

## Gravitons in a box

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Gravity and matter are universally coupled, and this unique universality provides us with an intriguing way to quantify quantum aspects of space-time in terms of the number of gravitons within a given box. In particular, we provide a limit on the number of gravitons if we trace out the matter degrees of freedom. We obtain the universal bound on the number of gravitons, which would be given by  $N_g \approx (m/M_p)^2$ . Since the number of gravitons also signifies the number of bosonic states they occupy, the number of gravitons indirectly constrain the system's gravitational entropy. We show that it saturates the Bekenstein bound on the gravitational area law of entropy. Based on these observations, we ascertain that the gravitons permeating in the *observable* Universe are always  $N_g \gg 1$ .

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### I. INTRODUCTION

Within the general theory of relativity, the dynamics of space-time is intertwined by both matter and the gravitational degrees of freedom. If both matter and the gravitational degrees of freedom are quantum in nature, then their interplay will pave an important role in any quantum theory of gravity [1]. However, there is no experimental proof yet to show that gravity is quantum in nature.

Like other forces of nature, the spin-2 *graviton* is thought to be the carrier of the gravitational interaction, and it is responsible for the gravitational attraction between two massive bodies. A graviton can be canonically quantized around a weak curvature background [2], and we can attribute both on-shell and off-shell degrees of freedom to the massless graviton. The former is responsible for describing independent dynamical modes, while the latter describes how the force is being mediated between two massive bodies. One of the interesting features of the gravitational interaction is that it is universal, and it is governed by Newton's constant  $G \sim 1/M_p^2$ , where  $M_p \sim 1.2 \times 10^{19}$  GeV.

Niels Bohr once argued that in a double-slit experiment with an electron that the photon which mediates electromagnetic interaction ought to be quantum if the electron is a bonafide quantum entity, see [3]. The purported weakness of the universal gravitational interaction with the matter precluded a similar argument favoring the quantum nature of a graviton. A similar weakness in the gravitational interaction was poignantly used by Dyson to point out that it would be extremely challenging to detect gravitons [4]. Although very tiny, a quantum graviton interaction with

matter can leave an indelible mark in classical/quantum systems [5–10].

In fact, just quantum mechanics, along with the special theory of relativity, suggests that a *graviton* exchange between two quantum superposed masses can provide a bonafide quantum feature if the two quantum systems are allowed to interact *solely* gravitationally. In fact, the two quantum systems can be entangled by a graviton exchange in a Feynman diagram, which can be tested via a scheme known as the quantum gravity induced entanglement of masses (QGEM) [9,11]. In any given experiment, the number of gravitons, either on shell or off shell, can leave a detectable mark on observations. In most typical cases, for general open systems, the number of quanta is also indicative of whether a quantum system can be approximated by the mean field or a statistical ensemble.<sup>1</sup>

This paper aims to point out that there is a universal bound implied by any quantum interaction of a graviton with the matter degrees of freedom. For the time being, we neglect any other interaction besides gravity. This universal behavior can be studied by *tracing out* the matter degrees of freedom; we show that the occupation number of gravitons is always proportional to the area law of such a gravitational system. Intriguingly, such a behavior can be thought to be universal from the point of view of Bekenstein's

<sup>1</sup>Number or density plays an important role. There are roughly 400 microwave photons per cubic centimeter, and their fluctuations have been discovered in terms of two-point temperature correlations by the latest Planck experiments [12]. However, we still do not know whether these fluctuations are classical or quantum [13,14].

bound on a gravitational entropy [15] (while it has been argued that physically realistic local quantum Hamiltonians in the ground state follow the area law of entanglement entropy [16,17]). Since in a thermodynamic system, the entropy is proportional to the number of gravitons, and entropy is saturated by the area law; it is no wonder that by tracing out all the matter degrees of freedom we obtain that the area law always bounds the occupation number of gravitons.

