UDC 621.396.96
DOI: 10.15587/1729-4061.2021.238155

DEVELOPMENT OF A METHOD FOR ESTIMATING THE EFFECT OF TRANSFORMATION OF THE NORMALIZED FREQUENCY MISMATCH FUNCTION OF A COHERENT BUNDLE OF RADIO PULSES ON THE QUALITY OF RADAR FREQUENCY RESOLUTION

Serhii Yevseiev
Corresponding author
Doctor of Technical Science, Professor*
E-mail: serhii.yevseiev@hneu.net
Oleksandr Kuznietsov
PhD, Associate Professor**
Oksana Biesova
PhD, Senior Researcher
Air Force Science Centre***
Dmytro Kyrychenko
Department of Combat Application of Anti-Aircraft Missile Systems***
Olena Lukashuk
PhD**
Stanislav Milevskyi
PhD, Associate Professor*
Serhii Pohasii
PhD, Associate Professor*
Iryna Husarova
PhD, Associate Professor
Department of Applied Mathematics
Kharkiv National University of Radio Electronics
Nauky ave., 14, Kharkiv, Ukraine, 61166
Anna Goloskokova
PhD, Associate Professor
Department of Software Engineering and Management Information Technologies
National Technical University «Kharkiv Polytechnic Institute»
Kyrpychova str., 2, Kharkiv, Ukraine, 61002
Volodymyr Sobchenko
PhD
National Academy of the State Border Guard Service of Ukraine
Named after Bohdan Khmelnitsky
Shevchenka str., 46, Khmelnytsky, Ukraine, 29003
*Department of Cybersecurity and Information Technology
Simon Kuznets Kharkiv National University of Economics
Nauky ave., 9-A, Kharkiv, Ukraine, 61166
Department of Physics and Radioelectronics*
***Ivan Kozhedub Kharkiv National Air Force University
Sumska str., 77/79, Kharkiv, Ukraine, 61023

The necessity of studying the influence of the transformation of the frequency mismatch function of a coherent bundle of radio pulses on the quality of solving the radar frequency resolution problem is substantiated. This solution determines the effectiveness of radar observation of high-speed and maneuvering individual and group aerodynamic objects. The method is based on explicit expressions for calculating the normalized frequency mismatch function of a coherent bundle of radio pulses, taking into account its transformation due to the radial motion of high-speed and maneuvering individual and group aerodynamic objects. The estimation of the potential frequency resolution of bundles with different numbers of radio pulses with typical parameters for a coherent pulse radar is carried out. Possible values of frequency resolution under the additive effect of uncorrelated internal noise of the radar receiver and the multiplicative effect of correlated phase fluctuations of the radar signal are estimated. With an insignificant multiplicative effect of correlated phase fluctuations, a twofold increase in the number of radio pulses in a bundle provides an improvement in the frequency resolution (reduction of the width of the normalized frequency mismatch function) by 100 %. With the predominant multiplicative effect of these fluctuations, a twofold increase in the number of radio pulses results in an improvement in the frequency resolution by about 40 %. The developed method is of great theoretical and practical importance for the further development of the radar theory of high-speed and maneuvering individual and group aerodynamic objects

Keywords: coherent bundle of radio pulses, resolution, mismatch function, phase fluctuations

Received date 18.06.2021
Accepted date 10.08.2021
Published date 30.08.2021

How to Cite: *Yevseiev, S., Kuznietsov, O., Biesova, O., Kyrychenko, D., Lukashuk, O., Milevskyi, S., Pohasii, S., Husarova, I., Goloskokova, A., Sobchenko, V. (2021). Development of a method for estimating the effect of transformation of the normalized frequency mismatch function of a coherent bundle of radio pulses on the quality of radar frequency resolution. Eastern-European Journal of Enterprise Technologies, 4 (4 (112)), 13–22. doi: https://doi.org/10.15587/1729-4061.2021.238155*

1. Introduction

One of the main requirements for modern radar systems is to ensure high-quality resolution of radar signals for independent radar observation of aerodynamic objects. Resolution of radar signals enables separate detection and measurement of informative parameters of signals reflected from aerodynamic objects with close coordinates and motion parameters.

In the traditional sense, the task of radar detection is to decide on the presence or absence of the reflected signal in the adopted implementation. However, this approach is quite simplified.

In real radar observation, an object is considered detected by the presence of a series of reflected signals, i. e. by the result of processing a bundle of radio pulses in the radar. The necessary condition is to take into account changes in the coordinates and parameters of the object during its detection. That is, the object coordinates must be taken into account when deciding on detection. This procedure is «detection-measurement», and if there are several objects with similar parameters – «detection-measurement-resolution».

If a sufficiently reliable resolution by the measured parameter is ensured, further detection-measurement can be performed separately for each radar object with the required quality indicators.

The ability to separate signals with similar parameters is determined by the sensitivity of the output effect of the signal processing device to the deviation of its parameters from the expected values. This sensitivity is characterized by mismatch functions that determine the dependence of the result of consistent processing of a coherent signal on the magnitude of mismatch between its expected and actual parameters.

Modern aerodynamic objects are capable of changing radial velocities over a wide range. Also, the ability of aerodynamic objects to perform sudden and complex maneuvers determines their movement with radial accelerations and the need to use higher range derivatives to characterize their motion. This significantly complicates the problem of separating aerodynamic objects by radial velocity.

A coherent bundle of pulses is widely used in coherent pulse radars to ensure high radial velocity resolution. Coherent pulse radars, which detect and track aerodynamic objects, typically have a centimeter wavelength range (3 cm to 10 cm). Since the resolution of aerodynamic objects by radial velocity is based on the Doppler frequency resolution of radar signals, the frequency mismatch function of a coherent bundle of radio pulses is subject to analysis.

The mismatch function of radar signals characterizes the quality of their resolution provided that there is no influence of interfering factors on the process of coordinated signal processing, which can lead to significant distortions of this function. Such factors are the additive effect of uncorrelated internal noise, which is always present in the receiver, and the multiplicative effect of correlated phase fluctuations caused by real conditions of propagation and reflection of the radar signal. In addition, the known expressions of the frequency mismatch function of a coherent bundle of radio pulses are inconvenient for practical calculations and do not take into account the influence of the radial motion of the radar object on the transformation of the mismatch function.

Therefore, it is necessary to obtain a frequency mismatch function of a coherent bundle of radio pulses in a user-friendly form, taking into account the possible transformation due to the radial motion of the radar object. This is an urgent problem for radar theory. Assessing the impact of factors, caused by the additive and multiplicative effect of the real conditions of performing tasks as intended by the coherent pulse radar, on the quality of signal frequency resolution is an important practical radar problem.

2. Literature review and problem statement

A significant number of works on radar theory are devoted to radar resolution of aerodynamic objects and noise effect on radar signal processing.

