals $(1 + \alpha/(1-\alpha))q_t \equiv mq_t$, where q_t is optimally set by insiders.[9] In summary, the managerial policy can be denoted by the tuple (l_t, q_t, f).

We will show that, under the optimal investment and payout policies, net worth N_t follows a continuous diffusion process in the absence of crashes. A crash causes, however, a discrete fall in the bank's net worth giving rise to two possible scenarios. Under scenario 1, the bank has a strictly positive net worth position following the shock, and optimally delevers by executing *asset sales* and using the proceeds to reduce outstanding debt. Under scenario 2, the bank's equity capital is wiped out and the bank is insolvent.

Consider first the scenario where net worth remains positive following a crash. If N_t and A_t are, respectively, the bank's net worth and risky assets before the shock then net worth immediately after the shock, N_t^+, is given by:

$$N_t^+ = N_t - f\,A_t = N_t\left(1 - f\frac{A_t}{N_t}\right) = N_t\,(1 - fl_t) \equiv N_t\Phi_s(l_t, f) \qquad (2)$$

Variables under the safe (i.e. solvency) regime are denoted by a subscript s. The bank's net worth remains non-negative if and only if $\Phi_s(l_t, f) \geq 0 \iff l_t \leq \frac{1}{f} \equiv \hat{l}$. Return on equity is given by return on the risky loans minus interest on debt and total payout. The process for the bank's net worth under the positive net worth condition

[8]The distinction between outside (α) and inside ($1-\alpha$) equityholders makes explicit that insiders' risk preferences matter for corporate decision making. Unlike outsiders, who are more diversified, insiders' fortunes are very much tied to the firm. Insiders' risk aversion generates an interior optimum for the bank's financial policies. The parameter α also matters for the design of a bailout fund financed by a tax on bank dividends to outside equityholders (see section 5).

[9]Insiders' payout, q_tN_t, could be interpreted more broadly than dividends to include any pecuniary payouts over and above what they could get outside the firm (see Lambrecht and Myers (2017)).

is:

$$
\begin{aligned}
dN_t &= A_t \frac{dP_t}{P_t} - \rho D_t dt - mq_t N_t dt = \frac{l_t N_t}{P_t} dP_t - \rho(l_t - 1) N_t dt - mq_t N_t dt \text{ , or :} \\
\frac{dN_t}{N_t} &= \left[(\mu + \lambda\kappa f - \rho)\, l_t + \rho - mq_t \right] dt + \sigma l_t dB_t + (\Phi_s(l_t, f) - 1)\, dy_t \\
&\equiv g_s(l_t, q_t, f)\, dt + \sigma l_t dB_t + (\Phi_s(l_t, f) - 1)\ dy_t \quad \text{for} \quad l_t \leq \frac{1}{f} \qquad (3)
\end{aligned}
$$

Consider next the scenario where the drop in the bank's risky assets during a crash exceeds its net worth, i.e. $fA_t > N_t$ (or $l_t > \hat{l}$) such that net worth is entirely wiped out. The bank is recapitalized under a bailout or bail-in but not in liquidation. Let $\Phi_j(l_t, f)$ be the adjustment of net worth under IRM j (indices referring to the liquidation -i.e. bankruptcy- bailout and bail-in regimes are denoted, respectively, by b, o and i). Bank debt may no longer be safe and the cost of debt becomes a function of the firm's gearing ratio and loan quality, i.e. $\rho_j(l_t, f)$ where $j \in b, o, i$. Similar to (3), the process for net worth is given by:

$$
\begin{aligned}
\frac{dN_t}{N_t} &= \left[(\mu + \lambda\kappa f - \rho_j(l_t, f))\, l_t + \rho_j(l_t, f) - mq_t \right] dt + \sigma l_t dB_t + (\Phi_j(l_t, f) - 1)\, dy_t \\
&\equiv g_j(l_t, q_t, f)\, dt + \sigma l_t dB_t + (\Phi_j(l_t, f) - 1)\ dy_t \quad \text{for} \quad l_t > \frac{1}{f} \qquad (4)
\end{aligned}
$$

We derive the functions for $\rho_j(l_t, f)$ and $\Phi_j(l_t, f)$ in the next section.

2.1 Definitions of the restructuring mechanisms

We will show that a restructuring mechanism j is fully characterized by the following four elements: (1) the adjustment in net worth due to the restructuring, denoted by $\Phi_j(l, f)$, (2) insiders' continuation probability p_j under mechanism j, (3) insiders' net worth recovery rate, $\phi_j(l, f)$, and (4) lenders' recovery rate on the debt, $\Omega_j(l, f)$. We adopt what we believe to be plausible assumptions regarding the specifications for Φ_j, ϕ_j and Ω_j. It should be clear, however, that our framework is sufficiently flexible and general to accommodate different assumptions.

2.1.1 Asset Sales.

Under the asset sales regime, net worth drops by a factor of $\Phi_s(l_t, f) = 1 - fl_t \leq 1$ as in (2) since $fl_t \leq 1$. The bank is always solvent and lenders incur no losses, i.e. $\Omega_s(l_t, f) = 1$. Therefore debt is safe ($\rho_s = \rho$). Managers are sure to continue, i.e. $p_s = 1$, but suffer a fractional loss on their net worth in the bank as reflected by the recovery rate $\phi_s(l_t, f) = 1 - fl_t$.

2.1.2 Liquidation.

Since the insolvent bank is not recapitalized in the liquidation regime, it follows that $\Phi_b(l_t, f) = 0$. There is a proportional bankruptcy cost c_b ($0 \leq c_b \leq 1$) such that lenders receive the value of the assets in liquidation minus the bankruptcy cost, $(1-f)(1-c_b)A_t$, with fully secured depositors having a prior claim over junior lenders. Inside (and outside) equityholders get nothing; therefore, $\phi_b(l_t, f) = 0$ and $p_b = 0$. The recovery rate on junior market debt is therefore $\left[(1-f)(1-c_b)A_t - D_{1t}\right]/D_{2t}$. Assuming the debt is priced competitively by risk neutral lenders, the bank faces the following after-tax interest rate on its debt:

$$
\begin{aligned}
\rho_b(l_t, f) &= \frac{D_{1t}}{D_t}\rho'(1-\tau) + \frac{D_{2t}}{D_t}(1-\tau)\left[\rho' + \lambda\left(1 - \frac{(1-f)(1-c_b)A_t - D_{1t}}{D_{2t}}\right)\right] \\
&= \rho + \lambda(1-\tau)\left(\frac{D_t - (1-f)(1-c_b)l_t N_t}{D_t}\right) \\
&= \rho + \lambda(1-\tau)\left(\frac{fl_t - 1 + (1-f)c_b l_t}{l_t - 1}\right) \\
&\equiv \rho + \lambda(1-\tau)\left[1 - \Omega_b(l_t, f)\right] \qquad (5)
\end{aligned}
$$

The amount of debt prior to default is $D_t = A_t - N_t = (l_t - 1)N_t$. Therefore, lenders' recovery rate is $\Omega_b(l_t, f) = (1-f)(1-c_b)A_t/D_t = (1-f)(1-c_b)l_t/(l_t-1)$. The bank's weighted average cost of borrowing $\rho_b(l_t, f)$ depends on the total amount of debt (D_t) but not on the split between senior (D_{1t}) and junior (D_{2t}) debt. The cost of borrowing, $\rho_b(l_t, f)$, increases in the bank's gearing (l_t) and crash risk exposure (f), and feeds back into insiders' optimization problem when they set l_t and f (see Section 2.2).

2.1.3 Bailout.

Following a crash, assets drop from A_t to $(1-f)A_t$. We assume that under the bailout regime, the government recapitalizes the entire bank which now has assets amounting to $(1-f)A_t$.[10] If the optimal asset to net worth ratio l_o is constant (which we prove below) then the bank's net worth before and after the bailout are, respectively, A_t/l_o and $(1-f)A_t/l_o$. It follows that the bank's net worth drops by a factor $1-f$, and therefore $\Phi_o(l_t, f) \equiv 1-f$ in the budget constraint (4).

In return for recapitalizing the bank, the government receives equity alongside existing shareholders whose share is diluted by an exogenously given factor $\xi_o (\leq 1)$. The fraction ξ_o depends on existing outside shareholders' bargaining power (as determined, for instance, by the degree of ownership concentration) and how crucial it is for the government to save the bank to avoid negative externalities for the wider economy.[11] Senior and junior lenders' claims are protected, and therefore $\Omega_o(l_t, f) = 1$. This implies that debt is risk free and therefore $\rho_o(l_t, f) \equiv \rho$.[12] Finally, we assume the government appoints new managers that replace some existing managers and dilute the stake of surviving managers by a factor $\xi_o (\leq 1)$. Hence, insiders' net worth recovery rate conditional upon continuation is $\phi_o(l_t, f) = (1-f)\xi_o$. Managers survive a bailout

[10] Our results are qualitatively the same if only a fraction of the bank is recapitalized, with the remaining assets being liquidated.

[11] Bailouts have been more prevalent and favorable to equityholders of systemically important financial institutions and of firms of strategic national interest (such as railways, airlines, car or plane manufacturers). E.g. during the recent financial crisis the UK government bailed out the Royal Bank of Scotland and Lloyds Banking Group and acquired an equity stake of 72% and 43%, respectively.

[12] Internet appendix 3.2 generalizes the model by assuming that bailouts occur with a probability less than 1, and that liquidation or bail-in are alternatives to a bailout. The model could also be extended to allow for the possibility that unsecured bondholders are not bailed out. The claim of all depositors should be protected, though, to avoid a bank run in the event of a crash.

with probability $p_o \in [0, 1]$. Bunkanwanicha et al. (2019) find that during the financial crisis (2008-2010), the vast majority (77%) of CEOs of firms in receipt of TARP funding remained at the helm of their banks, and were significantly (18%) more likely to retain their jobs compared with their matched non-TARP peers.

2.1.4 Bail-in.

In a bail-in, the claims of the creditors of the failed bank are converted into equity in order to absorb the losses and recapitalize the bank.[13] A bail-in is not negotiated (it is imposed upon the firm and its creditors by the authority responsible for resolution). The bail-in not only significantly changes the ownership structure of the firm but may coincide with restructurings (e.g. splitting up the bank) that alter the bank's investment and payout policy. Unfortunately, it is not known in advance exactly how the resolution authority will restructure the bank. This poses a real challenge for pricing bail-in bonds. We do not attempt to model the restructuring process but take its outcome as exogenously given. In particular, we assume that the optimal asset to net worth ratio after the bail-in is $l^* > 1$. The corresponding market to book value is denoted by w^*, which means that the market value of the total (inside plus outside) equity after the bail-in is given by: $F_t^+ \equiv w^*(1 - f)A_t/l^*$. In what follows we assume $w^* = 1$, which means that the restructuring process does not create (nor destroy) value.[14] Since the price of bail-in bonds depends on the bank's equity value after the bail-in, we need to know l^*. As previously explained, l^* is (exogenously) set by the resolution authority. As with the bailout we assume that just enough debt is converted into equity to achieve the optimal gearing ratio l^* of risky assets to net worth.

[13] A straight debt write-down (i.e. without equity conversion) is another loss absorption mechanism. We discuss bail-ins with debt write-down in section 6 and argue that, at least from a regulatory viewpoint, they are inferior to bail-ins with debt-to-equity conversion.

[14] An earlier version of the paper solves the model with $w^* > 1$ as a free parameter. The results for the comparisons between liquidation, bailout and debt-to-equity conversion bail-in are unaffected.

Assuming that the bank has a constant optimal gearing ratio l_i prior to the bail-in (a claim we verify below) the amount of debt prior to the shock is: $D = A - N = l_i N - N = (l_i - 1)A/l_i$. The bank's assets after the shock are $A^+ = (1-f)A$. Hence, the optimal amount of debt and net worth after the bail-in are, respectively: $D^+ = (l^* - 1)(1-f)A/l^*$ and $N^+ = (1-f)A/l^*$. To enable the bail-in we therefore require that:

$$D_1 = A\left[\frac{(l^*-1)(1-f)}{l^*}\right] \quad \text{and} \quad D_2 = D - D_1 = A\left[\frac{l_i - 1}{l_i} - \frac{(l^*-1)(1-f)}{l^*}\right]$$

D_1, as specified above, is the maximum amount of deposits the bank can adopt while ensuring they are fully protected. As $\rho_i(l_i, f)$ does not depend on the split between senior deposits and junior market debt (see equation (6)), any lower level of deposits combined with a higher level of market debt (such that $D_{1t} + D_{2t} = D_t$) is fine too.

To recapitalize the bank, the bail-in forcibly converts unsecured debt into equity and dilutes existing equityholders. We assume that unsecured lenders and existing equityholders (inside as well as outside) receive, respectively, a fraction $1 - \xi_i$ and ξ_i of the firm's equity. Secured lenders (e.g. depositors) earn the risk-free rate of interest ρ'. Assuming lenders are risk neutral and debt is issued competitively, the after-tax cost of debt is given by:

$$\begin{aligned} \rho_i(l_i, f) &= \frac{D_1}{D}\rho'(1-\tau) + \frac{D_2}{D}(1-\tau)\left\{\rho' + \lambda\left[1 - \frac{(1-\xi_i)(1-f)A/l^*}{D_2}\right]\right\} \\ &= \rho + \lambda(1-\tau)\left[1 - \frac{(1-f)l_i(1+h)}{l_i - 1}\right] \quad \textit{where} \quad h \equiv -\frac{\xi_i}{l^*} \end{aligned} \tag{6}$$

Or equivalently, the recovery rate on the total bank debt is:

$$\Omega_i(l_i, f) = \frac{D_1 + (1-\xi_i)(1-f)A/l^*}{D} = \frac{(1-f)(1+h)l_i}{l_i - 1} \tag{7}$$

Note again that $\rho_i(l_i, f)$ does not depend on the split between junior and senior debt, assuming deposits are fully secured by the assets in liquidation. Bail-ins only make economic sense if the junior debt is risky. This requires that $\rho_i(l_i, f) > \rho$, or equivalently:

$$\rho_i(l_i, f) > \rho \iff h < \frac{fl_i - 1}{(1-f)l_i} \iff l_i > \frac{1}{(1+h)f - h} \equiv \hat{l}_i(f; h) \tag{8}$$

We verify later (see Proposition 4) that junior debt is indeed risky. Next, for bail-ins with debt-to-equity conversion to go through, no creditor or shareholder should be worse off under the bail-in compared to what he or she would get under a hypothetical liquidation scenario (this is the so-called "No Creditor Worse off than under Liquidation" (NCWOL) test of the Bank Recovery and Resolution Directive (BRRD) in the European Union). The payoff to junior creditors in liquidation equals $(1-f)(1-c_b)A - D_1 = (1-f)(1-l^*c_b)A/l^*$, whereas their payoff in a bail-in equals $(1-\xi_i)(1-f)A/l^*$. Consequently, a bail-in is acceptable to junior creditors if and only if $(1-l^*c_b) \leq (1-\xi_i)$, or equivalently if $c_b \geq \xi_i/l^* \equiv -h$. Therefore, the liquidation costs should be sufficiently large for a bail-in to be acceptable. We assume this condition to be satisfied (ξ_i/l^* is small; typically well below 5%).