## II. GRAVITON COUPLING TO MATTER

To set up our computation, let us assume that we work in a world line of a particle in a Fermi-normal coordinate, and without loss of generality, let us assume that the particle motion is in the  $x$ -axis. Let us consider an ideal matter following a geodesic trajectory,  $x^\mu$  (the situation of an observer following a generic timelike curve can be analyzed similarly without affecting the final results). We start from the general relativistic point-particle Lagrangian,<sup>2</sup>

$$L = -m\sqrt{g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu}, \quad (1)$$

where  $m$  is the mass of the system, and  $g_{\mu\nu}$  is the metric expressed in Fermi normal coordinates. In particular, we write the metric as  $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ , where  $\eta_{\mu\nu}$  is the Minkowski metric, and  $h_{\mu\nu}$  is the space-time curvature perturbation near the geodesic up to order  $\mathcal{O}(x^2)$ .

Let us assume that the matter is moving slowly; the dominant contribution to the dynamics is given by [18],

$$g_{00} = -(1 + \ddot{h}_{11}x^2), \quad (2)$$

where  $\ddot{h}_{11} = 2R_{0101}$  is the “+” component of the gravitational waves usually discussed in the transverse-traceless (TT) coordinates, and  $R$  is the Riemann tensor (here  $\ddot{h}_{11} \equiv \ddot{h}_{11}(t, 0)$  denotes only a number evaluated on the reference geodesic).

Now, let us use Eqs. (1) and (2), and we can then readily find the interaction Lagrangian between graviton and matter degrees of freedom,

$$L_{\text{int}} = \frac{m}{4}\ddot{h}_{11}x^2. \quad (3)$$

Let us now expand the gravitational fluctuation in terms of Fourier modes,<sup>3</sup> see Refs. [19],

<sup>2</sup>We assume  $c = 1$  here, and we work with the  $(-, +, +, +)$  signature, and  $\mu = 0, 1, 2, 3$ .

<sup>3</sup>The notion of Fourier modes implicitly assumes that we are in asymptotically flat space-time and free from curvature singularities.

$$h_{ij}(t, \mathbf{x}) = \int d\mathbf{k} \sqrt{\frac{G\hbar}{\pi^2 \omega_k}} g_{k,\lambda} e_{ij}^\lambda(\mathbf{n}) e^{-i(\omega_k t - \mathbf{k}\cdot\mathbf{x})} + \text{H.c.}, \quad (4)$$

where  $G$  is the Newton’s constant,  $\omega_k = k$ ,  $k = \|\mathbf{k}\|$ ,  $\mathbf{n} = \mathbf{k}/\|\mathbf{k}\|$ , and  $g_{k,\lambda}$  is the annihilation operator. In Eq. (4), we also implicitly assume the summation over the polarizations,<sup>4</sup>  $\sum_\lambda$ , where  $e_{jk}^\lambda$  denote the basis tensors for the two polarizations,  $\lambda = 1, 2$ . From Eqs. (3) and (4), we however see that only  $e_{11}^\lambda(\mathbf{n})$  is relevant, and we can write also the corresponding kinetic term for the massless graviton field to be

$$\begin{aligned} H_{\text{grav}} &= \int d\mathbf{k} \hbar \omega_k g_{k,\lambda}^\dagger g_{k,\lambda} \\ &= \sum_\lambda \int d\mathbf{k} \frac{\hbar \omega_k}{4} [P_{k,\lambda}^2 + Y_{k,\lambda}^2], \end{aligned} \quad (5)$$

where

$$Y_{k,\lambda} = g_{k,\lambda} + g_{k,\lambda}^\dagger \quad P_{k,\lambda} = i(g_{k,\lambda}^\dagger - g_{k,\lambda}). \quad (6)$$