Thus, [1] considers the characteristics of radio-electronic systems that solve the problem of resolution of aerodynamic and ballistic radar objects. This paper presents mathematical expressions for the frequency mismatch function of the main types of radar signals, in particular, a coherent bundle of radio pulses. However, the proposed expressions have a general form and are inconvenient for the practical calculation of the frequency mismatch function. In addition, they do not take into account the transformation of the time scale of the radar signal due to the radial motion of the radar object. So, the disadvantage of this work is the lack of relationships that take into account the specified transformation of the time scale and provide a separate analysis of this frequency mismatch function within the main and side maxima.

Radar signal processing is usually considered in the presence of the internal noise of the receiver, which has an additive nature. The principles of construction of appropriate processing devices for modern radars are given in [2]. The mathematical apparatus that determines the features of radar signal processing taking into account only additive noise is considered in [3]. Peculiarities of coordinated radar signal processing during its multi-channel reception are considered in [4]. The feature of these works [2–4] is the theoretical results on the synthesis of devices for processing signals of modern radars. However, the works [2–4] do not take into account real environmental conditions. In real conditions, the radar performs tasks as intended under the multiplicative effect of the radio wave propagation medium. This causes correlated phase fluctuations of the radar signal, which can reduce the quality of frequency resolution of radar signals. In [5], the issue of statistical analysis of radar signal parameters in relation to the tasks of checking the quality of frequency resolution and monitoring the serviceability of radio devices is considered. However, the obtained results do not apply to the problem of radar resolution of aerodynamic objects by radial velocity.

Atmospheric inhomogeneities cause fluctuations in its refractive index, which in turn causes phase distortions of the radar signal and significantly complicates radar operation. Thus, the work [6] is devoted to problematic issues related to the use of modern radio systems under the influence of a perturbed atmosphere. This effect is enhanced in the presence of wind activity according to [7]. The results of [6, 7] should be extended to the features of solving the problem of radar signal resolution in these conditions. The corresponding mathematical apparatus, which is typical for solving this problem, is given in [8]. The disadvantage of [6–8] is the lack of consideration of the conditions of the multiplicative effect of the radio wave propagation medium, which causes correlated phase fluctuations of the radar signal.

The task of radar resolution becomes especially relevant for radars that monitor nearly invisible aircrafts capable of operating in a group. In this case, the substantiation of radio wave characteristics was carried out in [9] and the results need to be taken into account when conducting research on the resolution of aerodynamic objects that is missing.

In [10], relations were obtained that take into account the transformation of spectral and temporal representations of single and bundle radio signals reflected from an accelerating object. The practical application of this transformation in assessing the quality of detection of moving objects is considered in [11], which takes into account the differences in changes in the time scale of the carrier oscillations and their complex envelope. The results of these works should also be used to solve the frequency resolution problem. However, these expressions refer to signal transformation in an ionized radio wave propagation medium and can be used only for the means of locating space objects. In addition, this paper does not take into account the influence of fluctuations on the transformation of the phase structure of the bundle radio signal.

The influence of phase fluctuations caused by atmospheric instability is considered in the study of modern signal processing methods [12] and features of its implementation in Doppler measurement systems [13]. In this case, a necessary condition is to determine the appropriate assumptions about the statistical characteristics of correlated phase fluctuations. These assumptions about the normal distribution law and the oscillating form of the correlation function of phase fluctuations with their experimental confirmation are given in [14]. The disadvantage of [12–14] is the lack of consideration of the complexity of the shape of the aerodynamic object, which is quite relevant in the detection of nearly invisible highly maneuverable objects.

Numerical analysis of the measurement accuracy of the radial velocity of an aerodynamic object for the cases of exponential and alternating correlation laws of phase fluctuations of radio pulses of the received bundle was carried out in [15]. The work [16] considers the combined effect of the internal noise of the receiving device and correlated phase fluctuations of radio pulses of the received bundle with the oscillating correlation function. In addition, the corresponding values of the root mean square errors of Doppler frequency measurement are obtained. It is shown that for modern coherent pulse radars, these errors for bundles of (8...16) radio pulses in the troposphere can be (67.1...95.5) Hz, and in the ionosphere – (7.8...11.3) Hz. The disadvantage of [15, 16] is the use of relations that are inconvenient for practical calculations and do not take into account the effect of radial motion of the radar object on the transformation of the mismatch function. However, the obtained results are the starting point for further calculation of the fluctuation components of the frequency resolution of the coherent bundle of radio pulses.

In addition to this factor, the coherence of the received radio signal may be affected by the complexity of the shape of the aerodynamic object and noise of direct and reflected radio waves. This leads to Doppler noise. These phenomena cause additional phase fluctuations of the radio pulses of the received bundle and the corresponding additional transformation of its mismatch function. However, these phenomena have a less regular and stable effect on radar signal resolution than the constant impact of the radio wave propagation medium.

Thus, the issues of taking into account the transformation of the frequency mismatch function due to the radial motion of the radar object were not considered. The known expressions for the frequency mismatch function have a general form and are inconvenient for practical calculations. In addition, these works do not assess the impact of statistical characteristics of phase fluctuations, introduced by the radio wave propagation medium, on the radar frequency resolution.

Therefore, it is necessary to define practically convenient expressions for calculating the normalized frequency mismatch function of a coherent bundle of radio pulses, taking into account its transformation due to the radial motion of the object. It is necessary to study the effect of phase fluctuations of the radio pulses of the bundle on the frequency resolution. It is of practical use to estimate the frequency resolution in possible ranges of statistical characteristics of phase fluctuations of the radar signal, according to which their impact on the operation of coherent pulse radars is unacceptable.

3. The aim and objectives of the study

The aim of the study is to develop a method for determining the normalized frequency mismatch function of a coherent bundle of radio pulses, taking into account the transformation of its time scale due to the radial motion of the radar object. This will allow evaluating the combined additive effect of uncorrelated internal noise of the receiving device and the multiplicative effect of correlated phase fluctuations of the bundle radio pulses on the decrease in frequency resolution.

To achieve the aim, the following objectives were set:

– to estimate the decrease in resolution for the radial motion of the aerodynamic object;

– to evaluate the decrease in frequency resolution.

4. Materials and methods of research

The problem of Doppler frequency resolution of radar signals using a coherent bundle of radio pulses, which is a sensing signal of coherent pulse radars, is considered. The frequency of repetition of bundle radio pulses in coherent pulse surveillance radars can vary from 700 Hz to 1,700 Hz. A feature of coherent pulse tracking radars is the presence of auto-tracking systems in them, in particular, for range and radial velocity. Such radars use quasi-continuous sounding signals with a repetition rate of 20 kHz to 100 kHz.

Regarding the causes of phase fluctuations, the main one is the radio wave propagation medium, the influence of which takes place in any radar operating conditions. During the radar signal propagation in the turbulent troposphere, there are significant fluctuations in its refractive index, which causes phase fluctuations. The corresponding values of the variance of the signal phase fluctuations for the radar wavelengths from 3 cm to 1 m can be from 0.0044 rad2 to 19.4 rad2. That is, for centimeter-range waves, the specified variance can reach values up to ten or more radians squared. The correlation interval of phase fluctuations can be $\tau=(0.1...1)$ s.