The BRRD stipulates that management should be replaced following a bail-in. We assume that management receives with probability $p_i \in [0,1]$ a final severance claim $M_i(\phi_i(l,f)N)$ according to which managers' net worth stake is diluted by a factor ξ_i (< 1), i.e. $\phi_i(l,f) \equiv (1-f)\xi_i$ (the precise definition of M_i will be introduced in Section 2.2). The adjustment to net worth following a bail-in is $\Phi_i(l_t,f) \equiv (1-f)l_i/l^*$. Therefore, if the resolution authority reduces leverage ($l_i/l^* > 1$) then the bank's net worth actually increases if $(1-f)l_t/l^* > 1$.[15]

2.1.5 Summary of definitions.

Table 1 below summarizes our assumptions regarding Φ_j, p_j, ϕ_j and Ω_j across the four restructuring mechanisms.

Figure 1 illustrates the effect of each restructuring mechanism on the bank's balance sheet. The figure shows the bank's balance sheet 1) before the crash, 2) immediately

[15] We do not allow managers to freeride on such an increase as their claim relates to the bank's net worth after the shock, but assuming the original gearing is maintained, i.e. $\phi_i \equiv (1-f)\xi_i$ (and not $\phi_i = (1-f)\xi_i l_i/l^*$).

IRM j	$\Phi_j(l,f)$	p_j	$\phi_j(l,f)$	$\Omega_j(l,f)$
Asset sales $j=s$	$1-fl$	$p_s=1$	$1-fl$	1
Liquidation $j=b$	0	$p_b=0$	0	$\frac{(1-f)(1-c_b)l}{l-1}$
Bailout $j=o$	$1-f$	$0\leq p_0\leq 1$	$(1-f)\xi_o$	1
Bail-in $j=i$	$\frac{(1-f)l}{l^*}$	$0\leq p_i\leq 1$	$(1-f)\xi_i$	$\frac{(1-f)(1+h)l}{l-1}$

Table 1: Summary of definitions of different IRMs. Φ_j is the net worth adjustment due to restructuring, p_j is the probability of insiders receiving a continuation or (in the case of bail-in) severance claim, ϕ_j is insiders' net worth recovery rate, Ω_j is lenders' recovery rate on bank debt and ξ_j is the dilution factor of insiders' equity stake.

after the crash, and 3) after the restructuring. Under liquidation all assets are sold off. Notice how also the asset sale regime leads to a significant contraction in the firm's assets, whereas bailouts and bail-ins focus on restructuring the bank's liabilities. Deposits (D_1) are always fully protected.

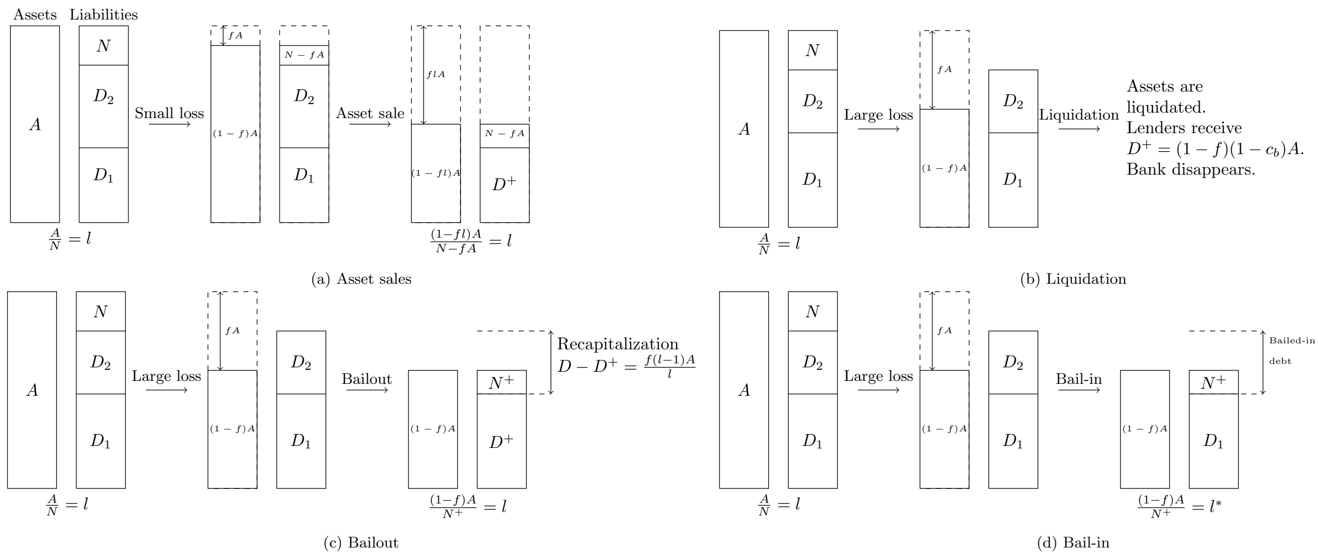

Figure 1: Balance sheet illustrations of each restructuring mechanism. Panel A shows that after a small loss in assets the bank rebalances its capital structure by selling assets and using the proceeds to pay down debt. After a large loss in assets, the bank becomes insolvent. Insolvency is resolved through liquidation (panel B), bailout (panel C) or bail-in (panel D).

2.2 Insiders' optimization problem

We assume insiders have a power utility function (and therefore constant relative risk aversion) with coefficient of risk aversion $\eta \in (0,1)$, i.e. $U(r) = \frac{r^{1-\eta}}{1-\eta}$.[16] Let $\delta > 0$ be the insiders' subjective discount rate. Their objective is to maximize the life-time utility of payouts.

Under the asset sales regime, the bank is always safe and operates perpetually. Managers' claim value is thus defined as:

$$M_s(N) \equiv \max_{q_t, f\in[0,1], l_t \leq 1/f} E\left[\int_0^\infty e^{-\delta t} U(q_t N_t) dt \middle| N_0 = N\right]$$

where the dynamics of N follow (3).

Under the liquidation regime, equityholders are entirely wiped out during a crash and hence payouts are only extracted up to T_1 the random arrival time of the first crash. This leads to the claim value definition:

$$M_b(N) \equiv \max_{q_t, f\in[0,1], l_t > 1/f} E\left[\int_0^{T_1} e^{-\delta s} U(q_t N_t) dt \middle| N_0 = N\right]$$

under net worth dynamics (4). As pointed out before, the net worth dynamics depend on the bank's cost of borrowing $\rho_b(l_t, f)$ which in turn is set by competitive, risk neutral lenders who price l_t and f into the interest rate at which the bank can roll over its debt. In equilibrium, insiders' policies l_t, f and q_t are optimal given the cost of borrowing, and the borrowing rate ρ_b set by lenders is competitive given insiders' policies.

Under the bailout regime, the bank operates forever because of the government subsidy but the value of payouts extracted will diminish after each shock due to potential

[16] We do not explicitly consider $\eta \geq 1$ in our analysis (the special case of $\eta = 1$ corresponds to log utility). When $\eta \geq 1$, insiders' utility goes to negative infinity as the payouts extracted approach zero. Insiders therefore avoid insolvency at all costs, and always adopt safe debt if there is a positive probability of them receiving no payouts after insolvency.

dismissal and stake dilution of the existing managers. The claim value is thus:

$$M_o(N) \equiv \max_{q_t, f\in[0,1], l_t>1/f} E\left[\int_0^{T_1} e^{-\delta t}U(q_tN_t)dt + \sum_{k=1}^{\infty}\int_{T_k}^{T_{k+1}} e^{-\delta t}[p_o\xi_o^{1-\eta}]^k U(q_tN_t)\Big| N_0 = N\right]$$

with p_o and ξ_o being the survival probability and stake dilution factor of the existing managers during a shock.[17] T_k is the random arrival time of the k^{th} shock.

Recall the index $j \in \{s, b, o, i\}$ for labeling the four regimes under consideration which are asset sales, liquidation, bailout and bail-in, respectively.[18] The dynamic programming equation of all regimes can be conveniently summarized as:

$$M_j(N) = \max_{q_t, l_t, f} E\left[\int_0^{T_1} e^{-\delta t}U(q_tN_t)dt + e^{-\delta T_1}p_jM_j(\phi_j(l_{T_1-}, f)N_{T_1-})\Big| N_0 = N\right] \quad (9)$$

subject to $0 \leq f \leq 1$, the (in)solvency constraint $l_t \leq (>) 1/f$ when $j = s$ (b, o, i), the intertemporal budget constraint:

$$\frac{dN_t}{N_t} = [(\mu + \kappa\lambda f - \rho_j(l_t, f))l_t + \rho_j(l_t, f) - mq_t]dt + \sigma l_t dB_t \quad \text{for } t < T_1 \quad (10)$$

and the transversality condition $\lim_{t\to\infty} E\left[e^{-\delta t}M_j(N_t)\right] = 0$.

From equation (9), managers' claim value M_j consists of two components. The first component is the expected discounted utility of payouts extracted up to the arrival of the first crash. The second component reflects the residual claim value to the managers after a crash, and can be understood as a continuation value originating from the dynamic programming principle. As explained previously, the residual claim value depends on (1) the probability p_j of managers having a continuation claim and (2) managers' net worth recovery rate $\phi_j(l_{T_1-}, f)$ following a shock.

We assume that managers know ex ante whether insolvency will be resolved through liquidation, bailout or bail-in (we relax this assumption in internet appendix 3.2). Man-

[17] The definition of $M_o(N)$ can be verified (see internet appendix 1.1) by introducing a sequence of i.i.d. random variables $(X_k)_{k\geq 0}$ where X_k takes on value ξ_o with probability p_o or value 0 otherwise, such that each X_k reflects the random effect of dismissal and dilution when the k^{th} shock arrives.

[18] As discussed in Section 2.1.4, under the bail-in regime the managers receive a severance claim of value $M_i(\phi_i N_{T_1-})$ when the shock arrives. Hence, (9) represents an implicit definition of $M_i(N)$.

agers solve for optimal policies and the corresponding claim values under the solvency and insolvency regimes respectively. They compare $M_s(N)$ and $M_j(N)$ (where $j = b, o$, or i is given) and ultimately adopt the policies that maximize their private value. This completes the formulation of managers' optimization problem. The proposition below gives a general characterization managers' optimal payout (q_t) policy , the optimal gearing ratio (l_t) and the optimal jump risk exposure (f) under the various restructuring mechanisms.

Proposition 1 *The optimal asset to net worth ratio (l_j) and crash risk exposure (f_j) are the constants solving the optimization problem:*

$$\max_{l,f} G_j(l,f) \equiv \max_{l,f} \left\{ [\mu + \kappa\lambda f - \rho_j(l,f)]l - \frac{\eta\sigma^2}{2} l^2 + \rho_j(l,f) + \frac{\lambda p_j}{1-\eta}[\phi_j(l,f)]^{1-\eta} \right\} \tag{11}$$

The optimal payout yield to insiders is given by

$$q_j = \frac{\lambda + \delta - (1-\eta)G_j(l_j, f_j)}{m\,\eta} \tag{12}$$

and the insiders' life-time utility is $M_j(N_t) = \frac{C_j N_t^{1-\eta}}{1-\eta}$ where

$$C_j \equiv \left[\frac{\eta}{\lambda + \delta - (1-\eta)G_j(l_j, f_j)} \right]^{\eta} m^{-(1-\eta)}$$

The subscript j takes value of either s (when it is optimal for the bank to stay solvent and the policy space in problem (11) *is restricted to $l \leq 1/f$) or $\{b, o, i\}$ (when it is optimal to put the bank at risk of insolvency and the policy space in problem* (11) *is restricted to $l > 1/f$).*

The general structure of the optimal policies is the same across mechanisms. The bank's optimal loan portfolio size $A_t \equiv l_j N_t$ is a constant multiple l_j of the bank's net worth N_t. We define and examine this constant l_j as well as the optimal loan quality determinant f_j in subsequent sections. Combined payout to insiders and outsiders are a constant fraction mq_j of net worth. The bank follows a constant debt to net worth ratio, which is given by $D_t/N_t = l_j - 1$. Finally, the private value of insiders' claim M_j is a concave increasing function of the bank's net worth N_t. The degree of concavity increases with insiders' coefficient of risk aversion η.

Proposition 1 does not tell us whether it is optimal to restructure through asset sales or whether it is optimal to put the bank at risk of insolvency instead. Managers' desire to expose loans to jump risk is driven by the reward κ for taking on crash risk. The following proposition proves that there exists a critical reward threshold $\underline{\kappa}_j$ such that for $\kappa \geq \underline{\kappa}_j$ managers prefer to issue low quality loans, which may lead to future insolvency under IRM j ($j \in \{b, o, i\}$).

Proposition 2 *There exists a critical risk premium $\underline{\kappa}_j \geq 1$ for IRM j (j=b,o,i) such that managers keep the bank solvent by adopting low leverage and issuing high quality loans if the reward for taking on crash risk exposure is sufficiently low ($1 \leq \kappa < \underline{\kappa}_j$). Managers put the bank at risk of insolvency by adopting high leverage and issuing low quality loans if the rewards are high (i.e. $\kappa \geq \underline{\kappa}_j$).*

The proposition shows that the solution to managers' optimization problem leads to two possible scenarios. If the crash risk premium is low ($\kappa < \underline{\kappa}_j$) then managers adopt an investment and payout policy that guarantees the bank's solvency in crashes. If the crash risk premium is high ($\kappa \geq \underline{\kappa}_j$) then managers' policies lead to bank insolvency in crashes. We examine these two scenarios in section 3 and section 4, respectively.

3 Optimal policies without insolvency: asset sales

Under the asset sale regime banks do not become insolvent. Following a crash, banks sell off assets to pay down debt and to delever. The following proposition characterizes the bank's optimal gearing ratio (l_s) and its optimal jump risk exposure (f_s). For the bank to take on debt, it is necessary that the Merton ratio is larger than one. We therefore impose the standing assumption $(\mu - \rho)/(\sigma^2\eta) > 1$ throughout the rest of the paper. Some of our results presented in the next section require a higher Merton ratio. Any additional assumptions will be explicitly stated when needed.

Proposition 3 *If the bank has to stay solvent in crashes then the optimal asset to net worth ratio, l_s and f_s are given by:*

$$l_s = \frac{\mu - \rho}{\sigma^2\eta} \quad \textit{and} \quad f_s \equiv \frac{\sigma^2\eta}{\mu - \rho}\left(1 - \kappa^{-\frac{1}{\eta}}\right) < 1$$

with $l_s f_s = 1 - \kappa^{-\frac{1}{\eta}} < 1$. Whenever the bank's asset base drops by a factor $(1 - f_s)$ from A to $(1 - f_s)A$ due to a crash, the bank's net worth drops by a factor $(1 - f_s l_s)$. The bank restores the optimal asset to net worth ratio, l_s, by selling an amount of assets equal to $Af_s(l_s - 1)$ and using the proceeds to pay off debt.