The interaction Hamiltonian can be derived from the interaction Lagrangian in Eq. (3), and by using Eq. (4), we obtain [20]

$$H_{\text{int}} = -m \sum_\lambda \int d\mathbf{k} e_{11}^\lambda(\mathbf{n}) C_k Y_{k,\lambda} x^2, \quad (7)$$

where  $x$  is the position operator of the particle, and

$$C_k^\lambda = \sqrt{\frac{G\hbar \omega_k^3}{16\pi^2}}. \quad (8)$$

The interaction in Eq. (7) assumes long-wavelength gravitons such that the cutoff frequency is  $\bar{\omega} = 2\pi/l$ . We note that by combining Eqs. (5) and (7), we find the relevant part of the gravitational Hamiltonian,

$$H = \sum_\lambda \int d\mathbf{k} \frac{\hbar \omega_k}{4} [P_{k,\lambda}^2 + Y_{k,\lambda}^2] - m \sum_\lambda \int d\mathbf{k} e_{11}^\lambda(\mathbf{n}) C_k Y_{k,\lambda} x^2. \quad (9)$$

The key points here are that we treat both matter and graviton on an equal footing, when it comes to being quantum in nature, and we are dealing with a self-gravitating quantum system of mass  $m$ , whose interaction with the graviton is determined by Eq. (7).

<sup>4</sup>The basis tensors satisfy the completeness relation:  $\sum_\lambda e_{ij}^\lambda(\mathbf{n}) e_{kl}^\lambda(\mathbf{n}) = P_{ik} P_{jl} + P_{il} P_{jk} - P_{ij} P_{kl}$ , where  $P_{ij} \equiv P_{ij}(\mathbf{n}) = \delta_{ij} - \mathbf{n}_i \mathbf{n}_j$ . For later convenience we write the integral  $\int d\mathbf{n} P_{11}(\mathbf{n}) P_{11}(\mathbf{n}) = 32\pi/15$ .

### III. TRACING OUT MATTER DEGREES OF FREEDOM

Suppose we consider the mean field approximation of the matter sector in Eq. (9), i.e.,  $x^2 \rightarrow \langle x^2 \rangle$ , which is applicable in our case, because the gravitational coupling to matter degrees of freedom is very weak, suppressed by  $G \sim 1/M_p^2$ . Furthermore, we are far away from any pathologies in space-time, which allows us to work within the leading order contribution in  $G \sim 1/M_p^2$ ; it is a good assumption that the states of the graviton and the matter sector are factorizable. From Eq. (9), we thus find a displaced harmonic trap for the gravitational field,

$$H = \sum_{\lambda} \int d\mathbf{k} \frac{\hbar\omega_k}{4} [P_{k,\lambda}^2 + (Y_{k,\lambda} - \alpha_{\lambda,k})^2], \quad (10)$$

where the center is given by  $\alpha_{\lambda,k} \equiv \frac{2m}{\hbar\omega_k} e^{\lambda}(\mathbf{n}) C_k \langle x^2 \rangle$ . Now, let us suppose that the gravitational field mode  $g_{k,\lambda}$  is in a ground state, centered around  $\alpha_k^{\lambda}$ , which can be described by a displaced coherent state,

$$|\alpha_k^{\lambda}\rangle = D(\alpha_k^{\lambda})|0\rangle = e^{\alpha_k^{\lambda}[g_{k,\lambda}^{\dagger} + g_{k,\lambda}]}|0\rangle. \quad (11)$$

By choosing the gravitational field to be in the ground state of the displaced harmonic trap, we envisage that the matter system and the gravitational field have reached a steady state; a different choice for the gravitational field state will not change significantly the final result as long as the state remains centered and confined around the same minimum. For such a displaced quantum state, we can compute the expectation values  $\langle \cdot \rangle$ ; here, we are interested in the mean value and fluctuations of the number operator  $N_{k,\lambda} \equiv g_{k,\lambda}^{\dagger} g_{k,\lambda}$ , where the gravitons follow the commutation relations  $[g_{k,\lambda}, g_{k',\lambda'}^{\dagger}] = \delta(\mathbf{k} - \mathbf{k}') \delta_{\lambda,\lambda'}$ .