To analyze the influence of statistical characteristics of phase fluctuations on the decrease in frequency resolution, the method of calculating the root mean square error of measuring the frequency of a bundle in the presence of phase fluctuations of its radio pulses was used. Taking into account this method, the degree of expansion of the main maximum of the normalized frequency mismatch function is determined and the decrease in frequency resolution is estimated. The analysis of this multiplicative effect is carried out depending on possible values of statistical characteristics of phase fluctuations.

The method for determining the angular resolution by the presence of a combined effect of random fluctuations of the received wavefront (multiplicative noise) and additive noise oscillations was used to estimate the frequency resolution.

To evaluate the results obtained in the study, computer simulation of the proposed expressions of the developed method was performed. The simulation was performed using the Mathcad application package. Typical parameters of coherent pulse radars were used in the simulation (relevant data are given when obtaining the simulation results).

5. Results of the development of a method for determining the normalized frequency mismatch function of a coherent bundle of radio pulses

5. 1. Estimation of resolution reduction for the case of radial motion of an aerodynamic object

The disadvantages of known methods (expressions) for determining the frequency mismatch function of a coherent bundle of radio pulses are given in Table 1.

To study the effect of phase fluctuations of the bundle radio pulses on the frequency resolution, the motion of an aerodynamic object is considered.

The motion of an aerodynamic object requires taking into account changes in all signal spectrum frequencies. The transformation of the signal spectrum frequency, including the carrier frequency, can be determined by the following ratio:

$$f' = \eta f, \tag{1}$$

where f' – frequency of the received signal; f – frequencies of the emitted signal.

The parameter η determines the transformation of the scale of the radar signal, which occurs due to the radial motion of the aerodynamic object and is determined by the expression [1]:

$$\eta = \frac{1 - V_r/c}{1 + V_r/c}, \tag{2}$$

where V_r – radial velocity of the object; c – propagation velocity of the electromagnetic wave.

Thus, the parameter η determines changes in the time scale or, what is the same, any of the spectrum frequencies f of the emitted signal during the radial motion of the aerodynamic object.

Taking into account the relationship of the radial velocity of the radar object with the Doppler shift of the frequency of the received signal [1, 10, 11, 17], the transformation parameter (2) can be determined as follows:

$$\eta = \frac{f_0 - F_d}{f_0}, \tag{3}$$

where f_0 – signal carrier frequency; F_d – Doppler frequency shift f_0.

In accordance with this change in the spectrum frequencies, all time parameters of the signal change: oscillation periods, signal duration, time intervals, modulation law. In addition, there are changes in the signal amplitude, which are associated with the preservation of its energy during stretching or compression and loss (receipt) of energy during reflection from the radar object.

Taking into account these factors leads to the expression for the instantaneous value of the reflected signal:

$$u'(t) = \eta u(\eta t), \tag{4}$$

where $u'(t)$ and $u(t)$ – received and emitted signals, respectively.

Thus, in the complex form, expression (4) can be defined by the relation:

$$u'(t) = \eta \operatorname{Re}\left[\dot{U}(\eta t) e^{j2\pi f_0 \eta t}\right]. \tag{5}$$

Numerical estimates indicate that changes in signal amplitude and energy due to the object motion can almost always be ignored.

Time scale changes can be neglected for a signal with a small bandwidth-duration product of the modulation law $W\tau_s \approx 1$ (W – width of the signal frequency spectrum, and τ_s – signal duration).

For signals with a large bandwidth-duration product $W\tau_s \gg 1$, the influence of changes in the time scale of the modulation law due to the radial motion of the object is significant.

Radio pulse bundles, both incoherent and coherent, are common types of radar signals for which changes in the time scale of the modulation law can significantly affect the appearance of the frequency mismatch function. This function determines the Doppler frequency resolution and measurement accuracy of this parameter.

When considering expressions for mismatch functions, changes in the amplitude and duration of a single pulse can be neglected due to the Doppler effect. However, it is necessary to take into account much more significant changes in the time intervals between pulses, which are associated with changes in the time scale during the radial motion of the aerodynamic object.

Table 1

Disadvantages of known methods (expressions) for determining the frequency mismatch function of a coherent bundle of radio pulses

Method for determining the frequency mismatch function of a coherent bundle of radio pulses	Disadvantages of the method
Calculation of the radar signal mismatch function provided there is no influence of interfering factors on coordinated signal processing	Expressions of the frequency mismatch function of a coherent bundle of radio pulses are inconvenient for practical calculations and do not take into account the influence of the radial motion of the radar object on the transformation of the mismatch function
Mathematical relations for estimating the frequency mismatch function of the main types of radar signals, in particular, a coherent bundle of radio pulses	Expressions have a general form and are inconvenient for the practical calculation of the frequency mismatch function. They do not take into account the transformation of the time scale of the radar signal due to the radial motion of the radar object
Radar signal processing is considered in the presence of additive effects of receiver internal noise	In real conditions, the radar performs tasks as intended under the multiplicative effect of the radio wave propagation medium, which causes correlated phase fluctuations of the radar signal. This reduces the quality of frequency resolution of radar signals
Relationships that take into account the transformation of spectral and temporal representations of single and bundle radio signals reflected from an accelerating object	Expressions refer to signal transformation in the ionized radio wave propagation medium and are of practical use only for the means of locating space objects
Method for determining the combined effect of internal noise of the receiving device and correlated phase fluctuations of the received radio pulses with the oscillating correlation function	These phenomena have a less regular and stable effect on the resolution of radar signals than the constant impact of the radio wave propagation medium. The coherence of the received radio signal is also affected by the complexity of the shape of the aerodynamic object

Taking into account the transformation factor, the complex correlation integral [1, 15, 16, 18] for i-th pulse of the bundle has the following form:

$$\dot{Z}_i(\hat{\eta},\eta)= =\frac{1}{2}\int_{-\infty}^{\infty}\dot{U}_i(t-\hat{\eta}t_i)\dot{U}_i^*(t-\eta t_i)\exp(j2\pi f_0\Delta\eta t)\mathrm{d}t, \qquad (6)$$

where $\dot{U}_i(t)$ – complex voltage envelope of the i-th pulse of the bundle; $\hat{\eta}$ – estimate of the value η; $\Delta\eta=\eta'-\eta=(F_d-\dot{F}_d)/f_0=F/f_0$ – mismatch by the transformation parameter of the expected and received signals; $F=F_d-\dot{F}_d$ – mismatch by the Doppler shift of the frequency of the expected and received signals; t_i – center shift of the i-th pulse relative to the start of the countdown.

For pulses with a rectangular envelope and amplitude U, expression (6) takes the following form:

$$\dot{Z}_i=\frac{U^2}{2}\int_a^b\exp(j2\pi Ft)\mathrm{d}t. \qquad (7)$$

All possible options for the limits a and b for $t_i<0$, $t_i>0$, $\Delta\eta<0$ and $\Delta\eta>0$ are presented in Fig. 1. They are determined by the time limits of the shaded areas for which the product of the expression factors under the integral of expression (6) differs from zero.

