ABSTRACT

Title of Dissertation: ESSAYS ON DECISION MAKING OVER RISK

Ibraheem Abdel-Rahman Catovic, Doctor of Philosophy, 2021

Dissertation directed by: Professor Erkut Y. Ozbay, Department of Economics

I make several contributions to the literature on decision making over risk-bearing products. Summarily, my research highlights the importance of non-distributional factors in the decision-making process for products bearing risk. I demonstrate that the majoritarian consumer behavior of the multi-billion-dollar Mega Millions lottery game is not rationalizable by existing preference-based models. Non-preference-based explanations are also unable to account for the observed behavior. I identify factors likely influencing choice behavior and incorporate them into a new model, which can account for the majoritarian choice behavior. In a separate project, I explore the relationship between risk preferences and preferences over winning number selection in games of chance. I provide novel experimental and empirical evidence of an interdependence of these preferences. Specifically, I find that at relatively reasonable odds, self-selection of winning numbers is most preferred. However, when odds worsen, allocation of selection to a random mechanism

becomes most preferred. In a third project, I explore consumer preferences over Prize Linked Savings (PLS) products, which have the principal guarantee of standard savings products coupled with the positive return skewness of games of chance. I provide experimental evidence that designing PLS products with gaming features, like those found in games of chance, increases the appeal of PLS and the creation of savings. Furthermore, PLS products so designed may serve as better substitutes for games of chance than PLS products without such features.

ESSAYS ON DECISION MAKING OVER RISK

by

Ibraheem Abdel-Rahman Catovic

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Advisory Committee:
Professor Erkut Y. Ozbay, Chair
Professor Emel Filiz-Ozbay
Professor Melissa Kearney
Professor Yusufcan Masatlioglu
Professor Barry Sopher
Professor Emmanuel Zur

Dedication

I dedicate this dissertation to my paternal grandparents Dr. Saffet Catovic and Dr. Sarah Catovic, both who earned their PhDs against the odds and dedicated their lives to education and improving the lives of others.

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Chapter 1: A Mega Millions Anomaly

Abstract

The interstate lottery game Mega Millions introduced a new product in October 2017 called Just the Jackpot. Sales of this product have been anemic. The Standard option accounts for over 90% of sales even though it is never the expected value maximizer for consumers among ticket options at any jackpot level. Several popular decision theoretic models predict Just the Jackpot should have strong appeal, while interest in the Standard option should be low. I show that consumers' choice of product is not due to inattentiveness, liquidity constraints or lags in the adjustment of consumption to new product introduction. I argue that the data trends are due to differences in ex post outcome feedback on foregone choices depending on which option is selected, as well as minimal winner regret, something not accounted for in most models. I propose a Feedback Weighted Regret Minimax model that incorporates a feedback parameter as well as a novel winner-loser regret feature that captures the data trends significantly better. It is puzzling that lottery managers chose to introduce Just the Jackpot, as existing decision models predict negligible increases in Mega Millions participation on the extensive margin. I show that inducing players to switch from another Mega Millions option to Just the Jackpot maximizes neither lottery revenue nor lottery profits. Finally, I argue that the seemingly irrational inverse relationship between jackpot size and the Just the Jackpot sales percentage can be explained by changes in player demographics, as a larger share of players at bigger jackpots are likely unaware of the existence of the Just the Jackpot option.

Introduction

This paper provides novel empirical evidence of the inability of the most popular models of decision making under risk to capture actual lottery player behavior. An analysis of the ticket options and sales for Mega Millions, the second-largest lottery game in the United States, shows that the actual extant choice pattern in the data is not generally rationalizable by any of these models, using commonly utilized functional forms employed in the literature over a wide range of parameter values. This is particularly intriguing, as the annual sales of Mega Millions tickets is in the range of a few billion dollars and therefore captures the choice behavior of millions of people, and there are only three Mega Million ticket options available. The data patterns are also not due to player unawareness of some of the available ticket options, nor a lag in lottery consumption adjustment, nor liquidity constraints binding on the more expensive ticket options. The paper argues that this apparent anomaly is due to the insufficiency of the (state space) payoff distribution for modeling the decision making process for this game. The paper proposes that the major driver of the choice behavior is a difference in the generation of ex post feedback on foregone choices, depending on which ticket option is purchased. Another factor likely impacting choice behavior is minimal winner regret. A model is proposed accounting for both of these phenomena, and this model is able to more closely align with the revealed preferences in the sales data.

The central motivation for the paper is the introduction of a third Mega Millions ticket option in October 2017, called Just the Jackpot. Just the Jackpot gives the consumer the best value of any ticket option for winning the jackpot. Sales of this product started quite low at about 2% of Mega Millions sales, and then proceeded to fall below 0.5% by the end of 2017 and remain below that percentage through the writing of this paper. The sales data is anomalous for a few reasons. First, lottery sales data from many games over decades demonstrates that jackpot size is the primary driver of sales for jackpot games, with revenues and profits often growing exponentially as jackpots get excessively large. Given that Just the Jackpot gives the best value of any ticket option at winning the jackpot, it is surprising its sales are so low, especially at high jackpot levels. Second, over 90% of the sales are of the Standard option, even though the Standard option is not the expected value maximizer at any jackpot level. Third, evaluation of the choice framework through a decision theory lens suggests that Just the Jackpot should be a highly desirable product, and the Standard option the least desirable option, across wide parameter ranges for commonly used parameterizations over a variety of models and jackpot levels. Essentially, mainstream decision theory would likely have endorsed the creation of this product from a consumer welfare perspective, yet it has been nothing short of a disaster. To its credit, mainstream decision theory models suggests that introducing Just the Jackpot does not bring in many new players, and that the demand for Just the Jackpot would be almost entirely due to existing players opting for a more preferred ticket type. Interestingly, this paper shows that such behavior is not revenue or profit maximizing from Mega Million's perspective, and therefore questions the decision to create such a product in the first place. This study shows that there is indeed still more to be learned about the factors individuals take into consideration when making choices over risk, and provides some policy implications relating to lottery product design.

Rationalizing lottery consumption within a unified model consistent with other common behavior over risk dates back to the birth of decision theory. Attempting to incorporate participation in unfair lotteries within an Expected Utility (EU) framework proved to be a tricky proposition. An initial attempt at reconciling simultaneous (risk loving) lottery participation and (risk averse) insurance purchasing within an EU framework involved a utility function with multiple inflection points (Friedman and Savage, 1948). While this approach technically could explain the lottery-insurance paradox at some wealth levels, it opened up bigger cans of worms, including the predicted depopulation of the middle class due to risk loving behavior (for a comprehensive critique of the Friedman-Savage approach, see Hirshleifer, 1966). An alternative approach was to remove lottery and gambling participation from evaluation by the utility of wealth function, by classifying such activities as *pleasure-oriented* gambling, discernible from *wealth-oriented* gambling by its repetitive, small stake nature (Hirshleifer, 1966). *Pleasure-oriented* gambling qualifies as a consumption good and therefore not subject to consideration under EU, whereas *wealth-oriented* gambling would be. There are various rationales consistent with the *pleasure-oriented* classification, including: the (short-lived) right to dream or fantasize about potential winnings (Clotfelter and Cook, 1990); contributing to socially-desirable causes that are funded by proceeds from such activities (Clotfelter and Cook, 1990); an escape from the routine, mundane and predictable nature of modern industrial life (Bloch, 1951); a mechanism for releasing tensions and registering non-disruptive protests against an inequitable capitalistic system (Devereux, 1949; Frey, 1984); a way to establish social cohesion and maintain friendships (Guillen et al., 2012). In spite of these reasons, it is unlikely that individuals partake in games of chance solely or even mostly for non-pecuniary purposes. About half of California lottery players polled stated that they played the lottery for the money more so than the fun, the share of which moved inversely with income (Los Angeles Times, 1986). More recent evidence further validates this, as a poll of over 1,000 US adults estimated that '21% of Americans, and 38% of those with incomes below $25,000, think that winning the lottery represents the most practical way for them to accumulate several hundred thousand dollars' (Consumer Federation of America, 2006). There is even evidence that lottery players place value on the means of lottery risk resolution, for instance, with preferences over self-selection of winning numbers for lottery entries (see Simon 2008 for a comprehensive discussion of number selection behavior in lotteries).

As the evidence against EU as a sufficient framework to model decision behavior over risk began to pile up, a number of models emerged as modifications or wholesale alternatives to EU. These models sought to maintain much of the normative appeal of EU while allowing behavior that was becoming increasingly viewed as common and even rational. Some of the more widely utilized and referenced models include: prospect theory (Kahneman and Tversky, 1979); anticipated utility (Quiggin, 1982); cumulative prospect theory (Tversky and Kahneman, 1992); disappointment aversion (Gul, 1991); regret theory (Loomes and Sugden, 1982; Bell 1982); salience theory (Bordalo et al. 2012). These models are able to handle the lottery-insurance paradox fully within their frameworks, in spite of differing axiomatic foundations and psychological motivations. These models maintain the sufficiency of the (state space) payoff distribution for decision making, with no need to attribute lottery or gambling play to consumption in any way, and without explicitly modeling any of the non-pecuniary motives mentioned above. However, these models are unable to explain the Mega Millions choice patterns present in the sales data, as this paper demonstrates.

The rest of the paper proceeds as follows. Section 2 introduces Mega Millions and the sales data. Section 3 reviews the decision theoretic models and their predictions regarding the Mega Millions ticketing options. Section 4 addresses non-decision theoretic explanations of the lack of interest in Just the Jackpot, namely player unawareness of its existence, slow adjustment of sales, and liquidity constraints due to its relative costliness. Section 5 proposes behavioral mechanisms that could be impacting Mega Millions choice behavior. Section 6 incorporates these mechanisms into a model that is able to explain Mega Millions choice behavior. Section 7 analyzes a few interesting counterfactuals with implications on lottery design. Section 8 concludes.

Mega Millions Data

Mega Millions is one of the two major interstate lottery games in the United States, the other being Powerball. To put the size of the US lottery industry into perspective, aggregate lottery revenue across states and games in fiscal year 2018 was $85.6 billion (NASPL, 2020). This is larger than the combined gambling revenues of $41.7 billion of commercial casinos (AGA, 2019) and $33.7 billion of Indian tribal gaming (NIGC, 2019), a total of \$75.4 billion. Aggregate lottery spending in the United States outpaces that of sports tickets, books, video games, movie box office tickets, and music - combined (CNNMoney, 2015). Powerball and Mega Millions are the two largest selling lottery games in the United States. In fiscal year 2018, Powerball recorded \$5.2 billion in sales, while Mega Millions recorded \$3.2 billion, combining for about 10% of aggregate lottery sales in the United States.

A Mega Millions entry requires the player to select five numbers between 1 and 70 without replacement and a sixth ball from 1 to 25. Players may choose their own numbers or opt for a random assignment of numbers. Drawings for each game are held twice a week, during which a large drum spits out five numbers from the larger range and a second drum spits out the sixth number from the smaller range. If a player's ticket has partial number matches with the numbers drawn, that ticket is eligible for certain prize amounts based on the number of balls matched. If a ticket matches all six balls (order independent), that ticket is eligible to collect the jackpot amount, albeit in installments over 30 years. Jackpot winners also have the option to cash out the jackpot in a single immediate payment, for an amount that is less than the advertised jackpot. If multiple tickets match all six balls within a drawing, the jackpot is split among the winners. All other lower prize amounts are given as advertised in a single payment, regardless of the number of winners at that prize tier.[1] If there is no ticket that matches all six numbers in a drawing, the jackpot rolls over into the next drawing and increases by an amount determined by the projected and actual sales for that drawing. The current iteration of Mega Millions features prizes ranging from \$2 to the jackpot, which starts at \$40 million. The odds of winning \$2 are 1 in 37, the odds of winning the jackpot are 1 in 302,575,350, and the overall odds of winning any prize are 1 in 24 (for the full prize-odds matrix, see the Appendix).[2]

There are three ticket types offered by the Mega Millions lottery game. It will be convenient to categorize Mega Millions prizes in two groups: all the prizes besides the jackpot are grouped into the lower tier (L), and the jackpot (J) is its own category. This is a natural classification as only the jackpot can roll over and is the only parimutuel prize.. The first is the Standard ticket, which increased from a price of \$1 to \$2 during

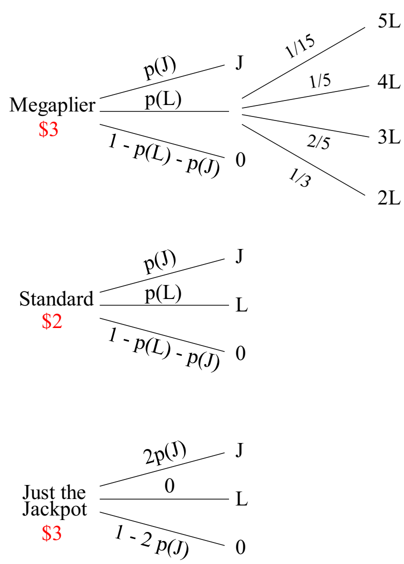
[1] The exception is California, which determines payouts for non-jackpot prizes on a parimutuel basis.

[2] Mega Millions started as the Big Game in September 1996, selling tickets in six states: Georgia, Illinois, Maryland, Massachusetts, Michigan and Virginia. In May 2002, the Big Game turned into Mega Millions. In January 2010, a cross-selling agreement was made between the Mega Millions consortium and the Multi State Lottery Association (MUSL, which runs Powerball). Up to that point, a state could not sell both Mega Millions and Powerball products. This agreement allowed states to offer both. Over the years more and more states began offering Mega Millions. Currently, Mega Millions is available for purchase in 45 states, Washington DC and the US Virgin Islands. The most recent addition to the Mega Millions family was the state of Mississippi, which began selling Mega Millions tickets in January 2020 (for even more Mega Millions history see Mega Millions, 2020).

the latest Mega Millions prize-odds changes in October 2017. This ticket amounts to a single entry into both the lower tier and the jackpot. The second is the Megaplier, which increased from a price of \$2 to \$3 during the October 2017 changes. The Megaplier ticket yields a single entry into the lower tier and the jackpot, just like the Standard. However, prospective lower tier prize amounts are at least doubled with a Megaplier ticket. During the drawing, a Megaplier value is displayed by a random number generator, in addition to the winning numbers selected from the drums. This Megaplier multiplier takes a value of 2, 3, 4 or 5 with probabilities of 1/3, 2/5, 1/5 and 1/15, respectively, amounting to a multiplier expected value of 3. So, if a ticket qualifies for a lower tier prize amount and the purchaser paid the extra dollar for the Megaplier option, the holder would be entitled to that prize amount multiplied by the Megaplier value. If the Megaplier option was not added, the holder would be entitled only to the prize amount. The Megaplier impacts all prize amounts except the jackpot. The third option is the Just the Jackpot ticket, which is a new option introduced during the October 2017 changes. It has no similar counterpart in Powerball, unlike the Megaplier. Just the Jackpot costs \$3 and entitles the purchaser to two entries into the jackpot drawing only, the two entries are not eligible for any lower tier prizes. Figure 1 displays the probability tree diagram for each of the ticket types.

It is worthwhile to briefly consider the choice framework of the current version of the Mega Millions game. A Standard ticket costs \$2, while the Megaplier and Just the Jackpot each cost \$3. The expected value of the lower tier for a Standard ticket is \$0.25, whereas it is \$0.75 for the Megaplier lower tier due to the expected Megaplier multiplier of 3. The expected value of the minimum jackpot of \$40 million, assuming no prize sharing, is \$0.13, and a jackpot of \$75 million yields about the same expected value of \$0.25 as the lower tier prizes for a Standard ticket. From an expected valued maximization perspective of a single ticket with a single jackpot winner, the Standard ticket is best for jackpots below \$378 million, and Just the Jackpot for jackpots in excess of that, with Megaplier never being the best option under any feasible jackpot. However, modeling the decision behavior based on the actual ticket costs introduces a cost effect between the Standard ticket and the other two options. For example, consider a half-Standard ticket, which is a Standard ticket with all of the win probabilities cut in half, and let the cost of this be \$1. If this payoff distribution is valued much more by a player than a dollar, then the decision problem pitting the three ticket options with their costs against each other would be under-valuing the Standard ticket option, and vice versa. Modifying the decision problem by adding this half-Standard ticket to the Standard ticket would yield one and a half Standard tickets at a cost of \$3. Comparing this with the other two options more appropriately captures the decision framework. Players are not limited to a single ticket purchase and can buy as many tickets as they want, and this cost neutral transformation better captures this reality. The choice problem is not so much which ticket option would a player prefer, but rather which payoff distribution a player would prefer at a given cost. The cost neutral approach will be the baseline approach for the model evaluations. Now, ranking tickets by expected value under the assumption of a single jackpot winner yields the Megaplier with the maximum expected value for jackpots below $223,870,250 and Just the Jackpot for jackpots above that: the cost neutral Standard option never maximizes the expected value at any feasible jackpot level.

Figure 1: Mega Millions Ticket Types



In 2018, the Mega Millions jackpot reached an enormous $1.6 billion, making it the largest lottery payout in US history to a single individual (Mega Millions, 2019). Figure 2 presents the Mega Millions sales data for this historic round, spanning from July 27 to October 23. A Mega Millions jackpot draw is a specific Tuesday or Friday on which the winning numbers are drawn, whereas a round consists of the set of draws beginning with the minimum jackpot up until the next jackpot reset. A key takeaway is the exponential climb in draw sales as the jackpot gets abnormally high. The sales data for the whole two years, including the historic round, is in Table A2 in the Appendix. The Total column represents the total Mega Millions sales for that draw of the three ticket types in DC and the 14 states that offer Just the Jackpot: total nationwide sales are approximately three times those amounts. The Standard, Megaplier and Just the Jackpot columns represent the percentage of sales in dollars (not tickets) of each ticket type. The percentage reported of the number of tickets would further elevate the Standard percentages over the other two options, as it costs one dollar less. There are a few things to note right off the bat. First, sales do not monotonically increase with jackpot at lower jackpot levels. This is due to the weekend effect: there is a drawing on Tuesday night and one on Friday night, the sales for Friday drawings will typically outdo those for the following Tuesday, even with the increase in jackpot. Second, as the jackpot rolls over and sales respond to the increased expected jackpots, the difference in successive jackpots increases. In this case, the difference start at $5 million and maxes out at $600 million.

Figure 2: Mega Millions Sales for $1.6 Billion Jackpot

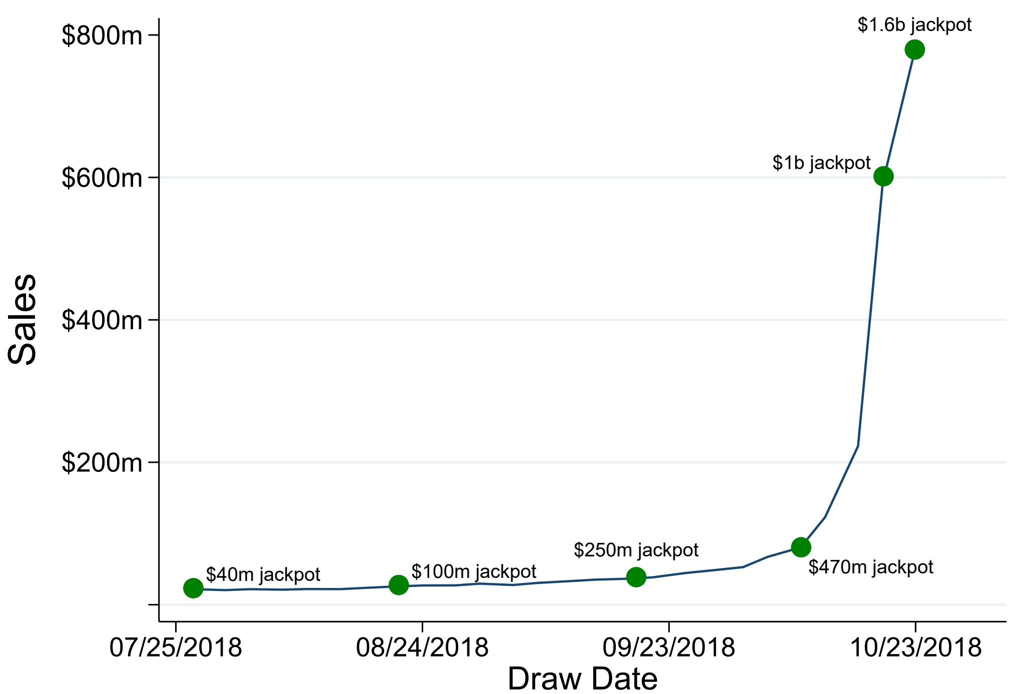
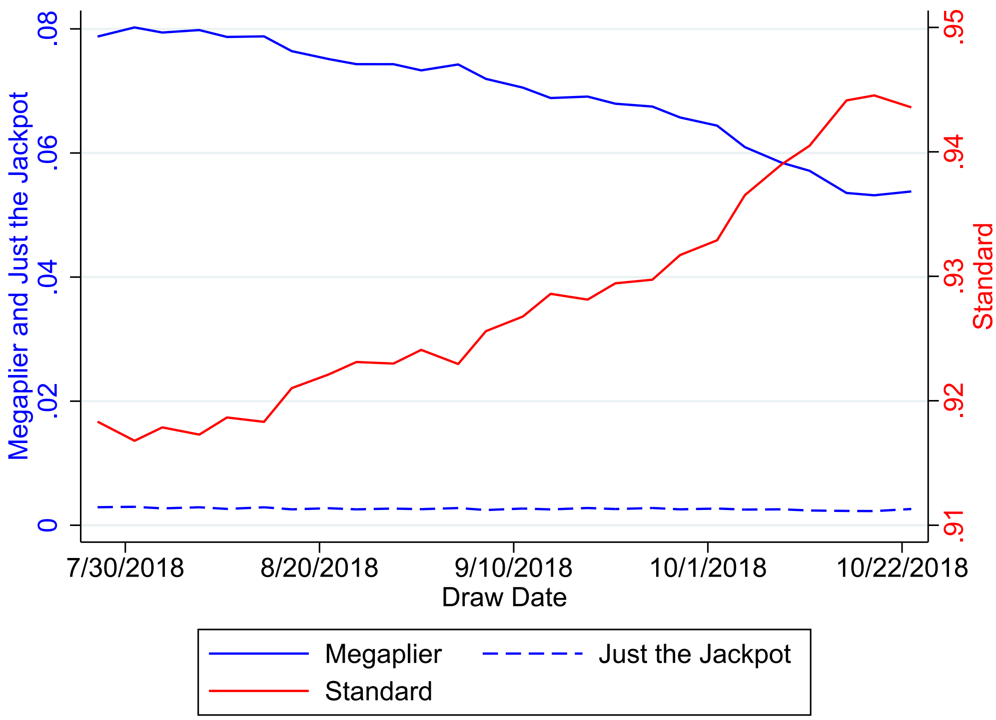


Figure 3 presents the sales percentages of each ticket type during the historic round, which are representative of the overall trends over the two years. The Standard ticket is selected in over 90% of transactions across all jackpot levels, the percentage increasing with jackpot size, in spite of it not being an expected value maximizer at any jackpot level under the cost neutral approach. The Megaplier seems to hold some niche appeal, with a range of 5% to 8% of sales, decreasing with jackpot size. This negative relationship between the Megaplier sales percentage and jackpot size is as expected:

from an expected value viewpoint, Megaplier holds its biggest relative appeal at the minimum jackpot level. As the jackpot increases, the share of expected value going to the jackpot increases, implying a reduction in the relative appeal of Megaplier. The Just the Jackpot sales numbers are particularly intriguing. Once again, the sales and percentages reported are only for those states that offer Just the Jackpot. The percentage of sales never goes above a measly 0.3% for any of the draws. Even more bizarre is the flat or perhaps even slightly negative relationship between the Just the Jackpot sales percentage and the jackpot amount. A positive relationship is expected, since Just the Jackpot holds minimal relative appeal at the minimum jackpot amount, and increases in relative appeal with each successive rollover. The diminutive sales of Just the Jackpot tickets are perhaps surprising, given the clear impact jackpot size is having on aggregate sales in Figure 2, let alone all the evidence highlighting the impact of jackpot sizes on sales and profitability.[3] This product is nothing short of a flop, to the point where the state of Wisconsin discontinued Just the Jackpot on October 30, 2018, citing a "lack of interest" in the product, per an email exchange the author of this paper had with a Wisconsin lottery official. The next section introduces a number of the more popular decision models and demonstrates their collective inability to explain the sales data.

[3]The positive impact of jackpot size on lottery sales is well-established in the lottery literature (DeBoer, 1990; Cook and Clotfelter, 1993; Forrest, Simmons and Chesters, 2002). There is even evidence of the positive impact of jackpot size on lottery sales and a negative impact of expected value, but the high correlation between expected value and jackpot size may be affecting the estimates (Cook and Clotfelter, 1993). Lottery sales decreases in New York in the 1980s were attributed to increased participation resulting in fewer rollovers and smaller jackpots, with a recommendation to reduce the odds of winning to generate larger jackpots (DeBoer, 1990). Many of the innovations to jackpot lottery games over the years have come about with the intent of increasing jackpot sizes: increased ticket costs, larger minimum starting jackpots, larger target player pools, and decreased odds of winning the jackpot. The development of interstate lotteries was due to the belief that larger player pools with decreased jackpot odds would allow jackpots to grow in size, with a disproportional effect on revenue and profitability. Mega Millions ran with this notion by introducing its Just the Jackpot ticket option, made available in 14 states and Washington, DC since the most recent Mega Millions iteration in October 2017.

Figure 3: Mega Millions Sales Proportions for $1.6 Billion Jackpot



Decision Theory Model Predictions

This section evaluates the Mega Millions choice problem in a number of decision models. These models require one or more functional specifications to concretely predict choice behavior over risk. For example, Expected Utility requires the specification of a utility function over wealth. Additionally, estimation of these models generally proceed by assuming a certain family of functions. For instance, the power function x^α is often used as the functional family for utility over wealth in a number of models. The majority revealed preference in the data is for the Standard option across jackpot levels. A minority of players prefer the Megaplier, and Just the Jackpot participation is effectively negligible. For a model to be realistically considered to rationalize the data, it should be able to demonstrate the primary trends in the data using the common functional forms assumed in the literature across a sufficiently wide range of parameter values. Specifically, the parameter ranges should be consistent with those estimated in the experimental model estimation literature. Additionally, at least some of those parameter values should predict a preference for the Standard option over the range of jackpots observed in the data, modeling the behavior of a frequent Mega Millions player who buys at all jackpot levels. Perhaps there should be more parameter-jackpot combinations consistent with Standard preferences than either of the two other Mega Millions options. However, simply using the percentage of parameter combinations predicting each preference as an indicator of population preferences implicitly assumes uniformity of the population across parameter values. This assumption is extremely tenuous and is a weakness of the proposed approach. It turns out that none of the models tested are able to demonstrate even a single parameter combination that can predict Standard selection across the range of feasible Mega Millions jackpots, so that the distribution of the population over parameter values effectively becomes a moot point. Additionally, there should be a good number of parameter-jackpot combinations that predict not participating in Mega Millions at all, since most people do not play Mega Millions. Even restricting the population to existing lottery players suggests that participation predictions should not be too high, particularly at low jackpot levels, as Mega Millions accounted for less than 5% of US lottery sales in 2018. Such a restriction is reasonable, since some people may have moral or religious objections to lottery play and therefore the choice problem becomes trivial.

An important assumption is that the choice framework utilized in model evaluation is selecting one of the three ticket distributions, or a fourth outside option of not buying any and therefore having the cost of the ticket with probability 1. In reality, players may opt to play another lottery game or do anything else with those unspent funds, but the analysis forthcoming essentially subsumes this within the option of not playing and thereby having those funds. This assumption essentially just sets a threshold for Mega Millions play. For example, consider \$3 that can be spent on Mega Millions or in some other way. It is possible that there is an outside option with a cost of \$3 with a higher utility than simply having those \$3. This would just raise the required utility to play Mega Millions. So, in situations where modeling a utility of \$3 implies just barely choosing a Mega Millions option, raising that threshold could result in the model predicting no participation. It turns out that across models, situations in which Standard is preferred sit disproportionately close to the participation threshold relative to the two other ticket types, so that increasing the threshold would result in even worse predictions of Standard preferences. Also, model implications are determined purely using distributional and state space payoff information: non-monetary concerns like entertainment utility or preferences over how lotteries are resolved are excluded from the analysis. Even if such considerations exist, assuming these non-monetary considerations impact each of the Mega Millions options equally would not impact results.

Expected Utility

In order for Expected Utility to explain participation in unfair lotteries, convex utility is required, at least over certain ranges of wealth, as in Friedman and Savage (1948). Jackpot games like Mega Millions sometimes offer positive expected returns when jackpots get excessively large. Under the assumption of a single jackpot winner, the requisite break-even jackpot amounts are about \$530 million for the \$2 Standard purchase, \$685 million for the Megaplier purchase, and \$453 million for the Just the Jackpot purchase (two jackpot only entries). Expected value calculation and the decision modeling that follows use the advertised jackpot amount, not the cash value (one-time payout) of the jackpot. This latter amount becomes more relevant for the revenue estimations that happen in Section 7. The minimum starting jackpot is \$40 million, jackpot amounts above \$400 million are relatively uncommon. Factoring in the likelihood of multiple jackpot winners based on the number of tickets sold further increases these break-even thresholds. Therefore, the general case for Mega Millions participation would require a convex utility function to meet the individual rationality constraint.

One thing to consider before making functional and parametric assumptions is whether any of these ticket options dominate any other. No option FOSD any other: while the Megaplier FOSD the Standard option at the lower tiers, the higher probability of winning the jackpot in the cost neutral Standard option negates FOSD over the whole distribution; both the Standard and Megaplier FOSD over Just the Jackpot is nullified by the higher probability of winning the jackpot with a Just the Jackpot entry. There is some conditional dominance in the second order: both the Standard and Megaplier options SOSD Just the Jackpot at jackpot levels below approximately \$224 million, above which the added value of the better jackpot odds under Just the Jackpot negates SOSD; there is no cost neutral SOSD between the Standard and Megaplier ticket at any jackpot level. The SOSD results imply that at jackpot levels below \$224 million, which the majority of latent jackpots meet, a risk averse EU maximizer will not buy Just the Jackpot, irrespective of parametric considerations. While this result is consistent with the lack of take up of Just the Jackpot in the data, risk averse EU maximizers would not buy any of the options at jackpot levels below \$224 million, as the expected return for each is negative. So dominance results under EU cannot rationalize the data.

The next step is to consider functional forms of utility with the appropriate parametric value ranges. As Mega Millions tickets usually offer negative expected returns, convex utility is required for individual rationality to hold. As was mentioned earlier, Mega Millions can be simplified into a game with a probability of getting a lower tier prize (L) and a smaller probability of getting the jackpot (J), which rolls over into the next jackpot if no one wins. Without loss of generality, Mega Millions can be reduced to a lower tier and jackpot two branch lottery, and the expected utility of each ticket can be represented as:

1. Standard: $1.5p_L u(L) + 1.5p_J u(J)$
2. Megaplier:$p_L(\sum_{m=2}^{5} p_m u(m * L)) + p_J u(J)$
3. Just the Jackpot: $2p_J u(J)$

Here the u(0) is normalized to 0, and the subscript m refers to the four possible Megaplier values. Regarding the Megaplier, a convex u(.) and a mean Megaplier value of 3 imply that $(\sum_{m=2}^{5} p_m u(m * L)) > 3u(L)$. These conditions reduce to:

$$0.5p_J u(J) > 1.5p_L u(L) \tag{1}$$

$$0.5p_J u(J) < 1.5p_L u(L) \tag{2}$$

$$1.5p_L u(L) + 1.5p_J u(J) > u(3) \tag{3}$$

where (1) is the preference condition for Standard over Megaplier, (2) is the preference condition for Standard over Just the Jackpot, and (3) is the rationality constraint. It is immediately apparent that (1) and (2) cannot simultaneously hold, meaning that under the assumption of a convex utility function, the Standard option can never be the most preferred option under EU, irrespective of the rationality constraint or wealth level. Given the argued necessity of a convex utility function to capture lottery participation at most jackpot levels, the strong conclusion is that EU is unable to account for the preference patterns in the data.

Prospect Theory

The theory that has gained the most traction as an alternative to EU theory is Prospect Theory (Kahneman and Tversky, 1979), abbreviated to PT, and its refined version Cumulative Prospect Theory (Tversky and Kahneman, 1992), which will be addressed in the next section. Among its advantages is its innate ability to explain the insurance-lottery paradox via the introduction of a probability weighting function $w(p)$, which over-weights small probabilities and under-weights large probabilities. A preference for participation in an unfair lottery no longer necessitates a convex utility of wealth function, as sufficient probability over-weighting can overcome even heavily concave utility and lead to risk loving behavior. Under PT, the three cost neutral probability-weighted expected utility representations of the simplified Mega Millions lottery options are:

1. Standard: $w(1.5p_L)u(L) + w(1.5p_J)u(J)$
2. Megaplier: $w(p_L)(\sum_{m=2}^{5} w(p_m)u(m * L)) + w(p_J)u(J)$
3. Just the Jackpot: $w(2p_J)u(J)$

The difference in representation here in relation to EU is the incorporation of the possibly non-linear $w(p)$, allowing $\sum_n w(p_n) \neq 1$. A general feature of $w(.)$ is the over-weighting of small probabilities and the under-weighting of large ones. The probabilities of winning any Mega Millions prize are much smaller than the typical inflection point probabilities both theoretically postulated (Prelec, 1998) and experimentally estimated (Tversky and Kahneman, 1992; Camerer and Ho, 1994; Wu and Gonzalez, 1996). Therefore, subadditivity is expected to hold at the Mega Millions win probabilities under PT: $w(xp) < xw(p), x > 1$. The two preference conditions and rationality constraint for the Standard option to be selected are:

$$[w(1.5p_J) - w(p_J)]u(J) > [\delta w(p_L) - w(1.5p_L)]u(L) \tag{4}$$

$$w(1.5p_L)u(L) > [w(2p_J) - w(1.5p_J)]u(J) \tag{5}$$

$$w(1.5p_L)u(L) + w(1.5p_J)u(J) > \lambda u(3) \tag{6}$$

Simplifications beyond these conditions are not as readily made without further assumptions, unlike EU. Condition (4) is the preference condition for Standard over Megaplier, (5) is the preference condition for Standard over Just the Jackpot, and (6) is the rationality constraint, with $\lambda > 1$ capturing the loss aversion associated with paying \$3 to play the lottery. The presence of loss aversion in the Mega Millions PT representation depends on whether each of the three ticket evaluations include the payment of \$3, or if not playing is a separate fourth lottery that gives \$3. Loss aversion does not impact preference between Mega Millions tickets and only raises the threshold for Mega Millions participation. Loss aversion serves the same role as an outside option with a higher threshold. Therefore, loss aversion will not be included as a model parameter ($\lambda = 1$), or equivalently, not playing and keeping \$3 is an outside fourth option.