We find the occupation number of  $g_{k,\lambda}$  gravitons in the ground state to be

$$\langle N_{k,\lambda} \rangle = |\alpha_k^{\lambda}|^2 = \left( \frac{2m}{\hbar\omega_k} e^{\lambda}(\mathbf{n}) C_k \langle x^2 \rangle \right)^2. \quad (12)$$

The total number of gravitons can be then computed by summing and integrating over all the graviton modes,

$$N_g \equiv \sum_{\lambda} \int d\mathbf{k} \langle N_{k,\lambda} \rangle = \sum_{\lambda} \int d\mathbf{k} \left[ \frac{2m}{\hbar\omega_k} e^{\lambda}(\mathbf{n}) C_k \langle x^2 \rangle \right]^2. \quad (13)$$

By using Eq. (8) and footnote 4, we note that

$$N_g = \int k^2 dk \frac{4m^2}{\hbar^2 \omega_k^2} \int d\mathbf{n} \sum_{\lambda} e^{\lambda}(\mathbf{n}) e^{\lambda}(\mathbf{n}) C_k^2 \langle x^2 \rangle^2 \quad (14)$$

reduces to

$$N_g = \frac{8m^2 G}{15\pi\hbar} \langle x^2 \rangle^2 \int_0^{\bar{\omega}} d\omega_k \omega_k^3, \quad (15)$$

where  $\bar{\omega}$  is the cutoff frequency for a box of side  $l$ .

### IV. NUMBER OF GRAVITONS

We can perform the integration in Eq. (15), and by using  $\bar{\omega} = 2\pi/l$ , we find a factor  $(2\pi/l)^4/4$ . Furthermore, we note that  $\langle x^2 \rangle^2 \leq l^4/2^4$ . We then find that the  $l$  dependence goes away from the numerator and the denominator in Eq. (15), leaving us with a straightforward relationship, where the number of gravitons is simply given by the mass  $m$  and Newton's constant  $G$ ,

$$N_g \leq \frac{4\pi^3 G m^2}{30 \hbar} = \frac{4\pi^2}{30} \left( \frac{\text{Area}}{4G\hbar} \right). \quad (16)$$

In the above, by area, we mean the area corresponding to the Schwarzschild radius of mass  $m$ , given by  $R_s = 2Gm$ , which is also the natural length scale over which the above number of gravitons are confined. The same result, aside minor numerical factors, is obtained for a relativistic energy-momentum tensor.<sup>5</sup>

Since the entropy of a black hole is proportional to the area of the hole, the number of gravitons found above naturally scales as Bekenstein's entropy  $N_g \sim S_g$ , or specifically, it is consistent with the bound set by Bekenstein, i.e.,  $N_g \sim S_g < 2\pi ER/\hbar$ , where  $E$  is total mass energy of the system, and  $R$  is the characteristic radius, in the black hole case  $ER \sim 2Gm^2$  [15]. At this point, it has not avoided our attention that the gravitational entropy is indeed holographic [21,22].

An intriguing feature of the above expression is that the number of gravitons signifies how we may be able to quantify the quantum nature of space-time itself. If we can squeeze the matter within the Schwarzschild radius, the number of gravitons saturates the black hole entropy. In general relativity, the Schwarzschild radius is the only length scale that appears if we place a mass  $m$  in an asymptotically flat space-time. In fact, from Eq. (16), we can roughly estimate the number of gravitons to be

$$N_g \approx (m/M_p)^2. \quad (17)$$

For a solar mass blackhole,  $m \sim 10^{33}$  g,  $M_p \sim 10^{-5}$  g, and  $N_g \sim 10^{76}$ . The large  $N_g \geq 1$  sets a barrier for any massive

<sup>5</sup>Suppose we had taken an example of a relativistic energy-momentum tensor  $T_{\mu\nu}$ , where the graviton-matter interaction would be dictated by the interaction  $\sqrt{G}h_{\mu\nu}T^{\mu\nu}$ , then quantizing the graviton in a transverse traceless (TT) gauge, we would get a similar answer barring the numerical factors,  $N_g \leq (32\pi^3/105)(Gm^2/\hbar)$ .

quantum system, whose mass exceeds that of the gravitational mass  $m \geq M_p \sim 10^{-5}$  g. From Bohr's correspondence principle, we may be able to treat such a self-gravitating system approaching toward a classical limit.