For all options, the integration result can be written as a single expression, which is a normalized mismatch function for the i-th pulse:

$$\frac{\dot{Z}_i(\hat{\eta},\eta)}{E_0}=\dot{\rho}_i(F)= =\begin{cases}\dfrac{\sin\left[\pi F\tau_i\left(1-|Ft_i|/f_0\tau_i\right)\right]}{\pi F\tau_i}\exp\left[j\pi F\tau_i(\hat{\eta}+\eta)\right] \\ \text{for } |Ft_i|\le f_0\tau_i, \\ 0 \ \text{ for } |Ft_i|>f_0\tau_i,\end{cases} \qquad (8)$$

where $E_0=U^2\tau_i/2$ – energy of a single pulse; τ_i – pulse duration.

Taking into account these assumptions, the complex mismatch function for the i-th pulse of the bundle has the following form:

$$\dot{\psi}_i(\hat{\eta},\hat{\eta})=\int_{-\infty}^{\infty}\dot{U}_i(t-\hat{\eta}t_i)\dot{U}_i^*(t-\eta t_i)\exp(j2\pi f_0\Delta\eta t)\mathrm{d}t. \qquad (9)$$

For a rectangular radio pulse, the relation (9) is as follows:

$$\dot{\psi}_i=U^2\int_a^b\exp(j2\pi Ft)\mathrm{d}t= =\frac{U^2}{j2\pi F}\left(e^{j2\pi Fb}-e^{j2\pi Fa}\right)=\frac{U^2\tau_i}{2}\frac{\sin\pi F(b-a)}{\pi F\tau_i}e^{j\pi F(b+a)}. \qquad (10)$$

For arbitrary values of t_i and $\Delta\eta$, the difference $b-a=\tau_i-|\Delta\eta t_i|$, sum $b+a=(\eta+\eta')t_i$, and formula (10) takes the following form:

$$\dot{\psi}_i=\frac{U^2\tau_i}{2}\frac{\sin\pi F(\tau_i-|\Delta\eta t_i|)}{\pi F\tau_i}e^{j\pi F(\eta+\hat{\eta})t_i}. \qquad (11)$$

Below we consider the mismatch function for a coherent bundle with an even number of rectangular pulses of duration τ_i with a rectangular bundle envelope and countdown $|t_i|=(2i-1)T/2$ from the center of the bundle (T – pulse period). For approximate numerical estimates, we can assume $(\eta+\eta')/2\approx 1$.

The result of combining two mismatch functions, symmetric relative to the start of the countdown after normalization and substitution $\eta+\eta'\approx 2$ is the function:

$$\dot{\rho}_i(F)=\begin{cases}\dfrac{\sin\pi F\tau_i\left(1-\dfrac{|F|T}{f_0\tau_i}\dfrac{2i-1}{2}\right)}{\pi F\tau_i}\cos\left[\pi FT(2i-1)\right] \\ \text{for } 0\le\dfrac{|F|T}{f_0\tau_i}\dfrac{2i-1}{2}\le 1, \\ 0 \ \text{ for } \dfrac{|F|T}{f_0T}\dfrac{2i-1}{2}\ge 1.\end{cases} \qquad (12)$$

For a coherent bundle of n pulses, the frequency mismatch function is determined by the expression:

$$\rho(F)=\frac{1}{n}\left|\sum_{i=1}^{n}\dot{\rho}_i(F)\right|. \qquad (13)$$

With an even number of pulses $n=2m$ and countdown $t_i=(2i-1)T/2$ from the center of the bundle, the frequency mismatch function of the coherent bundle of pulses is described by the system of equations:

$$\rho(F)=\frac{1}{m}\begin{cases}\left|\sum\limits_{i=1}^{m}\dfrac{\sin\pi F\tau_i\left[1-\dfrac{|F|T}{f_0\tau_i}\left(\dfrac{2i-1}{2}\right)\right]}{\pi F\tau_i}\cos\left[\pi FT(2i-1)\right]\right| \\ \text{for } 0\le\dfrac{|F|T}{f_0\tau_i}\le\dfrac{2}{2m-1}, \\ \left|\sum\limits_{i=1}^{m-1}\dfrac{\sin\pi F\tau_i\left[1-\dfrac{|F|T}{f_0\tau_i}\left(\dfrac{2i-1}{2}\right)\right]}{\pi F\tau_i}\cos\left[\pi FT(2i-1)\right]\right| \\ \text{for } \dfrac{2}{2m-1}\le\dfrac{|F|T}{f_0\tau_i}\le\dfrac{2}{2m-3}, \\ \left|\sum\limits_{i=1}^{m-k}\dfrac{\sin\pi F\tau_i\left[1-\dfrac{|F|T}{f_0\tau_i}\left(\dfrac{2i-1}{2}\right)\right]}{\pi F\tau_i}\cos\left[\pi FT(2i-1)\right]\right| \\ \text{for } \dfrac{2}{2m-(2k-1)}\le\dfrac{|F|T}{f_0\tau_i}\le\dfrac{2}{2m-(2k+1)}, \\ \left|\dfrac{\sin\pi F\tau_i\left[1-\dfrac{|F|T}{f_0\tau_i}\left(\dfrac{2i-1}{2}\right)\right]}{\pi F\tau_i}\cos\left[\pi FT(2i-1)\right]\right| \\ \text{for } \dfrac{2}{3}\le\dfrac{|F|T}{f_0\tau_i}\le 2.\end{cases}$$

Here m – the number of pairs of bundle pulses symmetric to the center.

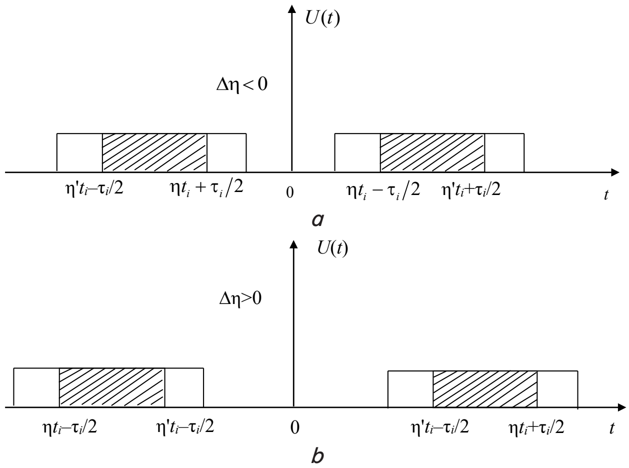

Fig. 1. Explanation of possible integration limits: *a* – for $t_i<0$, $\Delta\eta<0$; *b* – for $t_i>0$, $\Delta\eta>0$