According to the bank's optimal investment policy the amount of risky loans issued (A_t) equals a constant multiple l_s of the bank's net worth (N_t). In the absence of crashes, net worth follows a geometric Brownian motion under the optimal investment and payout policies. Therefore, absent jumps, net worth always stays positive. The optimal value for l_s under the asset sales regime is always strictly less than $1/f_s$ to ensure the bank remains solvent also when a crash occurs. Nevertheless, leverage amplifies the effect of a loss in the firm's loan portfolio on the bank's net worth and asset base. Consider a levered firm with $A = 100$, $N = 20$ and suppose $l_s = 5$ and $f_s = 0.1$. A 10% loss in assets due to a crash reduces net worth by a factor $1 - f_s l_s = 0.5$ from 20 to 10. This causes the asset to net worth ratio to jump to $l = 90/10 = 9$, making the bank too risky. Managers rebalance by selling off an amount $Af_s(l_s - 1) = 40$ in loans, and using the proceeds to pay off debt, restoring the asset to net worth ratio to its optimal level $l_s = 50/10 = 5$. The example illustrates how leverage amplifies contractions in the bank's balance sheet following losses in its loan portfolio. An initial loss $f_s A$ of the bank's loans leads to a subsequent loan sale of $(l_s - 1)f_s A$.[19] See panel A of Figure 1.

The optimal exposure to crashes increases in the premium κ associated with jump risk. A higher expected return μ and lower volatility σ associated with the diffusion risk reduce f_s. Under the optimal policies the fraction of net worth at risk in a crash equals $f_s\, l_s = 1 - \kappa^{-\frac{1}{\eta}}$. Consequently, as we approach risk neutrality (i.e. $\eta \to 0$) close to 100% of the bank's net worth is at risk if $\kappa > 1$. Risk averse managers, on the other hand, put significantly less net worth at stake. As the risk premium associated with crash risk disappears ($\kappa \to 1$) the bank's optimal exposure to crashes goes to zero ($f_s \to 0$), i.e. insiders issue loans of the highest quality. The optimal asset to net worth ratio l_s for the optimal jump exposure f_s equals the Merton (1969) investment policy. The optimal level of investment increases with the excess return $\mu - \rho$, decreases with volatility σ and insiders' risk aversion η, and is independent of the frequency λ with which crashes occur.

4 Optimal policies with insolvency

We now consider the case where the bank becomes insolvent when a crash occurs. We consider three IRMs (liquidation, bailout and bail-in) and examine how they affect the bank's optimal level of investment l and jump risk exposure f. The prevailing IRM is common knowledge. In what follows we impose the following parameter restrictions for, respectively, the bailout and bail-in cases:

$$\frac{\mu - \rho}{\sigma^2 \eta} > 1 + \frac{p_o \xi_o^{1-\eta}}{\kappa} \tag{13}$$

$$\frac{\mu - \rho + \lambda h(1-\tau)}{\sigma^2 \eta} > 1 + \frac{p_i \xi_i^{1-\eta}}{\kappa - (1+h)(1-\tau)} \quad \text{and} \quad \kappa > (1+h)(1-\tau) \tag{14}$$

[19] Asset sales happen in a frictionless manner in our model. Internet appendix 3.3 shows that proportional transaction costs associated with selling assets after a jump reduce the level of the optimal asset to net worth ratio. The qualitative properties associated with the bank's optimal policies remain, however, largely the same.

Conditions (13) and (14) ensure that managers' objective function ($M_j(N)$) has a unique interior maximum ((l_j, f_j)) under the bailout and bail-in regimes, respectively, when managers have a strictly positive probability of a continuation claim and a strictly positive residual equity stake after restructuring (i.e. if $p_o\xi_o, p_i\xi_i > 0$). We remind the reader that $h \equiv -\xi_i/l^* > -1$ and that therefore $0 < 1 + h \leq 1$.

Proposition 4 *For the insolvency regime (i.e. $\kappa \geq \underline{\kappa}_j$), the optimal investment policy (l_j) under the liquidation, bailout and debt-to-equity conversion bail-in regime is:*

$$l_b = l_b(f_b) = \frac{\mu - \rho}{\eta\sigma^2} + \frac{[\kappa f_b - (1-\tau)(1-(1-f_b)(1-c_b))]\,\lambda}{\eta\sigma^2} \tag{15}$$

$$= l_b(1) = \frac{\mu - \rho}{\eta\sigma^2} + \frac{[\kappa - (1-\tau)]\,\lambda}{\eta\sigma^2} \quad (liquidation) \tag{16}$$

$$l_o = l_o(f_o) = \frac{\mu - \rho}{\eta\sigma^2} + \frac{\kappa\lambda f_o}{\eta\sigma^2} \quad (bailout) \tag{17}$$

$$l_i = l_i(f_i) = \frac{\mu - \rho}{\eta\sigma^2} + \frac{[\kappa - (1-\tau)]\lambda f_i + \lambda h(1-\tau)(1-f_i)}{\sigma^2\eta} \quad (bail-in) \tag{18}$$

If managers have a zero continuation probability or zero residual equity stake ($p_j\xi_j = 0$), then they adopt maximum crash risk exposure ($f_j = 1$). Since managers have no claim in liquidation ($p_b = 0$) they adopt maximum exposure under the liquidation regime, i.e. $f_b = 1$. If $p_o\xi_o, p_i\xi_i > 0$, then the optimal exposure level under bailout or bail-in is given by some $f_o, f_i \in (0,1)$ which is the unique solution to the equation

$$l_o(f_o) - \frac{p_o\xi_o^{1-\eta}}{\kappa(1-f_o)^\eta} = 0 \qquad (for\ bailouts) \tag{19}$$

$$l_i(f_i) - \frac{p_i\xi_i^{1-\eta}}{[\kappa-(1+h)(1-\tau)](1-f_i)^\eta} = 0 \quad (for\ bail-ins) \tag{20}$$

Junior debt is always risky under bail-ins.

The asset to net worth ratio l exceeds the Merton solution ($l = (\mu-\rho)/(\eta\sigma^2)$) under all three IRMs.[20] Since firms adopt the Merton investment policy under the asset sale regime (see Proposition 3), it follows there is a discrete upward jump in investment

[20]This result is perhaps less trivial for the bail-in regime because of the negative h. We verify this claim in the proof of Proposition 4.

and leverage when we move at $\underline{\kappa}_j$ from the asset sale regime to one of the three IRMs. This discrete increase in loan issuance coincides with a drop in loan quality. Under the liquidation regime banks even adopt the maximum possible risk exposure ($f_b = 1$). Given managers' limited liability and zero payoff in liquidation, they do not care whether the bank ends up insolvent a little or a lot.[21] This creates a serious moral hazard problem from lenders' viewpoint.

Importantly, high (low) crash risk exposure is combined with high (low) leverage. In particular, if jump risk premiums are sufficiently high ($\kappa > \underline{\kappa}_j$) banks issue high volumes of low quality loans, which is a toxic combination of high financial risk (leverage) and high business risk (exposure to crashes).

Banks do not adopt maximum crash risk exposure under the bailout and bail-in regime (i.e. $f_o, f_i < 1$) if managers retain some "skin in the game" after the restructuring (i.e. if $p_o\xi_o, p_i\xi_i > 0$) because doing so would wipe out all the firm's assets and leave no bank to be bailed out (or to be bailed-in).[22] Only when managers are sure to lose everything ($p_o\xi_o, p_i\xi_i = 0$) do they adopt 100% exposure (i.e. $f_o = f_i = 1$) to crashes. Using Eq. (19) and Eq. (20) one can show that the optimal exposure to crashes (f_o and f_i) increases with μ, λ and κ, and decreases with ρ, σ, η and dilution adjusted probability of continuation ($p_o{\xi_o}^{1-\eta}$ and $p_i{\xi_i}^{1-\eta}$). If managers do not retain any skin in the game under the bail-in regime then it follows from propositions 1 and 4 that the restructuring is essentially the same as liquidation.

[21]Managers' objective function is essentially convex in f for large values of f because the continuation value is zero in liquidation if $f > 1/l$. This convexity leads to excessive risk taking of the managers and creates a corner solution ($f_b = 1$) (see internet appendix 1.2 for further details).

[22]If the IRM is bailout or bail-in with $v_j > 0$, the residual claim under the IRM creates a discontinuity in managers' objective function at the critical point of insolvency $f = 1/l$ and thus managers definitely prefer a marginally insolvent firm to a marginally solvent one (e.g. under bailout managers enjoy a free government rescue). However, the residual claim also creates local concavity near $f = 1$, which discourages extreme risk taking (see internet appendix 1.2 for further details).

Corollary 1 *If managers have zero continuation probability or no residual equity stake in a bail-in (i.e. $p_i \xi_i = 0$), then the bail-in is economically identical to a liquidation. In particular, $l_i = l_b$, $f_i = f_b = 1$, $\rho_i = \rho_b$, $q_i = q_b$ and $M_i = M_b$.*

Excessive risk taking (as in the liquidation regime) can be avoided for bail-ins by giving managers skin in the game (or by imposing penalties in bail-ins). Corollary 1 does not hold for bailouts because the cost of debt is always kept at the risk-free rate by the government. Managers who anticipate a bailout do not take the cost of financing into account regardless they retain skin in the game or not. Therefore, the leverage and payout decisions are always different across the bailout and liquidation regimes.

4.1 A comparison of the insolvency resolution mechanisms

Assume now that the reward κ for taking on crash risk varies across banks (due to differences in banks' operating efficiency or loan selection skills), and follows some distribution. From Proposition 2 it follows that all banks with a κ below (above) $\underline{\kappa}_j$ are (in)solvent following a crash. As such, an IRM with a lower threshold $\underline{\kappa}_j$ leads to a higher insolvency rate and default probability in the banking industry.

In what follows we compare the critical thresholds $\underline{\kappa}_j$, the cost of debt, the bank's optimal investment and payout policies, its exposure to crashes, and managers' claim value across the three IRMs.

Proposition 5 *If the parameters are such that conditions* (13) *and* (14) *hold, then we have the following comparison results across different IRMs where $v_j \equiv p_j \xi_j^{1-\eta}$ denotes the dilution-adjusted continuation probability of insiders:*

i) Loan quality is highest (lowest) under the bail-in (liquidation) regime, i.e.:
$f_b \geq f_o \geq f_i$ *if either* $v_i \geq v_o$ *or* $v_o > v_i > 0$ *and* $(1+h)(1-\tau) \leq \kappa \leq \frac{v_o(1+h)(1-\tau)}{v_o - v_i}$

ii) The cost of bank debt is highest (lowest) under the liquidation (bailout) regime: $\rho_b(l_b, f_b) \geq \rho_i(l_i, f_i) \geq \rho_o(l_o, f_o) = \rho$

iii) The asset to net worth ratio is highest (lowest) under the bailout (bail-in) regime: $l_o \geq l_b \geq l_i$

iv) Managers' claim value is highest (lowest) under the bailout (liquidation) regime: $M_o(N) \geq M_i(N) \geq M_b(N)$ *if* $v_o \geq v_i$

v) The payout yield is highest (lowest) under the liquidation (bailout) regime: $q_b \geq q_i \geq q_o$ *if* $v_o \geq v_i$

vi) The critical crash risk premium above which managers put the bank at risk of insolvency is highest (lowest) under the liqidation (bailout) regime, i.e.: $\underline{\kappa}_b \geq \underline{\kappa}_i \geq \underline{\kappa}_o$ *if* $v_o \geq v_i$

i) Managers face a tradeoff when setting the optimal loan quality. On the one hand, a high crash exposure f (i.e. low loan quality) improves the risk-adjusted performance of the leveraged equity.[23] On the other hand, managers' residual claim value after a resolution is proportional to $(1-f)^{1-\eta}$ and as such a low f preserves a larger fraction of residual value. Managers' (dilution-adjusted) continuation probability v_j acts as a weighting attached to the residual claim value.

If managers anticipate an insolvent bank is always liquidated then there is no trade-off involved. They simply expose 100% of the firm's assets to crash risk for maximum return (i.e. $f_b = 1$). Under bailout with $v_o > 0$, however, the residual claim and managers' infinite marginal utility near zero provide them with an incentive to keep

[23]The relevant criterion here is: $(\mu + \kappa\lambda f - \rho_j)l_j + \rho_j - \frac{\eta\sigma^2}{2}l_j^2$ which resembles the classical mean-variance performance measure adopted by a risk averse agent. In the cases of liquidation and bail-in, a higher f increases both the loan return and cost of debt, but the net effect on the performance measure is positive at the optimally chosen l_j.

some “skin” in the game. This explains why $f_o < 1 = f_b$ (and similarly why $f_i < 1$ for bail-in for $v_i > 0$). Therefore, the liquidation IRM leads to the lowest loan quality.

The comparison of f_o and f_i is more subtle and crucially depends on v_o and v_i. Consider first the case $v_i \geq v_o$. Increasing f generates a higher risk adjusted performance of the equity under bailouts than bail-ins. If $v_i \geq v_o$ the managers have a stronger incentive to retain a residual claim under bail-ins than under bailouts. Hence, the trade-off that determines the optimal f unambiguously implies that $f_i \leq f_o$.

If $v_o > v_i$, bailouts favor a higher f than bail-ins in terms of risk-adjusted performance but a lower f in terms of their residual claim value (since $v_o > v_i$). The ranking of f_i and f_o becomes ambiguous in general. We know from corollary 1 that $f_o < f_i = 1$ for $v_o > v_i = 0$. However, $f_i < f_o$ still holds under an additional condition $\kappa < \frac{v_o(1+h)(1-\tau)}{(v_o - v_i)}$. We remind the reader that $1 + h$ is positive and depends on l^*.

ii) The cost of debt is highest in the liquidation regime, because (1) lenders only receive the proceeds from liquidation and (2) there are no liquidation proceeds (since $f_b = 1$). In a bail-in lenders acquire a claim on the assets of the restructured bank. Creditors are better off than under the liquidation regime because (1) of the NCWOL test and (2) the bailed-in bank has a positive asset base ($f_i \leq 1$). Therefore, $\rho_i(l_i, f_i) < \rho_b(l_b, f_b)$. Finally, $\rho_o = \rho$ because debt is risk free in the bailout regime.

iii) Loan issuance (i.e. the asset to net worth ratio) is highest under the bailout regime because of the low cost of debt ($\rho_o = \rho$). Next, the amount of loans issued under the liquidation regime is higher than under the bail-in regime. Although the cost of debt is highest under the liquidation regime, crash risk exposure is much higher under the liquidation regime ($f_b = 1$) than under the bail-in regime. Since investment increases in f, the higher risk exposure f under the liquidation regime dominates and causes investment to be higher under liquidation than bail-in (i.e. $l_b \geq l_i$).

iv) Consider next managers’ claim value M_j. Managers are best (worst) off if insolvencies are resolved through bailouts (liquidation). Under the liquidation regime managers get nothing when the bank becomes insolvent. Under a bail-in, the restructuring creates extra value to creditors. This reduces the cost of debt. Furthermore, under a bail-in the bank carries on as a going concern, which creates space for a managerial severance claim. Finally, managers fare best under the bailout regime because the cost of debt is lowest, loan issuance is highest generating higher growth, and managers remain in post with some positive dilution-adjusted probability v_o.

v) Total payout equals mq_jN_t and is proportional to managers' payout yield q_j. From Proposition 1 it follows that $M_j = N^{1-\eta}/\left[(1-\eta)mq_j^{\eta}\right]$. Hence, payout is inversely related to managers' claim M_j. Therefore (see iv), payout is highest under the liquidation regime and lowest under the bailout regime. Under the former, managers want to milk the firm before it is liquidated. Under the latter, managers prefer to reinvest profits for long term growth, providing a novel argument in favor of bailouts.

vi) The ranking for $\underline{\kappa}_j$ shows that managers are most (least) likely to put the bank at risk of insolvency under the bailout (liquidation) regime. The explanation mirrors the previous argument why managers most (least) prefer the bailout (liquidation) regime. If one considers an industry of banks with different levels of κ then the critical threshold $\underline{\kappa}_j$ determines the insolvency rate in a crash. The bailout (liquidation) regime generates the highest (lowest) insolvency rate and default probability. Put differently, one could say that the bailout (liquidation) regime makes managers most (least) prone to put the bank at risk of insolvency, consistent with conventional wisdom.