The notion that large values of  $N_g$  can be associated with classical behavior is further reinforced by computing the quantum fluctuations,

$$\Delta N_g \equiv \sqrt{\langle N^2 \rangle - \langle N \rangle^2}, \quad (18)$$

of the total number operator  $N \equiv \sum_{\lambda} \int d\mathbf{k} N_{\mathbf{k},\lambda}$ . We have already calculated  $\langle N \rangle = N_g$ , and using the commutation relations  $[g_{\mathbf{k},\lambda}, g_{\mathbf{k}',\lambda'}^\dagger] = \delta(\mathbf{k} - \mathbf{k}')\delta_{\lambda,\lambda'}$  we can show  $\langle N^2 \rangle = N_g + N_g^2$  (see, for example, [23]). Inserting the expectation values back in Eq. (18), we then find that the fluctuations grow only as  $\Delta N_g = \sqrt{N_g}$ . Hence, we find that the relative quantum fluctuations follow as

$$\Delta N_g / N_g \sim 1 / \sqrt{N_g}, \quad (19)$$

in line with the expectation that the system appears classical for large values of  $N_g \gg 1$ .

A curious reader would wonder if we were to trace the graviton degrees of freedom instead of matter degrees of freedom. In such a case,  $Y_{\mathbf{k},\lambda} \rightarrow \langle Y_{\mathbf{k},\lambda} \rangle$  in Eq. (9), and we would be left with a simple harmonic oscillator potential. For such a system, the occupation number  $N_m$  will not follow the area law. We can further ask whether the area law is a generic property for all massless fields or specific to the matter-graviton coupling. We recall that the quadratic nature of the coupling  $\sim x^2$  in Eq. (3) was critical: we have shown  $N_g \propto \langle x^2 \rangle$ , where  $\langle x^2 \rangle$  is always nonzero, leading to the area law. In contrast, a linear coupling, e.g.,  $\sim x$ , can always be canceled by a change of the reference frame and should thus not play a role in the derivation of the area law. More generally, one can expect the emergence of an area law whenever the coupling gives rise to a length scale; while for other types of interactions (such as the electromagnetic), one is left without such an interpretation.

## V. APPLICATIONS AND FEATURES

Let us now provide another intriguing connection to what we have just found in Eqs. (16) and (17). In Refs. [24–27], see Ref. [28] for earlier discussions, a very interesting idea has been developed, a corpuscular nature of a black hole. A black hole is perceived to be a Bose-Einstein condensate of a large number of weakly interacting gravitons, where the authors have found exactly a similar scaling as in Eq. (17) but from a different point of view. They did not have to trace out the matter degrees of freedom explicitly. Instead, they argued that a black hole could be justly described by an ensemble of  $N_g \gg 1$  gravitons. The authors further argued that the number of

gravitons will suppress the effective interaction of a gravitons, i.e., given by  $\alpha_g \leq 1/N_g$ . For a large  $N_g \gg 1$ , the black hole behaves like a coherent system, albeit a leaky one. The graviton escape signifies the Hawking evaporation of a black hole. Such a gravitational system with  $N_g \gg 1$  can be perceived to be approaching a classicalization limit, where extracting quantum features will become extremely hard.