Condition (4) takes advantage of the fact that $(\sum_{m=2}^{5} w(p_m)u(m * L)) = \delta u(L)$ for some real number $\delta > 1$. The value of δ is determined by both the shapes of $u(.)$ and $w(.)$. The Megaplier value probabilities are much closer to typical inflection points than the prize win probabilities, so that δ may be more heavily influenced by $u(.)$ and similar but larger in value to a δ for the same utility function under EU. This is because the Megaplier values of 4 and 5, which are above the mean of 3, have probabilities that fall in the generally understood over-weighting portion of $w(.)$, whereas the Megaplier probability values of 2 and 3 are close to the usual inflection point range. A consequence of the subadditivity is that while both the LHS of (4) and the RHS of (5) represent a change in probability of $0.5p_J$, the LHS of (4) is larger than the RHS of (5). This allows a wider range of permissible behavior relative to the EU constraints.

Given expected population variation of preferences, a parametric modeling of PT should demonstrate a preference for the Standard option over a substantial range of parameter values and jackpot levels. The most commonly used utility function in the parameter estimation literature of PT is the CRRA power function

$$u(x) = x^{\alpha}, x \geq 0, \alpha > 0 \tag{7}$$

(see Abdellaoui, 2000 for a list of studies using this formulation). The PT estimations will test two of the more referenced $w(.)$ in the literature: ${}^{p^{\gamma}}/{}_{[p^{\gamma}+(1-p)^{\gamma}]^{\frac{1}{\gamma}}}$ (Tversky and Kahneman, 1992) and the more theoretically-motivated $exp(-(-lnp)^{\gamma})$ (Prelec, 1998). Tversky and Kahneman (1992) generate experimental data and estimate median values of $\alpha = 0.88$, $\gamma = 0.61$ for gains. All lottery options are framed as gains, so the loss side estimates of parameter values can be ignored, as symmetry is not an implicit assumption of PT. These estimates are consistent with diminishing sensitivity and strong probability distortion. They can also serve as starting points for determining parameter ranges that support a rational preference for the Standard ticket over various jackpot amounts.

At these starting points, the Standard option is highly individually rational, with a net PT utility of about 50 at the minimum jackpot of \$40 million, increasing to a net PT utility of about 700 for a jackpot of \$1 billion. This jump in net PT utility with jackpot change highlights the fact that a minimum of 75% of the PT utility value of the Standard option at these median parameter values is due to the jackpot PT utility valuation, share increasing with jackpot size. While Standard is rational, it is not preferred at these parameter values: for jackpots up to about \$100 million, Megaplier is preferred, while Just the Jackpot is preferred for all larger jackpots. Model predictions were generated for jackpot levels of \$40 million, \$100 million, \$200 million, \$300 million, \$400 million, \$500 million, \$750 million and \$1 billion, with lower jackpot levels occurring in reality with more frequency than higher ones. Using the Tversky and Kahneman (1992) functional specifications, utility power function parameter α is allowed to run from 0.71 to 1.10. This range is chosen because it is the interquartile range of parameter values estimated for subjects modeled under PT in the experimental estimation of Abdellaoui, Bleichrodt and L'Haridon (2008). While an experimental setting may arguably not constitute a representative sample of Mega Millions players, it is also difficult to find a reason why lottery player preference estimates would necessarily fall outside of this range. Furthermore, results are robust to the extension of the range of permissible α to $[0.5, 1.5]$, not just here but in all the popular models tested in this paper. Probability distortion parameter γ from both the Tversky and Kahneman (1992) and Prelec (1998) specifications run from 0.50 to 0.99. A γ of 1 reduces to EU, a γ above 1 is distortion in the opposite direction than the literature finds evidence for.

The total number of parameter-jackpot combinations in each PT specification is 16,000: 40 utility parameter values, 50 probability distortion parameter values, and 8 jackpots. The results reported in Table 1 are of the specification most favorable to a Standard preference, which happens to be the Prelec (1998) weighting function specification. It is generally more likely to predict lottery play given its relative steepness at low probability levels in comparison to the Tversky and Kahneman (1992) specification: 91.4% of the 16,000 parameter-jackpot tests predict playing some version of Mega Millions, much more than the 77.2% in the Tversky and Kahneman (1992) specification. Of these 16,000 combinations, only 104 predict a preference for the Standard option, about 0.7%. The number of jackpot tests for which Standard is preferred and rational is at most 2 of a possible 8 for any parameter pair, as indicated by the Max Jackpots Standard column in Table 1. No parameter pair predicts Standard preference at the minimum $40 million jackpot, and most of the predictions are for the rarer high jackpot amounts. Figure 4 gives a mapping of preferences over parameter values by jackpot amount for the Prelec PT specification. Overall, PT does very poorly in explaining the strong preference for Standard in the data. In fact, it actually predicts a strong preference for Just the Jackpot, contrary to the choice behavior in the data.

Table 1: Summary of Cost Neutral Model Predictions

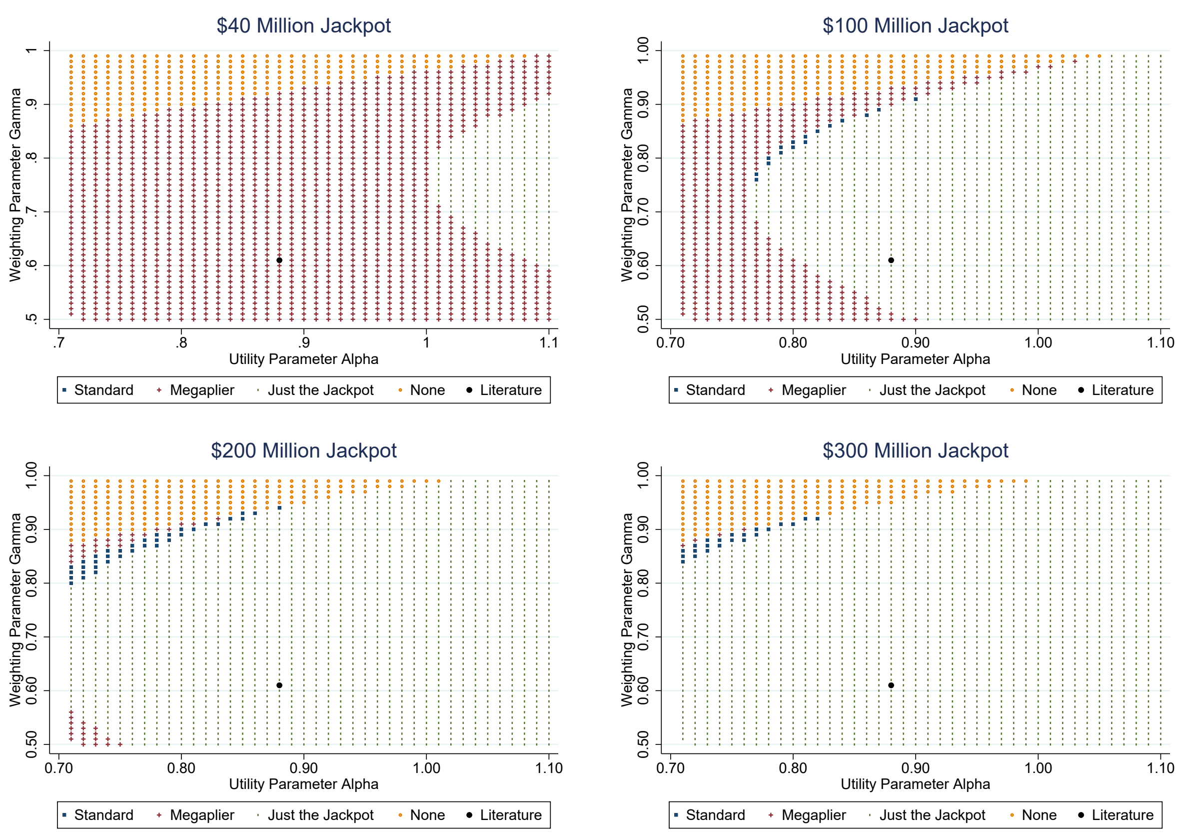
Model	Max Jackpots Standard
Expected Utility	0
Prospect Theory	2
Rank Dependent Utility	3
Disappointment Aversion	0
Regret Theory	1
Salience Theory	0

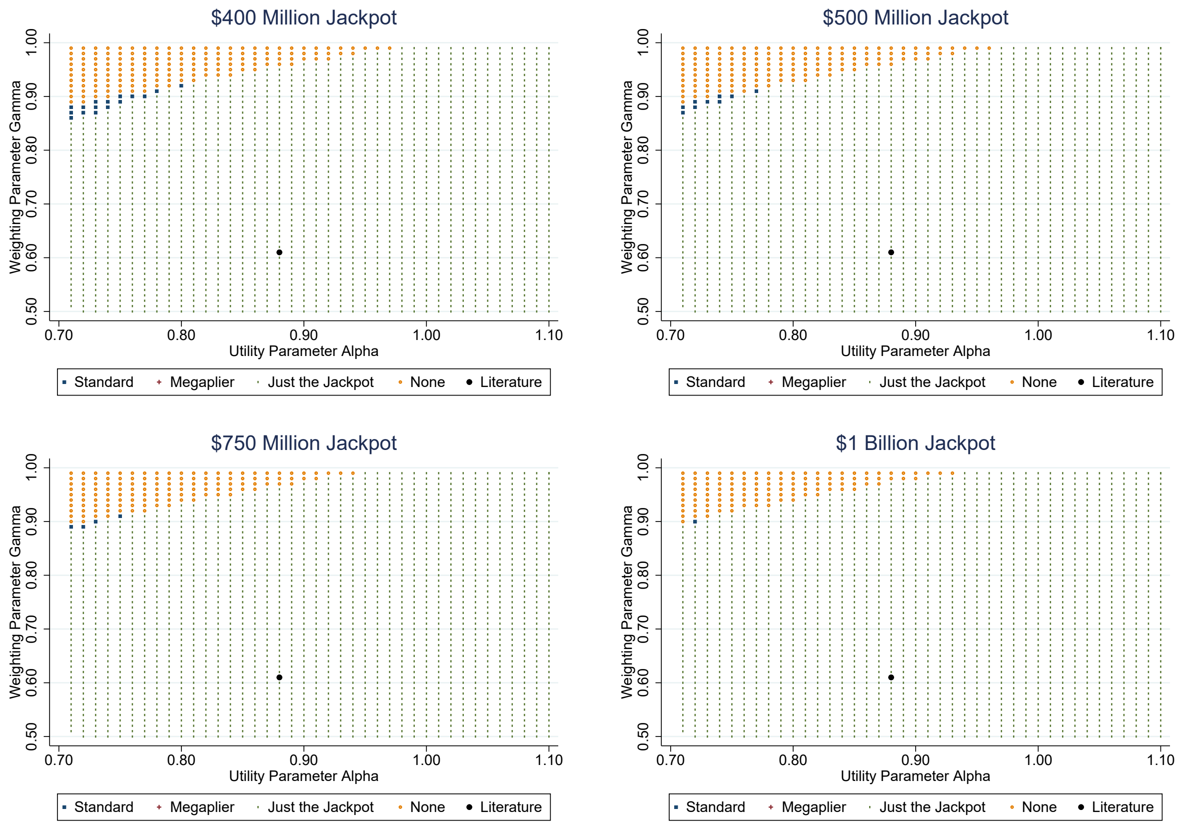
Table 1 shows the maximum number of jackpots that any one parameter set predicts a Standard preference for. Prospect Theory uses the Prelec (1998) weighting function; Rank Dependent Utility is Cumulative Prospect Theory using the Tversky and Kahneman (1992) weighting function; Disappointment Aversion with a certainty equivalent less than 4 specification; Regret Theory under pairwise aggregation and full overlap scenario; Salience Theory under a no overlap scenario with $\theta = 1,000,000$.

A major driver in the preference relationship between Mega Millions options under PT is subadditivity. The cost neutral comparison boils down to getting bigger lower tier prizes and the jackpot with probability p with the Megaplier, a shot at both the lower tier and jackpot with a probability of $1.5p$ with the Standard, and two shots at only the jackpot with Just the Jackpot. The effective non-linearity between these probabilities strongly impacts preference under PT. However, the intuitive motivation of subadditivity in the case of Mega Millions participation is not so clear cut. Subadditivity implies that if buying multiple Mega Millions tickets in sequence, the value of the first ticket is larger than that of the second, the value of the second larger than that of the third, etc. This results in every player only buying a finite number of Mega Millions entries, because at some point the subadditivity will drive ticket valuations below the cost of participation under the assumption of approximate linearity of $u(.)$ over small wealth intervals (Rabin, 2000). At some point, all existing players will drop out, and the only way to maintain participation is to bring in new players, which does not seem sustainable. It is also not consistent with the existence of regular players who play such games on a frequent basis for extended periods of time. There needs to be some "resetting" of subadditivity at some point. Is it with every transaction? Every change in jackpot? Resetting after every draw seems more reasonable, as technically the lottery changes after every draw and the previously purchased tickets are not valid for future draws. However, that would allow a player buying multiple tickets over consecutive drawings to value an earlier ticket at a lower jackpot over a later ticket at a higher jackpot, violating FOSD, a violation which PT does allow, but does not seem reasonable in this case. A Mega Millions scenario perhaps most consistent with PT and subadditivity is a regular player who purchases a single ticket for every draw.

Subadditivity coupled with the significant value added of a possibility of winning prizes, however small, ensure that only a single entry is purchased per draw. Subadditivity resetting with every draw allows this pattern to continue indefinitely. The Standard ticket is the cheapest option that gives entry into the Mega Millions, and the added perks of the Megaplier and Just the Jackpot do not offset the additional dollar cost. While this story may have descriptive appeal, it should be able to be validated within a parametric modeling of PT. Parametric estimations of PT utility comparisons for single purchases using both the Tversky and Kahneman (1992) and Prelec (1998) weighting functions result respectively in a paltry 2 and 22 parameter pairs predicting Standard preference, even worse than the cost neutral framework. The single purchase framework is discussed more thoroughly in Section 5 below.

Figure 4: Mapping of Prelec PT Preferences



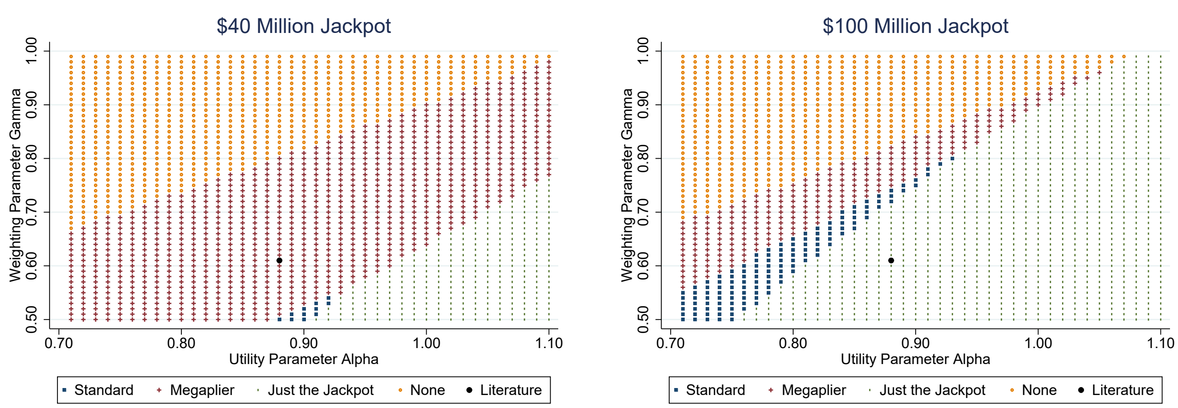


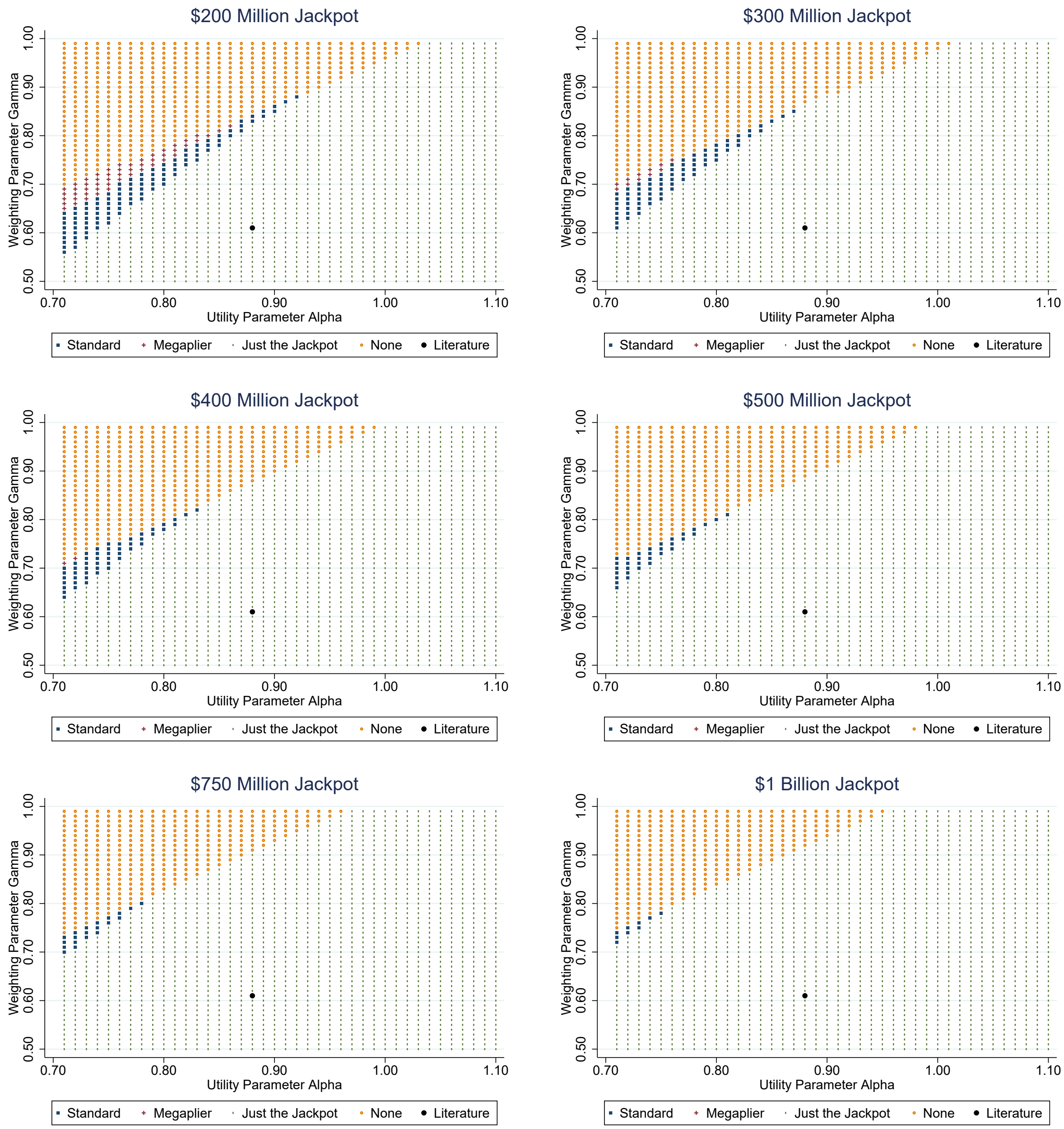
Rank Dependent Utility

One criticism of PT is its allowance of FOSD violations. Quiggin (1982) introduced Anticipated Utility (AU) theory, which uses the cumulative probability distribution to determine the probability weights and eliminates FOSD violations. Outcomes are ordered by size, and weights are determined by the marginal impact of the probability. The implication is that $w(p)$ can change depending on where in the distribution it falls, with higher distortions occurring when it falls closer to 0 or 1. Using the marginal probability contribution ensures that $\sum w(p) = 1$. Kahneman and Tversky modified PT into Cumulative Prospect Theory (CPT, 1992) by incorporating Quiggin's idea of marginal probability contributions. Models that use the cumulative probability distributions to determine weights have been termed Rank Dependent Utility (RDU) models. While CPT incorporates more structure than AU (loss aversion, separate weighting over gains and losses, reference point, etc.), in the case of Mega Millions, the only difference is how the weights are determined via the cumulative distribution. For example, consider a lottery with three outcomes $x_1 < x_2 < x_3$ with probabilities p_1, p_2 and p_3. Under AU, the weightsπ_1, π_2 and π_3 are $w(p_1), w(p_1 + p_2) - w(p_1), 1 - w(p_1 + p_2)$. Under CPT, the weights are $1 - w(p_3 + p_2), w(p_3 + p_2) - w(p_3), w(p_3)$. The preference and rationality constraints are the same as in (4), (5) and (6) from PT, except the weights are now the marginal weights derived from the cumulative probability distribution. Once again, the parametrization will employ the power function in (7) and the two weighting functions used in the PT modeling over the same ranges.

The two weighting functions and two rank dependent models yield four specifications. Once again, Table 1 only reports the specification most favorable to a Standard preference, which happens to be CPT using the Tversky and Kahneman (1992) weighting function. Under this specification, 12,215 parameter pairs are consistent with playing Mega Millions, of which 422 predict a preference for the Standard ticket, about 2.6%. The number of jackpot tests for which Standard is preferred and rational is 3 of a possible 8 for any parameter pair, one more than under PT modeling. Figure 5 gives a mapping of preferences over parameter values by jackpot amount for the Tversky Kahneman CPT specification. The preference for Standard is better distributed across jackpot levels here, although only 11 parameter pairs support a Standard preference at the most common jackpot amount of $40 million. The estimations were carried out applying the relevant probability distortion to the Megaplier probabilities. Forcing linear weighting over Megaplier probabilities does not substantially change the results of the predictions. The results of all four RDU specifications end up quite similar to those of PT: low preferences for Standard, high preferences for Just the Jackpot, and seemingly high rates of Mega Millions play. All of these run contrary to the choice behavior observed in the sales data.

Figure 5: Mapping of Tversky Kahneman CPT Preferences





Disappointment Aversion

Disappointment Aversion (DA) is an axiomatic model that swaps in an alternative to the independence axiom, with the intent of accommodating the Allais paradox while minimally straying from EU (Gul, 1991). Given a lottery p, there must exist a certainty equivalent for it. All outcomes larger than the certainty equivalent are considered elation outcomes, all outcomes below the certainty equivalent are considered disappointment outcomes (DA is a reference dependent model of sorts). Let a be the summed elation probabilities, q and r the normalized elation and disappointment sub-lotteries, so that $aq + (1-a)r = p$. The DA utility representation is

$$\left\{[\gamma(a)\sum_{x\in q}u(x) * q(x)] + [(1-\gamma(a))\sum_{x\in r}u(x) * r(x)] \quad \gamma(a) = \frac{a}{1+(1-a)\beta} \right. \tag{8}$$

with $\beta \in (-1, \infty)$. A negative β indicates an over-weighting of the elation outcomes and therefore elation loving, whereas a positive β indicates an over-weighting of the disappointment outcomes, hence disappointment averse, and $\beta = 0$ reduces to EU. The Allais paradox is consistent with disappointment averse preferences.

There has not been much work done on fitting individual decisions over risk using DA modeling. Abdellaoui and Bleichrodt (2007) propose an elicitation method for estimating DA parametrization using a power function as in (7) for utility. Estimation results from their experiment implementing the elicitation procedure are a median value for the power function parameter of $\alpha = 0.89$ for gains, and median DA parameter β ranging from about 0 to 3.5, depending on the probability of winning. They strongly reject the notion of a constant β for an individual, and that as the probability of winning increases, the higher β reflects the increased desire to avoid disappointment. When the probabilities of winning are low, the feeling of disappointment ex post will not be as intense compared to when the probability is higher, since the expectation is for a disappointing outcome (Abdellaoui and Bleichrodt, 2007). This result falls outside of the scope of the DA model as proposed by Gul (1991). The lowest win probability tested in Abdellaoui and Bleichrodt (2007) was 0.1, which gave a median β slightly below 0, indicative of elation loving. As the win probabilities for Mega Millions are exponentially smaller than 0.1, projecting the inverse relationship between win probability and elation loving onto the Mega Millions data should predict β estimates well in the elation loving range. Parameter values that are more likely to predict purchasing Mega Millions products will be lower values for β and higher α values for the power utility function.

A necessary prerequisite for DA modeling is determining certainty equivalents of the lotteries in question. As these may vary significantly for players of Mega Millions, DA modeling for Mega Millions executed here allows certainty equivalents to fall into two possible ranges that should cover the actual certainty equivalents for most players: certainty equivalents less than 4, and certainty equivalents between 4 and 10. This requires two separate estimations. Letting α to once again move between 0.71 and 1.10 in increments of 0.01 and β range from -0.9 to 4.0 in increments of tenths, with the same 8 jackpot levels, there are 16,000 possible parameter-jackpot combinations under each specification. There is a grand total of 0 parameter-jackpot combinations in which the Standard option is both rational and most preferred under either certainty equivalent specification. Results in Table 1 are for the certainty equivalent less than four specification, selected since the cost of tickets in the cost neutral framework is \$3. No mapping of parameters is given here since no tested parameter combination predicts Standard preference at any feasible jackpot level. The only DA parameter values that allow a rational preference for the Standard option are $\beta < -.95$, corresponding to absurd elation loving. Such extreme elation loving equates to a minimum of 30% weight going to elation outcomes under DA utility evaluation of the Standard option for certainty equivalents below 4 which have an actual probability of about 2% of occurring. In the case of certainty equivalents between 4 and 10, the weight is about 9% for outcomes with an objective probability of approximately 0.5%. These weights are somewhat incredulous, especially if they are to hold across decision problems. The conclusion must be that DA is not an appropriate model to account for the preferences for the Standard option displayed in the Mega Millions data.

Regret Theory

The models analyzed so far are structured so that lottery evaluation is independent of the set of lotteries available: evaluation of a lottery is wholly within-lottery. The remaining models require lottery evaluation to be contingent on the opportunity set, allowing for both within and between-lottery factors in lottery evaluation. Regret Theory (RT) was independently and simultaneously developed by Bell (1982) and Loomes and Sugden (1982). The environment for RT is in the mold of Savage (1954), in which the decision framework is choosing among acts that result in consequences depending on the state of the world that occurs. This native environment differs from those of the models discussed so far, which are choices between probability distributions over outcomes. The premise is that utility consists of an objective part, separate from the choice setting, as well as a part that depends on the other choices available. Specifically, an individual can make a choice or take a certain course of action between two available, and then some state of the world resolves, and the outcome associated with that action in that state of the world ensues. An individual could perhaps feel some elation or rejoice if that outcome is better than the outcome in the same state of the world under the other course of action, and perhaps some regret if the outcome observed is worse than what would have occurred under the other course of action. RT attempts to capture the anticipation of such feelings in the decision making process. Theories that allow for set-dependent lottery evaluation may have a better shot at explaining the Mega Millions choice behavior, due to the interdependence of the ticket types. With a given ticket in hand, a regret-influenced player may feel regret for playing at all if it is a losing ticket. In the case of the ticket winning a lower tier prize, such a player will feel no regret or maximal rejoice if the ticket is a Megaplier, a combination of regret and rejoice if it is Standard, and maximal regret if it is Just the Jackpot, even more so than not playing. If the ticket is a jackpot winner, there is no difference in outcome across ticket types, and rejoice is felt over not playing.

The method of action evaluation under RT is fundamentally between two actions only. Loomes and Sugden (1982) specify that

$$A_i \succeq A_k \Longleftrightarrow \sum_{j=1}^{n} p_j[c_{ij} - c_{kj} + R(c_{ij} - c_{kj}) - R(c_{kj} - c_{ij})] \geq 0 \qquad (9)$$

where c_{ij} is the choiceless utility that the outcome of action A_i yields in state j, and $R(.)$ is a strictly increasing regret-rejoice function. This condition equates to the modified expected utility of A_i being greater than that of A_k. Defining a function $Q(\xi) = \xi + R(\xi) - R(-\xi)$, the evaluation condition simplifies to

$$A_i \succeq A_k \Longleftrightarrow \sum_{j=1}^{n} p_j[Q(c_{ij} - c_{kj})] \geq 0 \qquad (10)$$

Loomes and Sugden (1982) demonstrate that a convex $Q(.)$ is consistent with typical EU violations like the common consequence effect, Allais paradox and the lottery-insurance paradox. One drawback of RT is that its extension to decisions between three or more actions is not straightforward. A keystone of the axiomatic foundation of RT is the relinquishing of transitivity and maintenance of the sure-thing principle. Therefore, as RT is built to compare two actions, invoking transitivity for decisions over three or more actions is not acceptable (Loomes and Sugden, 1982).

Bleichrodt et al. (2010) develop and perform the trade-off estimation procedure to experimentally estimate parametric forms of RT. The power function family in (7) was used to measure both choiceless utility c and the regret function $Q(.)$. Mean estimates of choiceless utility were effectively linear at 0.98 and 1.01 across two elicitation problems. However, there was significant variation at the individual level, with more subjects classified under concave utility than convex. Mean estimates of the power function parameter for $Q(.)$ yielded mean estimates of 1.73 and 1.89 across two elicitation problems. There is some variance at the individual level, but the overwhelming majority of subjects display a convex $Q(.)$ under RT. The convexity estimated is consistent with the functional requirements in RT for many commonly observed choice behaviors. The relevant set of actions are purchasing any three of the Mega Millions ticket types, or not playing at all, so four actions. As in Bleichrodt et al. (2010), both choiceless utility c and regret function $Q(.)$ will take a power function representation as in (7). The power parameter in c will run from 0.71 to 1.10 in hundredths intervals as in previous models, and the power parameter for $Q(.)$ will run from 0.6 to 3.0 in tenths. The Mega Millions choice problem is not pairwise, so a RT extension is necessary, and two different ones will be employed. For pairwise mean utility, a simple average of the pairwise utilities is used to predict preference, where the option yielding the maximum average is preferred (Loomes and Sugden, 1982). In the case of state-wise mean utility, the input corresponds to $c_{ij} - mean(c_{-ij})$, where c_{-ij} refers to the outcomes of all actions besides A_i (Loomes and Sugden, 1987; Sugden 1993). There does not seem to be any compelling reason to weigh certain actions and outcomes more so than others.

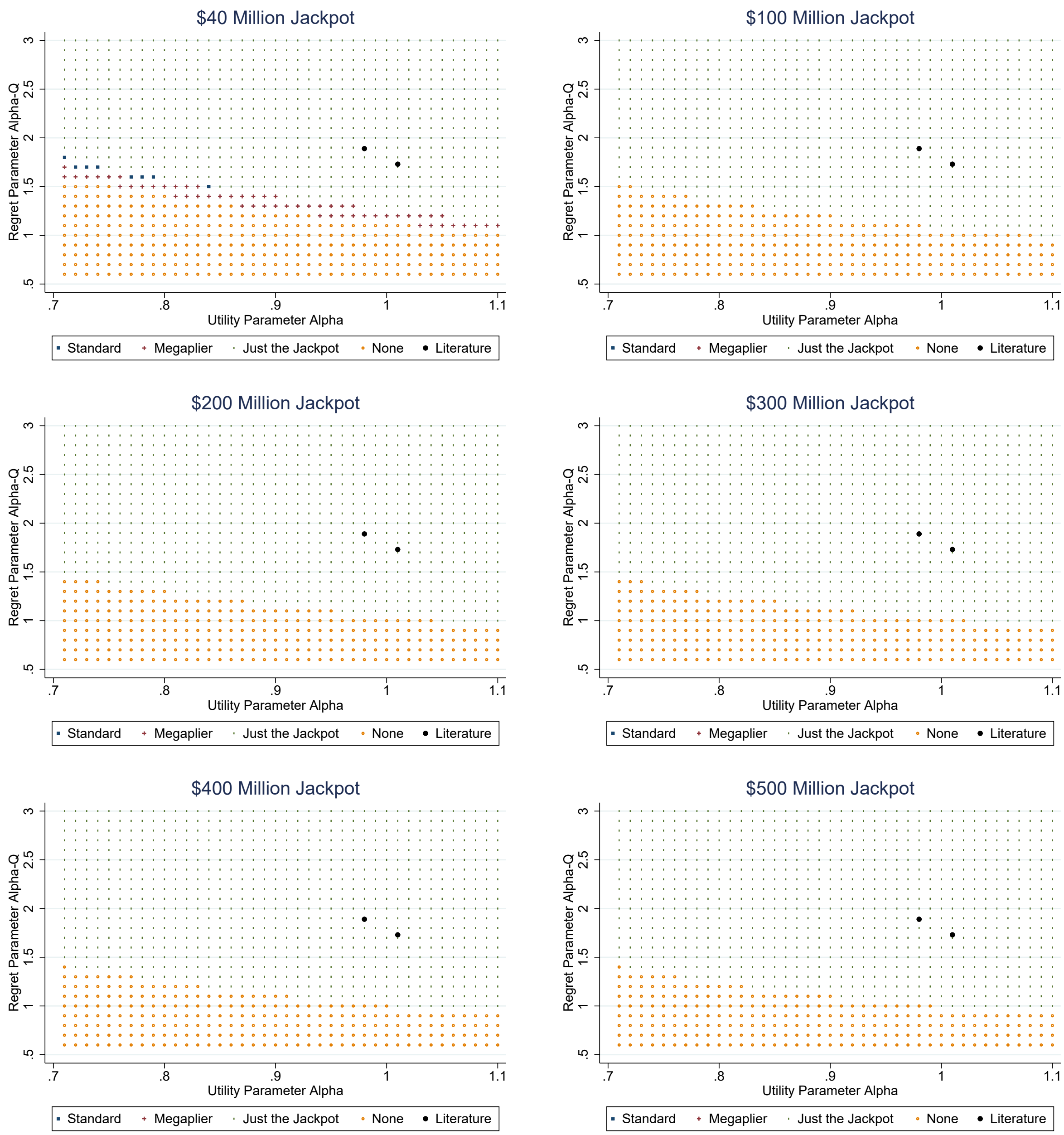
In order for RT to model a decision process, a matrix of state-contingent outcomes must be fully specified (Loomes and Sugden, 1982). With independence of the available lotteries assumed, this criterion is generally satisfied. However, the cost neutral framing of the decision problem may not satisfy independence. Consider the cost neutral pairwise choice between a Standard and Megaplier purchase. For any purchase amount, the Megaplier yields a smaller set of number combinations eligible for prizes, although winning combinations result in a higher payoff for lower tier prizes. Consider spending \$6 on either two Megapliers or three Standard entries, and assume the Megaplier value is 3, meaning lower tier prizes are multiplied by three. The number combinations on the first two tickets would be the same, regardless of Megaplier or Standard status. The third entry is what can cause a matrix uniqueness violation, as Table 2 displays. If the third entry is Ticket 3a, Ticket 3a can only win in states of the world that Tickets 1 and 2 do not. However, if the third entry is Ticket 3b, Ticket 3b and Ticket 1 have the same Mega Ball entry, entitling them both to the lowest possible prize if the Mega Ball value drawn is 25. Whether Ticket 3a or 3b is the third Standard ticket changes the set of possible outcomes pairs and breaks the outcome matrix uniqueness. One way to tackle this is to assume the case of Ticket 3a, of no overlapping numbers. With a given ticket, the probability of randomly drawing another ticket with no matching numbers is $(\frac{24}{25}) * \binom{65}{5} / \binom{70}{5} \approx 65\%$. Note that this probability will decrease greatly as the number of existing tickets required to not match a new ticket increases. The other option of considering Ticket 3b is more problematic in that there are other potential scenarios like Ticket 3b that result in further differing outcome matrices, such as sharing two numbers, three numbers, etc. Therefore, the polar overlap case of sharing all winning states for lower prize levels will be tested. While technically this scenario cannot manifest in reality, since matching the Mega Ball is a condition for some lower level prizes, it serves as a limiting case to test the RT model. Allowing the jackpot winning to overlap can only hurt the Standard option, since the jackpot is simply split among the winning tickets, so the jackpot entries from additional Standard entries will not overlap. Both the no overlap and full overlap cases are tested, as depending on the parameter values of c and $Q(.)$, one of these boundary cases will always make the Standard option as favorable as it can be under RT.