4.2 Comparative statics

Table 2 illustrates our model and shows the net debt ratio (D_j/A_j), the crash risk exposure (f_j), and insiders' payout yield (q_j) for the three IRMs. An asterix in the table indicates that the bank is safe and engages in asset sales when a crash happens.

		D_b/A_b	D_o/A_o	D_i/A_i	f_b	f_o	f_i	q_b	q_o	q_i
Benchmark		83.100	88.694	82.854	100.000	99.474	97.186	11.075	8.776	10.967
μ'	0.08	43.667*	86.934	78.317	18.709*	99.343	95.962	11.742*	9.931	11.698
	0.12	85.917	90.033	85.789	100.000	99.567	97.892	10.164	7.454	10.066
σ'	0.18	86.311	90.851	86.168	100.000	99.620	97.978	10.589	7.689	10.493
	0.22	79.551	86.303	79.147	100.000	99.293	96.197	11.435	9.578	11.314
κ'	1.8	66.200*	87.875	80.813	7.253*	99.311	95.319	11.429*	9.371	11.339
	2.2	84.636	89.408	84.479	100.000	99.589	98.167	10.640	8.139	10.547
ρ'	0.03	85.917	90.033	85.789	100.000	99.567	97.892	10.304	7.594	10.206
	0.07	43.667*	86.934	78.317	18.709*	99.343	95.962	11.602*	9.791	11.558
λ	0	66.200*	66.200*	66.200*	11.225*	11.225*	11.225*	11.440*	11.440*	11.440*
	0.1	88.733	93.227	88.617	100.000	99.761	98.502	9.675	1.999	9.501
τ	0.25	80.500	86.954	80.216	100.000	99.474	97.186	10.967	8.722	10.859
	0.45	85.700	90.433	85.492	100.000	99.474	97.186	11.183	8.830	11.074
η	0.5	87.000	91.330	86.963	100.000	99.936	99.443	11.700	6.090	11.672
	0.8	58.400*	85.961	58.400*	11.632*	98.155	11.632*	9.631*	9.159	9.631*

Table 2: Comparative statics. Optimal corporate policies under different model parameters. Base parameters used are $\mu' = 0.1$, $\sigma' = 0.2$, $\rho' = 0.05$, $\kappa' = 2$, $\tau = 0.35$, $\lambda = 0.05$, $\eta = 0.65$, $\delta = 0.4$, $\alpha = 0.8$, $p_o = p_i = 0.85$, $\xi_o = \xi_i = 0.1$, $l^* = 5$ and $w^* = 1$. Numerical results are all expressed in percentages. An asterisk $*$ indicates that the bank is safe and engages in asset sales when a crash arrives.

Our parameter values for μ', σ', ρ' and τ are standard. We choose a relatively low baseline coefficient of relative risk aversion ($\eta = 0.65$) because for $\eta \geq 1$ banks never become insolvent and always operate within the asset sales regime. Insiders' subjective discount rate ($\delta = 0.4$) is set high enough to ensure that insiders' claim value remains bounded for all parameter combinations. δ does not affect the optimal net debt ratio (NDR), nor the bank's crash risk exposure f. A lower δ does, however, reduce insiders' payout yield q as it makes insiders more patient.[24]

The base values for the crash arrival rate ($\lambda = 0.05$) and the risk premium parameter ($\kappa' = 2$) imply that for $f = 0.9$, the expected before-tax return on the bank's assets is $\mu' + \lambda f(\kappa' - 1) = 0.145$. For the bail-in regime we need to make additional assumptions regarding the gearing ratio l^* adopted by the resolution authority after the bail-in. Gearing (l) within the banking industry are empirically observed and reported. We set l^*, the (risky) asset to net worth ratio post bail-in, equal to 5 in line with empirical estimates.[25]

The table illustrates that insolvencies occur least (most) often in the liquidation (bailout) regime, reflecting our earlier result that $\underline{\kappa}_b > \underline{\kappa}_i > \underline{\kappa}_o$. For our parameter combinations, banks are safe under the bailout regime only if there is no crash risk ($\lambda = 0$). Under the bail-in regime, banks remain safe for zero crash risk, as well as for high insider risk aversion ($\eta = 0.8$). Under the liquidation regime, banks remain safe for zero crash risk, high insider risk aversion, low expected return on assets ($\mu' = 0.08$), a high interest rate ($\rho' = 0.07$), and a low crash risk premium ($\kappa' = 1.8$).

Under the asset sale regime banks adopt a low NDR and low crash risk exposure. The NDR of safe banks ranges under the three IRMs from 43.7% to 66.2%. Crash

[24]Reducing δ from 0.4 to 0.35 for the base parameter case reduces the payout yields in, respectively, the liquidation, bailout, and bail-in regimes from 11.1%, 8.8% and 11.0% to 9.5%, 7.2% and 9.4%.

[25]Banks have been delevering since the financial crisis. Cohen and Scatigna (2016) report that the ratio of capital to total risk-weighted assets for US commercial banks rose from 13.9% in 2009 to 17.4% in 2012.

risk exposure ranges from 7.3% to 18.7%. Payout yield ranges from 9.6% to 11.7%. Introducing insolvency risk causes a discrete upward shift in both the NDR and crash risk exposure, and generates a larger dispersion in the payout yield. The NDR now ranges from 78.3% to 93.2%. Crash risk exposure ranges from 95.3% to 100%. Insiders' payout yield ranges from 2.0% (for $\lambda = 0.1$ under the bailout regime) to 11.7% (for $\mu' = 0.08$ under the liquidation regime). Under the bailout regime, insiders prefer to reinvest income for future growth and therefore pay out very little. This has important implications for the bank's market value (see Section 5) and also implies less need for regulatory constraints on payouts.

The table allows us to gauge the effect of parameter changes on the control variables. Increasing the expected return on assets (μ'), the crash risk premium (κ'), or the crash arrival rate (λ) has a positive effect on the NDR and crash risk exposure, but a negative effect on insiders' payout yield. Increasing return volatility (σ') and risk aversion (η) decreases the NDR and crash risk exposure, but increases the payout yield. E.g., under the bail-in regime increasing volatility σ' from 0.18 to 0.22 reduces the NDR from 86.2% to 79.1%, and crash risk exposure from 98.0% to 96.2%, whereas insiders' payout yield increases from 10.5% to 11.3%. Taxes increase the NDR and payout yield but do not affect crash risk exposure. A higher pre-tax borrowing cost ρ' reduces the NDR and crash risk exposure, but increases payout. If the government wanted to reduce bank gearing under the bailout regime, it could reduce corporate taxes, or increase the pre-tax cost of borrowing ρ' by raising the interest rate or by imposing a fixed deposit insurance rate. For example, a 4% increase in ρ' or a 20% cut in τ reduce the NDR for the bailout regime by 3.1% and 3.5%, respectively.

5 IRMs, Bank Value and Bailout Funds

In this section we examine which IRM maximizes the bank's total market value (net of any recapitalizations). We show how to set up a self-financing bailout fund in which banks make contributions during good times that cover the expected costs of bailouts.

Under the optimal policies, the bank's total net worth has the dynamics defined in (4) (for $j = b, o, i$). Since debt is competitively priced, our analysis can be restricted to the value created for inside and outside equityholders. Recall that the combined payout to both outside and inside equityholders equals mq_jN_t. The net market value created is the expected discounted value of the payout flow net of any capital injections. We assume that market participants are well diversified and have a subjective discount rate $\delta(> \rho)$.[26] We calculate the net market value for each IRM using risk neutral valuation.

In liquidation all proceeds go to lenders. Hence, inside and outside equityholders only receive a combined payout mq_bN_t up to the arrival time of the first crash. The net market value created under the liquidation regime, W_b, is therefore:

$$W_b \equiv E\left[\int_0^{T_1} e^{-\delta t} mq_b N_t dt\right] - N_0$$

where T_1 is the random arrival time of the first shock.

Under the bailout regime the bank operates forever, but new capital is injected after every crash. The net market value created is given by:

$$W_o \equiv E\left[\int_0^{\infty} e^{-\delta t} mq_o N_t dt\right] - E\left[\sum_{k=0}^{\infty} e^{-\delta T_k} N_{T_k}\right] \equiv I_o - C_o$$

where T_k is the k-th arrival time of the Poisson shock and $T_0 \equiv 0$.

Under the bail-in regime inside and outside equityholders receive a combined payout flow mq_iN_t up to the first shock, after which they receive a fraction ξ_i of the bank's

[26] For example, if all market participants are subject to sudden death with Poisson arrival rate ω, then $\delta \equiv \rho + \omega$ (assuming sudden death is uncorrelated with other shocks in the economy).

post bail-in equity. The net market value created is given by:

$$W_i \equiv E\left[\int_0^{T_1} e^{-\delta t} m q_i N_t dt\right] + E\left[e^{-\delta T_1}\xi_i \frac{(1-f_i)A_{T_1-}}{l^*}\right] - N_0$$

where $\frac{(1-f_i)A_{T_1-}}{l^*}$ is the value of the equity after the bail-in. Note the change in net worth following a crash:

$$\frac{(1-f_i)A_{T_1-}}{l^*} = \frac{(1-f_i)A_{T_1-}}{l_i}\frac{l_i}{l^*} = (1-f_i)\frac{l_i}{l^*}N_{T_1-} \equiv \Phi_i(l_i, f_i)N_{T_1-} \tag{21}$$

We now state the net market value created under each IRM.

Proposition 6 *Suppose that $\delta + \lambda f_j - g_j > 0$,[27] and that $\kappa > \underline{\kappa}_j$ for $j = b, o, i$ then the net market value created by the bank under IRM j is given by:*

$$W_b = \left(\frac{mq_b}{\delta + \lambda - g_b} - 1\right) N_0, \tag{22}$$

$$W_o = I_o - C_o \equiv \left(\frac{mq_o}{\delta + \lambda f_o - g_o} - \frac{\delta + \lambda - g_o}{\delta + \lambda f_o - g_o}\right) N_0 \tag{23}$$

$$W_i = \left(\frac{mq_i + \lambda \Phi_i(l_i, f_i)\xi_i}{\delta + \lambda - g_i} - 1\right) N_0 \tag{24}$$

If the parameters are such that $\xi_i \approx 0$ (i.e. equityholders are almost wiped out in a bail-in), $\frac{\mu' - \rho'}{(\sigma')^2 \eta} > 2$ and $\delta > g_j - \lambda f_j$ for $j \in \{b, o, i\}$ then the bank's value creation is highest (lowest) under the bailout (bail-in) regime, i.e.:

$$W_o \geq \ W_b \ \geq \ W_i$$

Under managers' optimal risk exposure, bailouts and bail-ins create the highest and lowest net value, respectively. At first sight, $W_b \geq W_i$ might appear surprising considering that managers prefer bail-in to bankruptcy (i.e. $M_i(N) \geq M_b(N)$). This ranking is primarily driven by the fact that managers pick loans with 100% crash exposure in the liquidation regime, and the associated high return is more favorable under a risk-neutral market valuation criterion.

[27] Recall that g_j is the growth rate of the net worth under IRM j as defined in (4).

The highest net value is achieved under the bailout regime. The payout stream generated under the bailout regime more than compensates for the recapitalizations in crashes. Therefore, we can create a self-financing bailout fund. For instance, we can make the fund a recipient of dividends alongside outside equityholders by splitting the fraction α of free cash flows that currently accrue to outside equityholders into two components: a fraction α_1 going to outside equityholders and a fraction α_2 going to the bailout fund (with $\alpha_1 + \alpha_2 = \alpha$). The cashflows going into the fund could be raised through a tax on bank dividends. Since the optimal loan volume (l_o) and loan quality (reflected by f_o) do not depend on α, the creation of the fund does not alter insiders' lending incentives. How big does α_2 have to be to meet the expected costs of recapitalizations? Using equation (23), α_2 is the solution to $\alpha_2 m q_o = (\delta + \lambda - g_o)$. Solving for α_2, and using the equation (12) for q_o gives the following:

Corollary 2 *The expected costs of future bailouts can be covered by putting a fraction α_2 of total dividends into a bailout fund, where the tax rate α_2 is given by:*

$$\alpha_2 = \frac{(\delta + \lambda - g_o)\eta}{\lambda + \delta - (1-\eta)G_0(l_o, f_o)} \leq 1 \iff \delta \leq \rho_o + (\mu + \kappa\lambda f_o - \rho_o) l_o - \lambda \equiv \delta_o \quad (25)$$

The higher investors' discount rate δ, the larger the bank's required contribution to the bailout fund. As δ converges to δ_o (which corresponds to the bank's internal rate of return), the required dividend tax rate approaches 100% which means that all dividends are paid as a tax into the bailout fund (i.e. $\alpha_2(\delta_o) = 1$), leaving nothing for inside and outside equityholders. The corollary demonstrates that bailouts can be self-financing and need not rely on public money, provided that the bank generates a strictly positive NPV net of recapitalization costs. Considering that bailouts also generate the most value (compared to liquidation and bail-ins) and the highest internal reinvestment rate, our model suggests there is a strong case for retaining bailouts as a possible tool for resolving bank insolvencies.

6 Bail-in with Debt Write-down

So far our analysis has focused on bail-ins that convert unsecured debt into common equity. Although the first bail-in bonds were of the equity conversion type, more recently principal write-down bonds ("PWD bonds") have been on the rise. PWD bonds do not convert into equity when the firm becomes insolvent or when some trigger event occurs. Instead the firm is internally recapitalized by writing down the principal of the unsecured debt. In this section we briefly study PWD bonds, and compare their features with the bail-in bonds from previous sections.[28]

Suppose that the bank has an amount D_2 of PWD bonds. A proportion $\gamma \in [0,1]$ of the junior debt D_2 is written down when the bank becomes insolvent. Similar to the equity-conversion bail-in, we assume that the amount of debt to be written down is set in such a way that the leverage level after the restructuring is maintained at the same level l_d (where the subscript "d" refers to the debt write-down regime). Hence, we require:

$$D_1 + (1-\gamma)D_2 = A(1-f)\left(\frac{l_d - 1}{l_d}\right) \tag{26}$$

The total amount of debt before the restructuring is simply:

$$D = D_1 + D_2 = A\left(\frac{l_d - 1}{l_d}\right) \tag{27}$$

Solving the system of equations (26) and (27) for the two unknowns (D_1 and D_2) yields:

$$D_1 = \left(\frac{l_d - 1}{l_d}\right) A \left(1 - \frac{f}{\gamma}\right) \quad \text{and} \quad D_2 = \left(\frac{l_d - 1}{l_d}\right) A \left(\frac{f}{\gamma}\right) \tag{28}$$

It is natural (but not strictly necessary) for the balance sheet to include senior debt (i.e. $D_1 \geq 0$), which requires $f \leq \gamma$ (for $f > \gamma$ the bank holds a net cash position alongside the PWD bonds). The debt write-down IRM is graphically illustrated in Figure 2.