Given $\pi F\tau_i \ll 1$, this expression can be written as follows:

$$\rho(F)=\frac{1}{m}\begin{cases}\left|\sum_{i=1}^{m}\left(1-\frac{2i-1}{2f_0\tau_i}|F|T\right)\cos\left[\pi(2i-1)|F|T\right]\right| \\ \text{for } 0\le\frac{|F|T}{f_0\tau_i}\le\frac{2}{2m-1}, \\ \left|\sum_{i=1}^{m-1}\left(1-\frac{2i-1}{2f_0\tau_i}|F|T\right)\cos\left[\pi(2i-1)|F|T\right]\right| \\ \text{for } \frac{2}{2m-1}\le\frac{|F|T}{f_0\tau_i}\le\frac{2}{2m-3}, \\ \left|\sum_{i=1}^{m-k}\left(1-\frac{2i-1}{2f_0\tau_i}|F|T\right)\cos\left[\pi(2i-1)|F|T\right]\right| \\ \text{for } \frac{2}{2m-(2k-1)}\le\frac{|F|T}{f_0\tau_i}\le\frac{2}{2m-(2k+1)}, \\ \left|\left(1-\frac{1}{2f_0\tau_i}|F|T\right)\cos\left[\pi(2i-1)|F|T\right]\right| \\ \text{for } \frac{2}{3}\le\frac{|F|T}{f_0\tau_i}\le 2. \end{cases} \quad (14)$$

Thus, if the transformation of the time scale of a coherent bundle of *n* radio pulses is not taken into account, expression (14) takes the following form:

$$\rho(F)=\frac{1}{m}\begin{cases}\left|\sum_{i=1}^{m}\cos\left[\pi FT(2i-1)\right]\right| \\ \text{for } 0\le\frac{|F|T}{f_0\tau_i}\le\frac{2}{2m-1}, \\ \left|\sum_{i=1}^{m}\cos\left[\pi FT(2i-1)\right]\right| \\ \text{for } \frac{2}{2m-1}\le\frac{|F|T}{f_0\tau_i}\le\frac{2}{2m-3}, \\ \left|\sum_{i=1}^{m}\cos\left[\pi FT(2i-1)\right]\right| \\ \text{for } \frac{2}{2m-(2k-1)}\le\frac{|F|T}{f_0\tau_i}\le\frac{2}{2m-(2k+1)}, \\ \left|\cos\left[\pi FT(2i-1)\right]\right| \text{ at } \frac{2}{3}\le\frac{|F|T}{f_0\tau_i}\le 2. \end{cases} \quad (15)$$

Fig. 2 shows the graphs of the mismatch function $\rho(|F|T)$, which are obtained from expression (14) taking into account the transformation of its time scale. The graphs are constructed for bundles of 8 (curve 1), 12 (curve 2) and 16 (curve 3) radio pulses, which are widely used in modern coherent pulse surveillance radars and aerodynamic radars.

Fig. 3 shows similar graphs obtained by expression (15) for the case when the transformation of the time scale of the mismatch function is not taken into account.

The accepted value of the product of the bundle parameters $f_0\tau_i=5$ is not typical, but is chosen to clearly represent these functions on the same scale.

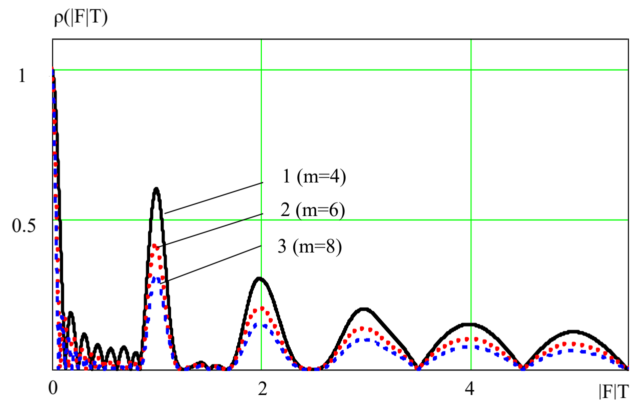

Fig. 2. Normalized frequency mismatch function taking into account time scale transformation

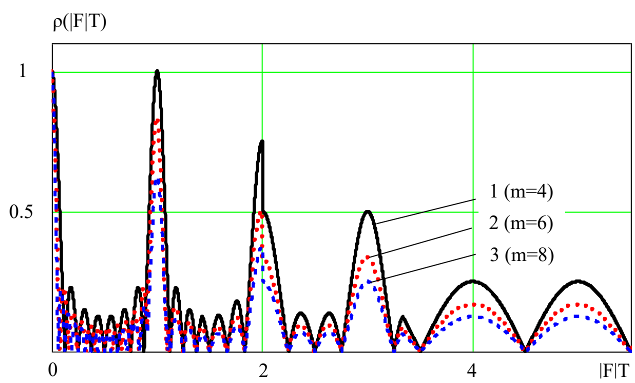

Fig. 3. Normalized frequency mismatch function without taking into account time scale transformation

From the obtained graphs, it is seen that at small frequency discrepancies $F<1/T$, the corresponding mismatch functions in Fig. 2, 3 are almost indistinguishable. In the case of large frequency discrepancies $F>1/T$, the difference becomes significant. That is, due to changes in the time scale of the bundle, with increasing frequency mismatch *F*, the envelope of the individual peaks of the mismatch function decreases much faster than in the traditional case. In the traditional case, the time scale transformation is not taken into account, and the peaks expand.

Thus, a significant difference in the peak amplitude at $F=0$ and $F=1/T$ in practical cases can significantly reduce, and in some cases remove the restriction on the pulse period value *T*. This is due to the condition of unambiguous Doppler frequency measurement.

It is useful to consider the results obtained with respect to bundles of radio pulses with parameters typical for coherent pulse radars. The following parameters are typical for such radars, which provide observation of aerodynamic objects [17]: radio pulse duration – units of microseconds; wavelength range – centimeter; radio pulse repetition period – from tens of microseconds to units of milliseconds. The standard errors of Doppler frequency measurement, caused by the motion of aerodynamic objects, can be tens to hundreds of Hz [16].

Thus, for coherent pulse radars, the condition $F<1/T$ is met, in which the effect of the time scale transformation of the normalized frequency mismatch function is not significant.

Fig. 4 shows the functions $\rho(F)$ obtained by expression (14) for the case of coherent bundles of 8 (curve 1) 12 (curve 2) and 16 (curve 3) radio pulses.