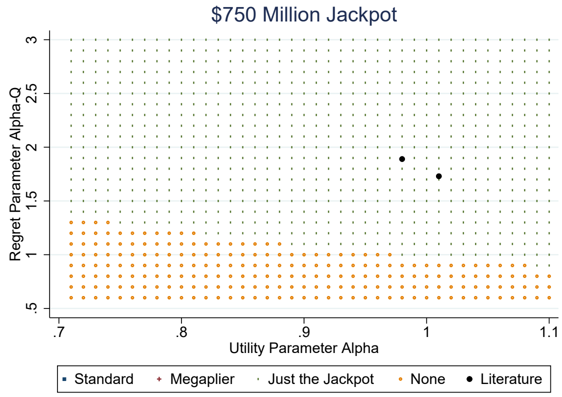
Table 2: Example of Outcome Matrix Differences

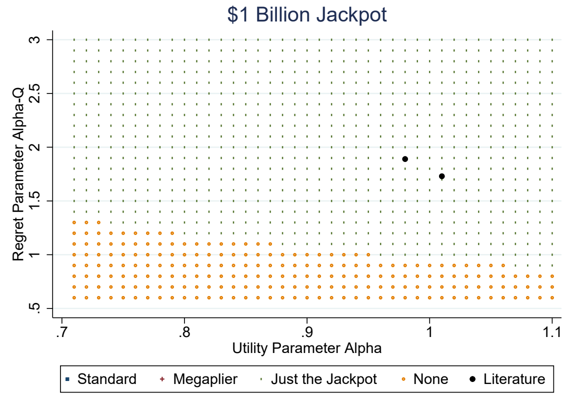
	Combination (#-#-#-#-#|Mega Ball)	Outcome Pairs with Ticket 1, Megaplier=3
Ticket 1	1-2-3-4-5|**25**	
Ticket 2	6-7-8-9-10|24	N/A
Ticket 3a	11-12-13-14-15|23	{(J,0);(0,J);(3L,0);(0,L);(6,0);(0,2);(0,0)}
Ticket 3b	16-17-18-19-20|**25**	{(J,2);(6,J);(3L,0);(0,L,);(3L,2);(6,L,);(6,2);(0,0)}

The total number of distinct parameter-jackpot combinations per specification comes out to 8,000: 40 parameter values for c, 25 parameter values for $Q(.)$, 8 jackpot levels. Note that there are also four specifications: 2 non-pairwise aggregation methods and 2 overlap scenarios. First, there is no parameter-jackpot combination that predicts a preference for the Standard option under the state-wise mean utility specification with both overlap scenarios. Under the pairwise mean utility specification, the choice of overlap scenario is essentially inconsequential to prediction. With no overlap of winning combinations, only a single parameter combination predicts Standard preference at a jackpot of $40 million, and none for all other jackpots. Under a full overlap scenario, 8 parameter combinations predict Standard preference at a jackpot of $40 million, and none for all other jackpots. The results of the pairwise mean specification under a full overlap scenario are presented in Table 1, as it is the least inconsistent with the actual choice behavior. Figure 6 gives a mapping of preferences over parameter values by jackpot amount for this specification. One potential critique is that the native problem of Mega Millions choice does not generate a unique outcome matrix, and therefore RT cannot make a prediction for this game. However, the polar cases of possible outcome matrices via overlap scenarios were both considered and result variation is negligible. Both are suggestive of a minimal preference for the Standard and Megaplier options, with preference behavior mostly being for Just the Jackpot or not participating at all.

Figure 6: Mapping of Pairwise RT Preferences under Full Overlap







Salience Theory

The most recent of the reviewed decision models in this paper is Salience Theory (ST), which allows the relative salience of payoffs within states of the world to impact the weight placed on those states of the world (Bordalo et al., 2012). The psychological motivation is that salient payoffs are over-weighted by the decision maker, and not so salient payoffs are under-weighted. ST is also set-dependent like RT. To apply ST to the Mega Millions problem, let's start with an example. Each of the ticket options gives some chance at the jackpot for \$3: Megaplier gives 1 entry, the Standard option 1.5 entries, and Just the Jackpot 2 entries. Since the native framework in ST is binary choice, just consider the Standard option and Just the Jackpot for now as the only two options available. The jackpot odds break down into 2 meaningful states of the world: the shared 1.5 entries, and the extra half entry of Just the Jackpot. Since the outcome of the 1.5 shared entries is the same, the salience in this state is the minimum of zero. This would lead to an under-weighting of this state relative to its objective probability. The bonus half entry of Just the Jackpot is highly salient, as the payoff difference is a minimum of \$40 million. This state would be over-weighted relative to its objective probability. This example highlights a major diversion between ST and PT/RDU: small probabilities only get over-weighted in ST if they are salient, whereas PT/RDU consistently over-weights small probabilities.

The notation used in Bordalo et al. (2012) is replicated here for model exposition. Consider a lottery L_i that has outcomes x_s^i across the states s$\in S$, and lottery L_j with outcomes x_s^{-i}. Let $\sigma(x_s^i,\, x_s^{-i})$ be the salience function determining the salience of a state. Like RT, the natural setting of ST is making binary comparisons. In the binary case, $\sigma(.)$ is symmetric, but not necessarily so when extended to non-pairwise decision making. They propose

$$\sigma(x_s^i, x_s^{-i}) = \frac{\mid x_s^i - x_s^{-i} \mid}{\mid x_s^i \mid + \mid x_s^{-i} \mid + \theta} \tag{11}$$

where $\theta > 0$. Salience increases with the difference in payoff between the two lotteries in a state, but decreases as the absolute average payoff deviates from 0. For example, consider L_A with payoffs {1,6} and L_B with payoffs {2,5}. The payoff difference between lotteries in each state is 1, but the salience in the first state is larger because those payoffs are closer to zero. States are then ranked by decreasing salience, so the most salient gets a rank of 1. The salience rank of state s for lottery L_i is denoted k_s^i. For a $\delta \in (0, 1]$ and objective state probabilities of π_s, the modified probability of state s is

$$\left\{ \pi_s^i = \pi_s * \omega_s^i \quad \omega_s^i = {}^{\delta^{k_s^i}}/_{\sum_r \delta^{k_r^i} * \pi_r} \right. \tag{12}$$

specified as such so that the modified probabilities sum to 1. The valuation of lottery L_i uses these modified weights: $V(L_i) = \sum_s \pi_s^i u(x_s^i)$. A state is over-weighted if and only if it is more salient than average, under-weighted otherwise. Notice that salience is determined independently of the objective state probabilities. Since the Mega Millions decision problem is non-pairwise, an extension of ST is required for proper evaluation. Bordalo et al. (2012) elaborate on a non-pairwise extension in their online appendix. They propose non-pairwise salience function $\hat{\sigma}(x_s^i, x_s^{-i}) = \sigma(x_s^i, f(x_s^{-i}))$, where $\sigma(.)$ is the salience function and $f(x_s^{-i}) = \frac{1}{N-1} \sum_{j \neq i} x_s^j$, a simple average. Once the composite alternative state outcomes are calculated, lottery valuation proceeds exactly as in the binary case. However, $\hat{\sigma}(.)$ is not necessarily symmetric, so valuations need to be calculated for each lottery under consideration. As in RT, calculations will be impacted depending on whether additional Standard tickets overlap in number combinations with existing counterfactual Megapliers. Therefore, both full overlap and no overlap scenarios will be tested.

While $\delta \in (0, 1]$ is the possible range for the distortion parameter, Bordalo et al. (2012) show that $\delta = 0.7$ is consistent with a number of observed EU violations. Konigsheim et al. (2019) experimentally estimate the distortion parameter δ and find that the Bordalo et al. (2012) assumptions of $\delta = 0.7$ and linear utility are reasonable and consistent with the results of their estimation, where δ is estimated to live in the range of 0.5 to 0.8 among subjects who behave according to ST. Bordalo et al. (2012) also show that a $\theta \geq 0.1$ is also consistent with observable EU violations that ST can account for; the choice of θ can impact the salience ranking within a lottery. Setting $\theta = 0.1$ in the Mega Millions choice problem yields some questionable salience rankings. For instance, from the perspective of Standard evaluation under a no overlap scenario, the state of the world in which the Standard ticket gives \$4 and the other tickets give nothing is more salient than the state of the world in which Standard and Just the Jackpot win the jackpot while the Megaplier wins nothing. Recall salience ranking is independent of the probabilities of states of the world. The choice problems in Bordalo et al. (2012) that justify $\theta = 0.1$ had payoffs thousands of times smaller than jackpot levels. A selection of $\theta = 1,000,000$ seems to yield significantly more reasonable salience rankings. As a robustness check, estimations are run for both overlap scenarios using both $\theta = 0.1$ and $\theta = 1,000,000$. The results reported in Table 1 are for the no overlap scenario with $\theta = 1,000,000$. For the parametrization run here, non-linearity of utility will be permitted, and the power function in (7) will once again be the functional form employed. The power parameter α will run from 0.71 to 1.10 in increments of 0.01, distortion parameter δ will run from 0.5 to 0.99 in increments of 0.01, and the 8 jackpot levels used in previous estimations will also be tested. This amounts to a total of 16,000 distinct parameter-jackpot combinations per specification. Not a single one of these corresponds to a preference for the Standard option under any specification, and a majority of them correspond to a preference for Just the Jackpot. Therefore, no figure will be provided displaying the preference distribution across parameters. The extreme salience of the additional jackpot entries for Just the Jackpot overpowers the relative advantages of the Standard and Megaplier options under ST, rendering it inept at matching the choice behavior in the data.

Non-Decision Theoretic Explanations

There are a few reasons to conjecture that certain aspects of the real world implementation of Mega Millions outside of the simple choice problem may be constraining consumers, leading to consumption choices that would differ absent such constraints. It is plausible that since Just the Jackpot is relatively new, many consumers may not be aware of its existence, and therefore attention is limited to the Megaplier and Standard options. It may be that newly offered lottery products take time to acquire a strong sales base, and Just the Jackpot has not had enough time to do so. This could be an effect of consumer inattention, or just due to slow adjustment in consumption. There could also be some liquidity constraints at play, since both Megaplier and Just the Jackpot cost \$3 relative to the Standard price of \$2. Severely constrained consumers could opt for the cheaper Standard option even if they prefer one of the other distributions. This section takes a closer look at all of these possibilities.

Limited Attention

Since Just the Jackpot has only been around since October 2017, it is plausible that many players are simply not aware of its existence. If inattentiveness or limited attention (LA) applies in this case, it could be that for some consumers, C(Megaplier, Standard, Just the Jackpot) = Just the Jackpot, and C_{LA}(Megaplier, Standard, Just the Jackpot) = C(Megaplier, Standard) = Standard. This could account for the main data patterns, since the Standard option comprises over 90% of sales, and all of the models tested above find Just the Jackpot to be the most appealing Mega Millions product overall. At face value, testing for limited attention is not really feasible. The author's correspondence with numerous lottery officials in various states suggests that advertising of Just the Jackpot can vary significantly at the retailer level (like signs outside the retail location mentioning the new option, a retailer bringing up the product in conversation with customers, etc.), and acquiring retail-level data is quite costly, let alone the effective impossibility of determining Just the Jackpot advertising intensity at each retailer, which is not part of any compiled data set the author is aware of.

Fortunately, the lottery commission of Kansas created a natural test of limited attention by running statewide promotions for Just the Jackpot. On November 2, 2018, a complementary Just the Jackpot ticket was given to customers who purchased \$6 worth of Mega Millions tickets of any type in a single transaction, resulting in \$15,975 in promotional sales, or 5,325 Just the Jackpot giveaways. Admittedly, there was \$155,454 Mega Millions in non-promotional sales purchased in Kansas for that drawing. In order to maximize the number of players who qualified for the promotion, 5,325 separate players would spend \$11 and thereby qualify for just one giveaway each, and non-qualifying players would each spend \$5. This would imply at best 22% of players during that drawing being exposed to the promotion; 10% is a more realistic expectation. Given that just 67 non-promotional Just the Jackpot tickets were purchased for that drawing, if players indeed had preferences in line with the model predictions, then the thousands of players who received the promotion would have been enough to produce a noticeable uptick in Just the Jackpot purchases for subsequent drawings.

To formally test the limited attention hypothesis, a difference in difference analysis can be implemented. The neighboring state of Nebraska displays similar sales levels and trends to those of Kansas, and also offers Just the Jackpot. Both states have a trend of diminishing Just the Jackpot sales with time. Ideally, every player in Kansas for the promotional drawing would have been eligible for the giveaway for the difference in difference analysis to be most convincing. However, the already paltry Just the Jackpot sales levels and the gross number of players who did qualify for the promotion increases the chances of identifying an impact of the promotion. The regression estimates for the difference in difference model presented below are listed in Table 3, with the dependent variable being the percentage of sales that are Just the Jackpot. Similar conclusions are reached using dependent variables of Just the Jackpot sales, as well as Just the Jackpot sales as a percentage of the jackpot.

$$JtJ\%Sales = \beta_0 + \beta_1 Period + \beta_2 Kansas + \beta_3 Period * Kansas + \varepsilon$$

The result of the difference in difference test is no evidence of an impact of the promotion on Just the Jackpot sales. The downward trend in Just the Jackpot sales is in no way stymied by the promotion. Therefore, it must be concluded that for at least habitual Mega Millions consumers, limited attention is not culpable for the dearth of Just the Jackpot sales. This restriction is due to the promotion being run at a low jackpot amount of $52 million. Most of the players qualifying for this promotion would have been regular players and not jackpot frenzy players, who generally only play when jackpots get excessively high. Since no such promotion was run at an unusually high jackpot level , this analysis makes no claim regarding limited attention among jackpot frenzy players.

Table 3: Difference in Difference Test of Limited Attention

	JtJ % Sales
Intercept (β_0)	.0009*** (.00005)
Period (β_1)	-.0005*** (.00005)
Kansas (β_2)	.001*** (.0002)
Promotion (β_3)	.0005 (.0009)

Standard errors are in parentheses below the estimates. *** indicates significance at better than 1%.

Slow Sales Adjustment

There is a possibility that there is a significant lag in consumption adjustment after the introduction of a new lottery product. More precisely, the introduction of an additional ticket option within an existing lottery game may require a long timeline to acquire its stable sales range. Data consistent with this hypothesis would show an increase in sales percentage of a new lottery product with the passage of time, controlling for appropriate factors like jackpot size. In the case of Just the Jackpot, there should be some indication of an increased share in the Mega Millions sales percentage as time passes. A quick glance at Table A2 in the Appendix, which displays the time series of Mega Millions sales for states that offer Just the Jackpot since its inception on October 31, 2017 up through September 2019, seems to indicate the opposite. The Just the Jackpot sales percentage starts off at 2.08% on the first day it is available for purchases, but quickly drops below 1% in a week's time, then settles below 0.5% about a month after that and further declining into the 0.2% to 0.4% range and never rebounding. This is in line with a consumption lag of no more than a month, and the adjustment runs opposite to what the hypothesis would require.

A linear regression of the Just the Jackpot sales percentage on time further validates this hypothesis rejection. Table 4 presents a couple of variations on this regression, where Time is a counting variable for each passing draw date, set equal to 1 for the first day Just the Jackpot was available for purchase. Also, since jackpots for a specific draw are due to a combination of the previous jackpot rolling over and the actual sales for that draw, using jackpot as a regressor in a sales regression introduces endogeneity: a lotto consumer likely takes into account the expected jackpot when deciding on lottery consumption, but that lottery consumption itself is impacting the jackpot size (Cook and Clotfelter, 1993). Therefore, two regressions are provided in Table 4: one regressing the sales percentage on time and the log of the rollover, which is the previous draw's jackpot amount, unless the previous jackpot was won, in which case the rollover is zero; the other using the log of the rollover and time as instruments for the log of the jackpot and regressing sales percentage on time and the predicted log jackpot. Both regressions produce essentially the same highly significant negative estimate of -0.000018. An additional puzzling result is the significant negative coefficients on Log Rollover and Log Jackpot, indicative of the trend highlighted above for the historic $1.6 billion Mega Millions run. As the jackpot increases, Just the Jackpot should increase in relative appeal, as the lower tier distribution remains unchanged. The regression results indicate the opposite, and this will be addressed in greater detail later on.

Table 4: Regressions of Just the Jackpot Sales Percentage

	(1)	(2)
Intercept	0.0062*** (.0005)	0.0218*** (.005)
Time	-0.000018*** (.000002)	-0.000018*** (.000002)
Log Rollover	-0.000087*** (.000024)	
Log Jackpot		-0.0009*** (.00026)

Time is a counting variable for passing draw dates. Log Rollover is the natural log of the rollover amount, which is equal to the previous draw's jackpot or zero, in case the previous jackpot was won. In this case, the natural log of the rollover of zero is also set equal to zero. Log Jackpot is the natural log of the jackpot amount. (1) presents linear regression results, whereas (2) presents two stage least squares estimates, in which Log Jackpot is instrumented for with Log Rollover and Time. Note that jackpot and rollover have a correlation coefficient of 0.97. Standard errors are in parentheses below the estimates. *** indicates significance at better than 1%.

Liquidity Constraints

Given that many lottery products costing a dollar or less, the Just the Jackpot price of \$3 may be too steep for some consumers. Lotteries are often considered to be a poor man's tax, regressive in nature. Data bears out the regressivity, but only in so much as lower income tiers spend a higher percentage of their income on lottery products, as aggregate spending is relatively steady across income levels (Clotfelter and Cook, 1990; Kearney, 2005). Since lotteries are consumed even at the lowest income levels, where liquidity constraints are most likely to bind, it is plausible that some constrained consumers may opt to purchase the cheaper Standard ticket at the price of \$2, even if Just the Jackpot is the preferred option without such constraints.

There are two reasons up front why liquidity concerns are likely not the primary determinant of the data patterns. First, regressivity varies significantly within lotteries: instant games are highly regressive, draw games with jackpots that rollover are much less so, even somewhat progressive when jackpots become exceedingly large (Clotfelter and Cook, 1987). Mega Millions is precisely a draw game with a rollover jackpot that can get exceedingly large, much larger than the Maryland jackpots of a few million dollars analyzed in their study, and therefore Mega Millions players are on average wealthier than many other lottery game players. Second, from the limited attention test on the Kansas Mega Millions data, at least 20% of sales during typical jackpots are due to ticket purchases of at least \$6, meaning multiple tickets were purchased in a single transaction. It is likely that a larger percentage than that abounds during atypically high jackpot draws, in which jackpot frenzy buyers join the fray and the lottery becomes more progressive.

The most recent changes to the Mega Millions game on October 31, 2017 involved both a change to the payoff-odds matrix and an increase in the cost of the ticket options. Standard tickets doubled in cost from \$1 to \$2, and Megaplier increased 50% in cost, from \$2 to \$3. Assessing the impact on Megaplier sales of this cost increase to \$3 can provide some further insight regarding liquidity constraints on Just the Jackpot, which also costs \$3 but was newly introduced on October 31, 2017. For the two years prior to the change, median nationwide Megaplier sales by draw amounted to about \$2.14 million. After the change, the median Megaplier sales remarkably remained within just a few hundred dollars of the median for the two years before the change. This implied an approximate one-third reduction in Megaplier entries purchased after the cost change was instituted. So the cost change was essentially revenue-neutral for Megaplier sales. This is not true for the Standard ticket. Median Standard ticket revenue before the cost change was about \$16.7 million, whereas after the cost change it rose to \$25.4 million, over a 50% increase in median revenue, implying about a 25% reduction in median Standard entries. Therefore, overall median Mega Millions revenue grew after the increased cost, reduced odds and resultant increased jackpots of the October 31, 2017 restructuring.

Is this evidence of the presence of liquidity constraints for Mega Millions consumption? The most recent changes implemented to Mega Millions altered the expected return and effective price of both the Standard and Megaplier options. Effective price of a lottery entry can be thought of as the normalized expected loss, so the cost minus the expected value of the ticket, divided by the cost. Expected values vary as jackpots increase and with the probability of sharing the jackpot, so here the minimum jackpot is used in expected value determination, along with the simplifying assumption of no jackpot sharing. Prior to the changes, the effective price of the Standard ticket was 77 cents, and for the Megaplier it was 63 cents. After the 2017 changes, effective prices increased to 81 cents for the Standard ticket and 71 cents for the Megaplier. The Standard effective price increase was about $\frac{0.81}{0.77}-1 \approx 5\%$, whereas for Megaplier it was about $\frac{0.71}{0.63}-1 \approx 13\%$. The Megaplier became relatively more pricey after the changes than the Standard ticket did, and this by itself can rationalize the shifts in sales behavior witnessed with the 2017 changes. An increase in effective price should lead to a reduction in quantity demanded, which the data confirms for both ticket types. The higher relative price increase on Megaplier is more consistent with a net move from Megaplier to Standard, and not the other way around. While it is not possible to track individual purchasing behavior to confirm this with certainty, a higher percentage reduction in Megaplier purchases relative to Standard entries is consistent with this idea. The price increase on the Megaplier was high enough to diminish Megaplier sales to the point of revenue-neutrality, whereas the price increase on the Standard option was low enough so that revenue actually increased.

Notice that this rationalization of the data does not require a liquidity constraint story. It is conceivable that with the cost increase, constrained consumers who had previously purchased Megaplier under the old cost regime could buy Megaplier less frequently, switch to buying Standard tickets, or stop playing Mega Millions. While liquidity constraints are consistent with the observed reduction in Megaplier entries observed, the price increase channel described above can also explain it. The pricing motive may arguably better explain the shifts in consumption than liquidity constraints, since after the change in cost regime, the Standard cost doubled, whereas the Megaplier only increased in cost by 50%, both increasing by the same amount of $1. It is plausible that a dollar increase in cost will cause liquidity constraints to bind for more individuals the smaller the original cost is, so that Standard sales would be more impacted than Megaplier sales by binding constraints. The point estimates of price elasticity are indicative of this as well: a 5% price increase in the Standard option led to about a 25% reduction in quantity consumed, whereas a 13% price increase of the Megaplier resulted in only a 33% reduction in quantity consumed. More tightly binding liquidity constraints will strongly reduce consumption when prices increase, implying a higher price elasticity absent those constraints. The Standard price elasticity is approximately double that of the Megaplier, suggesting that if liquidity constraints are even present, they are likely impacting Standard consumption more than Megaplier consumption. Furthermore, recall that the motivation for looking at Megaplier consumption shifts with a cost increase was to see if low Just the Jackpot sales percentages were due to liquidity constraints. Recall that a minimum purchase of Megaplier and Just the Jackpot both costs $3. At best, liquidity constraint arguments are confounded by the price channel predicting the same directional movements in the data. Even with the increase in cost, Megaplier sales account for between 5.5% to 8.5% of Mega Millions dollars sales, 20 to 30 times more than the paltry average 0.3% of Just the Jackpot, which costs the same as the Megaplier. For liquidity constraint arguments to have bite, there should have been a much larger reduction in Megaplier sales after the cost increase. Therefore, it must be concluded that liquidity constraints are not the primary driver of low Just the Jackpot sales.

Status Quo Bias

It is possible that consumers who played Mega Millions prior to the introduction of Just the Jackpot in October 2017 may not want to change their behavior after the introduction. The preference-based models as they were considered above did not have an allowance for status quo bias built in, although alternative formulations of some of those models can account for such a bias. The Mega Millions sales data prior to the October 2017 Just the Jackpot launch can help identify if Mega Millions players suffer from status quo bias in large numbers.

The Megaplier was a known and available option in most states by the time of the October 2017 Mega Millions changes. However, for a number of years, players in many states only had access to a single Mega Millions product: whatever version the Standard ticket was at that time. If status quo bias is present among Mega Millions players, it should be present for the introduction of both the Megaplier and Just the Jackpot options. For the January 18, 2011 drawing, five states that were already offering just the Standard option for Mega Millions introduced the Megaplier: Massachusetts, New Jersey, New York, Ohio, and Washington. The aggregate Megaplier sales percentage was 5.8%.

Of these five states, only New Jersey and New York offer Just the Jackpot. Those two states had a combined Megaplier sales percentage of 3.9%. Small but significant Megaplier sales percentages persist with time. For the November 18, 2011 drawing ten months later, they had a combined Megaplier sales percentage of 4.2%. For the launch of Just the Jackpot for the October 31, 2017 drawing, the combined Just the Jackpot sales percentage was 0.8%. For the same size jackpot drawing on July 27, 2018 ten months later, Just the Jackpot sales percentages drop to 0.1%. These Just the Jackpot percentages are more in line with the percentages observed for the time series outside the first few weeks after Just the Jackpot was introduced. Differences in sales percentages by a factor of about 40 suggests that responses to Megaplier and Just the Jackpot introduction varied widely even in the presence of status quo bias. One interpretation is that players do not have a status quo bias, but a small percentage prefers the Megaplier.

However, these discrepancies in sales percentages upon Megaplier and Just the Jackpot introduction are not necessarily enough to rule out status quo bias among most Mega Millions players. Perhaps if no status quo bias was present, the true Megaplier sales percentage would have been 50%. Return to the models and predictions presented in Table 1 and consider a reduced Mega Millions game with only Standard and Megaplier options. It turns out that running the reduced Mega Millions game in these models results in the Standard option replacing Just the Jackpot as the most preferred option across jackpot levels. There still remains a minority of parameter-jackpot combinations predicting Megaplier preference, in the same range of a few percentage points observed in the New Jersey and New York data. Therefore, the models are able to account for the observed behavior in the Megaplier-Standard-only Mega Millions game. Status quo bias is also capable of explaining the behavior, but without more compelling reasons to favor that hypothesis over the model predictions, the inclination is to go with the simplest explanation. There also does not seem to be a reason to think that status quo bias should only be present for the introduction of Just the Jackpot and not the Megaplier. Given the evidence, status quo bias is unlikely to be driving choice behavior in the Mega Millions game.

Behavioral Mechanisms

Feedback-Conditional Regret Theory

The various models analyzed share a general consensus that with jackpots relatively low, high concavity of utility and other model-specific parameter ranges, Megaplier or not playing at all are the preferred options. For larger jackpots, convex or even relatively linear utility and relevant model-specific parameter ranges, Just the Jackpot is the preferred option. Data consistent with any of these models should show a low percentage of Standard sales, with Megaplier and Just the Jackpot sales making up the majority of sales, and a decent level of non-participation. Furthermore, the non-decision model explanations of limited attention, slow sales adjustment and liquidity constraints have been ruled out. One hypothesis that would essentially bypass the entire conundrum is that lottery play is not rationalizable, especially given the inability of a wide range of models with varying rationality criteria to explain the Mega Millions data. One manifestations of this hypothesis is lottery and gambling play being classified as consumption goods (Hirshleifer, 1966), not subject to utility of wealth evaluations. However, sensitivity of lottery sales to expected returns across a variety of games (Kearney 2005) and surveys reporting that lottery players mainly view the lottery as a means to acquire wealth counter this hypothesis. Additionally, the attempts of many models in the literature at explaining the so called lottery-insurance paradox is a concession in the literature that lottery consumption is primarily a wealth concern that can be rationalizable via preference modeling.

The remaining possibility is that Just the Jackpot is simply not appealing to Mega Millions consumers. Decision theory suggests that the poor appeal is not justifiable via the payoff distribution or even the choice set. Perhaps the mechanics of actually playing the Mega Millions lottery will provide some insight into the choice behavior. Once a decision to play is made, a player must either pick a set(s) of numbers or have one(s) randomly assigned. The player must also determine which of the three ticket options to select. Consider spending \$3 on Mega Millions. If the player goes with the Megaplier, after the winning numbers are drawn, a player will feel no regret if the ticket is eligible for a prize. If the ticket wins a lower tier prize, that is more than would have been won had that ticket been a Standard or Just the Jackpot. In the case of winning the jackpot, there is no difference between ticket types. However, in the event of the Megaplier not winning a prize (a 96% probability), a player may regret not opting for one of the other ticket types. That original ticket would not have won regardless of the ticket type, but additional counterfactual entries may have been eligible for prizes. However, that ex post regret can only be felt in a probabilistic sense, since those additional tickets were never generated. Now consider originally opting for Just the Jackpot, which would give two different entries into only the jackpot portion. In the unlikely event of one of them winning the jackpot, no regret would be felt, and possibly some rejoice. This is because at any given cost, Just the Jackpot gives the most entries into the jackpot, and it may be that opting for another ticket type would not have generated the winning ticket. But in the nearly certain event of Just the Jackpot entries not being jackpot eligible, a partial ticket match may stir up regret, since if the ticket had been one of the other ticket types, it would have won a prize. This regret may be different from the regret engendered by the Megaplier, since in the case the Megaplier is not eligible for a prize, a player will not know with certainty the outcomes of the foregone Standard or Just the Jackpot entries. However, most models do not take such a distinction into account, including state and opportunity set-dependent models like Regret and Salience Theory. The ex post regret would be the same if additional voided entries were given for players who opted for the Megaplier or the Standard option, bringing the total of void and non-voided entries equal to the number of counterfactual Just the Jackpot entries.

There is precedence in the literature that this distinction in engendered regret ex post is contributing to the distaste for Just the Jackpot. Specifically, "feedback about what definitely would have occurred produces a greater potential for regret than pallid, abstract knowledge of what was statistically likely to occur" (Larrick, 1993). This description captures the regret difference here, since a Just the Jackpot holder has feedback about what definitely would have occurred if the Megaplier was chosen, whereas the Megaplier holder is missing definitive feedback for the foregone second Just the Jackpot ticket. The implication is that Megaplier buyers would prefer not being given the voided second Just the Jackpot ticket, and a similar extension can be made for

Standard buyers with respect to the Just the Jackpot alternative. While Regret Theory and its ilk may be said to account for anticipated regret, this is not the same as ex post regret or post-decision regret, which refers to having knowledge of the outcomes of foregone options in at least some states of the world. A number of studies in the psychology literature have detected effects on decision making of the presence of ex post regret in varying settings: choice between two risky gambles (Zeelenberg at al., 1996); consumer purchase decisions (Inman and Zeelenberg, 1998); an ultimatum game in which the proposer knows that he/she will be informed of the minimum acceptable offer after the proposal is made (Zeelenberg and Beatie, 1997); and most relevant to the problem at hand, influencing participation behavior and regret motivation in the Dutch postcode lottery, for which the entry is the player's postal code (Zeelenberg and Pieters, 2004). The Dutch postcode lottery has ex post regret built into it, since a player will know the outcome of the foregone option independent of participation. This is not true for lotteries in general: by entering, one will know whether one wins or loses, as well as the outcome of not entering (keeping the ticket cost); whereas by not entering, one will never know the outcome had they entered.

Given the evidence for ex post regret considerations in decision making, Humphrey (2004) attempts to add an ex post regret flavor to original Regret Theory (Loomes and Sugden, 1982; Loomes and Sugden, 1987). The specifics of the Humphrey (2004) modification to Regret Theory is provided in the Appendix. The main takeaway is that even incorporating differences in ex post feedback into Regret Theory does not yield a model capable of matching the strong preference for Standard in the data. The predictive power is comparable to that of the Rank Dependent Utility results listed in Table 1; detailed results of Feedback-Conditional Regret Theory (FCRT) are listed in Table A1 in the Appendix. These disappointing results do not necessarily mean that feedback consideration is not impacting choice behavior per se, as Regret Theory may also not be an appropriate base model to apply feedback considerations to. The structural interdependence of the Mega Millions ticket options engenders feedback variation based upon the option chosen, which at least descriptively seems to have the potential to impact choice behavior. Indeed, the model developed in the Section 6 does include a feedback parameter which contributes to its relatively strong predictive power.

Winner and Loser Regret

FCRT attempts to capture what is probably a major factor in the aversion to Just the Jackpot. The albeit unlikely possibility of purchasing a Just the Jackpot ticket that would have been eligible for a lower tier prize, particularly the million dollar prize level, may weigh heavily on some consumers. While FCRT is not able to rationalize the data, the variation in the elicitation of feedback on foregone actions ex post does seem to be a reasonable factor in the decision making process from a psychological and behavioral perspective. Returning to the mechanics of Mega Millions, any given number combination could be a Megaplier, Standard or Just the Jackpot entry. In the case of that ticket being jackpot eligible, ticket type is inconsequential. If it is not eligible for any prize, once again the type is inconsequential. However, if it is eligible for a lower prize, Megaplier gives the best possible outcome, Just the Jackpot gives the worst, and Standard is intermediary. In such a scenario, Standard and Just the Jackpot holders may experience ex post regret for not having opted for a Megaplier, with Just the Jackpot eliciting (perhaps disproportionately) more regret than the Standard option.

For example, consider the state of the world in which the ticket designated as a Megaplier would win \$5 million, in which case the Standard version of that ticket would win \$1 million, and Just the Jackpot would win nothing. For ease of exposition, assume linear utility of wealth and a state-specific regret magnitude, defined as the difference of the maximum attainable outcome in that state and the outcome of the chosen ticket type. In this state of the world, Megaplier yields no regret, Standard yields a regret of 4 million, and Just the Jackpot yields a regret of 5 million, so that the regret of a Standard designation is 80% of that of a Just the Jackpot designation. A linear RT framework also yields an 80% state-specific utility ratio of Standard foregoing Megaplier and Just the Jackpot foregoing Megaplier. A reduction of this ratio will generally result in the Standard option becoming relatively more favorable compared to the Megaplier without also making Just the Jackpot relatively more favorable.