The other limiting case, when  $N_g \ll 1$ , provides us with an intriguing possibility of extracting the quantum nature of a graviton. This limit arises when  $m \leq 10^{-5}$  g; for such a system, the number of gravitons is less than one irrespective of the size of the box. Therefore, it seems that nature provides us with a window of opportunity where it might be possible to construct experiments cleverly to study the quantum nature of gravity for  $m \leq 10^{-5}$  g. Indeed, if  $m \ll 10^{-5}$  g, it will be again harder to probe or extract any quantum features. It has been experimentally possible to create macromolecules  $m \sim 10^{-19}$  g over spatial superposition for  $\sim 0.25 \mu\text{m}$  or atoms  $m \sim 10^{-21}$  g over  $\sim 0.5$  m [29,30]. In all these cases,  $N_g \sim 10^{-28}$  or  $10^{-32}$ , way too small to observe any detectable features of quantum gravity.

The situation appears to be quite familiar in cosmology as well if we were to assume that a world line can describe the entire observable patch of the Universe. In that case, the mass contained in the Universe would be  $m \sim \rho \times V$ , where  $\rho = H^2(t)M_p^2$  is the constant energy density of the Universe, and  $V \sim H^{-3}(t)$  is the observable volume, which would scale with the Hubble expansion rate  $H(t)$  of the Universe. If we were now to compute the occupation number of gravitons within my observable patch of the Universe, we would find

$$N_g \sim (M_p/H(t))^2. \quad (20)$$

In the current Universe,  $H \sim 10^{-42}$  GeV, which gives  $N_g \sim 10^{121}$ , which indicates that the space-time is perhaps very close to a classical description, and the number of gravitons again scales as the area law,  $N_g \sim \text{Area}/4G$ , because the observable area of the Universe will scale as  $\sim H^{-2}(t)$ .

On the other hand, we already have a constraint on the largest scale in the Universe indirectly. Since we have not seen any primordial gravitational waves, or the B-mode polarization in the cosmic microwave background radiation, we believe that the scale of cosmic inflation cannot be arbitrarily large. The latest Planck data place an upper bound on  $H \sim H_{\text{inf}} \sim 10^{13}$  GeV [12]. Of course, we have invoked here primordial inflation as a mechanism to seed the fluctuations in the cosmic microwave background radiation. However, even for such a high energy probe, the number of gravitons present in the Hubble radius during



inflation would behave nearly classically, i.e.,  $N_g \sim 10^{12}$  and  $\Delta N_g/N_g \sim 10^{-6}$ .

Finally, we can ask how robust our analysis is with regard to the nature of classical gravity. Note that we have not assumed any specific form of gravitational action. The only assumption we have made here is that the gravitons can be described by the massless degrees of freedom in a harmonic oscillator state with the minimal coupling to matter degrees of freedom given by  $G \sim 1/M_p^2$ , which is valid for gravitational theories beyond three spatial dimensions. However, this argument will change if we insist on studying any higher derivative extension of gravity, which will bring inevitably a new scale in the problem, say  $M_s < M_p$  in four space-time dimensions, see [31,32]. For such class of theories of gravity, we have to tread the occupation number of gravitons carefully case by case.

## VI. CONCLUSION

We briefly conclude by highlighting that we have provided a rather model-independent constraint on the occupation number of gravitons in a quantum system determined by its mass and  $M_p$  by tracing out the matter

degrees of freedom. The occupation number is bounded by the area law, which is a reminiscence to Bekenstein's bound. Our bound suggests that the occupation number of gravitons in the black hole geometry will be bounded by the area of a black hole, which is also the gravitational entropy of the object. For mass  $m \geq 10^{-5}$  g, the number of gravitons occupied within the gravitational radius, i.e., the Schwarzschild radius, is much larger than one. In an optimal bound on mass,  $m \sim 10^{-5}$  g would be ideal for extracting any quantum behavior of a graviton. Furthermore, tracing out the matter degrees of freedom renders the space-time fairly classical within the observable patch of the Universe.

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