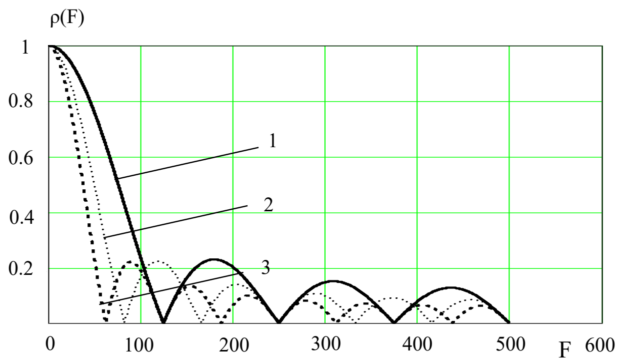

Fig. 4. Normalized mismatch functions of coherent bundles of 8 (curve 1) 12 (curve 2) and 16 (curve 3) radio pulses

The following typical parameters of coherent pulse radars are selected: carrier frequency – f_0=3 GHz (wavelength λ=10 cm), radio pulse duration – τ_i=3 μs, pulse period – T=1 ms. From the obtained graphs, it is seen that for the selected parameters of the coherent bundle of radio pulses, due to the considered time scale transformation, the condition of unambiguous Doppler frequency measurement is provided. As the frequency mismatch F in the region of real values increases, the envelope of the individual peaks of the mismatch function decreases significantly compared to the central peak.

The frequency resolution of radar signals can be determined directly from the expression of the normalized frequency mismatch function. If $\rho(F)$ assumes double differentiation, the frequency resolution at the level of 0.5 can be determined by the formula [1, 8]:

$$\Delta F_{0.5}=\frac{1}{\sqrt{\left|\rho''(0)\right|}}, \tag{16}$$

where $\rho''(0)$ – the second derivative of the normalized mismatch function in the area of the central peak.

Double differentiation of expression (14) within the central peak gives the following result:

$$\rho''(0)=-\frac{\pi^2T^2}{m}\sum_{i=1}^{m}(2i-1)^2. \tag{17}$$

Therefore, expression (16) with respect to (17) is converted to:

$$\Delta F_{0.5}=\frac{1}{\pi T\sqrt{\frac{1}{m}\sum_{i=1}^{m}(2i-1)^2}}. \tag{18}$$

The frequency resolution determined by (18) can be considered as the maximum possible, since it corresponds to the case of no external and internal noise.

The values of $\Delta F_{0.5}$ for the above bundles of 8, 12 and 16 radio pulses at T=1 ms are given in Table 2.

Table 2

Potential frequency resolution

n	8	12	16
$\Delta F_{0.5}$ (Hz)	70	46	35

Radar observation of air objects in real conditions is associated with the occurrence of fluctuations in the radar signal propagation, which in turn causes fluctuations in the normalized frequency mismatch function. Therefore, based on the obtained results, it is necessary to evaluate the decrease in frequency resolution, which occurs due to additive and multiplicative noise.

5. 2. Estimation of frequency resolution reduction

Additive noise means the internal noise of the radar receiver, which can be considered Gasusian [1, 8, 17].

The influence of inhomogeneities of the radio wave propagation medium is considered to be the main multiplicative noise. It is the multiplicative noise that causes the initial phase fluctuations of the radio pulses of the bundle radio signal, which leads to a decrease in its coherence and distortion of the normalized frequency mismatch function.

The experience of theoretical and experimental studies suggests that the distribution law of phase fluctuations of the radar signal is close to normal, and their correlation function can be approximated by the oscillating dependence [5, 14–16].

As shown in [15, 19, 20], the expression for the variance of the frequency measurement error of the radio pulse bundle due to the additive effect of internal noise of the receiving device has the following form:

$$D_F=\frac{12}{q^2T^2\left(4m^2-1\right)}, \tag{19}$$

where q^2 – signal-to-noise ratio by power.

According to [16, 21], the variance of the frequency measurement error of the bundle of radio pulses, which is due to the multiplicative effect of correlated phase fluctuations described by the oscillating correlation function, is determined by the expression:

$$D_{fl}=\frac{18\sigma_\varphi^2}{m^2\left(4m^2-1\right)^2T^2}\times$$
$$\times\left[\begin{aligned}&\sum_{j=1}^{m}(2j-1)^2\left\{1-e^{-\left[\frac{T}{\tau}(2j-1)\right]}\cos\left[(2j-1)\upsilon T\right]\right\}+\\&+2\sum_{k=1}^{m-j}e^{-\frac{T}{\tau}k}\sum_{j=1}^{m-1}(2j-1)(2j+2k-1)\times\\&\times\left\{\cos(j\upsilon T)-e^{-\left[\frac{T}{\tau}(2j-1)\right]}\cos\left[(2j+k-1)\upsilon T\right]\right\}\end{aligned}\right], \tag{20}$$

where σ_φ^2 – variance of phase fluctuations; τ – correlation interval of phase fluctuations; $\upsilon=2\pi/T_{fl}$ – oscillation frequency of the correlation coefficient of phase fluctuations; T_{fl} – oscillation period of the correlation coefficient of phase fluctuations.

The total variance of the frequency measurement error of the bundle of radio pulses, which is due to the influence of both these factors, is determined by the expression:

$$D = D_F + D_{fl}. \qquad (21)$$

As shown in [18], to determine the angular resolution $\Delta\theta$ in the presence of random wavefront distortions of the radar signal, the following ratio can be used:

$$\Delta\theta = q\sigma_\theta, \qquad (22)$$

where q – signal-to-noise voltage ratio at the output of the coordinated processing device; σ_θ – root mean square measurement error of the wave arrival angle.

By analogy with the problems to be solved, the frequency resolution in the presence of the additive effect of internal noise and the multiplicative effect of correlated phase fluctuations can be determined by the expression:

$$\Delta F_{0.5} = q\sqrt{D}, \qquad (23)$$

in which the variance of the frequency measurement error of the bundle of radio pulses D is determined by expressions (19)–(21).

According to [6, 14–16, 21], the influence of tropospheric inhomogeneities leads to phase fluctuations with a variance $\sigma_\varphi^2 = (0.01...10)$ rad^2 and a correlation interval τ=(0.1...1) s. The assessment should be carried out for cases where the resolution coefficient K_r=10lg(q^2/2) takes values from 17 dB to 27 dB, which are characteristic of surveillance and tracking radars according to the estimates of detection quality indicators provided in [1, 17].

The calculation is performed for the values of the resolution coefficient K_r=17 dB and K_r=27 dB, and two options of the values of statistical characteristics of phase fluctuations:

1. $\sigma_\varphi^2 = 0.01$ rad^2 and τ=1 s – the predominant additive effect of internal noise, the multiplicative effect of phase fluctuations is almost absent.

2. $\sigma_\varphi^2 = 10$ rad^2 and τ=0.1 s – the multiplicative effect of phase fluctuations far exceeds the additive effect of internal noise.

Comparison of the $\Delta F_{0.5}$ values listed in Table 2 indicates that the additive effect of the internal noise of the receiving device can lead to the expansion of the normalized frequency mismatch function and a 6-fold decrease in the frequency resolution. The multiplicative effect of correlated phase fluctuations can lead to an increase in this effect by 27...118 times.