There is precedence for thinking that this ratio should indeed be less than 80%, even with linear utility assumptions. In the million dollar state described above, Megaplier is the winner and Just the Jackpot the loser, while Standard falls in between the two. From a regret perspective, holding a Megaplier engenders no regret, while the other two may do so. Just the Jackpot can be said to engender loser regret, as it yields the worst possible state-specific outcome. Standard may be classified as engendering winner regret, as the holder walks away with \$1 million but could have walked away with \$5 million. Filiz-Ozbay and Ozbay (2007) theoretically propose and experimentally demonstrate that observed overbidding relative to the Risk Neutral Nash Equilibrium in first price auctions can be attributed to bidders anticipating loser regret. Loser regret would be felt ex post if a bidder loses an auction, is then told the winning bid, and realizes a higher but still rational bid could have won the auction. Their experiment also demonstrates that bidders do not strongly anticipate winner's regret. Winner's regret in a first price auction occurs if a bidder wins an auction and is told the second highest bid, and realizes a lower, more profitable bid would have still won the auction. Mean bid differences between treatment groups, as well as self-reported feelings of anticipated winner and loser regret, both suggest that only loser regret is significantly impacting bidding behavior in first price auctions. Engelbrecht-Wiggans and Katok (2008) run a similar test of winner and loser regret in first price auctions, but find significant effects of both winner and loser regret on bidding behavior. However, they note that winner regret becomes more prevalent in bidding behavior with successive auctions in a repeated experimental session, suggesting that winner regret is not as anticipated as loser regret and becomes more impactful with bidding experience.

The insights on winner and loser regret from the first price auction experiments are relevant to the Mega Millions choice problem. If the conclusion from Filiz-Ozbay and Ozbay (2007) carries over to the Mega Millions setting, the implication would be that players deciding on which ticket type to purchase would focus on the lower tier prize disparity between Megaplier and Just the Jackpot disproportionately more than that between Megaplier and Standard. Incorporating this into a model would improve the overall desirability of the Standard option. In order to determine where a given action's outcome in a certain state falls on the regret scale, the best and worst outcomes for the state must be known. A parameterized term that can capture the intensity of winner regret is

$$\gamma_{kj} = \begin{cases} \left(\frac{\max\limits_{A_i \in A} u(x_{ij}) - u(x_{kj})}{\max\limits_{A_i \in A} u(x_{ij}) - \min\limits_{A_i \in A} u(x_{ij})} \right)^{\rho} & \max\limits_{A_i \in A} u(x_{ij}) \neq \min\limits_{A_i \in A} u(x_{ij}) \\ 0 & \max\limits_{A_i \in A} u(x_{ij}) = \min\limits_{A_i \in A} u(x_{ij}) \end{cases} \tag{13}$$

where A represents the set of available actions, x_{kj} is the outcome of action k in state j, and $u(.)$ is the (choiceless) utility function, and $\rho > 0$ is a winner regret intensity parameter. The power functional form is used to allow for substantial variations in γ_{kj} depending on where x_{kj} falls within the range of state payoffs. The denominator of (13) is the maximum possible utility difference in state j, essentially the maximum regret. The numerator of (13) gives the action-specific regret in state j. If action k happens to be the best action in state j, then (13) reduces to 0, irrespective of ρ. If action k happens to be the worst action in state j, then (13) reduces to 1, irrespective of ρ. More generally, when there are only two actions in the choice set, (13) will always reduce to either 0 or 1. When there are only two possible outcomes in a certain state of the world, there can be only a max and a min outcome, and hence no potential for winner regret. Going back to the lower tier evaluation of Megaplier, Standard and Just the Jackpot, (13) equals 0 for Megaplier evaluation, it equals 1 for Just the Jackpot evaluation, and it equals something in between 0 and 1 for Standard evaluation. If not playing at all means having the ticket cost in a cost neutral framework, for instance the minimum cost of \$3 that can buy any type, not playing also will fall between 0 and 1, albeit much closer to 1 as the lower prize tier being evaluated increases. The distinction between winner and loser regret is more ambiguous with this definition in comparison to the first price auction setting. An action yielding the worst possible outcome in a state will engender loser regret, but getting a marginally better than worst outcome can hardly be considered as shifting to winner regret. This is exemplified by not playing Mega Millions with a counterfactual \$5 million Megaplier: the player still has the \$3 ticket cost relative to nothing from Just the Jackpot, but the difference in regret is negligible. A more appropriate description is that the designation in (13) allows for shades of winner and loser regret.

Demand Quota

The argument up to this point has been for the cost neutral choice framework. While that framework is more appropriate from a purely decision modeling perspective, the fact of the matter is that single purchases dominate Mega Millions sales. The Kansas Lottery Commission promotion of a free Just the Jackpot entry with a \$6 Mega Millions purchase shows that only 20% of the sales in that period qualified for the promotion, and therefore more than 80% of the transactions did not qualify. The only multi-entry transaction that would not qualify for the promotion is a purchase of two Standard tickets. It is difficult to come up with any defensible reasons as to why most of the transactions would be of pairs of Standard tickets. Based on the data, it seems reasonable that there is a large drop off in the number of transactions after single ticket purchases.

It is important to identify possible reasons for the dominance of single purchases. It actually is not so important to nail down a specific reason, but to rule out certain ones that would constrain consumers to choose a sub-optimal option. One constraining reason is liquidity constraints, but the argument provided earlier shows that the high levels of Standard sales are likely not primarily due to liquidity constraints forcing constrained players to settle for their less preferred Standard option. A non-constraining reason is that players set some sort of demand quota, either per draw(s) or by jackpot amount. Some players may just make one purchase for every draw, or once a week, or once the jackpot goes over \$200 million. This could be due to a force of habit, or even to regulate potentially addictive consumption. It could also be to avoid regret of not playing. A single entry into either the Standard or Megaplier would shield a player from large amounts of regret in states of the world with big winnings, even with the lack of ex post feedback that comes by not playing. It is therefore worthwhile to consider a single purchase framework, which differs from the cost neutral framework only in how the Standard option is distributed. The single purchase framework puts back an additional dollar to the Standard option in every state of the world, and takes away the additional half Standard entry. A single purchase analysis of the decision models tested in Section 3 is provided in Table 5. The predictions are with the cost neutral framework, except for Salience Theory, which now has a few instances of Standard preference prediction. However, factoring in the winner regret into the decision modeling within a single purchase framework should favorably impact the Standard standing, since relative to the Megaplier and Just the Jackpot, it leaves an additional dollar in the consumer's pocket in the likely event of not being prize eligible. The next section will show results from the proposed model in both the cost neutral and single purchase frameworks.

Table 5: Summary of Single Purchase Model Predictions

Model	Max Jackpots Standard
Expected Utility	0
Prospect Theory	2
Rank Dependent Utility	2
Disappointment Aversion	0
Regret Theory	1
Salience Theory	2

Table 5 shows the maximum number of jackpots that any one parameter set predicts a Standard preference for. Prospect Theory uses the Prelec (1998) weighting function; Rank Dependent Utility is Cumulative Prospect Theory using the Tversky and Kahneman (1992) weighting function; Disappointment Aversion with a certainty equivalent less than 4 specification; Regret Theory under pairwise aggregation and full overlap scenario; Salience Theory under a no overlap scenario with $\theta = 1,000,000$.

Data Rationalization

Model

The previous section argued for incorporation of feedback on foregone choices ex post and winner regret into a cost neutral or single purchase framework. A model incorporating these features would be more similar to models that incorporate between-lottery information, like Regret Theory and Salience Theory, than within-lottery only models like Expected Utility, Rank Dependent Utility, Disappointment Aversion, etc. Incorporating ex post feedback on foregone outcomes into a decision model requires information about other lotteries in the consideration set, in addition to the lottery being evaluated. The proposed winner regret coefficient requires knowing the span of outcomes over each state for evaluation purposes. A model of the Mega Millions game that considers these factors therefore implicitly holds that the choice evaluation process is a function of the available choices, which will allow for subsequent counterfactual analyses of reduced choice sets and removal of feedback and winner regret mechanisms from Mega Millions.

The process for developing a model to account for the Mega Millions choice data is behavioral, in the sense that psychological and behavioral motives are identified in the actual game, which then inform an appropriate model. The approach is not preference-based and therefore will not be presented with an axiomatic foundation. There may be no single "right" model given the approach, but the intent is to develop as simple and parsimonious a model as possible. Given the argued importance of regret in the Mega Millions game structure, some sort of regret model seems to be a good starting point for a model capturing the Mega Millions decision making process. There are a plethora of regret-based decision models that have developed over the past few decades: Minimax Regret (Savage, 1951); Regret Theory (Loomes and Sugden, 1982; Bell, 1982); Reference-Dependent Regret (Krahmer and Stone, 2005); Smooth Regret Aversion (Hayashi, 2008), to name a few. Some of these models are intended for pairwise choice scenarios with extensions to non-pairwise choice, like Regret Theory. Others, like Minimax Regret, fundamentally allow for non-pairwise choice settings. The Mega Millions choice setting has four options and would more naturally fit in a non-pairwise model.

Before introducing the model, the choice framework will be laid out. The framework is in the spirit of Savage (1954) and resembles the framework of most regret-based models, in particular Loomes and Sugden (1982). There are a finite number N states of the world, where each state of the world $j \in N$ corresponds to a unique vector of consequences $\overrightarrow{x_j}$ of finite length M. Each state j has an associated probability $p_j \in [0, 1]$ with $\sum_j p_j = 1$. There are M actions to choose from, each action $A_i \in A$ being an N-tuple of consequences. x_{ij} refers to the consequence of choosing action A_i if state j realizes.

A decision maker is tasked with choosing a single action from the set of available actions. The decision maker is aware of not only the probability of every consequence in each action, but also the vector of consequences $\overrightarrow{x_j}$ for each possible state of the world. A decision maker behaves according to Regret Weighted Feedback Minimax (RWFM) if the choice satisfies

$$\phi(A) = \begin{cases} \arg\min_A \sum_j \left[p_j * \gamma_{kj} * \left(\max_A u(x_{ij}) - u(x_{kj}) \right) \right] & x_{kj} \ Reveals \max_A x_{ij} \\ \arg\min_A \sum_j \left[p_j * \gamma_{kj} * \delta \left(\max_A u(x_{ij}) - u(x_{kj}) \right) \right] & otherwise \end{cases} \tag{14}$$

where γ_{kj} is as specified in (13), $\delta < 1$ captures the difference in regret engendered ex post if choosing action A_k and state j realizing does not reveal $\max_A x_{ij}$. If under every action and every state there is always either no feedback or feedback, then the decision criteria reduces to just the first line of (14), removing the need for the feedback parameter δ. The criteria in (14) is a probability-weighted Minimax, with an additional winner regret weight γ_{kj} and possible scaling down of the regret term by δ if action A_k and state j realizing does not reveal $\max_A x_{ij}$.

The method to proceed with evaluating (14) is to start with any of the available actions in A, which will take the role of A_k. Then for each state of the world, the $\max_A x_{ij}$ must be identified, so that with a given utility function, $\max_A u(x_{ij}) - u(x_{kj})$ can be evaluated. It then must be determined if in state j action A_k is revealing of the $\max_A x_{ij}$. If so, the first case of (14) is used for evaluating A_k in state j. If not, the individual-specific δ is applied using the second case of (14). γ_{kj} can also be calculated by identifying the $\min_A x_{ij}$ and applying the individual-specific ρ. p_j is the probability of state j, which is not the same as and will generally be smaller than the probability of getting the consequence x_k under A_k, as x_k may realize in more than one state. The valuation of A_k can then be established by summing over all states. Proceed in the same manner for all $A_i \in A$, and the action that yields the minimum is the indicated choice of action.

Since this is a newly proposed model, there is not really a precedent on what range of values δ and ρ can practically take. The utility function will be parameterized with the power function as in (7), and power parameter α will range from 0.71 to 1.10 in increments of hundredths as in the other model estimations above. δ will be allowed to run from 0.1 to 1 in increments of tenths, and ρ will run from 0 to 5 in increments of 1. To give an idea of how the extreme values of these parameters impact evaluation, return to the Mega Millions choice problem, and assume linear utility for ease of exposition. Consider evaluating the Standard option in the state of the world in which the Megaplier wins \$5 million and the Standard wins \$1 million. Since opting for Standard in this state of the world reveals the counterfactual Megaplier of \$5 million and state max, the first case of (14) is used and δ falls out of consideration. Notice that $\gamma_{kj} = \left(\frac{5{,}000{,}000-1{,}000{,}000}{5{,}000{,}000-0}\right)^{\rho} = 0.8^{\rho}$. At the allowable minimum, $0.8^0 * (5,000,000 - 1,000,000) = 4,000,000$. At the allowable maximum, $0.8^5 * (5,000,000 - 1,000,000) = 1,310,720$, resulting in a difference in regret by a factor of more than three, so that the Standard option will become more favorable in this state of the world as ρ increases. Now consider evaluating Megaplier in the state of the world in which Just the Jackpot wins the \$40 million jackpot with the additional jackpot entry it gets relative to the Megaplier. Holding a jackpot-losing Megaplier ticket does not reveal the additional counterfactual Just the Jackpot winning entry, bringing δ into play. For a given γ_{kj}, the extreme values of δ result in regrets of $\gamma_{kj} * 1 * (40,000,000) = \gamma_{kj} * 40,000,000$ and $\gamma_{kj} * 0.1 * (40,000,000) = \gamma_{kj} * 4,000,000$, a difference by a factor of ten. So, highly discounting non-feedback (low δ) and diminished winner regret (high ρ) has the potential to strongly alter choice prediction.

A cost neutral analysis of the Mega Millions choice problem using RWFM will ensue, followed by a single purchase analysis. Since this is still a regret-based model, predictions will be dependent on the overlap specification of additional Standard entries. Once again, both extremes of full and no overlap are considered, in order to set bounds on predictive power. The total number of parameter-jackpot combinations considered is 19,200: 40 utility parameter values, 10 values of feedback parameter δ, six values of winner regret parameter ρ, tested at eight jackpots. Table 6 presents the results. In the no overlap scenario, 1,956 parameter-jackpot combinations are consistent with a Standard preference, a substantial 10.2% of the combinations in this overlap scenario. Furthermore, Standard is the Mega Millions option with the largest number of combinations predicting its selection, Just the Jackpot at 6.3% and Megaplier at 1.8%. The estimates do not change substantially under a full overlap scenario, with 10.7% of parameter-jackpot combinations indicative of Standard selection and 81.6% predicting non-participation. The insensitivity to the overlap specification is an additional plus of this model, since some sort of mixing between these two extremes is most practical.

There is one downside to the estimation results of the cost neutral RWFM, as no parameter combination predicts Mega Millions participation of any type at the minimum jackpot of \$40 million, although there are numerous combinations that consistently predict Standard selection across the other seven tested jackpots. It should be noted that allowing utility convexity to go just a few hundredths above 1.1 will yield some combinations predicting Mega Millions play and specifically Standard selection. However, it may be a bit concerning that in the concavity-convexity range that experimental studies identify as holding much of the density there is no predicted Mega Millions play. If players approach Mega Millions with a self-imposed demand quota of a single minimum purchase per draw, week, jackpot threshold, etc., the cost neutral approach is no longer appropriate. Under a single purchase approach, all options but Standard remain the same as in the cost neutral approach, but Standard loses its additional entries. So a Standard ticket differs from a Megaplier in that it keeps an additional dollar per Megaplier in exchange for reduced payouts in lower tier eligible states. Since there are no additional Standard entries modeled, there is no longer a need to consider variation in overlap scenarios. The estimation results of both the cost neutral and single purchase approach can be seen side to side in Table 6. Using the same parameter-jackpot combinations as in the cost neutral case, 4,994 of the 19,200 combinations indicate Standard selection, or 26.0%. A majority of combinations still predict non-participation at 64.7%. The primary benefit of this approach over the cost neutral one is that now 318 of the 2,400 parameter combinations (13.3%) tested at the $40 million jackpot indicate Standard selection, with no other Mega Millions option favored at the $40 million jackpot. The single purchase approach predicts greater Mega Millions participation at each jackpot level relative to the cost neutral approach.

Table 6: Cost Neutral vs Single Purchase RWFM Predictions

Model	**Max Jackpots Standard**
No Overlap	7
Full Overlap	7
Single Purchase	8

Table 6 shows the maximum number of jackpots that any one parameter set predicts a Standard preference for. No overlap is the cost neutral specification for which additional Standard entries share no winning states of the world with Megaplier. Full overlap is the cost neutral specification for which additional Standard entries share all winning states of the world with Megaplier. Single purchase is the specification of a single $2 Standard purchase, $3 Megaplier purchase and $3 Just the Jackpot purchase.

From Table 6 alone it is not necessarily clear as to whether the cost neutral or single purchase approach is more appropriate to model the Mega Millions choice problem. It is possible that cost neutral better captures the problem for some players, while single purchase does better for others. It is even possible that players facing low jackpots restrict themselves to single purchases given the frequency of low jackpots, but switch to behavior more consistent with cost neutrality when jackpots become excessively high. Whatever the explanation may be, each RWFM specification differs with and does significantly better than all of the models analyzed in this paper in three key ways consistent with the sales data:

1. Standard is the Mega Millions ticket option most preferred over wide ranges of parameter combinations.

2. Standard preference is demonstrated for a large set of parameter values across the range of feasible jackpot values.

3. Not playing Mega Millions at all is the preference for most parameter-jackpot combinations.

As was noted earlier, there is effectively no precedent as to an appropriate range of parameter values capturing feedback and winner regret considerations in the population. A wide range of values were tested, perhaps excessively so, in order to see the full implications of the model. Therefore, results like a higher percentage of parameter-jackpot combinations predicting Just the Jackpot than Megaplier should be taken with a grain of salt. This does not imply that we should expect to see a similar pattern in the data, which we don't. This observation applies to a lesser degree to the common models tested above, since the parameter values tested in those cases are based on precedents established in the experimental literature. But the distribution of the population over the presumptive parameter values tested is most certainly non-uniform, so simply imposing the predicted percentages onto the population as expected behavior would be misleading. What the predictions can tell is if a product is expected to be preferred at all, and perhaps give some notion of the intensity of preference in the population. Literal interpretation of the magnitudes of the results in any model rests upon unrealistically strong assumptions.

In the pursuit of parsimony, RWFM can be constrained to remove either the feedback or winner regret parameter. The former is equivalent to setting $\delta = 1$ in (14), whereas the latter is equivalent to setting $\gamma_{kj} = 1$ for all k, j. Table 7 reports the parameter percentages with a no feedback restriction, and Table 8 reports the percentages for a no winner regret restriction. There are some trade offs relative to the unrestricted version of the model, results of which are in Table 6. Removing feedback distinctions renders the cost neutral framework inept at capturing the strong Standard preference in the data over the gamut of feasible jackpots, but the single purchase framework still is quite effective, although the Megaplier is now never predicted at any parameter-jackpot combination. Eliminating the winner regret parameter is marginally better for the cost neutral framework than removing the feedback parameter, but only predicts a Standard preference for a maximum of 4 of the 8 tested jackpots for any of the parameter values considered. Once winner regret is removed, the single purchase framework becomes about as effective in mimicking the data trends as the common models tested earlier. A case can be made that accounting for winner regret and dropping the feedback parameter from the model does well enough at predicting the general data patterns. Accepting this restriction de facto sets the single purchase framework as the appropriate modeling framework as well. Figure 7 displays the parameter distribution at each tested jackpot for the restricted RWFM single purchase model dropping the feedback parameter.

Table 7: RWFM Predictions with No Feedback

Model	Max Jackpots Standard
No Overlap	2
Full Overlap	2
Single Purchase	8

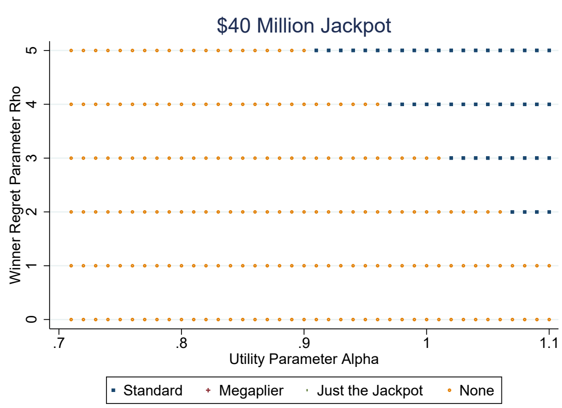
Table 7 shows the maximum number of jackpots that any one parameter set predicts a Standard preference for, under a RWFM model without a feedback differentiation parameter. No overlap is the cost neutral specification for which additional Standard entries share no winning states of the world with Megaplier. Full overlap is the cost neutral specification for which additional Standard entries share all winning states of the world with Megaplier. Single purchase is the specification of a single \$2 Standard purchase, \$3 Megaplier purchase and \$3 Just the Jackpot purchase.

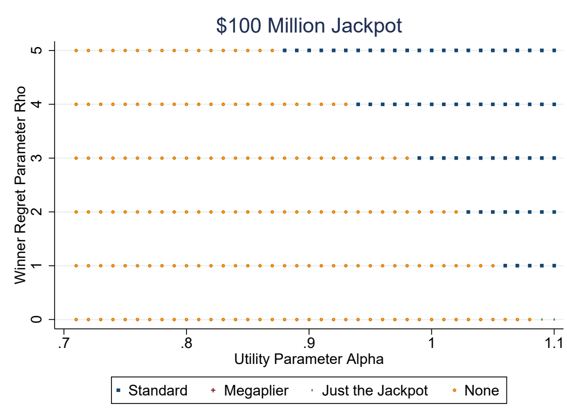
Table 8: RWFM Predictions with No Winner Regret

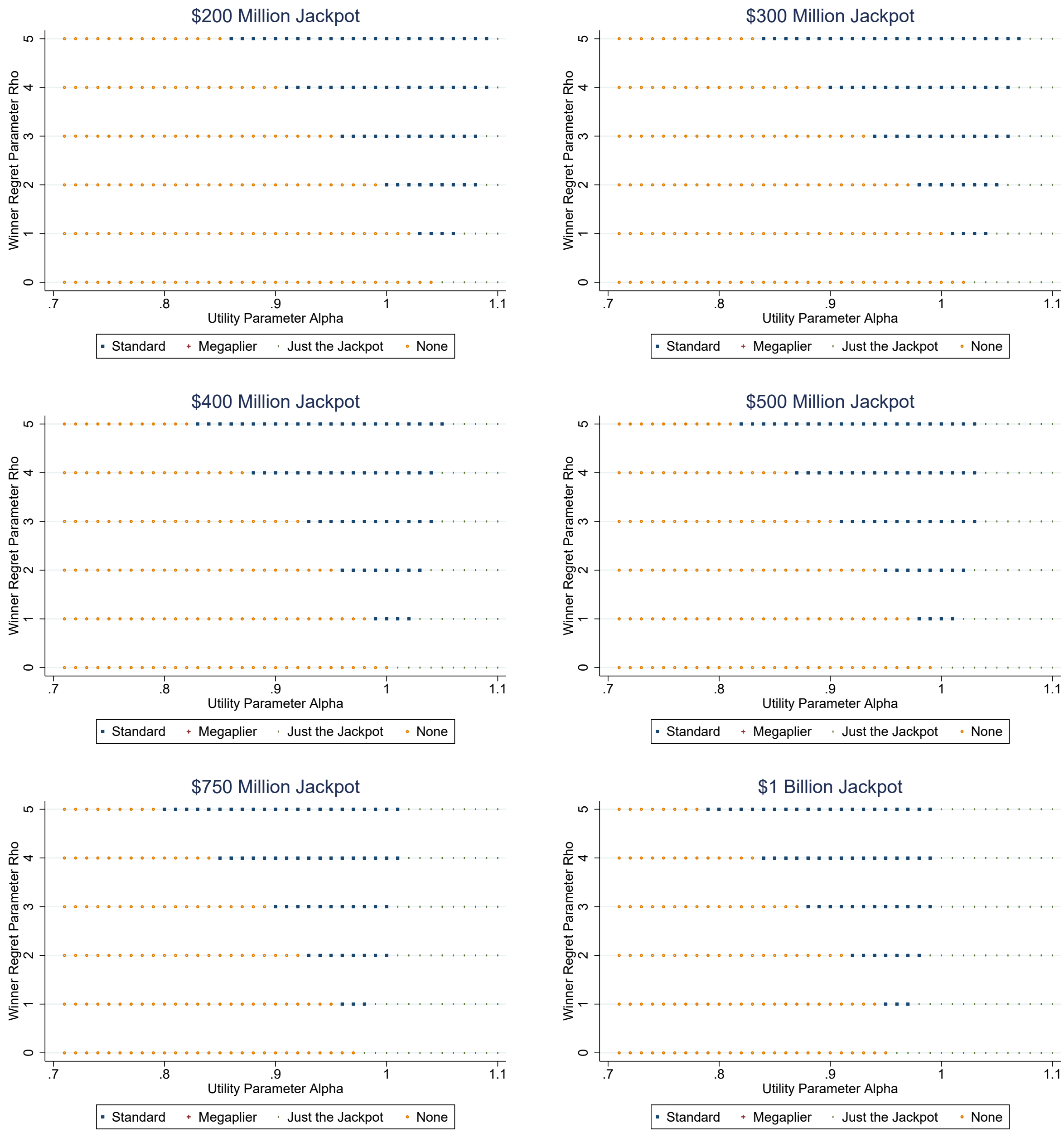
Model	Max Jackpots Standard
No Overlap	4
Full Overlap	4
Single Purchase	2

Table 8 shows the maximum number of jackpots that any one parameter set predicts a Standard preference for, under a RWFM model without a winner regret parameter. No overlap is the cost neutral specification for which additional Standard entries share no winning states of the world with Megaplier. Full overlap is the cost neutral specification for which additional Standard entries share all winning states of the world with Megaplier. Single purchase is the specification of a single \$2 Standard purchase, \$3 Megaplier purchase and \$3 Just the Jackpot purchase.

Figure 7: Mapping of Single Purchase RWFM with No Feedback







Limited Attention, Revisited

Another, perhaps even more puzzling facet of the sales data is the flat or even downward trend of Just the Jackpot sales as a percentage of total sales as jackpots grow in size. This is not consistent with RWFM predictions in any of its specifications analyzed above. As jackpot size increases, not only does the number of parameter combinations predicting Just the Jackpot selection increase, but also the percentage of the combinations predicting Mega Millions play that are Just the Jackpot. The regressions in Table 4 confirm the inverse relationship between Just the Jackpot sales percentage and jackpot size. Note that as the jackpot size increases, Just the Jackpot must become relatively more appealing under any decision framework taking probabilities into account. As the jackpot increases in size, lower tier prizes and probabilities remain unchanged. Therefore, Just the Jackpot sales percentages should increase with jackpot size, but the data indicates the opposite.

The Kansas Lottery Commission field experiment demonstrated that limited attention is not responsible for driving low aggregate sales levels of Just the Jackpot tickets. The jackpot amounts for the draws the promotions were active were \$52 million and \$252 million, both typical jackpot amounts for Mega Millions. There are two general types of players that play games with rolling jackpots. There are the frequent players who play at some frequency across all jackpot levels, with intensity of play perhaps increasing with the jackpot size. Then there are the jackpot frenzy players, who generally abstain from playing until jackpots become unusually high. What the Kansas experiment established is that limited attention is not the cause of poor Just the Jackpot sales among frequent players. The argument proposed here is that extrapolating that conclusion to jackpot frenzy players is not appropriate. Frequent lottery consumers should be expected to have increased awareness of the products available in that market, much more so than infrequent consumers. There is also evidence that as jackpots get excessively high, regressivity of the lottery becomes more proportional or even progressive (Clotfelter and Cook, 1987), meaning that wealthier and higher income individuals make up a much higher percentage of the consumer base of jackpot games at higher jackpots amounts. The sales base changes with the jackpot level: exponential sales increases at unusually high jackpots are not primarily due to regular Mega Millions consumers scaling up their purchases many times over. The change in income distribution at low and high jackpot levels indicates that the large jackpots are driving the participation of the relatively wealthier players. It is reasonable to presume that these wealthier, higher income players derive much less marginal benefit from lower tier prize winnings than poorer players. Therefore, it seems likely that they would be willing to give up the chances for lower tier prizes for additional entries into the jackpot at a given desired expenditure amount. There is also an increase in the pooling of funds for lottery ticket purchases at higher jackpot levels by groups of individuals, like co-workers, friends or family. For such pools that have the intent to split winnings of any tickets purchased, it is not conceivable that any of the lower tier prizes, maybe save the \$1 million, can hold any substantial bearing in the decision to purchase a bunch of tickets. These reasons collectively point to limited attention reducing Just the Jackpot selection at higher jackpot levels. Since there has not been a similar experiment to the Kansas one conducted at an excessively high jackpot level, these reasons highlighting the difference in the player composition at the very least allow for limited attention to remain a viable hypothesis at higher jackpot levels. The author of this paper holds it to be the only plausible explanation, given no theoretical or psychological explanation for the Just the Jackpot sales percentage to decrease with jackpot size.

Counterfactual Analysis

No Feedback Just the Jackpot

The first counterfactual analyzed is how behavior would look if Just the Jackpot tickets did not give ex post feedback on the Standard and Megaplier options. There is a very simple and practical way to accomplish this: require Just the Jackpot entries to be the selection of a single number between 1 and 302,575,350. Such an entry would give no feedback on the foregone Standard or Megaplier options, in the same way that those options don't give feedback on the foregone additional jackpot entries. There is an assumption of indifference between the current selection process and the proposed one, and that players are fully aware of the jackpot odds under both processes. It is possible that the convoluted selection mechanism currently in use allows players to be unaware of the actual odds, although they are displayed clearly on the Mega Millions website. They are not advertised in any way though, so the proposed change to the selection mechanism becomes an advertisement of the jackpot odds, which may influence behavior. This issue is set to the side for the counterfactual analysis.

Enacting the mechanism change for Just the Jackpot only makes it no longer revealing of the outcomes of the foregone Megaplier and Standard tickets. In the model, this means Just the Jackpot evaluation now utilizes δ in all states. Table 9 reports the RWFM predictions for this restructured Mega Millions game. The predictions give further credence to the single purchase specification over the cost neutral ones, as they are no longer able to rationalize a Standard preference over a sufficiently large range of jackpots. For the most part, some of the players opting for Standard or Megaplier in the original game now opt for Just the Jackpot. In fact, comparing the percentages not playing between Tables 6 and 10 indicate that removing the feedback on Just the Jackpot does not increase the overall play rate. This begs the question of whether or not cannibalization by Just the Jackpot of other ticket types is desirable for Mega Millions.

Table 9: RWFM Predictions with No Feedback Just the Jackpot

Model	Max Jackpots Standard
No Overlap	2
Full Overlap	2
Single Purchase	8

Table 9 shows the maximum number of jackpots that any one parameter set predicts a Standard preference for, under a RWFM model with a restructured Just the Jackpot option that provides no feedback on foregone Megaplier and Standard selections. No overlap is the cost neutral specification for which additional Standard entries share no winning states of the world with Megaplier. Full overlap is the cost neutral specification for which additional Standard entries share all winning states of the world with Megaplier. Single purchase is the specification of a single $2 Standard purchase, $3 Megaplier purchase and $3 Just the Jackpot purchase.

Optimal Sales and Profits

RWFM is not unique in its prediction of Just the Jackpot primarily cannibalizing existing Mega Millions sales. Table 10 reports the differences in the overall Mega Millions play rates using the various models tested earlier, with and without Just the Jackpot as an option. In spite of the inability of these models to justify the choice behavior in the sales data, these models consistently predict negligible impacts of introducing Just the Jackpot on the extensive margin. Such an analysis could have been undertaken leading up to the creation of Just the Jackpot and would have indicated that the benefit of its introduction would be increased consumer welfare. However, Mega Millions and other lottery products are not designed with consumer welfare as the primary optimization objective. Many states have legislation explicitly stating the objective of lotteries to be revenue maximization for the state. Therefore, inducing players to switch from another Mega Millions option to Just the Jackpot should result in increases in revenue, in order to comply with state legislation.

Consider three counterfactual Mega Millions games, each of which only has one of the three ticketing options available: Standard Mega Millions, Megaplier Mega Millions, and Just the Jackpot Mega Millions. The current Mega Millions game ends up most resembling Standard Mega Millions, given the high percentage of Standard sales. The approach then is to construct hypothetical jackpot and sales progressions in each of the counterfactual games, using estimated jackpot and sales growth in the existing game, along with the existing profitability rate, which is relatively stable across draws. With the hypothetical sales and jackpot progressions in each game, the long run average sales and profitability per draw in each game can be estimated, and by that determine the sales of which ticket type are most in line with state revenue objectives. This procedure implicitly assumes that consumption patterns in these counterfactual games mimic that of the existing game: constant initial sales at the minimum jackpot across games, the same predicted sales growth as a function of the cash value of the jackpot, and the same target profitability rate in each game. These assumptions are quite strong, but if accepted allow for clear predictions about Mega Millions revenue maximization.

Table 10: Cost Neutral Play Rates

	Mega Millions Play Rate		
Model	**All Options**	**No Just the Jackpot**	**Difference**
Expected Utility	20.3%	18.8%	**1.5%**
Prospect Theory	91.4%	91.2%	**0.2%**
Rank Dependent Utility	76.3%	76.1%	**0.2%**
Disappointment Aversion	14.3%	12.9%%	**1.4%**
Regret Theory	75.9%	75.6%	**0.3%**
Salience Theory	88.2%	87.2%	**1.0%**

Table 10 shows the predicted play rates under a cost neutral framework for each of the common models, both with and without Just the Jackpot as an available option. Prospect Theory uses the Prelec (1998) weighting function; Rank Dependent Utility is Cumulative Prospect Theory using the Tversky and Kahneman (1992) weighting function; Disappointment Aversion with a certainty equivalent less than 4 specification; Regret Theory under pairwise aggregation and full overlap scenario; Salience Theory under a no overlap scenario with $\theta = 1,000,000$.

The sales data over the two year period shows a remarkably consistent net revenue rate of 52% of gross sales. As an aside, net revenue is not the same as profits, which would be net revenue less retailer commission and costs of running Mega Millions. Further assume the same retailer commission and additional cost structure across counterfactual game types, so net revenue optimization translates directly to profit maximization. Net revenue for a given draw is defined as gross sales since the last jackpot reset, less lower tier prize payouts since the last jackpot reset, less the cash value of the jackpot for that draw. For example, a minimum jackpot of \$40 million may have a cash value of \$25 million, depending on interest rates and other factors. If the sales for that draw were \$20 million and lower tier payouts were \$2 million, potential net revenue would be $\$20,000,000 - \$2,000,000 - \$25,000,000 = -\$7,000,000$. Consider the jackpot rolling over to \$50 million with a cash value of \$30 million, with sales for that draw of \$20 million and lower tier payouts of \$2 million for that draw. Potential net revenue for that draw would be $(\$20,000,000 + \$20,000,000) - (\$2,000,000 + \$2,000,000) - \$30,000,000 = \$6,000,000$. Mega Millions essentially chooses the cash value of the jackpot for the coming round based on predicted sales and converts that into an advertised annuitized amount so that net revenue is at 52%, save for the first few draws of a given round of jackpots due to the relatively low sales compared to the jackpot size. This is true for all the jackpot rounds except the first one after the October 2017 changes, in which net revenue is lower and more volatile. There is an inherent endogeneity of jackpot and sales determination: sales are the sole influencer of jackpot size, while jackpot size will influence sales. Mega Millions announces a tentative jackpot for the next round at the end of the draw, which they determine based on sales projections. Mega Millions at times scales up the jackpot due to unexpectedly high sales, but does not reduce the jackpot if sales are abnormally low. Over the time range in the data, Mega Millions does reasonably well in setting jackpot amounts to keep expected net revenue stable at about 52%: the interquartile range over the two years of data for draws outside of the first few and the first round of jackpots is 51.8% to 52.3%. Note that expected net revenue in the data is calculated using expected and not actual lower tier payouts, and that it can be calculated for every draw, although it will only realize for a draw with a winning jackpot.