[28] We thank the referee for drawing our attention to debt write-downs.

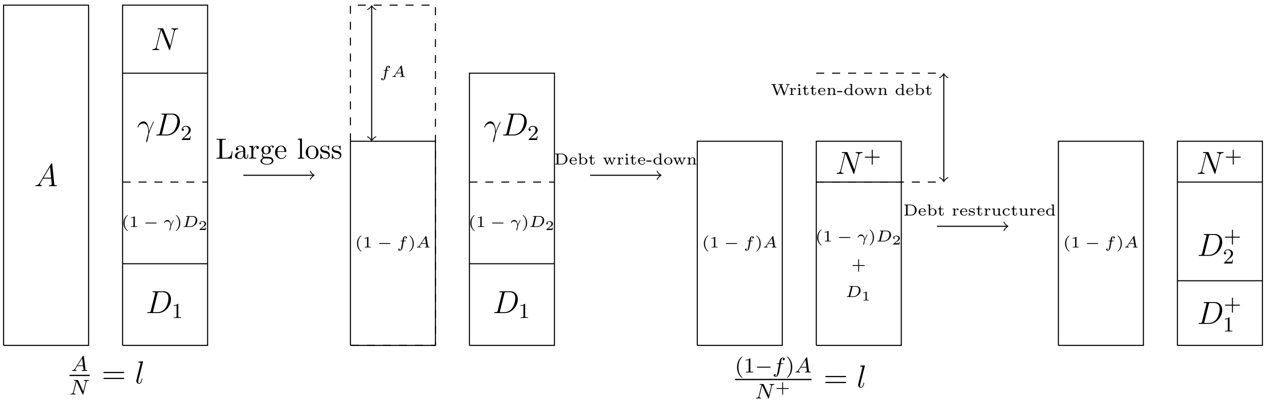

Figure 2: Illustration of a bail-in with debt write-down.

By construction the recovery rate of the junior debt D_2 is $1-\gamma$ in a crash. Therefore, risk-neutral junior debtholders require a (before-tax) return of $\rho' + \lambda\gamma$. The after-tax weighted average cost of debt across all debtholders is given by:

$$\begin{aligned}\rho_d(l_d, f) &= \frac{D_1}{D}\rho'(1-\tau) + \frac{D_2}{D}(\rho' + \lambda\gamma)(1-\tau) \\ &= \left(1 - \frac{f}{\gamma}\right)\rho + \frac{f}{\gamma}[\rho + \lambda\gamma(1-\tau)] \;=\; \rho + \lambda(1-\tau)f\end{aligned}$$

The overall recovery rate on the debt is $\Omega_d = \frac{D_1+(1-\gamma)D_2}{D} = 1 - f$. Note that the recovery rate and the cost of debt do not depend on the leverage level l_d nor on the fraction γ of junior debt to be written down.

$N^+ = (1-f)A/l_d = (1-f)N$ is the available net worth immediately after the debt write-down. The outstanding debt of the bank is then restructured and the new amount of senior and junior debt are determined again using equation (28) (where A equals the most recent asset level after the crash). As such, the bank operates perpetually but its capital stock shrinks by a factor $1-f$ after each crash. As before, we assume that after each crash existing managers' claim is diluted by a factor ξ_d with probability p_d. We summarize the debt write-down IRM using the notation of Table 1:

$$\Phi_d(l, f) = 1 - f, \quad p_d \in [0,1], \quad \phi_d(l, f) = (1-f)\xi_d, \quad \Omega_d = 1 - f. \tag{29}$$

The below proposition characterizes insiders' optimal policies.

Proposition 7 *The optimal investment policy (l_d) for bail-ins with debt write-down is:*

$$l_d \;=\; l_d(f_d) \;=\; \frac{\mu-\rho}{\eta\sigma^2} \;+\; \frac{[\kappa-(1-\tau)]\,\lambda f_d}{\eta\sigma^2} \tag{30}$$

If managers have a zero continuation probability or zero residual equity stake ($p_d\xi_d = 0$), then they adopt maximum crash risk exposure ($f_d = 1$). If $p_d\xi_d > 0$, then the optimal exposure level is the unique solution to the equation:

$$l_d(f_d) \;-\; \frac{p_d\xi_d^{1-\eta}}{[\kappa-(1-\tau)](1-f_d)^\eta} + \frac{1-\tau}{\kappa-(1-\tau)} = 0 \tag{31}$$

Junior debt is always risky.

We can now compare the bail-in with debt-to-equity conversion and the bail-in with debt write-down. Assume for this purpose that $\xi \equiv \xi_i = \xi_d$ and $p_i = p_d$. Therefore, the main differences between the two IRMs originate from the difference in their recovery rates. Recall that: $\Omega_i(l_i, f_i) = \frac{(1-f_i)(1+h)l_i}{l_i-1}$ and $\Omega_d(l_d, f_d) = 1 - f_d$. Therefore,

$$\frac{\partial\Omega_i(l_i,f_i)}{\partial f_i} = -(1+h)\frac{A_i}{D_i} < \frac{\partial\Omega_d(f_d)}{\partial f_d} = -1 \iff \frac{D_i}{A_i} < 1+h = \frac{l^*-\xi}{l^*} \iff l_i < \frac{l^*}{\xi} \tag{32}$$

Condition (32) is satisfied if inside equityholders' post bail-in stake, ξ_i, is not too high, which is normally the case in practice. For example, if $\xi_i = 0.1$ and $l^* = 5$ (or equivalently $(\frac{D_i}{A_i})^* = 0.8$) then the condition becomes $D_i/A_i < 1 + h = 0.98$. The condition implies that that the recovery rate (and therefore the cost of debt) is less sensitive to changes in crash risk exposure for debt write-downs, than for debt-to-equity conversion bail-ins. A sufficient (but not necessary) condition for (32) to be satisfied is that $l_d(f_d = 1) < l^*/\xi$. We use this sufficient condition in the proposition below.

Next, note that $\frac{\partial\Omega_i(l_i,f_i)}{\partial l_i} < \frac{\partial\Omega_d(f_d)}{\partial l_d} = 0$. Consequently, the cost of debt does not depend on leverage under the debt write-down regime, whereas the cost of debt increases with leverage under the debt-to-equity conversion bail-in regime.

Comparing proposition 7 with proposition 4 we infer that the first-order condition (18) for l_i and (30) for l_d are the same, provided that we set $h = 0$ in the former.

Likewise, the first-order condition (20) (with $h = 0$) for f_i and (31) for f_d are the same, except for the additional term $(1-\tau)/(\kappa-(1-\tau))$ in (31). This additional term results from the lower sensitivity of the cost of debt ρ_d to crash risk exposure and encourages insiders to take on more risk with debt write-downs than with debt-to-equity conversion. Higher crash risk exposure, in turn, leads to a higher asset to net worth ratio (see Eq. (30)). The following proposition formally compares both bail-in regimes.

Proposition 8 *Suppose $\xi \equiv \xi_i = \xi_d$ and $p \equiv p_i = p_d$. If $p\xi \neq 0$ and $l_d(f_d = 1) = \frac{\mu+[\kappa-(1-\tau)]\lambda-\rho}{\eta\sigma^2} < \frac{l^*}{\xi} \equiv -1/h$ then $f_i < f_d$, $l_i < l_d$, $\rho_i < \rho_d$, $q_i < q_d$ and $M_i > M_d$. In other words, comparing debt write-downs with debt-to-equity conversion bail-ins for equal dilution-adjusted continuation probability (i.e. we assume $v_d = v_i$), the former have a higher optimal asset to net worth ratio ($l_d > l_i$), a higher crash risk exposure ($f_d > f_i$), a higher cost of debt ($\rho_d > \rho_i$), a higher payout yield ($q_d > q_i$), and a lower managerial claim value ($M_d < M_i$).*

Debt write-downs encourage inside equityholders to adopt a higher crash risk exposure than bail-ins because the cost of borrowing under debt write-downs is less sensitive to changes in crash risk exposure. The higher crash risk exposure in the debt write-down regime raises l_d (see equation (30)) and encourages insiders to adopt a higher asset to net worth ratio. Equityholders are not penalized by a higher borrowing cost since ρ_d does not depend on l_d. Nevertheless, the cost of debt is higher for PWD bonds than for bail-in bonds because the recovery rate of the former is lower. Ex post the holders of the PWD bonds bear most of the cost of the crash, whereas equityholders take the hit in an equity-conversion bail-in.

Insiders achieve a higher life-time utility under a bail-in with equity conversion than under a debt write-down ($M_i \geq M_d$) because under the former the cost of debt is lower and a lower fraction of the assets is wiped out in crashes (which in turn leads to a higher insider recovery rate ϕ). We showed before that the payout yield q is inversely related to insiders' claim value M. Therefore if $M_i \geq M_d$, then one can prove that this implies that $q_i \leq q_d$.

The rankings in Proposition 8 may not hold if the condition $l_d(f_d = 1) < l^*/\xi$ is violated, which happens if insiders leave very little value on the table for unsecured creditors in an equity conversion bail-in. This is unlikely to happen in practice as insiders are typically largely or entirely wiped out (i.e. ξ_i is very small).[29]

We conclude that along several dimensions bail-ins with debt-to-equity conversion are superior to bail-ins with debt write-down because the former induce lower leverage and lower crash risk exposure, while at the same time generating a higher claim value for insiders. Given that PWD bonds encourage insiders to take more risk and adopt higher leverage, their increasing usage in recent years should be of some concern to regulators.

7 Policy implications and conclusion

In the wake of the financial crisis a new framework for resolving bank insolvencies is being developed. Some politicians have argued that governments must commit never to bail out banks again. This may be throwing out the baby with the bath water. Leaving aside the fact that bailouts are a quick way to contain systemic risk, our model shows that, from a micro-prudential perspective, banks adopt the lowest payout rate and create the most value net of any recapitalization costs under the bailout regime. On the downside, insiders have a stronger incentive to put banks at risk of insolvency, increasing the insolvency rate within the banking sector. The exposure of bank assets to crashes can, however, be kept low by giving insiders skin in the game in the event of a bailout. Excessive risk taking can be curbed by penalizing (rather than rewarding)

[29]Numerical simulations (see Section 4 of the internet appendix) show that for $l^* = 5$, ξ_i has to be approximately at least 0.58 before rankings for f and q are reversed.

managers for failure. To avoid bailouts with public money, a fraction of total bank payouts during good times can be put in a bailout fund to cover expected bailout costs. This can be implemented through a proportional tax on payouts, without distorting insiders' incentives. Such a bailout fund is viable if banking (net of recapitalization costs) is a positive NPV activity before and after the bailout. Furthermore, a bailout fund could resolve bank insolvency in a relatively speedy fashion.

Under the liquidation regime banks are least prone to insolvency but they incur the largest loss given default. A regulatory regime shift from bailout to liquidation therefore not only reduces the insolvency rate within the banking sector, but also increases the loss given default. This highlights a regulatory tradeoff that hitherto has not been recognized.

If the aim is to keep the amount of lending as well as the banks' exposure to crashes low then bail-ins with debt-to-equity conversion can be a superior alternative to liquidation or bailouts. The price to pay is that banks grow more slowly and generate less value under this bail-in regime. We show that bail-ins with straight debt write-down create incentives for higher leverage, higher crash risk exposure, higher cost of debt, higher payout, and lower insider claim values than their debt-to-equity conversion counterpart. The increasing usage of principal debt write-down (PWD) bonds in recent years might arguably be a cause for concern with regulators. Our model also highlights a number of caveats associated with bail-ins more generally. First, banks need a sufficient amount of unsecured creditors that can be bailed in to avoid a bank run. The bail-in unravels if depositors are at risk. Second, whether bail-ins mitigate managers' incentives to issue low quality loans, depends on managers' payoff in a bail-in. As with bailouts, it is important that managers' fortunes remain closely linked at all times to the state of the bank; managers may have to be punished in the event of heavy losses. The BRRD stipulates that management should in principle be replaced in a bail-in. Our model shows that merely replacing managers exacerbates moral hazard problems if managers have no liability and walk away scot-free. Finally, while a bail-in turns an insolvent bank into a solvent one, it does not inject any new capital (unlike bailouts). Bail-ins may therefore not resolve a bank's liquidity problems.

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8 Appendix

Proof of Proposition 1. The Hamilton-Jacobi-Bellman (HJB) equation associated with equation (9) has the following general form:

$$\begin{aligned}\delta M_j(N_t) = \max_{q_t,l_t,f}\Big\{ & u(q_tN_t) \ - mq_tN_t\frac{\partial M_j(N_t)}{\partial N_t} + [\mu+\kappa\lambda f-\rho_j(l_t,f)]l_tN_t\frac{\partial M_j(N_t)}{\partial N_t} \\ & + \frac{1}{2}\sigma^2 l_t^2 N_t^2 \frac{\partial^2 M_j(N_t)}{\partial N_t^2} + \rho_j(l_t,f)N_t\frac{\partial M_j(N_t)}{\partial N_t} \\ & + \lambda\left[p_j M_j(\phi_j(l_t,f)N_t) - M_j(N_t)\right]\Big\} \end{aligned} \tag{33}$$

Conjecturing the value function in form of $M_j(N) = \frac{C_jN^{1-\eta}}{1-\eta}$, the HJB equation can be written as:

$$\begin{aligned}\frac{\lambda+\delta}{1-\eta} = \max_{q>0,l,f}\Big\{ & \frac{q^{1-\eta}}{C_j(1-\eta)} - mq + [\mu+\kappa\lambda f-\rho_j(l,f)]l - \frac{\sigma^2\eta}{2}l^2 \\ & + \rho_j(l,f) + \frac{\lambda p_j}{1-\eta}[\phi_j(l,f)]^{1-\eta}\Big\} \end{aligned} \tag{34}$$

The right-hand-side of (34) decouples into:

$$\max_{q>0}\left\{\frac{q^{1-\eta}}{C_j(1-\eta)} - mq\right\} + \max_{l,f}\left\{[\mu+\kappa\lambda f-\rho_j(l,f)]l - \frac{\sigma^2\eta}{2}l^2 + \rho_j(l,f) + \frac{\lambda p_j}{1-\eta}[\phi_j(l,f)]^{1-\eta}\right\}$$

In every regime, the optimal q is given by a simple first order condition leading to $q_j = (mC_j)^{-1/\eta}$. Meanwhile, the feasible domain of (l,f) depends on whether we are in the solvency or insolvency regime. For $j = s$ the constraint is $l \leq 1/f$ where for $j = b, o, i$ we have $l > 1/f$ instead. The optimal (l, f) can then be obtained by maximizing the following investment objective function on the relevant regime:

$$\max_{l,f} G_j(l,f) \equiv \max_{l,f} \left\{ [\mu + \kappa\lambda f - \rho_j(l,f)]l - \frac{\sigma^2\eta}{2}l^2 + \rho_j(l,f) + \frac{\lambda p_j}{1-\eta}[\phi_j(l,f)]^{1-\eta} \right\}$$

Denote the optimizers by l_j and f_j, and the optimized investment function by $H_j \equiv G_j(l_j, f_j)$. The unknown claim value multiplier C_j can be solved by putting $q = q_j$, $l = l_j$ and $f = f_j$ in (34) which gives $\frac{\eta}{1-\eta}m^{1-\frac{1}{\eta}}C_j^{-\frac{1}{\eta}} + H_j - \frac{\lambda+\delta}{1-\eta} = 0$ and in turn $C_j = \left[\frac{\eta}{\lambda+\delta-(1-\eta)H_j}\right]^{\eta} m^{\eta-1}$. C_j is well-defined for as long as $H_j < \frac{\lambda+\delta}{1-\eta}$.[30] C_j is increasing in H_j since we work under $\eta < 1$. Thus to compare the managers' claim value under different regimes, it is sufficient to compare the H_j's. ■

Proof of Proposition 3. In the asset sales regime, $\rho_s = \rho$, $\phi_s(l,f) = 1 - fl$ and $p_s = 1$. Then the investment objective function is:

$$G_s(l,f) \equiv (\mu + \kappa\lambda f - \rho)l - \frac{\sigma^2\eta}{2}l^2 + \rho + \frac{\lambda}{1-\eta}(1-fl)^{1-\eta}$$

and our goal is to find the pair (l, f) satisfying $l \leq 1/f$ and $0 \leq f \leq 1$ which maximizes $G_s(l, f)$.