Fig. 5 shows a graphs of the dependence of frequency resolution $\Delta F_{0.5}$, obtained by (23) taking into account expressions (19)–(21), for bundles with the number of radio pulses n=8, 12 and 16 on the variance of phase fluctuations σ_φ^2. The graphs were obtained at K_r=27 dB for two values of the correlation interval of phase fluctuations τ=0.1 s (1 – n=8; 2 – n=12; 3 – n=16) and τ=1 s (1′ – n=8; 2′ – n=12; 3′ – n=16).

So, the proposed method for estimating the influence of transformation of the normalized frequency mismatch function of a coherent bundle of radio pulses on the quality of radar frequency resolution involves the following steps.

The first step is to evaluate the decrease in resolution for the case of radial motion of the aerodynamic object:

– determine the motion of the aerodynamic object taking into account changes in all spectrum frequencies of the radar signal by expression (1) taking into account formulas (2) and (3);

– determine the instantaneous value of the signal reflected from the aerodynamic object by expressions (4) or (5);

– calculate the normalized mismatch function for the i-th pulse by expression (8);

– estimate the resolution for the radial motion of the aerodynamic object in the absence of external and internal noise by expression (18).

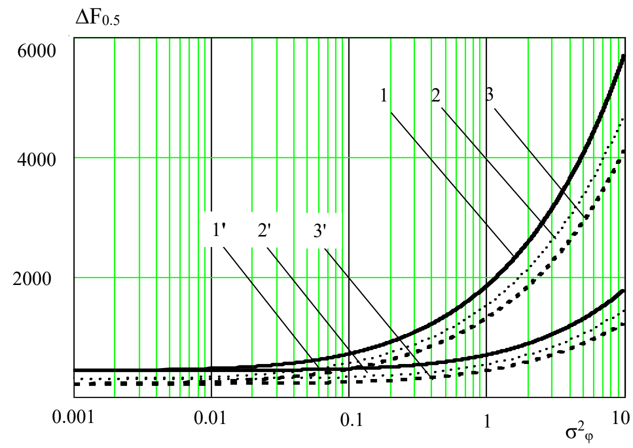

Fig. 5. Dependence of frequency resolution of coherent bundles of radio pulses on the variance of phase fluctuations

The second step is to assess the decrease in frequency resolution in the presence of external and internal noise:

– determine the variance of the frequency measurement error of the bundle of radio pulses due to the additive effect of the internal noise of the receiving device by expression (19);

– determine the variance of the frequency measurement error of the bundle of radio pulses due to the multiplicative effect of the correlated phase fluctuations described by the oscillating correlation function by expression (20);

– determine the total variance of the frequency measurement error of the bundle of radio pulses due to the effect of both these errors by expression (21);

– estimate the frequency resolution in the presence of the additive effect of the internal noise and the multiplicative effect of correlated phase fluctuations by expression (23).

6. Discussion of the results of evaluating the decrease in frequency resolution

The multiplicative effect of correlated phase fluctuations due to real observation conditions of coherent pulse radars of aerodynamic objects causes the expansion of the normalized frequency mismatch function and deterioration of the frequency resolution by up to hundreds and more times.

As follows from Table 2, under the insignificant multiplicative effect of correlated phase fluctuations ($\sigma_\varphi^2 = 0.01$ rad^2 and τ=1 s), with an increase in the number of radio pulses in the bundle, there is an obvious proportional improvement in the frequency resolution. Thus, doubling of the number of bundle radio pulses from 8 to 16 provides an improvement in the frequency resolution (reduction of the width of the normalized frequency mismatch function) almost twice (by 100 %). However, due to the multiplicative effect of phase fluctuations and partial loss of the bundle coherence, this improvement is significantly reduced.

With the predominant multiplicative effect of correlated phase fluctuations ($\sigma_\varphi^2=10$ and $\tau=0.1$ s), an increase in the number of radio pulses from 8 to 16 causes an improvement in the frequency resolution by approximately 1.4 times (40 %). That is, the effect of resolution improvement is reduced by 60 %.

The graphs in Fig. 5 show that in the case of almost complete correlations of phase fluctuations at $\tau=1$ s (graphs 1′, 2′, 3′), there is a slight deterioration of $\Delta F_{0.5}$ compared to the case of the effect of only the internal noise of the receiving device, even at large variance values of phase fluctuations.

However, at $\tau=0.1$ s (curves 1, 2, 3), there is a sharp increase in the width of the normalized frequency mismatch function and the corresponding deterioration of the frequency resolution.

At the variance values of phase fluctuations $\sigma_\varphi^2<0.1$ rad^2, they have almost no multiplicative effect on the frequency resolution and the value of this parameter is due to the influence of only the internal noise of the receiving device. However, with the variance of phase fluctuations in the range from 0.1 rad^2 to 10 rad^2, which can occur in real conditions when the radar performs tasks as intended, the contribution of the multiplicative effect of phase fluctuations in the deterioration of the frequency resolution quality increases significantly and can be ten times greater than the additive effect of internal noise.

Thus, the results directly determine the conditions under which the quality of frequency resolution of radar signals depends much more on the statistical characteristics of phase fluctuations than on the signal-to-noise ratio, which allows predicting the possible effectiveness of radar observation of aerodynamic objects in real operation conditions.

The limitations of this study are as follows.

When considering expressions for mismatch functions, changes in the amplitude and duration of an individual pulse are neglected due to the Doppler effect. However, it is necessary to take into account much more significant changes in the time intervals between pulses, which are associated with changes in the time scale during the radial motion of the aerodynamic object.

The frequency resolution determined by (18) can be considered the maximum possible (ideal), as it corresponds to the absence of external and internal noise.

The study assumes that the distribution law of phase fluctuations of the radar signal is close to normal, and the correlation function of phase fluctuations can be approximated by the oscillating dependence.

The obtained results can be further developed in the direction of optimizing algorithms of spatial processing of radar signals in order to increase the accuracy of measuring the angular coordinates of aerodynamic objects. This task is practically important for radars that provide altitude measurements under tropospheric refraction conditions. At the same time, taking into account the correlated phase fluctuations becomes especially important for phased-array radars, in which the phase method of determining angular coordinates is implemented.

7. Conclusions

1. An assessment of resolution reduction for the radial motion of an aerodynamic object was carried out. It is found that at small frequency discrepancies $F<1/T$, the transformation of the normalized frequency mismatch function is almost not manifested. However, the transformation of the normalized frequency mismatch function becomes significant when $F>1/T$ and is manifested in a rapid decrease in the amplitude and a significant expansion of the individual peaks of the mismatch function. This allows to significantly reduce, and in some practical cases, even remove restrictions on the bundle pulse period to comply with the condition of unambiguous Doppler frequency measurement.

2. An assessment of frequency resolution reduction was performed. It is found that the multiplicative effect of correlated phase fluctuations due to real observation conditions of coherent pulse radars of aerodynamic objects causes the expansion of the normalized frequency mismatch function and deterioration of the frequency resolution by one hundred and more times. In this case, the effect of a sharp increase in the width of the normalized frequency mismatch function and the corresponding deterioration of the frequency resolution is observed with increasing variance of phase fluctuations from 0.1 rad^2 to 10 rad^2 and decreasing correlation interval of phase fluctuations from 7 s to 0.1 s, which can practically be observed in the perturbed troposphere. The results of the numerical evaluation allow analyzing the quality of radar frequency resolution in ideal, real and complex conditions of radar observation of aerodynamic objects.