The sales data can be leveraged to establish a relationship between jackpots and sales to predict sales growth as a function of the natural log of the lagged cash values of the jackpot. A quadratic fit works much better than a linear fit at modeling the sales growth, highlighting convexity in growth as jackpots increase. In each of the counterfactual games the same minimum jackpot of $40 million and initial gross sales for the first draw of $22 million are assumed, which is approximately the average first draw sales across the time period. For the first few draws in the actual data, jackpots usually increase in increments of $5 million, with sales holding relatively constant with the increasing jackpots. However, profitability increases significantly with each draw, settling at the 52% after about 4 draws. This sales and jackpot initiation are copied to the three counterfactual games. After the first few draws the lagged cash jackpots are used to estimate sales growth and therefore predicted sales for the next draw. Using these predicted sales, a cash value of the jackpot can be determined by setting the profitability rate to 52%. Profitability is determined differently in each version: Megaplier Mega Millions has high expected lower tier payoffs, so that jackpot growth is relatively low draw to draw; Just the Jackpot Mega Millions has no lower tier payoffs, so all the payout is packed into the jackpot, which grows fastest in this version; Standard Mega Millions takes a middle path with moderate lower tier payments and jackpot growth. Note that Megaplier Mega Millions gives the worst odds at the jackpot, while Just the Jackpot Mega Millions gives the best. The result is that the expected length of a jackpot round (23) is longest under Megaplier Mega Millions, and lowest (13) under Just the Jackpot Mega Millions.

The construction of hypothetical sales and jackpot progressions also allows for calculating the probability of reaching a specific draw number in each counterfactual game via the binomial distribution. It would be misleading to simply choose a probability level and compare the net revenues between games, since this would not be giving an appropriate notion of comparable revenue, due to variation in the number of draws to reach that probability level across games. What can be done is to normalize the revenues in each draw by dividing revenue in a draw by the number of draws up to that point in the round. Once that is done, the probabilities of reaching a certain draw in each game can be used to construct CDFs of net revenue in each game. The convexity of net revenue progressions allows net revenue per draw to be increasing monotonically in draws. Figure 8 displays the CDFs of net revenue in each game, and Figures 8a and 8b are magnifications of Figure 8 for ease of visibility. There is no FOSD of any distribution, but the Megaplier CDF is below and to the right of the CDF of Just the Jackpot for probabilities larger than about 15%. Standard is also dominated by Megaplier for most of the distribution, although there is some crossover at the top few percent. Median net revenue occurs at the draw number for which the probability of reaching that draw number falls below the 50% threshold, which occurs at different draw numbers in each game. Median Megaplier net revenue is about $14.4 million, larger than median Standard net revenue of about $13.7 million and median Just the Jackpot net revenue of $13.4 million. These amounts can be seen visually by checking where the 50% probability line intersects each of the CDFs in Figure 8b. The same task is also undertaken for gross sales with similar patterns. Figure 9 displays the CDFs of gross sales for each counterfactual game, accompanied by Figures 9a and 9b as magnifications of Figure 9.

Figure 8: CDFs of Net Revenue

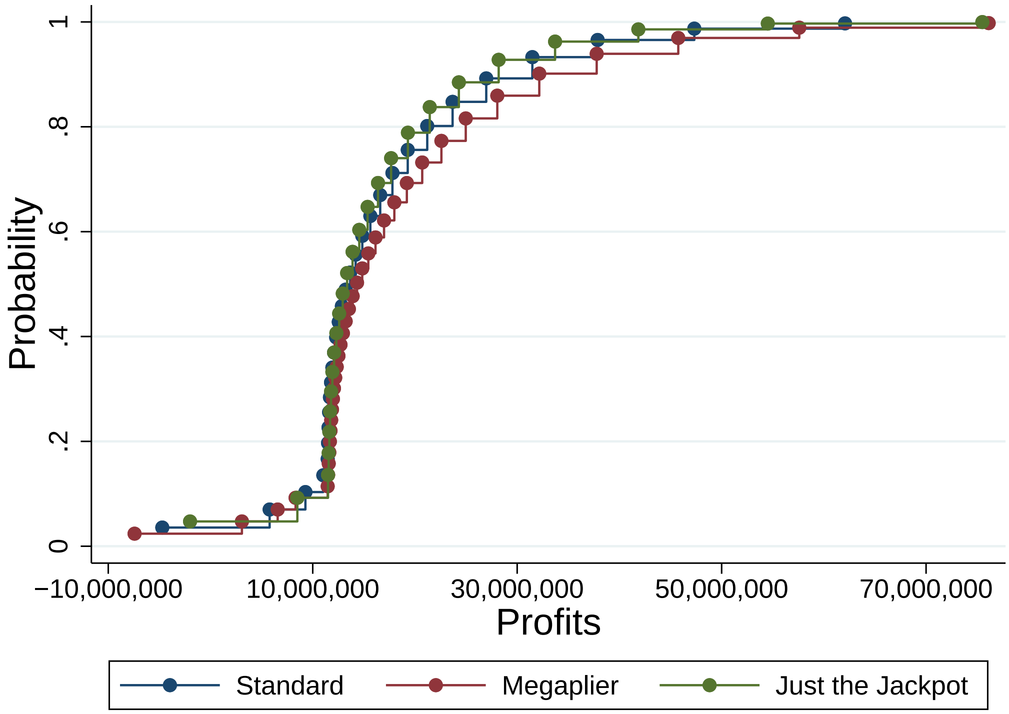


Figure 8a: Lower CDFs of Net Revenue

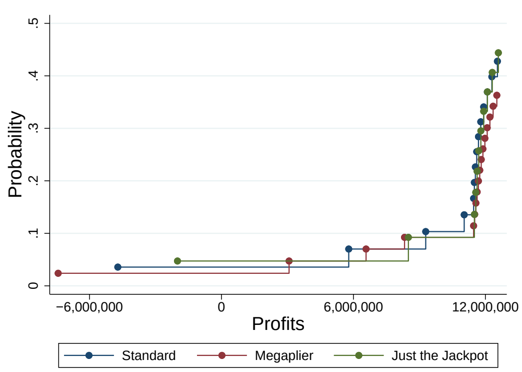


Figure 8b: Upper CDFs of Net Revenue

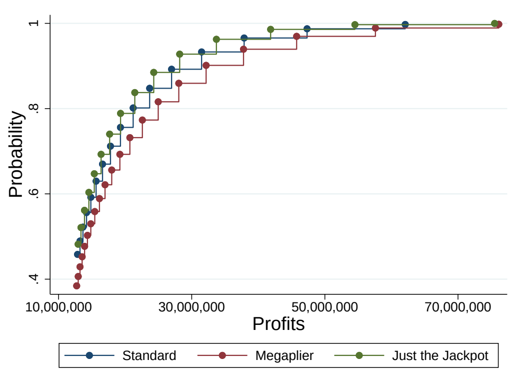


Figure 9: CDFs of Gross Sales

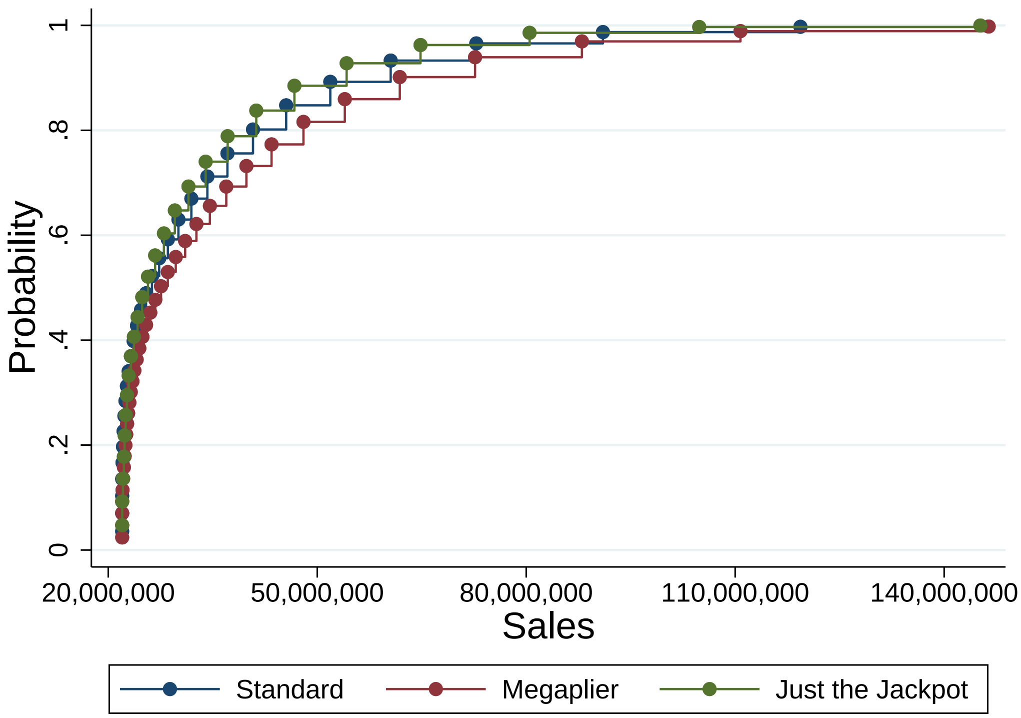


Figure 9a: Lower CDFs of Gross Sales

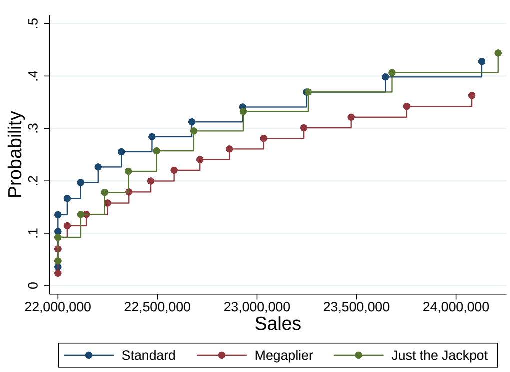
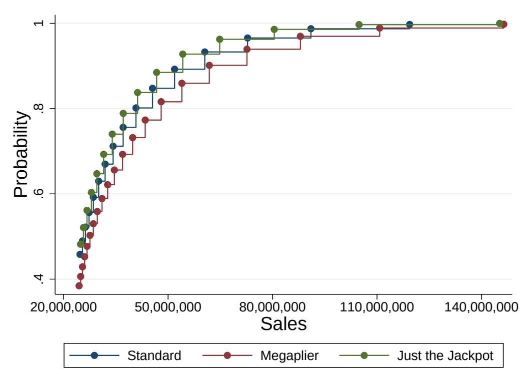


Figure 9b: Upper CDFs of Gross Sales



While the CDFs and median net revenue comparisons give some idea of product desirability from the point of view of revenue maximization for states, average net revenue is the most useful comparison, since it is something states could actually work into budget plans. The convexity of net revenue results in a right-skewed distribution, so that the median values do not give approximations to the means. Means are estimated by running through 1,000 concurrent simulations of each of the counterfactual games.

This results in about 70% more draws in the Megaplier version than the Just the Jackpot version, and about 35% more than the Standard version. The 1,000 simulations result in 12,904 draws in the Just the Jackpot game. The mean net revenue and sales for the Just the Jackpot game can be estimated by dividing the total net revenue and sales by 12,904. The simulations result in 16,271 draws of the Standard game. One way to estimate the mean would be to proceed in the same way as Just the Jackpot mean determination, by dividing aggregates by the number of rounds. An alternative way is to keep the number of draws fixed between games instead of the number of rounds, so that time is constant across games. This requires finding the simulation number in the Standard and Megaplier versions for which the number of draws crosses 12,904, the number of draws in the 1,000 Just the Jackpot simulations. Only aggregate Standard and Megaplier revenues and sales for the simulations before the 12,904th draw is crossed and normalize. Results are robust to either mean calculation method. The mean sales and net revenue estimates using the fixed draw approach are presented in Table 11. The Megaplier version not only has the largest median sales and net revenues, but also has the highest mean ones as well. The Just the Jackpot version only yields approximately 85% the average sales and net revenues of Megaplier, while the Standard version yields about 90% of the Megaplier values. These results indicate that attempting to get players to switch from Megaplier and Standard selections to Just the Jackpot is not revenue maximizing and therefore not in line with the objectives explicitly outlined in the legislation of many states that offer lottery products. Just the Jackpot introduction would only be consistent with state objectives if it brought in players who were either not playing any lotteries or switching from an even less profitable lottery product outside Mega Millions. However, the mainstream preference-based decision models do not predict gains on the extensive margin by introducing Just the Jackpot into the existing Mega Millions game. The analysis places some serious question marks on the introduction of Just the Jackpot as a Mega Millions option.

Table 11: Mean Sales and Net Revenues

	Sales	**Net Revenues**	**Sales % Megaplier**	**Net Revenues % Megaplier**
Megaplier	$46,923,590	$24,346,334	100%	100%
Standard	$42,224,822	$21,890,201	90.0%	89.9%
Just the Jackpot	$39,938,215	$20,705,705	85.1%	85.0%

Table 11 shows the mean sales and net revenues for each of the hypothetical games based on 1,000 simulations of the prospective jackpot and sales progressions, as well as each of the game's means as a percentage of Megaplier means.

Conclusion

This paper has used sales data to analyze the underlying choice behavior in the Mega Millions lottery game since the introduction of the Just the Jackpot ticket option in October 2017. Just the Jackpot had some initial interest on its opening draw of about 2% of Mega Millions sales, but levels dropped to the 0.2% to 0.3% range within a few months. A variety of the more popular and accepted decision theoretic models all predicted higher levels of interest in the Just the Jackpot option then the data demonstrated, as well as low interest in the Standard option. Alternative explanations outside of the theoretical decision modeling framework, like limited attention, liquidity constraints and slow sales adjustments, were also ruled out. Further investigation pinned down the likely culprits to differences in ex post feedback on foregone outcomes between the various choices, along with low amounts of winner regret that having a Standard entry engenders in lower prize tier-eligible states of the world. The Regret Weighted Feedback Minimax model that captures these two behavioral tendencies is proposed and demonstrated to unequivocally outperform existing models in rationalizing the data from all perspectives. The inverse relationship between Just the Jackpot sales percentage and jackpot size in the data, which no reasonable model can account for, is argued to be due to limited attention of jackpot frenzy players. These players only participate when jackpots become abnormally high and may not have much experience with or be fully aware of the options available to play. If such players were made fully aware of the Just the Jackpot option, it is presumed that the expected positive relationship between jackpot size and Just the Jackpot sales percentage would be observed in the data. Counterfactual analysis suggests that introducing Just the Jackpot into the existing Mega Millions structure will not bring in new players, but rather cause some existing players to switch to Just the Jackpot from another ticket option. Under reasonable assumptions, such switching is shown to not be net revenue maximizing and therefore not in line with state legislative mandates. These results pose serious doubts relating to the introduction of Just the Jackpot in the first place, independent of the actual disinterest in the product demonstrated by players.

Appendix

Mega Millions Prize-Odds Matrix[4]

Match	Prize*	Odds
○ ○ ○ ○ ○ + ●	Jackpot	1 in 302,575,350
○ ○ ○ ○ ○	$1,000,000	1 in 12,607,306
○ ○ ○ ○ + ●	$10,000	1 in 931,001
○ ○ ○ ○	$500	1 in 38,792
○ ○ ○ + ●	$200	1 in 14,547
○ ○ ○	$10	1 in 606
○ ○ + ●	$10	1 in 693
○ + ●	$4	1 in 89
●	$2	1 in 37
Overall chances of winning any prize:		1 in 24

[4] Taken from https://www.megamillions.com/How-to-Play.aspx on 09/17/20

Feedback-Conditional Regret Theory

Feedback-Conditional Regret Theory (FCRT) is an extension of Regret Theory proposed by Humphrey (2004) which allows for variation in feedback on outcomes of foregone choices to be included in decision modeling. The model maintains Regret Theory's native pairwise environment. The decision criteria is a generalization of (9)

$$A_i \succeq A_k \Longleftrightarrow \sum_{j=1}^{n} p_j[M(x_{ij}, x_{kj}) - M(x_{kj}, x_{ij})] \geq 0$$

For the purposes of the forthcoming analysis, (13) will be directly related to (9) by setting

$$M(x_{ij}, x_{kj}) = c_{ij} + R(c_{ij} - c_{kj})$$

where modified utility function $M(x_{ij}, x_{kj})$ can lose its symmetric character if the ex post regret status in state j differs between $A_i$and A_k. In the case that in state j, receiving x_{ij} fully reveals the state of the world and foregone outcome x_{kj}, $M(x_{ij}, x_{kj}) = m(x_{ij}, x_{kj})$. In the case that in state j, receiving x_{ij} does not fully reveal the state of the world and foregone outcome x_{kj}, $M(x_{ij}, x_{kj}) = \mu(x_{ij}, x_{kj})$. However, "the decision-maker has anticipated a state of the world under which they will receive x_{ij} and forego x_{kj}, but actually receiving x_{ij} does not reveal x_{kj} (as opposed to some other outcome, say, x_{kj*}) as the outcome of the foregone act" (Humphrey, 2004). Three conditions are imposed on $M(.,.)$ when $x_{ij} > x_{kj}$:

1. $m(x_{ij}, x_{kj}) > \mu(x_{ij}, x_{kj})$
2. $m(x_{kj}, x_{ij}) < \mu(x_{kj}, x_{ij})$
3. $\mu(x_{kj}, x_{ij}) - m(x_{kj}, x_{ij}) > m(x_{ij}, x_{kj}) - \mu(x_{ij}, x_{kj})$

The first two conditions highlight that an action fully revealing the state of the world and foregone outcome in that state has amplified utility when the outcome is larger than the foregone outcome relative to non-revelation, and diminished utility when the outcome is smaller than the foregone outcome relative to non-revelation. The third condition states that the magnitude of the difference in utilities between revelation and non-revelation is larger in the case of regret than rejoice. Humphrey (2004) does not provide a precise structural form or suggested parameterization to his FCRT. This paper will assume original RT, with a choiceless utility portion that is the same regardless of feedback, and an $R(.)$ function, where $R(.)$ is scaled up by a $\delta > 1$ if A_i will reveal the outcome of A_k in state j. With no feedback of the outcome of the foregone action ex post in state j, $R(c_{ij} - c_{kj})$ is used, and with feedback, $\delta * R(c_{ij} - c_{kj})$ is used. This functional choice of the relationship between $m(.,.)$ and $\mu(.,.)$ is chosen because of its simplicity and ease of interpretation, but is clearly not the only formulation consistent with the Humphrey (2004) stipulations. Notice that in order to fulfill the third condition on $M(.,.)$ using (7) to model $R(c_{ij} - c_{kj})$, $\beta > \alpha$ is necessary. To start, $\beta = \alpha$ will be forced so that the third condition is only weakly satisfied.

Notice that ex post feedback is not necessarily symmetric: A_i may reveal the outcome of A_k in state j, but A_k may or may not reveal the outcome of A_i in state j. For example, the actions of opting into a fair coin flip and not opting in display the asymmetry: flipping the coin reveals the foregone outcome of not playing in either resultant states of the world, whereas opting out does not reveal the foregone outcome, since the coin is never flipped. However, the choice could be modified so that the coin will be flipped and state of the world revealed, independent of choice of action. In that case, both actions would be fully revealing of the state of the world and outcomes of the foregone action. The implication is that in this modified game, rejoice would be larger in magnitude than in the original game for a favorable flip, and regret would also be larger in magnitude in the event of an unfavorable flip. It is also possible that an action may only reveal the outcomes of foregone actions in some but not all states of the world. Then, only in those states would $m(.,.)$ be used for utility evaluation, $\mu(.,.)$ would be used in all other states.

Mega Millions falls into this partial revelation framework. Not playing Mega Millions leaves one with the cost of the ticket with certainty, and there is no knowledge ex post of what the outcome of playing any ticket type would have been. Pairwise comparisons between not playing and each of the three ticket types results in utilizing $\mu(.,.)$ in every state when evaluating not playing, and utilizing $m(.,.)$ in every state when evaluating any ticket type. Pairwise comparisons between ticket types is different. At any given cost, Just the Jackpot gives the greatest number of distinct entries. For example, at a cost of \$6, Just the Jackpot gives four number combination entries, Standard gives three, and Megaplier gives two. So, consider evaluating the choice between Standard and Just the Jackpot. A choice of Just the Jackpot would be fully revealing of the counterfactual Standard in every resultant state: if one or more of those tickets are partial number matches, those would have been eligible for lower tier prizes had they been Standard. If none of them are partial matches, well then if they had been Standard tickets, they would not have been eligible. If one of them won the jackpot, it also would have won the jackpot had it been Standard. Now consider the perspective of choosing Standard. If any of those three tickets are lower tier eligible, that would indicate that they would be ineligible had they been Just the Jackpot. If any of them were jackpot winners, they also would be under Just the Jackpot designation. Now the asymmetry sets in. If none of the tickets are prize eligible, under Just the Jackpot designation a fourth ticket would have been generated, with two possibilities: that ticket wins the jackpot, or is a partial match or complete mismatch and wins nothing. Therefore, Standard is fully revealing of foregone outcomes of Just the Jackpot in some states, whereas Just the Jackpot is fully revealing of foregone Standard outcomes in every state. Revelation moves in increasing order respectively with not playing, Megaplier, Standard and Just the Jackpot. It may be that Just the Jackpot as the most revealing option is diminishing its appeal.

The framework of FCRT is the same as that of RT, so the same concerns and considerations highlighted in RT carry over to modeling FCRT. Two aggregation methods were used in RT modeling, pairwise and state-wise mean utilities. For FCRT, only pairwise mean utilities will be modeled. This is because with the factoring in of feedback into RT, calculating $c_{ij} - mean(c_{-ij})$ as the input to $R(.)$ becomes difficult to interpret. Relative to A_i in state j, some foregone actions may have their outcomes revealed whereas others may not. It becomes unclear as to whether or not $m(.,.)$ or $\mu(.,.)$ should be used in evaluation. Therefore this aggregation method is not modeled under FCRT, only pairwise is, in which averages are taken of pairwise utilities and the feedback parameter can be applied unequivocally. The two overlap scenarios from RT also carry over, and in this case there is no confounding that feedback provides in interpretation. Under FCRT, full overlap does impact the feedback channel relative to no overlap. With full overlap of lower tier prizes, the state-wise differentiation in feedback between Megaplier and Standard is effectively removed. Since Megaplier and Standard have the same sets of winning numbers under full overlap, Megaplier becomes fully revealing of foregone Standard outcomes in all states, whereas it does not in states in which a non-overlapping Standard ticket would be prize eligible.

The total number of tested parameter-jackpot combinations is 32,000 per overlap scenario: the choiceless utility power function parameter runs from .71 to 1.10 in increments of hundredths; $R(.)$ also takes the form of a power function, and its parameter runs from 0.6 to 3.0 in increments of tenths; feedback parameter δ runs from 2 to 5 in increments of one. Both boundary overlap scenarios are tested under pairwise mean aggregation for the same 8 jackpot levels tested in all other models. Only 8 parameter-jackpot combinations are consistent with a Standard preference under the no overlap scenario of a possible 32,000. Table A1 presents the test results of FCRT using the full overlap scenario, which finds 305 parameter-jackpot combinations indicative of a Standard preference. Still, no parameter combination predicts a Standard preference for the five highest of the eight jackpots under either overlap specification. FCRT does only marginally better than RT at capturing the choice behavior in the data, and cannot be said to be remotely consistent with it.

Table A1: Feedback-Conditional Regret Theory Predictions

	Percent of Parameter-Jackpot Combinations				
Model	Standard	Megaplier	Just the Jackpot	None	Max Jackpots Standard
FCRT	1.0%	0.9%	74.7%	23.4%	3
Regret-only FCRT	2.6%	0.7%	71.4%	25.3%	4

Table A1 shows the percentages of tested parameter-jackpot combinations that predict preferences for each of the four options, along with the maximum number of jackpots that any one parameter set predicts a Standard preference for. FCRT is Feedback-Conditional Regret Theory under the full overlap specification; Regret-only FCRT is Feedback-Conditional Regret Theory on regretful outcomes only under the full overlap specification.

As was noted above, the previous analysis set $\beta = \alpha$ and therefore only weakly satisfied condition three on $M(.,.)$, whereas Humphrey (2004) lists the condition in strict form. It is important to mention that this condition is essentially a behavioral assumption that regret is a more salient feeling than rejoice, although one might incline to agreement. The other extreme, which does satisfy condition three, is to assume that ex post feedback does not increase the feeling of rejoice, but only that of regret. This is consistent with the example given in Zeelenberg (1999), and amounts to

$$\begin{cases} m(x_{ij}, x_{kj}) = \mu(x_{ij}, x_{kj}) & x_{ij} > x_{kj} \\ m(x_{kj}, x_{ij}) = \delta * \mu(x_{kj}, x_{ij}) & x_{ij} > x_{kj}, \delta > 1 \end{cases}$$

This other extreme is also tested, with somewhat better results than the forced equivalency of rejoice and regret above. The same 64,000 parameter-jackpot-scenario combinations are run through, with no overlap yielding just 4 parameter-jackpot combinations at only the two lowest jackpots with a predicted Standard preference. The full overlap condition is much better, with 830 parameter-jackpot combinations consistent with a Standard preference, or 2.6% of the full overlap combinations. Results of this specification are presented in Table A1. These span seven of the eight jackpots, with high concentration at the lowest jackpots. Furthermore, no one set of parameters can explain a Standard preference for more than four jackpot levels. While the full overlap scenario fares somewhat decently relative to most of the models and scenarios tested, full overlap is again a boundary condition, with partial or no overlap much more realistic assumptions. Bolstering this conclusion is the fact that about 80% of Mega Millions tickets in Texas over the past ten years were randomly generated and not self-selected, and therefore no or partial overlap is unequivocally the better assumption for those 80%, let alone the likelihood of players purchasing multiple entries and self-selecting the same number combinations. One positive takeaway from full overlap FCRT is that this is the first model in which Standard does not hold the fewest parameter-jackpot predictions, as it just outperforms the Megaplier. However, the conclusion is that FCRT is also unable to account for the observed Mega Millions choice behavior.

Table A2: Mega Millions Sales for States Offering Just the Jackpot

Date	Jackpot	Total	Standard	Megaplier	Just the Jackpot
10/31/2017	$40,000,000	$7,282,759	91.15%	6.76%	2.08%
11/3/2017	$48,000,000	$8,037,966	91.89%	6.94%	1.17%
11/7/2017	$59,000,000	$7,435,573	91.83%	7.17%	0.99%
11/10/2017	$71,000,000	$7,980,197	92.01%	7.14%	0.85%
11/14/2017	$82,000,000	$7,741,920	92.09%	7.09%	0.82%
11/17/2017	$95,000,000	$8,517,323	92.31%	6.97%	0.71%
11/21/2017	$106,000,000	$8,847,507	91.95%	7.40%	0.65%
11/24/2017	$119,000,000	$8,353,300	91.78%	7.62%	0.60%
11/28/2017	$132,000,000	$9,239,587	92.01%	7.41%	0.58%
12/1/2017	$145,000,000	$10,199,466	92.07%	7.41%	0.52%
12/5/2017	$160,000,000	$9,870,955	92.52%	6.94%	0.54%
12/8/2017	$176,000,000	$10,157,419	92.75%	6.76%	0.49%
12/12/2017	$191,000,000	$10,455,870	92.77%	6.74%	0.49%
12/15/2017	$208,000,000	$11,932,542	92.90%	6.65%	0.45%
12/19/2017	$223,000,000	$12,930,848	93.20%	6.37%	0.43%
12/22/2017	$253,000,000	$16,356,936	93.57%	6.05%	0.38%
12/26/2017	$277,000,000	$14,952,197	93.50%	6.11%	0.39%
12/29/2017	$306,000,000	$27,881,874	93.93%	5.73%	0.33%
1/2/2018	$361,000,000	$32,219,824	94.15%	5.53%	0.32%
1/5/2018	$450,000,000	$50,135,632	94.14%	5.56%	0.30%
1/9/2018	$40,000,000	$8,934,178	92.37%	7.19%	0.44%
1/12/2018	$45,000,000	$8,435,116	92.14%	7.47%	0.40%
1/16/2018	$50,000,000	$7,680,835	92.21%	7.38%	0.40%
1/19/2018	$55,000,000	$8,413,244	92.17%	7.45%	0.38%
1/23/2018	$63,000,000	$8,010,639	92.16%	7.44%	0.40%
1/26/2018	$76,000,000	$8,799,877	92.23%	7.41%	0.35%
1/30/2018	$89,000,000	$8,265,817	92.15%	7.47%	0.39%
2/2/2018	$104,000,000	$9,644,471	92.25%	7.40%	0.35%
2/6/2018	$120,000,000	$9,245,950	92.29%	7.34%	0.37%
2/9/2018	$136,000,000	$9,875,370	92.41%	7.27%	0.33%
2/13/2018	$153,000,000	$9,801,786	92.43%	7.22%	0.35%
2/16/2018	$168,000,000	$10,701,619	92.63%	7.05%	0.32%

Date	Jackpot	Total	Standard	Megaplier	Just the Jackpot
2/20/2018	$185,000,000	$9,999,342	92.53%	7.12%	0.34%
2/23/2018	$204,000,000	$11,552,057	92.70%	6.99%	0.31%
2/27/2018	$222,000,000	$12,165,814	92.85%	6.83%	0.32%
3/2/2018	$243,000,000	$12,912,058	92.70%	6.99%	0.31%
3/6/2018	$265,000,000	$13,858,603	93.00%	6.69%	0.31%
3/9/2018	$290,000,000	$14,282,330	92.91%	6.80%	0.30%
3/13/2018	$318,000,000	$16,366,748	93.24%	6.47%	0.29%
3/16/2018	$345,000,000	$18,870,642	93.38%	6.35%	0.26%
3/20/2018	$377,000,000	$19,415,088	93.31%	6.38%	0.31%
3/23/2018	$421,000,000	$21,290,674	93.29%	6.40%	0.31%
3/27/2018	$458,000,000	$24,879,498	93.52%	6.17%	0.31%
3/30/2018	$521,000,000	$51,350,316	94.18%	5.58%	0.25%
4/3/2018	$40,000,000	$8,664,295	91.87%	7.80%	0.33%
4/6/2018	$45,000,000	$8,741,042	91.90%	7.80%	0.30%
4/10/2018	$50,000,000	$8,136,523	91.82%	7.86%	0.32%
4/13/2018	$55,000,000	$9,039,829	92.02%	7.70%	0.28%
4/17/2018	$67,000,000	$8,228,002	91.90%	7.78%	0.32%
4/20/2018	$80,000,000	$9,019,060	92.05%	7.66%	0.29%
4/24/2018	$96,000,000	$8,643,616	92.08%	7.62%	0.31%
4/27/2018	$111,000,000	$9,946,819	92.35%	7.38%	0.27%
5/1/2018	$126,000,000	$9,798,001	92.23%	7.48%	0.29%
5/4/2018	$143,000,000	$10,506,463	92.35%	7.37%	0.28%
5/8/2018	$40,000,000	$7,498,599	91.79%	7.91%	0.30%
5/11/2018	$45,000,000	$7,960,610	91.94%	7.78%	0.28%
5/15/2018	$50,000,000	$7,356,060	91.74%	7.96%	0.30%
5/18/2018	$55,000,000	$8,085,046	91.96%	7.77%	0.28%
5/22/2018	$60,000,000	$7,416,149	91.86%	7.85%	0.29%
5/25/2018	$73,000,000	$8,378,826	92.07%	7.66%	0.27%
5/29/2018	$84,000,000	$7,456,218	91.89%	7.82%	0.30%
6/1/2018	$97,000,000	$9,107,809	92.12%	7.61%	0.27%
6/5/2018	$110,000,000	$9,291,018	92.22%	7.49%	0.28%
6/8/2018	$127,000,000	$9,955,497	92.44%	7.30%	0.26%
6/12/2018	$144,000,000	$9,784,722	92.45%	7.27%	0.28%
6/15/2018	$161,000,000	$10,495,370	92.56%	7.18%	0.26%
6/19/2018	$175,000,000	$10,214,179	92.54%	7.18%	0.28%

Date	Jackpot	Total	Standard	Megaplier	Just the Jackpot
6/22/2018	$192,000,000	$11,325,558	92.60%	7.13%	0.27%
6/26/2018	$212,000,000	$12,108,530	92.66%	7.05%	0.29%
6/29/2018	$232,000,000	$13,578,444	92.71%	7.01%	0.28%
7/3/2018	$256,000,000	$14,206,276	92.63%	7.07%	0.30%
7/6/2018	$283,000,000	$14,250,322	92.69%	7.02%	0.29%
7/10/2018	$306,000,000	$16,881,618	93.01%	6.70%	0.30%
7/13/2018	$340,000,000	$20,728,668	93.17%	6.56%	0.27%
7/17/2018	$375,000,000	$21,488,624	93.30%	6.42%	0.28%
7/20/2018	$433,000,000	$34,642,408	93.88%	5.88%	0.24%
7/24/2018	$522,000,000	$60,161,128	94.29%	5.48%	0.23%
7/27/2018	$40,000,000	$8,131,869	91.83%	7.88%	0.29%
7/31/2018	$45,000,000	$7,665,313	91.68%	8.02%	0.30%
8/3/2018	$50,000,000	$8,271,744	91.79%	7.94%	0.27%
8/7/2018	$55,000,000	$7,805,630	91.73%	7.98%	0.29%
8/10/2018	$63,000,000	$8,363,887	91.87%	7.87%	0.26%
8/14/2018	$75,000,000	$8,013,807	91.83%	7.88%	0.29%
8/17/2018	$88,000,000	$8,825,262	92.10%	7.64%	0.26%
8/21/2018	$102,000,000	$9,315,947	92.21%	7.51%	0.27%
8/24/2018	$118,000,000	$10,036,574	92.31%	7.43%	0.26%
8/28/2018	$134,000,000	$9,822,489	92.30%	7.43%	0.27%
8/31/2018	$152,000,000	$10,976,064	92.41%	7.33%	0.26%
9/4/2018	$167,000,000	$10,000,861	92.30%	7.43%	0.28%
9/7/2018	$187,000,000	$11,436,642	92.56%	7.19%	0.25%
9/11/2018	$207,000,000	$11,832,820	92.68%	7.05%	0.27%
9/14/2018	$227,000,000	$12,810,338	92.86%	6.89%	0.25%
9/18/2018	$252,000,000	$12,799,656	92.81%	6.91%	0.28%
9/21/2018	$275,000,000	$13,730,861	92.94%	6.79%	0.26%
9/25/2018	$303,000,000	$15,337,032	92.97%	6.75%	0.28%
9/28/2018	$336,000,000	$17,082,332	93.17%	6.57%	0.26%
10/2/2018	$367,000,000	$18,566,592	93.29%	6.44%	0.27%
10/5/2018	$420,000,000	$24,050,072	93.65%	6.10%	0.25%
10/9/2018	$470,000,000	$28,247,306	93.90%	5.84%	0.26%
10/12/2018	$548,000,000	$43,123,228	94.05%	5.71%	0.24%
10/16/2018	$667,000,000	$78,702,176	94.41%	5.36%	0.23%
10/19/2018	$1,000,000,000	$207,492,976	94.45%	5.32%	0.23%