The solution strategy is the following sequential optimization approach which we will also adopt for the other regimes. In the first stage, we consider f as a given constant and we find l satisfying $l \leq 1/f$ which maximizes $G_s(l, f)$. Denote the maximizer by $l_s(f)$ which depends on the value of the fixed f. Then the second stage optimization involves finding $0 \leq f \leq 1$ which maximizes $G_s(l_s(f), f)$. Suppose the maximizer is f_s. Then the pair of optimizers to the original problem is given by $(l_s(f_s), f_s)$.

In the first stage problem, direct differentiation gives $\frac{\partial}{\partial l}G_s(l,f) = \mu + \kappa\lambda f - \rho - \sigma^2\eta l - \frac{f\lambda}{(1-fl)^{\eta}}$ and $\frac{\partial^2}{\partial l^2}G_s(l) = -\sigma^2\eta - f^2\lambda\eta(1-fl)^{-\eta-1} < 0$. Note that $\frac{\partial}{\partial l}G_s(l,f) \to -\infty$

[30] Indeed, $H_j < \frac{\lambda+\delta}{1-\eta}$ is the necessary and sufficient condition for the managers' claim to have finite value under regime j.

as $l \to \frac{1}{f}$, and since $\kappa > 1$ we have $\frac{\partial}{\partial l} G_s(l, f)\Big|_{l=0} = \mu + (\kappa - 1)\lambda f - \rho > \mu - \rho > 0$. The first order condition:

$$\mu + \kappa\lambda f - \rho - \sigma^2 \eta l - \frac{f\lambda}{(1 - fl)^\eta} = 0 \tag{35}$$

has exactly one root given by some $l_s(f) \in (0, 1/f)$ which is the maximizer of $G_s(l, f)$ on $l \leq 1/f$ under a fixed f.

Write $H_s(f) \equiv G_s(l_s(f), f)$. In the second stage problem we want to find $0 \leq f \leq 1$ maximizing $H_s(f)$. Since $l_s(f)$ satisfies the first order condition $\frac{\partial G_s}{\partial l}\Big|_{l=l_s(f)} = 0$, we have:

$$\begin{aligned} H_s'(f) = \frac{\partial G_s}{\partial l}\Big|_{l=l_s(f)} \times \frac{dl_s(f)}{df} + \frac{\partial G_s}{\partial f}\Big|_{l=l_s(f)} &= \lambda l_s(f) \left(\kappa - \frac{1}{(1 - fl_s(f))^\eta}\right) \\ &= \frac{l_s(f)}{f} \left(-\mu + \rho + \sigma^2 \eta l_s(f)\right) \end{aligned}$$

where the last equality is due to (35). The first order condition $H_s'(f) = 0$ gives $l_s(f) = \frac{\mu-\rho}{\sigma^2\eta}$,[31] and the associated f is obtained from $\kappa - \frac{1}{(1-fl_s(f))^\eta} = 0$ leading to a candidate solution $f = f_s \equiv \frac{\sigma^2\eta}{\mu-\rho}\left(1 - \kappa^{-\frac{1}{\eta}}\right)$. Note that $l_s(f_s)f_s = 1 - \kappa^{-\frac{1}{\eta}} < 1$. The condition $\frac{\mu-\rho}{\sigma^2\eta} > 1$ ensures $f_s < 1$.

We have shown that $H_s'(f) = 0$ has a unique root at some $0 < f_s < 1$. It remains to check this candidate solution f_s indeed corresponds to a maximum of $H_s(f)$. By considering (35), it can be easily verified that $l_s(0) = \frac{\mu-\rho}{\sigma^2\eta} > 0$ and $l_s(1) < 1$. Hence $H_s'(0) = (\kappa - 1)\lambda l_s(0) > 0$ and $H_s'(1) = l_s(1)\left(-\mu + \rho + \sigma^2\eta l_s(1)\right) < 0$. Then we must have $H_s'(f) \geq 0$ for $0 \leq f \leq f_s$ and $H_s'(f) \leq 0$ for $f_s \leq f \leq 1$. We conclude a maximum is attained at f_s. ■

Proof of Proposition 2 and 4 (Complemented by the internet appendix). In each of the following subsections, we will first prove for each IRM the form of the optimal l_j and f_j (i.e. Proposition 4), and then verify the existence of $\underline{\kappa}_j$ above which managers will put the bank at risk of insolvency (i.e. Proposition 2).

[31] It is easy to check that $l_s(f) = 0$, the alternative solution of the first order condition, will lead to a candidate optimizer $f = -\frac{\mu-\rho}{\lambda(\kappa-1)} < 0$ which is not feasible.

(i) Liquidation regime

In the liquidation regime, $\rho_b(l,f) = \rho + \lambda(1-\tau)\left[\frac{fl-1+(1-f)lc_b}{l-1}\right]$, $\phi_b = 0$ and $p_b = 0$. The investment objective function is then:

$$\begin{aligned} G_b(l,f) &\equiv (\mu + \kappa\lambda f - \rho_b(l,f))l - \frac{\sigma^2\eta}{2}l^2 + \rho_b(l,f) \\ &= [\mu + (\kappa - (1-\tau)(1-c_b))\lambda f - \rho - \lambda(1-\tau)c_b]l - \frac{\sigma^2\eta}{2}l^2 + \rho + \lambda(1-\tau) \end{aligned}$$

We first find the maximizer of the above function over $l > \frac{1}{f}$ under a fixed f. There are two possibilities. If $1/f < l_b(f) \equiv \frac{\mu+(\kappa-(1-\tau)(1-c_b))\lambda f-\rho-\lambda(1-\tau)c_b}{\sigma^2\eta}$, then since $l = l_b(f)$ solves the first order condition:

$$\frac{\partial}{\partial l}G_b(l,f) = \mu + (\kappa - (1-\tau)(1-c_b))\lambda f - \rho - \lambda(1-\tau)c_b - \sigma^2\eta l = 0 \tag{36}$$

and since G_b is concave in l, it must attain the maximum at $l = l_b(f)$ on $l > 1/f$. Otherwise if $1/f \geq l_b(f)$, then G_b is strictly decreasing in l on $l > 1/f$ and the maximum is attained at $1/f$. If we define $\hat{f}_b \in (0,1)$ as the unique solution to the equation $l_b(f) = 1/f$ or equivalently:

$$\zeta_b(f) \equiv \frac{\mu + (\kappa - (1-\tau)(1-c_b))\lambda f - \rho - \lambda(1-\tau)c_b}{\sigma^2\eta} - \frac{1}{f} = 0$$

then the condition $1/f < (\geq) l_b(f)$ is equivalent to $f > (\leq)\hat{f}_b$. The optimized value function is hence given by:

$$H_b(f) \equiv \begin{cases} G_b\left(\frac{1}{f}, f\right) = \frac{(\mu+(\kappa-(1-\tau)(1-c_b))\lambda f-\rho-\lambda(1-\tau)c_b)}{f} - \frac{\sigma^2\eta}{2f^2} + \rho + \lambda(1-\tau), & f \leq \hat{f}_b \\ G_b\left(l_b(f), f\right) = \frac{(\mu+(\kappa-(1-\tau)(1-c_b))\lambda f-\rho-\lambda(1-\tau)c_b)^2}{2\sigma^2\eta} + \rho + \lambda(1-\tau), & f > \hat{f}_b \end{cases}$$

In the second stage of the optimization problem we differentiate $H_b(f)$ on $f \leq \hat{f}_b$ and $f > \hat{f}_b$ respectively. On $f > \hat{f}_b$:

$$\begin{aligned} H_b'(f) &= \lambda(\kappa - (1-\tau)(1-c_b))\frac{\mu + (\kappa - (1-\tau)(1-c_b))\lambda f - \rho - \lambda(1-\tau)c_b}{\sigma^2\eta} \\ &> \lambda(\kappa - (1-\tau)(1-c_b))\frac{\mu + (\kappa - (1-\tau)(1-c_b))\lambda \hat{f}_b - \rho - \lambda(1-\tau)c_b}{\sigma^2\eta} \\ &= \frac{\lambda(\kappa - (1-\tau)(1-c_b))}{\hat{f}_b} > 0 \end{aligned}$$

On $f \leq \hat{f}_b$:

$$H_b'(f) = \frac{d}{df} G_b\left(\frac{1}{f}; f\right) = \frac{\partial}{\partial l} G_b(l,f)\Big|_{l=1/f} \times \frac{d}{df}\left(\frac{1}{f}\right) + \frac{\partial}{\partial f} G_b(l,f)\Big|_{l=1/f}$$
$$= -\frac{\partial}{\partial l} G_b(l,f)\Big|_{l=1/f} \times \frac{1}{f^2} + \lambda(\kappa - (1-\tau)(1-c_b))\frac{1}{f} \geq 0$$

since $G_b(l,f)$ is decreasing for all $l \geq 1/f$ when $f \leq \hat{f}_b$ and hence $\frac{\partial}{\partial l} G_b(l,f)\Big|_{l=1/f} \leq 0$. In both cases, H_b is increasing in f such that it is maximized at $f = f_b \equiv 1$. The corresponding investment level is $l_b(f_b) = \frac{\mu + (\kappa - 1 + \tau)\lambda - \rho}{\sigma^2 \eta}$.

To show the existence of $\underline{\kappa}_b$ above (below) which managers will prefer a risky (safe) bank and engage in liquidation (asset sales) in a crash, view $H_b = G_b(l_b(f_b), f_b)$ and $H_s = G_s(l_s(f_s), f_s)$ as functions of κ and let $J_b(\kappa) = H_b - H_s = H_b(\kappa) - H_s(\kappa)$. The general strategy of the proof, which we will also adopt for the other regimes, is to show that the function J_b is increasing in κ and thus there exists critical $\underline{\kappa}_b \geq 1$ such that $H_b \geq (<) H_s$ when $\kappa \geq \underline{\kappa}_b$ $(1 \leq \kappa < \underline{\kappa}_b)$.

Since f_s and $l_s(f_s)$ are available in closed-form from Proposition 3, we can compute:

$$H_s = G_s(l_s(f_s), f_s) = \frac{(\mu - \rho)^2}{2\sigma^2 \eta} + \kappa\lambda + \rho + \frac{\lambda\eta}{1-\eta}\kappa^{-\frac{1-\eta}{\eta}}$$

and then we obtain $\frac{dH_s}{d\kappa} = \lambda - \lambda\kappa^{-\frac{1}{\eta}}$. On the other hand:

$$\frac{dH_b}{d\kappa} = \frac{d}{d\kappa}\left(\frac{(\mu + (\kappa - (1-\tau))\lambda - \rho)^2}{2\sigma^2\eta} + \rho + \lambda(1-\tau)\right)$$
$$= \frac{\mu + (\kappa - (1-\tau))\lambda - \rho}{\sigma^2 \eta}\lambda = l_b \lambda$$

such that $J_b'(\kappa) = \lambda\kappa^{-\frac{1}{\eta}} + \lambda(l_b - 1) > 0$. Hence $J_b(\kappa)$ is strictly increasing and there exists $\underline{\kappa}_b \geq 1$ such that $J_b(\kappa) < (\geq) 0$ for $1 \leq \kappa < \underline{\kappa}_b$ $(\kappa \geq \underline{\kappa}_b)$.[32]

(ii) Bailout regime

[32]Strictly speaking, to rule out the case of $\underline{\kappa}_b = \infty$ we should also verify that $J_b(\infty) > 0$. This result is not hard to be established, and can be done by making use of the analytical expression of H_s and observing that H_b has a quadratic growth for large κ. Note that it is possible to have $\underline{\kappa}_b = 1$ and in this case $H_b \geq H_s$ for all $\kappa \geq 1$.

In the bailout regime we have $\rho_o = \rho$, $\phi_o(l, f) = (1-f)\xi_o$ and $p_o \in [0, 1]$. If we define $v_o \equiv p_o \xi_o^{1-\eta}$ as the dilution-adjusted continuation probability of insiders (the same notation we have used in Proposition 5), the investment objective function is:

$$G_o(l, f) \equiv (\mu + \kappa\lambda f - \rho)l - \frac{\sigma^2\eta}{2} l^2 + \rho + \frac{\lambda v_o}{1-\eta}(1-f)^{1-\eta}$$

Similar to the analysis of the liquidation regime, we can define $\hat{f}_o \in (0, 1)$ as the unique solution to the equation:

$$\zeta_o(f) \equiv \frac{\mu + \kappa\lambda f - \rho}{\sigma^2\eta} - \frac{1}{f} = 0$$

Then under a fixed f the maximizer of $G_o(l, f)$ on $l > 1/f$ is given by $l = l_o(f) \equiv \frac{\mu+\kappa\lambda f-\rho}{\sigma^2\eta}$ when $f > \hat{f}_o$, or $l = 1/f$ when $f \leq \hat{f}_o$. Substituting the maximizer into $G_o(l, f)$ gives optimized value function under a fixed f as:

$$H_o(f) \equiv \begin{cases} G_o(1/f, f) = \frac{\mu+\kappa\lambda f-\rho}{f} - \frac{\sigma^2\eta}{2f^2} + \rho + \frac{\lambda v_o}{1-\eta}(1-f)^{1-\eta}, & 0 \leq f \leq \hat{f}_o \\ G_o(l_o(f), f) = \frac{(\mu+\kappa\lambda f-\rho)^2}{2\sigma^2\eta} + \rho + \frac{\lambda v_o}{1-\eta}(1-f)^{1-\eta}, & \hat{f}_o < f \leq 1 \end{cases}$$

If $v_o = 0$, the optimization problem then resembles the one in the liquidation regime and it is straightforward to verify that $H_o(f)$ is increasing such that the maximum is attained at $f_o \equiv 1$. We only outline the strategy of the proof here for the case of $v_o > 0$ and defer the technical details to the internet appendix. The main complication here originates from the piecewise definition of $H_o(f)$ on $f \leq \hat{f}_o$ and $f > \hat{f}_o$ respectively leading to two different first order conditions. Under condition (13) on the Merton ratio $\frac{\mu-\rho}{\sigma^2\eta} > 1 + \frac{v_o}{\kappa}$, we can show that $H_o(f)$ is indeed monotonically increasing on $f \leq \hat{f}_o$ and attains a global interior maximum on $f > \hat{f}_o$. Hence $f_o \in (\hat{f}_o, 1)$ is given by the first order condition derived over the second regime of $\hat{f}_o < f \leq 1$:

$$H_o'(f) = \frac{\kappa\lambda(\mu + \kappa\lambda f - \rho)}{\sigma^2\eta} - \frac{v_o\lambda}{(1-f)^\eta} \equiv \lambda\kappa\Theta_o(f) = 0 \tag{37}$$

and the corresponding investment level is $l_o(f_o) = \frac{\mu+\kappa\lambda f_o-\rho}{\sigma^2\eta}$.