Acknowledgments

The obtained results are based on the scientific achievements of scientists in the field of radar theory, Doctor of Technical Sciences Professor Minervin Mykola and Doctor of Technical Sciences Professor Shirman Yakov, who worked at the Kharkiv National University of Radio Electronics.

The author's team wishes to express special gratitude to the Deputy Head of the Ivan Kozhedub Kharkiv National Air Force University, Honored Worker of Science and Technology of Ukraine, Doctor of Technical Sciences, Professor Vasyuta Konstantyn for reliable research results on taking into account phase fluctuations on the quality of spatial processing of radar signals and consulting assistance during the processing of research materials.

Reference

1. Shirman, Ya. D. (Ed.) (2007). Radioelektronnye sistemy: osnovy postroeniya i teoriya. Moscow: Radiotekhnika, 512.
2. Melvin, W. L., Scheer, J. A. (2012). Principles of Modern Radar: Advanced techniques. IET. doi: https://doi.org/10.1049/sbra020e
3. Zohuri, B. (2020). Fundaments of Radar. Radar Energy Warfare and the Challenges of Stealth Technology, 1–110. doi: https://doi.org/10.1007/978-3-030-40619-6_1
4. Klemm, R., Nickel, U., Gierull, C., Lombardo, P., Griffiths, H., Koch, W. (Eds.) (2017). Novel Radar Techniques and Applications Volume 1: Real Aperture Array Radar, Imaging Radar, and Passive and Multistatic Radar. IET. doi: https://doi.org/10.1049/sbra512f

5. Herasimov S., Roshchupkin E., Kutsenko V., Riazantsev, S., Nastishin, Yu. (2020). Statistical analysis of harmonic signals for testing of Electronic Devices. International Journal of Emerging Trends in Engineering Research, 8 (7), 3791–3798. doi: https://doi.org/10.30534/ijeter/2020/143872020
6. Kovalchuk, A., Oleshchuk, M., Karlov, V., Karpenko, O., Biesova, O., Lukashuk, O. (2021). Analysis of sensitivity of target tracking systems to external interference in multichannel radars with fixed parameters. Advanced Information Systems, 5 (1), 82–86. doi: https://doi.org/10.20998/2522-9052.2021.1.11
7. Savchenko, V., Laptiev, O., Kolos, O. et. al. (2020). Hidden Transmitter Localization Accuracy Model Based on Multi-Position Range Measurement. 2020 IEEE 2nd International Conference on Advanced Trends in Information Theory (IEEE ATIT 2020) Conference Proceedings. Kyiv, 246–251.
8. Barton, D. K. (2012). Radar Equations for Modern Radar. Artech House, 264.
9. Herasimov, S., Belevshchuk, Y., Ryapolov, I., Volkov, A., Borysenko, M., Tokar, O. (2020) Modeling technology of radar scattering of the fourth generation EF-2000 Typhoon multipurpose aircraft model. International Journal of Emerging Trends in Engineering Research, 8 (9), 5075–5082. doi: https://doi.org/10.30534/ijeter/2020/30892020
10. Minervin, N. N., Karlov, D. V., Konovalov, V. M. (2013). Features of influencing the ionosphere on radar signals at accelerated motion of space objects. Applied Radio Electronics, 12 (4), 530–532.
11. Karlov, V., Kuznietsov, O., Belousov, V., Tuzikov, S., Oleschuk, M., Petrushenko, V. (2021). Accuracy of measurement of aerodynamic objects angular coordinates under tropospheric refraction conditions. Control, Navigation and Communication Systems, 1 (63), 146–152. doi: https://doi.org/10.26906/sunz.2021.1.146
12. Volosyuk, V. K., Gulyaev, Y. V., Kravchenko, V. F., Kutuza, B. G., Pavlikov, V. V., Pustovoit, V. I. (2014). Modern methods for optimal spatio-temporal signal processing in active, passive, and combined active-passive radio-engineering systems. Journal of Communications Technology and Electronics, 59 (2), 97–118. doi: https://doi.org/10.1134/s1064226914020090
13. Klochko, V. K. (2016). Algorithms of 3D radio-wave imaging in airborne Doppler radar. Radioelectronics and Communications Systems, 59 (8), 335–343. doi: https://doi.org/10.3103/s0735272716080021
14. Karlov, V. D., Radiukov, A. O., Pichuhin, I. M., Karlov, D. V. (2015). Statistical descriptions of radio-location signals, reflected from local objects in the conditions of anomalous refraction. Science and Technology of the Air Force of Ukraine, 4 (21), 71–74.
15. Karlov, V., Kuznietsov, O., Artemenko, A., Karlov, A. (2019). Evaluation of the accuracy of measuring the radial velocity of a target with an exponential and alternating decrease in phase correlation of the burst radio signal. Advanced Information Systems, 3 (1), 71–75. doi: https://doi.org/10.20998/2522-9052.2019.1.12
16. Kuznietsov, O., Karlov, V., Karlov, A., Kiyko, A., Lukashuk, O., Biesova, O., Petrushenko, M. (2020). Estimation of the Dispersion of the Error in Measuring the Frequency of a Pack with Correlated Fluctuations in the Initial Phases of its Radio Pulses. 2020 IEEE Ukrainian Microwave Week (UkrMW). doi: https://doi.org/10.1109/ukrmw49653.2020.9252588
17. Sedyshev, Yu., Atamanskiy, D. (2010). Radioelektronnye sistemy. Kharkiv: Kharkivskyi unyversytet Povitrianykh Syl, 418.
18. Minervin, N. N., Vasyuta, K. S. (2013). Measure of angular resolution capability and measuring accuracy of a wave arrival corner in the presence of irregular distortions of its front and additive noise. Applied Radio Electronics, 12 (4), 484–486.
19. Mogyla, A. A. (2014). Application of stochastic probing radio signals for the range-velocity ambiguity resolution in doppler weather radars. Radioelectronics and Communications Systems, 57 (12), 542–552. doi: https://doi.org/10.3103/s0735272714120036
20. Herasimov, S. (2020). Aircraft flight route search method with the use of cellular automata. International Journal of Advanced Trends in Computer Science and Engineering, 9 (4), 5077–5082. doi: https://doi.org/10.30534/ijatcse/2020/129942020
21. Yevseiev, S., Kuznietsov, O., Herasimov, S., Horielyshev, S., Karlov, A., Kovalov, I. et. al. (2021). Development of an optimization method for measuring the Doppler frequency of a packet taking into account the fluctuations of the initial phases of its radio pulses. Eastern-European Journal of Enterprise Technologies, 2 (9 (110)), 6–15. doi: https://doi.org/10.15587/1729-4061.2021.229221