Date	Jackpot	Total	Standard	Megaplier	Just the Jackpot
10/23/2018	$1,600,000,000	$270,925,600	94.36%	5.38%	0.26%
10/26/2018	$40,000,000	$13,036,618	91.83%	7.87%	0.30%
10/30/2018	$45,000,000	$10,551,508	91.71%	8.02%	0.27%
11/2/2018	$52,000,000	$10,219,383	91.52%	8.08%	0.40%
11/6/2018	$70,000,000	$10,557,753	92.38%	7.38%	0.24%
11/9/2018	$90,000,000	$9,887,921	91.85%	7.92%	0.23%
11/13/2018	$106,000,000	$9,892,155	91.94%	7.81%	0.24%
11/16/2018	$122,000,000	$10,390,820	91.89%	7.89%	0.22%
11/20/2018	$139,000,000	$10,697,782	92.06%	7.71%	0.23%
11/23/2018	$155,000,000	$9,695,451	91.81%	7.97%	0.23%
11/27/2018	$172,000,000	$10,518,311	92.08%	7.70%	0.22%
11/30/2018	$190,000,000	$11,379,043	92.15%	7.64%	0.21%
12/4/2018	$208,000,000	$11,760,428	92.25%	7.52%	0.23%
12/7/2018	$226,000,000	$11,971,119	92.35%	7.43%	0.21%
12/11/2018	$245,000,000	$11,941,937	92.37%	7.41%	0.22%
12/14/2018	$262,000,000	$12,661,328	92.46%	7.32%	0.22%
12/18/2018	$284,000,000	$12,616,857	92.45%	7.32%	0.23%
12/21/2018	$305,000,000	$14,181,272	92.64%	7.14%	0.22%
12/25/2018	$321,000,000	$20,179,728	93.32%	6.48%	0.19%
12/28/2018	$370,000,000	$22,724,544	93.07%	6.74%	0.20%
1/1/2019	$425,000,000	$35,750,088	93.40%	6.40%	0.21%
1/4/2019	$40,000,000	$9,123,219	91.73%	8.05%	0.22%
1/8/2019	$45,000,000	$8,562,782	91.66%	8.11%	0.23%
1/11/2019	$50,000,000	$8,886,587	91.62%	8.17%	0.21%
1/15/2019	$55,000,000	$8,388,774	91.62%	8.16%	0.22%
1/18/2019	$68,000,000	$9,132,415	91.80%	8.00%	0.20%
1/22/2019	$82,000,000	$8,193,440	91.56%	8.22%	0.22%
1/25/2019	$96,000,000	$9,172,900	91.78%	8.02%	0.20%
1/29/2019	$109,000,000	$9,492,464	92.00%	7.79%	0.21%
2/1/2019	$125,000,000	$10,061,010	91.86%	7.93%	0.21%
2/5/2019	$139,000,000	$10,369,302	92.03%	7.76%	0.21%
2/8/2019	$157,000,000	$10,528,767	92.07%	7.73%	0.20%
2/12/2019	$173,000,000	$10,018,091	91.92%	7.86%	0.22%
2/15/2019	$190,000,000	$11,229,604	92.22%	7.58%	0.20%
2/19/2019	$206,000,000	$11,180,536	92.22%	7.56%	0.22%

Date	Jackpot	Total	Standard	Megaplier	Just the Jackpot
2/22/2019	$224,000,000	$12,400,790	92.39%	7.42%	0.20%
2/26/2019	$245,000,000	$12,685,180	92.45%	7.34%	0.21%
3/1/2019	$267,000,000	$14,024,939	92.39%	7.41%	0.21%
3/5/2019	$40,000,000	$8,244,622	91.61%	8.17%	0.22%
3/8/2019	$45,000,000	$8,944,821	91.81%	7.99%	0.20%
3/12/2019	$50,000,000	$8,670,861	91.74%	8.04%	0.21%
3/15/2019	$40,000,000	$9,088,265	91.94%	7.86%	0.20%
3/19/2019	$45,000,000	$9,401,534	92.02%	7.76%	0.22%
3/22/2019	$50,000,000	$10,410,177	92.12%	7.67%	0.21%
3/26/2019	$57,000,000	$10,683,040	92.09%	7.68%	0.23%
3/29/2019	$75,000,000	$10,570,955	92.05%	7.73%	0.22%
4/2/2019	$88,000,000	$9,339,371	91.75%	8.04%	0.21%
4/5/2019	$104,000,000	$10,642,903	91.96%	7.84%	0.20%
4/9/2019	$120,000,000	$10,459,799	92.06%	7.73%	0.21%
4/12/2019	$140,000,000	$11,029,933	92.15%	7.66%	0.19%
4/16/2019	$157,000,000	$10,935,413	92.13%	7.66%	0.21%
4/19/2019	$175,000,000	$11,457,466	92.20%	7.60%	0.19%
4/23/2019	$192,000,000	$11,350,473	92.19%	7.60%	0.21%
4/26/2019	$212,000,000	$12,572,596	92.34%	7.47%	0.20%
4/30/2019	$229,000,000	$12,952,414	92.36%	7.44%	0.21%
5/3/2019	$252,000,000	$14,050,305	92.37%	7.34%	0.29%
5/7/2019	$273,000,000	$13,644,412	92.48%	7.31%	0.21%
5/10/2019	$295,000,000	$14,407,691	92.57%	7.23%	0.20%
5/14/2019	$316,000,000	$15,046,406	92.67%	7.12%	0.20%
5/17/2019	$339,000,000	$15,925,653	92.74%	7.07%	0.19%
5/21/2019	$367,000,000	$16,168,593	92.72%	7.08%	0.21%
5/24/2019	$393,000,000	$17,346,194	92.73%	7.08%	0.19%
5/28/2019	$418,000,000	$18,204,460	92.88%	6.92%	0.20%
5/31/2019	$444,000,000	$23,855,912	93.08%	6.73%	0.19%
6/4/2019	$475,000,000	$25,635,468	93.12%	6.68%	0.20%
6/7/2019	$530,000,000	$32,804,596	93.44%	6.36%	0.20%
6/11/2019	$40,000,000	$8,313,913	91.47%	8.32%	0.21%
6/14/2019	$45,000,000	$8,669,905	91.47%	8.32%	0.20%
6/18/2019	$50,000,000	$8,134,308	91.38%	8.41%	0.21%
6/21/2019	$55,000,000	$8,676,327	91.52%	8.30%	0.18%

Date	Jackpot	Total	Standard	Megaplier	Just the Jackpot
6/25/2019	$60,000,000	$8,174,943	91.44%	8.36%	0.21%
6/28/2019	$71,000,000	$8,766,333	91.47%	8.34%	0.19%
7/2/2019	$83,000,000	$8,580,800	91.43%	8.37%	0.21%
7/5/2019	$95,000,000	$8,544,161	91.41%	8.39%	0.19%
7/9/2019	$107,000,000	$8,979,822	91.67%	8.13%	0.20%
7/12/2019	$121,000,000	$9,703,712	91.79%	8.02%	0.19%
7/16/2019	$137,000,000	$9,908,994	91.82%	7.97%	0.20%
7/19/2019	$154,000,000	$10,679,969	91.95%	7.86%	0.20%
7/23/2019	$168,000,000	$10,430,294	91.97%	7.83%	0.21%
7/26/2019	$40,000,000	$7,675,488	91.48%	8.32%	0.20%
7/30/2019	$45,000,000	$7,227,490	91.35%	8.44%	0.21%
8/2/2019	$50,000,000	$7,928,006	91.40%	8.41%	0.19%
8/6/2019	$55,000,000	$7,479,508	91.42%	8.37%	0.21%
8/9/2019	$60,000,000	$7,850,505	91.51%	8.31%	0.18%
8/13/2019	$65,000,000	$7,472,842	91.43%	8.36%	0.21%
8/16/2019	$70,000,000	$7,926,022	91.54%	8.27%	0.18%
8/20/2019	$79,000,000	$7,714,175	91.56%	8.23%	0.20%
8/23/2019	$90,000,000	$8,595,792	91.75%	8.06%	0.19%
8/27/2019	$103,000,000	$8,902,318	91.91%	7.89%	0.20%
8/30/2019	$113,000,000	$9,777,008	91.89%	7.91%	0.19%
9/3/2019	$127,000,000	$9,092,326	91.72%	8.06%	0.22%
9/6/2019	$139,000,000	$10,153,715	91.87%	7.94%	0.19%
9/10/2019	$154,000,000	$10,299,151	91.90%	7.89%	0.21%
9/13/2019	$172,000,000	$11,530,413	92.07%	7.73%	0.20%
9/17/2019	$192,000,000	$11,325,551	92.09%	7.70%	0.21%
9/20/2019	$211,000,000	$12,694,116	92.33%	7.47%	0.20%
9/24/2019	$227,000,000	$12,992,502	92.38%	7.40%	0.22%

Chapter 2: Distribution-Dependent Utility of Gaming

Abstract

There is existing evidence that decision making over risk is impacted by factors like whether or not the decision maker can self-select numbers, a type of gaming utility, a feature common in lottery games. Such preferences would violate the fundamental properties of reflexivity and FOSD, but is nevertheless within the bounds of what the current literature accounts for. This paper provides experimental evidence that the estimated financial value of such factors is non-negligible, as subjects on average are willing to forego 10% to 30% of potential winnings. A novel result is the significant variation in self-selection preferences by payoff distribution. Sales data from lottery games played in Texas are adduced to further confirm the experimental findings. In order to ascertain the dependency of the gaming utility on the payoff distribution, an additional experiment is run to control for differences in non-distributional gaming factors between the Texas lottery games. The variation persists, leading to the conclusion that a preference for self-selection of numbers is distribution-dependent for many individuals. Reasons for these apparent inconsistencies with the existing literature and decision theoretic model predictions are discussed, and a possible regret-salience motive is proposed.

Keywords: Experiment, lottery, preferences, gaming
JEL Codes: C91, D81, D91

Declarations

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Code Availability: Fully available upon request of the author.
Author's Contributions: Provides some of the first evidence of distribution-dependent preferences over processes for risk resolution.

Introduction

Researchers have spent the past few decades modifying and refining both the underlying psychological motivations and representative theoretic modeling of decision behavior over risk. Many models maintain that if the payoff distributions of two lotteries are equivalent, a decision maker must display indifference. Other models in which lotteries are evaluated only in comparison to one another, like Regret Theory, may have the additional requirement of state space equivalence to necessitate indifference, a violation of the so-called equivalence axiom (Table 6 in Loomes and Sugden 1982). However, when comparing two lotteries with equivalent payoff distributions and only two distinct payoffs, distributional equivalence is sufficient to ensure indifference (see the appendix for a proof under Regret Theory). A question then abounds of whether these model predictions reflect real world choice behavior or even choice behavior in an experimental setting.

A scenario satisfying the distributional equivalence and two outcome sufficiency condition is as follows: Lottery A pays the subject \$10 if a computer randomly selects the same integer from 0-9 twice, each integer having an equal probability of being selected, \$0 otherwise. Lottery B requires the subject to choose an integer from 0-9, and if that matches the integer from 0-9 that a computer randomly selects, the subject receives \$10, \$0 otherwise. Both lotteries give \$10 with a 10% probability and \$0 with a 90% probability. These two lotteries differ only in a procedural sense: the "how" of the lottery resolution differs. Any decision maker that strictly prefers A or B would be violating reflexivity under the standard models. The procedure for risk resolution under Lottery A is one that does not involve the decision maker in any way once the decision to play Lottery A is made. Lottery B on the other hand requires further input from the decision maker even after the decision to play Lottery B is made, although this input is inconsequential probabilistically. This difference in participation of the decision maker in the resolution of the lotteries could drive a strict preference for either lottery if there is a process preference.

While standard decision theoretic models have little to say about the processes of risk resolution, there are numerous studies that look into preferences over such processes. Of particular relevance to this paper are those studies that assess the rationale behind the selection of numbers in lottery games (see Simon 1998 for a comprehensive discussion of number selection in lotteries). Some lottery players select lucky or personally significant numbers, or numbers that have contemporary or cultural significance, and often play such numbers repeatedly over periods of time (Clotfelter and Cook 1989; Clotfelter and Cook 1991). Some are superstitious, consulting dream books, lottery "experts" and astrologers to aid in number selection (Clotfelter and Cook 1991). This motive is an example of the well-documented illusion of control phenomenon, an experimental example of which is subjects who had a choice of a specific lottery ticket reporting significantly higher willingness to sell amounts than subjects who were assigned a lottery ticket (Langer 1975). Superstition not only impacts which numbers are chosen, but where lottery tickets are purchased, with increased sales at lottery retail locations which recently sold a winning ticket (Guryan and Kearney 2008). Representative biases (Kahneman and Tversky 1972) also impact number selection. Both the gambler's fallacy (negative autocorrelation) and hot hand (positive autocorrelation) have been observed in lottery games (Riedwyl 1990; Clotfelter and Cook 1993; Henze 1997) and roulette play (Croson and Sundali 2005). Another selection mechanism relates not so much to the numbers themselves as much as how the numbers are set on the lottery ticket grid. Many players select numbers to make certain patterns on the grids, such as: horizontal, vertical or diagonal lines; symmetric images across some reflection line; evenly or very nearly evenly-spaced number selections that result in seemingly patterned grids (Riedwyl 1990; Henze 1997). These evidences suggest that lottery players that harbor any of these considerations would likely not indicate indifference between lotteries A and B described above.

Preferences over risk resolution processes like self-selection of winning numbers in games of chance is part of the literature on process or procedural utility. Amartya Sen (1995) calls for economics to take more seriously procedural concerns, as consequence-only approaches would imply implausible conclusions like "whether a particular utility redistribution is caused by charity, or taxation, or torture" to be immaterial. However, procedural utility is often not readily quantifiable, which contributes to its typical lack of inclusion in modeling. Accounting for procedural utility can also lead to violating certain foundational desiderata of choice theory. For example, assume someone prefers mangoes to apples. When at a gathering, this individual will take a mango from a basket of mangoes and apples as long that mango is not the last mango in the basket, as that behavior may be considered rude. So, c(mango, mango, apple, apple) = mango and c(mango, apple, apple) = apple (Sen 1997). Such choice behavior is a violation of Sen's α and consequentially WARP. It is also conceivable that if the individual had been offered the last mango, instead of having to choose it, the exhibited behavior would have been consistent with theory. Such a complication highlights the difficulty of incorporating procedural utility into modeling. Sen (1997) gives an example of procedural preference over risky choices, via the story of a doctor with a single antidote for a deadly disease that has infected two children. The doctor knows that one of the children has a slightly higher probability of survival if given the antidote than the other. However, the doctor prefers a randomization device to determine which of the two children gets the antidote over administering the antidote to the child with a slightly higher chance of survival. This behavior is a violation of the independence axiom and therefore inconsistent with Expected Utility theory.

In this paper, simple choice experiments like the one described above between lotteries A and B are proposed to subjects. The results confirm that most subjects are not indifferent between two outcome distributionally-equivalent lotteries, and furthermore that subjects are willing to sacrifice 10% to 30% of their potential winnings to enact their preferred process. A truly novel result of the experiment is that the proportion of subjects with certain process preferences changes as the payoff distribution of the equivalent lotteries changes. Sales data from Texas lottery games are adduced to validate the experimental finding of a correlation between payoff distributions and a preference over risk resolution processes, which in this case is whether or not to self-select winning numbers. However, the sales data does not allow for a determination of whether the correlation is due to game characteristics that vary between lottery games, or if the payoff distributions of the games themselves are impacting the process preferences. An additional experiment is conducted which seeks to control for the game characteristics in the lottery data, only allowing payoff distributions to differ while more closely resembling real world lottery distributions than the first experiment. The experimental results suggest that while such controls may mitigate the relationship in the lottery data, a large and significant effect persists, implying that for many individuals the payoff distributions themselves impact the preference for self-selection. The author is not aware of any previous study with a similar finding of distribution-dependent process preferences.

The paper proceeds as follows: Section 2 recounts the relevant literature to the issues of decision making over risk and process utility. Section 3 describes the experimental approach and procedures. Section 4 looks at the experimental and empirical results. Section 5 includes a discussion of the potential driving factors behind the experimental and empirical results. Section 6 concludes.

Literature Review

There is a thorough literature documenting the purported non-pecuniary benefits of games of chance, like lotteries and gambling. Hirshleifer (1966) proposes a classification of such activities to account for the common simultaneous lottery and insurance participation that Expected Utility cannot readily account for. Rather than change the utility of wealth function, he takes the lottery and gambling behavior in question out of the utility of wealth consideration by classifying such activities as *pleasure-oriented* gambling, which would make such activities consumption goods and therefore not subject to evaluation by a utility of wealth function. *Wealth-oriented* gambling, which he defines as the "deliberate attempt to change wealth status", would be subject to evaluation by a utility of wealth function. *Wealth-oriented* gambling would be the kind of risky wealth growth options available in financial markets, *pleasure-oriented* gambling including those activities that some view with moral disapprobation. He suggests that a distinction between the two types of gambling can be easily observed in that *pleasure-oriented* gambling is repetitive small stakes gambling, whereas *wealth-oriented* gambling would be of the large stakes kind. His hypothesis is that at all wealth levels, *wealth-oriented* gambling will be predominantly risk aversive and *pleasure-oriented* gambling will be present, allowing for the simultaneous preference for unfair insurance and lotteries. This conclusion rests on the exclusion of *pleasure-oriented* gambling from evaluation by the utility of wealth function, which would suggest that such activities provide mostly consumption utility and negligible wealth utility, or that the potential wealth upside is insignificant in the decision to purchase such lotteries.

While such a distinction saves Expected Utility from a harrowing critique, the assumption that the distinction rests on is questionable. Assigning *pleasure-oriented* gambling value as a consumption good instead of a monetary good is appealing, but is it reasonable to do so? Rationales that are consistent with such an assignment include the (short-lived) right to dream or fantasize about potential winnings (Clotfelter and Cook 1990); contributing to socially-desirable causes that are funded by proceeds from such activities (Clotfelter and Cook 1990); an escape from the routine, mundane and predictable nature of modern industrial life (Bloch 1951); a mechanism for releasing tensions and registering non-disruptive protests against an inequitable capitalistic system (Devereux 1949; Frey 1984); and a way to establish social cohesion and maintain friendships (Guillén, Garvía and Santana 2012). While any or all of these reasons may play a role in the decision to purchase lottery or gambling products for certain individuals, classifying such products as consumption goods negates the possibility that any of these reasons may vary in intensity based upon the payoff distribution (Forrest, Simmons and Chesters 2002). Furthermore, the classification of such products as consumption goods is itself suspect; for one, it is quite hard to digest the argument that individuals purchase such products only for the non-pecuniary benefits they provide. Self-reported consumer evidence also validates this: about half of California lottery players polled stated that they played the lottery for the money more so than the fun, the share of which moved inversely to income (Los Angeles Times 1986). More recent evidence further validates this, as a poll of over 1,000 US adults estimated that '21%

of Americans, and 38% of those with incomes below $25,000, think that winning the lottery represents the most practical way for them to accumulate several hundred thousand dollars' (Consumer Federation of America 2006). So, in spite of the non-pecuniary benefits of lottery play, designating it solely as a consumption good does not seem to be appropriate.

In their seminal work laying the axiomatic foundations of EU, Von Neumann and Morgenstern assert that "concepts like a 'specific utility of gambling' cannot be formulated free of contradiction...anybody who has seriously tried to axiomatize that elusive concept, will probably concur in it" (Von Neumann and Morgenstern 1944). Therefore, attempts to account for the utility of gambling would have to depart from EU. A number of models have been proposed that attempt to capture both the monetary and non-monetary motivations of gambling-type activities (Royden, Suppes and Walsh 1959; Tversky 1967; Fishburn 1980; Dyer and Sarin 1982; Conlisk 1993; Schmidt 1998; Diecidue, Schmidt and Wakker 2001; Bleichrodt and Schmidt 2002; Luce, Ng, Marley and Aczél 2008; for a comparison of some of these models see Bleichrodt and Schmidt 2002). The primary features underlying these models are the assumption of a standard decision making model, such as EU, and the addition of a term that captures the (dis)utility of gambling, which is of consequence only when comparing risky options/gambles to riskless amounts/certainties. This latter term can be constructed in a number of ways: as a constant, if the utility is thought of as being a fixed amount independent of the distribution of the gamble (Fishburn 1980); as a function of the risky option; as a function of the riskless option (for a comparison of the construction of gambling utility using the risky or riskless option see Diecidue, Schmidt and Wakker 2001). In fact, some of these gambling utility models are a special case of the Expected Cardinality-Specific Utility proposed by Neilson (1992), in which a different utility function is allowed for lotteries depending on the number of outcomes n the lottery has. In the case of gambling utility, degenerate lotteries ($n = 1$) are evaluated with a utility function, and non-degenerate lotteries ($n > 1$) are evaluated with another utility function (Bleichrodt and Schmidt 2002). Gambling utility models with a base of EU can account for additional behaviors that EU cannot, including the lottery-insurance paradox and the Allais paradox. In that sense it succeeds in much the same way that models that incorporate probability weighting do. The basic probability weighting models allow for first order stochastic dominance violations, but using weights derived from the cumulative probability distribution ensures dominance compliance (Quiggin 1982; Tversky and Kahneman 1992). However, no such "fix" exists in the case of utility of gambling models, insofar as dominance compliance is a desirable consequence of a model. Diecidue, Schmidt and Wakker (2001) show that utility of gambling models necessarily violate either dominance or transitivity, two characteristics that many in the field view as indispensable to sound models of decision making under risk. Therefore, utility of gambling models have not received anywhere near the attention that other non-EU models have that can explain EU-inconsistent behavior while maintaining normatively desirable characteristics.

All the models discussed so far, and the majority of models of decision making under risk, can be classified as outcome-oriented. In relation to games of chance, a number of proposed non-pecuniary benefits have been listed above, but these stem from the mere presence of products with certain payoff distributions. In addition to payoff distributions and non-pecuniary benefits, a third possible source of utility from games of chance is the method by which risk is resolved. “Procedural utility means that there is something beyond instrumental outputs as they are captured in a traditional economic utility function. People may have preferences about how instrumental outcomes are generated. These preferences about processes generate procedural utility” (Frey, Benz and Stutzer 2004). Procedural or process utility has seen limited attention in economic theory, but there are a number of experimental and observational studies that conclude that individuals have procedural preferences in certain cases. One good example that clarifies the concept of procedural utility is legal arbitration: litigants who found the arbitration process to be fair were more likely to accept the court-mandated award, irrespective of the outcomes, although the outcomes themselves played a somewhat smaller role (Lind, Kulik, Ambrose and Park 1993). Many studies of organizational structure and protocols, as well as legal systems, provide evidence of procedural utility (see Frey, Benz and Stutzer 2004 for a review of studies that are suggestive of procedural utility). In this paper, the procedural preferences and utility will be restricted to self-selection or random generation of winning numbers in lotteries. Also, it is important to highlight that the process utility for games of chance is what this paper refers to as the utility of *gaming,* in contrast to the utility of *gambling* models in the literature.

Le Menestrel (2001) takes a procedural approach to the utility of gambling by defining an observable behavior as composed of both a consequence and a process. In the case of a gamble or lottery, which once again is defined as a lottery that has positive probability on more than one outcome, a process (dis)utility can be considered independent of the (dis)utility of consequences. Behavioral preferences are composed of consequence preferences and process preferences, where consequence preferences can abide by EU. The author axiomatizes the three preference types and provides conditions under which observed behavior can lead to a revelation of the underlying process and consequence preferences. The author notes a situation in which a mountain climber prefers a route with a 95% chance of survival over one with 100% chance of survival over one with 80% chance of survival. The monotonicity violation is unable to be explained by consequence-only approaches like EU. However, allowing the consequential monotonic rankings of 100% $\succ^c$ 95% $\succ^c$ 80% and a process in which risk adds to the excitement, so that Risk (survival below 100%) $\succ^p$ No Risk (100% survival) could lead to the observed behavior of 95% $\succ^b$ 100% $\succ^b$ 80% ($\succ^c$ is the preference relation over consequences, $\succ^p$ is the preference relation over processes, and $\succ^b$ is the preference relation over observed behavior). However, in the same vein as the utility of gambling models mentioned above, the process Le Menestrel (2001) identifies is whether or not the lottery is degenerate, and would predict indifference between non-degenerate lotteries that only differ in how risk is resolved.

Experimental Procedure

I implement a simple experimental design in order to determine if number selection matters to individuals in the resolution of risk within a controlled experimental setting. The experiment was conducted during the winter of 2019 on Amazon Mechanical Turk (MTurk), an online workplace that has seen increased usage by experimentalists in recent years. A number of classical laboratory experimental economic results have been replicated on MTurk (Horton, Rand and Zeckhauser 2011). Additional benefits of MTurk include the relative cheapness of subjects, along with access to much larger samples than are available in most traditional laboratory settings, and ease of implementation of static, non-interactive designs. The experimental design attempts to determine if there is some experimental evidence of a process utility of gaming, separate from the utility of gambling mentioned in the literature. The experimental designs are simple, static and non-interactive, only requiring a few minutes of a subject's time. The approach is within-subject, since even if an effect was found with a between-subject approach, definitively attributing the effect to a process utility would be difficult, as the argument that a certain factor was not controlled for could always be levied. The subject pool was restricted to those located in the United States.

The experiment consists of two questions, each offering subjects a choice between two lotteries. The first lottery option in question one is "Picking any number you want from 0-9 and then letting a computer randomly pick a number from 0-9. If the numbers match, you receive a \$10 prize amount; otherwise you receive \$0." The second lottery option in question one is "Letting a computer randomly pick a number between 0-9 two separate times. If the numbers match, you receive a \$10 prize amount; otherwise, you receive \$0." Both of these lotteries offer \$10 with a 10% chance and \$0 with a 90% chance. As stated in the introduction, consequential models predict indifference between these two lotteries. The first option will be called the "Self" option, the second option the "Computer" option. Subjects are asked to indicate which lottery they prefer, and are also given the option to indicate indifference. If a subject indicates strict preference, the subject is then asked to provide the minimum prize amount so that the subject still prefers the option initially selected, but with the new prize amount for that option only. The subject then plays out the preferred option with the new minimum prize amount. If the subject indicates indifference, the subject gets randomly assigned one of the two options to play out at the initial \$10 prize amount. The Appendix demonstrates how this first question was presented to subjects with the appropriate instructions.

The second question offers two more options, but with a starkly different distribution than the first question. The first lottery option is "Picking any number you want from 0-9 and then letting a computer randomly pick a number from 0-9. If the numbers do not match, you receive a \$10 prize amount; otherwise you receive \$0." The second lottery option is "Letting a computer randomly pick a number between 0-9 two separate times. If the numbers do not match, you receive a \$10 prize amount; otherwise, you receive \$0." Both of these lotteries offer \$10 with a 90% chance and \$0 with a 10% chance. The second question proceeds in the same manner as the first question once a player indicates preference. Subjects received a fixed payment of 10 cents for participating, and after a second randomization done outside of the experiment, subjects were eligible for up to an additional \$10 based on their responses and luck. The average time to complete the experiment was about five minutes, the median time was closer to three minutes. 400 subjects participated in the experiment; however, only 298 of the responses were fully consistent with rational behavior and the experiment instructions. For example, if a subject selected a preference for Self but resolved the risk according to the instructions for a Computer preference, such a response was dropped. Results do not change substantially for the complete, unfiltered data.

The decision theoretic model predictions and the evidence for number self-selection in lotteries allow for the formation of two hypotheses: the former implies indifference in both questions, whereas the latter implies a strict preference for Self in both questions. Results consistent with either of these hypotheses would be viewed as in line with the existing literature. Given that the time difference in resolution for the Self and Computer options in the experiment is negligible, a third possible hypothesis of a preference for Computer in both questions is ruled out. Such a hypothesis would be appropriate if the Computer option took less time to play out and subjects were consequentially indifferent but preferred to finish the experiment quickly and perhaps move on to other paid tasks on MTurk. Therefore, the two hypotheses are:

1. **The Decision Theory (DT) Hypothesis:** Subjects will be indifferent between

the Self and Computer options in both questions.

2. **The Utility of Gaming (UG) Hypothesis:** Subjects will display a strict preference for the Self option in both questions.

Results

Experiment I

Table 1 shows the distribution of preferences across the two questions. What is immediately apparent is that the results are not consistent with the DT hypothesis: fewer than one-quarter of subjects indicate indifference in both questions, while nearly two-thirds never indicate indifference. Over two-thirds of subjects are consistent in their preferences across questions, but about one-third of subjects demonstrate that the distribution is somehow impacting preferences over processes. 71 subjects are consistent with the DT Hypothesis, and 86 are consistent with the UG Hypothesis. Subjects consistent with either of the hypotheses correspond to the two largest response groups, amounting to about half of the total responses. However, 46 subjects prefer Computer for both questions, and 39 prefer Self when the win probability is 10% but switch to Computer when the win probability is 90%. There seems to be some evidence for relinquishing of "control" at the 90% win probability relative to the 10% win probability. Using a Wilcoxon matched-pairs signed rank test, the probability of equivalence of response between questions is p = .001. While this is quite a strong significance level by typical standards, it is worthy to note that 203 of the 298 subjects indicated process consistency across questions, while 63 indicated a relinquishing of control at the higher win probability relative to 32 in the opposite direction. The difference here is what is driving the significance, but it is important to note the difference corresponds to only about 10% of subjects. The stronger conclusion is that since nearly half of subjects do not behave in accordance with either hypothesis, there may be other factors at play in the decision calculus for many subjects, including payoff distribution considerations.

Table 1: Preference Reports of Experiment I

		10% win			
		Self	Indifferent	Computer	Total
90% win	Self	86	15	15	**116**
	Indifferent	11	71	2	**84**
	Computer	39	13	46	**98**
	Total	**136**	**99**	**63**	**298**

While the preference responses from the experiment run strongly against the DT Hypothesis, it is important to determine the economic significance of gaming utility, if any. Charness and Gneezy (2010) run an experiment in which subjects are endowed with \$10 and are asked to make investment decisions. One of the treatments they employ is an illusion of control (Langer 1975), in which subjects could roll a dice to determine the outcome of an investment. 25 of the 37 subjects elected to roll the dice themselves; however, when subjects were required to give up 5% of their endowment to roll the dice themselves, only 2 of 22 subjects pay the price. They conclude that while there is evidence of an illusion of control, it is not economically meaningful. So, requiring subjects with a strict preference to state their minimum prize amount to maintain their strict preference in my experiment aims to see if a similar inconsequential illusion of control effect is present here.

The minimum prize amounts reported in both questions are spread out, with a couple of values receiving large responses. In the question offering a 10% win probability, of those demonstrating a strict preference one way or the other, 59 subjects stated a minimum prize amount of \$5 to maintain their reported preference. These subjects were willing to give up half of their potential winnings, or half of the expected value (50 cents) to implement their preferred process. 30 subjects stated a \$9.99 minimum, a response which is in line with the findings of Charness and Gneezy (2010). In the 90% win probability question, 56 subjects state \$5 (a \$4.50 reduction in expected value) and 36 subjects state \$9.99. These are the two most frequent responses in each question. There are no significant differences in mean or median reported minimum prize amounts between the Self and Computer strict preference groups for each question. In fact, the means are within a few cents of each other for each question.

What is perhaps suspect is that for each question, 43 of the respondents who indicated a strict preference reported a minimum prize amount less than \$5, with 15 subjects reporting a minimum prize amount of \$1 in each question. While there is nothing inherently wrong with such responses, it could be argued that some of these subjects may have misunderstood the task, or instead reported how much money they would be willing to have removed from the \$10 prize amount (so an input of \$1 would correspond to a minimum of \$9). Table 2 reports a few median and mean minimum prize amounts: unfiltered, only those reporting \$5 or above, and amounts below \$5 transformed to \$10 minus that amount. For those who indicated indifference, a minimum of \$10 is imputed for mean and median calculations.

One method to ascertain the significance of the reported minimums is to determine how the data compares to the consequential prediction of a \$10 minimum. This can be done via a Mann-Whitney U Test, pitting the subject data against a constant of

$10 for the same number of observations. For both win probability questions and all of the three minimum aggregation methods, the test strongly rejects the hypothesis that the data is generated from the same distribution as the $10 prediction, p<.00001. In order to determine an estimate of how significant the difference is for each question and aggregation type, the constant amount of $10 can be incrementally reduced and tested against the data, up to the point where the test loses significance, say at the 5% level. These amounts are reported in Table 2. The unfiltered and transformed data yield minimums between $9 and $10, while the truncated data yields $9.99. The truncated data test results are more in line with those of Charness and Gneezy (2010), while the other two methods reveal economically significant reductions in prize minimums. So there is some evidence of economically significant valuations of the process utility in this case. It is also worthy noting that the Charness and Gneezy (2010) experiment required subjects to give up 5% of their endowment, while this experiment is asking for a reduction in potential winnings. The reduction of an endowment would be subject to loss aversion, whereas a reduction in potential winnings would not, if lotteries are all evaluated independently. Five percent of the Charness and Gneezy (2010) endowment amounts to 50 cents. In the 90% win question, more than half the subjects report a willingness to lose more than 50 cents in expected value to pursue their preferred process. Loss aversion could perhaps be causing the difference of conclusions between the two experiments.

Table 2: Minimum Prize Amounts of Experiment I

		Unfiltered	$5 or Above	Transformed
10% win	Mean	$7.37	$8.33	$8.33
	Median	$9	$9.99	$9
	n	296	253	296
	Minimum	$9	$9.99	$9.90
90% win	Mean	$7.39	$8.33	$8.30
	Median	$9	$9.50	$9
	n	298	255	298
	Minimum	$9.50	$9.99	$9.50

Lottery Data

The experimental results suggest significant heterogeneity in number selection preferences, even between different payoff distributions. While these results are not fully in line with the predictions based on the literature, an additional source with similar patterns would make a more compelling case. Many lottery games in the United States are draw games, requiring players to select a few numbers from a set of numbers, awarding prizes to players who get full or partial matches. A feature of these games is the option for players to be given a random set of numbers, an option appropriately called Quick Pick (QP), as it only requires making a single selection on the lottery ticket. Players can also choose their own numbers, or Self Pick (SP), which will require filling out the appropriate number of selections on the lottery ticket, usually between three and six number selections per entry.