Finally, similar to the proof of the liquidation regime, the existence of $\underline{\kappa}_o$ can be verified by showing that $J_o(\kappa) \equiv H_o(\kappa) - H_s(\kappa)$ is increasing. We give the proof for the case of $v_o > 0$ as an illustration. The case of $v_o = 0$ is easier since $f_o = 1$ which leads to an analytical expression of H_o.

Note that:

$$H_o = G_o(l_o, f_o) = G_o(l_o(f_o(\kappa); \kappa), f_o(\kappa); \kappa)$$

which depends on κ explicitly via the definition of G_o as well as implicitly via $f_o = f_o(\kappa)$ and $l_o(f_o) = l_o(f_o(\kappa); \kappa)$. But since f_o and l_o satisfy the first order conditions $\frac{\partial G_o}{\partial l}\Big|_{l=l_o, f=f_o} = \frac{\partial G_o}{\partial f}\Big|_{l=l_o, f=f_o} = 0$ when $v_o > 0$, envelope theorem leads to $\frac{dH_o}{d\kappa} = \frac{\partial G_o}{\partial \kappa}\Big|_{l=l_o, f=f_o} = \lambda f_o l_o$. Then:

$$J_o'(\kappa) = H_o'(\kappa) - H_s'(\kappa) = \lambda \kappa^{-\frac{1}{\eta}} + \lambda(f_o l_o - 1) > 0$$

as $f_o l_o > 1$ on the insolvency regime. Hence $J_o(\kappa)$ is strictly increasing.

(iii) Bail-in regime

Under bail-in, ρ_i is given by (6), $\phi_i = (1 - f)\xi_i$ and $p_i \in [0, 1]$. If we define $v_i \equiv p_i \xi_i^{1-\eta}$, the investment objective function is thus:

$$\begin{aligned} G_i(l, f) &= [\mu + \kappa\lambda f - \rho_i(l, f)]l - \frac{\sigma^2\eta}{2}l^2 + \rho_i(l, f) + \frac{\lambda v_i}{1-\eta}(1 - f)^{1-\eta} \\ &= \{\mu + [\kappa - (1-\tau)(1+h)]\lambda f - \rho + \lambda h(1-\tau)\}l - \frac{\sigma^2\eta}{2}l^2 + \rho + \lambda(1-\tau) \\ &\quad + \frac{\lambda v_i}{1-\eta}(1-f)^{1-\eta} \end{aligned} \tag{38}$$

As before, we first solve for the l maximizing $G_i(l, f)$ over $l > 1/f$ under a fixed f which can be derived using the exact same argument as in the bailout case. In particular, the optimizer is given by:

$$l = \begin{cases} \frac{1}{f}, & f \leq \hat{f}_i \\ l_i(f) \equiv \frac{\mu + [\kappa - (1-\tau)(1+h)]\lambda f + \lambda h(1-\tau) - \rho}{\sigma^2\eta}, & f > \hat{f}_i \end{cases}$$

where $\hat{f}_i \in (0, 1)$ is the solution to the equation:

$$\zeta_i(f) \equiv \frac{\mu + [\kappa - (1-\tau)(1+h)]\lambda f + \lambda h(1-\tau) - \rho}{\sigma^2\eta} - \frac{1}{f} = 0$$

In the second stage problem, we are solving for f which maximizes $H_i(f)$ where:

$$H_i(f) \equiv \begin{cases} G_i(1/f, f) = \frac{\mu+[\kappa-(1+h)(1-\tau)]\lambda f-\rho+\lambda h(1-\tau)}{f} - \frac{\sigma^2\eta}{2f^2} + \rho + \lambda(1-\tau) + \frac{\lambda v_i}{1-\eta}(1-f)^{1-\eta}, & f \leq \hat{f}_i \\ G_i(l_i(f), f) = \frac{\{\mu+[\kappa-(1+h)(1-\tau)]\lambda f-\rho+\lambda h(1-\tau)\}^2}{2\sigma^2\eta} + \rho + \lambda(1-\tau) + \frac{\lambda v_i}{1-\eta}(1-f)^{1-\eta}, & f > \hat{f}_i \end{cases}$$

When $v_i = 0$, it is easy to verify that $H_i(f)$ is increasing under the condition (14) that $\kappa > (1+h)(1-\tau)$ such that the maximizer is given by $f = f_i \equiv 1$, and then $l_i = l_i(f_i) = \frac{\mu+(\kappa-1+\tau)\lambda-\rho}{\sigma^2\eta} > 1 = \frac{1}{(1+h)f_i-h} = \hat{l}_i(f_i; h)$ such that $\rho_i(l_i, f_i) > \rho$.

Suppose $v_i > 0$. Under condition (14) that $\frac{\mu-\rho+\lambda h(1-\tau)}{\sigma^2\eta} > 1 + \frac{v_i}{\kappa-(1+h)(1-\tau)}$, we show in the internet appendix that H_i is increasing on $f \leq \hat{f}_i$ and attains an interior maximum at $f = f_i$ on $f > \hat{f}_i$ where f_i is given by the solution to the first order condition:

$$\Theta_i(f) \equiv \frac{\mu + [\kappa - (1-\tau)(1+h)]\lambda f - \rho + \lambda h(1-\tau)}{\sigma^2\eta} - \frac{v_i}{[\kappa - (1-\tau)(1+h)](1-f)^\eta} = 0 \quad (39)$$

Finally, since $h \leq 0$ we have $l_i > 1/f_i \geq \frac{1}{(1+h)f_i-h} = \hat{l}_i(f_i; h)$ and as such $\rho_i(l_i, f_i) > \rho$.

Now we verify the claim in footnote 20 that $l_i > \frac{\mu-\rho}{\sigma^2\eta}$ which is equivalent to showing $f_i > -\frac{h(1-\tau)}{\kappa-(1-\tau)(1+h)}$. But:

$$\begin{aligned} \Theta_i\left(-\frac{h(1-\tau)}{\kappa-(1-\tau)(1+h)}\right) &= \frac{\mu-\rho}{\sigma^2\eta} - \frac{v_i}{[\kappa-(1-\tau)(1+h)]^{1-\eta}[\kappa-(1-\tau)]^\eta} \\ &= \frac{\mu-\rho}{\sigma^2\eta} - \frac{v_i}{\kappa-(1-\tau)(1+h)}\left(\frac{\kappa-(1-\tau)(1+h)}{\kappa-(1-\tau)}\right)^\eta \\ &> \frac{\mu-\rho}{\sigma^2\eta} - \frac{v_i}{\kappa-(1-\tau)(1+h)} \\ &> \frac{\mu-\rho+\lambda h(1-\tau)}{\sigma^2\eta} - \frac{v_i}{\kappa-(1-\tau)(1+h)} > 1 > 0 \end{aligned}$$

by assumption (14), which establishes the claim as f_i is given by the down-crossing of $\Theta_i(f) = 0$.

The existence of $\underline{\kappa}_i$ can be proven using the same method as in the case of bailout.

■

Proof of Proposition 5. i) Since $f_b = 1$ we must have $f_b \geq f_o$ and $f_b \geq f_i$. Further recall again that under conditions (13) and (14), f_o and f_i are the unique roots of the equations in (37) and (39):

$$\Theta_o(f) \equiv \frac{\mu + \kappa\lambda f - \rho}{\sigma^2\eta} - \frac{v_o}{\kappa(1-f)^\eta} = 0$$
$$\Theta_i(f) \equiv \frac{\mu + [\kappa - (1+h)(1-\tau)]\lambda f - \rho + \lambda h(1-\tau)}{\sigma^2\eta} - \frac{v_i}{[\kappa - (1+h)(1-\tau)](1-f)^\eta} = 0$$

respectively. Note that:

$$\begin{aligned}\mu + [\kappa - (1+h)(1-\tau)]\lambda f - \rho + \lambda h(1-\tau) &= \mu + \kappa\lambda f - \rho - \lambda(1-\tau)[(1+h)f - h] \\ &< \mu + \kappa\lambda f - \rho\end{aligned}$$

for any f with $h \leq 0$. Moreover, $\frac{v_o}{\kappa} \leq \frac{v_i}{\kappa-(1+h)(1-\tau)}$ provided that either $v_i \geq v_o$, or $v_o > v_i > 0$ and $(1+h)(1-\tau) \leq \kappa \leq \frac{v_o(1+h)(1-\tau)}{v_o - v_i}$. Then $\Theta_o(f) \geq \Theta_i(f)$ and hence $f_i \leq f_o$ since again each root f_j is given by a down-crossing of $\Theta_j(f) = 0$ $(j = o, i)$.

ii) We have shown in the bail-in regime that $\rho_i(l_i, f_i) > \rho$. Moreover, it can be easily verified from construction of ρ_i that $\rho_i(l, f) \leq \rho + \lambda(1-\tau)$ for any $l > 1/f$. Then the result follows since $\rho_o = \rho$ and $\rho_b(l_b, f_b) = \rho_b(l_b, 1) = \rho + \lambda(1-\tau)$.

iii) On the one hand, we have:

$$\begin{aligned}l_i &= \frac{\mu + [\kappa - (1+h)(1-\tau)]\lambda f_i - \rho + \lambda h(1-\tau)}{\sigma^2\eta} \\ &\leq \frac{\mu + [\kappa - (1+h)(1-\tau)]\lambda \times 1 - \rho + \lambda h(1-\tau)}{\sigma^2\eta} = \frac{\mu + (\kappa - 1 + \tau)\lambda - \rho}{\sigma^2\eta} = l_b\end{aligned}$$

On the other hand, we want to show $l_o \geq l_b$ which is:

$$\frac{\mu + \kappa\lambda f_o - \rho}{\sigma^2\eta} \geq \frac{\mu + (\kappa - 1 + \tau)\lambda f_b - \rho}{\sigma^2\eta} = \frac{\mu + (\kappa - 1 + \tau)\lambda - \rho}{\sigma^2\eta}$$

or equivalently $f_o \geq \frac{\kappa-1+\tau}{\kappa}$. We make use of the function $\Theta_o(f)$ in (37) where f_o is defined as the solution to $\Theta_o(f) = 0$. Check that:

$$\Theta_o\left(\frac{\kappa-1+\tau}{\kappa}\right) = \frac{\mu+(\kappa-1+\tau)\lambda-\rho}{\sigma^2\eta} - \upsilon_o\kappa^{\eta-1}(1-\tau)^{-\eta}$$
$$\geq \frac{\mu-\rho}{\sigma^2\eta} - \frac{\upsilon_o}{\kappa}\left(\frac{\kappa}{1-\tau}\right)^{\eta} = \frac{1}{1-\tau}\left(\frac{\mu'-\rho'}{\sigma'^2\eta} - \upsilon_o(\kappa')^{\eta-1}\right) \geq 0 = \Theta_o(f_o)$$

since $\frac{\mu'-\rho'}{\sigma'^2\eta} \geq 1$ (recall each parameter with a prime symbol is its pre-tax value). The result follows as f_o is a down-crossing of $\Theta_o(f) = 0$.

iv) For $j \in \{b, o, i\}$, $G_j(l, f) = [\mu + \kappa\lambda f - \rho_j(l, f)]l - \frac{\sigma^2}{2}\eta l^2 + \rho_j(l, f) + \frac{\lambda \upsilon_j}{1-\eta}(1-f)^{1-\eta}$ with $\upsilon_b = 0$ and $0 \leq \upsilon_o, \upsilon_i \leq 1$. Now we verify that $\rho_i(l, f) \leq \rho_b(l, f)$. This is equivalent to:

$$\rho + \lambda(1-\tau)\left[1 - \frac{(1-f)(1+h)l}{l-1}\right] \leq \rho + \lambda(1-\tau)\left[1 - \frac{(1-f)(1-c_b)l}{l-1}\right]$$
$$\Longleftrightarrow\ c_b \geq -h \ \Longleftrightarrow\ c_b \geq \frac{\xi_i}{l^*}$$

where the last condition is our standing assumption. Thus we have $\rho = \rho_o(l, f) \leq \rho_i(l, f) \leq \rho_b(l, f)$. Hence we can establish that on $l \geq \hat{l} = 1/f \geq 1$ we have $G_o(l, f) \geq G_i(l, f) \geq G_b(l, f)$ for as long as $\upsilon_o \geq \upsilon_i$. This translates into the ordering of $G_j(l_j, f_j)$, in turn C_j and finally $M_j(N)$.

v) and vi) These follow immediately from the ranking of $M_j(N)$ for $j = b, o, i$. ■

Proof of Proposition 6 (Complemented by the internet appendix). We split the proof into three parts. In the first part, we verify the expressions of W_j for each IRM j. In the second part, we define the concept of internal rate of return (IRR) associated with the market value and prove the ranking of the IRRs under different IRM. Finally, we show how the ranking of the IRRs translate to that of the W_j's.