One prediction of consequential models would be an indifference between QP and SP. The expectation could then be that the percentage of QP would be around 50% across all draw games. Alternatively, as QP takes less time to complete, there could be an expectation of around 100% across all draw games, if players prefer to spend less time filling out entries. Incorporating the evidence that many players prefer to self-select numbers implies that QP would be 0% across games, if all players are assumed to have such preferences. Convex combinations of these homogeneous extremes would effectively cover any observed QP percentage, provided that percentage was relatively constant between games. Notice how the processes available essentially mimic the processes in the experiment: one process allocates the risk resolution totally to a computer, and another allows the player to pick numbers that are to be matched to numbers randomly selected via computer or lottery drum.

Table 3 presents the aggregates sales and QP percentages for the draw games offered in Texas from August 2011 through July 2019. The games are listed in descending odds for the top prize in each game. The odds for the top prize for Pick 3 is 1:1,000, whereas the odds for the Mega Millions jackpot are 1:302,575,350. All the games except Powerball and Mega Millions are games only available for purchase in Texas. First, the QP percentages are starkly different across games, inconsistent with the range of hypotheses permitted by the literature. Second, there seems to be an inverse relationship between the QP percentage and the overall odds. Games with better odds have lower prize amounts, as lottery tickets are similarly priced across games. Third, QP percentages are correlated with the parimutuel nature of the top prize: higher QP percentages occur within parimutuel games. There are a few competing explanations for the wide range of QP percentages across games.

The first explanation revolves around different utilities of gaming for the various lottery games. The two games with low QP percentages are the Pick 3 and Daily 4. Both of these games require selecting (either three or four) numbers from 0-9. All of the other games require selecting numbers from a larger number pool, such as 35 or 69. It is certainly easier to construct personally important numbers, like area codes or birthdays, using a few digits from 0-9 than from a pool of larger, two digit numbers. Another differentiating factor between Pick 3 and Daily 4 and the rest of the draw games is that they are the only two games that allow selecting with replacement. This would allow selecting a number like 777 in Pick 3 or repeating numbers, which other draw games would not allow. The single digit selection design and drawing with replacement make Pick 3 and Daily 4 ripe for playing lucky or important numbers. Selecting numbers to form patterns or designs on the playing board is also a motive, but this motive is arguably stronger in games with larger playing boards that require more number selections, which are the games with worse odds. Another related explanation is that the games with worse odds of desirable prizes require more numbers to select and a larger pool to choose from. Many players apparently find choosing six numbers for a worse odds game a daunting task (Clotfelter and Cook 1989). This could cause added mental stress in self-selecting for those games, reducing the impact of the utility of gaming from self-selecting. The two games with QP percentages near 50% are Cash 5 and All or Nothing. These games have poorer odds than Pick 3 and Daily 4, but are also not parimutuel. They also have a different number selection mechanism of selection without replacement of single and double digit integers, relative to the single digit integer selection with replacement of Pick 3 and Daily 4. The mental stress motive would be stronger for these games as well. These factors all hinder the appeal of self-selection and would therefore provide a lower utility of gaming for these games. The four parimutuel games have even larger pools of numbers to choose from without replacement. The added mental stress would further depress the gaming utility and result in the observed lower QP percentages.

Table 3: Texas Lottery Data

	Sales	QP Percentage	Parimutuel Top Prize
Pick 3	\$2,083,839,142	12.41%	No
Daily 4	\$753,942,261	13.01%	No
Cash 5	\$404,839,457	55.84%	No
All or Nothing	\$270,035,914	55.88%	No
Texas Two Step	\$457,981,345	76.40%	Yes
Lotto Texas	\$1,158,055,791	69.61%	Yes
Powerball	\$2,438,867,623	80.54%	Yes
Mega Millions	\$1,808,619,562	79.39%	Yes

Pick 3 requires selecting three numbers from 0-9; Daily 4 requires selecting four numbers from 0-9; All or Nothing requires selecting twelve numbers from 1-24; Cash 5 requires selecting five numbers from 1-35; Texas Two Step requires selecting five numbers from 1-35; Lotto Texas requires selecting six numbers from 1-54; Mega Millions requires selecting five numbers from 1-70 and one number from 1-25; Powerball requires selecting five numbers from 1-69 and one number from 1-26.

The second explanation for the varying QP percentages between games is the maximization of the expected return to playing. For fixed odds games, each entry yields exactly the same expected return, as there is no sharing of prizes in the case of multiple winning entries for any prize level. Games with a parimutuel top prize would only share this equality of expectation if all entries were determined in an effectively random manner. Bosch (1994) lists 2,588 popular number combinations of a German lottery, each being selected at least 50 times more often than by random expectation. These correspond to 0.038% of tickets sold, whereas the expected percentage by random assignment is 0.00018%. Under such circumstances, a player aware of which combinations are popular would increase the expected return by avoiding such combinations for this parimutuel game. Since a player cannot be reasonably aware of a comprehensive set of combinations that are popular for a given game, a player could opt to QP to increase the likelihood of drawing an unpopular combination. A study of the UK National Lottery estimates that 18% of the combinations are popular, in that about half of the players who self-select numbers choose from those 18% (Simon 1998). So, opting to QP would amount to an 82% chance of drawing an unpopular combination, substantially increasing the expected return of self-selecting, assuming self-selection would more likely yield a popular combination. This could explain the large gap in QP percentages between parimutuel and fixed odds games, and adding the utility of gaming explanation would account for the differences within each game type.

The first two explanations are wholly contained within the explanatory power of the existing literature. A third explanation is that the differences in QP percentages between games is somehow due to the payoff distributions themselves. Games with better odds and lower prizes, like Pick 3 and Daily 4, have low QP percentages, whereas games with poorer odds and higher prizes, like Powerball and Mega Millions, have high QP percentages. It is worthwhile to determine if payoff distributions impact preferences over self-selection of winning numbers. Unfortunately, in addition to highly variant payoff distributions, the lottery games differ in number selection mechanisms, potential mental stress of number selection, and parimutuel nature of the top prize. Returning to the experimental setting and controlling for the rationales consistent with the literature should yield some insight regarding the driver of the observed choice behavior in the lottery data.

Experiment II

The second experiment is essentially a modification of the first, with the intent being to determine the true culprit behind the QP percentage distribution across Texas lottery games. The first experiment provided some evidence that the choice behavior of many individuals cannot be accounted for with the decision theory and number selection literatures. However, the probability levels of 90% and 10% of winning are not representative of typical lottery odds. To better recreate the real-life setting, the odds of winning in the second experiment are reduced significantly. Once again, subjects are asked two questions about lottery preference and are told to choose their preferred option in each or report indifference. The first lottery option in question one is "Picking any number you want from 0-999 and then letting a computer randomly pick a number from 0-999. If the numbers match, you receive a $10 prize amount; otherwise you receive $0." The second lottery option in question one is "Letting a computer randomly pick a number between 0-999 two separate times. If the numbers match, you receive a $10 prize amount; otherwise, you receive $0." The second question reduces the odds of winning by a factor of 1,000. The first lottery option in question two is "Picking any number you want from 0-999,999 and then letting a computer randomly pick a number from 0-999,999. If the numbers match, you receive a $10 prize amount; otherwise you receive $0." The second lottery option in question one is "Letting a computer randomly pick a number between 0-999,999 two separate times. If the numbers match, you receive a $10 prize amount; otherwise, you receive $0." This experiment does not require disclosure of minimum prize amounts and therefore took less time and had no possible subject inconsistency issues, all 400 responses are included, although this was run as a separate session from the first experiment and these are not the same 400 respondents. Otherwise, it was similarly incentivized and ran in the same fashion as the first experiment.

The results of the second experiment are presented in Table 4. The 1:1,000 odds bears some similarity to Pick 3, as those are the exact odds for the top prize of $500. The 1:1,000,000 odds is typical of the odds size for some of the larger prizes in games like Lotto Texas, Powerball and Mega Millions. Unlike these games though, the number selection in the experiment more closely resembles Pick 3, which requires players to choose three digits from 0-9 with replacement. The 0.1% win question essentially requires the same, as picking three single digits from 0-9 with replacement and order sensitivity is equivalent to choosing a three digit number from 0-999. The 0.0001% win question extends the selection from three to six digits, 0-999,999. This design attempts to remove the number selection differences noted above between games like Pick 3 and Mega Millions. In addition to controlling for the number selection mechanism, if there are multiple winners in the experiment, each gets the promised prize of $10, so the parimutuel feature is controlled for. The possible mental stress of picking a three digit vs a six digit number is assumed to be equivalent or negligible. The time difference for completion of the experiment between Self and Computer is negligible, assuming players self-selecting are not mulling over which number to choose. Implementing these controls leaves the payoff distributions themselves as the variable of interest. If a variation in preferences is found between the two questions, the conclusion would be that the distributions themselves affect the preference for self-selection.

This indeed is what the experimental results suggest. In the 0.1% win question, Self is the most preferred option (165 subjects), whereas in the 0.0001% win question, allowing the computer to select is most preferred (184 subjects). A Wilcoxon matched-pairs signed rank test testing changes in response between questions is highly significant ($p<.00001$). 106 subjects opt for less "control" as the win probability drops, with only 37 subjects moving in the opposite direction. Less than 40% of subjects behave according to either hypothesis supported by the literature, namely selecting Self or Indifferent for both questions. In fact, the largest preference group is Computer for both questions. When moving from the experimental to the empirical setting, Self closely corresponds to SP and Computer to QP. However, there is no Indifferent option in the empirical setting, so individuals who are indifferent ultimately choose either to SP or QP. To make the experimental data more comparable to the empirical, the Indifferent option needs to be dealt with. One way is to drop the results reporting indifference. If so, 147 subjects prefer Self in the 0.1% win question and 113 prefer Computer, meaning 43% QP. In the 0.0001% question, 102 prefer Self and 158 prefer Computer, meaning 60% QP. One extreme case is to assign the indifference to Self. Doing so yields 31% QP in the 0.1% win question and 46% QP in the 0.0001% question. The other extreme is to assign indifference to Computer, which perhaps is more justifiable. QP is indeed quicker than SP in real life, while Self and Computer do not have much of a time difference in the experiment. Doing so yields 58% QP in the 0.1% win question and 71% QP in the 0.0001% question. Using the Texas lottery data, games more similar to the 0.1% win question have 10%-15% QP, while the other games more similar to the 0.0001% question have 70%-80% QP. While assigning Indifference to Computer gets the 0.0001% win QP percentage into the appropriate empirical range, even assigning Indifference to Self doesn't lower the 0.1% win QP percentage into the appropriate empirical range. The takeaway is that the controlled experiment is generating a significant gap in purported QP percentages in the same direction as the lottery data, but a smaller magnitude.

Table 4: Preference Reports of Experiment II

		0.0001% win			
		Self	Indifferent	Computer	**Total**
0.1% win	Self	85	18	62	**165**
	Indifferent	11	72	26	**109**
	Computer	17	9	96	**122**
	Total	**113**	**99**	**184**	**396**

Discussion

The experiment confirms an interaction between the payoff distribution and preferences over risk resolution for a significant percentage of subjects. A couple of psychological studies provide the only evidence the author is aware of in which a preference for the process of risk resolution varies by payoff distribution. Experimenters assessed subjects with a Desirability of Control scale (Burger and Cooper 1979) and find that high desire for control subjects bet more money when allowed to throw the dice themselves in a dice game (Burger and Cooper 1979), and this effect on dice games is more prominent when the odds of winning are relatively better (Wolfgang, Zenker and Viscusi 1984). These studies were between-subject studies with fewer than 100 subjects, and critically did not control for risk preferences in any way, so the results could also be attributable to more risk tolerance by subjects randomly assigned into treatments instead of an illusion of control. The experimental design in this paper is within-subject and can face no such criticism. Summarizing the results from the first two experiments, about 40% to 50% of subjects act in accord with the decision theory or utility of gaming literatures. There is a stronger preference for Computer in the second experiment in which win probabilities are extremely low. The preference for Computer regardless of payoff distribution could perhaps be rationalized as slightly less time consuming or mentally taxing than Self. Another 30% to 40% of subjects change their preferences for risk resolution between questions. The implication is that there is significant heterogeneity in preferences for self-selection of numbers, and for many agents this preference is dependent on the payoff distribution itself. The key contribution of this paper is evidence of the latter. The question remains as to what is driving such a dependency.

The experiments keep the prize winnings fixed at \$10 and vary the win probabilities and therefore the expected values of the lotteries. In the 90% and 10% win probability experiment, 63 subjects indicated a relinquishing of "control" when moving from the low win probability to the high, relative to 32 in the opposite direction. In the 0.1% and 0.0001% win probability experiment, 37 subjects indicated a relinquishing of "control" when moving from the lower win probability to the high, relative to 106 in the opposite direction. Combining these findings indicates a U-shaped behavior by the average agent displaying distribution-dependent risk resolution preferences: at very low and high probabilities of winning Computer is preferred, while at moderately low probabilities of winning Self is preferred. This non-monotonic behavior requires some creativity to rationalize. One attempt is to frame the discussion in terms of regret.

Consider defining two types of regret, one resulting from a poor outcome from Self, the other from a poor outcome from Computer. Once a player is made aware of the winning numbers post risk resolution, a player selecting Self could retroactively choose those numbers and win, while a player choosing Computer would be no better off with that information retroactively, since numbers are assigned randomly. Therefore, playing Self elicits a tangible regret when a poor outcome obtains, whereas playing Computer elicits a weaker abstract regret in the event of a poor outcome. If minimizing the pain of regret was the only concern outside of an evaluation of the payoff distribution, opting for Computer would be optimal for all distributions. Define a poor outcome as one that is less than the expected value of the lottery, and the magnitude of regret as the difference between the expected value and the poor outcome. Regret therefore has both probability and magnitude aspects. For a fixed magnitude of regret, the pain of regret would be higher if that regret was tangible instead of abstract. Referring back to the experiments, the lowest win probability of 0.0001% gives the highest chance of a poor outcome but simultaneously the smallest magnitude of regret, $\$10 * 0.000001 - 0 = \$0.00001$ cents. The highest win probability of 90% gives the lowest chance of a poor outcome but simultaneously the largest magnitude of regret, $\$10 * 0.9 - 0 = \$9$. A low probability of winning like 10% gives a magnitude of regret of $\$10 * 0.1 - 0 = \$1$.

One way to explain the U-shape is by the relative salience of either the probability or magnitude of regret. At the win probability of 0.0001%, the probability of a regretful outcome is close to one and is a more salient feature of the lottery than the minuscule magnitude of regret of $0.00001 cents. On the other hand, at the win probability of 90%, the probability of a regretful outcome is close to zero but the magnitude of regret of $9 is the more salient feature of the lottery. At the 10% win probability, the probability and magnitude features are not nearly as different in salience compared to the other two distributions. Therefore, when either of the regret aspects is highly salient, the regret motive becomes more powerful. When the regret motive is strong, namely when either of the regret features is highly salient, self-selecting and incurring tangible regret may overpower the benefit of self-selecting to the point that the net harm is more than the pain of abstract regret by assigning risk resolution to a computer. A weaker regret motive would not hinder the benefit of self-selection enough to dissuade from self-selection. This underlying mechanism is able to account for the observed U-shaped behavior by subjects that have distribution-dependent preferences over risk resolution.

It must be admitted that the regret and gaming utility story is not conclusive, but merely rationalizes the observed behavior by a large number of subjects in the experiment. There may be other explanations which account for the behavior equally well. Also, given that lottery questions were paired together in the experiment, it is not clear as to if the observed behavior is due to the absolute payoff distributions or the relative differences in paired payoff distributions. The U-shaped behavior does not seem to be symmetric: based on the four lotteries proposed in the two experiments, the distribution dependency of risk resolution preferences is stronger at extremely low win probabilities relative to high win probabilities. Future studies may be able to tease out a better understanding of this U-shaped phenomenon through risk resolution preference elicitation over more win probability values, and could even be expanded to lotteries with more than two branches if the effect is indeed found to be robust.

Conclusion

This paper provides novel evidence that preferences over how risk is resolved is dependent on the payoff distributions of lotteries for a large number of individuals. The risk resolution method considered is whether winning numbers are self-selected or delegated to a random number generator. While there is existing evidence that individuals may prefer to self-select even between lotteries that standard decision models would evaluate as equivalent, this paper goes further by providing evidence that the payoff distributions themselves impact the self-selection preference. A regret-salience motive is offered to explain the choice behavior of subjects displaying distribution-dependent self-selection preferences. Lottery sales data from Texas is adduced to strengthen the experimental findings. Evidence in both the controlled experimental setting and the real world lottery market point to considerations and mechanisms that the extant literature does not sufficiently address. More studies are welcomed to to better understand the scope and motivations of such behavior, and perhaps to even lay down some behavioral principles or theoretical foundations to account for distribution-dependent process preferences.

Appendix

Proof of the Sufficiency of Distributional Equivalence for Two Outcome Lotteries under Regret Theory

The native environment for Regret Theory is the choice between two actions that result in certain events occurring in specific states resulting in appropriate outcomes: there is no single correct way to extend the decision process to three or more outcome, although a few intuitive ones have been proposed. The actions for the purposes of this paper and proof correspond to undertaking either of two lotteries, A or B, with equivalent outcome distributions: receiving (x, y) with probabilities (p, 1-p), $x \neq y \in \mathbb{R}$. Notice that there are up to four possible outcome pairs for {A, B} over all consolidated states, listed 1 to 4: {x, y}; {x, x}; {y, x}; {y, y}. The first two states correspond to the probability of getting x under A, or p. Therefore, let $p_x \leq p$, so probability of State 1 is p_x and the probability of State 2 is $p - p_x$. Similarly, let $p_y \leq 1 - p$, so probability of State 3 is p_y and the probability of State 4 is $1- p - p_y$.Under Regret Theory, $A \succeq B \Leftrightarrow p_x Q(x - y) + (p - p_x)Q(x - x) + p_y Q(y - x) + (1 - p - p_y)Q(y - y) \geq 0 \Leftrightarrow p_x Q(x - y) + p_y Q(y - x) \geq 0 \Leftrightarrow (p_x - p_y)Q(x - y) \geq 0$, since under Regret Theory $Q(0) = 0$ and the symmetry of Q(.) means $Q(-\xi) = -Q(\xi)$. Therefore, indifference will hold iff $p_x = p_y$. Assume $p_x \neq p_y$. The distribution of A remains (x, y) with probabilities (p, 1-p). The distribution of B is (x, y) with probabilities $(p - p_x + p_y, p_x + 1- p - p_y)$. A and B are distributionally equivalent iff $p_x = p_y$ as premised, so $p_x = p_y$ and Regret Theory predicts indifference. □

Screenshots of Experiment I

Question 1
Consider the following options:

Option 1. Letting a computer randomly pick a number between 0-9 two separate times. If the numbers match, you receive a $10 prize amount; otherwise, you receive $0.
Option 2. Picking any number you want from 0-9 and then letting a computer randomly pick a number from 0-9. If the numbers match, you receive a $10 prize amount; otherwise you receive $0.

Which option would you prefer?

- Option 1
- Option 2
- I value both options equally

If you chose Option 1, please proceed to Section 1. If you chose Option 2, please proceed to Section 2. If you value both options equally, please proceed to Section 3. You will only complete one of the sections.

Section 1

Please enter the lowest prize amount you are willing to accept in Option 1, so that you still choose Option 1 over Option 2

Enter the minimum amount in the dollars and cents format "x.xx", it must be less than 10.00

Enter minimum amount

You will now play your new Option 1 with the prize amount you just entered above. Click on the two buttons below to see if the numbers drawn match

first draw
second draw

Now proceed directly to Question 2, do not complete Sections 2 or 3

Section 2

Please enter the lowest prize amount you are willing to accept in Option 2, so that you still choose Option 2 over Option 1

Enter the minimum amount in the dollars and cents format "x.xx", it must be less than 10.00

Enter minimum amount

You will now play your new Option 2 with the prize amount you just entered above. Please input your lucky number between 0 and 9

Enter your lucky number | click to lock your number and have the computer draw a number

Now proceed directly to Question 2, do not complete Section 3

Section 3

Since you value both options equally, it was randomly determined that you will play out Option ___ ONLY.

If you were assigned Option 1, click on the two buttons below to see if the numbers drawn match

first draw
second draw

If you were assigned Option 2, please input your lucky number between 0 and 9

Enter your lucky number | click to lock your number and have the computer draw a number

Chapter 3: Behavioral Modifications to Prize Linked Savings

Abstract

Prize Linked Savings (PLS) is a unique savings product with the ability to appeal to individuals in ways that standard savings products cannot. PLS combines the principal guarantee of traditional savings products with the return skewness of lottery products. In this paper, it is shown that the existing implementation method of uncertainty resolution in PLS products may not be maximizing the uptake of PLS. An experiment demonstrates that by switching from the extant raffle setup to a more interactive gaming one, PLS appeal is increased. Furthermore, doing so may also enable PLS to serve as a more effective substitute for games of chance.

Introduction

There are numerous ways that governments, firms and organizations attempt to induce individuals to save at higher rates than they would without any external influence or incentives acting upon them. Banks typically offer savings accounts and other similar assets whereby depositors deposit savings into an account, often with withdrawal restrictions, and are promised a fixed rate of interest on their deposit, independent of the ex-post profitability of those assets to the bank (although presumably the rate of interest chosen may certainly be correlated to the bank's ex-ante expected profits). Many companies provide incentives for their employees to save more of their income for retirement by offering limited matching contributions to a 401k or pension plan (for instance, matching every dollar an employee invests in a plan up to 5% of an employee's income), which is equivalent to a significant increase in the return on investment. In this case, if an employee without such an incentive would have selected to place 5% of his income in the plan, and with the incentive his behavior remains unchanged, he would reap an instantaneous return of 100% on his investment. Governments may directly incentivize individuals to save more, for instance, by creating various tax incentives, such as those offered on 401k accounts and IRAs.

Prize Linked Savings (PLS) is an investment vehicle that may also provide additional incentives to save. The PLS vehicle can take the form of a savings account, a bond, or another similar asset, where one party is providing some sort of loan to another party. The key difference between the PLS design and the standard design is the return profile: typically returns on bonds, loans, savings accounts and the like are stated as a fixed percentage of the deposit procured or the capital invested. However, the PLS design adds more risk and variability to the structure with a non-constant ex-post rate of return. The payment profile is set up as a lottery, typically a highly skewed lottery, where the probability of winning a significantly large amount of money is small, while the probability of winning little to no money is quite large. Notice, however, that this is a lottery in a purely economic sense: the PLS payoff profile ensures the preservation of the initial principal invested, whereas the more commonly understood concept of lottery, which I will differentiate from the economic lottery via the word 'lotto', requires relinquishing the principal as the price to be paid to participate in the lottery. So, PLS can be viewed as a lottery where the principal is guaranteed.

PLS is employed and has been employed by various banks and institutions across the world for decades, and in recent years has even seen some limited implementation in the United States. The late entry of PLS into American markets has been largely due to extant federal and state bans on private lotteries. However, a major legal hurdle was overcome with the passing of the American Savings Promotion Act in 2014, which removed federal barriers to bank and thrift participation in PLS products, although the process of revising state laws that prevent PLS introduction continues in various states (Commonwealth, 2016a). There has been some limited research on PLS, both empirical and experimental, that overall reaches a few conclusions. PLS is quite a popular product when introduced, and the source of the demand is partly due to substitution from other non-saving activities (consumption, lottos, gambling) and partly from cannibalization of existing savings (moving money from a savings account to a PLS account, for instance). There is also compelling evidence that PLS demand is positively related to the skewness of the PLS product and the size of the largest prize. However, a forthcoming study in the Journal of Financial Economics on PLS utilization at American credit unions seems to consolidate the existing evidences within a single narrative: if the substitution effect works through gambling preferences, PLS and gambling/lottos should be weaker substitutes the more differentiated they are on other dimensions (Cookson, forthcoming). This study ultimately concludes that the substitution effect does indeed work through gambling preferences, that PLS can serve as a substitute for gambling and lottos, and that the substitutability decreases the more differentiated the products become. For example, the author finds that PLS is a strong substitute for local gambling but not destination gambling, as destination gambling includes additional non-gambling factors that are part of the overall utility. He further finds that PLS is a strong substitute for gambling at casinos without a nightlife, but not for casinos with a nightlife.

Given the convincing conclusions of this study on PLS at American credit unions (which are currently where nearly all of the PLS products available in the United States are offered), I believe there is more that can be done to PLS to make it more substitutable for gambling and lottos, i.e. by making PLS even less differentiated when compared to gambling and lottos. Currently, the primary similarity between PLS and gambling/lottos is the positive skewness of the payoff distribution, something that traditional loss-protected savings methods do not offer. Other savings tools that do not guarantee principal maintenance but do have positive skewness features, such as stock investing, have been shown to serve as substitutes for lottos. There is a negative relationship between lotto prize size and stock market trading volume in the United States, and trading in assets that are more like lottos (stocks and options) are affected by changing lotto prize amounts, whereas those less like lotto are not (bonds and mutual funds) (Dorn, Dorn and Sengmueller, 2014). Given the evidence, it would behoove designers of PLS products to attempt to make PLS like gambling and lottos as much as possible to increase the substitutability effect. One way to do so that has not yet been implemented en masse is to “superficially” make PLS mimic gambling and lottos: incorporate a gaming aspect, as is common in many gambling activities, and/or incorporate a pick-your-numbers format, as is common in many lottos. Based upon the substitutability conjecture, ceteris paribus, these modifications should allow the modified PLS to perform at least weakly better than PLS as it is practiced today. Even without the substitutability conjecture, if there is positive value placed on the gaming or selection aspects of gambling and lottos, cetris paribus, this should also lead to modified PLS weakly outperforming standard PLS. It is these ideas that I seek to find evidence for via a simple experiment.

Motivation

There are numerous conceptual reasons why having savings may be desirable from an individual’s utilitarian perspective. Many large purchases may require a substantial amount of savings in order to be carried out: for instance, in order to purchase a house a significant down payment often needs to be secured in order for a lender to provide a loan for the remaining balance. Curbing some short-term spending to generate such savings could be utility-maximizing if the value of acquiring a house is large enough. It may be desirable to have precautionary savings to serve as a buffer in case an uninsurable event happens that causes significant financial loss, which without such savings may cause extreme losses to utility. It may also be desirable to have some savings even after one’s passing, if the utility of one’s progeny is incorporated into individual utility. These are just a few reasons why creating and growing savings may be utility improving behavior.

Despite the various potential utilitarian benefits to saving, in addition to certain policy goals and incentives that may seek to increase savings, savings rates are often not too high among those who may need savings the most, namely the poor (those with low wealth levels) and those with relatively lower incomes. Numerous studies have documented that savings rates increase with income (for a list and classification of such studies see Dynan, Skinner, and Zeldes, 2004). However, those with the lowest lifetime incomes may be the ones who most desperately need to save, as these savings would allow an increase in living standards and potentially a permanent move out of the poverty trap they may find themselves in. The incentive for the poor to save may be lacking, as whatever small level of savings they could muster, and whatever correspondingly small fixed returns, would not be viewed as a sufficient stock for them to escape poverty (Banerjee and Mullainathan, 2010). When the Consumer Federation of America and the Financial Planning Association asked 1,000 Americans about the most practical way for them to accumulate several hundred thousand dollars, 21%

replied with "win the lotto", and among the poorest respondents the rate of that answer was a much higher 38% (CFA, 2006). Perhaps this is why annual spending on lottos is similar across income strata in the United States, implying that the lowest income households spend a significantly larger portion of their incomes on lottos (Kearney, 2005). This same intuition may also be a reason to expect that PLS would be appealing and effective in encouraging saving among the most financially vulnerable, and there is historical precedence that this reasoning is sound.

PLS is by no means a new concept; there is evidence that investment tools that can be classified as PLS existed as early as the seventeenth century, although well after the appearance of lottos. Perhaps the first instance of PLS was the 'Million Adventure' appeal of 1694, which was established to help supplement the British treasury funds that were fast diminishing due to the piling expenses of the Nine Years War. The 'Million Adventure' offered 100,000 tickets at a cost of £10 each, with a number of winning prize amounts, ranging from £1,000 to £10 per year for 16 years, while those who did not win received £1 for per year for 16 years (Murphy, 2005). In this case, the PLS took the form of an annuity where participants invested £10 up front and were paid a fixed amount ranging from £1 to £1,000 per year depending on whether a given ticket was a winning one, and if so, to what extent. One interesting feature of the 'Million Adventure' was that the price to participate of £10 was quite low, so that participation was not limited to the wealthiest of society, and even those who were not able to afford a ticket individually could pool resources with others to participate in the lottery (Murphy, 2005). This wide appeal to members of varying economic strata in society is now a standard feature of modern PLS instruments.

So, at face value PLS seems to have characteristics that may induce an increase in savings from the baseline savings that would prevail without the presence of PLS, given its positively skewed payoff structure with no risk of loss (assuming away default probability, deposit insurance concerns, etc.). A few questions require answering though before PLS is proven effective at increasing savings levels. First, is there a demand at all for PLS, and if so, how much? Second, an apparent increase in savings after implementation of PLS is not necessarily a net increase, as it is possible that PLS may be just siphoning off savings from other savings vehicles. For PLS to be deemed an effective savings creator, it needs to be demonstrated that new savings is actually being generated, perhaps by substituting from other non-saving tools that provide a similar risk structure and payout, such as lottos or gambling activities. I will focus on examining existing evidence that addresses these questions, and based on the analysis, design an experiment to determine if further modifications to existing PLS product structures will engender an increased demand for PLS, particularly if part of that increased demand is derived from new, non-cannibalized savings.

Literature

The earliest evidence of PLS is the descriptive account of the 'Million Adventure' cited above. However, PLS instruments have been in use in many countries, including private utilization by commercial banks in Latin America, as well as public utilization in the UK via the issuance of Premium Bonds (Kearney, Tufano, Guryan, and Hurst, 2010). UK Premium Bonds have been offered since 1956, and there is strong evidence that they are held by individuals across the income distribution. Looking at data from 2005-2006, while there are a higher percentage of households in the upper income tiers that hold UK Premium Bonds when compared to households in lower income tiers (as is common with most financial assets), the relative share of Premium Bonds in asset composition seems to be slightly higher in lower income tiers than higher tiers, save the highest income levels, perhaps due to tax considerations, suggesting it is a popular instrument among lower income households (Kearney et al., 2010). Furthermore, there is evidence that sales of these bonds respond positively to the size of the largest prize offered, although this value contributes to only about 2 percent of the expected return, as there are other smaller prize amounts available with higher probabilities of winning (Tufano, 2008). Also, increasing skewness and reducing the amount and number of smaller prizes for the sake of one large prize also seems to increase PLS demand (Pfiffelmann, 2013). This may provide some insight into optimal PLS design incorporating one very large top prize instead of multiple smaller prizes when deciding how to divvy up the winnings.

Probably the most well-known and studied privately offered PLS instrument is that of the First National Bank of South Africa's Million a Month Account (MaMA). They began offering a PLS account in 2005 but were forced to close the program in 2008. The program was highly popular and saw massive take-up among all parts of society. The introduction of this PLS product did not cannibalize other modes of saving: rather, there is evidence that it may have increased other modes of saving, such as utilization of standard savings accounts, and that the increase in total savings by participants represented a 38% increase from mean savings levels (Cole, Iverson, and Tufano, 2014). There is also evidence that some of these newly created savings were generated as individuals substituted away from lottos and other similar games, as the authors noted that there were significant decreases in MaMA accounts created around the times when larger lotto jackpots were available. Perhaps further anecdotal evidence of this relationship is confirmed in the closure of the program itself, as the National Lotteries Board of South Africa sued the Bank. The Bank was found in violation of the Lottery Act of 1997, since the South African government has a legal monopoly on lottos, and the PLS was deemed to be a lotto (Cole et al., 2014). The conclusion from this study is that PLS undoubtedly can create new savings: its popularity did not merely represent a shift in savings from other instruments. A weaker conclusion is that PLS can serve as an imperfect substitute for lottos and perhaps other forms of gambling.

On the experimental side, there is even more evidence of the attractiveness and effectiveness of PLS products. In an online experiment run primarily through Amazon's MTurk, the introduction of PLS is found to increase total savings levels, in the range of 12 percent, as well as induce non-savers to save. The experimental results also suggest that the funds invested in PLS are taken from existing lottery and consumption expenditures, and not current savings holdings (Atalay, Bakhtiar, Cheung, and Slonim, 2012). This online study is perhaps the most similar in intent to the one I run, as we both share the same subject population of MTurk, and both attempt to determine if PLS uptake is not just extant savings cannibalization, but rather includes substitution from other activities. However, as my ultimate purpose is to determine if making PLS mimic gambling and lottos from an entertainment perspective will increase the draw of PLS, I do have something new to offer that their study does not address. Furthermore, the expected returns on their savings options are not typical of the returns on these products in the real world. Specifically, their treatments included interest rates on savings of 5, 10 and 20 percent, all much higher than the sub-one percent interest rates on standard savings accounts as of their paper's publication year of 2012. Also, the expected returns per dollar invested in their lotto options range from \$.90 to \$1.10, much higher than the overall Unites States average lotto return of \$.52 (Kearney, 2005). Given that the expected returns on their savings options are quite divorced from reality, it is hard to make strong inferences on real world behavior of introducing PLS based on their experimental findings.

In a lab experiment run at the University of Maryland, experimental results suggest that PLS appeal is greatest among males, self-reported lottery players, and those with low levels of deposits in their bank accounts, which serves as a measure of savings (Filiz-Ozbay, Guryan, Hyndman, Kearney, and Ozbay, 2015). The authors further conclude that individuals seem to exhibit more patience when they are awaiting payment from a risky investment than when the return is certain, given the expected returns are the same. The authors also attempt to model the risk preferences, as their experimental results could be explained by convex (risk-loving) utility, as well as non-linear probability weighting, and determine that non-linear probability weighting is the more plausible explanation. Specifically, men tend to overweight small probabilities more so than women, and this may provide an explanation as to why in their sample men tended to more strongly prefer PLS than women did, and why a higher percentage of men than women in the sample were gamblers (Filiz-Ozbay et al., 2015). Given that those at the lower income spectrum tend to spend a higher percentage of their wealth on lottos (Kearney, 2005), this is further evidence that PLS may have a stronger appeal among the poor, who often do not have the desire and lack the right incentives to save, due to their precarious situation.

There is some evidence, however, that simply introducing PLS into an environment will not automatically create new savers and savings. In order to respond to the low savings and asset acquisition of low income families in the United States, Congress introduced the Assets for Independence Act in 1988, which included the establishment of Individual Development Accounts (IDAs), which provide government-funded matching of savings for families with household income below 200% of the poverty line (Assets for Independence, 2014). These are exactly the types of people that the aforementioned studies hypothesize are most in need for savings incentives and will likely benefit the most from the introduction of a PLS instrument. However, the success of the IDA program to date is mixed, as there is not much evidence of a significant increase in savings. In order to modify the IDA incentive structure, a field experiment consisting of four different treatments was implemented: a reminder and follow-up phone call before and after deposit deadlines; an increase in deposit frequency from monthly to bi-weekly; an introduction of a PLS instrument; and increasing the match rate from \$2 to \$4 if half of the savings goal was reached (Haisley, Jones, Loibl, and Loewenstein, 2016). The authors find that none of these modifications significantly increased savings when compared to their respective control groups. They suggest that the ineffectiveness of the incentives is likely due to the liquidity constrained nature of the target population, mainly that they don't have free funds to properly respond to the incentives. In the case of PLS, the treatment included all the treatments except the increased match rate, while the control group had none of the four treatments. The treatments were done in such a compounded way so as to maximize the chance of increasing savings (Haisley, et al., 2016). There were 42 participants in the treatment group, 45 in the control group, amounting to a similar number of participants as the lab experiment run at the University of Maryland. The authors here actually find that those in the control group saved more than those in the PLS treatment, although the difference is not statistically significant. These recent findings, at the very least, cast some doubt on the notion that PLS inherently is a savings creator. This suggests that there is room for further analysis of PLS, including the modification and development of PLS instruments in order for them to become additionally effective, especially on the lower income target population. For instance, if part of the consumption of the target population is in lotto and gambling activities, the liquidity constraint could effectively be loosened if PLS could serve as a substitute for lotto and gambling activities. The implication of this study is that PLS was not an effective incentive to increase savings and is not suitable enough of a substitute for lotto and gambling activities when liquidity constraints are present. This study used a standard PLS design, where winners are just randomly selected from a pool of participants. Perhaps modifying the standard PLS design could make PLS a more suitable substitute for lotto, gambling or other uses of funds, even when liquidity constraints are present.

When considering optimal PLS design, in addition to issues of payment structure, such as expected value, skewness and variance of payments, there are two further behavioral implications that should be considered; namely, entertainment value and the illusion of control. When analyzing lottos in the United States, even though the average expected value of each dollar invested in the lotto is about 52 cents, lotto participants are not necessarily irrational if they participate in lottos, even if they are risk averse, if entertainment value is incorporated into the utility. Specifically, when gaming and entertainment features are considered, it is found that lotto participants act rationally by showing an increased demand for lottos when the expected value increases, consistent with the view that lotto players are not necessarily irrational or risk loving, but rather derive an entertainment value that is not captured when only considering expected monetary returns (Kearney, 2005). The illusion of control is a documented psychological phenomenon where individuals confound luck and skill, and exhibit increased confidence or belief in their ability to control outcomes in situations purely determined by luck (Langer, 1975). For instance, in an experiment where members of a treatment group were allowed to select their lottery ticket, while control group members were assigned a random ticket, individuals who chose their lottery ticket demonstrated a willingness to sell significantly higher than those who were randomly assigned a ticket, an effect due perhaps to the inclusion of the skill-associated task of selection in a situation of pure randomness and luck (Langer, 1975). These two behavioral aspects have essentially been unaddressed in the context of PLS to date, and given their purported existence specifically in the realm of lottos, for which there is anecdotal and empirical evidence that PLS serves as a substitute for, it would behoove researchers to look more closely at how these effects relate to PLS itself, in order to design PLS to be as effective as possible at increasing savings.

Experiment

Currently, standard PLS prize selection involves participants being entered into a drawing and the winner(s) being randomly selected from the pool of participants. There are numerous variations of this setup. One is that the number of entries (and hence the probability of winning) increases with every \$x deposited in the account. Another could be that just opening an account with an initial minimum deposit enters the account holder into the drawing. If there are y winners guaranteed, these structure types make the probability of winning depend on the total amounts deposited or the number of accounts created. Alternatively, winners could be determined similarly to how many lotto winners are determined: PLS participants receive certain sets of numbers, and if the numbers match the winning numbers drawn, the holder of those matching numbers wins. In this case, the probability of winning is quite independent of the number of participants. However, as the number of participants increases, there is an increased probability of multiple winners, meaning that the prize amount would be split among the winners.

The flagship pilot PLS program in the United States is the Save to Win program, launched in 2009 in Michigan (this is the same program that is studied in Cookson's forthcoming JFE article). The Save to Win program claims the majority of the PLS accounts and savings at depository institutions in the United States. As of early 2018, 112 credit unions in 11 states service over 21,000 PLS accounts with \$52 million in savings, and \$2.4 million in prizes awarded to date (Save to Win, 2018a).[5] As of the end of 2017, the overall PLS market spans 165 credit unions in 13 states that service 32,191 accounts with \$73 million in savings, and \$2.7 million in prizes awarded to date (Build Commonwealth, 2018). Save to Win PLS accounts are structured so that an initiation of an account is an entry into the drawing, and every \$25 deposit is an additional entry, up to ten entries per month, prizes awarded monthly and quarterly (Save to Win, 2018b). I had a correspondence with a customer service staff member of Save to Win, in which I asked if "Members in no way pick numbers or play a game for each 25 dollars they add, correct? Its just an additional entry into the contest?", to which she replied "That is correct".[6] Furthermore, I had a correspondence with Caezilia Loibl, one of the authors of the IDA empirical study. Regarding their PLS treatment, she confirmed that "We assigned each study participant a randomly drawn,

[5]Note that the amount of money deposited in accounts to date is likely much more than \$52 million, that is just the current amount of savings in accounts. This point is clear to see given that 2.4/52 yields an implied interest rate of 4.6%, way above any market interest rates on savings.

[6]Email correspondence with Jessica.Thelen@cusolutionsgroup.com on June 12 and 13, 2017.

2-digit number."[7] So, the conclusion is that standard PLS as it is currently designed includes neither the ability to choose winning numbers (as is common in lottos) nor a gaming aspect (as is common in gambling activities).

The main purpose of my experiment is to determine if including behaviorally-inspired modifications to PLS will increase demand for PLS, in terms of cannibalizing existing savings but also in creating new savings. I attempt to combine both the illusion of control and entertainment value in my modified PLS product. Experimentally, I propose a PLS product where participants are first told to choose a number within a set range, say between 1 and 1,000 (getting at the illusion of control). Once they choose a number, they are then told to spin a bingo cage in which there are 1,000 balls numbered between 1 and 1,000 (getting at entertainment value). If they pull the number they chose out of the bingo cage, they win the prize amount; otherwise, they keep their principal. Probabilistically speaking, there is no difference between this PLS product and a standard one where the probability of winning is set at 0.1%, and the drawing is done by some mechanism. However, if this standard PLS product can be viewed as providing no entertainment value and no illusion of control, while the modified PLS product has some entertainment value and may allow for the illusion of control, and if a participant places any positive value on either entertainment value or the illusion of control, a participant should strictly prefer the modified PLS over the standard PLS, ceteris paribus. A strict preference would need to translate into a significant increase in savings for this to have any practical value. If the modified PLS is shown to be more successful at creating new savings, this would be a major plus in the column of modified PLS. However, even if it is shown to generate an increased demand in PLS that is driven by a switchover from existing savings, this may be of interest to PLS providers if this is shown to happen even when the implied interest rate on the PLS instrument is less than the interest rate on the standard fixed return savings product.

The experimental design is a relatively short and simple one, in order to gauge if the modifications to PLS significantly impact savings more so than a standard PLS product. I run the experiment on Amazon MTurk, an online workplace where workers get paid fixed amounts for successfully performing tasks posted by requesters. Recently, many experiments and surveys have been run on MTurk, as the pay rates are quite cheap (often no more than the equivalent of a few dollars per hour's work), and they allow researchers to reach a somewhat diverse pool of subjects. One potential issue is that MTurk pays per task completed, so enacting a BDM mechanism or some other potential incentive compatible payment scheme is not possible. Results rely on the truth-telling of the subjects. While this at first glance seems quite sketchy, MTurk has a system

[7] Email correspondence with loibl.3@osu.edu on December 20, 2016.

set up so that a requester has the right to reject the work submitted, so workers are likely primed to do an honest job out of fear of rejection and loss of payment. MTurk also allows requesters to filter allowed workers based on their rejection rate, so being rejected not only affects the immediate payout of the task at hand, but can also exclude workers from many other profitable opportunities down the road. Furthermore, an MTurk replication of a prisoner's dilemma game, a priming game and a framing game all yielded comparable results to those firmly established within a lab setting (Horton, Rand and Zeckhauser, 2011). So, online workplaces such as MTurk are great sources of diverse research subjects at low costs for relatively simple experiments and surveys (for a detailed discussion on the pros and cons of the online workplace, see Horton, Rand and Zeckhauser, 2011).

The experiment consists of three questions posed to each subject, with the instructions that the subject choose the option most preferred in each question. I create three treatment groups, each treatment consisting of three questions. In each treatment, the three questions are exactly the same, only the options presented change. The questions are "Given that you have ($20, $100, $1,000) dollars, how would you prefer to spend those ($20, $100, $1,000) dollars?". Each question therefore represents a different reference wealth level. The first group, which can be viewed as the control group, sees a lotto, gambling and savings option in each question. The second group, which is the standard PLS group, sees a lotto, gambling, savings and standard PLS option in each question. The third group, which is the behavioral PLS group, sees a lotto, gambling, savings and behavioral PLS option in each question. The methodology then is to compare the proportions of respondents who select certain options both within and between groups.

Given the three treatment groups, each treatment was deployed on MTurk at least four hours apart, so as to reduce the number of participants who ended up in more than one group. Each group's maximum participation limit was set at 200, and each limit was reached in about an hour after deployment. MTurk provides the unique worker IDs with the respondent data, so I was able to remove participants who participated in more than one treatment (they were still paid for their work, as I made no specification of treatment groups in the instructions). This reduced the relevant sample size to 189 for the control group, 188 for the standard PLS group, and 184 for the behavioral PLS group. Each participant was paid ten cents for completing the three questions, the average response rate in each group was approximately 90 seconds, so the pay rate was about $4.50/hr. The exact layout of the experiment with the brief instructions is included in Appendix A. There is no lengthy exposition explaining PLS or a set of detailed instructions: the experiment was meant to be kept extremely simple. Examples of the exact wording of the five possible options (lotto, gambling, savings, standard PLS, behavioral PLS) in the treatments are listed below, recalling that the control group had three of the options and each treatment group had four of the options, and that these options correspond to the question where \$20 is the given wealth level:

1. Choosing a 4-digit number (any number between 1 and 10,000). If the number exactly matches the randomly drawn lucky number, you receive a \$99,980 return (\$100,000 in total); otherwise, you receive -\$20 return (\$0 in total).

2. Playing a game in which you spin a bingo cage, in which there are 100 balls numbered from 1-100. If you get a number between 1 and 40 you receive a \$20 return (\$40 in total); otherwise, you receive a -\$20 return (\$0 in total).

3. Receiving a \$1 return (\$21 in total) with certainty.

4. Letting a computer randomly pick a number from 1-1,000. If it picks the number 1, you receive a \$980 return (\$1,000 in total); otherwise, you receive no return (\$20 in total).

5. Playing a game in which you choose a number between 1 and 1,000, then spin a bingo cage, in which there are 1,000 balls numbered from 1-1,000. If you get the number you chose, you receive a \$980 return (\$1,000 in total); otherwise, you receive no return (\$20 in total).

As can be seen in Appendix A, as the wealth level changes, some of the payoffs and odds change, but within a wealth level across all treatments, options that are available over multiple treatments are the same. For instance, in the example above, the lotto, gambling and savings options for the \$20 wealth level are exactly as presented above in each treatment.

Hypotheses

Given the experimental design and results within the literature, there are several hypotheses that can be tested via experimentation. Under the assumption that there exist positive preferences in the population for either or both entertainment value and illusion of control, ceteris paribus, behavioral PLS will be a more appealing product than standard PLS. The general format of the hypotheses below is that having PLS as an option changes the distribution of preferences compared to not having PLS as an option, with behavioral PLS impacting the distributions more than standard PLS.

H1: There exists a demand for both standard PLS and behavioral PLS. This amounts to a positive number of participants who prefer the PLS option in each treatment group.

H1a: Additionally, given the added benefits of behavioral PLS, at each wealth level there will be a difference in the number of participants who choose PLS in the standard PLS treatment when compared with the number of participants who choose PLS in the behavioral PLS treatment.

H2: Both PLS types will serve as partial substitutes for both lottos and gambling, substitutability increasing with wealth.

H2a: Behavioral PLS will serve as a better substitute than standard PLS at each wealth level.

H3: Both PLS types will serve as partial substitutes for standard savings, substitutability decreasing with wealth.

H3a: Behavioral PLS will serve as a better substitute than standard PLS at each wealth level.

These hypotheses represent the combination of three ideas. First, PLS has a highly differentiated risk profile when compared with lottos, gambling or savings, but is more similar to lottos and gambling than savings is. It combines the risky upside of lottos and gambling with the loss protection of savings. This would suggest that introducing PLS may result in partial substitution away from each of these other activities. Second, behavioral PLS yields entertainment value and an illusion of control, so it should lead to even further substitution and appeal when compared with standard PLS. Third, the hypotheses also make predictions of how demand and substitutability will change as the initial wealth level changes. These predictions are primarily drawn from the reference-dependent loss aversion model of Tversky and Kahneman. The wealth level proposed in each of the three questions corresponds to a reference point. The relevant parts of the model include: diminishing sensitivity, or that the closer a fixed wealth change (loss or gain) is to the reference point, the larger the impact on utility; and loss aversion, or that losses loom larger than gains, for all $x > 0$, $|u(-x)| > u(x)$ (Tversky and Kahneman, 1991). Furthermore, if for all $x > 0$ $|u'(-x)| > u'(x)$, then for all $y > x$, $||u(-x)| - u(x)| < ||u(-y)| - u(y)|$. I also assume that the utility of entertainment value and the illusion of control are independent of the initial wealth level, as these are functions of the gaming structure. Therefore, for lotteries involving prospects of both losses and gains (in our case the lotto and gambling options), ceteris paribus, the preference for such lotteries decreases with the initial wealth level, which is also the principal that could be lost. This accounts for why the hypotheses predict substitutability from lottos and gambling to PLS increases with wealth, as these become less desirable as wealth increases. Additionally, when looking at lotteries involving prospects over gains (in our case savings), a decision maker who compares savings and PLS can take the savings option as a reference point, as it is an option that yields a fixed payment with certainty Equating the expected return of savings and PLS, a decision maker under reference dependent loss aversion will prefer PLS less as the amount of the return increases. Based on how I structured returns, the return amount on savings is increasing in wealth (although the percentage is decreasing), so I expect that substitution from savings to PLS will decrease with wealth.

Results

Table 1 provides the frequency table for the responses to the three questions in each of the three treatment groups. I will use a few naming conventions going forward: L for lotto, G for gamble, S for savings, SPLS for standard PLS, and BPLS for behavioral PLS. A few things are worth noting. First, in each of the nine questions, the frequency is such that L < G < PLS < S. This is reasonable, especially given the expected values on each of these products. For the \$20 question, E(L) = -50%, E(G) = -20%, E(SPLS) = E(BPLS) = 4.9%, E(S) = 5%. For the \$100 question, E(L) = -50%, E(G) = -20%, E(SPLS) = E(BPLS) = 1.99%, E(S) = 2%. For the \$1,000 question, E(L) = -50%, E(G) = -20%, E(SPLS) = E(BPLS) = 0.999%, E(S) = 1%. The returns of these products are selected to be typical of the expected returns of these products in the real world. For instance, expected returns over American lottos tends to be around -50% (Kearney, 2005), and as there are a large variety of gambling games, there is also a large variety of expected returns, but they are typically negative but better than those on lottos.[8] Second, as initial wealth increases, the frequency of L, G and BPLS are weakly decreasing in each treatment, while the frequency of S is increasing (the frequency of SPLS is non-monotone). Third, there is a higher frequency of BPLS when compared with SPLS at each wealth level. Also, there is a lower frequency of S at each wealth level in the SPLS treatment when compared with S in the BPLS treatment.

I will address each hypothesis one at a time. In order to test H1, I use a t-test to determine if the rates of SPLS and BPLS are significantly different from zero at each wealth level. The demand for both SPLS and BPLS at each wealth level are significantly different from zero at any reasonable significance level (each p-value < .000001).[9] This

[8]See https://wizardofodds.com/gambling/house-edge/ for typical expected returns/losses on a variety of casino games.

[9]I would typically use Pearson's Chi-Squared Test to determine if two samples are statistically different, but this test does not work well when some of the categories have very low frequencies. In this case, I am testing the observed PLS frequencies against a theoretical frequency of 0 PLS users, so

is unsurprising given the real-world demand for PLS and other experimental conclusions that PLS satisfies an existing demand. H1a is a more interesting hypothesis and one that is of clear importance to the premise of this paper. To determine if BPLS is utilized at a higher rate than SPLS at each wealth level, I use a Pearson's Chi-Squared Test, where the non-PLS options are all aggregated into one category. Interestingly, at each wealth level a p-value $< .01$ is generated, suggesting that behavioral modifications to PLS make it a more appealing tool, leading to an increased demand for BPLS when compared to SPLS. The issue of the source of the demand for PLS, along with the increased demand for BPLS, are addressed in the analysis of the remaining hypotheses.

Table 1

n=189	Lotto	Gamble	Savings	
$20	16.4%	28.0%	55.6%	
$100	10.1%	21.7%	68.3%	
$1,000	7.9%	14.8%	77.2%	
n=188	**Lotto**	**Gamble**	**Savings**	**Standard PLS**
$20	14.4%	22.3%	45.7%	17.6%
$100	9.6%	16.0%	52.7%	21.8%
$1,000	6.4%	10.1%	65.4%	18.1%
n=184	**Lotto**	**Gamble**	**Savings**	**Behavioral PLS**
$20	13.6%	19.6%	34.8%	32.1%
$100	9.8%	15.8%	44.0%	30.4%
$1,000	9.8%	11.4%	52.7%	26.1%

Considering substitutability of PLS for the other options, comparing the frequency distributions between each PLS treatment and the control treatment can identify the source(s) of the demand for PLS. To first test if PLS serves as a substitute for lottos, I aggregate all the non-lotto options into one category then use Pearson's Chi-Squared Test, comparing each PLS treatment to the control group. Of the six tests run (three wealth levels, 2 PLS treatments), not a single one yielded a remotely significant result, suggesting that neither SPLS nor BPLS serve as a partial substitute for lottos. Now consider gambling, and doing a similar aggregation and testing, a different result ensues. For SPLS, p-values of (.08, .06, .07) are yielded in increasing wealth order. For BPLS, p-values of (.01, .05, .19) are yielded in increasing wealth order. These values also suggest

I opt for a t-test here. Either test yields the same strong conclusion in this case.

that there is no clear relationship between the substitutability of PLS for gambling and the wealth level. Regarding H2a, the frequency and p-values suggest that BPLS was a better substitute for gambling than SPLS at the low wealth level, both were about the same at the intermediate wealth level, and SPLS was a better substitute at the higher wealth level. However, it is important to note that for SPLS at the high wealth level, the p-value of .07 would become insignificant at the 10% level had one more person selected gambling. Therefore, I am inclined to only conclude with confidence that BPLS at the low wealth level was a better substitute than SPLS. Also, since the expected return of PLS is decreasing with wealth, and while theoretically the gambling option should become less appealing with wealth, these two forces are working in opposite directions, making it hard to confidently assess the relationship between substitutability of PLS for gambling and wealth.

A similar approach to that taken above is used to see if PLS cannibalizes existing savings. Aggregating all non-savings into one category and using Pearson's Chi-Squared Test, a p-value $< .01$ is given for all six tests, providing evidence that PLS serves as a partial substitute for savings, suggesting validation of H3. Also, this evidence is even stronger given that the interest rates varied over wealth level (5% for \$20, 2% for \$100, 1% for \$1,000), suggesting this relationship holds over a range of interest rates. For the SPLS treatment, the differences in savings frequency between the control and SPLS groups in increasing wealth order are (9.9%, 15.6%, 11.8%). For the BPLS treatment, the differences in savings frequency between the control and BPLS groups in increasing wealth order are (20.8%, 24%, 24.5%). So, it looks like the conjecture that PLS substitutability for savings decreasing in wealth does not have strong evidence in support. It is also a possibility that the substitutability result has not been tested ideally, as neither the ratio nor the difference between the expected returns of savings and that of PLS is constant over the different wealth levels. Another similar experiment with return values chosen appropriately may allow for more careful analysis on how substitutability changes with wealth.

The most important conclusion of the experiment is reached via analysis of the data in addressing H3a. Testing H3a gives the reason for the overall increase in demand that BPLS has over SPLS: existing savings is the primary source for the added demand. Comparing the savings levels between the SPLS and BPLS treatments, aggregating all non-savings into a category and using the Pearson's Chi-Squared Test yields highly significant p-values in increasing wealth order of ($<.01$, .02, $<.01$). It seems that for a significant number of individuals, the entertainment and/or illusion of control value of BPLS is enough to compensate for the lower expected return that PLS has when compared to savings, providing experimental evidence that many individuals have a non-zero entertainment and/or illusion of control value.

Analysis

There are a few primary conclusions that the experimental results suggest. First, there exists a demand for PLS; however, this is not surprising given the existing success PLS products have had for literally centuries, making this result not controversial in any way. The more controversial issue is the source of that demand: is there significant amounts of new savings being created, or is the demand simply the result of the cannibalization of savings. My experiment suggests a combination of these two reasons. I don't find proof of PLS serving as a substitute for lottos, but it may serve as one for gambling, but most of the PLS demand seems to be sourced by existing savings. What does existing empirical and experimental evidence about PLS indicate about the source of PLS demand, and how (and even why) do they relate or differ from my experimental evidence?

A number of studies conclude that PLS does not merely cannibalize savings but creates new levels of savings. Atalay et al. (2012) and Filiz-Ozbay et al. (2015) provide experimental evidence of this, Cole et al. (2014) provide empirical evidence in their study of MoMa accounts in South Africa, and Cookson (forthcoming) provides empirical evidence in his study of Save to Win accounts in the Unites States. The source of this new savings is another important question, but one that is much harder to answer. Atalay et al. (2012) proceed to address this by providing experimental subjects the ability to split \$100 between a set of options: consumption, a lotto, PLS and standard savings. They conclude that most of the demand for PLS is sourced by substitution from lottos and then from consumption, with savings cannibalization contributing the least. I find quite the opposite, that savings cannibalization seems to be the primary source of the demand for PLS. Given my cited concerns with the expected returns on the assets in the Atalay et al. (2012) paper, I turn to Cookson (forthcoming) as a baseline to sort out the conflicting results.

Cookson (forthcoming) empirically analyzes the Save to Win Program PLS accounts at credit unions in Nebraska. Using cash withdrawal casino data, the author estimates that PLS introduction at the credit unions accounted for about a \$200,000 to \$400,000 substitution away from casino gambling, while the total amount of additional savings at the participating credit unions was about \$2 million. The author further analyzes data on lotto spending and gets a smaller but significant substitution effect away from lotto spending. Still, the majority of the increase in savings does not seem due to substitution away from lottos and gambling. In terms of the options available in my study, I basically have a gambling option where Atalay et al. (2012) have consumption, the other three options are the same, although their returns are starkly different. Cookson (forthcoming) uses credit card cash withdrawals that incur high cash advance fees and casino ATM requests of funds in excess of the account balance as a proxy for low self control. The author finds that the substitution effect is only large and significant for high self control individuals. This suggests that PLS introduction does not affect individuals uniformly; I believe this finding can provide insight into the conflicting results between my experiment and that of Atalay et al. (2012). It may be the case that many individuals who would be classified as high self control gamblers opted into the gambling option in my study and not the lotto option, and my results show that there was some evidence of substitution for gambling. Perhaps if Atalay et al. (2012) included a gambling option in their study, a similar sorting would have happened and the substitution witnessed would have happened from the gambling category instead of the lotto category. In light of the Cookson (forthcoming) study, my results can be placed appropriately within their overall findings. Given that I find no substitution effect from lottos, I am inclined to think that those who sorted into lottos may be low self control individuals or have a very strong preference for extreme positive skewness that PLS can't offer. Regardless, the Cookson (forthcoming) result that PLS serves heterogeneously across individuals as a substitute for lottos and gambling is quite strong, and the first glance conclusion from my experimental result of PLS not serving as a substitute for lottos needs to be taken within the context of all the existing evidence.

My major finding of BPLS cannibalizing savings much more than SPLS can be explained by looking at utility and risk preferences. Individuals who preferred savings over PLS are consistent with risk neutral and risk averse agents maximizing expected utility. In the case of BPLS, a much higher percentage of individuals preferred PLS than in the case of SPLS. Assuming a fixed utility value for entertainment and illusion of control, independent of wealth levels, expected returns, etc., the added utility that BPLS provides over SPLS seems to be enough to encourage a significant additional switchover from savings. A risk neutral or slightly risk averse agent who is a standard expected utility maximizer but also places some level of utility on entertainment and/or illusion of control strictly prefers BPLS over SPLS. For such an individual considering savings vs BPLS, if the added utility from entertainment and/or illusion of control is enough to compensate for the lower expected return and higher risk of BPLS, this individual will prefer BPLS. No such condition applies for this individual when considering savings vs SPLS, as there is no entertainment and/or illusion of control.

Given the limited scale of the experiment, as each individual only saw three questions, there is most certainly room for extension, improvement and robustness checks.

For instance, lottery and gambling uncertainty realization and payment happens immediately those products are purchased, while for savings there is no uncertainty but payment happens in the future, and for PLS uncertainty realization and payment both happen in the future. Framing the experiment within a time context may lead to relatively less demand for both savings and PLS when compared to the timeless version, given future discounting. There may also be room for estimating the value of entertainment and the illusion of control by asking more questions that vary the expected returns on PLS relative to the other assets. I also do not disentangle the impact of entertainment value and that of the illusion of control, as BPLS incorporates both. I implicitly assumes that these two effects do not cancel out each other, which is an assumption that could be tested in future research. However, given the strong evidence of the initial experiment, coupled with the corroborating conclusion of Cookson (forthcoming) that decreased differentiation between PLS and lottos/gambling increases substitutability of PLS, there is little reason to doubt that making behaviorally-inspired modifications to PLS will increase demand in PLS relative to PLS without such changes. Filiz-Ozbay et al. (2015) find that PLS requires lower returns than standard savings to induce savings. If so, even if this increase in demand is sourced solely from existing savings, this can allow providers of savings products to accrue savings at lower costs. For instance, banks can offer PLS at a slightly lower expected return than their standard savings account. If this only amounts to some individuals switching from savings to PLS, and the maintenance costs of PLS accounts are equal to those of standard savings, this will amount to cost savings by the banks. Therefore, regardless of the source of PLS demand, banks and similar institutions should be eager to provide PLS products as a lower cost source of funds. If they offer BPLS products, my experimental results suggest that it may result in additional switchover from savings and further cost savings when compared to offering SPLS products.

Conclusion

PLS is a unique savings product that combines the principal guarantee of standard savings with the upside risk of gambling and lottos. It would therefore seem to be a substitute for both the riskless savings and the risky gambling and lottos. The experimental and empirical evidence validates this supposition. Assuming that substitutability increases the less differentiated two products are, making PLS more similar to gambling and lottos would increase the substitution effect from gambling and lottos to PLS. Furthermore, if gambling and lottos provide an inherent gaming or entertainment value that in independent of their payoff distribution, along with an illusion of control value, capturing these effects in PLS will only make PLS more appealing. Currently, PLS products do not systematically seek to incorporate gaming aspects or illusion of control facets; rather they focus on the uniqueness of the payoff distribution when compared to existing savings products. It would behoove proponents and providers of PLS to gamify PLS: this could be by allowing participants to choose winning numbers, play some game to determine if they are a winner or advance into the next round of winner determination, etc. These could be done on site of the institutions that provide PLS, or this could be done remotely via online platforms. Many banks and savings institutions have existing online platforms for their customers to do their banking business. Banks and the like could incur the fixed costs of setting up an electronic PLS winner determination system within their existing platforms and reap the rewards of increased sources of funds at lower costs to them.

Appendix A

Instructions for All Treatments

Survey Instructions (Click to collapse)

Enter instructions for your survey:

- I am interested in the types of savings tools that people prefer.
- I ask three questions with a number of options, please choose the one you prefer the most in each question.

Savings Treatment

1. Given that you have $20 dollars, how would you prefer to spend those $20 dollars?

- ◯ Playing a game in which you spin a bingo cage, in which there are 100 balls numbered from 1-100. If you get a number between 1 and 40 you receive a $20 return ($40 in total); otherwise, you receive a -$20 return ($0 in total)
- ◯ Receiving a $1 return ($21 in total) with certainty
- ◯ Choosing a 4-digit number (any number between 1 and 10,000). If the number exactly matches the randomly drawn lucky number, you receive a $99,980 return ($100,000 in total); otherwise, you receive -$20 return ($0 in total)

2. Given that you have $100 dollars, how would you prefer to spend those $100 dollars?

- ◯ Playing a game in which you spin a bingo cage, in which there are 100 balls numbered from 1-100. If you get a number between 1 and 40 you receive a $100 return ($200 in total); otherwise, you receive a -$100 return ($0 in total)
- ◯ Receiving a $2 return ($102 in total) with certainty
- ◯ Choosing a 5-digit number (any number between 1 and 100,000). If the number exactly matches the randomly drawn lucky number, you receive a $4,999,900 return ($5,000,000 in total); otherwise, you receive -$100 return ($0 in total)

3. Given that you have $1,000 dollars, how would you prefer to spend those $1,000 dollars?

- ◯ Playing a game in which you spin a bingo cage, in which there are 100 balls numbered from 1-100. If you get a number between 1 and 40 you receive a $1,000 return ($2,000 in total); otherwise, you receive a -$1,000 return ($0 in total)
- ◯ Receiving a $10 return ($1,010 in total) with certainty
- ◯ Choosing a 6-digit number (any number between 1 and 1,000,000). If the number exactly matches the randomly drawn lucky number, you receive a $499,999,000 return ($500,000,000 in total); otherwise, you receive -$1,000 return ($0 in total)

Submit

SPLS Treatment

1. Given that you have $20 dollars, how would you prefer to spend those $20 dollars?

- ◯ Playing a game in which you spin a bingo cage, in which there are 100 balls numbered from 1-100. If you get a number between 1 and 40 you receive a $20 return ($40 in total); otherwise, you receive a -$20 return ($0 in total)
- ◯ Letting a computer randomly pick a number from 1-1,000. If it picks the number 1, you receive a $980 return ($1,000 in total); otherwise, you receive no return ($20 in total)
- ◯ Receiving a $1 return ($21 in total) with certainty
- ◯ Choosing a 4-digit number (any number between 1 and 10,000). If the number exactly matches the randomly drawn lucky number, you receive a $99,980 return ($100,000 in total); otherwise, you receive -$20 return ($0 in total)

2. Given that you have $100 dollars, how would you prefer to spend those $100 dollars?

- ◯ Playing a game in which you spin a bingo cage, in which there are 100 balls numbered from 1-100. If you get a number between 1 and 40 you receive a $100 return ($200 in total); otherwise, you receive a -$100 return ($0 in total)
- ◯ Letting a computer randomly pick a number from 1-10,000. If it picks the number 1, you receive a $19,900 return ($20,000 in total); otherwise, you receive no return ($100 in total)
- ◯ Receiving a $2 return ($102 in total) with certainty
- ◯ Choosing a 5-digit number (any number between 1 and 100,000). If the number exactly matches the randomly drawn lucky number, you receive a $4,999,900 return ($5,000,000 in total); otherwise, you receive -$100 return ($0 in total)

3. Given that you have $1,000 dollars, how would you prefer to spend those $1,000 dollars?

- ◯ Playing a game in which you spin a bingo cage, in which there are 100 balls numbered from 1-100. If you get a number between 1 and 40 you receive a $1,000 return ($2,000 in total); otherwise, you receive a -$1,000 return ($0 in total)
- ◯ Letting a computer randomly pick a number from 1-100,000. If it picks the number 1, you receive a $999,000 return ($1,000,000 in total); otherwise, you receive no return ($1,000 in total)
- ◯ Receiving a $10 return ($1,010 in total) with certainty
- ◯ Choosing a 6-digit number (any number between 1 and 1,000,000). If the number exactly matches the randomly drawn lucky number, you receive a $499,999,000 return ($500,000,000 in total); otherwise, you receive -$1,000 return ($0 in total)

Submit

BPLS Treatment

1. Given that you have $20 dollars, how would you prefer to spend those $20 dollars?

- ◯ Playing a game in which you spin a bingo cage, in which there are 100 balls numbered from 1-100. If you get a number between 1 and 40 you receive a $20 return ($40 in total); otherwise, you receive a -$20 return ($0 in total)
- ◯ Playing a game in which you choose a number between 1 and 1,000, then spin a bingo cage, in which there are 1,000 balls numbered from 1-1,000. If you get the number you chose, you receive a $980 return ($1,000 in total); otherwise, you receive no return ($20 in total)
- ◯ Receiving a $1 return ($21 in total) with certainty
- ◯ Choosing a 4-digit number (any number between 1 and 10,000). If the number exactly matches the randomly drawn lucky number, you receive a $99,980 return ($100,000 in total); otherwise, you receive -$20 return ($0 in total)

2. Given that you have $100 dollars, how would you prefer to spend those $100 dollars?

- ◯ Playing a game in which you spin a bingo cage, in which there are 100 balls numbered from 1-100. If you get a number between 1 and 40 you receive a $100 return ($200 in total); otherwise, you receive a -$100 return ($0 in total)
- ◯ Playing a game in which you choose a number between 1 and 10,000, then spin a bingo cage, in which there are 10,000 balls numbered from 1-10,000. If you get the number you chose, you receive a $19,900 return ($20,000 in total); otherwise, you receive no return ($100 in total)
- ◯ Receiving a $2 return ($102 in total) with certainty
- ◯ Choosing a 5-digit number (any number between 1 and 100,000). If the number exactly matches the randomly drawn lucky number, you receive a $4,999,900 return ($5,000,000 in total); otherwise, you receive -$100 return ($0 in total)

3. Given that you have $1,000 dollars, how would you prefer to spend those $1,000 dollars?

- ◯ Playing a game in which you spin a bingo cage, in which there are 100 balls numbered from 1-100. If you get a number between 1 and 40 you receive a $1,000 return ($2,000 in total); otherwise, you receive a -$1,000 return ($0 in total)
- ◯ Playing a game in which you choose a number between 1 and 100,000, then spin a bingo cage, in which there are 100,000 balls numbered from 1-100,000. If you get the number you chose, you receive a $999,000 return ($1,000,000 in total); otherwise, you receive no return ($1,000 in total)
- ◯ Receiving a $10 return ($1,010 in total) with certainty
- ◯ Choosing a 6-digit number (any number between 1 and 1,000,000). If the number exactly matches the randomly drawn lucky number, you receive a $499,999,000 return ($500,000,000 in total); otherwise, you receive -$1,000 return ($0 in total)

Submit

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