(i) Expression of W_j

The results under each IRM can be established somewhat similarly. We provide the proof of the bailout case as an illustration. On the one hand:

$$\begin{aligned} I_o &= E\left[\int_0^\infty e^{-\delta t} m q_o N_t dt\right] = m q_o \int_0^\infty e^{-\delta t} E(N_t) dt \\ &= m q_o N_0 \int_0^\infty \exp\left[-(\delta - g_o + \lambda f_o) t\right] dt = \frac{m q_o}{\delta + \lambda f_o - g_o} N_o \end{aligned}$$

On the other hand:

$$\begin{aligned} E[e^{-\delta T_k} N_{T_k}] &= E[e^{-\delta T_{k-1}} e^{-\delta(T_k - T_{k-1})} N_{T_k}] = E\left[E_{T_{k-1}}\left[e^{-\delta T_{k-1}} e^{-\delta(T_k - T_{k-1})} N_{T_k}\right]\right] \\ &= E\left[e^{-\delta T_{k-1}} N_{T_{k-1}} E_{T_{k-1}}\left[e^{-\delta(T_k - T_{k-1})} \frac{N_{T_k}}{N_{T_{k-1}}}\right]\right] \end{aligned}$$

where we have used the law of iterated expectation. But conditioning on the information up to time T_{k-1}, $e^{-\delta(T_k - T_{k-1})} \frac{N_{T_k}}{N_{T_{k-1}}} \overset{d}{=} e^{-\delta T} \frac{N_T}{N_0}$ where T is an $Exp(\lambda)$ random variable due to the stationary properties of the underlying Brownian motion and the Poisson process. Hence using an identity proven in the internet appendix:

$$E_{T_{k-1}}\left[e^{-\delta(T_k - T_{k-1})} \frac{N_{T_k}}{N_{T_{k-1}}}\right] = E\left[e^{-\delta T} \frac{N_T}{N_0}\right] = \frac{(1 - f_o)\lambda}{\delta + \lambda - g_o} \equiv \theta$$

and $E[e^{-\delta T_k} N_{T_k}] = \theta E[e^{-\delta T_{k-1}} N_{T_{k-1}}]$. Then we can deduce iteratively that $E[e^{-\delta T_k} N_{T_k}] = \theta^k N_0$. Finally:

$$C_o = E\left[\sum_{k=0}^{\infty} e^{-\delta T_k} N_{T_k}\right] = N_0 \sum_{k=0}^{\infty} \theta^k = \frac{1}{1 - \theta} N_0 = \frac{\delta + \lambda - g_o}{\delta + \lambda f_o - g_o} N_0$$

as $\theta = 1 - \frac{\delta + \lambda f_o - g_o}{\delta + \lambda - g_o} < 1$, and we obtain $W_o = \left[\frac{m q_o - (\delta + \lambda f_o - g_o)}{\delta + \lambda f_o - g_o}\right] N_0$.

The IRR is given by the value of δ leading to $W_o = 0$. The result can be obtained after substituting g_o by its analytical formula and $m = \frac{1}{1-\alpha}$.

(ii) Definition and ranking of the internal rate of returns

The internal rate of return (IRR) under a particular IRM j is defined as the value of $\delta = \delta_j$ such that $W_j = 0$. With the expressions of W_j derived in the first part of this proof and g_j defined in (4), one can show that

$$\delta_j = \rho_j + (\mu + \kappa\lambda f_j - \rho_j) l_j - \lambda \quad \text{for} \quad j = b, o \tag{40}$$

$$\delta_i = \rho_i + (\mu + \kappa\lambda f_i - \rho_i) l_i - \lambda\left[1 - \Phi_i(l_i, f_i)\xi_i\right] \tag{41}$$

We first verify that $\delta_i \leq \delta_b \leq \delta_o$. To establish the ranking of $\delta_i \leq \delta_b$ with small ξ_i, consider first:

$$\begin{aligned}\delta_i &= \rho_i + (\mu + \kappa\lambda f_i - \rho_i)l_i - \lambda + \lambda\Phi_i\xi_i \\ &= \frac{\{\mu + [\kappa - (1+h)(1-\tau)]\lambda f_i - \rho + \lambda h(1-\tau)\}^2}{\sigma^2\eta} + \rho - \lambda\tau + \lambda\Phi_i\xi_i\end{aligned}$$

Recall that $h = -\frac{\xi_i}{l^*}$ with $-1 < h \leq 0$. If we view δ_i as a function of ξ_i, then for any $\xi_i > 0$ we have:

$$\begin{aligned}\delta_i(\xi_i) &\leq \frac{\{\mu + [\kappa - (1-\tau)]\lambda f_i - \rho\}^2}{\sigma^2\eta} + \rho - \lambda\tau + \lambda\Phi_i\xi_i \\ &= \frac{\{\mu + [\kappa - (1-\tau)]\lambda f_i - \rho\}^2}{\sigma^2\eta} + \rho - \lambda\tau + \lambda\frac{l_i}{l^*}(1-f_i)\xi_i \\ &< \frac{\{\mu + [\kappa - (1-\tau)]\lambda - \rho\}^2}{\sigma^2\eta} + \rho - \lambda\tau + \lambda\frac{l_b}{l^*}(1-f_i)\xi_i \\ &= \delta_b + \lambda\frac{l_b}{l^*}(1-f_i)\xi_i\end{aligned}$$

where we have used the facts that $l_b > l_i$ and $f_i < 1$ for $p_i\xi_i \neq 0$, and that:

$$\delta_b = \frac{[\mu + (\kappa - 1 + \tau)\lambda - \rho]^2}{\sigma^2\eta} + \rho + \lambda(1-\tau) - \lambda$$

Since $\lambda\frac{l_b}{l^*}(1-f_i)\xi_i \to 0$ as $\xi_i \to 0$, we conclude $\delta_i(\xi_i) < \delta_b$ for small $\xi_i \approx 0$. Note that we have $\delta_i(0) = \delta_b$ because $f_i(\xi_i = 0) = 1$. Hence this result also implies $\delta_i'(0) \leq 0$.

Now we are going to establish that $\delta_o \geq \delta_b$ which is equivalent to:

$$\begin{aligned}&\frac{(\mu + \kappa\lambda f_o - \rho)^2}{\sigma^2\eta} + \rho \geq \frac{(\mu + (\kappa - 1 + \tau)\lambda - \rho)^2}{\sigma^2\eta} + \rho + \lambda(1-\tau) \\ &\iff (\mu + \kappa\lambda f_o - \rho)^2 - (\mu + (\kappa - 1 + \tau)\lambda - \rho)^2 \geq \lambda\eta\sigma^2(1-\tau) \\ &\iff \left(2\frac{\mu-\rho}{\sigma^2\eta} + \frac{\kappa\lambda f_o + (\kappa - 1 + \tau)\lambda}{\sigma^2\eta}\right)\left(f_o - \frac{\kappa - 1 + \tau}{\kappa}\right) \geq \frac{1-\tau}{\kappa} \\ &\Longleftarrow 2\left(f_o - \frac{\kappa - 1 + \tau}{\kappa}\right) \geq \frac{1-\tau}{\kappa}\end{aligned}$$

It is hence sufficient to show $f_o \geq 1 - \frac{1-\tau}{2\kappa}$. This can be done using the same argument in the proof of part iii) of Proposition 5 under a stronger condition that $\frac{\mu' - \rho'}{(\sigma')^2\eta} > 2$.

(iii) Ranking of the market values

Finally, for the comparison of market value, we will assume that the discount rate δ used for computation of W_j is restricted to $\delta_o \geq \delta \geq \delta_i$. The net market value associated with an IRM under arbitrary discount rate δ can be expressed in terms of its IRR. For bailout versus liquidation, note that:

$$W_o = \frac{\delta_o - \delta}{\delta - \lambda(1 - f_o) - \delta_o + mq_o} N_0 \geq \frac{\delta_o - \delta}{\delta - \delta_o + mq_o} N_0 \geq \frac{\delta_b - \delta}{\delta - \delta_b + mq_b} N_0 = W_b$$

for $\delta \leq \delta_o$ since $\delta_o \geq \delta_b$ and $q_o \leq q_b$ (recall part v of Proposition 5).

For liquidation versus bail-in, it is clear that $W_b \geq W_i$ on $\delta_b \geq \delta \geq \delta_i$ since on this range we have $W_b \geq 0 \geq W_i$. It remains to show that $W_b \geq W_i$ on $\delta > \delta_b$ for sufficiently small ξ_i. Now:

$$W_i = \frac{\delta_i - \delta}{\delta - \delta_i + mq_i - \lambda \Phi_i \xi_i} \leq \frac{\delta_i - \delta}{\delta - \delta_i + mq_b} = -\frac{1}{1 + \frac{mq_b}{\delta - \delta_i}} \equiv \Gamma(\xi_i)$$

where the inequality holds because $\delta_i - \delta < 0$ and $q_b \geq q_i$. Using the equivalence of liquidation and bail-in under $\xi_i = 0$ as per Corollary 1, it is easy to verify that $\Gamma(0) = W_b$. Moreover:

$$\Gamma'(\xi_i) = \frac{1}{\left(1 + \frac{mq_b}{\delta - \delta_i}\right)^2} \frac{mq_b}{(\delta - \delta_i)^2} \delta_i'(\xi_i)$$

In turn $\Gamma'(0) \leq 0$ since $\delta_i'(0) \leq 0$. Then for small ξ_i we deduce $W_i \leq \Gamma(\xi_i) \leq \Gamma(0) = W_b$. ■

Proof of Proposition 7. We just provide a sketch of proof since the idea is very similar to the proof of Proposition 4. Using the expressions in (29), the managers' objective function can be written as:

$$\begin{aligned} G_d(l, f) &= [\mu + \kappa \lambda f - (\rho + \lambda(1 - \tau) f)] l - \frac{\sigma^2 \eta}{2} l^2 + \rho + \lambda(1 - \tau) f + \frac{\lambda p_d \xi_d^{1-\eta}}{1 - \eta} (1 - f)^{1-\eta} \\ &= [\mu + (\kappa - 1 + \tau) \lambda f - \rho] l - \frac{\sigma^2 \eta}{2} l^2 + \rho + \lambda(1 - \tau) f + \frac{\lambda v_d}{1 - \eta} (1 - f)^{1-\eta} \end{aligned}$$

If $v_d = 0$, then $G_d(l, f)$ is obviously increasing in f and hence $f_d = 1$ is optimal. The optimal l can then be solved by maximizing a quadratic function in l where the optimizer has the form of (30).

For the more general case of $v_d > 0$, the first order conditions with respect to l and f are found as:

$$\mu + (\kappa - 1 + \tau)\lambda f - \rho - \sigma^2 \eta l = 0$$
$$(\kappa - 1 + \tau)\lambda l + \lambda(1 - \tau) - \lambda v_d (1 - f)^{-\eta} = 0$$

These lead to the expressions in (30) and (31). Moreover, provided that $\frac{\mu-\rho}{\eta\sigma^2} > \frac{v_d}{\kappa-(1-\tau)}$ we can follow the same argument in the internet appendix to verify that the first order condition in f admits a unique solution and the local extremum (l_d, f_d) indeed correspond to a global maximum. Finally, the condition $\frac{\mu-\rho}{\eta\sigma^2} > \frac{v_d}{\kappa-(1-\tau)}$ ensures the left-hand-side of (31) to be strictly positive at $f_d = 0$, and hence the optimal f_d must be strictly positive as well. Thus $\rho_d = \rho_d(l_d, f_d) = \rho + \lambda(1 - \tau)f_d > \rho$, i.e. the junior debt is risky. ■

Proof of Proposition 8. Write $v \equiv v_i = v_d$ for notional convenience. We focus on the case $v > 0$. If $v = 0$, then $f_d = f_i = 1$ and it is then straightforward to verify that the stated inequalities become equalities as special corner cases.

Let $v \equiv p\xi^{1-\eta}$. Given that $v \neq 0$, recall from Proposition 4 that the first order condition of f_i is given by:

$$\begin{aligned} l_i(f) - \frac{v}{[\kappa - (1-\tau)(1+h)](1-f)^\eta} = 0 &\iff [\kappa - (1-\tau)(1+h)]l_i(f) - v(1-f)^{-\eta} = 0 \\ &\iff [\kappa - (1-\tau)]l_i(f) - (1-\tau)hl_i(f) - v(1-f)^{-\eta} = 0 \\ &\iff \Xi_i(f) = 0 \end{aligned}$$

where:

$$\Xi_i(f) \equiv [\kappa - (1-\tau)]l_i(f) - (1-\tau)hl_i(f) - v(1-f)^{-\eta}$$
$$l_i(f) \equiv \frac{\mu + [\kappa - (1-\tau)]\lambda f - \rho + \lambda h(1-\tau)(1-f)}{\sigma^2 \eta}$$

Likewise, from Proposition 7 the first order condition of f_d is given by:

$$l_d(f) - \frac{v}{[\kappa - (1-\tau)](1-f)^\eta} + \frac{1-\tau}{\kappa - (1-\tau)} = 0 \iff [\kappa - (1-\tau)]l_d(f) + (1-\tau) - v(1-f)^{-\eta} = 0$$
$$\iff \Xi_d(f) = 0$$

where:

$$\Xi_d(f) \equiv [\kappa - (1-\tau)]l_d(f) + (1-\tau) - v(1-f)^{-\eta} l_d(f) \quad \equiv \frac{\mu + [\kappa - (1-\tau)]\lambda f - \rho}{\sigma^2 \eta}$$

From the assumption as well as the fact that $h < 0$, we deduce for any $f < 1$ that:

$$l_i(f) \equiv \frac{\mu + [\kappa - (1-\tau)]\lambda f - \rho + \lambda h(1-\tau)(1-f)}{\sigma^2 \eta} < \frac{\mu + [\kappa - (1-\tau)]\lambda f - \rho}{\sigma^2 \eta}$$
$$= l_d(f) < l_d(1) = \frac{\mu + [\kappa - (1-\tau)]\lambda - \rho}{\sigma^2 \eta} < \frac{l^*}{\xi} \equiv -1/h$$

Hence we have $l_i(f) < l_d(f)$ and $-hl_i(f) < 1$ for all $f < 1$. Therefore:

$$0 = \Xi_i(f_i) = [\kappa - (1-\tau)]l_i(f_i) - (1-\tau)hl_i(f_i) - v(1-f_i)^{-\eta}$$
$$< [\kappa - (1-\tau)]l_d(f_i) + (1-\tau) - v(1-f_i)^{-\eta} = \Xi_d(f_i)$$

The result $f_i < f_d$ follows immediately because f_d is given by a down-crossing of $\Xi_d(f) = 0$. We can also easily establish that $l_i \equiv l_i(f_i) < l_d(f_i) < l_d(f_d) \equiv l_d$ because $l_d(f)$ is strictly increasing in f.

To verify that $\rho_i \equiv \rho_i(l_i, f_i) < \rho_d(f_d) \equiv \rho_d$, it is sufficient to show the ordering of the debt recovery rate that:

$$\Omega_i(l_i, f_i) > \Omega_d(f_d) \iff \frac{(1-f_i)(1+h)l_i}{l_i - 1} > 1 - f_d$$

Given $f_i < f_d$, a sufficient condition for the above inequality is:

$$\frac{(1+h)l_i}{l_i - 1} > 1 \iff -hl_i < 1$$

which is true because $-hl_i(f) < 1$ for any $f < 1$ which includes the optimally endogenous $f = f_i < 1$.

Following the above argument, we can also prove that $\rho_i(l, f) < \rho_d(f)$ for any exogenously given (l, f) provided that $-hl < 1$. Then following the proof of Proposition 5, we can show that $G_i(l, f) > G_d(l, f)$ on $-hl < 1$. But since $-hl_i(f) < 1$ and $-hl_d(f) < 1$ for any $f < 1$ (the latter follows trivially from the assumption that $\frac{\mu+[\kappa-(1-\tau)]\lambda-\rho}{\sigma^2\eta} < -1/h$), we conclude $G_i(l_i, f_i) \geq G_i(l_d, f_d) > G_d(l_d, f_d)$ which leads to $M_i > M_d$ and in turn $q_i < q_d$. ■